

ABE 301

Deep Frying an Oreo Cookie

Final Project

Lauren Nicholson
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IV. Background

a. Deep Frying

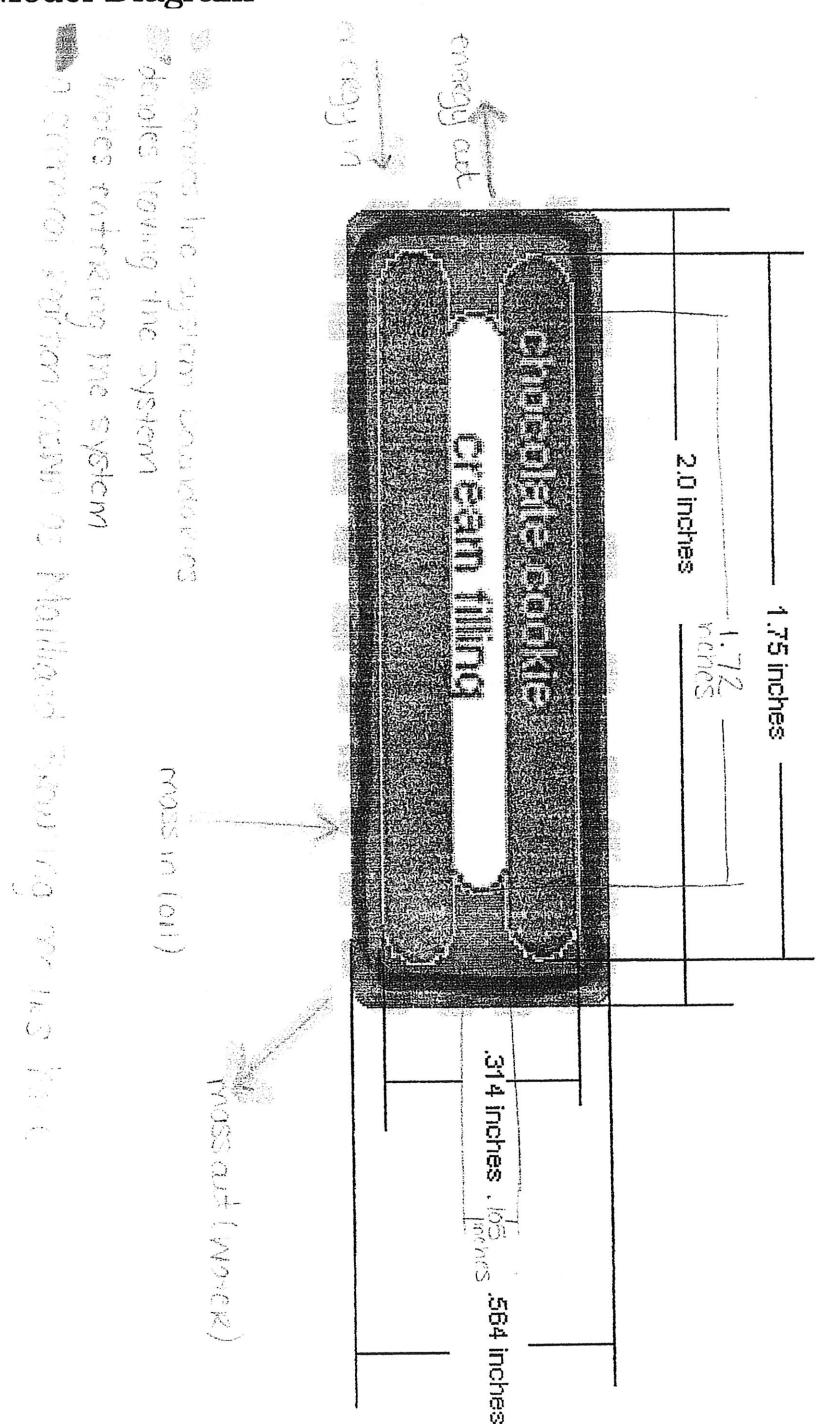
Deep frying is a form of cooking in which the food to be cooked is completely submerged in hot oil. Many foods are deep fried. The most common of these foods include French fries, chicken, onion rings, and fish. Although these foods are commonly deep fried the popularity of deep frying other foods has increased over the years. Today, many other foods are fried as well. Fried ice cream, fried dough, and fried candy are all staples at county fairs across the United States. Frying Oreo cookies has also become tremendously popular.

The fact that fried foods are completely submerged in hot oil to be cooked gives them a reputation for being very greasy. This is not true when fried foods are cooked properly. The minimum temperature oil should be heated to when used for deep frying is 325°F. This high temperature along with the fact that the food is completely surrounded by the oil means that fried foods have fairly short cook times, usually no longer than six minutes. It is common for a fried food to be coated in some sort of batter before frying.

b. Oreo Cookie

Oreo cookies are a wide known cookie in the United States. These cookies are a sandwich cookie composed of a cream filling between two crunchy chocolate cookies. It is known as "Milk's Favorite Cookie" in the United States by its manufacturer, Nabisco. It is not surprising that this favorite cookie is very commonly dipped in batter and deep fried.

Model Diagram



V. Model Diagram

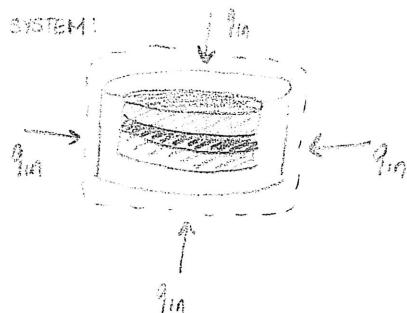
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VI. Final Model Derivation

a. Temperature

Final Temperature Iteration

SYSTEM:



PURPOSE: THE PURPOSE OF THIS ITERATION
IS TO MODEL THE TEMPERATURE
OF THE SYSTEM WITH RESPECT
TO X, R, + T TAKING THE
CHANGE IN COMPOSITION INTO
ACCOUNT.

