# Differential Energy Balance Equatn.

$$U + M\left(\frac{v^2}{2} + \Psi\right)$$

internal energy — mass — kinetic energy — potential energy

$$\underbrace{\frac{d}{dt}\left[U + M\left(\frac{v^2}{2} + \Psi\right)\right]}_{\text{Accumulation}} = \begin{pmatrix}\text{Rate energy} \\ \text{enters the system}\end{pmatrix} - \begin{pmatrix}\text{Rate energy} \\ \text{exits the system}\end{pmatrix}$$

1) Energy flow coming in w/ mass flow -

per unit mass

$$\sum_{k=1}^{K} \dot{M}_k \left(\hat{U} + \frac{v^2}{2} + \Psi\right)_k$$

2) Heat.

$$\dot{Q} = \sum \dot{Q}_j$$

3) Work –

Divide total Work into different forms. Defined as work done <u>on</u> system

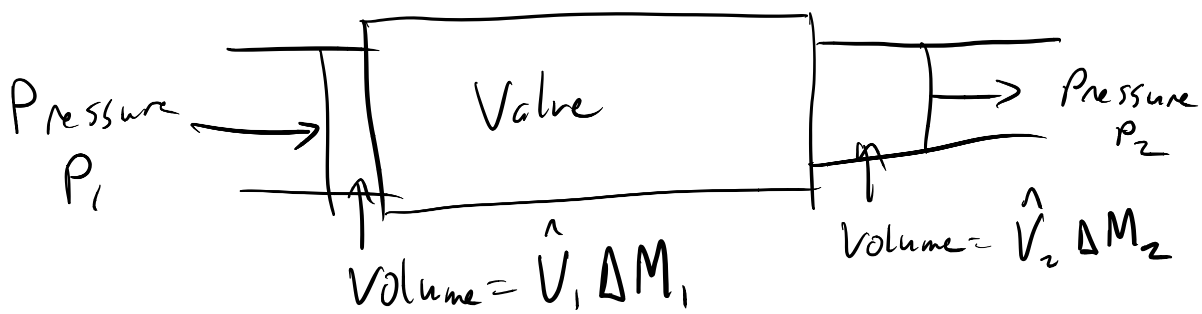
$\dot{W}_s$: shaft work – mechanical energy flow into system.

$$\boxed{\dot{W} = -P\frac{dV}{dt}}$$ – pressure volume work.

why negative?

$P$ is pressure

$\frac{dV}{dt}$ change in volume/time.




$$\begin{pmatrix}\text{Work done by surrounding} \\ \text{fluid to push } M_1 \text{ into} \\ \text{valve.}\end{pmatrix} = P_1 \hat{V}_1 \Delta M_1$$

$$\begin{pmatrix}\text{work done on surrounding} \\ \text{by movement of fluid mass} \\ (\Delta M_2) \text{ out of the valve}\end{pmatrix} = -P_2 \hat{V}_2 \Delta M_2$$

$$\begin{pmatrix}\text{Net work done on} \\ \text{the system due to moving} \\ \text{fluid}\end{pmatrix} = P_1 \hat{V}_1 \Delta M_1 - P_2 \hat{V}_2 \Delta M_2$$

→ for multiple streams →

$$\sum_{k=1}^{K} \Delta M_k P \hat{V}_k$$

$$\frac{d}{dt}\left\{U + M\left(\frac{V^2}{2} + \Psi\right)\right\} = \sum_{k=1}^{K} \dot{M}_k \left(\hat{U} + \frac{V^2}{2} + \Psi\right)_k + \dot{Q}$$

$$+ \dot{W}_s - P\frac{dV}{dt} + \sum_{k=1}^{K} \dot{M}_k (P\hat{V})_k$$

$\hat{U}$ $\wedge \rightarrow$ means per unit mass

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Now we simplify.

It's convenient to define

$$\boxed{H = U + PV}$$

Enthalpy is another equivalent energy - very convenient form of energy -

$$\star \quad \frac{d}{dt}\left\{U + M\left(\frac{V^2}{2} + \Psi\right)\right\} = \sum_{k=1}^{K} \left(\hat{H} + \frac{V^2}{2} + \Psi\right)_k \dot{M}_k + \dot{Q} + \dot{W}$$

$\dot{W}_s - P\frac{dV}{dt}$.

from this point on

we don't care about potential energy or kinetic.

$$\frac{dU}{dt} = \sum_{k=1}^{K} (\dot{M}\hat{H})_k + \dot{Q} + \dot{W}$$

per mass basis

$$\frac{dU}{dt} = \sum_{k=1}^{K} (\dot{N}\underline{H})_k + \dot{Q} + \dot{W}$$

per mol basis

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Special cases

(i) Closed system
(ii) adiabatic system
(iii) open, steady-state system
(iv) Uniform system.

(i) $\frac{dM}{dt} = 0, \quad \dot{M}_k = 0$

because we do not care

$$\frac{dU}{dt} + \cancel{\frac{d}{dt}\left(M\frac{v^2}{2} + M\Psi\right)} = \dot{Q} + \dot{W}$$

$$\frac{dU}{dt} = \dot{Q} + \dot{W}$$

(ii) Whenever you see $\dot{Q}$, eliminate it

(iii) $\frac{dM}{dt} = 0 \quad \frac{dV}{dt} = 0 \quad \frac{d}{dt}\left(U + M\left(\frac{v^2}{2} + \Psi\right)\right) = 0$

$$0 = \sum_{k=1}^{K} \dot{M}_k \left(\hat{H} + \frac{v^2}{2} + \Psi\right)_k + \dot{Q} + \dot{W}_s$$

(iv) Uniform system

$$U = M\hat{U}$$