2018

Modeling the Sweat Production of a Soccer Player

FINAL PROJECT

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Contents

Project Objective Statement

The objective of this project is to create a model that will output the rate of sweat production as well as total sweat volume of a professional soccer player over a 90-minute game.

Background Information

The human body can only operate normally between 36.5°C and 37.5°C and temperatures outside this range impair the body's regular function. Additionally, the body produces heat as a byproduct of metabolic reactions (Vella & Kravitz). In converting chemical energy into mechanical energy, 75% of the energy is lost as heat through radiation, convection, conduction, and sweat evaporation (Vella & Kravitz). During exercise, heat is generated at a higher rate and the body adjusts to this by increasing blood flow to sweat glands at the skin's surface, producing sweat so that it evaporates and cools the body (Vella & Kravitz). As sweat is mostly composed of water, sweating causes the body to lose its water content. A loss of more than 5% of the body's water content causes mild dehydration symptoms, including fatigue, dizziness, and thirst (Dehydration).

Final Iteration

Assumptions:

1. Ambient temperature and humidity are constant during the game
2. Throughout the game, the body temperature is constant throughout the body at 37°C
3. There is no wind or precipitation to affect the body temperature
4. Uniform has no effect on heat loss
5. Emotional stress of the game has no effect on sweat production
6. The player starts playing at time t=0
7. There is no extra time added at the end of either half
8. There are no pauses in game play
9. The player plays the full 90 minutes
10. Water is not consumed by the player during the game time
11. Dehydration occurs at a loss of 5% of water content of the body (“Water Purification”, 2008)
12. The body is composed of 60% water by mass (Perlman, 2016)
13. The Katch-McArdle Formula determines the normal body metabolism of the player (Gupta, et. al., 2017)
14. Sweat is the only means by which the body loses heat
15. Water content in the air slows the rate of evaporation.
16. The player is cylindrical in shape
17. The player’s shoulder width is ¼ the height of the player (Ida, 2012)
18. The player moves at a constant speed throughout the game.
19. Average speed of the player is not affected by the position played.
20. Oxygen is the limiting reactant in metabolism to create energy
21. The energy required to kick/dribble/tackle etc. is negligent to the energy required to run/sprint/jog
22. Volumes of other gases inhaled (e.g. nitrogen, hydrogen, etc.) is exhaled, meaning the net intake of these gases is zero; the only gas inhalation measured in this model is oxygen, the only gas exhalation measured in this model is carbon dioxide
23. Oxygen and carbon dioxide mass balances are at steady state
24. There is no change in elevation on the field
25. Conversion from chemical energy to kinetic energy is 50% efficient

System Diagram:

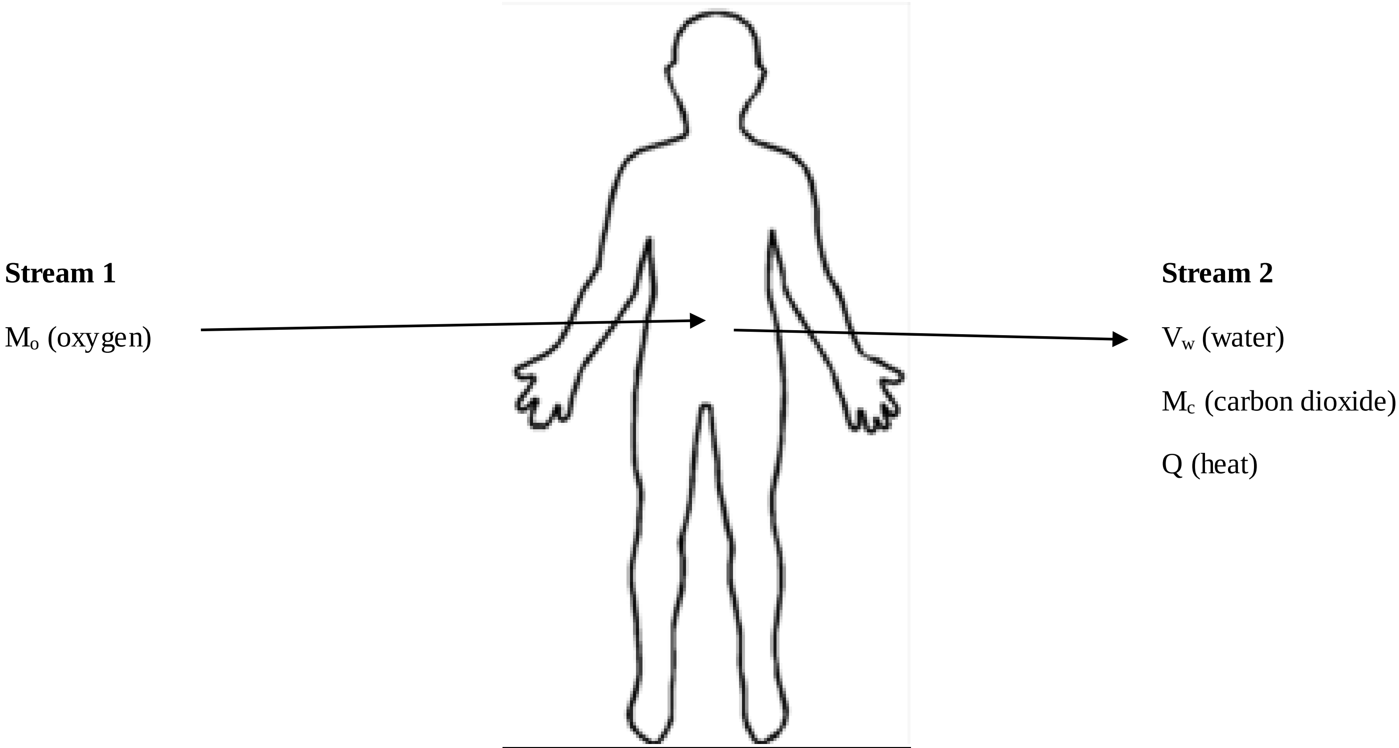

Figure 1: System diagram. A soccer player's body defines the system boundaries. The net components entering and leaving the system are oxygen, carbon dioxide, water, and heat. Environmental conditions and system specifications (listed below) affect how these inputs and outputs move between the system and environment.

