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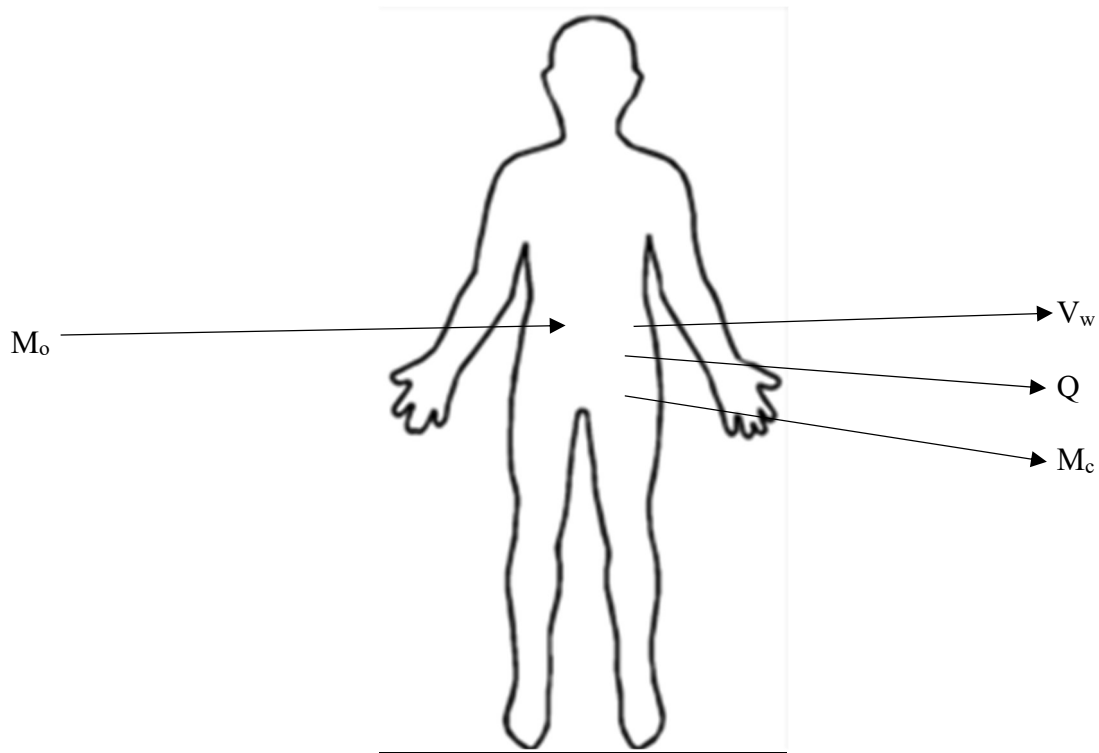
ABE 301

Deliverable 3

Nomenclature Table:

Symbol	Meaning	Units
A	surface area of player	[m <sup>2</sup> ]
c <sub>p</sub>	specific heat	[kJ/kg.K]
ΔH <sub>rxn</sub>	enthalpy of cellular respiration	[kJ]
dV <sub>w</sub> /dt	rate of sweat production	[L/min]
E <sub>ATP</sub>	energy of one mol of atp	[kJ/mol]
E <sub>K</sub>	kinetic energy	[kJ]
E <sub>P</sub>	potential energy	[kJ]
H	enthalpy	[kJ]
h <sub>conv,air</sub>	convective heat transfer coefficient of air	[W/m <sup>2</sup> K]
h <sub>player</sub>	height of player	[m]
h <sub>vap,water</sub>	heat of vaporization of water	[kJ/mol]
M <sub>A</sub>	mass of ATP	[kg]
M <sub>C</sub>	Mass of Carbon Dioxide	[kg]
M <sub>O</sub>	Mass of Oxygen	[kg]
M <sub>player</sub>	Mass of player	[kg]
MW <sub>O</sub>	molecular weight of oxygen	[kg]
MW <sub>W</sub>	molecular weight of water	[kg]
π	mathematical constant pi	[-]
Q	heat	[kJ]
r	rate of reaction of cellular respiration	[s <sup>-1</sup> ]
ρ <sub>w</sub>	density of water	[kg/m <sup>3</sup> ]
t	time	[s]
T <sub>air</sub>	Ambient temperature	[°C]
t <sub>dehydration</sub>	time of dehydration	[min]
T <sub>player</sub>	player temperature	[°C]
T <sub>ref</sub>	normal player temperature	[°C]
U	internal energy	[kJ]
v	speed of player	[m/s]
V <sub>dehydration</sub>	volume of water in player at which player is dehydrated	[L]
v <sub>G</sub>	stoichiometric coefficient of glucose	[-]
v <sub>O</sub>	stoichiometric coefficient of oxygen	[-]
V <sub>W</sub>	Volume of Water	[L]
V <sub>W,i</sub>	initial volume of water in player	[L]
V <sub>W,out</sub>	volume of water sweat out	[L]
W	work	[kJ]

