Final Report

THE INACTIVATION OF ESCHERICHIA COLI IN APPLE CIDER USING ULTRASOUND

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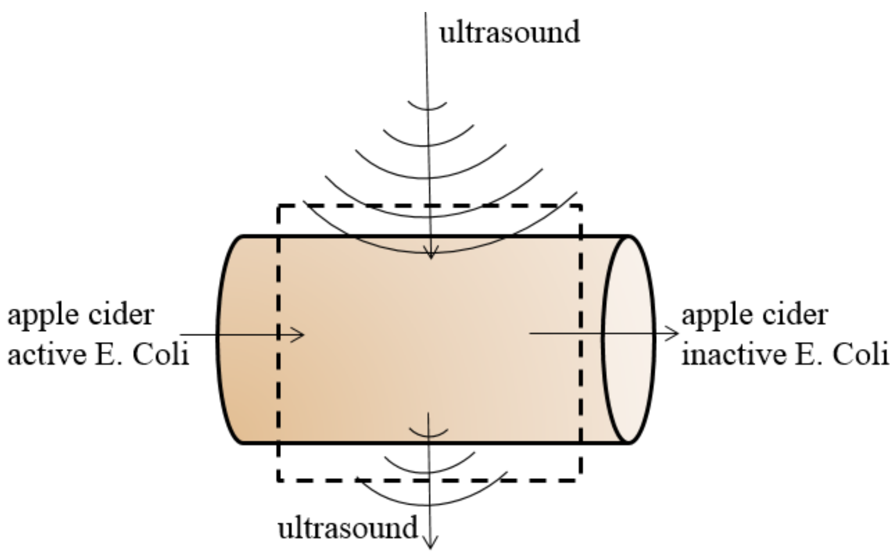
BACKGROUND

Ultrasound is a useful method of deactivating bacteria in food products because it does not generate a significant amount of heat. Applying too much heat to a food product can cause degradation of nutrients, such as ascorbic acid and vitamin C, and changes the mouthfeel and flavor profile of the food product (Ercan, 2013). There are five main theories on the mechanism of cell death caused by ultrasound. This model will focus on the theory that the transfer of mechanical energy from the soundwaves has a bactericidal effect (Cullen, 2012). The deactivation of the gram negative bacteria E. coli in apple cider using ultrasound will be modeled in order to find the amount of time needed to achieve a 5log reduction of E. coli at user inputted system dimensions.

SYSTEM DEFINITION



Note: The labeled arrows represent what is crossing the system boundaries.

The system consists of a section of pipe with apple cider flowing in with E. coli bacteria and apple cider flowing out with inactive E.coli bacteria. Ultrasound crosses the system boundaries in order to rupture the E. coli cells. After a certain amount of the energy from the ultrasound is expended the residual ultrasound leaves the system. The system is not a batch reactor, it is an open system with a residence time required to ensure E. coli deactivation.

Final Model

Objective

This model calculates the time it takes to reach a 5log reduction of E. coli bacteria in apple cider at a specific sound energy. The user inputs the inlet and outlet temperatures, the distance of the ultrasonic device from the apple cider, and the length of the pipe. After running the model, the user will know how long the system will run before a 5log reduction is reached. If a 5log reduction is not reached, the sound energy that was calculated from the inputted values is not large enough to cause the inactivation of E. coli in a system of the inputted dimensions.

Parameters

Notation	**Description**	**Dimensions**
F_i	Flow stream in	$\frac{grams}{second}$
F_o	Flow stream out	$\frac{grams}{second}$
ω_l	Mass percent of live cells in	-
ω_r	Mass percent of ruptured cells in	-
m_l	Mass of live cells	grams
m_r	Mass of ruptured cells	grams
t	time	seconds
S	Sound Energy	$\frac{Joules}{sec}$
α	Relationship between reaction constant and sound energy	$\frac{1}{s}$
β	Proportionality constant for the energy balance	$\frac{1}{s}$
Q	Heat generation	Joules
Q_{kill}	Energy needed to rupture one cell	$\frac{Joules}{sec * gram}$
E	Total energy of the system	Joules
C_p	Heat capacity of apple cider	$\frac{J}{g * C}$
T_i	Initial temperature of the apple cider	C
T_{kill}	Temperature at which E. coli is heated to per industry standards to inactivate it	C
T_o	Exit temperature of the apple cider	C
v	Speed of sound in apple cider	m/s
SI	Sound intensity	$\frac{Watt}{m^2}$
d	Distance from the ultrasonic source to the apple cider	m
L	Length of the system	m
t_{res}	Residence time	s

Assumptions

Several assumptions were made in order to create a logical model:

1) The apple cider and E. coli combination is a homogenous mixture
2) Fully developed one dimensional flow
3) E. coli is the only pathogen in the apple cider
4) There are no other particulates in the apple cider that will obstruct the soundwaves
5) The mass of an active E. coli bacteria is equal to that on an inactive one
6) The E. coli are not reproducing inside of the system
7) No inactive E. coli enter the system
8) Ultrasonic waves behave as normal mechanical waves
9) The apple cider and E. coli are entering and leaving the system at a constant rate
10) There is no height change between the inlet and outlet
11) Kinetic energy of the apple cider and the cells is negligible
12) Sound energy is dependent on distance from the source, sound intensity, and the speed of sound.
13) Sound energy is dissipated by: heat generation, rupturing of cells,
14) Once a sound wave has expended 90% of its energy it can no longer cause any changes to the system
15) The mass of an E. coli cell is 10^{-12} grams (Key Numbers, n.d.).
16) The speed of sound in apple cider is 1542.7 meters per second (Rao, 2005).
17) The heat capacity of apple cider is 3.55 Joule per gram Celsius (Rao, 2005).
18) Cell death is dependent on the amount of energy in the system related to how much energy it takes it kill a single cell

$$live\ cells \xrightarrow{E(t)} dead\ cells$$

Equations

Sound energy, heat generation, and the energy needed to kill an E. coli cell were all essential to the mass and energy balances. The energy needed to kill an E. coli cell was estimated based on