Assumptions:

- ① Geometry is a cylinder
- ② 3 compositions - batter, cookie, cream
- ③ unsteady - static
- ④ heating in $x + r$ directions
- ⑤ cooling through conduction

Model Derivation:

BOTH COMPOSITION HAS ITS OWN K-VALUE

$$\text{BATTER} = K_b \quad \text{COOKIE} = K_c \quad \text{CREAM} = K_c$$

$$\rho = \rho_b = \rho_c$$

$$\rho_b C_p \frac{\partial T}{\partial t} + q_{in} - q_{out} + q_{x,batter} + q_{r,batter} - q_{x,cookie} - q_{r,cookie} = \rho C_p \frac{\partial T}{\partial t}$$

like iteration #3 the
variables to

$$\frac{d}{dr} \left(r \frac{dT}{dr} \right) + K \frac{dT}{dx^2} = \rho C_p \frac{dT}{dt}$$

this equation becomes

$$\frac{K_b}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) \Big|_{r=0}^{r=.25} + K_c \frac{dT}{dx^2} \Big|_{x=.125}^{x=.1092} = \rho C_p \frac{dT}{dt}$$

This is for the batter phase, so
 $K = K_b$. Also, the batter only goes
from X -position of .282 cm to
.1092 cm at R -position of 0 cm to
.125 cm.

next will do the cookie phase

$$\frac{K}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) + K \frac{d^2T}{dx^2} = \rho C_p \frac{dT}{dt}$$

for the cookie composition X Ranges

from .1092 cm to .0504 cm \rightarrow r Ranges

from .125 cm to 1 cm

so we get

$$\frac{K_c}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) \Big|_{r=.125}^{r=1} + K_c \frac{d^2T}{dx^2} \Big|_{x=.1092}^{x=.0504} = \rho C_p \frac{dT}{dt}$$

the last phase is the cream phase

$$\frac{K}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) + K \frac{d^2T}{dx^2} = \rho C_p \frac{dT}{dt}$$

X-Range: .0504 cm \rightarrow 0 cm

R-Range: .14 cm \rightarrow 1 cm

$$\frac{K_c}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) \Big|_{r=.14}^{r=1} + K_c \frac{d^2T}{dx^2} \Big|_{x=.0504}^{x=0} = \rho C_p \frac{dT}{dt}$$

combining these equations we get a piecewise system
of equations for the temperature profile

$$\rho C_p \frac{dT}{dt} = \begin{cases} \frac{K_b}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) + K_b \frac{d^2T}{dx^2}, & 0 \leq r \leq .125 \quad .1092 \leq x \leq .282 \\ \frac{K_c}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) + K_c \frac{d^2T}{dx^2}, & .125 < r \leq 1 \quad .0504 \leq x \leq .1092 \\ \frac{K_c}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) + K_c \frac{d^2T}{dx^2}, & .14 \leq r \leq 1 \quad 0 \leq x \leq .0504 \end{cases}$$

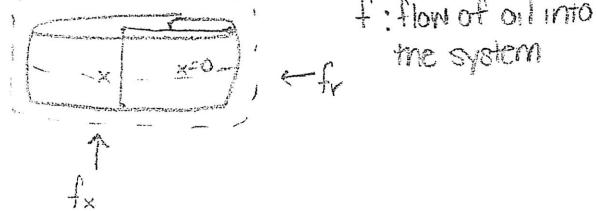


Final Temperature Model Equation

b. Oil Diffusion

FINAL OIL DIFFUSION ITERATION

SYSTEM:



f : flow of oil into the system

PURPOSE: To profile oil absorption into the system with respect to time, axial, + radial positions

ASSUMPTIONS:

- ① Geometry is a cylinder
- ② oil is entering from x & r -directions
- ③ oil diffusion is small, so it diffuses into the batter only

MODEL DERIVATION:

$$\text{in} - \text{out} = \text{acc}$$

$$f \cdot A|x_x - f \cdot A|x_{x+\Delta x} + f \cdot A|r - f \cdot A|r_{\Delta r} = \frac{d}{dt}(C \cdot V) \quad \text{from iteration 2. see appendix.}$$

$$f(2\pi r \Delta r)|_x - f(2\pi r \Delta r)|_{x+\Delta x} - f(2\pi r \Delta x)|_r + f(2\pi r \Delta x)|_{r+\Delta r} = \frac{d}{dt}(C[2\pi r \Delta x \Delta r])$$

$\div 2\pi r \Delta x \Delta r$

$$\frac{f_x - f_{x+\Delta x}}{\Delta x} + \frac{f_r|_r - f_r|r_{\Delta r}}{r \Delta r} = \frac{dC}{dt}$$

$$-\frac{\Delta f}{\Delta x} + -\frac{\Delta(f_r)}{r \Delta r} = \frac{dc}{dt} \quad \text{taking lim } \Delta x \rightarrow 0$$

$$-\frac{df}{dx} + -\frac{d(f_r)}{r dr} = \frac{dc}{dt} \quad \text{subbing in } f \text{ (known from iteration 1)}$$

$$\beta \frac{dc}{dx} + \frac{c}{r} \frac{d}{dr} \left(r \frac{dc}{dr} \right) = \frac{dc}{dt} \quad \text{where } \beta = \text{diffusivity constant}$$

$c = \text{concentration of oil}$

This is the final equation
for oil diffusion into the system

Finite difference for oil diffusion

$$C_{X,t}^P \left[1 - \frac{2\beta A t}{\Delta V^2} + \frac{\beta A t}{\Delta V} - \frac{2\beta A t}{\Delta X^2} \right] + \beta A t \left[\frac{C_{X,t+1}^P + C_{X,t-1}^P}{\Delta V^2} - \frac{C_{X,t+1}^P + C_{X,t-1}^P}{\Delta X^2} \right] = C_{X,t}^{P+1}$$

use these constant for easy solving

c. Maillard Browning

MAILLARD BROWNING ITERATION 3 (FINAL)

System: Same as Iteration 1

Assumptions:

- ① A chemical reaction is taking place
- ② Reaction is dependent on temperature of the system.