### System Diagram:



**Figure 1:** System diagram. A soccer player's body defines the system boundaries. The net components entering and leaving the system are oxygen, carbon dioxide, water, and heat. Environmental conditions and system specifications (listed below) affect how these inputs and outputs move between the system and environment.

#### Environmental conditions:

- Ambient Temperature (model input) [ $^{\circ}\text{C}$ ]

#### System specifications:

- Player mass (model input) [kg]

## **Iteration 1**

### **Assumptions:**

1. Ambient temperature and humidity are constant during the game
2. Throughout the game, the body temperature remains at 37°C
3. There is no wind or precipitation to affect the body temperature
4. Uniform has no effect on heat loss
5. Emotional stress of the game has no effect on sweat production
6. The player starts playing at time  $t=0$
7. There is no extra time added at the end of either half
8. There are no pauses in game play
9. The player plays the full 90 minutes
10. Water is not consumed by the player during the game time
11. Dehydration occurs at a loss of 5% of water content of the body (“Water Purification”, 2008)
12. The body is composed of 60% water by mass (Perlman, 2016)
13. The player’s normal body metabolism is negligent, does not produce heat to warm the body up to normal temperature
14. Sweat is the only means by which the body loses heat
15. Humidity has no effect on evaporation
16. The player is cylindrical in shape
17. The player’s shoulder width is  $\frac{1}{4}$  the height of the player (Ida, 2012)
18. The player does not move throughout the game

### **Model Inputs:**

- $T_{\text{air}}$  [°C]
  - This parameter may change the rate at which the player sweats.
  - If the temperature is greater than room temperature, the player’s sweat rate will increase as the body must produce even more sweat to get the body down to a normal, functional temperature.

- If the temperature is less than room temperature, the player's sweat rate will decrease as some of the heat produced by the body is used to bring the body temperature up to a normal, functional temperature.
- Stays constant over time (Assumption 1)
- For model testing purposes, this value was set at 23°C (room temperature)
- $M_{\text{player}}$  [kg]
  - This parameter determines how much work a player must do to move himself.
  - Additionally, the parameter helps to determine how much water the player can lose before becoming dehydrated.
  - For model testing purposes, this value was set at 80 kg (mass of Jordan Henderson, my favorite player)
- $h_{\text{player}}$  [m]
  - This parameter helps determine the surface area of the player which is used in determining the convective heat transfer with the air. (Assumption 15)
  - For model testing purposes, this value was set at 1.8 m (height of Jordan Henderson, my favorite player)

#### Model Outputs:

- $dV_w/dt$  [L/min]
  - As the player runs and adjusts to the game environment, the rate of sweat (water) leaving the body changes.
  - This output will be shown as a graph of rate of sweat production versus time.
- $V_w$  [L]
  - By integrating the graph of rate of sweat production versus time, the total volume of sweat produced in a unit of time (e.g. one minute, one half, one game, etc.) will be calculated.
  - This output will be shown as a graph and a numerical value after each half of the game (volume = x L at  $t = 45$  minutes,  $t = 90$  minutes).
- $t_{\text{dehydration}}$  [min]
  - By comparing the total amount of sweat produced by the player and the amount of water in the body at time  $t = 0$ , the time at which the player is dehydrated can be

found. If a player loses more than 5% of the water in their body, they will begin to show signs of dehydration.

