

Final Report

Decay of Toluene in Sol-Gel Encapsulated *Pseudomonas putida*

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Table of Contents

Background.....	3
Model Derivation.....	5
Model Analysis.....	6
Nomenclature Summary.....	11
References.....	12
Appendix.....	13

Background

Toluene is a solvent found in gasoline and toluene exposure has been linked to central nervous system dysfunction (ATSDR, 2000) for both short-term and long-term exposure.

Pseudomonas putida is a gram-negative bacterium that naturally breaks down toluene and incorporates the products into its metabolic cycle.

Once transported into *P. putida* by membrane protein TodX, toluene catalyzes the phosphorylation of TodS, the first protein in a two-component regulatory pathway (Wang et al., 2994). TodS then transphosphorylates TodT, which serves as an activator of the PTodX promoter (Lacal et al., 2006). Up to five phosphorylated TodT proteins bind to the promoter, causing increasing levels of activation of the gene (Lacal et al., 2008). The PTodX promoter controls transcription of the Tod cassette, which houses both the TodX membrane transport protein and the enzymes responsible for the degradation and incorporation of toluene into the citric acid cycle. The first enzyme in this pathway is toluene dioxygenase.

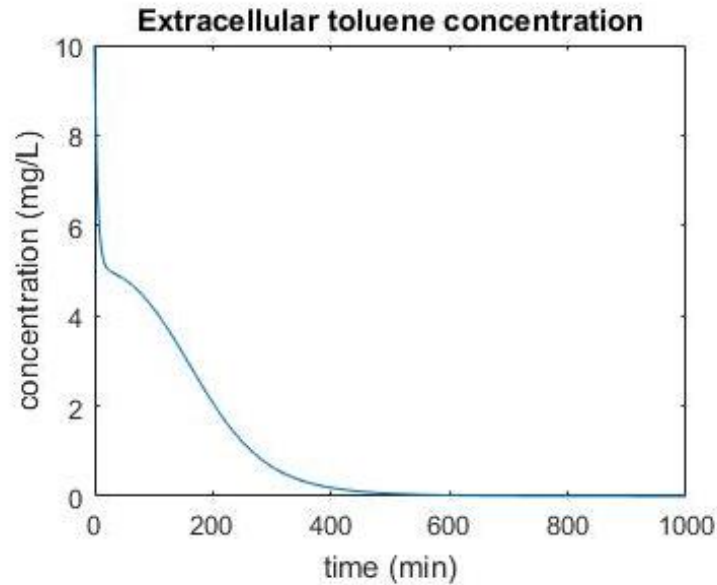
To act as a biosensor and bioreporter, *P. putida* TVA8 was engineered by inserting a lux gene cassette behind a copy of the PtodX reporter (Applegate et al., 1998). The assembly was then inserted via mini-transposon delivery. Thus, when the PtodX promoter is active, luciferase, a fluorescent protein, is expressed along with the enzymes of the toluene degradation pathway. A shortcoming of biosensors is that their lifetime is contingent upon the organisms remaining alive. TVA8 cells have been encapsulated in various formulations to determine the method that maintains functionality while extending cell viability. It has been determined that a TMOS alcohol-free sol-gel encapsulation is the formulation that best serves this goal (Zhang, 2008).

Previous studies have measured fluorescence of a colony of TVA8 cells when exposed to a finite inducement of toluene solution (Zhang 2008) but not model has been developed to describe the degradation of toluene by encapsulated TVA8 cells.

Model Derivation

The first mass component of the system is the toluene. It was assumed that the toluene solution was induced instantaneously and uniformly throughout the sol-gel slab. To account for the diffusion of the toluene from the sol-gel matrix into the cells, a pooled model was used. One variable, T_{out} (units of mg/L), represented the concentration of toluene outside of the cells. Another variable, T_{in} (also units of mg/L), represented the concentration of toluene inside the cells. As toluene requires the TodX transport protein to move into the cell, the rate of transport into the cells was defined as the difference in concentrations multiplied by the concentration of TodX. As this system is closed and contains a finite amount of toluene, the accumulation of T_{out} was set equal to the negative of this rate (Equation 1). The constant of proportionality k_5 was parametrized and set to 0.1 for the final model.

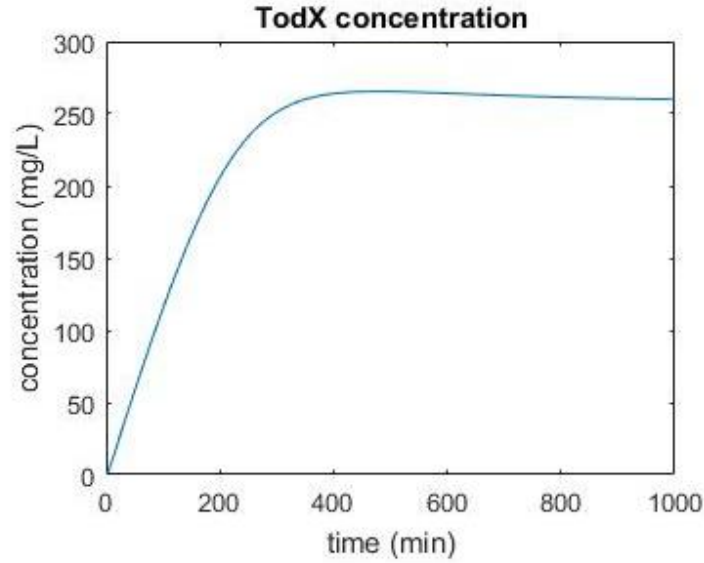
$$\frac{dT_{out}}{dt} = -k_5(T_{out} - T_{in})Todx \quad (1)$$



The generation of TodX (in mg/L) was modeled using a hill function and it was assumed that TodX decays at a constant rate (Equation 2). The constants of the hill function were assumed to

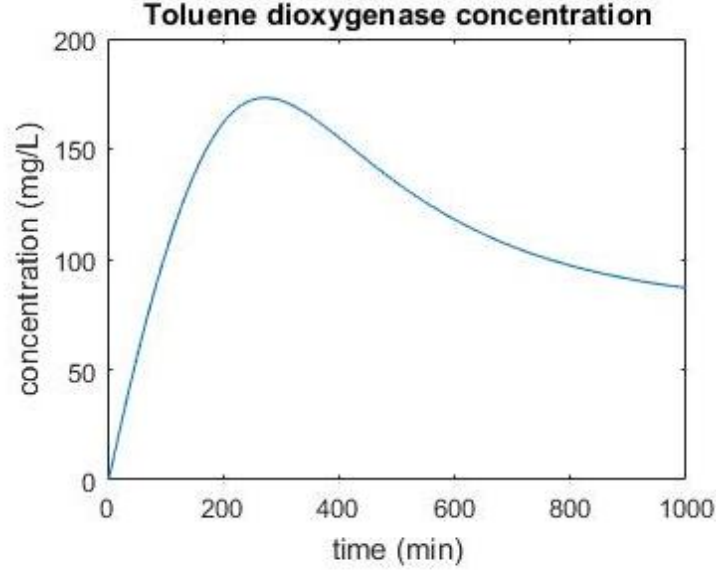
be the same as that of luciferase which were researched values (Kelly et al. 2000). The decay constant was calculated assuming a protein half-life of 1000 minutes ($k_1 = \log(2)/1000$).

$$\frac{dT_{odx}}{dt} = \frac{V_{m,tol}T}{K_{m,tol}+T} - k_1 T_{odx} \quad (2)$$



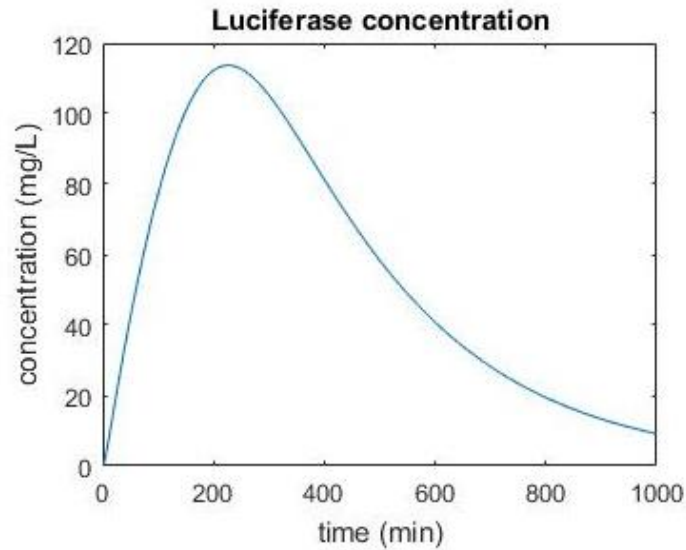
As toluene dioxygenase, E (mg/L), is expressed under the same promoter, it was assumed the same hill function could describe the generation and it was assumed that toluene dioxygenase decays at a constant rate (Equation 3) equal to that of luciferase

$$\frac{dE}{dt} = \frac{V_{m,tol}T}{K_{m,tol}+T} - k_1 E \quad (3)$$



Similarly, luciferase, L (mg/L), is expressed under the P_{tox} promoter so it was assumed to follow the same generation kinetics. The half-life of luciferase was found to be 180 minutes and so the decay rate was set to $\log(2)/180$ (Equation 4).

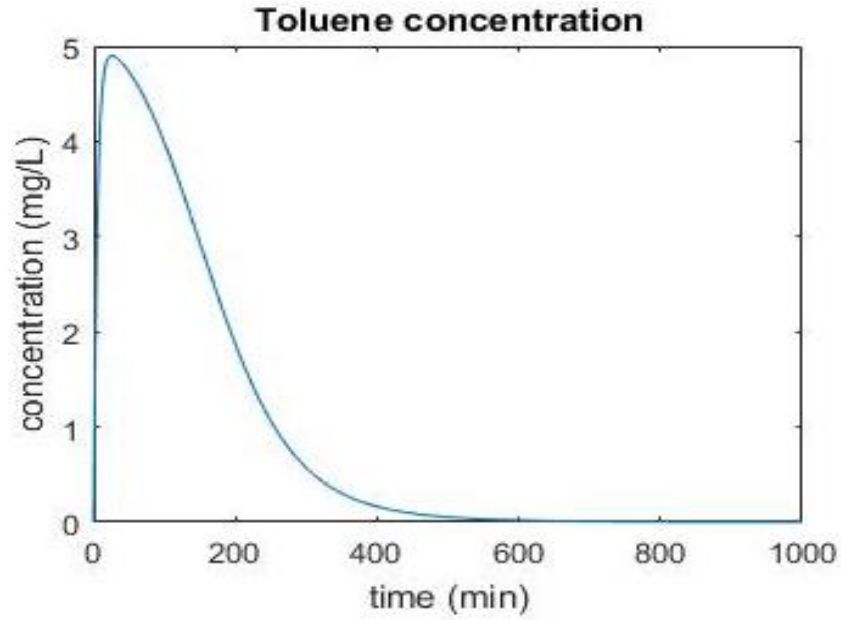
$$\frac{dL}{dt} = \frac{V_{m,lucif}T}{K_{m,lucif} + T} - k_1 L \quad (4)$$



Toluene is consumed by the toluene dioxygenase enzyme so its consumption rate was set proportional to the concentration of enzyme and the concentration of itself, the substrate. This

