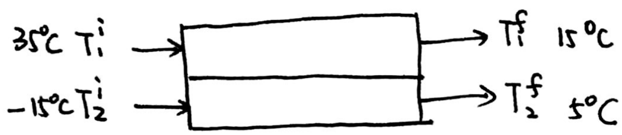
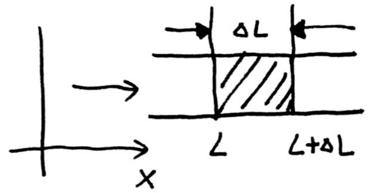
ABE 202 HW5

I co-current



1) Mass balance



$$\text{Mass}(t+\Delta t) - \text{Mass}(t) = \dot{m}_{in}\Big|_{L}^{\Delta t} - \dot{m}_{out}\Big|_{L+\Delta L}^{\Delta t}$$

at steady-state $0 = (\dot{m}_L - \dot{m}_{L+\Delta L})\,\Delta t.$

2) Energy Balance

$$\underset{@t+\Delta t}{\text{Energy}} - \underset{@\Delta t}{\text{Energy}} = \underset{\text{in }\Delta t}{\dot{E}_{in}\Big|_{L}} - \underset{\text{in }\Delta t}{\dot{E}_{out}\Big|_{L+\Delta L}} + \text{Energy flow due to heat in } \Delta t$$

at steady-state $0 = \dot{m}_L \underline{H}_L \cancel{\Delta t} - \dot{m}_{L+\Delta L}\underline{H}_{L+\Delta L}\cancel{\Delta t} + \underline{\dot{Q}}\Delta L\cancel{\Delta t}$

$\hookrightarrow \dot{m}(\underline{H}_{L+\Delta L} - \underline{H}_L) = \dot{m}\,C_p\,(T_{L+\Delta L} - T_L) = \dot{Q}\Delta L$

for flow I : $\dot{m}_1 C_{p1} \dfrac{dT_1}{dL} = \dot{Q} = k(T_2 - T_1)$ ...... ①

" 2 : $\dot{m}_2 C_{p2} \dfrac{dT_2}{dL} = -\dot{Q} = -k(T_2 - T_1)$ ...... ②

$\dot{m}_1$ & $\dot{m}_2$ are both positive for cocurrent flow

① + ② $\rightarrow \dfrac{d(T_1+T_2)}{dL} = 0 \rightarrow \int d(T_1+T_2) = \int 0\, dL$

$\boxed{T_1 + T_2 = C}$ .... ③

constant

Sub ③ into ①

$$\frac{dT_1}{dL} = \frac{k}{\dot{m}c_p}(T_2 - T_1) = \frac{k}{\dot{m}c_p}(c - 2T_1)$$

$$\int_{T_1^i}^{T_1^f} \frac{dT_1}{c-2T_1} = \int \frac{k}{\dot{m}c_p} dL$$

$$-\frac{1}{2}\ln(c-2T_1)\Big|_{T_1^i}^{T_1^f} = \frac{kL}{\dot{m}c_p}$$

$$\ln\left(\frac{c-2T_1^i}{c-2T_1^f}\right) = \frac{2kL}{\dot{m}c_p} \Rightarrow \ln\left(\frac{c-2T_1^f}{c-2T_1^i}\right) = -\frac{L}{L_0}$$

where $c = T_1^i + T_2^i = T_1^f + T_2^f = 20°C$

at length L : $T_1^i = 35°C$ $T_1^f = 15°C$ $L_0 = \frac{\dot{m}c_p}{2k}$

$$\ln\left(\frac{20-2\times35}{20-2\times15}\right) = \frac{2kL}{\dot{m}c_p}$$

$$\underset{\uparrow}{L} = \ln(5)\cdot L_0 = \underline{1.61 L_0}$$

= length of the whole exchanger

at any position in heat exchanger

$$\frac{c-2T_1(L)}{c-2T_1^i} = e^{-\frac{L}{L_0}}$$

$T_1 = 35°C$

$$\underline{T_1(L) = 25e^{-\frac{L}{L_0}} + 10} \quad \cdots\cdots ④$$

$$\underline{T_2(L) = c - T_1(L) = 10 - 25e^{-\frac{L}{L_0}}} \quad \cdots\cdots ⑤$$