Sound Energy

$$S = \frac{v * SI * d}{t_{res}}$$

Heat Generation

$$Q = F_i * C_p * (T_i - T_o)$$

The energy needed to kill an E. coli cell was based off of industry standards for thermal processing and research done on the death of E. coli over exposure time (FDA, 2010).

Kill Energy

$$Q_{kill} = \frac{initial\ mass\ of\ cells}{time\ exposed\ to\ T_{kill}} * C_p * (T_{kill} - T_i)$$

Model Derivation

The model was formed by performing component mass and energy balances on each stream that crosses the boundary of the system. Previous iterations of the model can be found in the appendix. The components of the streams that crossed the boundary of the system are apple cider, live cells, ruptured cells, and sound energy. The component mass balance of apple cider is not relevant because, based on the assumptions, there is no change in the mass of the apple cider. However, the properties of apple cider are relevant to the transfer of sound energy. The properties of apple cider relevant to the model have been defined in the assumptions. The mass an energy balances depend on each other and needed to be solved simultaneously.

Mass Balance

$$\frac{dm_l(t)}{dt} = F_i * \omega_l - \alpha * \frac{E(t)}{Q_{kill}} * 10^{-12}$$

$$\frac{dm_r(t)}{dt} = -F_o * \omega_r + \alpha * \frac{E(t)}{Q_{kill}} * 10^{-12}$$

Energy Balance

$$\frac{dE(t)}{dt} = S - Q - \left(Q_{kill} * \beta * m_r(t)\right) - .1S$$

COMPUTATIONAL PROGRAM

A computational program was developed to test the validity of the final model. The model was programed in Mathcad. Reasonable values for distance, length, and temperature have been inputted to demonstrate how the model works.

User Inputs

$d := .1\ m \qquad L := 1\ m \qquad T_i := 25\ C \qquad T_o := 30\ C$

$$F_i := 20\ \frac{g}{sec} \qquad \omega_1 := .1 \qquad \alpha := 10^9\ \frac{1}{s} \qquad \beta := 10\ \frac{1}{s} \qquad v := 1543.7\ \frac{m}{s}$$

$$t_{res} := \frac{L}{F_i}\ s \qquad F_o := F_i \qquad \omega_r := \omega_1 \qquad SI := 10^4\ \frac{watt}{m^2}$$

$$S := \frac{v \cdot SI \cdot d}{t_{res}}\ \frac{J}{s} \qquad Cp := 3.55\ \frac{J}{g \cdot C}$$

$$Q := F_i \cdot Cp \cdot (T_i - T_o)$$

$$a := 14$$

$$Q_{kJ} := \frac{.1}{a} \cdot Cp \cdot (68.1 - 25) \cdot 1000$$

$$Q_{kill} := \frac{Q_{kJ} \cdot 1000}{a} = 7.806 \times 10^4\ \frac{J}{g \cdot s}$$

Given

$$\frac{d}{dt} m(t) = F_i \cdot \omega_1 - \alpha \frac{E(t)}{Q_{kill}} \cdot 10^{-12}$$

$$m(0) = F_i \cdot \omega_1$$

$$\frac{d}{dt} m_r(t) = 0 - F_o \cdot \omega_r + \alpha \frac{E(t)}{Q_{kill}} \cdot 10^{-12}$$

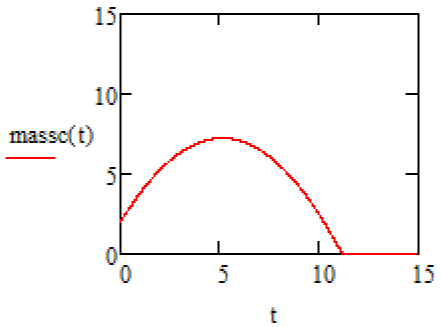
$$m_r(0) = 0$$

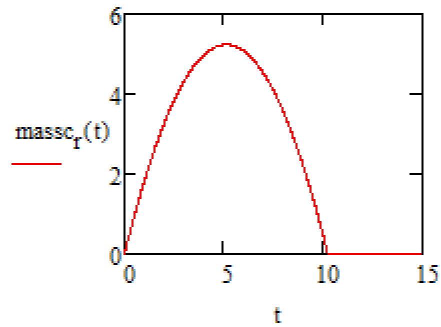
$$\frac{d}{dt} E(t) = S - Q - \left(Q_{kill} \cdot \beta \cdot m_r(t)\right) - .1 \cdot S$$

$$E(0) = 0$$

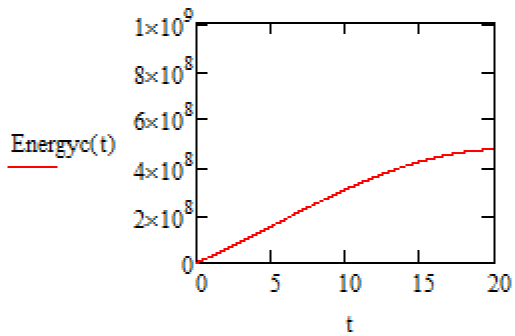
$$\begin{pmatrix} mass \\ Energy \\ mass_r \end{pmatrix} := \mathrm{Odesolve}\left[\begin{pmatrix} m \\ E \\ m_r \end{pmatrix}, t, 100\right]$$