Purpose: The purpose of this iteration is to model the change in concentration with time with the reaction being dependent on temperature

Model Derivation:

from Iteration 2

$$\frac{dC}{dt} = -2C$$

Gibbs free energy tells us

$$\Delta G = \Delta H - T\Delta S \quad \text{where } \Delta H = \text{change in enthalpy}$$

T = temperature

$\Delta S = \text{change in entropy}$

$\Delta G = \text{change in Gibbs free energy}$

another way to write the above equation is

$$\Delta G = -RT\ln z \quad \text{where } R \text{ is the universal gas constant}$$

equating the two equations

$$\Delta H - T\Delta S = -RT\ln z$$

$$\frac{\Delta H - T\Delta S}{-RT} = R\ln z \quad \text{becomes} \rightarrow \frac{-\Delta H}{RT} + \frac{\Delta S}{R} = \ln z$$

for simplicity, we will assume change in entropy is 0

$$\frac{-\Delta H}{RT} = \ln z \quad \text{take derivative of both sides with respect to T}$$

$$\frac{d}{dT}\left(-\frac{\Delta H}{RT}\right) = \frac{d}{dT}(\ln z) \rightarrow \frac{\Delta H}{R T^2} = \frac{d}{dT}(\ln z)$$

$$\frac{d(\ln z)}{dT} T^2 = \frac{\Delta H}{R} \quad \text{rearranging we get} \quad \frac{d(\ln z)}{dT} = \frac{-\Delta H}{R}$$

Solving for z

$$d(\ln z) = -\frac{\Delta H}{R} dT$$

$$z = A e^{-\Delta H / RT} \text{ where } A \text{ is some constant}$$

$$\frac{dc}{dt} = -z \cdot C$$

plugging in value for z

$$\frac{dc}{dt} = -A e^{-\Delta H / RT} \cdot C \leftarrow \text{final equation for Maillard Browning Iteration 3 (Final)}$$

see figure **3** for a graphical model of this equation

VII. Computational Program

a. Temperature

The computational program for temperature was done in Microsoft Excel. The electronic copy is available for viewing. Here are some of the elements from that program.

Table 1

Enter Properties Here		
oil temp	191	<i>Temperature of oil to be used</i>
initial temp	0	<i>initial temperature of system</i>
batter	0.003	<i>alpha value for the batter</i>
cookie	0.002	<i>alpha value for the cookie</i>
cream	0.0035	<i>alpha value for the cream</i>

Table 2

Constant Values and May Not be Changed			
delta x	0.0564	delta x2	0.00318096
delta r	0.125	delta r2	0.015625
delta t	0.5		

The values within the matrices are color coded by value of alpha. Because each k value is different for each composition, the resulting values of alpha differ. The colors help to visualize what is going on in the system. The red color represents the batter, the blue the cookie, and the green the cream.

Obtaining Results

Plots of the system temperature have been provided next to each matrix for the time steps. Temperature values can either be read off the plots or from the matrices.

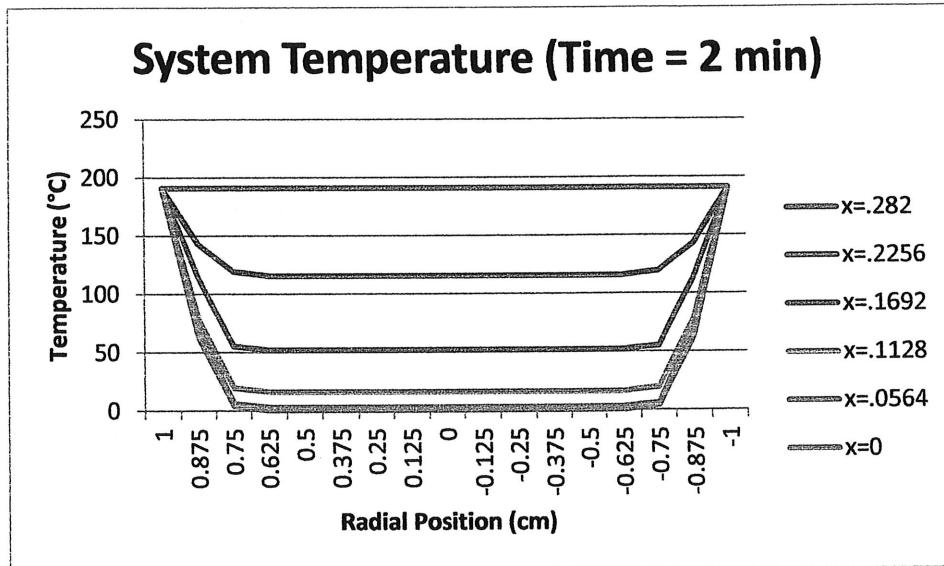
Example Results

Say we wanted to know the temperature at the very center of the system at 4 minutes. Scroll down to "System Temperature (Time=4min). Locate the x=0 curve. Then find where on that line radial position is also 0. Read straight over to find the temperature. In this case it is approximately 16°C.

Table 3

t=2	x											
r	0.282	0.2256	0.1692	0.1128	0.0564	0	0.0564	0.1128	0.1692	-0.2256	0.282	-
0	191	191	191	191	191	191	191	191	191	191	191	191
0.125	191	142.976	114.230	79.19	72.88	63.44	72.88	79.19	114.2	142.976	191	
0.25	191	119.379	55.2200	19.76	6.137	4.144	6.137	19.76	55.22	119.379	191	
0.375	191	115.495	52.1982	16.10	2.913	0.179	2.913	16.10	52.19	115.495	191	
0.5	191	115.343	52.0264	16.01	2.799	0.006	2.799	16.01	52.02	115.343	191	
0.625	191	115.326	52.0252	16.01	2.798	0	2.798	16.01	52.02	115.326	191	
0.75	191	115.326	52.0252	16.01	2.798	0	2.798	16.01	52.02	115.326	191	
0.875	191	115.326	52.0252	16.01	2.798	0	2.798	16.01	52.02	115.326	191	
1	191	115.326	52.0252	16.01	2.798	0	2.798	16.01	52.02	115.326	191	
0.875	191	115.326	52.0252	16.01	2.798	0	2.798	16.01	52.02	115.326	191	
0.75	191	115.326	52.0252	16.01	2.798	0	2.798	16.01	52.02	115.326	191	
0.625	191	115.326	52.0252	16.01	2.798	0	2.798	16.01	52.02	115.326	191	
0.5	191	115.343	52.0264	16.01	2.799	0.006	2.799	16.01	52.02	115.343	191	
0.375	191	115.495	52.1982	16.10	2.913	0.179	2.913	16.10	52.19	115.495	191	
0.25	191	119.379	55.2200	19.76	6.137	4.144	6.137	19.76	55.22	119.379	191	
0.125	191	142.9	114.230	79.19733	72.88639	63.44	72.88	79.19	114.2	142.976	191	
0	191	191	191	191	191	191	191	191	191	191	191	191

Figure 1



b. Oil Diffusion

The computational model for oil diffusion was also done on Microsoft Excel. The electronic file is available for viewing. Here are some elements from that file.