Model Inputs:

- T_{air} [°C]
- M_{player} [kg]
- h_{player} [m]
- v [m/s]
- H_{air} [kg water/ kg dry air]
  - The humidity affects how quickly the water can evaporate from the skin.

Model Outputs:

- dV_w/dt [L/min]
- V_w [L]
- $t_{dehydration}$ [min], if applicable

Fundamental Relationships:

- Cellular respiration
- Steady state mass balance
- Unsteady state mass balance
- Steady state energy
- Kinetic energy
- Internal energy
- Surface area of a cylinder
- Heat
- Basic body metabolism
- Humidity

$$Q_h = \Delta H_{vap}\rho_{air}AH_{air} \qquad [25]$$

Calculations:

- Mass balances
- Energy balance:

$$\frac{dQ}{dt} = -h_{conv,air}A\left(T_{air} - T_{player}\right) + \Delta H_{rxn}M_g\frac{M_{player}}{\rho_{player}{}^2c_p} + \frac{1}{2}M_AE_A + Q_m + Q_h \qquad [25]$$

Nomenclature:

Symbol	Meaning	Units
A	surface area of player	[m²]
ΔH_{rxn}	enthalpy of cellular respiration	[kJ]
E_A	energy of one mole of ATP	[kJ/mol]
$h_{conv,air}$	convective heat transfer coefficient of air	[W/m²K]
$h_{vap,water}$	heat of vaporization of water	[kJ/mol]
M_A	mass of ATP	[kg]
M_C	mass of carbon dioxide	[kg]
M_O	mass of oxygen	[kg]

M_{player}	mass of player	[kg]
MW_O	molecular weight of oxygen	[kg/mol]
MW_W	molecular weight of water	[kg/mol]
π	mathematical constant pi	[-]
Q_m	basic body metabolism	[J/s]
r	rate of reaction of cellular respiration	[s^{-1}]
ρ_W	density of water	[kg/m^3]
t	time	[s]
T_{air}	ambient temperature	[°C]
T_{player}	player temperature	[°C]
T_{ref}	reference temperature of player	[°C]
v	speed of player	[m/s]
$V_{dehydration}$	volume of water in player at which player is dehydrated	[L]
ν_G	stoichiometric coefficient of glucose	[-]
ν_O	stoichiometric coefficient of oxygen	[-]
V_W	volume of water	[L]

Model and Output:

```
M_player = 80; % kg
V_w = 0.6 * M_player; % kg, Assumption 12
V_wdehyd = V_wi - 0.05 * V_wi; % kg, Assumption 11
% k_player = 0.21; % W/m.K

% water parameters
MWw = 18.01/1000; % kg/mol
hvw = 43345 * 1000; % J/mol
rhow = 1; % kg/L

tspan = [0 90];
cT0 = T_ref;

[t,V] = ode45(@heat,tspan,cT0);
dV_outdt = V(:,1) * MWw/(hvw*rhow);
plot(t,dV_outdt);
xlabel('Time [min]');
ylabel('Sweat Rate [L/min]');
xlim([0 90]);

n = size(dV_outdt);
i = 1;
min = n/90;
total = 0;
while i <= n
    total = total + dV_outdt(i);
    if total > V_wdehyd
        time = i / min;
        time = round(time);
        fprintf('Player becomes dehydrated at %f minutes.', time);
    end
    i = i + 1;
end
```

```
fprintf('The total volume of sweat is %f L.\n',total);


function dydt = heat(t,V)
     h = 20; % W/m^2.K
     cp = 3470; % J/kg.K
     h_player = 1.8; % m
     w_player = h_player / 4; % m
     A = pi * w_player * h_player; % m^2
     M_player = 80; % kg
     rho_player = 985; % kg/m^3
     v_player = 6; % m/s
     Eatp = 70000 / 0.50718; % J/kg
     MWatp = 0.50718; % kg/mol
     Eatp = Eatp * MWatp; % J/mol
     H_rxn_atpsynth = 2880 * 1000; % J/mol glucose
     vg = -1; % from fundamental relationships
     vo = -6; % from fundamental relationships


    va = 36; % from fundamental relationships
    vc = 6; % from fundamental relationships
    vw = 6; % from fundamental relationships
    hvw = 43345 * 1000; % J/mol
    rho_air = 1.225; % kg/m^3
    H_air = 23 / (1000 * rho_air); % g water / m^3 dry air
    Qmet = (370 + (21.6 * M_player * (1-0.12))) * 4184 / 86400;

    ATPcons = M_player * v_player^2 / (Eatp); % mol ATP

    % Mass Balance
    ATPrxn = ATPcons;
    O2rxn = ATPrxn * -1 * vo / va;
    O2in = O2rxn;
    CO2rxn = ATPrxn * vc / va;
    CO2out = CO2rxn;
    Waterrxn = ATPrxn * vw / va;
    Glucoserxn = -1 * ATPrxn * vg / va;

    % Energy Balance
    q = H_rxn_atpsynth * Glucoserxn + 1/2 * ATPcons * Eatp;
    Qh = hvw * rho_air * H_air * A;

    T_air = 23; % degrees C

    T_player = V(1);

    dQdt = -1 * h * A * (T_player - T_air) + q * M_player /
 (rho_player^2 * cp) + Qmet + Qh;

    dydt = [dQdt];
end
```