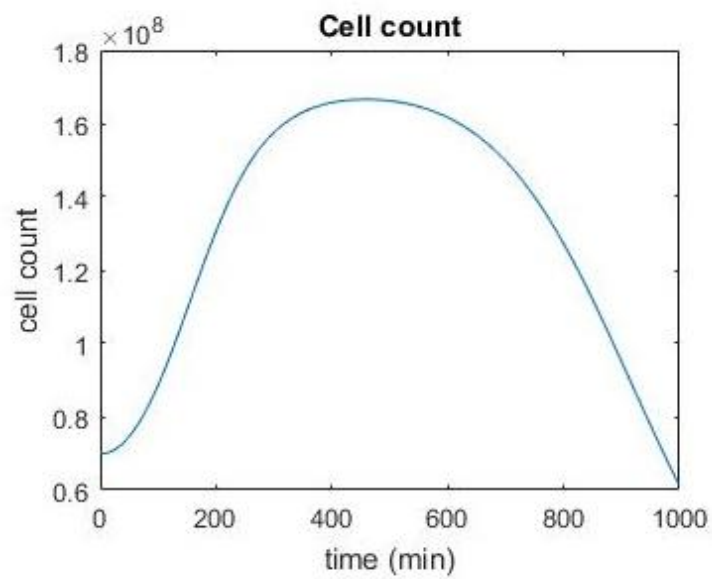
value was multiplied by the cell count to account for all the cells consuming toluene (Equation 5). The constant k_2 was parametrized to 10^{-12} .

$$\frac{dT_{in}}{dt} = k_5(T_{out} - T_{in})T_{odx} - k_2CET \quad (5)$$



As toluene is the sole carbon source for the cells in the gel, their growth rate is dependent upon the consumption rate of toluene. The cell death rate is also inversely proportional to the toluene concentration (Equation 6). The constants k_3 and k_4 were both parametrized to 10^{-5} .

$$\frac{dC}{dt} = k_3CET - k_4\frac{C}{T} \quad (6)$$



Analysis of Model

In comparing the output of the model to recorded data (Figure 1), it is seen that the general curve of the luciferase production and then decay is accounted for by the model.

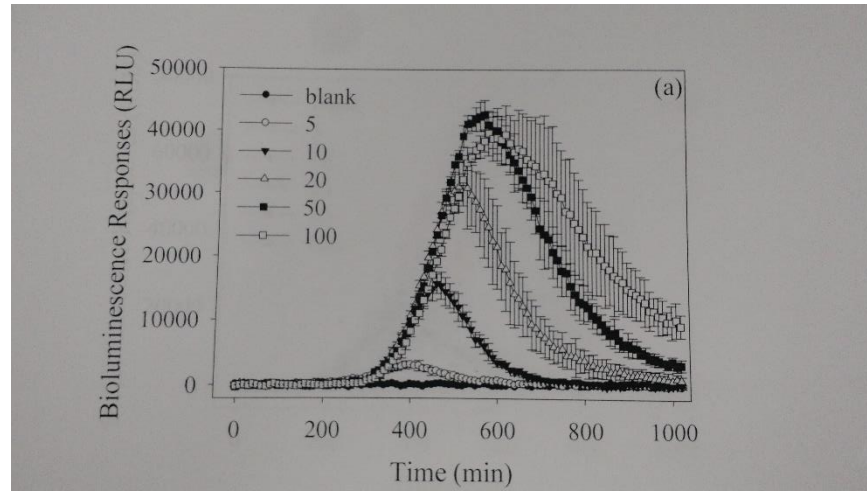


Figure 1: Measured data of toluene degradation in TVA8 cells (Zhang 2008). The data represented by the

white circles at 100 μ M is the concentration used in the model presented in this paper (9.2mg/L).

The starkest difference between the two is the noticeable time lag in the real data while the model does not account for this lag. Future improvements to the model to account for this lag include modeling the diffusion of the toluene solution throughout the gel or modeling the phosphorylation of the two-component regulatory system composed of TodS and TodT.

Nomenclature Summary

Variables

<i>Parameter Symbol</i>	<i>Meaning</i>	<i>Units</i>
T_{in}	concentration of toluene inside the cells	[mg/L]
T_{out}	concentration of toluene outside of the cells	[mg/L]
C	number of cells	[cell number]
L	concentration of luciferase	[mg/L]
E	concentration of toluene dioxygenase	[mg/L]
T_{odx}	concentration of todx promoter	[mg/L]

Constants

<i>Symbol</i>	<i>Meaning</i>	<i>Value</i>	<i>Source</i>
k_3	growth constant	10^{-5}	parametrized
k_4	death constant	10^{-5}	parametrized
$klucif$	max luciferase transcription rate	2	Kelly et al. 2000
$Klucif$	half-saturation luciferase transcription rate	5	Kelly et al. 2000
$T_{1/2}$	luciferase half life	180	Thompson et al. 1991
k_5	toluene degradation constant	0.1	parametrized
$ktol$	max tod transcription rate	2	Kelly et al. 2000
$Ktol$	half-saturation tod transcription rate	5	Kelly et al. 2000
$T_{1/2, Todx}$	Todx half life	50	parametrized

References

- Agency for Toxic Substances and Disease Registry (ATSDR). Toxicological Profile for Toluene. U.S. Public Health Service, U.S. Department of Health and Human Services, Atlanta, GA. 2000.
- Applegate, BM, Kehrmeier, SR, Sayler, GM. A Chromosomally Based tod-luxCDABE Whole-Cell Reporter for Benzene, Toluene, Ethylbenzene, and Xylene (BTEX) Sensing. *Applied and Environmental Microbiology* 1998;64(7):2730-2735.
- Kelly CJ, Bienkowski PR, Sayler, GS. Kinetic Analysis of a tod-lux Bacterial Reporter for Toluene Degradation and Trichloroethylene Cometabolism. *Biotechnology and Bioengineering* 2000;69(3):256-265.
- Lacal, J, Busch, A, Guazzaroni, ME, Krell, T, Ramos, JL. The TodS-TodT two-component regulatory system recognizes a wide range of effectors and works with DNA-bending proteins. *PNAS* 2006;103(21):8191-8196.
- Lacal, J, Guazzaroni, ME, Busch, A, Krell, T, Ramos, JL. Hierarchical Binding of the TodT Response Regulator to Its Multiple Recognition Sites at the tod Pathway Operon Promoter. *Molecular Biology* 2008;376:325-337.
- Reardon KF, Mosteller DC, Rogers JDB. Biodegradation kinetics of benzene, toluene and phenol as single and mixed substrates for *Pseudomonas putida* F1. *Biotechnol Bioeng* 2000;69:385–400.
- Thompson JF, Hayes LS, Lloyd DB. Modulation of firefly luciferase stability and impact on studies of gene regulation. *Gene*. 1991 Jul 22 103(2):171-7.
- Wang, Y, Rawlings, M, Gibson, DT, Labbe, D, Bergeron, H, Brousseau, R, Lau, PCK. Identification of a membrane protein and a truncated LysR-type regulator associated with the toluene degradation pathway in *Pseudomonas putida* F1. *Mol Gen Genet* 1995;246:570-579.
- Zhang, S (2008). Sol-gel Immobilization of Bioreporter Cells and CN1.4 for Biosensing and Bioactuation (Unpublished Masters Thesis). Purdue University, West Lafayette, IN.

Appendix A

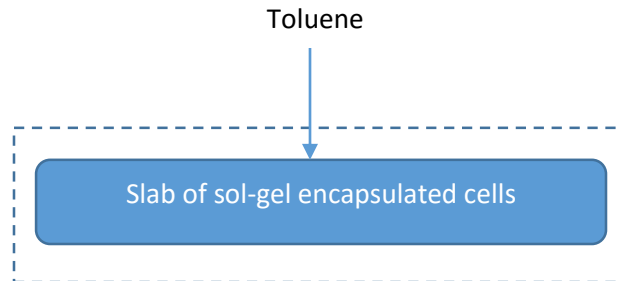
This section contains the history of iterations that led to the development of this model. Each iteration is described with a diagram of the system, a list of the variables modeled, the core assumptions that are modeled, the equations that were used to model the variables, the initial conditions, the parameter values, and graphs that represent the output of that iteration of the model.

For sake of simplicity, the iterations have been summarized in Table 1 and been given page numbers for reference.

<i>Iteration</i>	<i>Significant Changes</i>	<i>Page Number</i>
0	base model	14
0.1	nonconstant toluene degradation	17
0.2	adds luciferase concentration variable	20
0.3	adds luciferase decay term	23
1	adds hill function for generation of luciferase	26
2	cell growth rate dependent on toluene consumption rate	30
3	adds cell death term	34
4	cell death inversely proportional to toluene	38
5	adds toluene dioxygenase variable	42
6	toluene degradation proportional to toluene dioxygenase	46
7	adds T_{out} variable	50
8	models toluene diffusion with Fickian diffusion	55
9 (final)	adds $TodX$ variable	60

Iteration 0

Picture



Parameters

Parameter Symbol	Meaning	Units
T	concentration of toluene	[mg/L]
C	number of cells	[cell number]

Assumptions

1. Instantaneous, homogenous distribution of toluene throughout the system
2. No cell growth or death
3. Degradation of toluene is proportional to amount of toluene

Equations

$$\frac{dT}{dt} = -k_1 T$$

$$\frac{dC}{dt} = 0$$

Initial Conditions

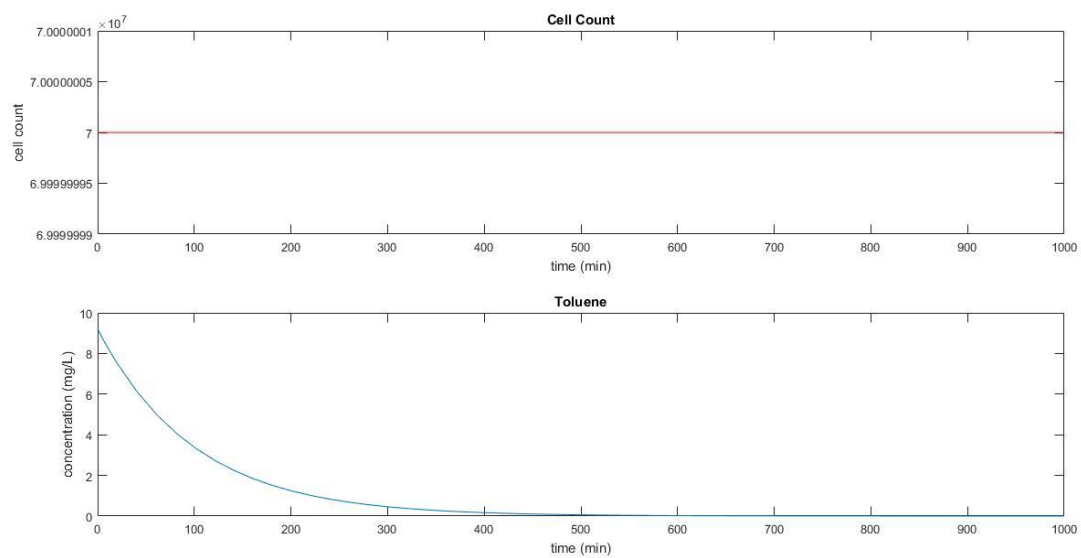
$$T(0) = 9.2 \text{ mg/L}$$

$$C(0) = 7 \times 10^8 \text{ cells}$$

Parameter Values

$$K1 = 1\text{E-}2 \text{ (parametrized value)}$$

Model Output



Main Code

```

clc
clear

Time = [0 1000];
Toluene = 9.2; % initial Toluene concentration [mg/L]

[T, Y] = ode45('LuciferaseModelFunctions0', Time, [70000000, 9.2]);

subplot(2,1,1);
plot(T,Y(:,1),'r');
title('Cell Count');
xlabel('time (min)');
ylabel('cell count');
subplot(2,1,2);
plot(T,Y(:,2));
hold on
title('Toluene');
xlabel('time (min)');
ylabel('concentration (mg/L)');