Table 4

Diffusivity Constant
beta 0.0001

Constant Values Not to be Changed			
delta t	0.5	delta r	0.2
delta x2	0.003181	delta r2	0.04
Oil Concentration (Outer)			1
Oil Concentration (Inner)			0

Obtaining Results

Oil diffusion into the system is so little that it is not easy to view on a graph. For this reason only one graph is shown. In order to determine the amount of oil diffused into the system, use the matrices. For example. At 1 minte the oil concentration at $x=.2256$ and $r = .2 \text{ cm}$ is $.00157 \text{ g/cm}^3$.

Figure 2

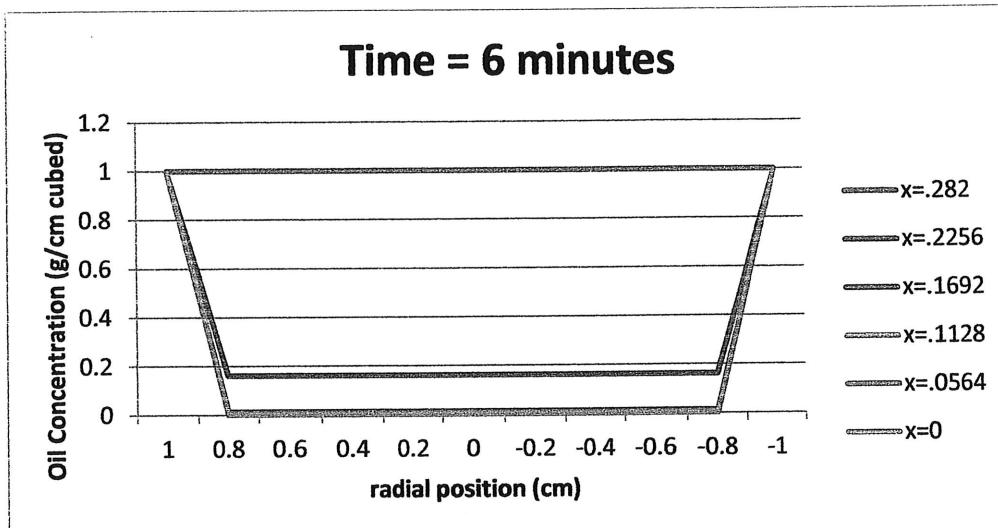


Table 5

c. Maillard Browning

The computation program for Maillard browning was done using MathCad. The electronic copy is available for viewing. Here is an image of the code.

```
constant := 1          deltaH := 1000t    temp := 46°      gas := 8.31t
```

These values can be changed depending on the temperature and material being reacted.

Giver

$$\frac{d}{dt} C464(t) = -\text{constant} \cdot e^{\left(\frac{-\text{deltaH}}{\text{gas temp}}\right)} \cdot C464(t) \quad C464(0) = .0434' \\ C464 := \text{Odesolve}(t, 60, 60) \quad t := 0, .1.. 60$$

Giver

$$\frac{d}{dt} C400(t) = -\text{constant} \cdot e^{\left(\frac{-\text{deltaH}}{\text{gas } 400}\right)} \cdot C400(t) \quad C400(0) = .0434' \\ C400 := \text{Odesolve}(t, 60, 60) \quad t := 0, .1.. 60$$

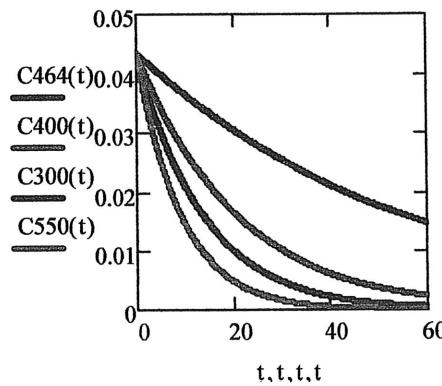
Giver

$$\frac{d}{dt} C300(t) = -\text{constant} \cdot e^{\left(\frac{-\text{deltaH}}{\text{gas } 300}\right)} \cdot C300(t) \quad C300(0) = .0434' \\ C300 := \text{Odesolve}(t, 60, 60) \quad t := 0, .1.. 60$$

Giver

$$\frac{d}{dt} C550(t) = -\text{constant} \cdot e^{\left(\frac{-\text{deltaH}}{\text{gas } 550}\right)} \cdot C550(t) \quad C550(0) = .0434' \\ C550 := \text{Odesolve}(t, 60, 60) \quad t := 0, .1.. 60$$

Figure 3



At any specific time or temperate the amount of sugars remaining can be determined. Just enter a value for time where it reads "enter time here" and the amount of sugars in g/cm³ will be calculated.

C300(enter time here) :=

C400(enter time here) :=

C464(enter time here) :=

C550(enter time here) :=

Example Calculations

$$C300(15) = 0.033$$

$$C400(15) = 0.021$$

The higher the value, the lighter in color the fried Oreo is.

$$C464(15) = 0.014$$

$$C550(15) = 8.07 \times 10^{-3}$$

VIII. Analysis/Summary

a. Analysis

Overall, the model appears to be very reasonable. Values for most of the constants were not known, but were estimated. Even with estimating these numbers the results obtained appeared similar to those that would occur in a realistic situation.

Some of the drawbacks of the model are that it is not as complex as it should be. This model was derived using my knowledge base which may not be as strong as others. There are probably assumptions and parameters of the system that I did not interpret correctly. Even though this may have been the case, the model obtained appeared accurate. Sample results obtained from the model are explained next.

b. Output Examples

i. Temperature Model

Using the "Final Temperature Computer Model" one can determine the temperature of any point within the system at any moment in time. For example, what is the temperature when the fried Oreo has been in the deep fryer for three minutes at $X=.2256$ and $r=.125$? One could use either the graph proved or the matrix.

