Exam 2

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ABE 30100

Problem A

Part 1

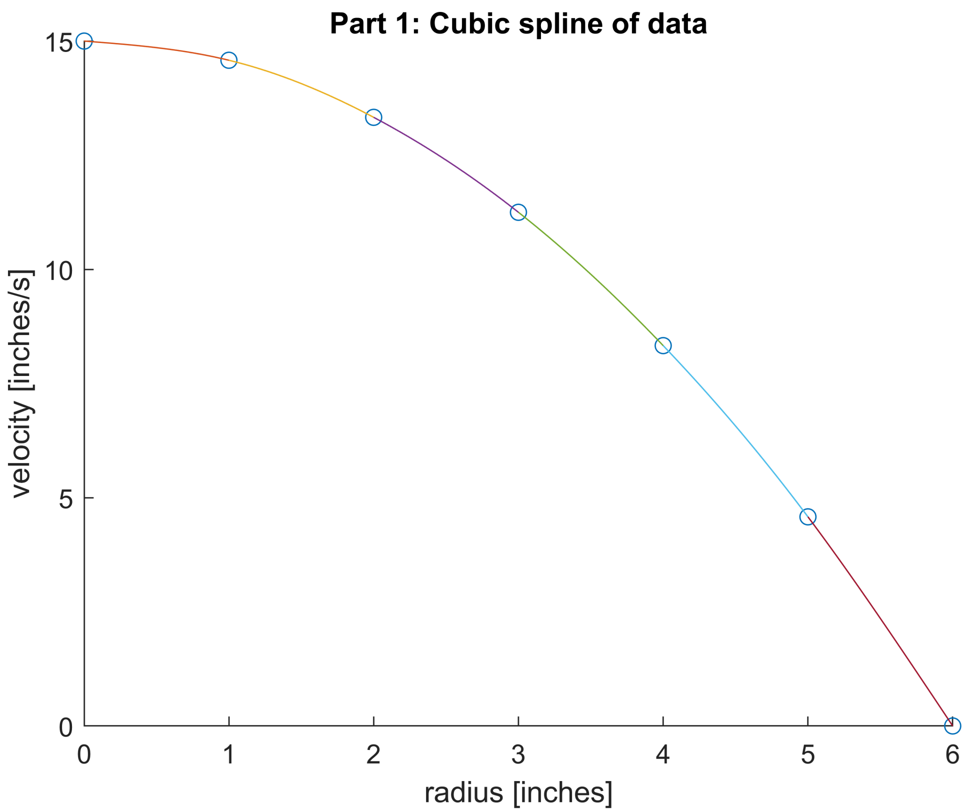
Create a cubic spline model for velocity as a funciton of radial position. Provide an appropriate plot.

```
% Input Data
r = [0, 1, 2, 3, 4, 5, 6]; % radius, independent variable, [inches]
v = [15, 14.58, 13.33, 11.25, 8.33, 4.58, 0]; % velocity, dependent variable, % [inches/s]

S = cubic_spline(r, v);
```

```
Valid from x = 0.0000 to 1.0000
```
f = $-0.17577\ x^3 - 0.24423\ x + 15.0$
```
Valid from x = 1.0000 to 2.0000
```
f = $0.048846\ (x - 1.0)^3 - 0.52731\ (x - 1.0)^2 - 0.77154\ x + 15.352$
```
Valid from x = 2.0000 to 3.0000
```
f = $16.689 - 0.38077\ (x - 2.0)^2 - 0.019615\ (x - 2.0)^3 - 1.6796\ x$
```
Valid from x = 3.0000 to 4.0000
```
f = $0.019615\ (x - 3.0)^3 - 0.43962\ (x - 3.0)^2 - 2.5\ x + 18.75$
```
Valid from x = 4.0000 to 5.0000
```
f = $21.612 - 0.38077\ (x - 4.0)^2 - 0.048846\ (x - 4.0)^3 - 3.3204\ x$
```
Valid from x = 5.0000 to 6.0000
```
f = $0.17577\ (x - 5.0)^3 - 0.52731\ (x - 5.0)^2 - 4.2285\ x + 25.722$

```
title('Part 1: Cubic spline of data')
xlabel('radius [inches]')
ylabel('velocity [inches/s]')
```