Figure 2: Final Iteration MATLAB model code

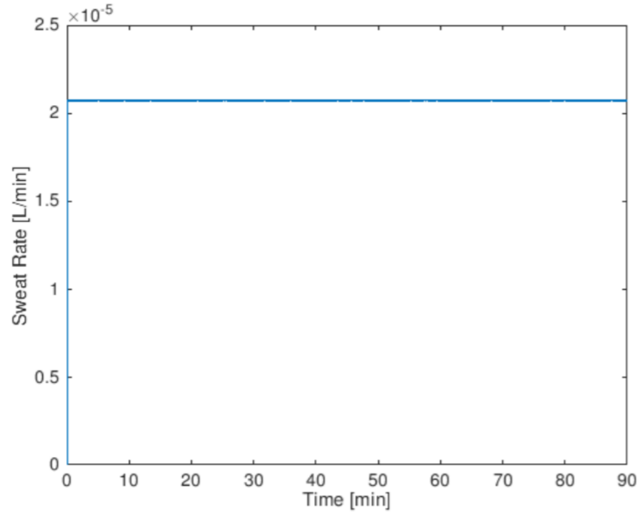

Figure 3: Final Iteration MATLAB output, graphical

```
The total volume of sweat is 0.111654 L.
```

Figure 4: Final Iteration MATLAB output, numerical

Model Analysis

The final model was a vast improvement upon the results of the previous iterations, taking into account a variety of parameters which affect the rate of perspiration of a soccer player. In the first iteration, perspiration due to the ambient temperature was the only parameter evaluated, assuming that the player did not move throughout the entirety of the game. After five iterations, the final model was able to take into account the player moving at a constant velocity, the inefficiency of the conversion from chemical energy to mechanical energy, the production of heat via the body's resting metabolism, and the effect of humidity on the rate of evaporation of water from the surface of the skin. These parameters changed the total sweat volume over the 90 minute game from 0 L to 0.1 L.

Each iteration calculated the heat produced by the soccer player as a differential equation with respect to time, evaluated with MATLAB's ode45 function over the 90 minute timespan with time intervals of 1 second. The model predicts that the rate of sweat production will be constant throughout the game, save for the initial few seconds of the game when the player's body adjusts to the conditions of the environment. The final iteration of the model outputs a sweat production rate of about 2.1×10^{-5} L/s and a total sweat volume of 0.111654 L.

Unfortunately, there are several limitations to this model, the first of which is the difficulty of modeling the random changes in velocity that a soccer player performs, in short periods of time, throughout a game. To simplify, this model assumed that the player moved at a constant velocity throughout the entirety of the game, which is completely unrealistic. Additionally, the model assumes that the body's thermoregulatory system is efficient enough so that the player's core body temperature remains constant when in reality, the core temperature of the body is raised during exercise due to inefficiencies such as diffusion of water through the dermis and the inability for sweat to evaporate in humid conditions. Finally, the model kept the geometry of the body very simple, which affected the calculation of the surface area of the player which in turn affected the effects of ambient temperature and humidity on perspiration rate.

In future iterations, this model could improve upon the limitations aforementioned as well as investigate the effects of the uniform on the body's resistance to heat and mass transfer, use smaller time steps to make calculations more accurate, and take into consideration the effect of the halftime break on the player's sweat rate and tendency to become dehydrated, as this allows the player a 15 minute break to become more rested and regain some of the fluids lost during the first 45 minutes. Additionally, factors such as blood flow and vascodilation, which greatly affect the rate at which water is allowed to leave the body, could also be added to the model.

This model is applicable to athletes and trainers wanting to improve their performance. The model's assumptions and equations could theoretically be adjusted so as to predict the sweat production rate of athletes of a variety of sports, not just soccer. While this model is a rough approximation of a professional soccer player's sweat production, improvements can make it more accurate and allow for a better understanding of which conditions have the greatest effects on an athlete's perspiration rate and time of dehydration during a game.

Iteration One

Assumptions:

1. Ambient temperature and humidity are constant during the game
2. Throughout the game, the body temperature is constant throughout the body at 37°C
3. There is no wind or precipitation to affect the body temperature
4. Uniform has no effect on heat loss
5. Emotional stress of the game has no effect on sweat production
6. The player starts playing at time t=0
7. There is no extra time added at the end of either half
8. There are no pauses in game play
9. The player plays the full 90 minutes
10. Water is not consumed by the player during the game time
11. Dehydration occurs at a loss of 5% of water content of the body ("Water Purification", 2008)
12. The body is composed of 60% water by mass (Perlman, 2016)
13. The player's normal body metabolism is negligent, does not produce heat to warm the body up to normal temperature
14. Sweat is the only means by which the body loses heat
15. Humidity has no effect on evaporation
16. The player is cylindrical in shape
17. The player's shoulder width is ¼ the height of the player (Ida, 2012)
18. The player does not move throughout the game

Model Inputs:

- T_{air} [°C]
  - This parameter may change the rate at which the player sweats.
  - If the temperature is greater than room temperature, the player's sweat rate will increase as the body must produce even more sweat to get the body down to a normal, functional temperature.
  - If the temperature is less than room temperature, the player's sweat rate will decrease as some of the heat produced by the body is used to bring the body temperature up to a normal, functional temperature.
  - Stays constant over time (Assumption 1)
  - For model testing purposes, this value was set at 23°C (room temperature)