```

Function Code

```

function A = LuciferaseModelFunctions0(~,Y)
A = zeros(2,1);

C = Y(1);
T = Y(2);

% Cell count is constant
A(1) = 0;

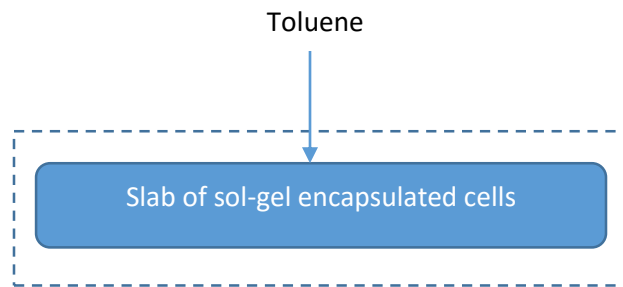
% Toluene level
k2 = 1E-2;
A(2) = -k2*T;

end

```


Iteration 0.1

Picture



Parameters

Parameter Symbol	Meaning	Units
T	concentration of toluene	[mg/L]
C	number of cells	[cell number]

Assumptions

1. Instantaneous, homogenous distribution of toluene throughout the system
2. No cell growth or death
3. Degradation of toluene is proportional to amount of toluene **and cell count**

Equations

$$\frac{dT}{dt} = -k_1 TC$$

$$\frac{dC}{dt} = 0$$

Initial Conditions

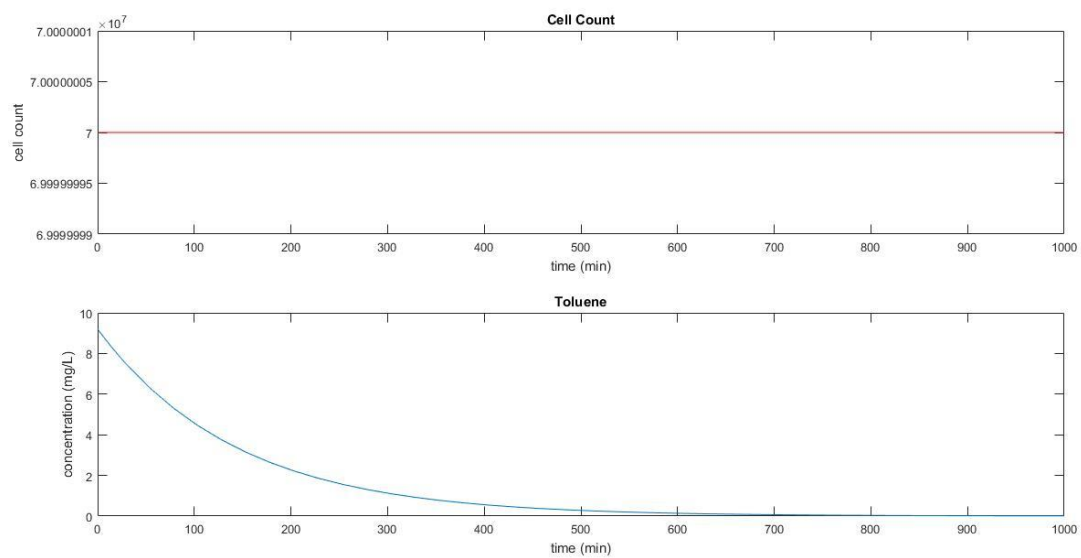
$$T(0) = 9.2 \text{ mg/L}$$

$$C(0) = 7 \times 10^8 \text{ cells}$$

Parameter Values

$$K1 = 1\text{E-}10 \text{ (parametrized value)}$$

Model Output



Main Code

```

clc
clear

Time = [0 1000];
Toluene = 9.2; % initial Toluene concentration [mg/L]

[T, Y] = ode45('LuciferaseModelFunctions01', Time, [70000000, 9.2]);

subplot(2,1,1);
plot(T,Y(:,1),'r');
title('Cell Count');
xlabel('time (min)');
ylabel('cell count');
subplot(2,1,2);
plot(T,Y(:,2));
hold on
title('Toluene');
xlabel('time (min)');
ylabel('concentration (mg/L)');

```

Function Code

```

function A = LuciferaseModelFunctions01(~,Y)
A = zeros(2,1);

C = Y(1);
T = Y(2);

% Cell count is constant
A(1) = 0;

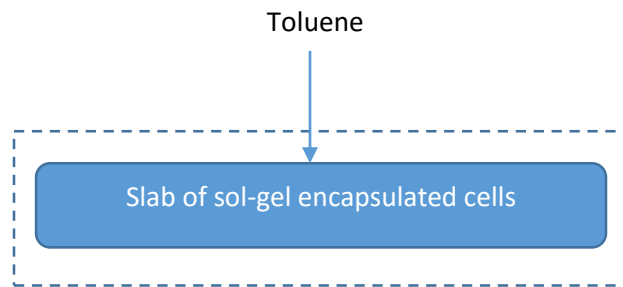
% Toluene level
k2 = 1E-10;
A(2) = -k2*T*C;

end

```

Iteration 0.2

Picture



Parameters

Parameter Symbol	Meaning	Units
T	concentration of toluene	[mg/L]
C	number of cells	[cell number]
L	concentration of luciferase per cell	[mg/L]

Assumptions

1. Instantaneous, homogenous distribution of toluene throughout the system
2. No cell growth or death
3. Degradation of toluene is proportional to amount of toluene and cell count
4. Luciferase generation rate is proportional to toluene concentration
5. No luciferase degradation

Equations

$$\frac{dT}{dt} = -k_1 TC$$

$$\frac{dC}{dt} = 0$$

$$\frac{dL}{dt} = k_2 T$$

Initial Conditions

$$T(0) = 9.2 \text{ mg/L}$$

$$C(0) = 7 \times 10^8 \text{ cells}$$

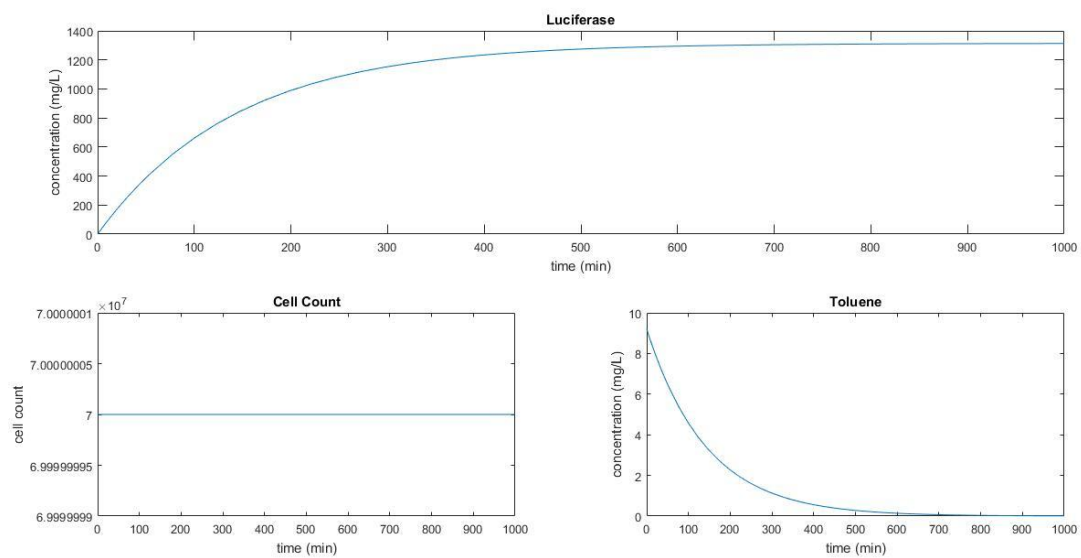
$$L(0) = 0 \text{ mg/L}$$

Parameter Values

$K1 = 1$

$K2 = 1E-10$ (parametrized value)

Model Output



Main Code

```

clc
clear

Time = [0 1000];
Toluene = 9.2; % initial Toluene concentration [mg/L]

[T, Y] = ode45('LuciferaseModelFunctions02', Time, [0, 70000000, 9.2]);

subplot(2,2,[1 2]);
plot(T,Y(:,1));
title('Luciferase');
xlabel('time (min)');
ylabel('concentration (mg/L)');
subplot(2,2,3);
plot(T,Y(:,2));
hold on
title('Cell Count');
xlabel('time (min)');
ylabel('cell count');
subplot(2,2,4);
plot(T,Y(:,3));
hold on
title('Toluene');
xlabel('time (min)');
ylabel('concentration (mg/L)');

```

Function Code

```

function A = LuciferaseModelFunctions02(~,Y)
A = zeros(3,1);

L = Y(1);
C = Y(2);
T = Y(3);

% Luciferase generation is proportional to Toluene concentration
k1 = 1E0;
A(1) = k1*T;

% Cell count is constant
A(2) = 0;

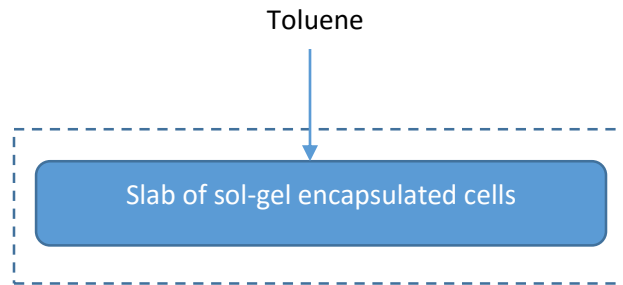
% Toluene level
k2 = 1E-10;
A(3) = -k2*T*C;

end

```

Iteration 0.3

Picture



Parameters

Parameter Symbol	Meaning	Units
T	concentration of toluene	[mg/L]
C	number of cells	[cell number]
L	concentration of luciferase per cell	[mg/L]

Assumptions

1. Instantaneous, homogenous distribution of toluene throughout the system
2. No cell growth or death
3. Degradation of toluene is proportional to amount of toluene and cell count and **luciferase count**
4. Luciferase generation rate is proportional to toluene concentration
5. **Luciferase degradation is proportional to the luciferase concentration and is equal to $\log(2)$ divided by the half life of the protein**

