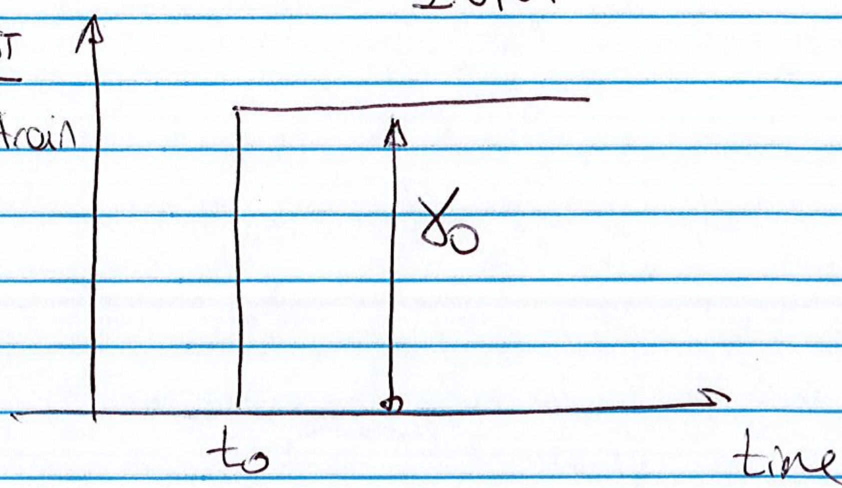
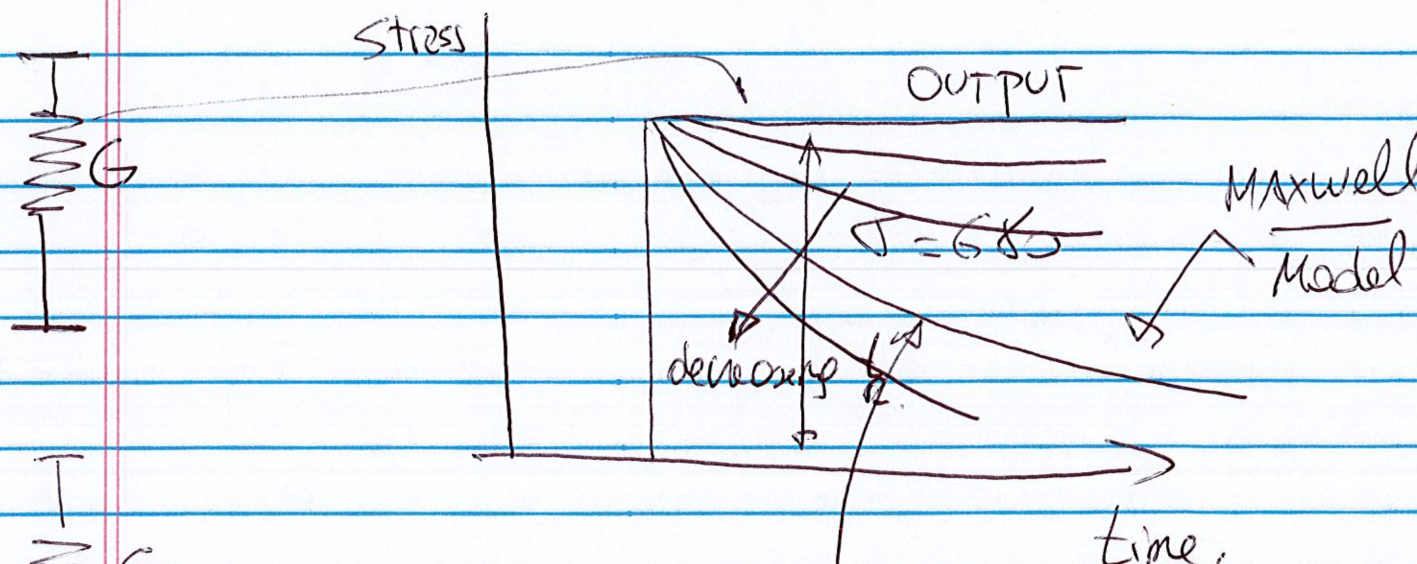
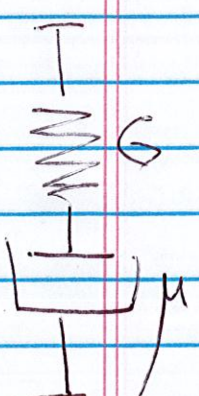
Question 2

RELAXATION TEST

Deformation → Strain







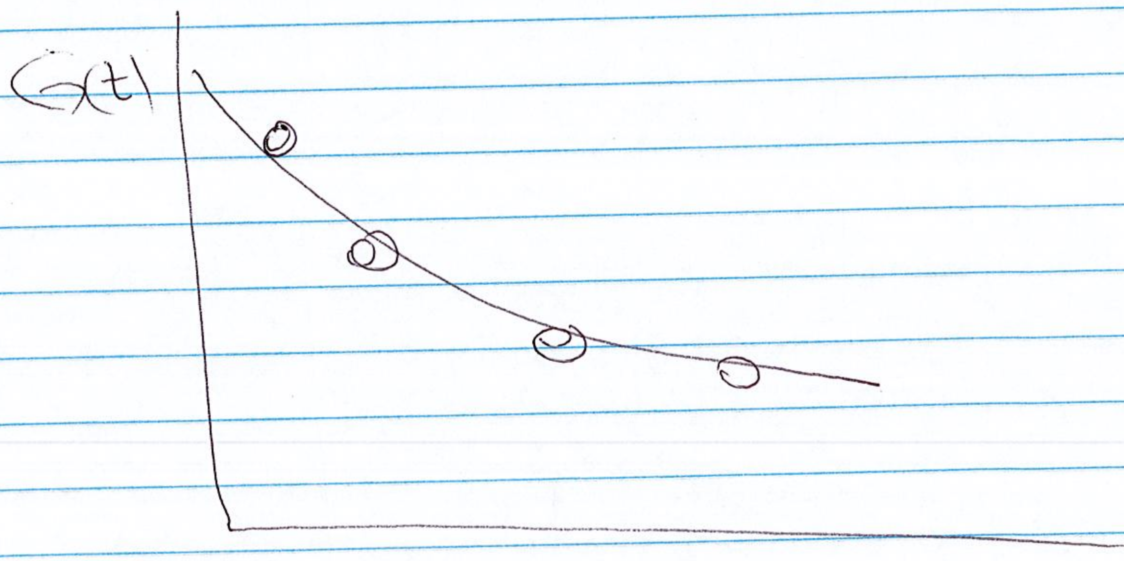
?