- This output will be shown as a numerical value ( $t = x$  minutes).
- Does not exceed 90 minutes (Assumption 9)

#### Fundamental Relationships:

- Steady state energy balance: internal energy + kinetic energy + potential energy = heat + work + enthalpy
  - $U + E_K + E_P = Q + W + H$
- Internal energy = mass \* specific heat \* change in temperature
  - $U = Mc_p \Delta T$
  - $C_p$  of human body = 3470 J/kg.K (SPECIFIC HEAT CAPACITIES, n.d.)
- Convective heat = convective heat transfer coefficient \* surface area \* difference in temperature \* time
  - $Q = h_c A \Delta T t$ 
    - $h_c = 20 \text{ W/m}^2\text{K}$  (Convective Heat Transfer, n.d.)
- Surface area of a cylinder =  $\pi$  \* diameter \* height
  - $A = \pi(h/4)h$
- Heat of evaporation = heat of vaporization of water \* volume of water \* density / molecular weight of water
  - $Q = h_{\text{vap,water}} V_w \rho_w / MW_w$

#### Calculations:

- Mass balances
  - Water:  $dV_w/dt = -V_{w,\text{out}}$ 
    - Change in water volume = – Volume sweat out
    - Principle: unsteady state, open system mass balance
- Energy balance
  - $M_{\text{player}} c_p (T_{\text{player}} - T_{\text{ref}}) = h_{\text{conv,air}} * A * (T_{\text{air}} - T_{\text{player}}) * t$ 
    - Heat of player = heat transferred from surroundings

- Principles: steady state, open system energy balance, convective heat transfer
- $M_{\text{player}} c_p (T_{\text{player}} - T_{\text{ref}}) = h_{\text{conv,air}} * A * (T_{\text{air}} - T_{\text{player}}) * t = h_{\text{vap,water}} * V_w * \rho_w / MW_w$ 
  - Heat of player = heat transferred from surroundings = heat of vaporization of water
  - Principles: evaporation, convective heat transfer
- $V_w = h_{\text{conv,air}} * A * (T_{\text{air}} - T_{\text{player}}) * t * MW_w / h_{\text{vap,water}} * \rho_w$ 
  - $h_{\text{vap,water}} = 43345 \text{ kJ/mol}$  (Water – Heat of Vaporization, n.d.)
- $dV_w/dt = h_{\text{conv,air}} * A * (T_{\text{air}} - T_{\text{player}}) * MW_w / h_{\text{vap,water}} * \rho_w$
- $V_{\text{dehydration}} = V_{w,i} - 0.05 * V_{w,i}$  (Assumption 11)
  - $V_{w,i} = 0.6 * M_p$  (Assumption 12)
  - $V_{w,i} - 0.05 * V_{w,i} = h_{\text{conv,air}} * A * (T_{\text{air}} - T_{\text{player}}) * t_{\text{dehydration}} * MW_w / h_{\text{vap,water}} * \rho_w$
  - $t_{\text{dehydration}} = (V_{w,i} - 0.05 * V_{w,i}) * h_{\text{vap,water}} * \rho_w / h_{\text{conv,air}} * A * (T_{\text{air}} - T_{\text{player}}) * MW_w$

## Iteration 2

### Assumptions:

18. ~~The player does not move throughout the game~~ The player moves at a constant speed throughout the game.
19. Average speed of the player is not affected by the position played.
20. Oxygen is the limiting reactant in metabolism to create energy
21. The energy required to kick/dribble/tackle etc. is negligent to the energy required to run/sprint/jog
22. Volumes of other gases inhaled (e.g. nitrogen, hydrogen, etc.) is exhaled, meaning the net intake of these gases is zero; the only gas inhalation measured in this model is oxygen, the only gas exhalation measured in this model is carbon dioxide
23. Oxygen, carbon dioxide, and ATP mass balances are at steady state
24. There is no change in elevation on the field

### Model Inputs:

- $T_{\text{air}}$  [°C]
- $M_{\text{player}}$  [kg]
- $h_{\text{player}}$  [m]
- $v$  [m/s]
  - This input is used to determine how much kinetic energy is needed to be produced by the player.
  - For model testing purposes, this value was set to 6 m/s.