Equations

$$\frac{dT}{dt} = -k_1 T C L$$

$$\frac{dC}{dt} = 0$$

$$\frac{dL}{dt} = k_2 T - \frac{\log(2)}{T_{1/2}} L$$

Initial Conditions

$$T(0) = 9.2 \text{ mg/L}$$

$$C(0) = 7 \times 10^8 \text{ cells}$$

$$L(0) = 0 \text{ mg/L}$$

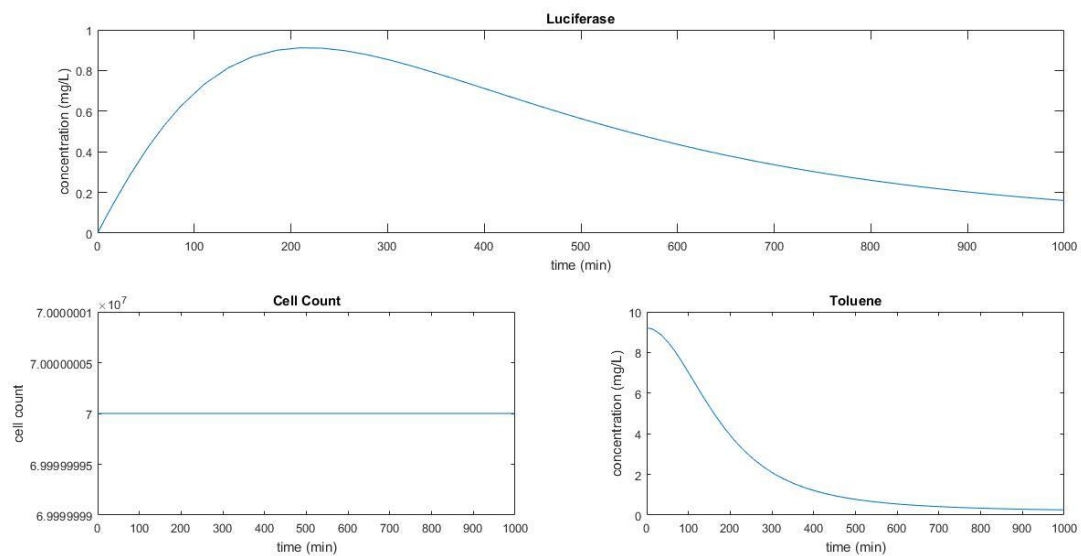
Parameter Values

$$K1 = 1\text{E-}3$$

$$K2 = 1\text{E-}10 \text{ (parametrized value)}$$

$$T_{1/2} = 180 \text{ min}$$

Model Output



Main Code

```

clc
clear

Time = [0 1000];
Toluene = 9.2; % initial Toluene concentration [mg/L]

[T, Y] = ode45('LuciferaseModelFunctions03', Time, [0, 70000000, 9.2]);

subplot(2,2,[1 2]);
plot(T,Y(:,1));
title('Luciferase');
xlabel('time (min)');
ylabel('concentration (mg/L)');
subplot(2,2,3);
plot(T,Y(:,2));
hold on
title('Cell Count');
xlabel('time (min)');
ylabel('cell count');
subplot(2,2,4);
plot(T,Y(:,3));
hold on
title('Toluene');
xlabel('time (min)');
ylabel('concentration (mg/L)');

```

Function Code

```

function A = LuciferaseModelFunctions03(~,Y)
A = zeros(3,1);

L = Y(1);
C = Y(2);
T = Y(3);

% Luciferase generation is proportional to Toluene concentration
k1 = 1E-3;
HalfLife = 180;
AlphaL = log(2)/HalfLife;
A(1) = k1*T - AlphaL*L;

% Cell count is constant
A(2) = 0;

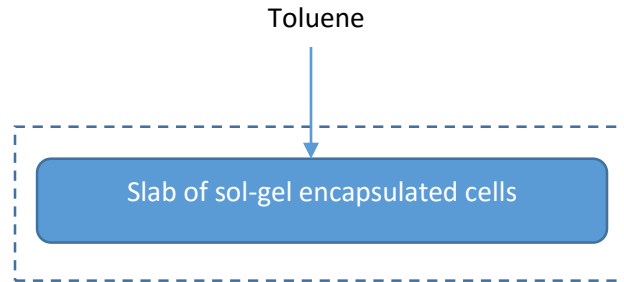
% Toluene level
k2 = 1E-10;
A(3) = -k2*T*C*L;

end

```

Iteration 1

Picture



Parameters

Parameter Symbol	Meaning	Units
T	concentration of toluene	[mg/L]
C	number of cells	[cell number]
L	concentration of luciferase	[mg/L]

Assumptions

1. Instantaneous, homogenous distribution of toluene throughout the system
2. No cell growth or death
3. Luciferase concentration is equal to the concentration of toluene dioxygenase enzyme
4. Generation of luciferase follows MM kinetics
5. Degradation of toluene is directly proportional to luciferase concentration and toluene concentration
6. Luciferase decays at a constant rate

Equations

$$\frac{dT}{dt} = -k_1 CLT$$

$$\frac{dC}{dt} = 0$$

$$\frac{dL}{dt} = \frac{V_m T}{K_m + T} - k_2 L$$

Initial Conditions

$$T(0) = 9.2 \text{ mg/L}$$

$$C(0) = 7 \times 10^8 \text{ cells}$$

$$L(0) = 0 \text{ mg/L}$$

Parameter Values

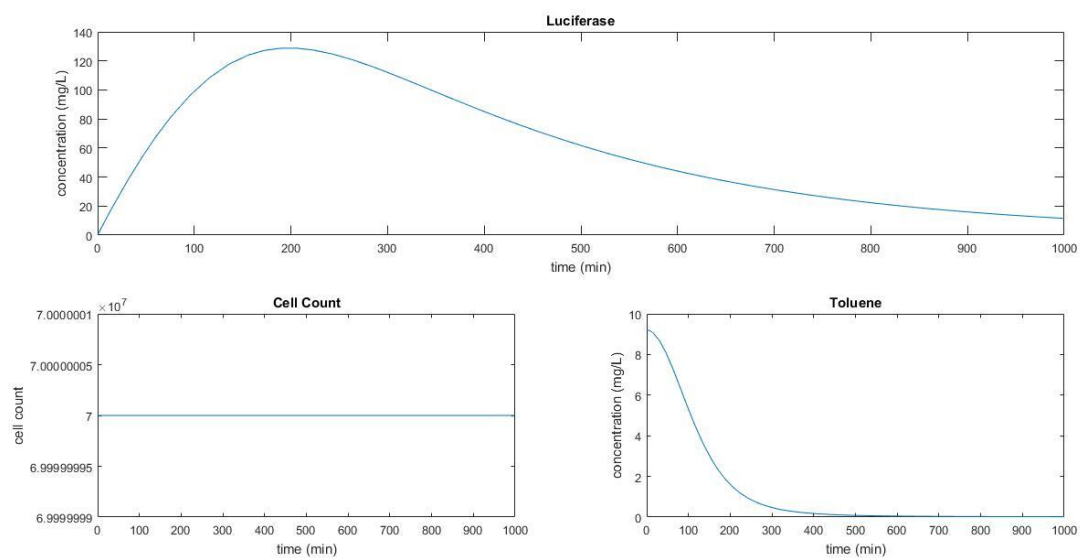
$V_m = 2$ (researched value)

$K_m = 5 \text{ mg toluene/L}$ (researched value)

$K_1 = \log(2)/\text{toluene half life} = .00167$

$K_2 = 1\text{E-}10 - 1\text{E-}12$ (parametrized value)

Model Output



Main Code

```
clc
clear

Time = [0 1000];
Toluene = 9.2; % initial Toluene concentration [mg/L]

[T, Y] = ode45('LuciferaseModelFunctions1', Time, [0, 70000000, 9.2]);

subplot(2,2,[1 2]);
plot(T,Y(:,1));
title('Luciferase');
xlabel('time (min)');
ylabel('concentration (mg/L)');
subplot(2,2,3);
plot(T,Y(:,2));
title('Cell Count');
xlabel('time (min)');
ylabel('cell count');
subplot(2,2,4);
plot(T,Y(:,3));
title('Toluene');
xlabel('time (min)');
ylabel('concentration (mg/L)');
```

Function Code

```

function A = LuciferaseModelFunctions1(~,Y)
A = zeros(3,1);

L = Y(1);
N = Y(2);
T = Y(3);

% Luciferase level
klucif = 2;    % [unitless]
Klucif = 5;    % [mg/L]

% calculate degradation rate
HalfLife = 180;    % [minutes]
AlphaL = log(2)/HalfLife;

A(1) = klucif*T/(Klucif+T)-AlphaL*L;

% Cell count is constant
A(2) = 0;

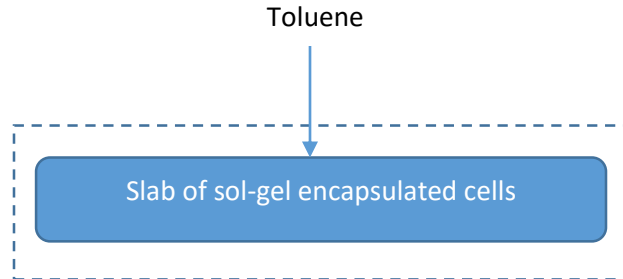
% Toluene level
k2 = .0001;
A(3) = -k2*L*T;

end

```

Iteration 2

Picture



Parameters

Parameter Symbol	Meaning	Units
T	concentration of toluene	[mg/L]
C	number of cells	[cell number]
L	concentration of luciferase	[mg/L]

Assumptions

1. Instantaneous, homogenous distribution of toluene throughout the system
2. **Cell growth is proportional to toluene consumption rate**
3. No cell death
4. Luciferase concentration is equal to the concentration of toluene dioxygenase enzyme
5. Degradation of toluene is directly proportional to luciferase concentration and toluene concentration
6. Luciferase decays at a constant rate

Bolded assumption indicates what is changed for this iteration

Equations

$$\frac{dT}{dt} = -k_2 CLT$$

$$\frac{dC}{dt} = k_3 CLT$$

$$\frac{dL}{dt} = \frac{V_m T}{K_m + T} - k_1 L$$

Initial Conditions

$$T(0) = 9.2 \text{ mg/L}$$

$$C(0) = 7 \times 10^8 \text{ cells}$$

$$L(0) = 0 \text{ mg/L}$$

Parameter Values

$$V_m = 2 \text{ (researched value)}$$

$$K_m = 5 \text{ mg toluene/L (researched value)}$$

$$K_1 = \log(2)/\text{toluene dioxygenase half life} = .00167$$

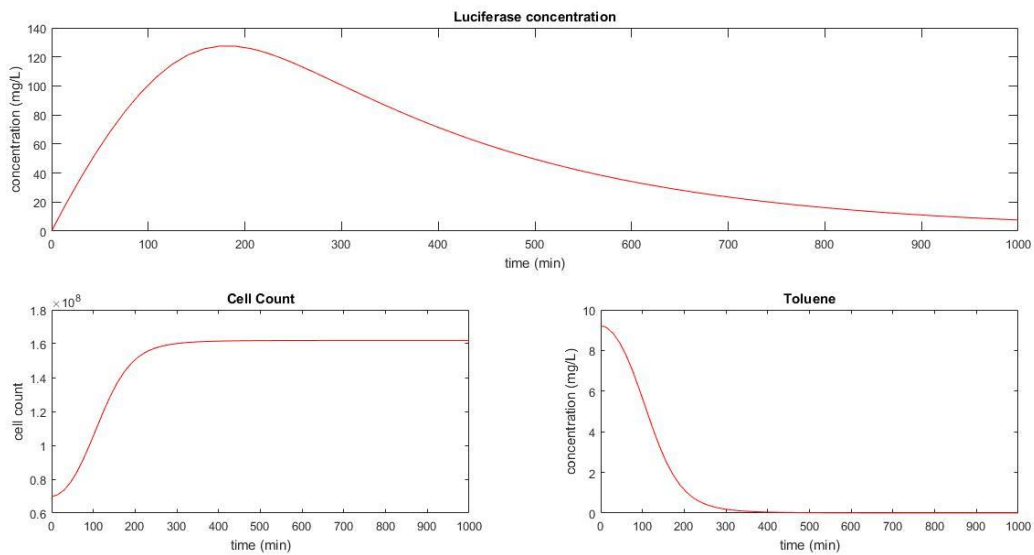
$$K_2 = 1\text{E-}10 - 1\text{E-}12 \text{ (parametrized value)}$$

K2 was set to 1E-12 for the output below

$$K_3 = 1\text{E-}4 - 1\text{E-}6 \text{ (parametrized value)}$$

K3 was set to 1E-5 for the output below

Model Output



Main Code

```
clc
clear

Time = [0 1000];
Toluene = 9.2; % initial Toluene concentration [mg/L]

[T, Y] = ode45('LuciferaseModelFunctions2', Time, [0, 70000000, 9.2]);

subplot(2,2,[1 2]);
plot(T,Y(:,1),'r');
hold on
title('Luciferase concentration');
xlabel('time (min)');
ylabel('concentration (mg/L)');
subplot(2,2,3);
plot(T,Y(:,2),'r');
hold on
title('Cell Count');
xlabel('time (min)');
ylabel('cell count');
subplot(2,2,4);
plot(T,Y(:,3),'r');
hold on
title('Toluene');
xlabel('time (min)');
ylabel('concentration (mg/L)');
```