$massc(t) := if(mass(t) < 0, 0, mass(t))$ $\qquad$ $massc_r(t) := if(-mass_r(t) < 0, 0, -mass_r(t))$





$Energyc(t) := if(Energy(t) < 0, 0, Energy(t))$



$$fivelog(t) := massc(t) - \frac{F_i \cdot \omega_1}{10^5}$$

$t_{guess} := 11$

$ans := root(fivelog(t_{guess}), t_{guess}) = 11.165$

$Energy(ans) = 3.373 \times 10^8$

It will take 11.165 seconds for a 5log reduction of E. coli to be achieved.

ANALYSIS AND SUMMARY

Reasonability of Output

Using the dimensions inputted above, both the mass of live cells and the mass of ruptured cells are reasonable. There is an initial increase in the mass of live cells in the system, this can be explained by calculated Q_{kill} value. Until enough energy has built up in the system, cells will not be ruptured immediately upon entering the system. Once steady state has been reached the model states that there are no live cells in the system. This means that E. coli are being ruptured the instant they cross the system boundary. The mass of ruptured E. coli in the system starts at 0, which correlates with the assumption that no inactive E. coli enter the system. There is an initial buildup of ruptured cells in the system that corresponds with the increase in the amount of live cells in the system. Once a steady state has been reached the model says there are no ruptured cells in the system. This makes sense with the model as it was defined, but does not make logical sense because if both the live and dead cells have a mass of 0 then there are no cells in the system. If there are no cells in the system this model is moot. The energy in the system increases consistently until steady state is reached. However, the value of energy required to achieve a 5log reduction is very high. This could mean that the theory of the transfer of mechanical energy from the soundwaves having a bactericidal effect is not completely accurate and that other factors are needed to achieve a 5log reduction using a reasonable amount of energy.

Benefits

An important feature of this model is the low temperature increase from the input temperature to the output temperature. Another benefit of this model is how simply it can be manipulated. The dimensions of the system are easily changed. Based on these changes a user can see if their system design will work without having to spend money to perform experiments.

Limitations

The final model is not accurate at low sound energies. At low sound energies, ultrasound can be used to stimulate bacterial growth. The increase in cell growth at low sound energies is thought to be caused by increased membrane permeability, which allows for higher nutrient transfer (Cullen, 2012).

A major limitation of this model occurs at long time periods. After steady state is held for a certain period of time the energy in the system approaches 0. This limitation occurs because the energy equation is dependent on the mass of cells in the system and the model states that the mass of cells eventually reaches 0.

Another limitation of the model is the proportionality constants. These constants were obtained using a guess and check method and it is not known if the final guess is the physical proportionality constant, or just one that happened to work. A small change in the proportionality constant can have a major effect on the model.

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Cullen, P. J., Tiwari, B. K., & Valdramidis, V. P. (2012). *Novel thermal and non-thermal technologies for fluid foods*. London: Academic.

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Rao, M. A., Rizvi, S. S., & Datta, A. K. (2005). *Engineering properties of foods*. Boca Raton: Taylor & Francis.

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APPENDIX

Each iteration is followed by the computational model created to check the validity of the items changed in that iteration.

Iteration 1

Assumptions:

-The apple cider and E. coli is a homogenous mixture

-Fully developed one dimensional flow

-E. coli is the only pathogen in the apple cider

-The mass of an active E. coli bacteria is equal to that on an inactive one

-The E. coli are not reproducing inside of the system

-No inactive E. coli enter the system

-Ultrasonic waves behave as normal mechanical waves

-The apple cider and E. coli are entering and leaving the system at a constant rate

-There is no height change between the inlet and outlet

-Kinetic energy of the apple cider and the cells is negligible

-Cell death occurs in a first order manner, where the reaction constant, k, is proportional to the sound energy, S, needed to rupture E. Coli

$$live\ cells \xrightarrow{k} dead\ cells$$

Mass Balance:

$$\frac{dm_l(t)}{dt} = F_i * \omega_l - \alpha S m_l(t)$$

Energy Balance:

$$\frac{dE(t)}{dt} = S$$

Values for sound energy, flow rate, mass percent, and alpha were assumed. A 5log reduction was achieved at 0.1 seconds. The sound energy at this point was 20 J/s. The energy in the system was constantly increasing, which is not logical because if energy is not dissipated the system eventually will build up too much energy and could potentially explode.

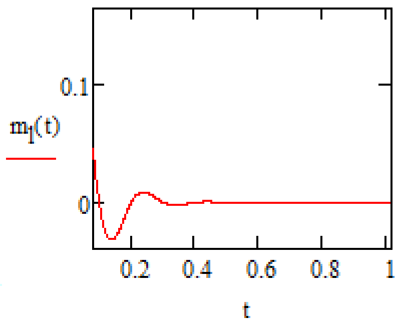
$S := 200 \ \frac{J}{sec} \qquad F_i := 20 \ \frac{g}{sec} \qquad \omega_1 := .02 \qquad \alpha := 10 \ \frac{1}{J}$

Given

$$\frac{d}{dt} m_1(t) = F_i \cdot \omega_1 - \alpha \cdot S \cdot m_1(t)$$

$$m_1(0) = F_i \cdot \omega_1$$

$$m_1 := \text{Odesolve}(t, 100)$$



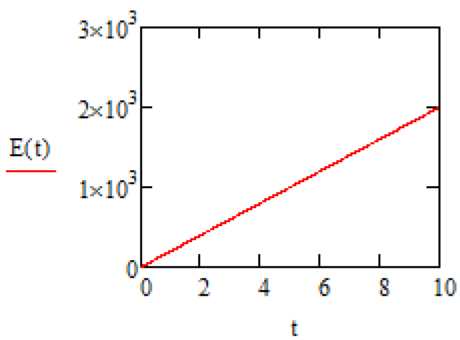
Cell mass goes negative, which is not possible.