This example will use the matrix. Locating these two values of x and r on the graph one comes to find that the temperature at that point is 139.2°C . This number is a believable number when comparing it to a real situation.

ii. Concentration Model

Using the "Final Concentration Computer Model" one can determine the concentration of oil that has diffused into the system. Oil, unlike temperature, does not change drastically. Because of this the numbers are too small to be plotted. The only option is through reading from the matrices. What is the concentration of oil after the Oreo has been in the fryer for two minutes at $x=.2256$ and $r=.2$? By reading the chart one comes to find the concentration of oil that has diffused into the system is .06 grams of oil per centimeter cubed. This is also a realistic number because when cooked properly, oil does not diffuse into the system very quickly.

iii. Maillard Browning Model

This model is different from the others because it relates a chemical reaction to a change in color; the less sugar that remain unreacted, the darker the color of the system. The batter starts off a light tan color and when all the sugars have been reacted, the color is a charred black. Example calculations and results are in the "Final Browning Computer Model." For example, at 300°C at 15 minutes $.033 \text{ g/cm}^3$ of sugars remain resulting in a light brown

color. This answer makes sense because 300°C is very low for a frying temperature.

c. Limitations

Unfortunately not every model is perfect. The concentration model was limited in the fact that it did not depend on temperature. In a real situation the hotter the oil, the slower the diffusion into the system is. The browning model is very arbitrary when it comes to the color scale. It was beyond my knowledge how to correlate color to sugars being reacted on a scientific basis.

d. Conclusion

This model has been derived from basic ideas and concepts to a developed model that can be used in multiple situations. The equations and computer models provided can answer questions regarding temperature, oil diffusion, and browning.

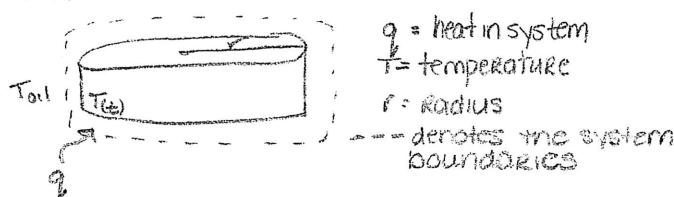
IX. Appendix

a. Temperature Iterations

i. Iteration 1

TEMPERATURE ITERATION 1

SYSTEM:



q = heat in system

T = temperature

r = radius

--- denotes the system boundaries

Purpose: The purpose of this iteration is to predict the temperature of the system as it changes with respect to time only.

ASSUMPTIONS:

① Geometry is a cylinder of the following dimensions

Radius (r) = 1 inch

height (h) = .564 inches

② steady-state

③ composition is the same throughout

④ heating of system through conduction

⑤ heating is non-directional

Model DERIVATION:

Through the conservation of energy it is known that $dU = dQ$ where U = internal energy & Q = heat entering the system

so we know that $q = U$

U is internal energy so we can assume $U = H$

it is known that $H = E_{\text{system}}$

so now we have $E_{\text{system}} = q$

it is known that $H = mC_p \Delta T$ substituting this in for q we get

$$q = E_{\text{system}} = mC_p \Delta T \quad \text{where } m = \text{mass}$$

C_p = heat capacity

ΔT = Temperature difference

We want to know how E_{system} changes with time, so we take the derivative with respect to time

$$\Rightarrow \frac{dE_{\text{system}}}{dt} = mC_p \frac{dT}{dt}$$

mass = density \cdot volume

$$m = \rho \cdot V$$

$$\frac{dE_{\text{SYSTEM}}}{dt} = \rho V C_p \frac{dT}{dt} \quad \text{where } \rho = \text{density}$$

$V = \text{system volume}$

It is known that $q = hA(T_{\text{out}} - T(t))$ where $h = \text{heat transfer coefficient}$
 $A = \text{area}$

subbing this in to the previous equation yields

$$\rho V C_p \frac{dT}{dt} = hA(T_{\text{out}} - T(t))$$

area (A) of a cylinder

$$\rightarrow 2\pi r^2 + 2\pi rh$$

Volume (V) of a cylinder

$$\rightarrow \pi r^2 h$$

$$\frac{dT}{dt} = \frac{h[2\pi r^2 + 2\pi rh][T_{\text{out}} - T(t)]}{\rho C_p [\pi r^2 h]}$$

This equation then becomes

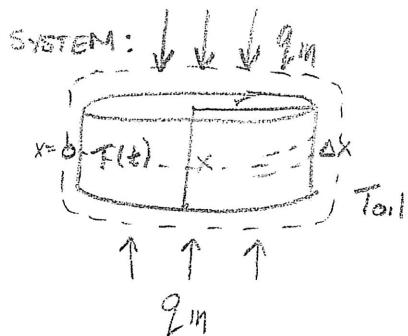
$$\frac{dT}{dt} = \frac{h}{\rho C_p} \left[\frac{(T_{\text{out}} - T(t))(2r + 2h)}{rh} \right] \quad \text{let } \frac{h}{\rho C_p} = \alpha \text{ for ease of computation purposes}$$

$$\frac{dT}{dt} = \alpha \left[\frac{(T_{\text{out}} - T(t))(2r + 2h)}{rh} \right] \quad \text{now solving this differential}$$

$$T(t) = T_{\text{out}} - \exp \left[\frac{-2r - 2h}{rh} \cdot t \right] \quad \leftarrow \text{final equation for TEMPERATURE ITERATION 1}$$

see figure **4** for the temperature profile associated with this model

ii. Iteration 2

TEMPERATURE ITERATION 2

q = heat entering system
 x = height

Purpose: The purpose of this iteration is to model the change in temperature of the system with respect to time & axial position (x)

ASSUMPTIONS:

- ① Geometry is a cylinder with dimensions
 $r = 1$ inch
 $x = .564$ inches
- ② Composition is the same throughout
- ③ heating through conduction
- ④ unsteady state
- ⑤ heating in axial direction

MODEL DERIVATION:

from Iteration = 1 we know

$$dq = \rho V C_p \frac{dT}{dt}$$

$$dq = q \cdot A|_x - q \cdot A|x+\Delta x = \rho V C_p \frac{dT}{dt}$$

$$q(\pi r^2)|_x - q(\pi r^2)|_{x+\Delta x} = \rho C_p (\pi r^2 \Delta x) \frac{dT}{dt}$$

$$\pi r^2 \Delta x$$

$$\frac{q|x - q|x+\Delta x}{\Delta x} = \rho C_p \frac{dT}{dt}$$

$$\text{taking } \lim_{\Delta x \rightarrow 0} \text{ yields } -\frac{dq}{dx} = \rho C_p \frac{dT}{dt}$$

