we have now —

3 balance equations

Differential forms.

0th: Mass

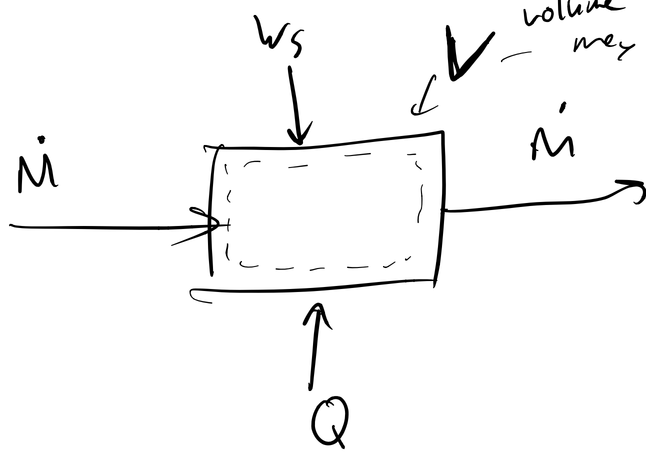
1st: Energy

2nd: Entropy.

1) Refresh + stretch – + see if we can make life a little easier yet.

Consider a system — w/ flow

volume — may be it can change.



Mass and heat flows occur @ Temperature T

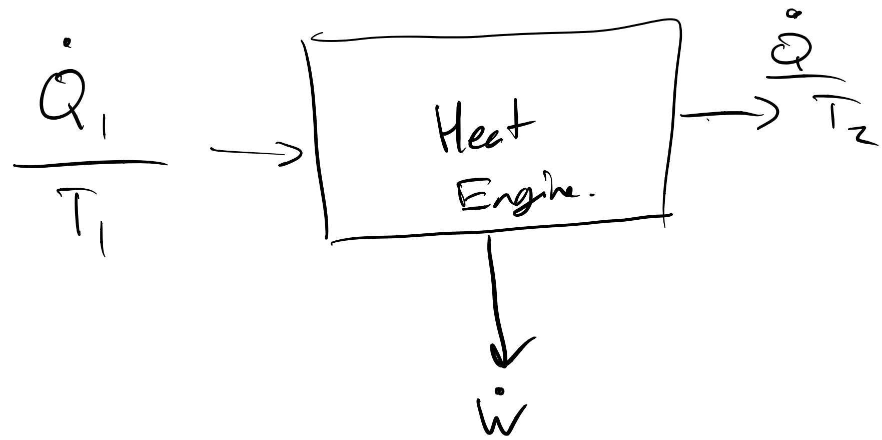
→ Write down the mass, energy, & entropy balance equations here, differential form

$$M: \quad \frac{dM}{dt} = \dot{M}$$

$$E: \quad \frac{dU}{dt} = \dot{M}\hat{H} + \dot{Q} - P\frac{dV}{dt} + \dot{W}_s$$

$$S: \quad \frac{dS}{dt} = \dot{M}\hat{S} + \frac{\dot{Q}}{T} + \dot{S}_{gen}$$

now - consider special cases



→ from this, write down energy + entropy balance equations

Steady state
no mass flow → either differential or integrated form.
2 Qs

$$\boxed{\begin{aligned} E: \quad & 0 = Q_1 + Q_2 + W \\ S: \quad & 0 = \frac{Q_1}{T_1} + \frac{Q_2}{T_2} + S_{gen} \end{aligned}}$$

$$Q_1 = \int_{t_1}^{t_2} \dot{Q}_1 \, dt \qquad W = \int_{t_1}^{t_2} \left(\dot{W}_s - P\left(\frac{dV}{dt}\right) \right) dt$$

$$Q_2 = \int_{t_1}^{t_2} \dot{Q}_2 \, dt$$

Work done by the engine $-W =$

↑ in terms of Q_1, T_1, T_2, S_{gen}

eliminate Q_2

$$0 = Q_1 + Q_2 + W$$

$$0 = \frac{Q_1}{T_1} + \frac{Q_2}{T_2} + S_{gen}$$

$$Q_2 = -Q_1 - W$$

Rearrange for $-W$

$$0 = \frac{Q_1}{T_1} + \frac{(-Q_1 - W)}{T_2} + S_{gen}$$

$$-W = -\frac{Q_1}{T_1} T_2 - T_2 S_{gen} + Q_1$$

combine Q_1's together.