Given

$$\frac{d}{dt} E(t) = S$$

$$E(0) = 0$$

$$E := \text{Odesolve}(t, 100)$$



Not logical: Energy should not constantly increase. Innacurate because this is a simple model with made up parameters

$$\text{root5log}(t) := m_1(t) - \frac{F_i \cdot \omega_1}{10^5}$$

$$t_{guess} := .02$$

$$ans := \text{root}\left(\text{root5log}\left(t_{guess}\right), t_{guess}\right) = 0.1$$

$$E(.1) = 20$$

Sound energy at 5log reduction= 20 J/s

Iteration 2

Changes from Iteration 1:

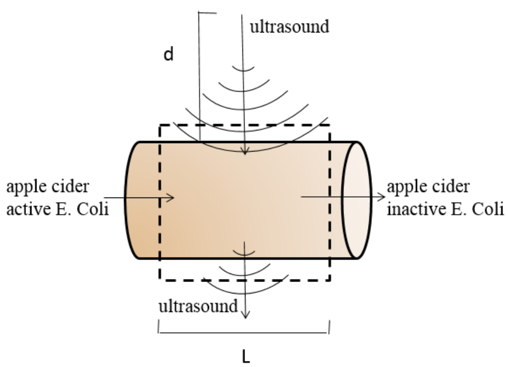
Sound energy is dependent on distance from the source, sound intensity, and the speed of sound. The speed of sound in apple cider was found to be 1543.7 m/s (Rao, 2005). The sound intensity of ultrasound is greater than that of non-ultrasonic sound waves. The distance to the system and the amount of time spent in the system also affect how much sound energy is applied to the apple cider.

Mass Balance:

$$\frac{dm_l(t)}{dt} = F_i * \omega_l - \alpha S m_l(t)$$

Energy Balance:

$$\frac{dE(t)}{dt} = \frac{v * SI * d * L}{t_{res}}$$



At the 5log reduction time the sound energy was found to be 1.54×10^7 J/s. The calculated value now represents the amount of energy actually needed to kill a cell. However, energy in the system is still constantly increasing.

Changing how S is defined: S is dependent on distance from source, intensity, and speed of sound

Speed of sound in cider as found in literature

$v := 1543.7 \frac{m}{s}$ $SI := 10^{4} \frac{watt}{m^{2}}$ $d := 1\ m$ $L := 2\ m$

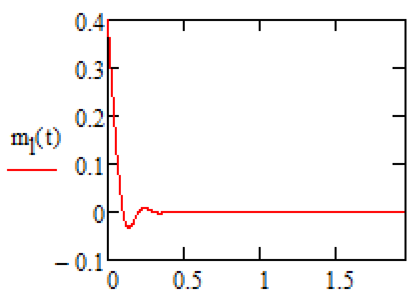
$S := \frac{v \cdot SI \cdot d}{t_{res}} = 1.544 \times 10^{8} \frac{J}{s}$ $t_{res} := \frac{L}{F_i}$

Given

$$\frac{d}{dt} m_1(t) = F_i \cdot \omega_1 - \alpha \cdot S \cdot m_1(t)$$

$$m_1(0) = F_i \cdot \omega_1$$

$$m_1 := \mathrm{Odesolve}(t, 100)$$



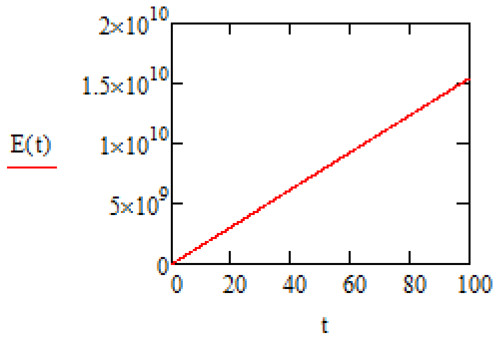
Cells all die, but mass still goes negative, which is not possible

Given

$$\frac{d}{dt}E(t) = S$$

$$E(0) = 0$$

$$E := \mathrm{Odesolve}(t, 100)$$



$$\mathrm{root5log}(t) := m_1(t) - \frac{F_i \cdot \omega_1}{10^5}$$

$$t_{guess} := .02$$

$$ans := \mathrm{root}\left(\mathrm{root5log}\left(t_{guess}\right), t_{guess}\right) = 0.1$$

$$E(.1) = 1.544 \times 10^7 \; \frac{J}{s}$$

Iteration 3

Changes from Iteration 2: Heat generation caused by the sound waves was added to the energy balance to account for a loss of energy from the system. The heat capacity of apple cider is 3.55 J/gC.