Function Code

```

function A = LuciferaseModelFunctions2(~,Y)
A = zeros(3,1);

L = Y(1);
N = Y(2);
T = Y(3);

% Luciferase level
klucif = 2;    % [unitless]
Klucif = 5;    % [mg/L]

% calculate degradation rate
HalfLife = 180;    % [minutes]
AlphaL = log(2)/HalfLife;

A(1) = klucif*T/(Klucif+T)-AlphaL*L;

% Cells grow proportionally to toluene concentration
k3 = 1E-5;
A(2) = k3*N*L*T;

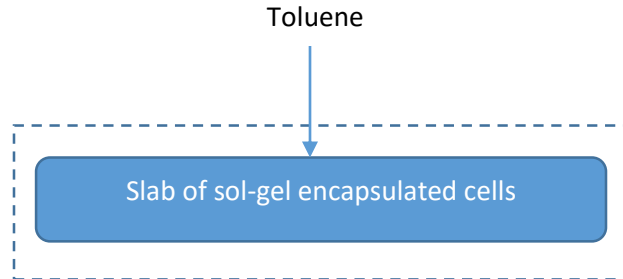
% Toluene level
k2 = 1E-12;
A(3) = -k2*N*L*T;

end

```

Iteration 3

Picture



Parameters

Parameter Symbol	Meaning	Units
T	concentration of toluene	[mg/L]
C	number of cells	[cell number]
L	concentration of luciferase	[mg/L]

Assumptions

1. Instantaneous, homogenous distribution of toluene throughout the system
2. Cell growth is proportional to toluene consumption rate
3. **Cells die at a rate proportional to the population count**
4. Luciferase concentration is equal to the concentration of toluene dioxygenase enzyme
5. Degradation of toluene is directly proportional to luciferase concentration and toluene concentration
6. Luciferase decays at a constant rate

Bolded assumption indicates what is changed for this iteration

Equations

$$\frac{dT}{dt} = -k_2CLT$$

$$\frac{dC}{dt} = k_3CLT - k_4C$$

$$\frac{dL}{dt} = \frac{V_m T}{K_m + T} - k_1 L$$

Initial Conditions

$$T(0) = 9.2 \text{ mg/L}$$

$$C(0) = 7 \times 10^8 \text{ cells}$$

$$L(0) = 0 \text{ mg/L}$$

Parameter Values

$$V_m = 2 \text{ (researched value)}$$

$$K_m = 5 \text{ mg toluene/L (researched value)}$$

$$K_1 = \log(2)/\text{toluene dioxygenase half life} = .00167$$

$$K_2 = 1E-10 - 1E-12 \text{ (parametrized value)}$$

K2 was set to 1E-12 for the output below

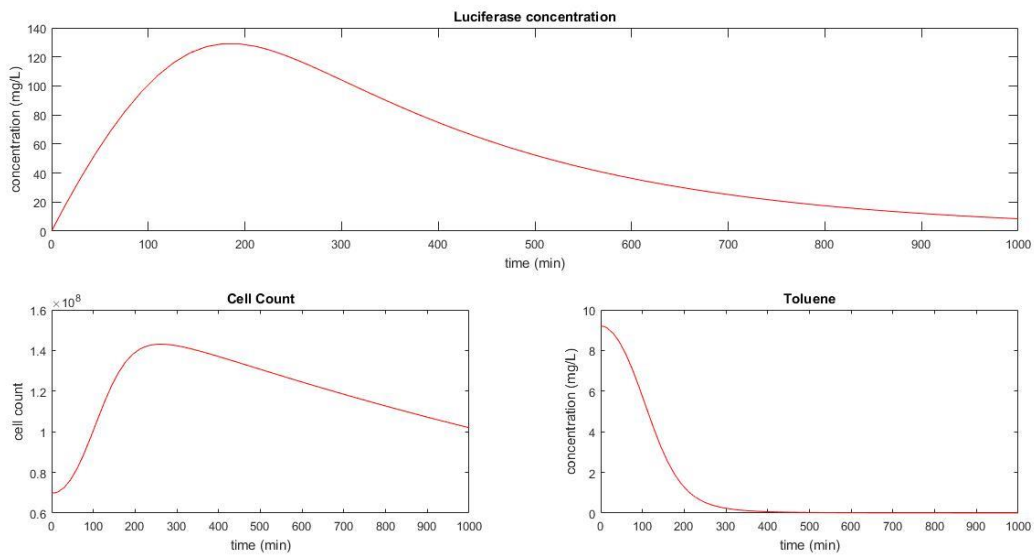
$$K_3 = 1E-4 - 1E-6 \text{ (parametrized value)}$$

K3 was set to 1E-5 for the output below

$$K_4 = 1E-3 - 1E-5 \text{ (parametrized)}$$

K4 was set to 5E-4 for the output below

Model Output



Main Code

```

clc
clear

Time = [0 1000];
Toluene = 9.2; % initial Toluene concentration [mg/L]

[T, Y] = ode45('LuciferaseModelFunctions3', Time, [0, 70000000, 9.2]);

subplot(2,2,[1 2]);
plot(T,Y(:,1),'r');
hold on
title('Luciferase concentration');
xlabel('time (min)');
ylabel('concentration (mg/L)');
subplot(2,2,3);
plot(T,Y(:,2),'r');
hold on
title('Cell Count');
xlabel('time (min)');
ylabel('cell count');
subplot(2,2,4);
plot(T,Y(:,3),'r');
hold on
title('Toluene');
xlabel('time (min)');
ylabel('concentration (mg/L)');

```

Function Code

```

function A = LuciferaseModelFunctions3(~,Y)
A = zeros(3,1);

L = Y(1);
C = Y(2);
T = Y(3);

% Luciferase level
klucif = 2;    % [unitless]
Klucif = 5;    % [mg/L]

% calculate degradation rate
HalfLife = 180;    % [minutes]
AlphaL = log(2)/HalfLife;

A(1) = klucif*T/(Klucif+T)-AlphaL*L;

% Cells grow proportionally to toluene concentration
k3 = 1E-5;
k4 = 5E-4;
A(2) = k3*C*L*T-k4*C;

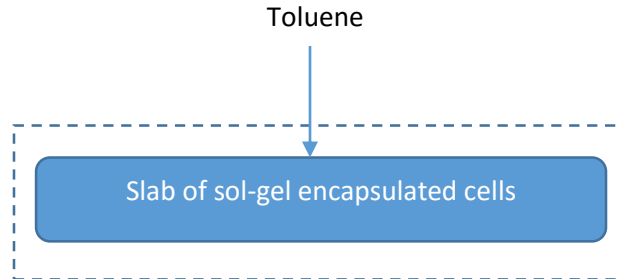
% Toluene level
k2 = 1E-12;
A(3) = -k2*C*L*T;

end

```

Iteration 4

Picture



Parameters

Parameter Symbol	Meaning	Units
T	concentration of toluene	[mg/L]
C	number of cells	[cell number]
L	concentration of luciferase	[mg/L]

Assumptions

1. Instantaneous, homogenous distribution of toluene throughout the system
2. Cell growth is proportional to toluene consumption rate
3. Cells die at a rate proportional to the population count **and inversely proportional to the toluene concentration**
4. Luciferase concentration is equal to the concentration of toluene dioxygenase enzyme
5. Degradation of toluene is directly proportional to luciferase concentration and toluene concentration
6. Luciferase decays at a constant rate

Bolded assumption indicates what is changed for this iteration

Equations

$$\frac{dT}{dt} = -k_2 CLT$$

$$\frac{dC}{dt} = k_3 CLT - k_4 \frac{C}{T}$$

$$\frac{dL}{dt} = \frac{V_m T}{K_m + T} - k_1 L$$

Initial Conditions

$$T(0) = 9.2 \text{ mg/L}$$

$$C(0) = 7 \times 10^8 \text{ cells}$$

$$L(0) = 0 \text{ mg/L}$$

Parameter Values

$$V_m = 2 \text{ (researched value)}$$

$$K_m = 5 \text{ mg toluene/L (researched value)}$$

$$K_1 = \log(2)/\text{toluene dioxygenase half life} = .00167$$

$$K_2 = 1\text{E-}10 - 1\text{E-}12 \text{ (parametrized value)}$$

K2 was set to 1E-12 for the output below

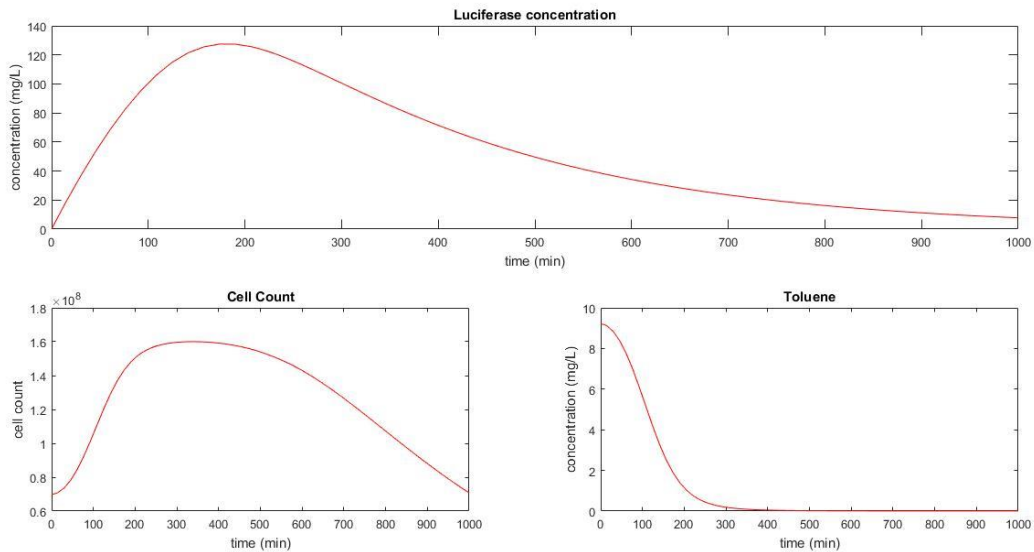
$$K_3 = 1\text{E-}4 - 1\text{E-}6 \text{ (parametrized value)}$$

K3 was set to 1E-5 for the output below

$$K_4 = 1\text{E-}3 - 1\text{E-}5 \text{ (parametrized)}$$

K4 was set to 1E-5 for the output below

Model Output



Main Code

```
clc
clear

Time = [0 1000];
Toluene = 9.2; % initial Toluene concentration [mg/L]

[T, Y] = ode45('LuciferaseModelFunctions4', Time, [0, 70000000, 9.2]);

subplot(2,2,[1 2]);
plot(T,Y(:,1),'r');
hold on
title('Luciferase concentration');
xlabel('time (min)');
ylabel('concentration (mg/L)');
subplot(2,2,3);
plot(T,Y(:,2),'r');
hold on
title('Cell Count');
xlabel('time (min)');
ylabel('cell count');
subplot(2,2,4);
plot(T,Y(:,3),'r');
hold on
title('Toluene');
xlabel('time (min)');
ylabel('concentration (mg/L)');
```