### Model Outputs:

- $dV_w/dt$  [L/min]
- $V_w$  [L]
- $t_{\text{dehydration}}$  [min]

### Fundamental Relationships:

- Cellular respiration:  $C_6H_{12}O_6 + 6O_2 \rightarrow 6CO_2 + 6H_2O + 36 \text{ ATP}$
- Steady state mass balance:  $\text{in} - \text{out} + \text{generation} - \text{consumption} = 0$

- Unsteady state mass balance:  $\text{in} - \text{out} + \text{generation} - \text{consumption} = \text{accumulation}$
- Steady state energy balance  $\text{internal energy} + \text{kinetic energy} + \text{potential energy} = \text{heat} + \text{work} + \text{enthalpy}$ 
  - $U + E_K + E_P = Q + W + H$
- Kinetic energy:  $\frac{1}{2} * \text{mass} * \text{velocity}^2$  (Assumption 21)
  - $\frac{1}{2} * M_p * v^2$
- Internal energy
- Surface area of a cylinder
- Heat
  - Heat = heat of reaction + heat transfer due to convection + energy from ATP
    - Heat of reaction = enthalpy of reaction \* mass of glucose reacted
    - Heat transfer due to convection = heat transfer coefficient of air \* surface area of player \* difference in temperature between player and air
    - Energy from ATP = moles of ATP \* energy released by hydrolysis of ATP

#### Calculations:

- Mass balances
  - Oxygen:  $M_{O,\text{in}} - M_{O,\text{reaction}} = 0$  (Assumption 22, 23)
    - Mass in = Mass used by reaction
    - Principle: steady state, open system mass balance
  - Carbon Dioxide:  $M_{C,\text{reaction}} - M_{C,\text{out}} = 0$  (Assumption 22, 23)
    - Mass created by reaction = Mass out
    - Principle: steady state, open system mass balance
  - ATP:  $M_{A,\text{reaction}} - M_{A,\text{consumed}} = 0$  (Assumption 23)
    - Mass created by reaction = Mass out
    - Principle: steady state, closed system mass balance
  - Water:  $dV_w/dt = -V_{w,\text{out}}$
- Energy balance
  - $\frac{1}{2} * M_p * v^2 + M_{\text{player}} c_p (T_{\text{player}} - T_{\text{ref}}) = h_{\text{conv,air}} * A * (T_{\text{air}} - T_{\text{player}}) * t + \Delta H_{\text{rxn}} * v_g / v_o$   
 $M_o / MW_o * r * t + M_{\text{ATP}} * E_{\text{ATP}}$



- Kinetic energy of player + heat of player = heat transferred from surroundings + heat created by reaction + energy from ATP
  - Principles: steady state, open system energy balance, enthalpy of reaction, convective heat transfer
- $M_{\text{player}} * v^2 / 2 = M_{\text{ATP}} * E_{\text{ATP}}$ 
  - Kinetic energy = energy of ATP
- $M_{\text{player}} c_p (T_{\text{player}} - T_{\text{ref}}) = h_{\text{conv,air}} * A * (T_{\text{air}} - T_{\text{player}}) * t + \Delta H_{\text{rxn}} * v_g / v_o * M_o / MW_o * r * t = h_{\text{vap,water}} * V_w * \rho_w / MW_w$ 
  - Heat of player = heat transferred from surroundings + heat of reaction = heat of vaporization of water
  - Principles: evaporation, enthalpy of reaction, convective heat transfer
- $V_w = (h_{\text{conv,air}} * A * (T_{\text{air}} - T_{\text{player}}) * t + \Delta H_{\text{rxn}} * v_g / v_o * M_o / MW_o * r * t) * MW_w / h_{\text{vap,water}} * \rho_w$
- $dV_w / dt = (h_{\text{conv,air}} * A * (T_{\text{air}} - T_{\text{player}}) * MW_w + \Delta H_{\text{rxn}} * v_g / v_o * M_o / MW_o * r) / h_{\text{vap,water}} * \rho_w$
- $V_{\text{dehydration}} = V_{w,i} - 0.05 * V_{w,i}$ 
  - $V_{w,i} - 0.05 * V_{w,i} = (h_{\text{conv,air}} * A * (T_{\text{air}} - T_{\text{player}}) * t_{\text{dehydration}} * MW_w + \Delta H_{\text{rxn}} * v_g / v_o * M_o / MW_o * r * t_{\text{dehydration}}) / h_{\text{vap,water}} * \rho_w$
  - $t_{\text{dehydration}} = (V_{w,i} - 0.05 * V_{w,i}) * h_{\text{vap,water}} * \rho_w / (h_{\text{conv,air}} * A * (T_{\text{air}} - T_{\text{player}}) * MW_w + \Delta H_{\text{rxn}} * v_g / v_o * M_o / MW_o * r)$