Mass Balance:

$$\frac{dm_l(t)}{dt} = F_i * \omega_l - \alpha S m_l(t)$$

Energy Balance:

$$\frac{dE(t)}{dt} = \frac{v * SI * d * L}{t_{res}} - F_i * Cp * (T_i - T_o)$$

Accounting for the energy lost to heating up the apple cider was not significant enough to decrease the energy build up in the system. Energy is still constantly increasing and never reaches a steady state.

Itteration 3: Changes only made to energy balance

The apple cider is heated up by the sound waves

$T_i := 25\ C \quad T_o := 30\ C \quad Cp := 3.55$

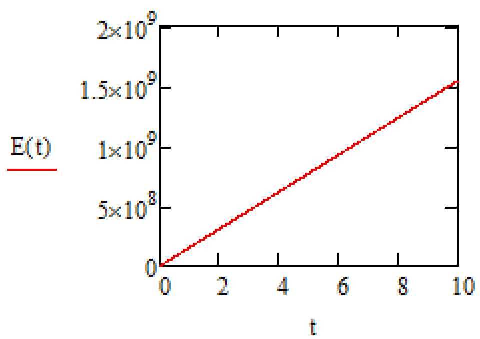
$Q := F_i \cdot Cp \cdot (T_i - T_o)$

Given

$$\frac{d}{dt}E(t) = S - Q$$

$$E(0) = 0$$

$E := \text{Odesolve}(t, 100)$



The model still does not make sense because energy is constantly increasing

<u>Iteration 4</u>

Changes from Iteration 3: Energy lost to rupturing E. coli cells was taken into account. Pasteurization time values were used to calculate how much energy was needed per gram to kill E. coli.

Mass Balance:

$$\frac{dm_l(t)}{dt} = F_i * \omega_l - \alpha S m_l(t)$$

$$\frac{dm_r(t)}{dt} = F_i * \omega_l - m_l(t)$$

Energy Balance:

$$\frac{dE(t)}{dt} = \frac{v * SI * d * L}{t_{res}} - F_i * Cp * (T_i - T_o) - [Q_{kill} * m_r(t)]$$

The energy required to rupture the weight of cells that is flowing through the system was not significant enough to stop the energy from constantly increasing.

Iteration 4:Changes only made to energy balance

Energy lost to rupturing cells

$Cp := 3.55$

$a := 14$

$$Q := \frac{.1}{a} \cdot Cp \cdot (68.1 - 25) \cdot 1000$$

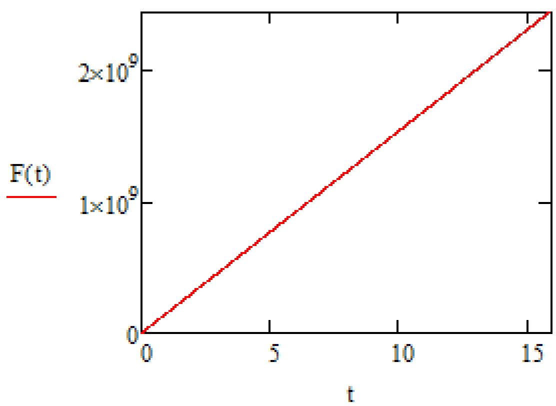
$$Q_{kill} := \frac{Q \cdot 1000}{a} = 7.806 \times 10^{4} \qquad \frac{\frac{k}{s}}{gram}$$

Given

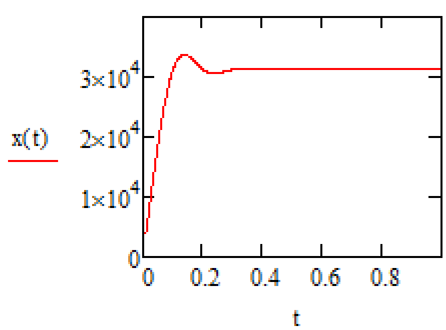
$$\frac{d}{dt}E(t) = S - Q - \left[Q_{kill} \cdot \left(F_i \cdot \omega_1 - m_1(t)\right)\right] \qquad m_r(t) := \left(F_i \cdot \omega_1 - m_1(t)\right)$$

$$E(0) = 0$$

$$F := \text{Odesolve}(t, 30)$$



$$x(t) := \left[Q_{kill} \cdot \left(F_i \cdot \omega_1 - m_1(t)\right)\right]$$



<u>Iteration 5:</u>

Changes from Iteration 4: After a certain amount of sound energy is lost from the sound wave, the sound wave will no longer have enough energy to rupture cells anymore and the sound will exit the system in the form of sound waves with decreased energy. Previous iterations only accounted for energy of a sound wave and how it is being dissipated, so the equation from previous iterations can be used to calculate how energy the sound waves have at a certain point. The value at which the sound waves no longer have enough energy to rupture cells was set to 10% of the original energy of the sound waves.

Mass Balance:

$$\frac{dm_l(t)}{dt} = F_i * \omega_l - \alpha S m_l(t)$$

$$\frac{dm_r(t)}{dt} = F_i * \omega_l - m_l(t)$$

Energy Balance:

$$\frac{dE(t)}{dt} = \frac{v * SI * d * L}{t_{res}} - F_i * Cp * (T_i - T_o) - [Q_{kill} * m_r(t)] - .1 * S(t)$$

Where

$$\frac{dS(t)}{dt} = \frac{v * SI * d * L}{t_{res}} - F_i * Cp * (T_i - T_o) - [Q_{kill} * m_r(t)]$$

These changes greatly improved the energy balance. The energy in the system no longer increases toward infinity, which creates a much better representation of how the energy from the sound wave is being used and dissipated.