Function Code

```

function A = LuciferaseModelFunctions4(~,Y)
A = zeros(3,1);

L = Y(1);
C = Y(2);
T = Y(3);

% Luciferase level
klucif = 2;    % [unitless]
Klucif = 5;    % [mg/L]

% calculate degradation rate
HalfLife = 180;    % [minutes]
AlphaL = log(2)/HalfLife;

A(1) = klucif*T/(Klucif+T)-AlphaL*L;

% Cells grow proportionally to toluene concentration
k3 = 1E-5;
k4 = 1E-5;
A(2) = k3*C*L*T-k4*C/T;

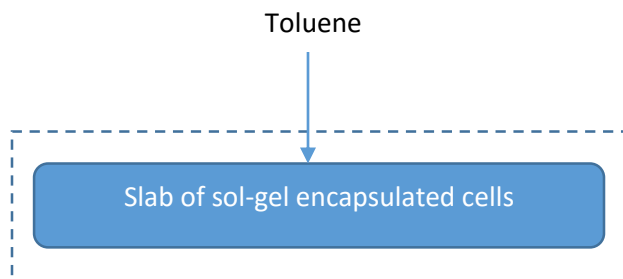
% Toluene level
k2 = 1E-12;
A(3) = -k2*C*L*T;

end

```

Iteration 5

Picture



Parameters

Parameter Symbol	Meaning	Units
T	concentration of toluene	[mg/L]
C	number of cells	[cell number]
L	concentration of luciferase	[mg/L]
E	concentration of toluene dioxygenase	[mg/L]

Assumptions

1. Instantaneous, homogenous distribution of toluene throughout the system
2. Cell growth is proportional to toluene consumption rate
3. Cells die at a rate proportional to the population count and inversely proportional to the toluene concentration
4. **Luciferase and toluene dioxygenase follow the same generation and degradation kinetics**
5. Degradation of toluene is directly proportional to luciferase concentration and toluene concentration
6. Luciferase decays at a constant rate

Bolded assumption indicates what is changed for this iteration

Equations

$$\frac{dT}{dt} = -k_2 CLT$$

$$\frac{dC}{dt} = k_3 CLT - k_4 \frac{C}{T}$$

$$\frac{dL}{dt} = \frac{V_m T}{K_m + T} - k_1 L$$

$$\frac{dE}{dt} = \frac{V_m T}{K_m + T} - k_1 L$$

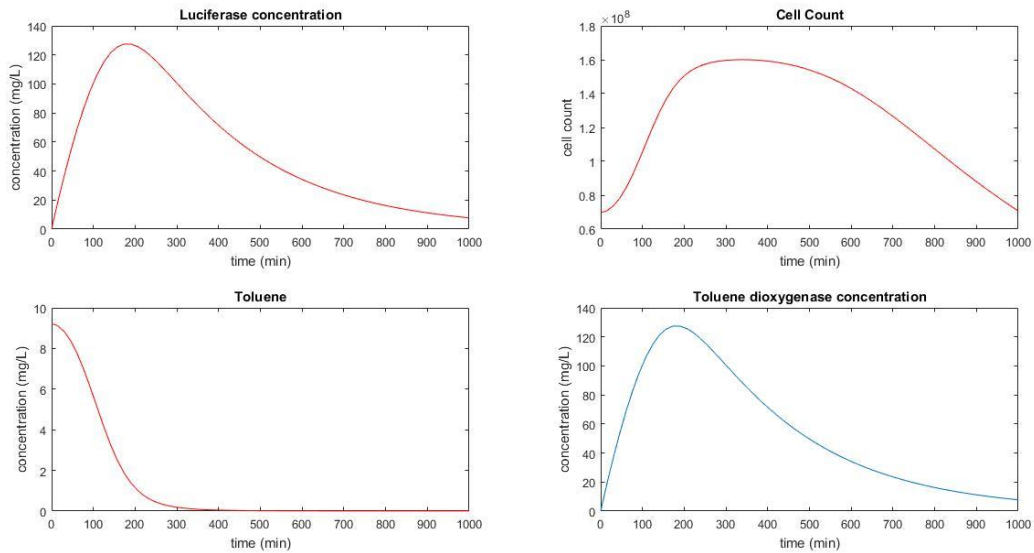
Initial Conditions

$T(0) = 9.2 \text{ mg/L}$
 $C(0) = 7 \times 10^8 \text{ cells}$
 $L(0) = 0 \text{ mg/L}$
 $E(0) = 0 \text{ mg/L}$

Parameter Values

$V_m = 2$ (researched value)
 $K_m = 5 \text{ mg toluene/L}$ (researched value)
 $K_1 = \log(2)/\text{toluene dioxygenase half life} = .00167$
 $K_2 = 1\text{E-}10 - 1\text{E-}12$ (parametrized value)
 K_2 was set to $1\text{E-}12$ for the output below
 $K_3 = 1\text{E-}4 - 1\text{E-}6$ (parametrized value)
 K_3 was set to $1\text{E-}5$ for the output below
 $K_4 = 1\text{E-}3 - 1\text{E-}5$ (parametrized)
 K_4 was set to $1\text{E-}5$ for the output below

Model Output



Main Code

```

clc
clear

Time = [0 1000];
Toluene = 9.2; % initial Toluene concentration [mg/L]

[T, Y] = ode45('LuciferaseModelFunctions5', Time, [0, 70000000, 9.2, 0]);

subplot(2,2,1);
plot(T,Y(:,1),'r');
hold on
title('Luciferase concentration');
xlabel('time (min)');
ylabel('concentration (mg/L)');
subplot(2,2,2);
plot(T,Y(:,2),'r');
hold on
title('Cell Count');
xlabel('time (min)');
ylabel('cell count');
subplot(2,2,3);
plot(T,Y(:,3),'r');
hold on
title('Toluene');
xlabel('time (min)');
ylabel('concentration (mg/L)');
subplot(2,2,4);
plot(T,Y(:,4));
hold on
title('Toluene dioxygenase concentration');
xlabel('time (min)');
ylabel('concentration (mg/L)');

```

Function Code

```

function A = LuciferaseModelFunctions5(~,Y)
A = zeros(4,1);

L = Y(1);
C = Y(2);
T = Y(3);
E = Y(4);

% Luciferase level
klucif = 2;    % [unitless]
Klucif = 5;    % [mg/L]

% calculate degradation rate
HalfLife = 180;    % [minutes]
AlphaL = log(2)/HalfLife;

A(1) = klucif*T/(Klucif+T)-AlphaL*L;

% Cells grow proportionally to toluene concentration
k3 = 1E-5;
k4 = 1E-5;
A(2) = k3*C*L*T-k4*C/T;

% Toluene level
k2 = 1E-12;
A(3) = -k2*C*L*T;

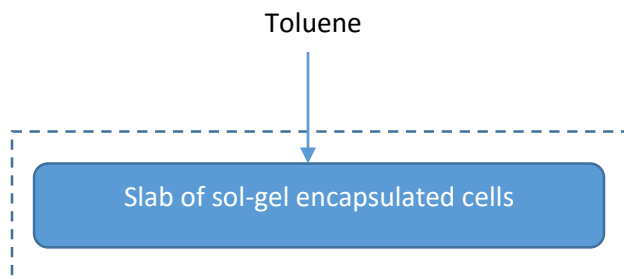
% Toluene dioxygenase level
A(4) = klucif*T/(Klucif+T) - AlphaL*L;

end

```

Iteration 6

Picture



Parameters

Parameter Symbol	Meaning	Units
T	concentration of toluene	[mg/L]
C	number of cells	[cell number]
L	concentration of luciferase	[mg/L]
E	concentration of toluene dioxygenase	[mg/L]

Assumptions

1. Instantaneous, homogenous distribution of toluene throughout the system
2. Cell growth is proportional to toluene consumption rate
3. Cells die at a rate proportional to the population count and inversely proportional to the toluene concentration
4. Luciferase and toluene dioxygenase follow the degradation kinetics
5. **Luciferase and toluene dioxygenase do not follow the same generation kinetics**
6. Degradation of toluene is directly proportional to **toluene dioxygenase concentration** and toluene concentration
7. Luciferase decays is proportional to the net cell change (protein is diluted when a cell divides)

Bolded assumption indicates what is changed for this iteration

Equations

$$\frac{dT}{dt} = -k_2 CET$$

$$\frac{dC}{dt} = k_3 CLT - k_4 \frac{C}{T}$$

$$\frac{dL}{dt} = \frac{V_{m,lucif}T}{K_{m,lucif} + T} - k_1L$$

$$\frac{dE}{dt} = \frac{V_{m,tol}T}{K_{m,tol} + T} - k_1\frac{dC}{dt}L$$

Initial Conditions

$$T(0) = 9.2 \text{ mg/L}$$

$$C(0) = 7 \times 10^8 \text{ cells}$$

$$L(0) = 0 \text{ mg/L}$$

$$E(0) = 0 \text{ mg/L}$$

Parameter Values

$$V_{m,lucif} = 2 \text{ (researched value)}$$

$$K_{m,lucif} = 5 \text{ mg toluene/L (researched value)}$$

$$V_{m,tol} = 2$$

$$K_{m,tol} = 5$$

$$K1 = \log(2)/\text{toluene dioxygenase half life} = .00167$$

$$K2 = 1E-10 - 1E-12 \text{ (parametrized value)}$$

K2 was set to 1E-12 for the output below

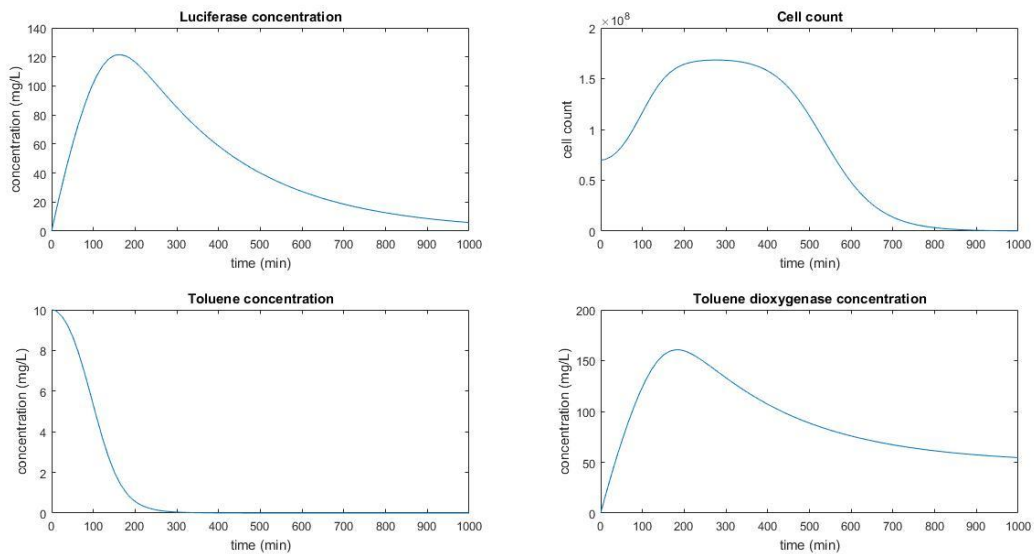
$$K3 = 1E-4 - 1E-6 \text{ (parametrized value)}$$

K3 was set to 1E-5 for the output below

$$K4 = 1E-3 - 1E-5 \text{ (parametrized)}$$

K4 was set to 1E-5 for the output below

Model Output



Main Code

```

clc
clear

Time = [0 1000];
Toluene = 9.2; % initial Toluene concentration [mg/L]

[t, Y] = ode45('LuciferaseModelFunctions6', Time, [0, 700000000, 10, 0]);

L = Y(:,1);
C = Y(:,2);
T = Y(:,3);
E = Y(:,4);

subplot(2,2,1);
plot(t,L);
title('Luciferase concentration');
xlabel('time (min)');
ylabel('concentration (mg/L)');
hold on
subplot(2,2,2);
plot(t,C);
title('Cell count');
xlabel('time (min)');
ylabel('cell count');
hold on
subplot(2,2,3);
plot(t,T);
title('Toluene concentration');
xlabel('time (min)');
ylabel('concentration (mg/L)');
hold on
subplot(2,2,4);
plot(t,E);
title('Toluene dioxygenase concentration');
xlabel('time (min)');
ylabel('concentration (mg/L)');
hold on

