ABE 202 Homework 1, 2017

1. Show that if variables X and Y are extensive, then X/Y is intensive
Answer: See lecture notes week 1

2. What is the balance equation for anything
Answer: See lecture notes week 1

3. As a result of a chemical spill in the BioLab, benzene is evaporating at the rate of 1 gram per minute into a room that is 8m X 4m X 3m in size and a ventilation rate of 10 cubic meters/min
a) compute the steady state concentration of benzene in the room

Volume of Room = 8x4x3 = 96 m^3 room. (estimated at 100 m3 for remainder of solution)

Ventilation rate = 10 m^3 of room air exchanged per minute.

If 10 m^3 air cleared out of a 100 m^3 room = 10% of room air cleared per minute
evaporated benzene = benzene gas, so clearing 10% of room air = clearing 10% of benzene

so rate of benzene ventilation is **-0.1(M_B)** where **M_B** is Mass of Benzene

rate of benzene evaporation is **1 gram/minute**

Mass balance equation:

$$\frac{dM_B}{dt} = (\dot{M_B})_{in} + (\dot{M_B})_{out}$$

$$\frac{dM_B}{dt} = (\dot{M_B})_{evaporation} + (\dot{M_B})_{ventilation}$$

$$\frac{dM_B}{dt} = 1\frac{g}{min} - 0.1(\dot{M_B})\text{g/min}$$

Steady state is when you get equal amounts coming in and going out at the same time
Mathematically we can write that as:

$$\frac{dM_B}{dt} = 0$$

$$\frac{dM_B}{dt} = 0 = 1 - 0.1(\dot{M_B})$$

$$0 = 1\frac{g}{min} - 0.1(\dot{M_B})\ g/min \ \rightarrow \ M_B = 10\text{ g}$$

If you divide by the room volume you get a final concentration 10g/100m^3 or 0.1 g/m^3

b) assuming that benzene = 0 at time = 0 (beginning of spill), compute the time needed for benzene to reach 95 percent of the steady state concentration

We have a useful formula for figuring out how the amount of Benzene gas changes over time…

$$\frac{\boldsymbol{dM_B}}{\boldsymbol{dt}} = (\boldsymbol{\dot{M_B}})_{\boldsymbol{in}} + (\boldsymbol{\dot{M_B}})_{\boldsymbol{out}}$$

The question asks for a length of time…to solve for t in the above equation we need to integrate

To make things cleaner lets alter the above equation.

Lets rename $(\boldsymbol{\dot{M_B}})_{\boldsymbol{in}}$ as φ_B

Lets rename $(\boldsymbol{\dot{M_B}})_{\boldsymbol{out}}$ as → λM_B

$$\frac{\boldsymbol{dM_B}}{\boldsymbol{dt}} = \varphi_B - \lambda M_B$$

$$\frac{1}{\varphi_B - \lambda M_B} dM_B = dt$$

$$\int_{M_B(0)}^{M_B(t)} \frac{1}{\varphi_B - \lambda M_B} dM_B = \int dt$$

$$-\frac{1}{\lambda}(\ln(\varphi_B - \lambda M_B(t)) - \ln(\varphi_B - \lambda M_B(0))) = t$$

Remember benzene =0 at time =0 so $M_B(0) = 0$

$$(\ln(\varphi_B - \lambda M_B(t)) - \ln(\varphi_B)) = -\lambda t$$

$$\ln\left(\frac{\varphi_B - \lambda M_B(t)}{\varphi_B}\right) = -\lambda t$$

Now lets substitute values back in. $M_B(t)$ at the t we care about is 95% of steady state benzene that we solved for in problem 3a. So $M_B(t)$ = 9.5g

$\ln\left(\frac{1-0.1(9.5)}{1}\right) = -.1t$ → t = (ln(0.05))/(-.1) → t = -10 ln (0.05) →

t ≈ 30 minutes

3B alternative method

Method 2

Easiest: steady-state and constant

Define $M_B^* = M_{BSS} - M_B$

Distance or deviation from steady state.

Mass Balance:

$$\frac{dM_B}{dt} = \Phi_B - \lambda \cdot M_B$$

$$\frac{d(M_{BSS} - M_B^*)}{dt} = \Phi_B - \lambda(M_{BSS} - M_B^*)$$

from a, $M_{BSS} = \Phi_B / \lambda \equiv$ a constant.

$$-\frac{dM_B^*}{dt} = \lambda \cdot M_B^*$$

$$\text{or} \quad M_B^* = M_B^*(0) \cdot \exp(-\lambda \cdot t)$$

Since M_B^* is disance away from steady-state, then 95% to steady state is equivalent to 5% away from steady state.

$$\text{so} \quad \frac{M_B^*}{M_B(0)^*} = 0.05 = \exp(-\lambda \cdot t)$$

$$\text{or} \quad \frac{\ln(0.05)}{\lambda} = t$$

$$= \frac{-2.99}{-0.1} \approx 30 \text{ min.}$$

4. The insecticide DDT has a half-life in the human body of approximately 7 years. (In 7 years its concentration decreases to half its initial concentration). Although DDT is no longer used in the United States, 25 years ago that average farmer had a body DDT concentration of 22 ppm (parts per million by weight) Estimate what the farm workers present concentration is.

For half-life related equations the following equation is useful

$$\ln \frac{N_t}{N_o} = \lambda t$$

where $\mathbf{N_t}$ = mass at time $\mathbf{t}$;

$\mathbf{N_0}$ = original mass

λ = decay constant

$t_{1/2}$ is the half-life

By substituting in values we know we can solve for the decay constant

$N_t = \frac{N_o}{2}$

$\ln (0.5) = \lambda(7 years)$

$-.693 = \lambda(7)$

$\boldsymbol{\lambda} \approx$ **-0.1**

Now that we have the decay constant we can use the other equation

$$N(t)=N_0 e^{\lambda t}$$

Substitute in

$N(25) = 22\ e^{(-.1)(25)}$

$N(25) \approx 1.8$ppm