decay proportional to G & μ

→ Proportional $\frac{\mu}{G} = t_R$

Question 3

t	$G(t)$
—	—
—	—
—	—
—	—

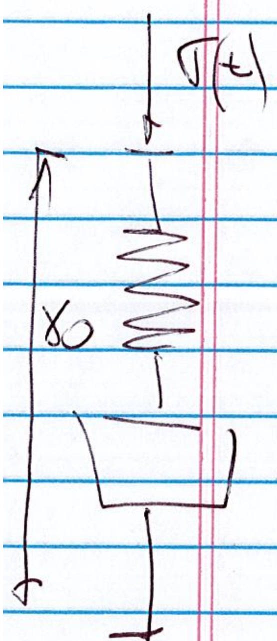


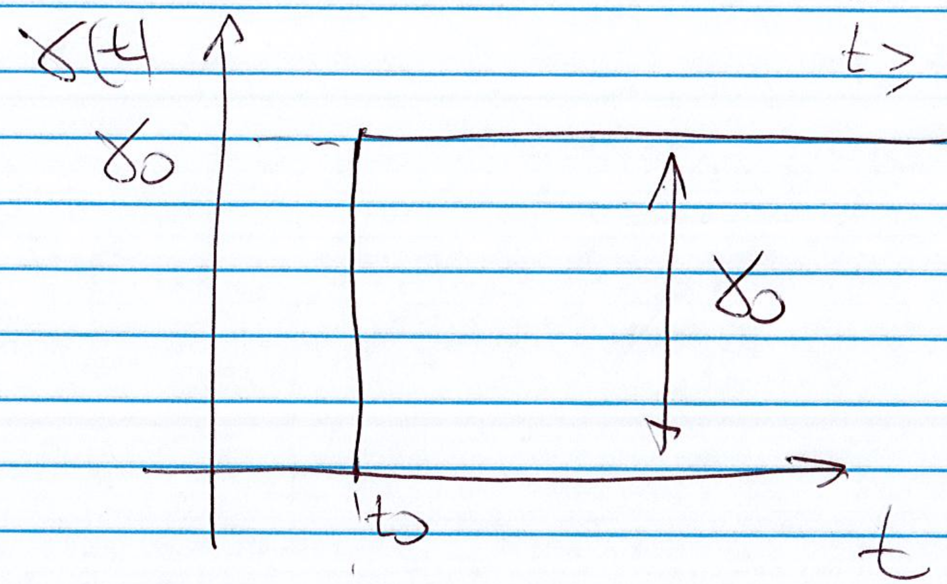
Fitting Experimental Data

Going back to to question 2 TRANSIENT EXP.

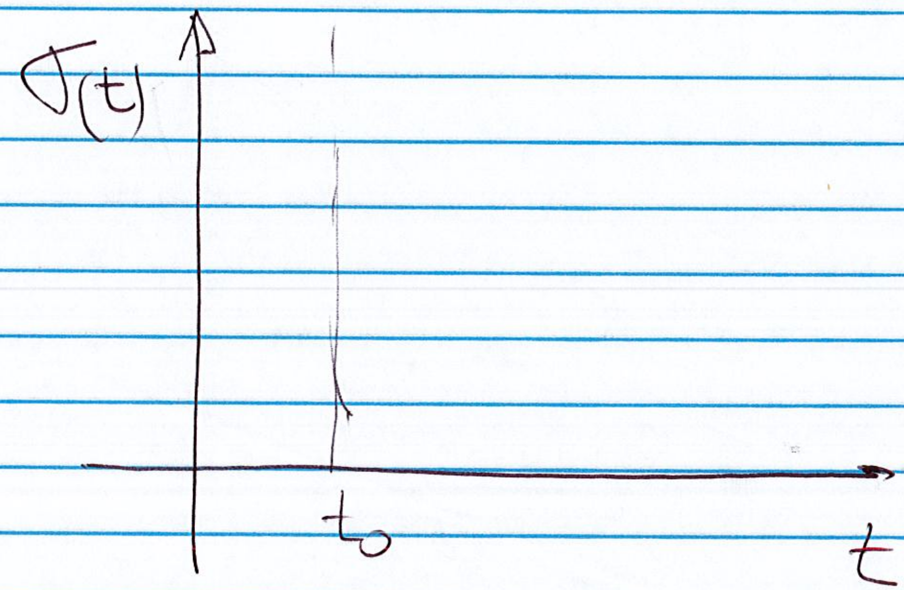
$t < t_0 \quad \gamma(t) = 0$

$t \geq t_0 \quad \gamma(t) = \gamma_0$





INPUT



OUTPUT

For a Maxwell element

$$\sigma(t) + t_R \frac{d\sigma(t)}{dt} = \mu \underbrace{\cancel{\dot{\gamma}(t)}}_{0} \qquad \frac{d\gamma(t)}{dt}$$

$$\sigma(t) + t_R \frac{d\sigma(t)}{dt} = 0$$

(4)

