1 D radial transient heat flow

Note Title 12/5/2007

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2\frac{dT}{dr}\right] = \underbrace{\frac{\rho C_p}{K}}_{\alpha}\frac{dT}{dt}$$

expanding the derivative

$$\frac{2}{r}\frac{dT}{dr} + \frac{d^2T}{dr^2} = \alpha\frac{dT}{dt}$$

Approximating the derivatives w/ finite differences (S index refers to radial direction, p index refers to time)

$$\frac{dT}{dr} \simeq \frac{T_S^p - T_{S-1}^p}{\Delta r}$$

$$\frac{d^2T}{dr^2} \simeq \frac{T_{S+1}^p - 2T_S^p + T_{S-1}^p}{\Delta r^2}$$

$$\frac{dT}{dt} \simeq \frac{T_S^{p+1} - T_S^p}{\Delta t}$$

Inserting these into ODE

$$\frac{2}{r}\frac{dT}{dr} + \frac{d^2T}{dr^2} = \alpha\frac{dT}{dt}$$

$$\frac{2}{r}\left(\frac{T_S^p - T_{S-1}^p}{\Delta r}\right) + \frac{T_{S+1}^p - 2T_S^p + T_{S-1}^p}{\Delta r^2} = \alpha\left(\frac{T_S^{p+1} - T_S^p}{\Delta t}\right)$$

Solving for T_S^{p+1}

$$T_S^{p+1} = \frac{\Delta t}{\alpha}\left[\frac{T_{S+1}^p + T_{S-1}^p}{\Delta r^2} - \frac{2}{r\Delta r}T_{S-1}^p\right] + \left[1 + \frac{2\Delta t}{\alpha r\Delta r} - \frac{2\Delta t}{\alpha\Delta r^2}\right]T_S^p$$

This eqn can be used to calculate $T(r,t)$ given step sizes and initial & boundary conditions