Iteration 5: Changes only made to energy balance

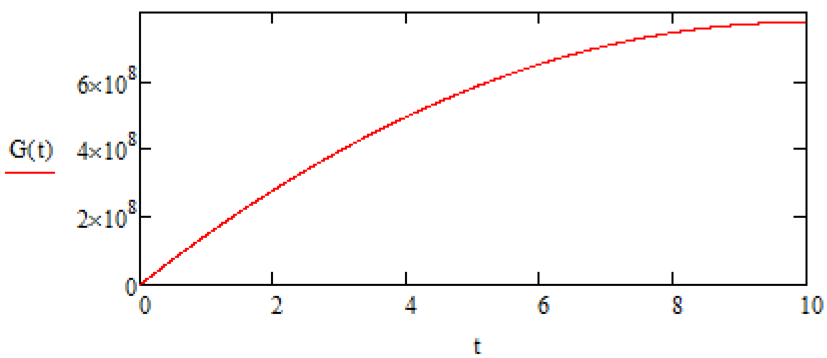
After a certain amount of energy is lost from the sound wave the soud wave won't be able to rupture cells anymore and the sound will exit the system in the form of sound waves with decreased energy.

Given

$$\frac{d}{dt}G(t) = S - Q - \left[Q_{kill}\cdot\left(F_i\cdot\omega_1 - m_1(t)\right)\right] - .1\,F(t)$$

$$G(0) = 0$$

$$G := \text{Odesolve}(t, 10)$$



$$G(.1) = 1.536 \times 10^{7} \quad \frac{J}{s}$$

Iteration 6

Changes from Iteration 5: The proportionality constant for the mass balance, α, was altered to create a more logical 5log reduction time. In order to like the mass and energy balance, E(t) was added to the mass balance. S(t) in the energy balance was changed to be solely equal to the initial value of sound energy, S. This change was made because S(t) represented how the energy in the sound wave was changing with respect to time, not how much sound energy was present initially.

Mass Balance:

$$\frac{dm_l(t)}{dt} = F_i * \omega_l - 10^{-6} S m_l(t) H(t)$$

$$\frac{dm_r(t)}{dt} = F_i * \omega_l - m_l(t)$$

Energy Balance:

$$\frac{dE(t)}{dt} = \frac{v * SI * d * L}{t_{res}} - F_i * Cp * (T_i - T_o) - [Q_{kill} * m_r(t)] - .1 * S$$

While a more realistic 5log time was reached, the mass balance now oscillates. The oscillations cause the cell mass to increase so that a 5log reduction is no longer met. Oscillations are most likely caused by the way the energy balance was added into the mass balance.

Iteration 6

Changes alpha to be much smaller to create a more logical 5log time. Changed F(t) to S because using F(t) was incorrect. F(t) is the change in how much energy is in the sound wave.

Given

$$\frac{d}{dt}m(t) = 5 - .000001 \cdot S \cdot H(t)\, m(t) \qquad T_i := 25\ C \quad T_a := 100\ C \quad Cp := 3.55$$

$$m(0) = 5$$

$$Q = 1.093 \times 10^{3}$$

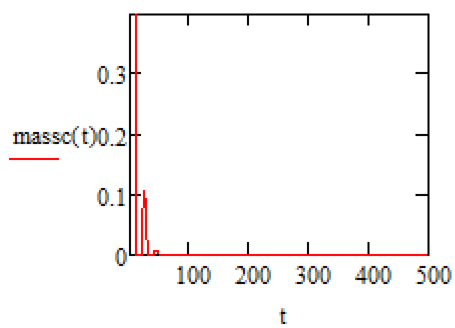
$$\frac{d}{dt}H(t) = S - Q - \left[Q_{kill} \cdot (5 - m(t))\right] - .1 \cdot S$$

$$Q_{kill} = 7.806 \times 10^{4}$$

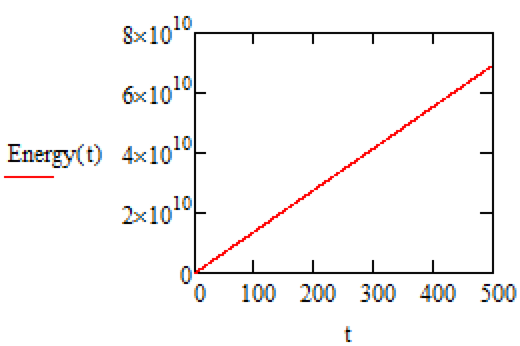
$$H(0) = 0$$

$$\begin{pmatrix} mass \\ Energy \end{pmatrix} := \text{Odesolve}\left[\begin{pmatrix} m \\ H \end{pmatrix}, t, 10000\right]$$

$$massc(t) := if(mass(t) < 0, 0, mass(t))$$



$$mass(10000) = 5.244 \times 10^{-11}$$



$$fivelog(t) := massc(t) - \frac{5}{10^{5}}$$

$$t_{guess} := .002$$

$$ans := root\left(fivelog\left(t_{guess}\right), t_{guess}\right) = 10$$

$$\frac{5}{10^{5}} = 5 \times 10^{-5}$$

<u>Iteration 7</u>

Changes from Iteration 6: Alpha was altered and m(t) was removed from the mass balance in an attempt to find the source of the oscillation

Mass Balance

$$\frac{dm_l(t)}{dt} = F_i * \omega_l - 10^{-6} * S * H(t)$$

$$\frac{dm_r(t)}{dt} = F_i * \omega_l - m_l(t)$$

Energy Balance

$$\frac{dE(t)}{dt} = \frac{v * SI * d * L}{t_{res}} - F_i * Cp * (T_i - T_o) - [Q_{kill} * m_r(t)] - .1 * S$$

The source of the oscillation was found to be m(t). However, after removing m(t) cell death appears to be linear. Since the method of cell death was assumed to be proportional to a first order reaction (see Iteration 1), a linear cell death does not make since.

