Intro to Mass, Energy, Entropy Balance Equations

$$\boxed{\text{In} - \text{Out} + \text{Generation} - \text{Consumption} = \text{Accumulation}}$$

How much water in a lake?

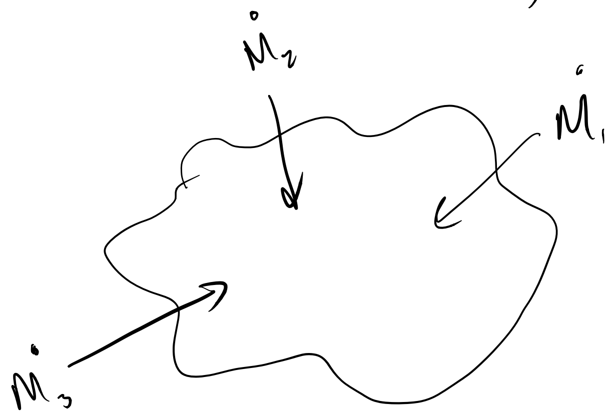
$$\left(\begin{array}{c}\text{Rate of}\\ \text{change of}\\ \text{water in the}\\ \text{lake}\end{array}\right) = \left(\begin{array}{c}\text{Rate at}\\ \text{which water}\\ \text{flows into}\\ \text{the lake}\end{array}\right) - \left(\begin{array}{c}\text{Rate at}\\ \text{which water}\\ \text{flows out}\\ \text{of the}\\ \text{lake}\end{array}\right)$$

What would the unit be?

$\frac{\text{Mass}}{\text{time}}$ $\frac{\text{Volume}}{\text{time}}$

$$\left(\begin{array}{c}\text{Change in}\\ \text{amount of}\\ \text{water in the}\\ \text{lake during}\\ \text{the month}\\ \text{of January}\end{array}\right) = \left(\begin{array}{c}\text{In}\\ \text{during}\\ \text{January}\end{array}\right) - \left(\begin{array}{c}\text{Out}\\ \text{during}\\ \text{January}\end{array}\right)$$

Dimensions? Volume, Mass



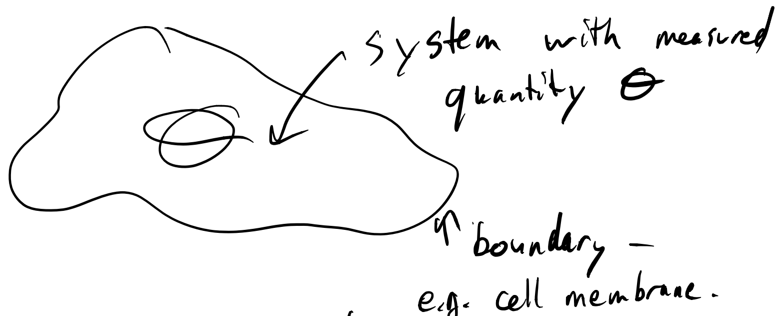
1) In general, we consider a system as 1) open or 2) closed.

2) System has to be defined by some boundary → boundaries can move or be stationary.

When describing systems that change over time, we need dimensions.

t: time
Δt: small interval of time
Θ: some measured conserved quantity in the system.

Day 1: "Time" by Pink Floyd
Day 2: "Take it to the limit" - Eagles.



Write a balance equation on Θ

Open:

$$\begin{pmatrix}\text{Amount of } \Theta \text{ in the} \\ \text{system @ time } t+\Delta t\end{pmatrix} - \begin{pmatrix}\text{Amount of } \Theta \text{ in} \\ \text{the system @} \\ \text{time } t\end{pmatrix} =$$

$$\begin{pmatrix}\text{Amount of } \Theta \text{ that} \\ \text{entered across boundary} \\ \text{between } t \text{ and } t+\Delta t\end{pmatrix} - \begin{pmatrix}\text{Amount of } \Theta \text{ that} \\ \text{exited across the} \\ \text{boundary between } t \text{ and } t+\Delta t\end{pmatrix}$$

$$+ \begin{pmatrix}\text{Amount generated in} \\ \text{the system between } t \text{ and } t+\Delta t\end{pmatrix}$$

Closed → same as above without inflow/outflow

these are all total quantities →

$$\text{Rate} \cdot \underbrace{\text{time interval}}_{\Delta t}$$

$$\Theta(t+\Delta t) - \Theta(t) = \Delta t \cdot (\text{Rate in}) - \Delta t \cdot (\text{Rate out}) + \Delta t \cdot (\text{Generation})$$

divide by Δt

then apply "Fundamental theorem of Calculus"

and get →

$$\lim_{\Delta t \to 0} \frac{\Theta(t+\Delta t) - \Theta(t)}{\Delta t} = \frac{d\Theta}{dt}$$

$$\frac{d\Theta}{dt} = \text{rate in} - \text{rate out} + \dot{\text{gen}} - \dot{\text{consumption}}$$

indicates time rate of change on flow

Ok! Here we go

Conservation of mass for a pure fluid →

$M(t+\Delta t) - M(t) =$ amount entered in Δt − amount exited in Δt.

$$\boxed{\frac{dM}{dt} = \sum_{K=1}^{K} \dot{M}_K}$$

total mass.

We can also do molar quantities

Convert $\underset{\text{mass}}{M}$ to $\underset{\text{moles}}{N}$

MW = molecular weight $\equiv \frac{\text{mass}}{\text{mol}}$

pure component

$$\frac{M}{MW} \rightarrow \frac{[\cancel{mass}]}{[\cancel{mass}]/[mol]} \longrightarrow N \ [mol]$$

$$\boxed{\frac{dN}{dt} = \sum_{k=1}^{K} \dot{N}_k}$$

molar flow rate.

what if you want total mass?

integrate:

$$\int_{t_1}^{t_2} \left(\frac{dM}{dt}\right) dt =$$

$$\frac{dM}{dt} = \sum_{k=1}^{K} \dot{M}_k$$

$$\int_{t_1}^{t_2} \underbrace{\sum_{k=1}^{K} \dot{M}_k}_{} \, dt$$

in time variable is a constant.

$$\sum_{k=1}^{K} \dot{M}_k \int_{t_1}^{t_2} dt$$

total change in system

is $(t_2 - t_1) \sum_{k=1}^{K} \dot{M}_k = M(t_2) - M(t_1)$