```


Function Code

```

function A = LuciferaseModelFunctions6(~,Y)
A = zeros(4,1);

L = Y(1);
C = Y(2);
T = Y(3);
E = Y(4);

% Cells grow proportionally to toluene concentration
k3 = 1E-5;
k4 = 1E-5;
A(2) = k3*C*E*T-k4*C/T;

% Luciferase level
klucif = 2;    % [unitless]
Klucif = 5;    % [mg/L]

% calculate degradation rate
HalfLife = 180;    % [minutes]
AlphaL = log(2)/HalfLife;

A(1) = klucif*T/(Klucif+T)-AlphaL*L;

% Toluene level
k2 = 1E-12;
A(3) = -k2*C*E*T;

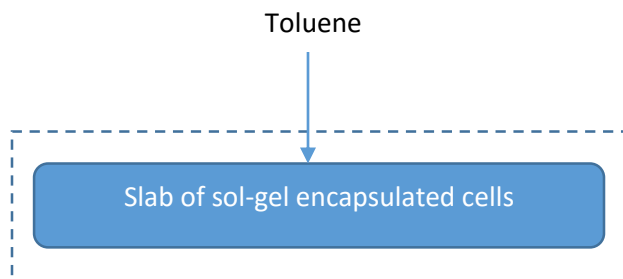
% Toluene dioxygenase level
ktol = 2;
Ktol = 3;
A(4) = ktol*T/(Ktol+T) - AlphaL*L;

end

```

Iteration 7

Picture



Parameters

Parameter Symbol	Meaning	Units
T_{in}	concentration of toluene inside the cells	[mg/L]
T_{out}	concentration of toluene outside of the cells	[mg/L]
C	number of cells	[cell number]
L	concentration of luciferase	[mg/L]
E	concentration of toluene dioxygenase	[mg/L]

Assumptions

1. Instantaneous, homogenous distribution of toluene throughout the system
2. Cell growth is proportional to toluene consumption rate
3. Cells die at a rate proportional to the population count and inversely proportional to the toluene concentration
4. Luciferase and toluene dioxygenase follow the degradation kinetics
5. Luciferase and toluene dioxygenase do not follow the same generation kinetics
6. Degradation of toluene is directly proportional to toluene dioxygenase concentration and toluene concentration
7. Luciferase decays is proportional to the net cell change (protein is diluted when a cell divides)
8. **Toluene diffuses from outside the cells to inside the cells at a rate proportional to extracellular toluene concentration**

Bolded assumption indicates what is changed for this iteration

Equations

$$\frac{dT_{out}}{dt} = -k_5 T_{out}$$

$$\frac{dT_{in}}{dt} = k_5 T_{out} - k_2 CET$$

$$\frac{dC}{dt} = k_3 CET - k_4 \frac{C}{T}$$

$$\frac{dL}{dt} = \frac{V_{m,lucif} T}{K_{m,lucif} + T} - k_1 L$$

$$\frac{dE}{dt} = \frac{V_{m,tol} T}{K_{m,tol} + T} - k_1 L$$

Initial Conditions

Tout(0) = 9.2 mg/L

C(0) = 7×10^8 cells

L(0) = 0 mg/L

E(0) = 0 mg/L

Tin(0) = .001 mg/L

Parameter Values

Vm,lucif = 2 (researched value)

Km,lucif = 5 mg toluene/L (researched value)

Vm,tol = 2

Km,tol = 5

K1 = log(2)/toluene dioxygenase half life = .00167

K2 = 1E-10 – 1E-12 (parametrized value)

K2 was set to 1E-12 for the output below

K3 = 1E-4 – 1E-6 (parametrized value)

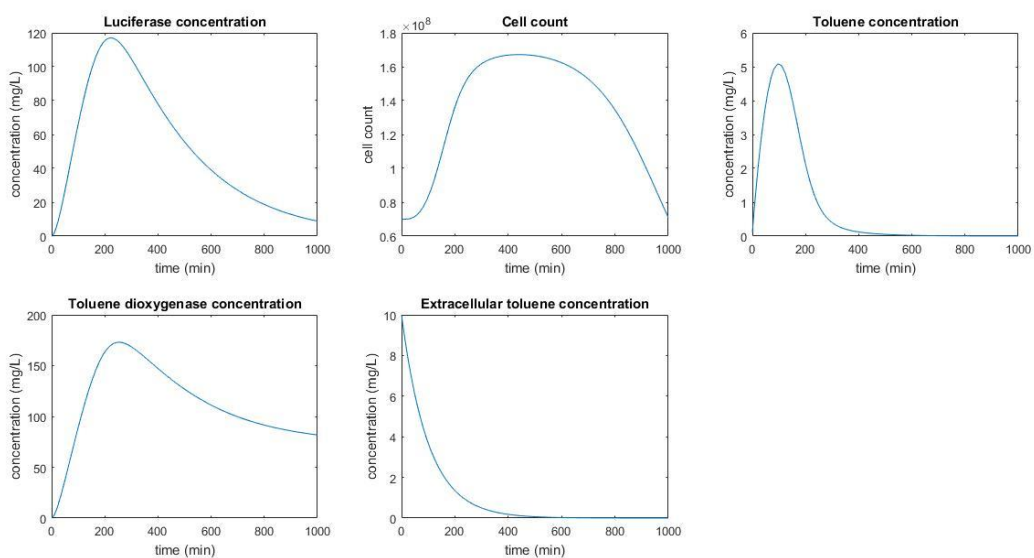
K3 was set to 1E-5 for the output below

K4 = 1E-3-1E-5 (parametrized)

K4 was set to 1E-5 for the output below

K5 = 1E-2

Model Output



Main Code

```

clc
clear

Time = [0 1000];
Toluene = 9.2; % initial Toluene concentration [mg/L]

[t, Y] = ode45('LuciferaseModelFunctions7', Time, [0, 700000000, 1E-1, 0,
10]);

L = Y(:,1);
C = Y(:,2);
T = Y(:,3);
E = Y(:,4);
Tout = Y(:,5);

subplot(2,3,1);
plot(t,L);
title('Luciferase concentration');
xlabel('time (min)');
ylabel('concentration (mg/L)');
hold on
subplot(2,3,2);
plot(t,C);
title('Cell count');
xlabel('time (min)');
ylabel('cell count');
hold on
subplot(2,3,3);
plot(t,T);
title('Toluene concentration');
xlabel('time (min)');
ylabel('concentration (mg/L)');
hold on
subplot(2,3,4);
plot(t,E);
title('Toluene dioxygenase concentration');
xlabel('time (min)');
ylabel('concentration (mg/L)');
hold on
subplot(2,3,5);
plot(t,Tout);
title('Extracellular toluene concentration');
xlabel('time (min)');
ylabel('concentration (mg/L)');
hold on

```

Function Code

```

function A = LuciferaseModelFunctions7(~,Y)
A = zeros(5,1);

L = Y(1);
C = Y(2);
T = Y(3);
E = Y(4);
Tout = Y(5);

% Cells grow proportionally to toluene concentration
k3 = 1E-5;
k4 = 1E-5;
A(2) = k3*C*E*T-k4*C/T;

% Luciferase level
klucif = 2;    % [unitless]
Klucif = 5;    % [mg/L]

% calculate degradation rate
HalfLife = 180;    % [minutes]
AlphaL = log(2)/HalfLife;

A(1) = klucif*T/(Klucif+T)-AlphaL*L;

% Toluene level
k5 = 1E-2;
ExtoIn = k5*Tout;

k2 = 1E-12;
A(3) = ExtoIn -k2*C*E*T;

% Toluene dioxygenase level
ktol = 2;
Ktol = 3;
A(4) = ktol*T/(Ktol+T) - AlphaL*L;

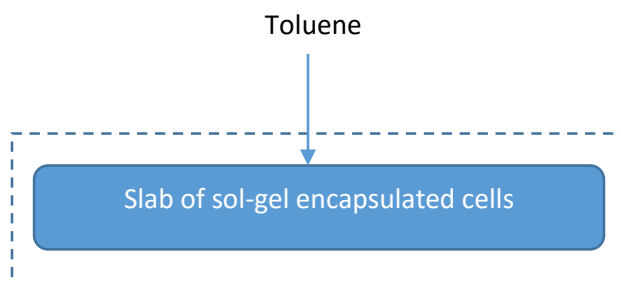
% Extracellular toluene
A(5) = -ExtoIn;

end

```

Iteration 8

Picture



Parameters

Parameter Symbol	Meaning	Units
T_{in}	concentration of toluene inside the cells	[mg/L]
T_{out}	concentration of toluene outside of the cells	[mg/L]
C	number of cells	[cell number]
L	concentration of luciferase	[mg/L]
E	concentration of toluene dioxygenase	[mg/L]

Assumptions

1. Instantaneous, homogenous distribution of toluene throughout the system
2. Cell growth is proportional to toluene consumption rate
3. Cells die at a rate proportional to the population count and inversely proportional to the toluene concentration
4. Luciferase and toluene dioxygenase follow the degradation kinetics
5. Luciferase and toluene dioxygenase do not follow the same generation kinetics
6. Degradation of toluene is directly proportional to toluene dioxygenase concentration and toluene concentration
7. Luciferase decays is proportional to the net cell change (protein is diluted when a cell divides)
8. **Toluene diffuses from outside the cells to inside the cells via Fickian diffusion**

Bolded assumption indicates what is changed for this iteration

Equations

$$\frac{dT_{out}}{dt} = -k_5(T_{out} - T_{in})$$

$$\frac{dT_{in}}{dt} = k_5(T_{out} - T_{in}) - k_2 CET$$

$$\frac{dC}{dt} = k_3 CET - k_4 \frac{C}{T}$$

$$\frac{dL}{dt} = \frac{V_{m,lucif} T}{K_{m,lucif} + T} - k_1 L$$

$$\frac{dE}{dt} = \frac{V_{m,tol} T}{K_{m,tol} + T} - k_1 L$$

Initial Conditions

$T_{out}(0) = 9.2$ mg/L

$C(0) = 7 \cdot 10^8$ cells

$L(0) = 0$ mg/L

$E(0) = 0$ mg/L

$T_{in}(0) = .001$ mg/L

Parameter Values

$V_{m,lucif} = 2$ (researched value)

$K_{m,lucif} = 5$ mg toluene/L (researched value)

$V_{m,tol} = 2$

$K_{m,tol} = 5$

$K_1 = \log(2)/\text{toluene dioxygenase half life} = .00167$

$K_2 = 1E-10 - 1E-12$ (parametrized value)

K2 was set to 1E-12 for the output below

$K_3 = 1E-4 - 1E-6$ (parametrized value)