- M_{player} [kg]
  - This parameter determines how much work a player must do to move himself.
  - Additionally, the parameter helps to determine how much water the player can lose before becoming dehydrated.
  - For model testing purposes, this value was set at 80 kg (mass of Jordan Henderson, my favorite player)
- h_{player} [m]
  - This parameter helps determine the surface area of the player which is used in determining the convective heat transfer with the air. (Assumption 15)
  - For model testing purposes, this value was set at 1.8 m (height of Jordan Henderson, my favorite player)

Model Outputs:

- dV_w/dt [L/min]
  - As the player runs and adjusts to the game environment, the rate of sweat (water) leaving the body changes.
  - This output will be shown as a graph of rate of sweat production versus time.
- V_w [L]
  - By integrating the graph of rate of sweat production versus time, the total volume of sweat produced in a unit of time (e.g. one minute, one half, one game, etc.) will be calculated.
  - This output will be shown as a graph and a numerical value after each half of the game (volume = x L at t = 45 minutes, t = 90 minutes).
- $t_{dehydration}$ [min], if applicable
  - By comparing the total amount of sweat produced by the player and the amount of water in the body at time t = 0, the time at which the player is dehydrated can be found. If a player loses more than 5% of the water in their body, they will begin to show signs of dehydration.
  - This output will be shown as a numerical value (t = x minutes).
  - Does not exceed 90 minutes (Assumption 9)

Fundamental Relationships:

- Steady state energy balance:

$$U + KE + PE = Q + W + H \tag{1}$$

- Heat

$$Q = M_{player} c_{p,player} \Delta T \quad [2]$$

- Convective heat

$$Q = h_c A \Delta T \quad [3]$$

- Surface area

$$A = \pi \left(\frac{h}{4}\right) h \quad [4]$$

- Heat of evaporation

$$Q = \frac{h_{vap,w} V_W \rho_W}{MW_W} \quad [5]$$

Calculations:

- Mass balances
  - Water:

$$\frac{dV_W}{dt} = -V_{W,out} \quad [6]$$

- Energy balance:

$$\frac{dQ}{dt} = -h_{conv,air} A \left(T_{player} - T_{air}\right) \quad [7]$$

$$\frac{dV_W}{dt} = -\frac{dQ}{dt} * \frac{MW_W}{h_{vap,w} \rho_w} \quad [8]$$

Nomenclature:

Symbol	Meaning	Units
A	surface area of player	[m^2]
c_p	specific heat	[kJ/kg.K]
KE	kinetic energy	[kJ]
PE	potential energy	[kJ]
$h_{conv,air}$	convective heat transfer coefficient of air	[W/m^{2}K]
h	height of player	[m]
$h_{vap,w}$	heat of vaporization of water	[kJ/mol]
M_{player}	mass of player	[kg]
MW_W	molecular weight of water	[kg]
π	mathematical constant pi	[-]
Q	heat	[kJ]
ρ_W	density of water	[kg/m^3]
t	time	[s]
T_{air}	ambient temperature	[°C]
$t_{dehydration}$	time of dehydration	[min]
T_{player}	player temperature	[°C]
U	internal energy	[kJ]
$V_{dehydration}$	volume of water in player at which player is dehydrated	[L]
V_W	Volume of Water	[L]

Model and Output:

```
  M_player = 80; % kg
  V_w = 0.6 * M_player; % kg, Assumption 12
  V_wdehyd = V_wi - 0.05 * V_wi; % kg, Assumption 11

  % water parameters
  MWw = 18.01/1000; % kg/mol
  hvw = 43345 * 1000; % J/mol
  rhow = 1; % kg/L

  tspan = [0 90];
  cT0 = T_ref;

  [t,V] = ode45(@heat,tspan,[cT0]);
  dV_outdt = V(:,1) * MWw/(hvw*rhow);
  plot(t,dV_outdt);
  xlabel('Time [s]');
  ylabel('Sweat Rate [L/s]');
  xlim([0 90]);

  n = size(dV_outdt);
  i = 1;
  min = n/90;
  total = 0;
  while i <= n
      total = total + dV_outdt(i);
      if total > V_wdehyd
          time = i / min;
          time = round(time);
          fprintf('Player becomes dehydrated at %f minutes.', time);
      end
      i = i + 1;
  end

fprintf('The total volume of sweat is %f L.\n',total);


function dydt = heat(t,V)
    h = 20; % W/m^2.K
    h_player = 1.8; % m
    w_player = h_player / 4; % m
    A = pi * w_player * h_player; % m^2

    T_air = 23; % degrees C

    T_player = V(1);

    dQdt = -1 * h * A * (T_player - T_air);

    dydt = [dQdt];
end
```

Figure 5: Iteration 1 MATLAB model code

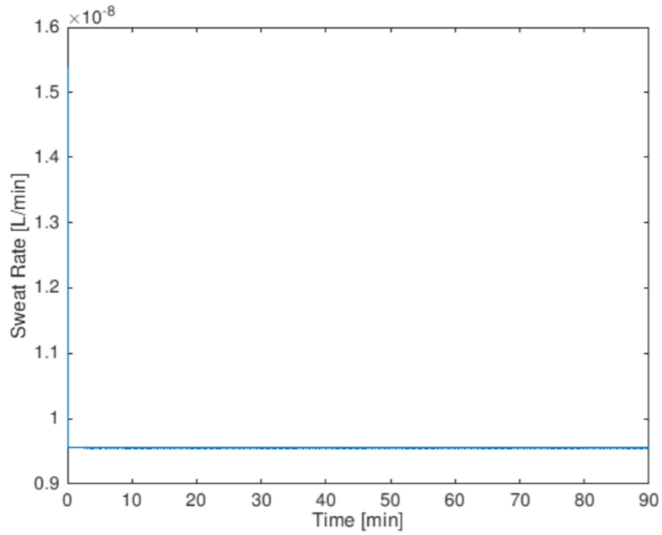

Figure 6: Iteration 1 MATLAB output, graphical

```
The total volume of sweat is 0.000000 L.
```

Figure 7: Iteration 1 MATLAB output, numerical

Justification for Next Iteration:

This iteration was incredibly simple, not even incorporating the player's movement into the heat generation. Because of this, the sweat rate was unrealistically small. The next iteration will assume the player moves at a constant speed.