### Iteration 3

#### Assumptions:

25. Conversion from chemical energy to kinetic energy is 25% efficient

#### Model Inputs:

- $T_{\text{air}}$  [ $^{\circ}\text{C}$ ]
- $M_{\text{player}}$  [kg]
- $h_{\text{player}}$  [m]
- $v$  [m/s]

#### Model Outputs:

- $dV_w/dt$  [L/min]
- $V_w$  [L]
- $t_{\text{dehydration}}$  [min]

#### Fundamental Relationships:

- Cellular respiration
- Steady state mass balance
- Unsteady state mass balance
- Steady state energy
- Kinetic energy
- Internal energy
- Surface area of a cylinder
- Heat

#### Calculations:

- Mass balances
  - Oxygen:  $M_{\text{O},\text{in}} - M_{\text{O},\text{reaction}} = 0$
  - Carbon Dioxide:  $M_{\text{C},\text{reaction}} - M_{\text{C},\text{out}} = 0$
  - ATP:  $M_{\text{A},\text{reaction}} - M_{\text{A},\text{consumed}} = 0$

- Water:  $dV_w/dt = -V_{w,out}$
- Energy balance
  - $\frac{1}{2} * M_p * v^2 + M_{player} c_p (T_{player} - T_{ref}) = h_{conv,air} * A * (T_{air} - T_{player}) * t + \Delta H_{rxn} * v_g / v_o * M_o / MW_o * r * t + M_{ATP} * E_{ATP}$
  - $M_{player} * v^2 / 2 = 0.25 * M_{ATP} * E_{ATP}$ 
    - Kinetic energy = 25% \* energy of ATP
  - $M_{player} c_p (T_{player} - T_{ref}) = h_{conv,air} * A * (T_{air} - T_{player}) * t + \Delta H_{rxn} * v_g / v_o * M_o / MW_o * r * t + 0.75 * M_{ATP} * E_{ATP} = h_{vap,water} * V_w * \rho_w / MW_w$ 
    - Heat of player = heat transferred from surroundings + heat of reaction + 75% \* energy of ATP = heat of vaporization of water
    - Principles: evaporation, enthalpy of reaction, convective heat transfer
  - $V_w = (h_{conv,air} * A * (T_{air} - T_{player}) * t + \Delta H_{rxn} * v_g / v_o * M_o / MW_o * r * t + 0.75 * M_{ATP} * E_{ATP} * r * t) * MW_w / h_{vap,water} * \rho_w$
  - $dV_w/dt = (h_{conv,air} * A * (T_{air} - T_{player}) * MW_w + \Delta H_{rxn} * v_g / v_o * M_o / MW_o * r + 0.75 * M_{ATP} * E_{ATP} * r) / h_{vap,water} * \rho_w$
- $V_{dehydration} = V_{w,i} - 0.05 * V_{w,i}$ 
  - $V_{w,i} - 0.05 * V_{w,i} = (h_{conv,air} * A * (T_{air} - T_{player}) * t_{dehydration} * MW_w + \Delta H_{rxn} * v_g / v_o * M_o / MW_o * r * t_{dehydration} + 0.75 * M_{ATP} * E_{ATP} * r * t_{dehydration}) / h_{vap,water} * \rho_w$
  - $t_{dehydration} = (V_{w,i} - 0.05 * V_{w,i}) * h_{vap,water} * \rho_w / (h_{conv,air} * A * (T_{air} - T_{player}) * MW_w + \Delta H_{rxn} * v_g / v_o * M_o / MW_o * r + 0.75 * M_{ATP} * E_{ATP} * r)$

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