$$\sigma(t) = -t_R \frac{d\sigma(t)}{dt}$$

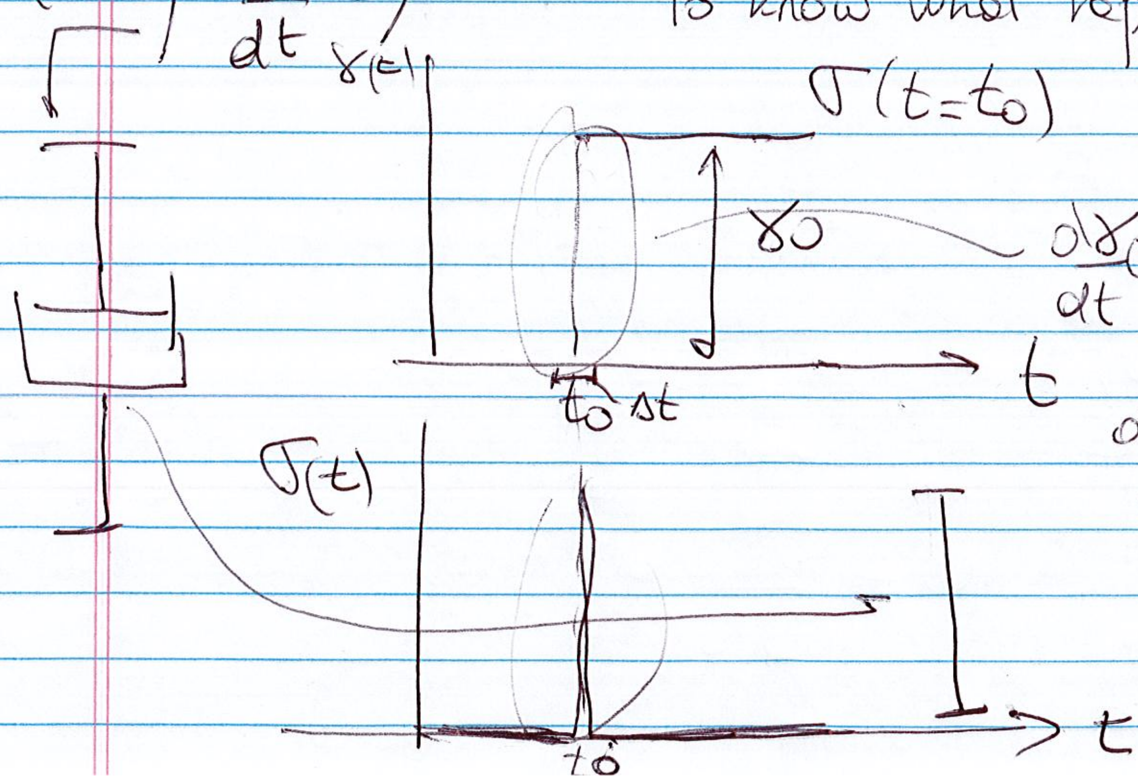
$$\int_{\sigma(t_0)}^{\sigma(t)} \frac{d\sigma(t)}{\sigma(t)} = -\int_{t=t_0}^{t} \frac{dt}{t_R}$$

$$\ln \frac{\sigma(t)}{\sigma(t=t_0)} = -\frac{1}{t_R}\int_{t_0}^{t} dt = -\left(\frac{t-t_0}{t_R}\right)$$

$$\boxed{\sigma(t) = \sigma(t=t_0) \cdot e^{-\frac{t-t_0}{t_R}}}$$

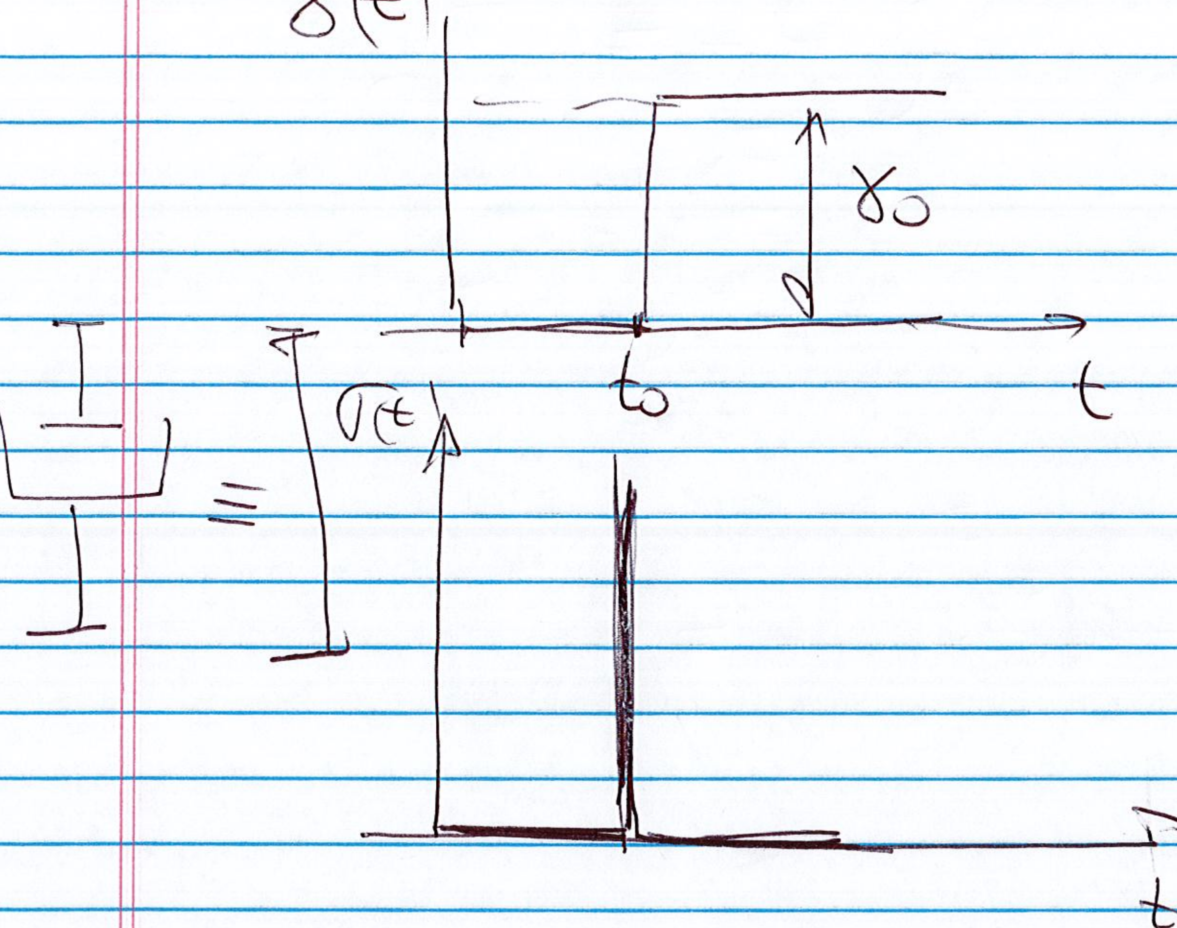
$$\sigma(t) = \mu \frac{d\gamma(t)}{dt} = \mu\,\dot{\gamma}(t)$$