Iteration Two

Assumptions:

18. ~~The player does not move throughout the game~~ The player moves at a constant speed throughout the game.
19. Average speed of the player is not affected by the position played.
20. Oxygen is the limiting reactant in metabolism to create energy
21. The energy required to kick/dribble/tackle etc. is negligent to the energy required to run/sprint/jog
22. Volumes of other gases inhaled (e.g. nitrogen, hydrogen, etc.) is exhaled, meaning the net intake of these gases is zero; the only gas inhalation measured in this model is oxygen, the only gas exhalation measured in this model is carbon dioxide
23. Oxygen and carbon dioxide mass balances are at steady state
24. There is no change in elevation on the field
25. The conversion from chemical energy to mechanical energy is 100% efficient

Model Inputs:

- T_{air} [°C]
- M_{player} [kg]
- h_{player} [m]
- v [m/s]
  - This input is used to determine how much kinetic energy is needed to be produced by the player.
  - For model testing purposes, this value was set to 6 m/s.

Model Outputs:

- dV_w/dt [L/min]
- V_w [L]
- $t_{dehydration}$ [min], if applicable

Fundamental Relationships:

- Cellular respiration

$$C_6H_{12}O_6 + 6\ O_2\ \rightarrow 6\ CO_2 + 6\ H_2O + 36\ ATP \qquad [9]$$

- Steady state mass balance

$$0 = in - out + generation - consumption \quad [10]$$

- Unsteady state mass balance

$$accumulation = in - out + generation - consumption \quad [11]$$

- Steady state energy balance (Equation 1)
- Kinetic energy (Assumption 21)

$$KE = \frac{1}{2} M_{player} v^2 \quad [12]$$

- Heat (Equation 2)
- Convective Heat (Equation 3)
- Surface area of a cylinder (Equation 4)
- Heat of Evaporation (Equation 5)
- Heat of reaction

$$Q = \Delta H_{rxn} * M_{g,rxn} \quad [13]$$

- Chemical energy (Assumption 25)

$$CE = KE = M_{ATP} * E_{ATP} \quad [14]$$

Calculations:

- Mass balances
  - Oxygen:

$$M_{O,in} - M_{O,rxn} = 0 \quad [15]$$

  - Carbon Dioxide:

$$M_{C,rxn} - M_{C,out} = 0 \quad [16]$$

  - ATP:

$$M_{A,rxn} - M_{A,cons} = 0 \quad [17]$$

  - Water:

$$\frac{dV_W}{dt} = -V_{W,out} + \frac{M_{W,rxn}}{\rho_W} \quad [18]$$

  - Glucose:

$$\frac{dM_G}{dt} = -M_{G,rxn} \quad [19]$$

- Energy balance:

$$\frac{1}{2} M_{player} v^2 = M_A E_A \quad [20]$$

$$\frac{dQ}{dt} = -h_{conv,air}A\left(T_{air} - T_{player}\right) + \Delta H_{rxn}M_g\frac{M_{player}}{\rho_{player}{}^2 c_p} \qquad [20]$$

[Equation 8]

Nomenclature:

Symbol	Meaning	Units
A	surface area of player	[m^2]
ΔH_{rxn}	enthalpy of cellular respiration	[kJ]
E_{ATP}	energy of one mole of ATP	[kJ/mol]
$h_{conv,air}$	convective heat transfer coefficient of air	[W/m^{2}K]
$h_{vap,water}$	heat of vaporization of water	[kJ/mol]
M_A	mass of ATP	[kg]
M_C	mass of carbon dioxide	[kg]
M_G	mass of glucose	[kg]
M_O	mass of oxygen	[kg]
M_{player}	mass of player	[kg]
MW_O	molecular weight of oxygen	[kg/mol]
MW_W	molecular weight of water	[kg/mol]
π	mathematical constant pi	[-]
ρ_W	density of water	[kg/m^3]
t	time	[s]
T_{air}	ambient temperature	[°C]
T_{player}	player temperature	[°C]
v	speed of player	[m/s]
$V_{dehydration}$	volume of water in player at which player is dehydrated	[L]
ν_G	stoichiometric coefficient of glucose	[-]
ν_O	stoichiometric coefficient of oxygen	[-]
V_W	volume of water	[L]

