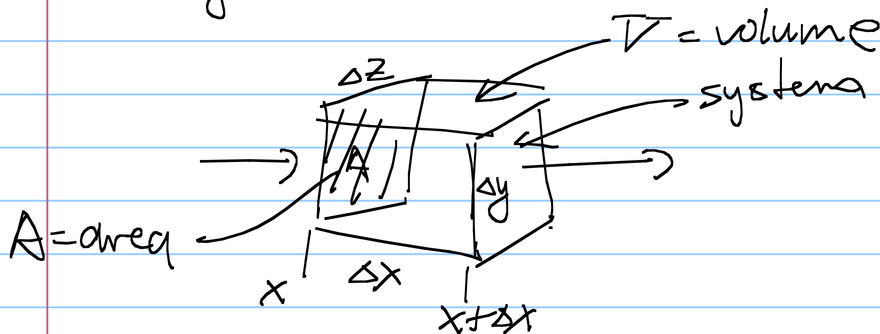
Shell/differential element system balances in different geometries (1-D)

rectangular (x direction)



IN − OUT = ACC   Conservation Eqn

quantity being conserved

$$(\;)A\Big|_{x} - (\;)A\Big|_{x+\Delta x} = \frac{d}{dt}(\;)V$$

Area $= \Delta y \Delta z$

$V = \Delta x \Delta y \Delta z$

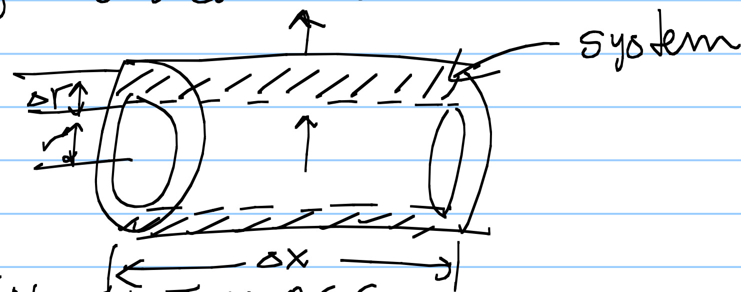
$$(\;)\Delta y \Delta z\Big|_{x} - (\;)\Delta y \Delta z\Big|_{x+\Delta x} = \frac{d}{dt}(\;)\Delta x \Delta y \Delta z$$

Divide by V, take limit as $\Delta x \to 0$

$$\frac{d(\;)}{dx} = \frac{d(\;)}{dt}$$

Note Title 10/10/2007

Cylindrical coordinates (r direction)



IN − OUT = ACC

$$(\ )A\Big|_r - (\ )A\Big|_{r+\Delta r} = \frac{d}{dt}(\ )V$$

Area $= 2\pi r \Delta x$

$$V = \pi r^2 \Delta x\Big|_{r+\Delta r} - \pi r^2 \Delta x\Big|_r$$

$$= 2\pi r \Delta r \Delta x \quad (\text{assume } \Delta r^2 \cong 0)$$

$$(\ )2\pi r \Delta x\Big|_r - (\ )2\pi r \Delta x\Big|_{r+\Delta r} = \frac{d}{dt}(\ )2\pi r \Delta r \Delta x$$

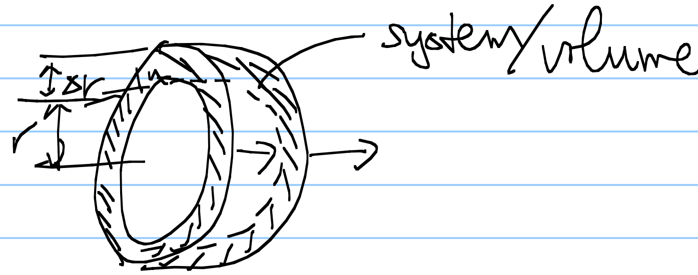
Divide by $2\pi \Delta r \Delta x$, take limit as $\Delta r \to 0$

$$\lim_{\Delta r \to 0} \frac{(\ )r\Big|_r - (\ )r\Big|_{r+\Delta r}}{} = \frac{d(\ )}{dt} r$$

$$\frac{1}{r}\frac{d[(\ )r]}{dr} = \frac{d(\ )}{dt}.$$

Note Title 10/10/2007

Spherical coordinates (r direction)



IN − OUT = ACC

$$(\ )A\Big|_{r} - (\ )A\Big|_{r+\Delta r} = \frac{d}{dt}(\ )V$$

Area = $4\pi r^2$

Volume = $V = \frac{4}{3}\pi(r+\Delta r)^3 - \frac{4}{3}\pi(r)^3$

$$= 4\pi r^2 \Delta r$$

(Assuming $\Delta r^2, \Delta r^3 \cong 0$)

$$(\ )4\pi r^2\Big|_{r} - (\ )4\pi r^2\Big|_{r+\Delta r} = \frac{d}{dt}(\ )4\pi r^2 \Delta r$$

Divide by $4\pi\Delta r$, take limit as $\Delta r \to 0$

$$\lim_{\Delta r \to 0} \frac{(\ )r^2\Big|_{r} - (\ )r^2\Big|_{r+\Delta r}}{\Delta r} = \frac{d(\ )}{dt} r^2$$

$$\frac{1}{r^2}\frac{d[(\ )r^2]}{dr} = \frac{d(\ )}{dt}$$