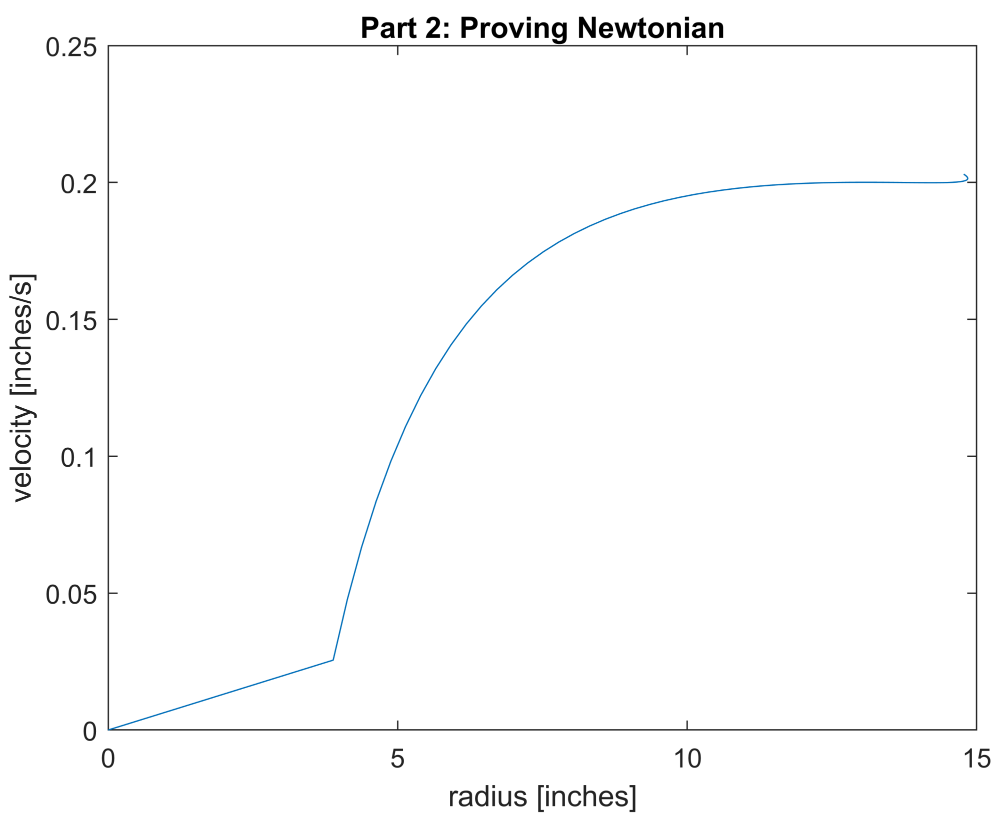
Part 2

Calculate the value of the fluid viscosity, mu, and prove that this is a Newtonian fluid.

```
r = 0:0.1:6; % radius, [inches]
mu = zeros(length(r),1); % viscosity, [lbm/in-s]
vr = zeros(length(r),1); % velocity, [inches/s]
delP = 40; % pressure, [lbf/in^2]
r_total = 6; % total radius of the pipe, [inches]
L = 120; % length, [inches]
i = 1; % counter

while r(i) < r_total
    if x < 1
        vr(i) = -0.17577 * r(i)^3 - 0.24423 * r(i) + 15;
        mu(i) = delP * (r_total ^ 2 - r(i) ^ 2) / (4 * vr(i) * L);
    elseif x < 2
        vr(i) = 0.048846 * (r(i) - 1) ^ 3 - 0.52731 * (r(i) - 1) ^ 2 - 0.77154 * r(i) + 15.352
        mu(i) = delP * (r_total ^ 2 - r(i) ^ 2) / (4 * vr(i) * L);
    elseif x < 3
        vr(i) = 16.689 - 0.38077 * (r(i) - 2) ^ 2 - 0.019615 * (r(i) - 2) ^ 3 - 1.6796 * r(i);
        mu(i) = delP * (r_total ^ 2 - r(i) ^ 2) / (4 * vr(i) * L);
    elseif x < 4
        vr(i) = 0.019615 * (r(i) - 3) ^ 3 - 0.43962 * (r(i) - 3) ^ 2 - 2.5 * r(i) + 18.75;
        mu(i) = delP * (r_total ^ 2 - r(i) ^ 2) / (4 * vr(i) * L);
    elseif x < 5
        vr(i) = 21.612 - 0.38077 * (r(i) - 4) ^ 2 - 0.048846 * (r(i) - 4) ^ 3 - 3.3204 * r(i);
```

```
        mu(i) = delP * (r_total ^ 2 - r(i) ^ 2) / (4 * vr(i) * L);
    else
        vr(i) = 0.17577 * (r(i) - 5) ^ 3 - 0.52731 * (r(i) - 5) ^ 2 - 4.2285 * r(i) + 25.722;
        mu(i) = delP * (r_total ^ 2 - r(i) ^ 2) / (4 * vr(i) * L);
    end
    i = i + 1; % adds to counter to continue while loop
end
figure
plot(vr,mu)
title('Part 2: Proving Newtonian')
xlabel('radius [inches]')
ylabel('velocity [inches/s]')
```