Model and Output:

```
M_player = 80; % kg
V_w = 0.6 * M_player; % kg, Assumption 12
V_wdehyd = V_wi - 0.05 * V_wi; % kg, Assumption 11
% k_player = 0.21; % W/m.K

% water parameters
MWw = 18.01/1000; % kg/mol
hvw = 43345 * 1000; % J/mol
rhow = 1; % kg/L

tspan = [0 90];
cT0 = T_ref;

[t,V] = ode45(@heat,tspan,cT0);
dV_outdt = V(:,1) * MWw/(hvw*rhow);
plot(t,dV_outdt);
xlabel('Time [min]');
ylabel('Sweat Rate [L/min]');
xlim([0 90]);

n = size(dV_outdt);
i = 1;
min = n/90;
total = 0;
while i <= n
    total = total + dV_outdt(i);
    if total > V_wdehyd
        time = i / min;
        time = round(time);
        fprintf('Player becomes dehydrated at %f minutes.', time);
    end
    i = i + 1;
end

fprintf('The total volume of sweat is %f L.\n',total);


function dydt = heat(t,V)
    h = 20; % W/m^2.K
    cp = 3470; % J/kg.K
    h_player = 1.8; % m
    w_player = h_player / 4; % m
    A = pi * w_player * h_player; % m^2
    M_player = 80; % kg
    rho_player = 985; % kg/m^3
    v_player = 6; % m/s
    Eatp = 70000 / 0.50718; % J/kg
    MWatp = 0.50718; % kg/mol
    Eatp = Eatp * MWatp; % J/mol
    H_rxn_atpsynth = 2880 * 1000; % J/mol glucose
    vg = -1; % from fundamental relationships
    vo = -6; % from fundamental relationships
```

```
    va = 36; % from fundamental relationships
    vc = 6; % from fundamental relationships
    vw = 6; % from fundamental relationships

    ATPcons = M_player * v_player^2 / (2 * Eatp); % mol ATP

    % Mass Balance
    ATPrxn = ATPcons;
    O2rxn = ATPrxn * -1 * vo / va;
    O2in = O2rxn;
    CO2rxn = ATPrxn * vc / va;
    CO2out = CO2rxn;
    Waterrxn = ATPrxn * vw / va;
    Glucoserxn = -1 * ATPrxn * vg / va;

    % Energy Balance
    q = H_rxn_atpsynth * Glucoserxn;

    T_air = 23; % degrees C

    T_player = V(1);

    dQdt = -1 * h * A * (T_player - T_air) + q * M_player /
  (rho_player^2 * cp);

    dydt = [dQdt];
end
```

Figure 8: Iteration 2 MATLAB model code

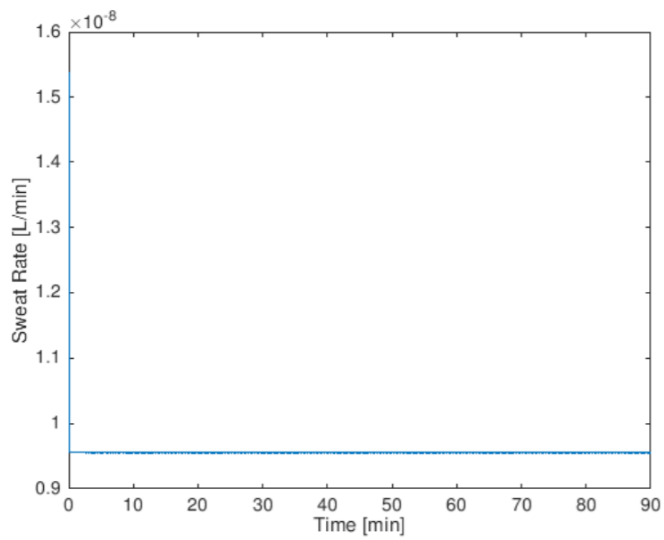

Figure 9: Iteration 2 MATLAB output, graphical

```
The total volume of sweat is 0.000000 L.
```

Figure 10: Iteration 2 MATLAB output, numerical

Justification for Next Iteration:

Again, the iteration was incredibly simple, assuming a constant speed and no heat generation by the player besides the heat of the reaction to create movement. Because of this, the sweat rate was again unrealistically small. The next iteration will assume the conversion of chemical energy to mechanical energy is not 100% efficient.

Iteration Three

Assumptions:

25. ~~Conversion from chemical energy to kinetic energy is 100% efficient~~ Conversion from chemical energy to kinetic energy is 50% efficient

Model Inputs:

- T_{air} [°C]
- M_{player} [kg]
- h_{player} [m]
- v [m/s]

Model Outputs:

- dV_w/dt [L/min]
- V_w [L]
- $t_{dehydration}$ [min], if applicable

Fundamental Relationships:

- Cellular respiration
- Steady state mass balance
- Unsteady state mass balance
- Steady state energy
- Kinetic energy
- Internal energy
- Surface area of a cylinder
- Heat

Calculations:

- Mass balances
    - Equations 15 - 19
- Energy balance:

$$\frac{1}{2}M_{player}v^2 = \frac{1}{2}M_A E_A \quad [21]$$

$$\frac{dQ}{dt} = -h_{conv,air}A\left(T_{air} - T_{player}\right) + \Delta H_{rxn}M_g\frac{M_{player}}{{\rho_{player}}^2 c_p} + \frac{1}{2}M_A E_A \quad [22]$$

Nomenclature:

Symbol	Meaning	Units
A	surface area of player	[m^2]
ΔH_{rxn}	enthalpy of cellular respiration	[kJ]
E_A	energy of one mole of ATP	[kJ/mol]
$h_{conv,air}$	convective heat transfer coefficient of air	[W/m^{2}K]
$h_{vap,water}$	heat of vaporization of water	[kJ/mol]
M_A	mass of ATP	[kg]
M_C	mass of carbon dioxide	[kg]
M_O	mass of oxygen	[kg]
M_{player}	mass of player	[kg]
MW_O	molecular weight of oxygen	[kg/mol]
MW_W	molecular weight of water	[kg/mol]
π	mathematical constant pi	[-]
r	rate of reaction of cellular respiration	[s^{-1}]
ρ_W	density of water	[kg/m^3]
t	time	[s]
T_{air}	ambient temperature	[°C]
T_{player}	player temperature	[°C]
T_{ref}	reference temperature of player	[°C]
v	speed of player	[m/s]
$V_{dehydration}$	volume of water in player at which player is dehydrated	[L]
ν_G	stoichiometric coefficient of glucose	[-]
ν_O	stoichiometric coefficient of oxygen	[-]
V_W	volume of water	[L]