$A|_x$ = cross-sectional area where q is entering
 $\rightarrow A = \pi r^2$

V = differential volume
 $\rightarrow V = \pi r^2 \Delta x$

$$q_{\text{conduction}} = -K \frac{dT}{dx}$$

subbing into above equation

$$-\frac{d}{dx} \left(-K \frac{dT}{dx} \right) = \rho C_p \frac{dT}{dt} \quad \text{assume } K \text{ is constant so it can be pulled out of the differential}$$

$$K \frac{d^2T}{dx^2} = \rho C_p \frac{dT}{dt}$$

$$\frac{K}{\rho C_p} \frac{d^2T}{dx^2} = \frac{dT}{dt} \quad \leftarrow \text{final equation for temperature iteration 2}$$

* This equation is difficult to solve so it is approximated using the finite difference method in Microsoft Excel. The derivation for this method is as follows:

$$\frac{dT}{dt} = \frac{T_x^{P+1} - T_x^P}{\Delta t} \quad \leftarrow \frac{d^2T}{dx^2} = \frac{T_{x+1}^P - 2T_x^P + T_{x-1}^P}{\Delta x^2} \quad \text{let } \frac{K}{\rho C_p} = \bar{g}$$

plugging these into $\frac{dT}{dt} = \bar{g} \frac{d^2T}{dx^2}$

$$\frac{T_x^{P+1} - T_x^P}{\Delta t} = \bar{g} \frac{T_{x+1}^P - 2T_x^P + T_{x-1}^P}{\Delta x^2}$$

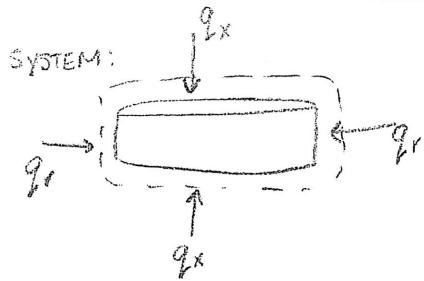
solving for T_x^{P+1} & simplifying yields

$$T_x^{P+1} = \frac{\bar{g} \Delta t}{\Delta x^2} [T_{x+1}^P + T_{x-1}^P] + \left[1 - \frac{2\bar{g} \Delta t}{\Delta x^2} \right] T_m^P$$

plugging these values into excel

see figure 5 for the temperature profile associated with this model.

iii. Iteration 3

TEMPERATURE ITERATION 3

ASSUMPTIONS:

only change is heating taking place in axial & radial directions

MODEL DERIVATION

$$\text{in-out} = \text{ACC}$$

same idea from Iteration 2

$$q_r A|_r - q_r A|_{r+\Delta r} + q_x A|_x - q_x A|_{x+\Delta x} = \frac{dE_{sys}}{dt}$$

$$\frac{dE_{sys}}{dt} = \rho V C_p \frac{dT}{dt}$$

$$q_r A|_r - q_r A|_{r+\Delta r} + q_x A|_x - q_x A|_{x+\Delta x} = \rho V C_p \frac{dT}{dt}$$

$$V = 2\pi r \Delta r \Delta x \quad A_r = 2\pi r \Delta x \quad A_x = 2\pi r \Delta r$$

$$q_r(2\pi r \Delta x)|_r - q_r(2\pi r \Delta x)|_{r+\Delta r} + q_x(2\pi r \Delta r)|_x - q_x(2\pi r \Delta r)|_{x+\Delta x} = \rho C_p (2\pi r \Delta r \Delta x) \frac{dT}{dt}$$

$$\frac{q_r(2\pi r \Delta x)|_r - q_r(2\pi r \Delta x)|_{r+\Delta r}}{2\pi r \Delta r \Delta x} + \frac{q_x(2\pi r \Delta r)|_x - q_x(2\pi r \Delta r)|_{x+\Delta x}}{2\pi r \Delta r \Delta x} = \rho C_p \frac{dT}{dt}$$

$$\frac{q_r \cdot r|_r - q_r \cdot r|_{r+\Delta r}}{r \Delta r} + \frac{q_x|_x - q_x|_{x+\Delta x}}{\Delta x} = \rho C_p \frac{dT}{dt}$$

$$\frac{-\Delta q \cdot r}{r \Delta r} + \frac{-\Delta q}{\Delta x} = \rho C_p \frac{dT}{dt}$$

taking $\lim_{r,x \rightarrow 0}$

$$-\frac{dq_r}{r dr} + -\frac{dq}{dx} = \rho C_p \frac{dT}{dt}$$

$$\text{we know } q_r = -K \frac{dT}{dr} + q_x = -K \frac{dT}{dx}$$

subbing in

$$\frac{K}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) + K \frac{d^2T}{dx^2} = \rho C_p \frac{dT}{dt} \quad \text{let } \frac{K}{\rho C_p} = \alpha$$

$$\frac{\alpha}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) + \alpha \frac{d^2T}{dx^2} = \frac{dT}{dt} \quad \leftarrow \text{final equation for temperature iteration 3}$$

in order to solve this equation the finite difference method must be used

first we must expand the differential

$$\frac{\alpha}{r} \left[r \cdot \frac{d^2T}{dr^2} + 1 \cdot \frac{dT}{dr} \right] + \alpha \frac{d^2T}{dx^2} = \frac{dT}{dt}$$

$$\alpha \frac{d^2T}{dr^2} + \frac{\alpha}{r} \frac{dT}{dr} + \alpha \frac{d^2T}{dx^2} = \frac{dT}{dt}$$

$$\frac{dT}{dt} = \frac{T_{x,r}^{p+1} - T_{x,r}^p}{\Delta t} \quad \frac{dT}{dr} = \frac{T_{x,r}^p - T_{x,r-1}^p}{\Delta r} \quad \frac{d^2T}{dr^2} = \frac{T_{x,r+1}^p - 2T_{x,r}^p + T_{x,r-1}^p}{\Delta r^2}$$