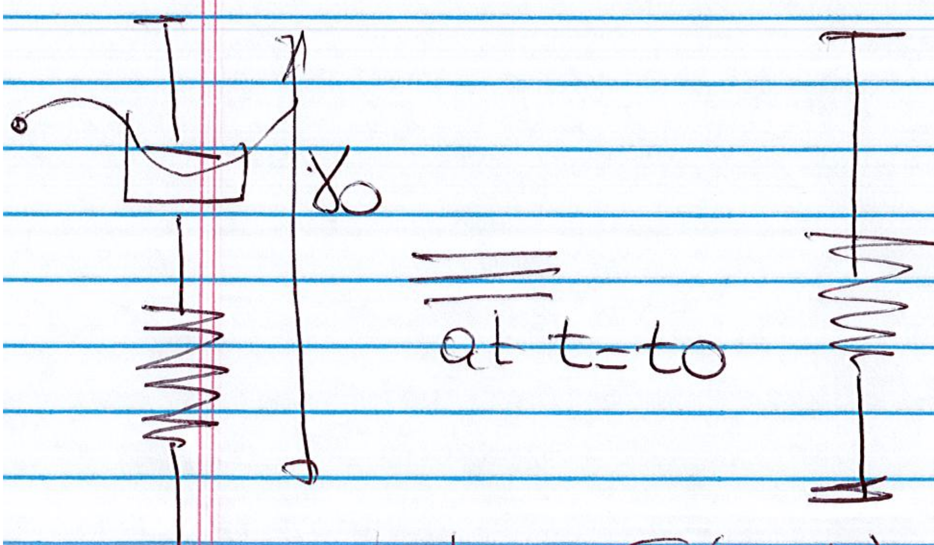
To know what happens with $\sigma(t=t_0)$



$$\frac{d\gamma(t)}{dt} \underset{\Delta t \to 0}{\cong} \frac{\gamma_0 - 0}{\Delta t}$$

$$\frac{d\gamma(t)}{dt} \Rightarrow \infty$$





$\sigma_0 = G\gamma_0$

what is $\sigma(t = t_0) = \sigma_0 = G\gamma_0$

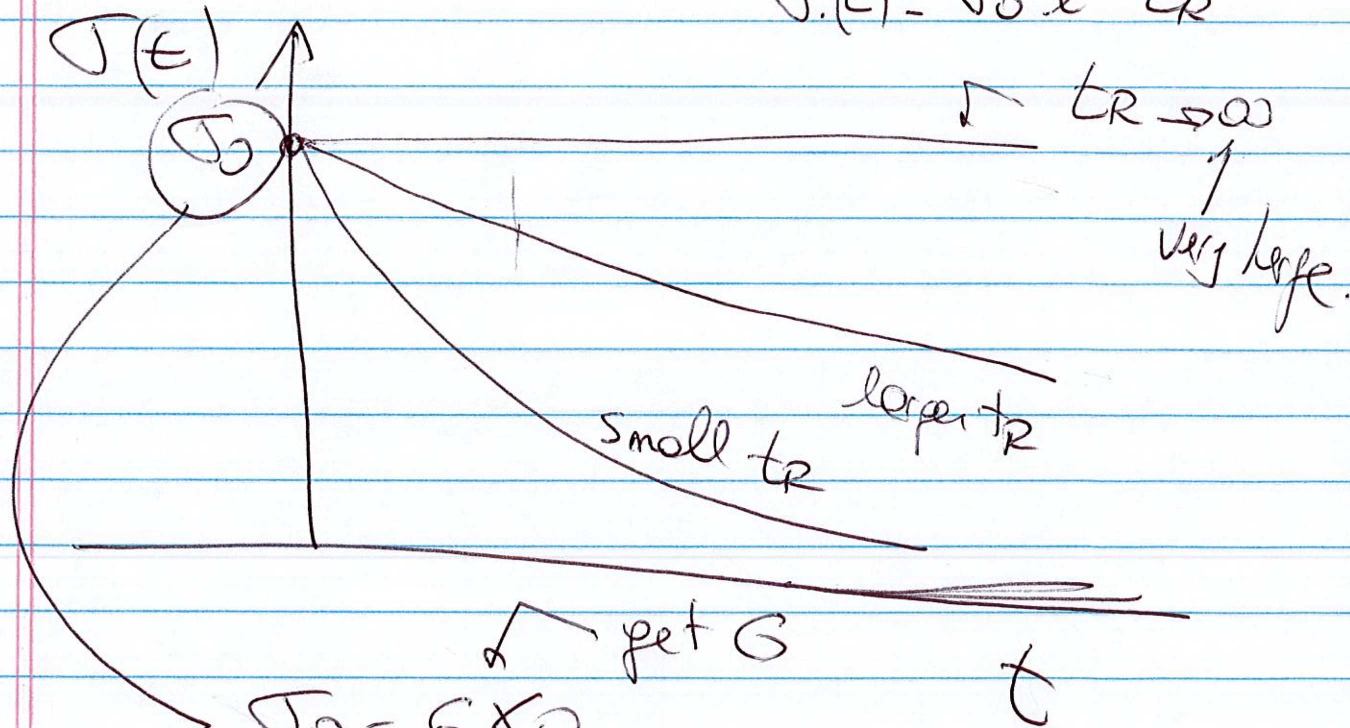
$$\sigma(t) = \sigma_0 \, e^{-\frac{t-t_0}{t_R}} \qquad (6)$$

small t_R

$G\gamma_0$

Let's assume $t_0 = 0$

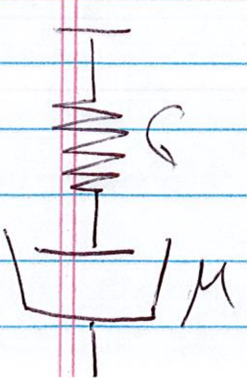
$$\sigma(t) = \sigma_0 \, e^{-\frac{t}{t_R}}$$