3) Entropy Balance.

$$0 = \dot{m}_L \underline{S_L} \cancel{\Delta t} - \dot{m}_{L+\Delta L}\underline{S_{L+\Delta L}}\cancel{\Delta t} + \frac{\dot{Q}}{T_1}\Delta L \cancel{\Delta t}$$

$$\dot{m}\frac{[S_{L+\Delta L} - S_L]}{\Delta L} = \frac{k(T_2 - T_1)}{T_1}$$

$\lim \Delta L \to 0$

$$\dot{m}\frac{dS_1}{dL} = \frac{k(T_2 - T_1)}{T_1} \rightarrow \int_0^L \frac{k}{\dot{m}}\left(\frac{T_2(L) - T_1(L)}{T_1(L)}\right) dL = \int_0^L dS_1$$

Sub T_1 T_2 from ④ & ⑤

$$\underline{S}_1(L) - \underline{S}_1(L=0) = \frac{k}{\dot{m}}\int\left(\frac{-50e^{-\frac{L}{L_0}}}{25e^{-\frac{L}{L_0}}+10}\right) dL$$

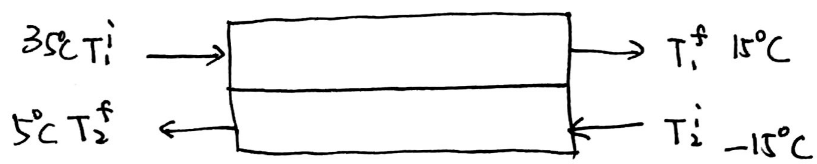
$$= \frac{k}{\dot{m}}\left(2 \cdot L_0 \ln\left(5e^{-\frac{L}{L_0}}+2\right)\right)\Big|_0^L$$

since $L_0 = \dfrac{\dot{m}C_p}{2k}$

$$\Rightarrow = \frac{\cancel{k}}{\cancel{\dot{m}}}\,\cancel{2}\,\frac{\cancel{\dot{m}}C_p}{\cancel{2k}} \ln\left(\frac{5e^{-\frac{L}{L_0}}+2}{5+2}\right)$$

$$\underline{S}_1(L) - \underline{S}_1(L=0) = C_p \ln\left(\frac{5e^{-\frac{L}{L_0}}+2}{7}\right)$$

(II) Counter - current flow



1) Mass balance.

for each flow same with co-current flow

But $\dot{M}_2 = -\dot{M}_1$ since different direction

2) Energy Balance

$$\dot{M}_1 C_{p1}\frac{dT_1}{dL} = k(T_2 - T_1)$$

$$\dot{M}_2 C_{p2}\frac{dT_2}{dL} = k(T_2 - T_1)$$

Since $\dot{M}_1 = -\dot{M}_2$

$$\frac{d(T_1 - T_2)}{dL} = 0 \quad \Rightarrow \quad \boxed{T_1 - T_2 = \text{constant} = C}$$

$$T_1 - T_2 = T_1^i - T_2^i = T_1^f - T_2^f = 30°C$$

for stream 1

$$\dot{m}_1 C_p \frac{dT_1}{dL} = k(\overset{-30}{T_2 - T_1})$$

$$= -kC$$

$$\Rightarrow T_1(L) = -\frac{kC}{\dot{m}C_p} \cdot L + T_1^i$$

$$= -\frac{30kL}{\dot{m}C_p} + 35 = \underline{-\frac{15L}{L_0} + 35}$$

$$T_2(L) = T_1 - C = -\frac{30kL}{\dot{m}C_p} + 5 = \underline{-\frac{15L}{L_0} + 5}$$

3) Entropy

$$\dot{m}\frac{dS_1}{dL} = \frac{k(\overset{C}{T_2 - T_1})}{T_1} = -\frac{kC}{T_1}$$

$$\int dS = \int_0^L \frac{-k \cdot 30}{\dot{m}\left(\frac{-kL}{\dot{m}C_p} \cdot L + 35\right)} \, dL$$

$$S(L) - S(L=0) = C_p \ln\left(1 - 0.048\frac{L}{L_0}\right)$$

at $T_1(L) = 15$

$$15 = -\frac{15L}{L_0} + 35$$

$$\underline{L_2 = 1.3L_0}$$

Counter-current is shorter