Iteration 7

altered alpha and took out m(t) to try and find source of oscilation. m(t) is the source

Given

$$\frac{d}{dt}m(t) = 5 - .001\,H(t) \qquad T_i := 25\ C \quad T_a := 100\ C \quad Cp := 3.55$$

$$m(0) = 5$$

$$Q = 1.093 \times 10^{3}$$

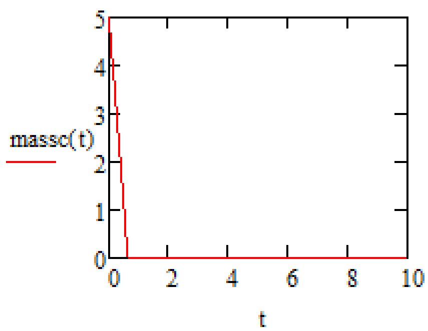
$$\frac{d}{dt}H(t) = S - Q - \left[Q_{kill}\cdot(5 - m(t))\right] - .1\cdot S$$

$$Q_{kill} = 7.806 \times 10^{4}$$

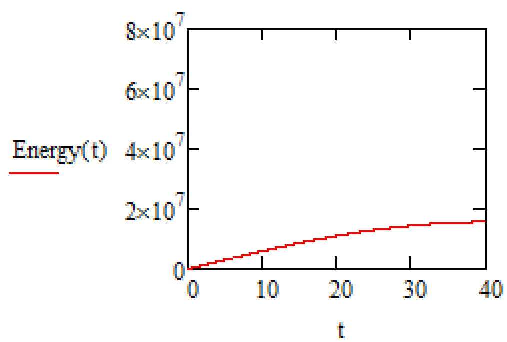
$$H(0) = 0$$

$$\begin{pmatrix} mass \\ Energy \end{pmatrix} := \text{Odesolve}\left[\begin{pmatrix} m \\ H \end{pmatrix}, t, 10000\right]$$

$$massc(t) := \text{if}(mass(t) < 0, 0, mass(t))$$



$$mass(.6) = 0.275$$



$$Energy(.636) = 3.891 \times 10^{5}$$

$$fivelog(t) := massc(t) - \frac{5}{10^{5}}$$

$$t_{guess} := .002$$

$$\frac{5}{10^{5}} = 5 \times 10^{-5}$$

$$ans := \text{root}(fivelog(t_{guess}), t_{guess}) = 0.636$$

Iteration 8

Changes from Iteration 7: Cell death is now dependent on the amount of energy in the system and the amount of energy needed to kill a single E. coli cell. This changes the assumption that cell death is proportional to a first order reaction. After the method of cell death was changed, a new alpha was defined to make the mass balance logical.

Mass Balance

$$\frac{dm_l(t)}{dt} = F_i * \omega_l - 10^9 \frac{H(t)}{Q_{kill}} * 10^{-12}$$

$$\frac{dm_r(t)}{dt} = F_i * \omega_l - m_l(t)$$

Energy Balance

$$\frac{dE(t)}{dt} = \frac{v * SI * d * L}{t_{res}} - F_i * Cp * (T_i - T_o) - [Q_{kill} * m_r(t)] - .1 * S$$

These changes improved the mass balance. The mass of live cells in the system is no longer oscillates and is now decreasing gin a linear fashion.

Iteration 8

Changed alpha to make mass balance logical. changed the assumption on how to model cell death. Now cell death is dependant on the amount of energy in the system related to how much energy it takes it kill a single cell. Changes assumption of first order cell death

Given

$$\frac{d}{dt}m(t) = 5 - 10^{9}\cdot\frac{H(t)}{Q_{kill}}\cdot 10^{-12}$$

$T_i := 25\ C$ $T_a := 100\ C$ $Cp := 3.55$

$$m(0) = 5$$

$$\frac{d}{dt}H(t) = S - Q - \left[Q_{kill}\cdot(5 - m(t))\right] - .1\cdot S$$

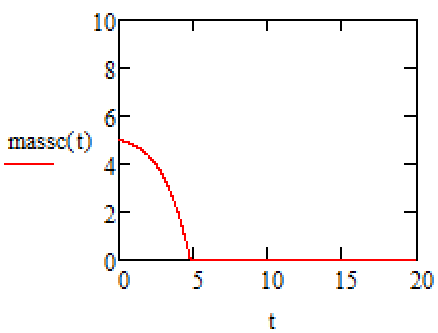
$Q = 1.093 \times 10^{3}$

$$H(0) = 0$$

$Q_{kill} = 7.806 \times 10^{4}$

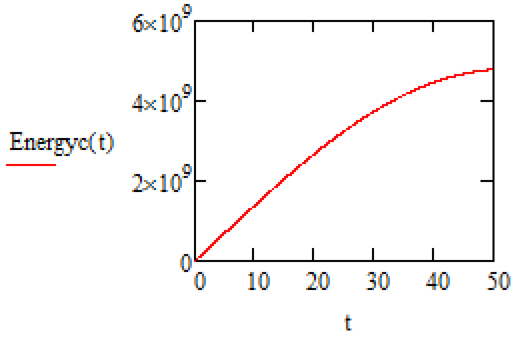
$$\begin{pmatrix} mass \\ Energy \end{pmatrix} := Odesolve\left[\begin{pmatrix} m \\ H \end{pmatrix}, t, 10000\right]$$