Model and Output:

```
M_player = 80; % kg
V_w = 0.6 * M_player; % kg, Assumption 12
V_wdehyd = V_wi - 0.05 * V_wi; % kg, Assumption 11
% k_player = 0.21; % W/m.K

% water parameters
MWw = 18.01/1000; % kg/mol
hvw = 43345 * 1000; % J/mol
rhow = 1; % kg/L

tspan = [0 90];
cT0 = T_ref;

[t,V] = ode45(@heat,tspan,cT0);
dV_outdt = V(:,1) * MWw/(hvw*rhow);
plot(t,dV_outdt);
xlabel('Time [min]');
ylabel('Sweat Rate [L/min]');
xlim([0 90]);

n = size(dV_outdt);
i = 1;
min = n/90;
total = 0;
while i <= n
    total = total + dV_outdt(i);
    if total > V_wdehyd
        time = i / min;
        time = round(time);
        fprintf('Player becomes dehydrated at %f minutes.', time);
    end
    i = i + 1;
end

fprintf('The total volume of sweat is %f L.\n',total);


function dydt = heat(t,V)
    h = 20; % W/m^2.K
    cp = 3470; % J/kg.K
    h_player = 1.8; % m
    w_player = h_player / 4; % m
    A = pi * w_player * h_player; % m^2
    M_player = 80; % kg
    rho_player = 985; % kg/m^3
    v_player = 6; % m/s
    Eatp = 70000 / 0.50718; % J/kg
    MWatp = 0.50718; % kg/mol
    Eatp = Eatp * MWatp; % J/mol
    H_rxn_atpsynth = 2880 * 1000; % J/mol glucose
    vg = -1; % from fundamental relationships
    vo = -6; % from fundamental relationships
```

```
    va = 36; % from fundamental relationships
    vc = 6; % from fundamental relationships
    vw = 6; % from fundamental relationships

    ATPcons = M_player * v_player^2 / (Eatp); % mol ATP

    % Mass Balance
    ATPrxn = ATPcons;
    O2rxn = ATPrxn * -1 * vo / va;
    O2in = O2rxn;
    CO2rxn = ATPrxn * vc / va;
    CO2out = CO2rxn;
    Waterrxn = ATPrxn * vw / va;
    Glucoserxn = -1 * ATPrxn * vg / va;

    % Energy Balance
    q = H_rxn_atpsynth * Glucoserxn + 1/2 * ATPcons * Eatp;

    T_air = 23; % degrees C

    T_player = V(1);

    dQdt = -1 * h * A * (T_player - T_air) + q * M_player /
 (rho_player^2 * cp);

    dydt = [dQdt];
end
```

Figure 11: Iteration 3 MATLAB model code

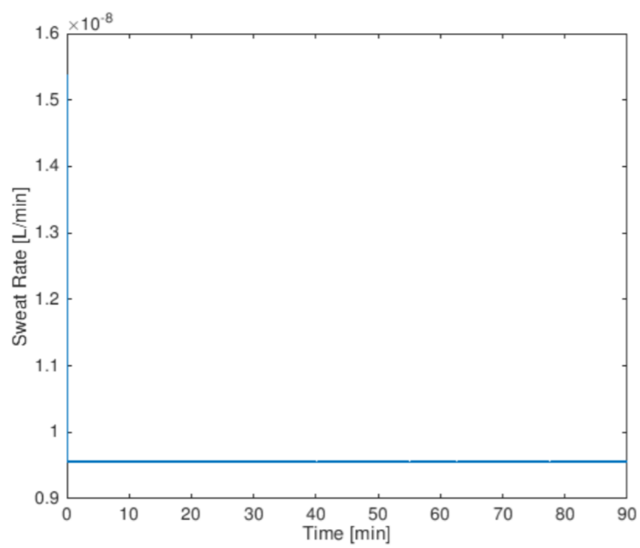

Figure 12: Iteration 3 MATLAB output, graphical

```
The total volume of sweat is 0.000000 L.
```

Figure 13: Iteration 3 MATLAB output, numerical

Justification for Next Iteration:

The final sweat volume is still too low to allow the model to be considered adequate. For the next iteration, I will consider the human base metabolism, as that energy is also still being produced by the body.

Iteration Four

Assumptions:

13. ~~The player's normal body metabolism is negligent, does not produce heat to warm the body up to normal temperature~~ The Katch-McArdle Formula determines the normal body metabolism of the player (Gupta, et. al., 2017)

Model Inputs:

- T_{air} [°C]
- M_{player} [kg]
- h_{player} [m]
- v [m/s]

Model Outputs:

- dV_w/dt [L/min]
- V_w [L]
- $t_{dehydration}$ [min], if applicable

Fundamental Relationships:

- Cellular respiration
- Steady state mass balance
- Unsteady state mass balance
- Steady state energy
- Kinetic energy
- Internal energy
- Surface area of a cylinder
- Heat
- Basic body metabolism

$$Q_m = (370 + \left(21.6 * M_{player} * (0.88)\right)) * 4184 \frac{kcal}{J} * \frac{day}{86400\ s} \quad [23]$$

Calculations:

- Mass balances

- Energy balance:

$$\frac{dQ}{dt} = -h_{conv,air}A\left(T_{air} - T_{player}\right) + \Delta H_{rxn} M_g \frac{M_{player}}{{\rho_{player}}^2 c_p} + \frac{1}{2} M_A E_A + Q_m \qquad [24]$$