$$\frac{d^2T}{dx^2} = \frac{T_{x+1,r}^p - 2T_{x,r}^p + T_{x-1,r}^p}{\Delta x^2} \quad \begin{aligned} r &- \text{radial direction} \\ x &- \text{axial direction} \\ p &- \text{time step} \end{aligned}$$

plug these into the derived heat equation

$$\begin{aligned}
 & \frac{\partial}{\partial r} \left[\frac{T_{x,r+1}^P - T_{x,r-1}^P}{\Delta r^2} \right] + \frac{\partial}{\partial r} \left[\frac{T_{x,r}^P - T_{x,r-1}^P}{\Delta r} \right] + \alpha \left[\frac{T_{x,r+1,r}^P - 2T_{x,r}^P + T_{x,r-1,r}^P}{\Delta x^2} \right] = \frac{T_{x,r}^{P+1} - T_{x,r}^P}{\Delta t} \\
 & \frac{\partial \Delta t}{\partial r} \left(\frac{T_{x,r+1}^P + T_{x,r-1}^P}{\Delta r^2} \right) - \frac{2\alpha \Delta t}{\Delta r^2} \left(T_{x,r}^P \right) + \frac{\alpha \Delta t}{r \Delta r} \left(T_{x,r-1}^P \right) + \frac{\alpha \Delta t}{\Delta x^2} \left(T_{x,r+1,r}^P \right) + \frac{2\alpha \Delta t}{\Delta x^2} \left(T_{x,r-1,r}^P \right) + T_{x,r}^P = T_{x,r}^{P+1}
 \end{aligned}$$

$$T_{x,r}^P \left[1 - \frac{2\alpha \Delta t}{\Delta r^2} + \frac{\alpha \Delta t}{r \Delta r} - \frac{2\alpha \Delta t}{\Delta x^2} \right] + \alpha \Delta t \left[\frac{T_{x,r+1}^P + T_{x,r-1}^P}{\Delta r^2} - \frac{T_{x,r+1,r}^P + T_{x,r-1,r}^P}{\Delta x^2} \right] = T_{x,r}^{P+1}$$

use excel to solve for this

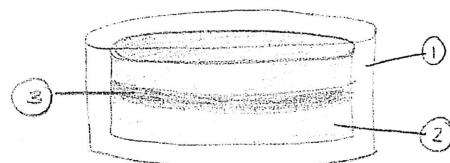
see figure 6 for a profile of this equation

iv. Iteration 4

TEMPERATURE ITERATION 4

Want to model temperature as a function of time + position
 → assume constant geometry is a cylinder

- temperature not dependent on position (steady state)
- system is in steady state (heat transfer = 0)

COMPOSITIONS

1. batter

2. coffee cookie

→ phase change does not occur at varying temperatures

3. cream filling

→ phase change (solid to liquid) occurs at 40°C (approximate)

The outer layer of the cookie is being heated via convection. The inner layers via conduction.

$$\text{Area of batter layer} \rightarrow 2\pi rh + 2\pi r^2 \\ = 2\pi(1 \text{ in})(.564) + 2\pi(1 \text{ in})^2 = 9.81 \text{ in}^2$$

 K_b = batter K_c = cookie K_f = cream

$$\text{Area of cookie} \rightarrow 2\pi rh + 2\pi r^2$$

$$= 2[\pi r^2 + 2\pi rh]$$

$$= 2[\pi(.8)^2] + 2\pi(.8)(.105) = 5.97 \text{ in}^2$$

$$\text{Area of cream} \rightarrow 2\pi rh + 2\pi r^2$$

$$= 2\pi(1.8)(.105) + 2\pi(1.8)^2 = 5.21 \text{ in}^2$$

assuming q is steady state (constant for all compositions)

$$\text{From } q = hA(T(t) - T_0)$$

also know $h = \frac{K}{\Delta r}$ in this case Δr is the distance between composition changes

$$q = \frac{K}{\Delta r} A (T(t) - T_0)$$

assuming q is constant through all compositions

$$q = \frac{K_b}{\Delta r_b} A_b (T_0 - T_{b(t)}) = \frac{K_c}{\Delta r_c} A_c (T_{b(t)} - T_{c(t)}) = \frac{K_f}{\Delta r_f} (T_{c(t)} - T_{f(t)})$$

$$q = \frac{\partial E_{sys}}{\partial t} = mC_p \frac{dT}{dt} \quad \text{wt } \frac{1}{mC_p} = \alpha$$

$$\frac{dT}{dt} = \frac{\alpha K_b}{\Delta r_b} A_b (T_0 - T_{b(t)}) = \frac{\alpha K_c}{\Delta r_c} A_c (T_{b(t)} - T_{c(t)}) = \frac{\alpha K_f}{\Delta r_f} (T_{c(t)} - T_{f(t)})$$

$$\frac{dT}{dt} = \frac{\alpha K_b}{\Delta r_b} A_b (T_0 - T_{b(t)})$$

$$\frac{dT}{T_0 - T_{b(t)}} = \frac{\alpha K_b}{\Delta r_b} A_b dt$$

$$\ln(T_0 - T_{b(t)}) = \frac{\alpha K_b}{\Delta r_b} C_b t$$

$$T_{b(t)} = T_0 - \exp\left(-\frac{\alpha K_b}{\Delta r_b} C_b t\right)$$

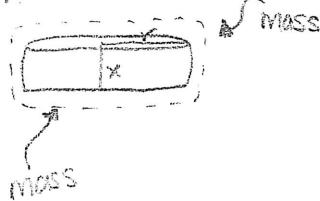
} THIS can be done for temperature & the other two compositions as well. Mathcad plots of the equations are on the following page.

b. Oil Diffusion

i. Iteration 1

OIL DIFFUSION ITERATION 1

SYSTEM:



Purpose: The purpose of this iteration is to profile the diffusion of oil into the system.