$\sigma_0 = G\gamma_0$

OUTPUT   INPUT

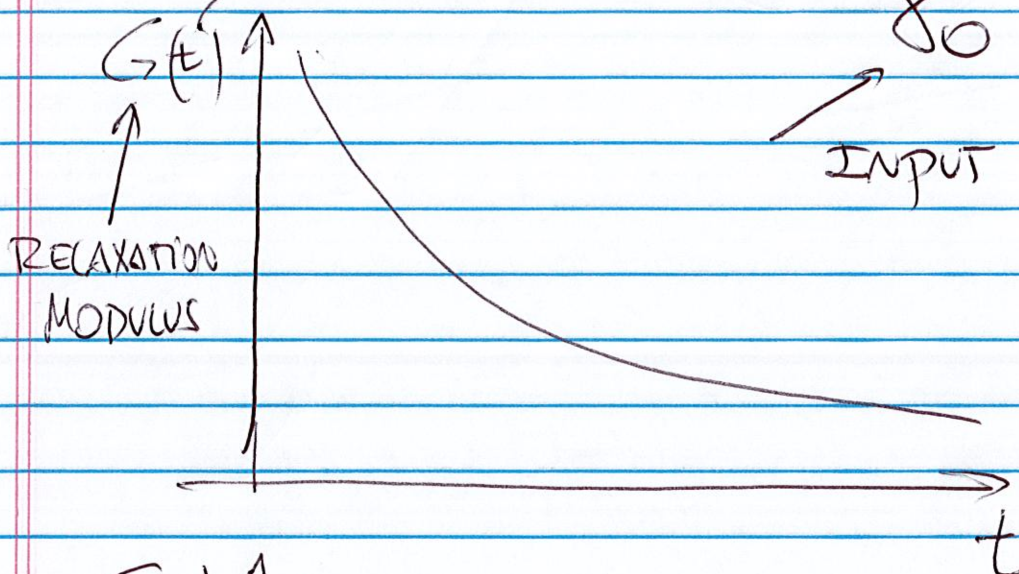
From plot you get $t_R = \frac{\mu}{G}$

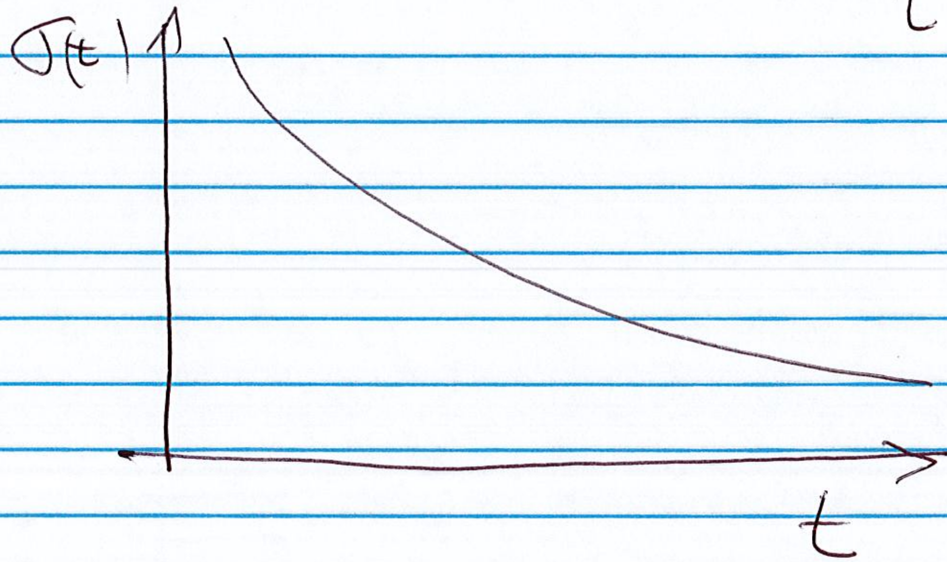


Problem 3 (7)