Nomenclature:

Symbol	Meaning	Units
A	surface area of player	[m^2]
ΔH_{rxn}	enthalpy of cellular respiration	[kJ]
E_A	energy of one mole of ATP	[kJ/mol]
$h_{conv,air}$	convective heat transfer coefficient of air	[W/m^2K]
$h_{vap,water}$	heat of vaporization of water	[kJ/mol]
M_A	mass of ATP	[kg]
M_C	mass of carbon dioxide	[kg]
M_O	mass of oxygen	[kg]
M_{player}	mass of player	[kg]
MW_O	molecular weight of oxygen	[kg/mol]
MW_W	molecular weight of water	[kg/mol]
π	mathematical constant pi	[-]
Q_m	basic body metabolism	[J/s]
r	rate of reaction of cellular respiration	[s^{-1}]
ρ_W	density of water	[kg/m^3]
t	time	[s]
T_{air}	ambient temperature	[°C]
T_{player}	player temperature	[°C]
T_{ref}	reference temperature of player	[°C]
v	speed of player	[m/s]
$V_{dehydration}$	volume of water in player at which player is dehydrated	[L]
ν_G	stoichiometric coefficient of glucose	[-]
ν_O	stoichiometric coefficient of oxygen	[-]
V_W	volume of water	[L]

Model and Output:

```
M_player = 80; % kg
V_w = 0.6 * M_player; % kg, Assumption 12
V_wdehyd = V_wi - 0.05 * V_wi; % kg, Assumption 11
% k_player = 0.21; % W/m.K

% water parameters
MWw = 18.01/1000; % kg/mol
hvw = 43345 * 1000; % J/mol
rhow = 1; % kg/L

tspan = [0 90];
cT0 = T_ref;

[t,V] = ode45(@heat,tspan,cT0);
dV_outdt = V(:,1) * MWw/(hvw*rhow);
plot(t,dV_outdt);
xlabel('Time [min]');
ylabel('Sweat Rate [L/min]');
xlim([0 90]);

n = size(dV_outdt);
i = 1;
min = n/90;
total = 0;
while i <= n
    total = total + dV_outdt(i);
    if total > V_wdehyd
        time = i / min;
        time = round(time);
        fprintf('Player becomes dehydrated at %f minutes.', time);
    end
    i = i + 1;
end

fprintf('The total volume of sweat is %f L.\n',total);


function dydt = heat(t,V)
    h = 20; % W/m^2.K
    cp = 3470; % J/kg.K
    h_player = 1.8; % m
    w_player = h_player / 4; % m
    A = pi * w_player * h_player; % m^2
    M_player = 80; % kg
    rho_player = 985; % kg/m^3
    v_player = 6; % m/s
    Eatp = 70000 / 0.50718; % J/kg
    MWatp = 0.50718; % kg/mol
    Eatp = Eatp * MWatp; % J/mol
    H_rxn_atpsynth = 2880 * 1000; % J/mol glucose
    vg = -1; % from fundamental relationships
    vo = -6; % from fundamental relationships
```

```
    va = 36; % from fundamental relationships
    vc = 6; % from fundamental relationships
    vw = 6; % from fundamental relationships
    Qmet = (370 + (21.6 * M_player * (1-0.12))) * 4184 / 86400;

    ATPcons = M_player * v_player^2 / (Eatp); % mol ATP

    % Mass Balance
    ATPrxn = ATPcons;
    O2rxn = ATPrxn * -1 * vo / va;
    O2in = O2rxn;
    CO2rxn = ATPrxn * vc / va;
    CO2out = CO2rxn;
    Waterrxn = ATPrxn * vw / va;
    Glucoserxn = -1 * ATPrxn * vg / va;

    % Energy Balance
    q = H_rxn_atpsynth * Glucoserxn + 1/2 * ATPcons * Eatp;

    T_air = 23; % degrees C

    T_player = V(1);

    dQdt = -1 * h * A * (T_player - T_air) + q * M_player /
 (rho_player^2 * cp) + Qmet;

    dydt = [dQdt];
end
```

Figure 14: Iteration 4 MATLAB model code

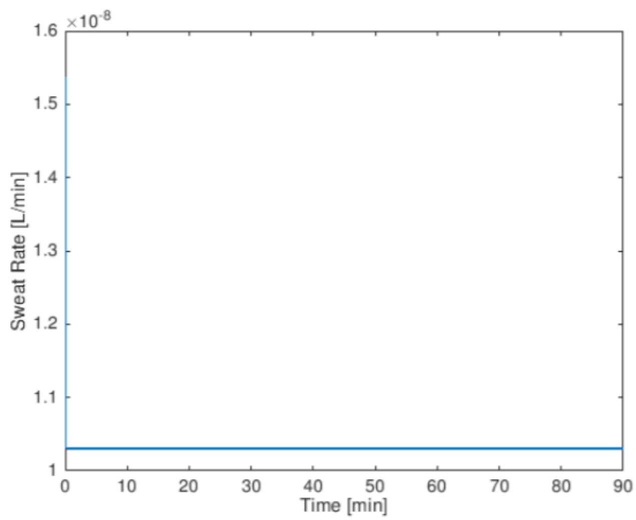

Figure 15: Iteration 4 MATLAB output, graphical

```
The total volume of sweat is 0.000056 L.
```

Figure 16: Iteration 4 MATLAB output, numerical

Justification for Next Iteration:

The sweat volume finally outputs a value greater than 0 L! However, this is still nowhere near a realistic value. For the next iteration, I will consider humidity affecting the rate of evaporation of water.

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