Part 3

Calculate the flowrate, Q.

```
Q = pi * delP * r_total ^ 4 / (8 * mean(mu) * L) % flowrate, [in^3/s]
```

```
Q = 968.3916
```

Problem B

```
V = 100; % volume, [L]
q = 5; % [L/h]
CA0 = 100; % [mol/L]
```

```
vm1 = 3.3; % [mol/L-h]
km1 = 0.05; % [mol/L-h]
vm2 = 5; % [mol/L-h]
km2 = 200; % [mol/L-h]

syms C
f = q * CA0 - q * C - (vm1 * C / (km1 + C) + vm2 * C / (km2 + C)) * V;

[x_root, i] = newton_raphson(f, CA0, 0.001);
x_root
```

```
x_root = 23.5893
```

```
V = 150; % volume, [L]

syms C
f = q * CA0 - q * C - (vm1 * C / (km1 + C) + vm2 * C / (km2 + C)) * V;

[x_root, i] = newton_raphson(f, CA0, 0.001);
x_root
```

```
x_root = -1.4477
```

Functions

Root-Finding Functions

```
function [x_root, i] = newton_raphson(f, x1, error_tol)
    C      = x1;
    zero   = double(subs(f));                         % sets zero to the value of the
                                                          % function at the given x
                                                          % point

    x_root = x1;                                      % renames input x value
    i      = 0;                                       % sets iteration counter to zero
    while abs(zero) > error_tol                       % checks to see if another
                                                          % iteration should be
                                                          % performed

        C     = x_root;
        slope = double(subs(diff(f)));                % finds the slope of the functio
                                                          % at the given point

        if slope == 0                                 % checks for a minimum or maximu
            fprintf('Error: stuck at minimum or maximum of function.\n')
            zero    = 0;                              % breaks the while loop so that
                                                          % function doesn't go on for

            x_root = 'N/A';
        else
            b      = zero - slope * x_root;           % finds the b of the function
                                                          % y = mx + b

            x_root = double(-b / slope);              % finds the new x where y = 0 fo
                                                          % the linear function

            C      = x_root;
```

```
            zero    = double(subs(f));                          % finds the value of the functi
                                                                    % at the x found above
            i       = i + 1;                                    % adds iteration to counter
        end
    end
    if x_root == 'N/A'                                          % changes the zero value to N/A
                                                                    % the case that a maximum wa
                                                                    % found after loop break
        zero  = 'N/A';
    end
end
```