$$G(t) = \frac{\sigma(t)}{\gamma_0}$$

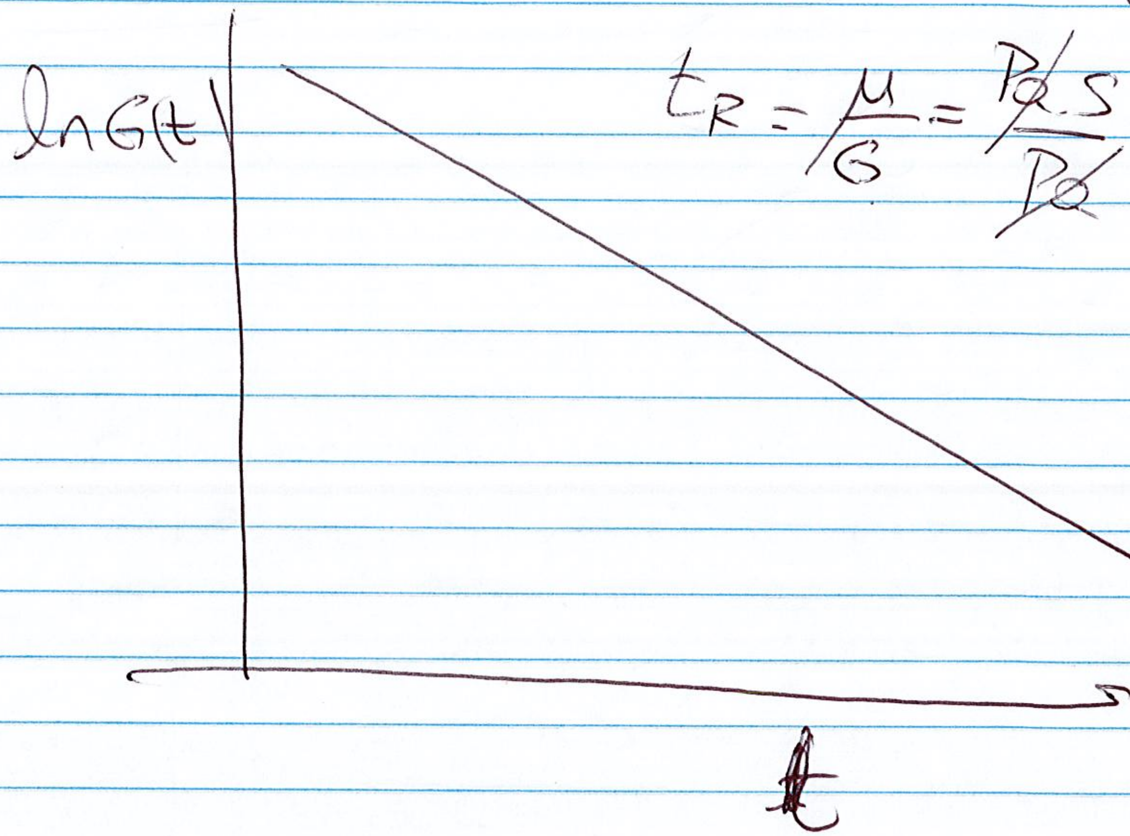
INPUT





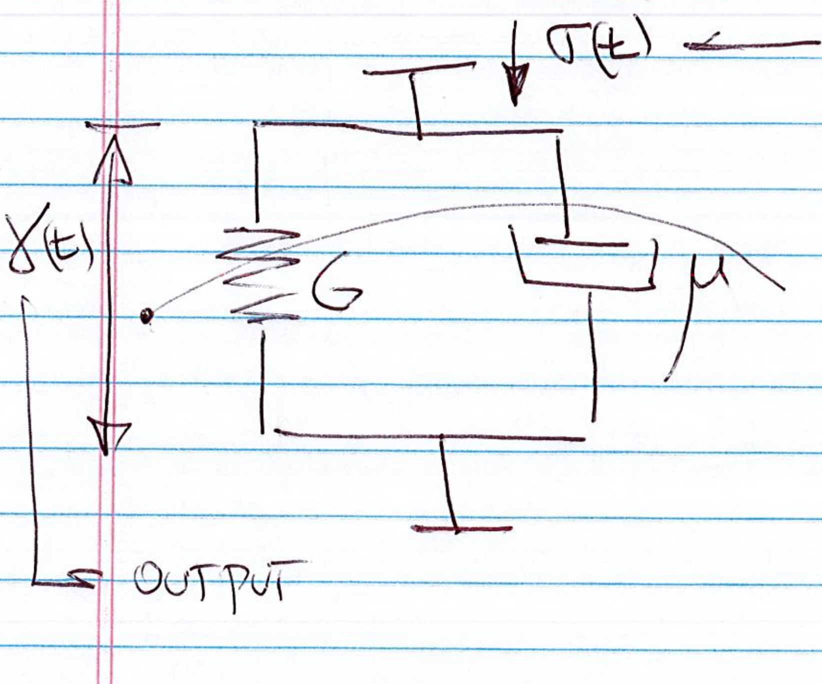
$$G(t) = G_0 e^{-t/t_R}$$

$$\boxed{\ln G(t) = \ln G_0 - t/t_R}$$



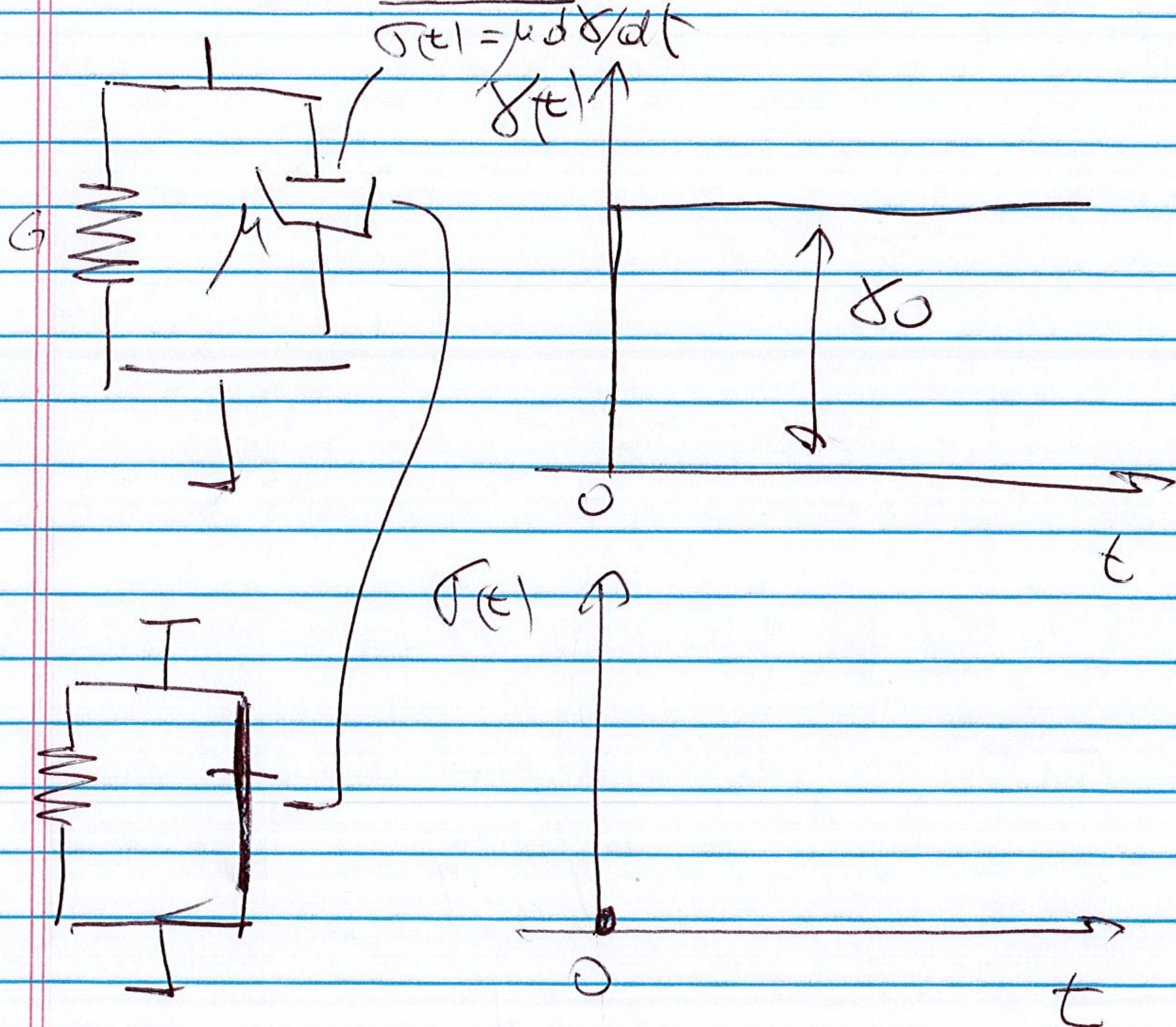
$$t_R = \frac{\mu}{G} = \frac{\cancel{Pa}\,S}{\cancel{Pa}}$$