Assumptions:

- ① Oil diffusion is small so concentration of oil increases in the batter only (phase is consistent)
- ② Geometry is a cylinder with $r = 1 \text{ inch}$
- ③ Diffusion is in the x -direction only
- ④ Steady-state system

MODEL DERIVATION:

$$\text{in} - \text{out} = \text{acc}$$

$\text{acc} = \text{Rate of oil in} \cdot \text{Volume}$ let the Rate = J (flux)

$$\text{acc} = J \cdot V$$

$$\text{in} = \text{oil in} \cdot \text{area} \quad \text{out} = \text{oil out} \cdot \text{area} \quad \text{let oil} = g$$

$$J \cdot V = g \cdot A|_x - g \cdot A|_{x+\Delta x}$$

$$J \cdot \pi r^2 \Delta x = g(\pi r^2)|_x - g(\pi r^2)|_{x+\Delta x}$$

$$J = \frac{g(\pi r^2)|_x - g(\pi r^2)|_{x+\Delta x}}{\Delta x}$$

$$J = \frac{-\Delta g|_x}{\Delta x} \quad \text{take lim } x \rightarrow 0$$

$$J = -\frac{dg}{dx} \quad \text{We know this must occur at some rate, so an arbitrary rate constant must be utilized}$$

$$J = -\beta \frac{dg}{dx} \Rightarrow \frac{dg}{dx} = -\frac{\beta}{J} \quad \leftarrow \text{Final equation for oil diffusion iteration 1}$$

see figure 10 for a graphical representation of this model.

ii. Iteration 2

OIL DIFFUSION ITERATION 2

SYSTEM: same as iteration 1

ASSUMPTIONS:

- The only assumption changing from iteration 1 is that the system is not at steady state, diffusion changes with time

PURPOSE: The purpose of this iteration is to profile the concentration of oil diffusing into the system with respect to time + position

Model Derivation:

We can use the same idea from iteration 1

$$\text{in-out} = \text{acc}$$

$$\text{acc} = J \cdot A|_x - J \cdot A|_{x+\Delta x}$$

$$\text{in-out} = (g|_{t+\Delta t} - g|_t) \cdot V \quad \text{differential volume } V = A \cdot \Delta x$$

$$J \cdot A|_x - J \cdot A|_{x+\Delta x} = (g|_{t+\Delta t} - g|_t) \cdot A \cdot \Delta x$$

$$\frac{J \cdot A|_x - J \cdot A|_{x+\Delta x}}{A \cdot \Delta x} = g|_{t+\Delta t} - g|_t$$

this becomes

$$\frac{-\Delta J}{\Delta x} = \frac{\Delta g}{\Delta t} \quad \text{taking } \lim_{x,t \rightarrow 0}$$

$$-\frac{dJ}{dx} = \frac{dg}{dt} \quad \text{from Iteration 1 we know } J = -\beta \frac{dg}{dx}$$

subbing this in

$$-\frac{d}{dx} \left(-\beta \frac{dg}{dx} \right) = \frac{dg}{dt} \rightarrow \text{becomes } \beta \frac{d^2g}{dx^2} = \frac{dg}{dt} \quad \leftarrow \text{final equation for oil diffusion iteration 2}$$

- will use finite difference method to solve this equation

$$\frac{d\bar{g}}{dt} = \frac{\bar{g}_{x+1}^P - \bar{g}_x^P}{\Delta t} \quad \frac{d^2\bar{g}}{dx^2} = \frac{\bar{g}_{x+1}^P - 2\bar{g}_x^P + \bar{g}_{x-1}^P}{\Delta x^2}$$

$$\frac{\bar{g}_x^{P+1} - \bar{g}_x^P}{\Delta t} = \frac{\bar{g}_{x+1}^P - 2\bar{g}_x^P + \bar{g}_{x-1}^P}{\Delta x^2} \cdot \beta$$

$$\bar{g}_x^{P+1} = \Delta t \left(\frac{\bar{g}_{x+1}^P - 2\bar{g}_x^P + \bar{g}_{x-1}^P}{\Delta x^2} \right) \cdot \beta + \bar{g}_x^P$$

simplifying

$$\bar{g}_x^{P+1} = \Delta t \beta \left(\frac{\bar{g}_{x+1}^P + \bar{g}_{x-1}^P}{\Delta x^2} \right) + \left(1 - \frac{\beta \Delta t}{\Delta x^2} \right) \bar{g}_x^P$$

see figure for the graphical interpretation of
this model

c. Maillard Browning

i. Iteration 1

Maillard Browning ITERATION 1

SYSTEM :



The system is the baked Oreo cookie. The sugars within the batter are what are being reacted.

PURPOSE: The purpose of this iteration is to predict the concentration of sugar in the system with respect to time.

ASSUMPTIONS:

- ① A chemical reaction is taking place
- ② The reaction is a zero order reaction (rate does not depend on concentration)
- ③ The brown color of fried dough is directly proportional to the amount of sugar remaining in the system

MODEL DERIVATION:

We can say that the reaction occurs at some rate,

$$r = \frac{dc}{dt} = \text{change in concentration over time}$$

Since concentration is decreasing over time so we can say it is negative

$r = -\frac{dc}{dt}$ This is happening at a rate. This rate depends on the reactant. In our case we will call this Z .

$$-\frac{dc}{dt} = Z \rightarrow \frac{dc}{dt} = -Z$$

This says that the rate of reaction is not dependent on concentration of sugar in the system

Solving for C yields

$$C_0 \int_{t=0}^{t=t} dc = \int_{t=0}^{t=t} -Z dt$$

$$C|_{C_0}^C = -Zt|_{t=0}^{t=t} \rightarrow C = C_0 - Zt \leftarrow \text{final equation for MAILLARD BROWNING ITERATION 1}$$

See figure for a concentration vs. time plot of this model

ii. Iteration 2

MAILLARD BROWNING ITERATION 2

SYSTEM: same as iteration 1

ASSUMPTIONS:

- ① A chemical reaction is taking place
- ② The reaction is dependent on concentration
- ③ same assumption 3 as iteration 1

PURPOSE: The purpose of this iteration is to predict the amount of sugar (concentration) in the system with respect to time in a more accurate way than iteration 1

MODEL DERIVATION:

Know from Iteration 1

$$r = -\frac{dc}{dt}$$

$-\frac{dc}{dt} = z \cdot c$ where c is concentration
 z is Reaction constant \rightarrow Rate is dependent on concentration in the system

solving for C

$$\int_{C_0}^c \frac{dc}{c} = \int_{t_0}^t -z dt$$

$$\ln \frac{c}{C_0} = -zt$$

$$\ln C = -zt + \ln C_0$$

$$C = C_0 e^{-zt} \leftarrow \text{final equation for Maillard Browning Iteration 2}$$

see figure for a graphical model of this equation

d. Iteration Figures