$massc(t) := if(mass(t) < 0, 0, mass(t))$



$massc(.01) = 4.998$

$\text{Energyc}(t) := \text{if}(\text{Energy}(t) < 0, 0, \text{Energy}(t))$



$\text{Energy}(5) = 6.984 \times 10^{8}$

$\text{fivelog}(t) := \text{massc}(t) - \frac{5}{10^{5}}$

$\frac{5}{10^{5}} = 5 \times 10^{-5}$

$t_{guess} := .002$

$\text{ans} := \text{root}\left(\text{fivelog}\left(t_{guess}\right), t_{guess}\right) = 27.646$

Iteration 9

Changes from Iteration 8: After further consideration, it was determined that the equation for mass of ruptured cells was not accurately represented in the model. An equation for the mass or ruptured cells was added. The energy balance was altered to be dependent on how many Joules were spend rupturing these cells. A new proportionality constant, β, was defined for the energy balance to fix an error with units.

Mass Balance

$$\frac{dm_l(t)}{dt} = F_i * \omega_l - \alpha * \frac{E(t)}{Q_{kill}} * 10^{-12}$$

$$\frac{dm_r(t)}{dt} = -F_o * \omega_r + \alpha * \frac{E(t)}{Q_{kill}} * 10^{-12}$$

Energy Balance

$$\frac{dE(t)}{dt} = \frac{v * SI * d * L}{t_{res}} - F_i * Cp * (T_i - T_o) - [Q_{kill} * \beta * m_r(t)] - .1 * S$$

These changes allowed the energy balance to reach steady state at an earlier time point. This model is more accurate than previous versions because the previous iterations had the energy of cell death accounted for by the number of live cells that had entered the system instead of the number of cells in the system that had ruptured.

Iteration 9

added an equation for mass of ruptured cells and made energy dependant on how many Joules were spent rupturing these cells. defined a new constant β that is the proportionality constant for energy.

Given

$$\frac{d}{dt}m(t) = 5 - 10^9 \cdot \frac{H(t)}{Q_{kill}} \cdot 10^{-12}$$

$T_i := 25\ C \quad T_e := 100\ C \quad Cp := 3.55$

$$m(0) = 5$$

$$\frac{d}{dt}m_r(t) = 0 - 5 + 10^9 \cdot \frac{H(t)}{Q_{kill}} \cdot 10^{-12}$$

$Q = 1.093 \times 10^3$

$$m_r(0) = 0$$

$Q_{kill} = 7.806 \times 10^4$

$$\frac{d}{dt}H(t) = S - Q - \left(Q_{kill} \cdot 10\, m_r(t)\right) - .1 \cdot S$$

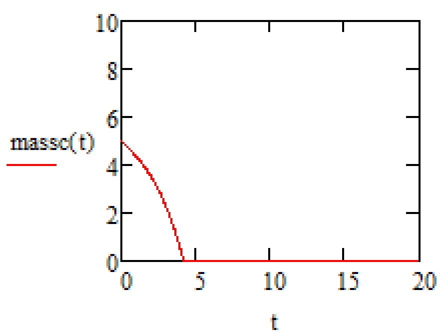
$\beta := 10$

$$H(0) = 0$$

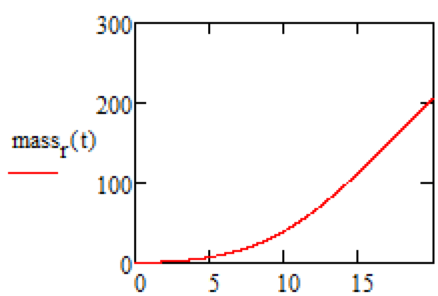
$\alpha := 10^9$

$$\begin{pmatrix} mass \\ Energy \\ mass_r \end{pmatrix} := \text{Odesolve}\left[\begin{pmatrix} m \\ H \\ m_r \end{pmatrix}, t, 10000\right]$$

$$massc(t) := \text{if}(mass(t) < 0, 0, mass(t))$$



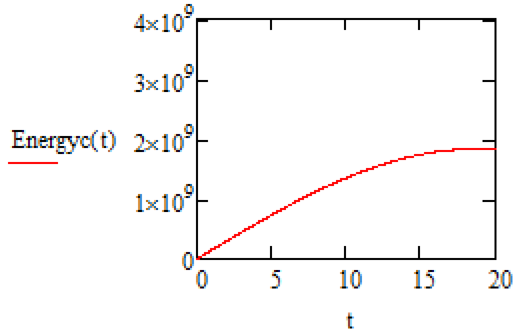
$massc(.01) = 4.994$



$mass_r(.01) = 6.096 \times 10^{-3}$

Mass of ruptured cells is equal to cells in-current grams of live cells until live cell equilibrium is reached.

$\text{Energyc}(t) := \text{if}(\text{Energy}(t) < 0, 0, \text{Energy}(t))$



$\frac{5}{10^5} = 5 \times 10^{-5}$

$\text{fivelog}(t) := \text{massc}(t) - \frac{5}{10^5}$

$t_{guess} := 5$

$\text{Energyc}(5) = 7.322 \times 10^8$

$\text{ans} := \text{root}\left(\text{fivelog}\left(t_{guess}\right), t_{guess}\right) = 5.005$