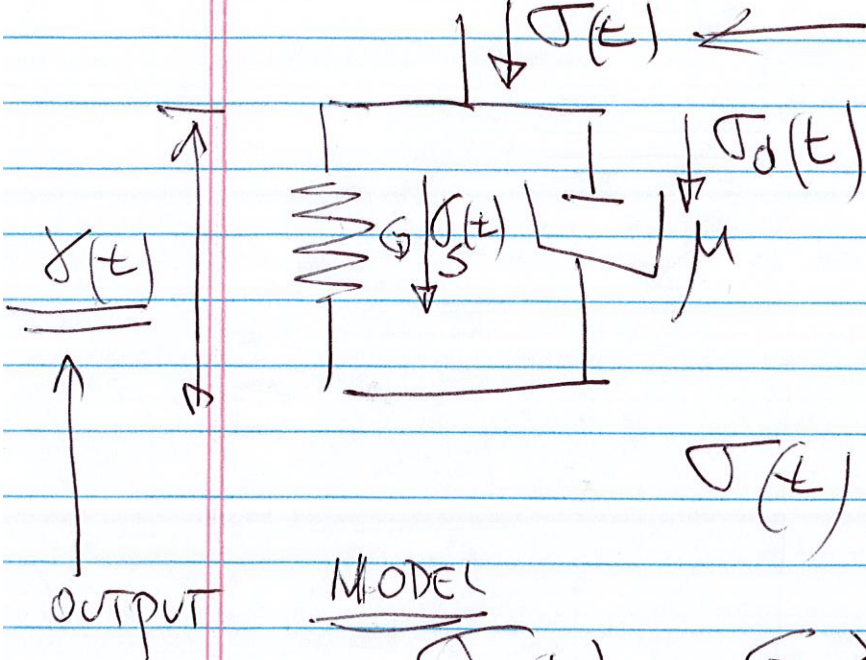
Kelvin-Void Model



Creep Test

RELAXATION TEST ARE NOT GOOD
FOR <u>SOLID</u> VISCOELASTIC MATERIALS





$$\sigma(t) = \sigma_s(t) + \sigma_d(t)$$

MODEL

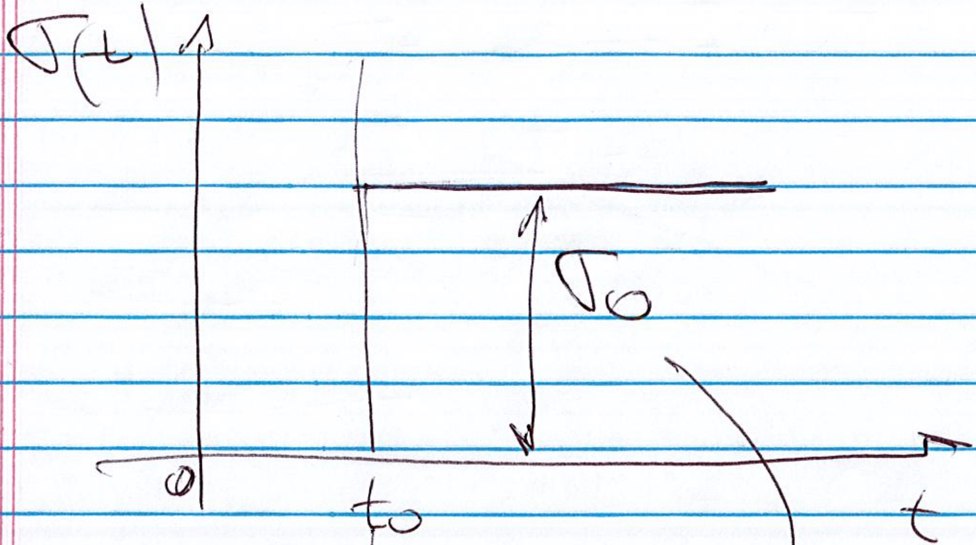
$$\sigma(t) = G\,\gamma(t) + \mu \frac{d\gamma(t)}{dt}$$