$$-W = Q_1 \frac{T_1}{T_1} - \frac{Q_1 T_2}{T_1} - T_2 S_{gen}$$

$$\boxed{-W = Q_1 \left(\frac{T_1 - T_2}{T_1} \right) - T_2 S_{gen}}$$

Maximum $S_{gen} = 0$

$$\boxed{-W = Q_1\left(\frac{T_1 - T_2}{T_1}\right)}$$

efficiency

$$\boxed{\frac{-W}{Q_1} = \frac{T_1 - T_2}{T_1}}$$

Fraction of heat supplied converted to work

Carnot Efficiency –

Also impossible –

upper design limit

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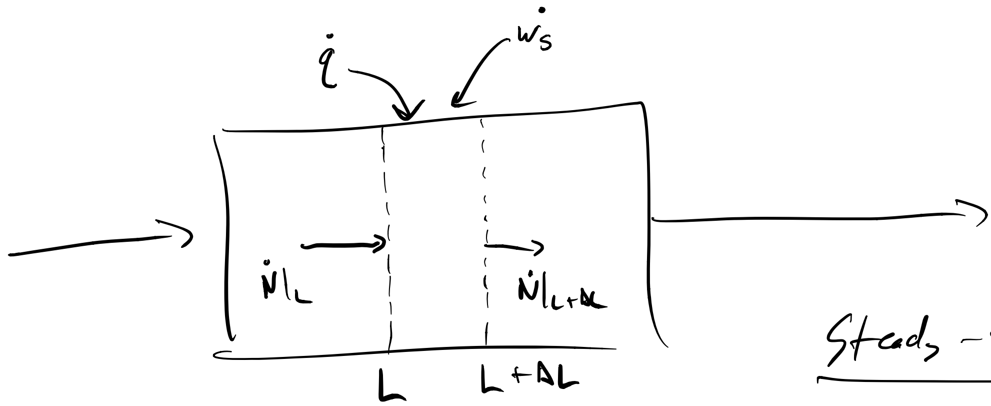
5 minute break.

– System that changes over space



Steady-state

Mass/Mol: $\dfrac{dN}{dt} = 0 = \dot{N}|_L - \dot{N}|_{L+\Delta L}$

Energy: $\dfrac{dU}{dt} = 0 = \dot{N}\underline{H}|_L - \dot{N}\underline{H}|_{L+\Delta L} + \dot{q}\Delta L + \dot{w}_s \Delta L$

Entropy: $\dfrac{dS}{dt} = 0 = \dot{N}\underline{S}|_L - \dot{N}\underline{S}|_{L+\Delta L} + \dfrac{\dot{q}}{T}\Delta L$

$+ \dot{\sigma}_{gen} \Delta L$

what we want →

Equations for state changes from one end of device to other end.

Total work, Total Q, etc.

1) Mass: $\lim\limits_{\Delta L \to 0} \dfrac{\dot{N}_{L+\Delta L} - \dot{N}|_L}{\Delta L} = \dfrac{0}{\Delta L} \to \boxed{\dfrac{d\dot{N}}{dL} = 0}$

$\dot{N} = \text{constant.}$

Energy: $\dot{N} \lim\limits_{\Delta L \to 0} \left(\dfrac{H_{L+\Delta L} - H_L}{\Delta L} \right) = \boxed{\dot{N}\dfrac{dH}{dL} = \dot{q} + \dot{w}_s}$

Entropy:
$$\dot{N} \lim_{\Delta L \to 0} \left(\frac{\underline{S}|_{L+\Delta L} - \underline{S}|_{L}}{\Delta L} \right) = \boxed{\dot{N} \frac{d\underline{S}}{dL} = \frac{\dot{q}}{T} + \dot{\sigma}_{gen}}$$

iff Reversible

$$\boxed{\begin{aligned} \dot{N} \frac{dS}{dL} &= \frac{\dot{q}}{T} \\ w_s &= \dot{N} \left(\frac{d\underline{H}}{dL} - T \frac{d\underline{S}}{dL} \right) \end{aligned}}$$

:)

March 1 @ 6:00p

WLSR

rudin@purdue.edu