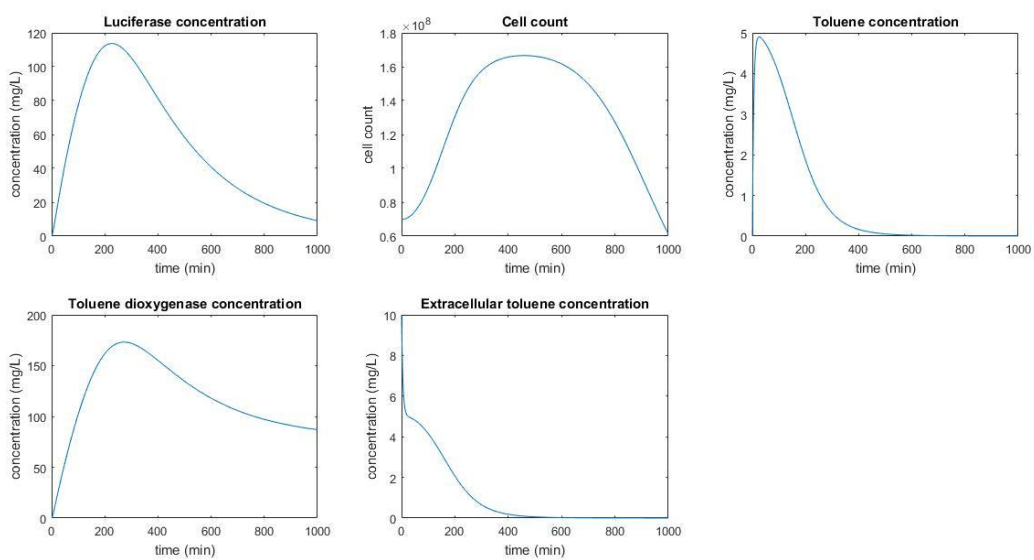
K3 was set to 1E-5 for the output below

$K_4 = 1E-3 - 1E-5$ (parametrized)

K4 was set to 1E-5 for the output below

$K_5 = 1E-2$

Output



Main Code

```

clc
clear

Time = [0 1000];
Toluene = 9.2; % initial Toluene concentration [mg/L]

[t, Y] = ode45('LuciferaseModelFunctions8', Time, [0, 700000000, 1E-3, 0,
10]);

L = Y(:,1);
C = Y(:,2);
Tin = Y(:,3);
E = Y(:,4);
Tout = Y(:,5);

subplot(2,3,1);
plot(t,L);
title('Luciferase concentration');
xlabel('time (min)');
ylabel('concentration (mg/L)');
hold on
subplot(2,3,2);
plot(t,C);
title('Cell count');
xlabel('time (min)');
ylabel('cell count');
hold on
subplot(2,3,3);
plot(t,Tin);
title('Toluene concentration');
xlabel('time (min)');
ylabel('concentration (mg/L)');
hold on
subplot(2,3,4);
plot(t,E);
title('Toluene dioxygenase concentration');
xlabel('time (min)');
ylabel('concentration (mg/L)');
hold on
subplot(2,3,5);
plot(t,Tout);
title('Extracellular toluene concentration');
xlabel('time (min)');
ylabel('concentration (mg/L)');
hold on

```

Function Code

```

function A = LuciferaseModelFunctions8(~,Y)
A = zeros(5,1);

L = Y(1);
C = Y(2);
Tin = Y(3);
E = Y(4);
Tout = Y(5);

% Cells grow proportionally to toluene concentration
k3 = 1E-5;
k4 = 1E-5;
A(2) = k3*C*E*Tin-k4*C/Tin;

% Luciferase level
klucif = 2;    % [unitless]
Klucif = 5;    % [mg/L]

% calculate degradation rate
HalfLife = 180;    % [minutes]
AlphaL = log(2)/HalfLife;

A(1) = klucif*Tin/(Klucif+Tin)-AlphaL*L;

% Toluene level
k5 = 1E-1;
ExtoIn = k5*(Tout-Tin);

k2 = 1E-12;
A(3) = ExtoIn -k2*C*E*Tin;

% Toluene dioxygenase level
ktol = 2;
Ktol = 3;
A(4) = ktol*Tin/(Ktol+Tin) - AlphaL*L;

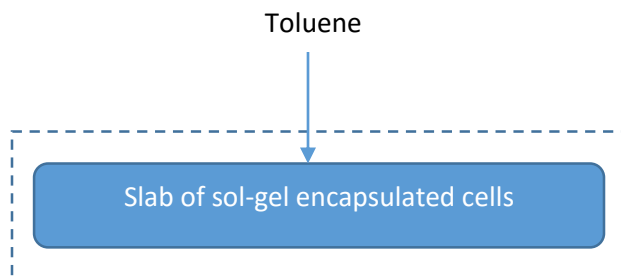
% Extracellular toluene
A(5) = -ExtoIn;

end

```

Iteration 9

Picture



Parameters

Parameter Symbol	Meaning	Units
T_{in}	concentration of toluene inside the cells	[mg/L]
T_{out}	concentration of toluene outside of the cells	[mg/L]
C	number of cells	[cell number]
L	concentration of luciferase	[mg/L]
E	concentration of toluene dioxygenase	[mg/L]
T_{odx}	concentration of todx promoter	[mg/L]

Assumptions

1. Instantaneous, homogenous distribution of toluene throughout the system
2. Cell growth is proportional to toluene consumption rate
3. Cells die at a rate proportional to the population count and inversely proportional to the toluene concentration
4. Luciferase and toluene dioxygenase follow the degradation kinetics
5. Luciferase and toluene dioxygenase do not follow the same generation kinetics
6. Degradation of toluene is directly proportional to toluene dioxygenase concentration and toluene concentration
7. Luciferase decays is proportional to the net cell change (protein is diluted when a cell divides)
8. Toluene diffuses from outside the cells to inside the cells via Fickian diffusion **at a rate proportional to the concentration of transport protein todx**

Bolded assumption indicates what is changed for this iteration

Equations

$$\frac{dT_{out}}{dt} = -k_5(T_{out} - T_{in})T_{odx}$$

$$\frac{dT_{in}}{dt} = k_5(T_{out} - T_{in})T_{odx} - k_2CET$$

$$\frac{dC}{dt} = k_3CET - k_4\frac{C}{T}$$

$$\frac{dL}{dt} = \frac{V_{m,lucif}T}{K_{m,lufic} + T} - k_1L$$

$$\frac{dE}{dt} = \frac{V_{m,tol}T}{K_{m,tol} + T} - k_1E$$

$$\frac{dT_{odx}}{dt} = \frac{V_{m,tol}T}{K_{m,tol} + T} - k_1T_{odx}$$

Initial Conditions

$$T_{out}(0) = 9.2 \text{ mg/L}$$

$$C(0) = 7 \cdot 10^8 \text{ cells}$$

$$L(0) = 0 \text{ mg/L}$$

$$E(0) = 0 \text{ mg/L}$$

$$T_{in}(0) = .001 \text{ mg/L}$$

$$T_{odx}(0) = .01 \text{ mg/L}$$

Parameter Values

$$V_{m,lucif} = 2 \text{ (researched value)}$$

$$K_{m,lucif} = 5 \text{ mg toluene/L (researched value)}$$

$$V_{m,tol} = 2$$

$$K_{m,tol} = 5$$

$$K_1 = \log(2)/\text{toluene dioxygenase half life} = .00167$$

$$K_2 = 1E-10 - 1E-12 \text{ (parametrized value)}$$

K2 was set to 1E-12 for the output below

$$K_3 = 1E-4 - 1E-6 \text{ (parametrized value)}$$

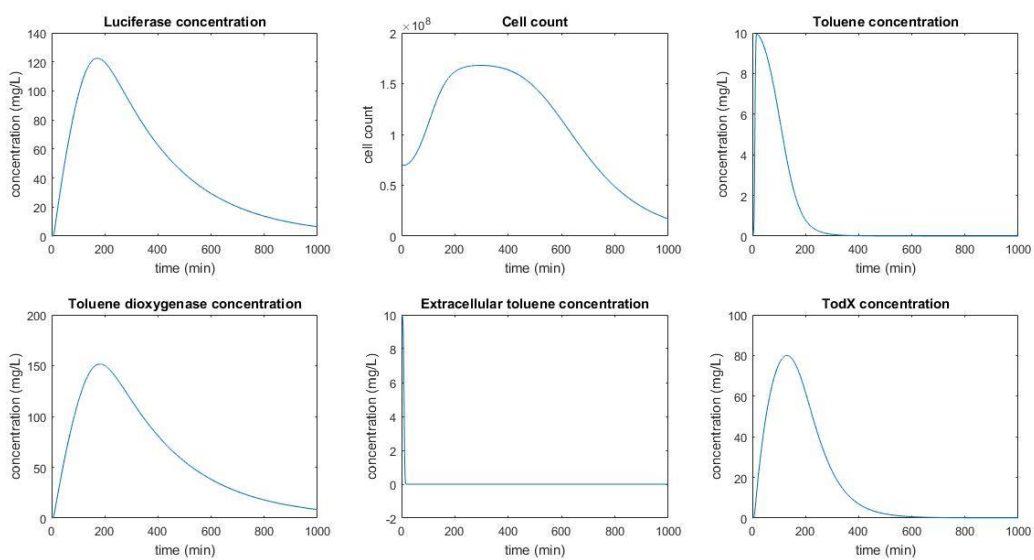
K3 was set to 1E-5 for the output below

$$K_4 = 1E-3 - 1E-5 \text{ (parametrized)}$$

K4 was set to 1E-5 for the output below

$$K_5 = 1E-2$$

Output



Main Code

```

clc
clear

Time = [0 1000];
Toluene = 9.2; % initial Toluene concentration [mg/L]

[t, Y] = ode45('LuciferaseModelFunctions9', Time, [0, 700000000, 1E-3, 0, 10, 1E-2]);

L = Y(:,1);
C = Y(:,2);
Tin = Y(:,3);
E = Y(:,4);
Tout = Y(:,5);
Todx = Y(:,6);

subplot(2,3,1);
plot(t,L);
title('Luciferase concentration');
xlabel('time (min)');
ylabel('concentration (mg/L)');
hold on
subplot(2,3,2);
plot(t,C);
title('Cell count');
xlabel('time (min)');
ylabel('cell count');
hold on
subplot(2,3,3);
plot(t,Tin);
title('Toluene concentration');
xlabel('time (min)');
ylabel('concentration (mg/L)');
hold on
subplot(2,3,4);
plot(t,E);
title('Toluene dioxygenase concentration');
xlabel('time (min)');
ylabel('concentration (mg/L)');
hold on
subplot(2,3,5);
plot(t,Tout);
title('Extracellular toluene concentration');
xlabel('time (min)');
ylabel('concentration (mg/L)');
hold on
subplot(2,3,6);
plot(t,Todx);
title('TodX concentration');
xlabel('time (min)');
ylabel('concentration (mg/L)');
hold on

```

Function Code

```

function A = LuciferaseModelFunctions9(~,Y)
A = zeros(6,1);

L = Y(1);
C = Y(2);
Tin = Y(3);
E = Y(4);
Tout = Y(5);
Tidx = Y(6);

% Cells grow proportionally to toluene concentration
k3 = 1E-5;
k4 = 1E-5;
A(2) = k3*C*E*Tin-k4*C/Tin;

% Luciferase level
klucif = 2; % [unitless]
Klucif = 5; % [mg/L]

% calculate degradation rate
HalfLife = 180; % [minutes]
AlphaL = log(2)/HalfLife;

A(1) = klucif*Tin/(Klucif+Tin)-AlphaL*L;

% Toluene level
k5 = 1E-1;
ExtoIn = k5*Tout*Tidx;

k2 = 1E-12;
A(3) = ExtoIn -k2*C*E*Tin;

% Toluene dioxygenase level
ktol = 2;
Ktol = 3;
A(4) = ktol*Tin/(Ktol+Tin) - AlphaL*E;

% Extracellular toluene
A(5) = -ExtoIn;

% Tidx level
TidxHalfLife = 50;
AlphaX = log(2)/TidxHalfLife;
A(6) = ktol*Tin/(Ktol+Tin) - AlphaX*Tidx;

end

```