Cubic Spline Functions

```
function S = cubic_spline(x1, y)
m = length(x1);                                                 % finds the length of the x vec
n = length(y);                                                  % finds the length of the y vec

if m ~= n                                                       % checks that x and y are the sa
                                                                    % length
    error('Error: x and y have different dimensions.');
elseif m < 3                                                    % checks that there are more tha
                                                                    % points
    error('Error: not enough points to create a cubic spline.');
else
    scatter(x1,y);                                              % plots the data as a scatter pl
    hold on;
    [A, B, C, D] = spline_coeff(x1,y);                          % calls the spline coefficient-
                                                                    % function
    S = zeros(m, 1);                                            % makes a vector of zeros to ho
                                                                    % spline functions
    for i = 1:m-1                                               % iterates through the spline
                                                                    % functions (one less than t
                                                                    % number of points)
        a = double(A(i));                                       % assigns a variable to the A c
        b = double(B(i));                                       % assigns a variable to the B c
        c = double(C(i));                                       % assigns a variable to the C c
        d = double(D(i));                                       % assigns a variable to the D c
        digits(5)                                               % assigns the number of signifi
                                                                    % displayed when the functi
                                                                    % printed
        syms x                                                  % assigns x to be a symbolic va
        fprintf('Valid from x = %.4f to %.4f', x1(i), x1(i + 1))
                                                                % prints the calculated spline
                                                                    % its valid range
        f = vpa(a) + (vpa(b) * (x - vpa(x1(i)))) + (vpa(c) * ((x - vpa(x1(i))) ^ 2)) + ...
            (vpa(d) * ((x - vpa(x1(i))) ^ 3))
        vals = x1(i):0.01:x1(i+1);                              % makes an array of x values in
                                                                    % range
        x = vals;
        plot(vals, subs(f));                                    % plots the calculated spline f
        hold on;
```

```
    end
end
end

function H = h_matrix(x)
n = length(x);                                        % finds the length of the x vect
H = zeros(n,n);                                       % creates a square vector with 
                                                          % the size of the x vector
H(1,1) = 1;                                           % assigns the first and last ele
                                                          % the diagonal to be 1

H(n,n) = 1;
for i = 2:n-1                                         % iterates through the rest of t
    for j = 1:n                                       % iterates through the columns
        if j == i                                     % finds the element on the diago
                                                          % assigns the elements on ar
                                                          % and after the appropriate
                                                          % functions
            H(i, j) = double(2 * ((x(i) - x(i - 1)) + (x(i + 1) - x(i))));
            H(i, j - 1) = double(x(i) - x(i - 1));
            H(i, j + 1) = double(x(i + 1) - x(i));
        end
    end
end
end

function C = k_matrix(x, y)
m = length(x);                                        % finds the length of the x vect
n = length(y);                                        % finds the length of the y vect
if m ~= n                                             % checks that x and y have the s
    error("Error: x and y have different dimensions.");
else
    K = zeros(m,1);                                   % makes a vector of zeros of the
    for i = 2:m-1                                     % iterates through the middle el
                                                          % the first and last) and ca
                                                          % the appropriate variables

        h1 = double(x(i + 1) - x(i));
        h0 = double(x(i) - x(i - 1));
        a2 = double(y(i + 1));
        a1 = double(y(i));
        a0 = double(y(i - 1));
                                                      % calculates the K-function
        K(i,1) = double(((3 * (a2 - a1)) / h1) - ((3 * (a1 - a0)) / h0));
    end
    H = h_matrix(x);                                  % calls the H matrix function
    C = H\K;                                          % obtains the C coefficients by
                                                          % the H matrix by the K vect

end
end

function [A, B, C, D] = spline_coeff(x, y)
m = length(x);                                        % finds the length of x
n = length(y);                                        % finds the length of y
if m ~= n                                             % checks that x and y are the sa
```

```
    error('Error: x and y have different dimensions.');
else
    B = zeros(length(y)-1,1);                               % makes an appropriately sized B
                                                                % vector
    D = B;                                                  % makes an appropriately sized D
                                                                % vector
    C = k_matrix(x,y);                                      % calls the K matrix function to
                                                                % coefficients
    A = y;                                                  % assigns the y values to the A
    for i = 1:m-1                                           % iterates through the lengths o
                                                                % D vectors and finds the B
                                                                % coefficients from A, C, an
                                                                % h variable
        h = (x(i + 1) - x(i));
        B(i,1) = double(((A(i + 1) - A(i)) / h) - (((C(i + 1) + 2 * C(i)) * h) / 3));
        D(i,1) = double((C(i + 1) - C(i)) / (3 * h));
    end
end
end
```