Using Euler and Heun methods solve the following differential equation

y’(t) = 5\*y(t)\*e-t \*sin(t) y(0)=1

for step sizes 1 and 0.1

Plot your solutions for 0<t<10

Note: This ODE can be solved analytically by simple separation of variables.

What do you observe about the step size and various methods?

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Euler stepsize=1 |  |  |  |  |  |  |  |  |
| y'=5\*y\*sin(t)\*exp(-t) | | | y(0)=1 |  |  |  |  |  |
| Euler |  |  |  | |  | | --- | |  | |  |  |  |  |
| t | y | y' | analytical |  |  |  |  |  |
| 0 | 1 | 0 | 1 |  |  |  |  |  |
| 1 | 1 | 1.547799 | 3.418442 |  |  |  |  |  |
| 2 | 2.547799 | 1.567661 | 10.31035 |  |  |  |  |  |
| 3 | 4.115461 | 0.144575 | 13.54012 |  |  |  |  |  |
| 4 | 4.260036 | -0.29525 | 12.99524 |  |  |  |  |  |
| 5 | 3.964787 | -0.12809 | 12.32186 |  |  |  |  |  |
| 6 | 3.836701 | -0.01329 | 12.13121 |  |  |  |  |  |
| 7 | 3.823415 | 0.011453 | 12.14337 |  |  |  |  |  |
| 8 | 3.834868 | 0.006364 | 12.17388 |  |  |  |  |  |
| 9 | 3.841231 | 0.000977 | 12.18437 |  |  |  |  |  |
| 10 | 3.842208 | -0.00047 | 12.18441 |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |

Euler step size = 0.1

Heun step size = 1

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Heun |  |  |  |  |  |  | |  | | --- | |  | |  |  |  |  |
| t | y | y'1 | y1 | y'1 | avg y' | analytical |  |  |  |  |  |
| 0 | 1 | 0 | 1 | 1.547799 | 0.7739 | 1 |  |  |  |  |  |
| 1 | 1.7739 | 2.745641 | 4.519541 | 2.780874 | 2.763257 | 3.418442 |  |  |  |  |  |
| 2 | 4.537157 | 2.791713 | 7.32887 | 0.257461 | 1.524587 | 10.31035 |  |  |  |  |  |
| 3 | 6.061744 | 0.212948 | 6.274692 | -0.43488 | -0.11096 | 13.54012 |  |  |  |  |  |
| 4 | 5.950779 | -0.41243 | 5.538351 | -0.17892 | -0.29567 | 12.99524 |  |  |  |  |  |
| 5 | 5.655105 | -0.18269 | 5.472411 | -0.01895 | -0.10082 | 12.32186 |  |  |  |  |  |
| 6 | 5.554282 | -0.01923 | 5.535048 | 0.01658 | -0.00133 | 12.13121 |  |  |  |  |  |
| 7 | 5.552955 | 0.016634 | 5.569589 | 0.009243 | 0.012938 | 12.14337 |  |  |  |  |  |
| 8 | 5.565893 | 0.009236 | 5.57513 | 0.001418 | 0.005327 | 12.17388 |  |  |  |  |  |
| 9 | 5.57122 | 0.001417 | 5.572637 | -0.00069 | 0.000364 | 12.18437 |  |  |  |  |  |
| 10 | 5.571585 | -0.00069 | 5.578465 | 0 | -0.00034 | 12.18441 |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |

Heun step size = 0.1

Observations

With the Euler method, note that even with significantly smaller step size, the solution can still lead to significant error. It this particular case, the smaller step size results in a very poor solution.

With a higher order method, the solution is much better, even at larger step size.