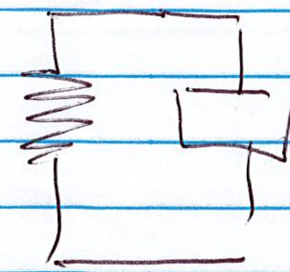
INPUT: $\sigma(t)$

OUTPUT: $\gamma(t)$, $\frac{d\gamma(t)}{dt}$

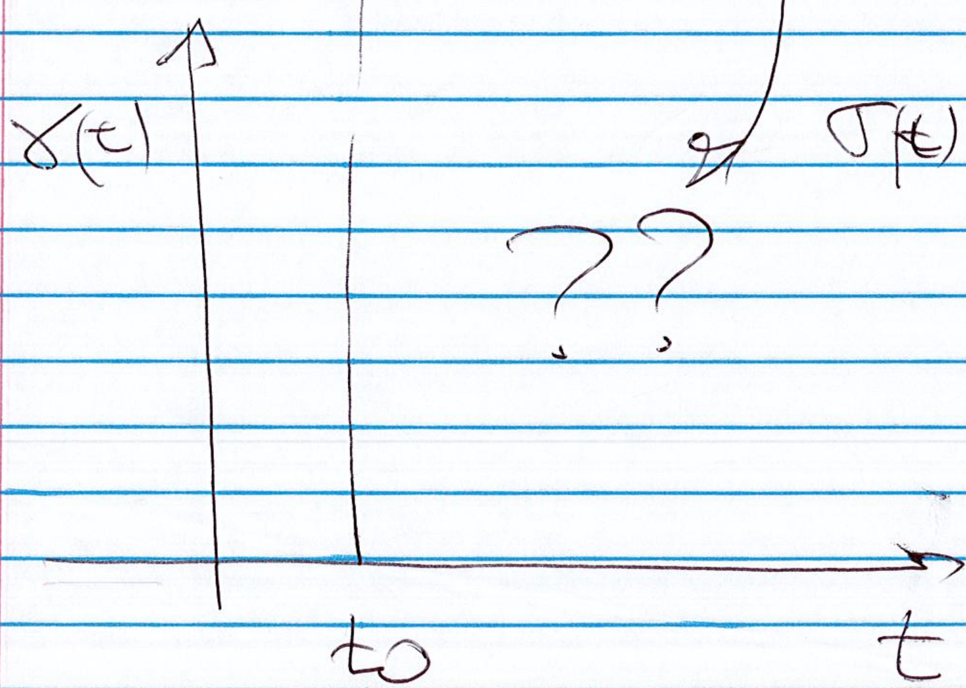
CREEP TEST

INPUT $\sigma(t)$
OUTPUT $\gamma(t)$ $\Longrightarrow$ We get properties of the materials





$$\sigma(t) = G\,\gamma(t) + \mu \frac{d\gamma(t)}{dt}$$



$$\sigma_0 = G\,\gamma(t) + \mu \frac{d\gamma(t)}{dt}$$

$$\sigma_0 - G\,\gamma(t) = \mu \frac{d\gamma(t)}{dt}$$

$$\int_{\gamma(t=t_0)}^{\gamma(t)} \frac{d\gamma(t)}{\sigma_0 - G\gamma(t)} = \int_{t_0}^{t} \frac{1}{\mu}\, dt \qquad (12)$$

$$y(t) = \sigma_0 - G\gamma(t)$$

$$dy(t) = -G\, d\gamma(t) \Longrightarrow d\gamma(t) = -\frac{1}{G}\, dy(t)$$

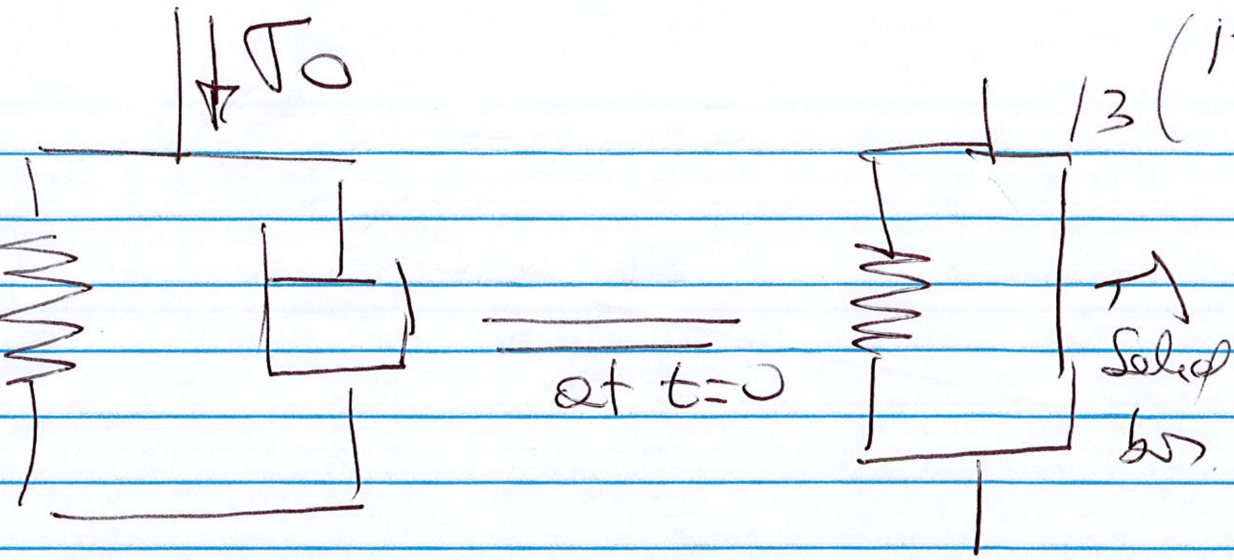
$$-\frac{1}{G} \int_{y_0}^{y(t)} \frac{dy(t)}{y(t)} = \frac{1}{\mu}(t - t_0)$$

$$-\frac{1}{G} \ln \frac{y(t)}{y(t=t_0)} = \frac{1}{\mu}(t - t_0)$$

$$\ln \frac{\sigma_0 - G\gamma(t)}{\sigma_0 - G\gamma(t_0)} = -\frac{G}{\mu}(t - t_0)$$

??

$$\ln \underline{\sigma_0 - G\gamma(t)}$$



at $t = t_0$ $\quad \gamma(t_0) = 0$

$$\ln \frac{\sigma_0 - G\,\gamma(t)}{\sigma_0 - G\,\cancel{\gamma(t_0)}} = -\frac{1}{t_{ret}}\, t$$

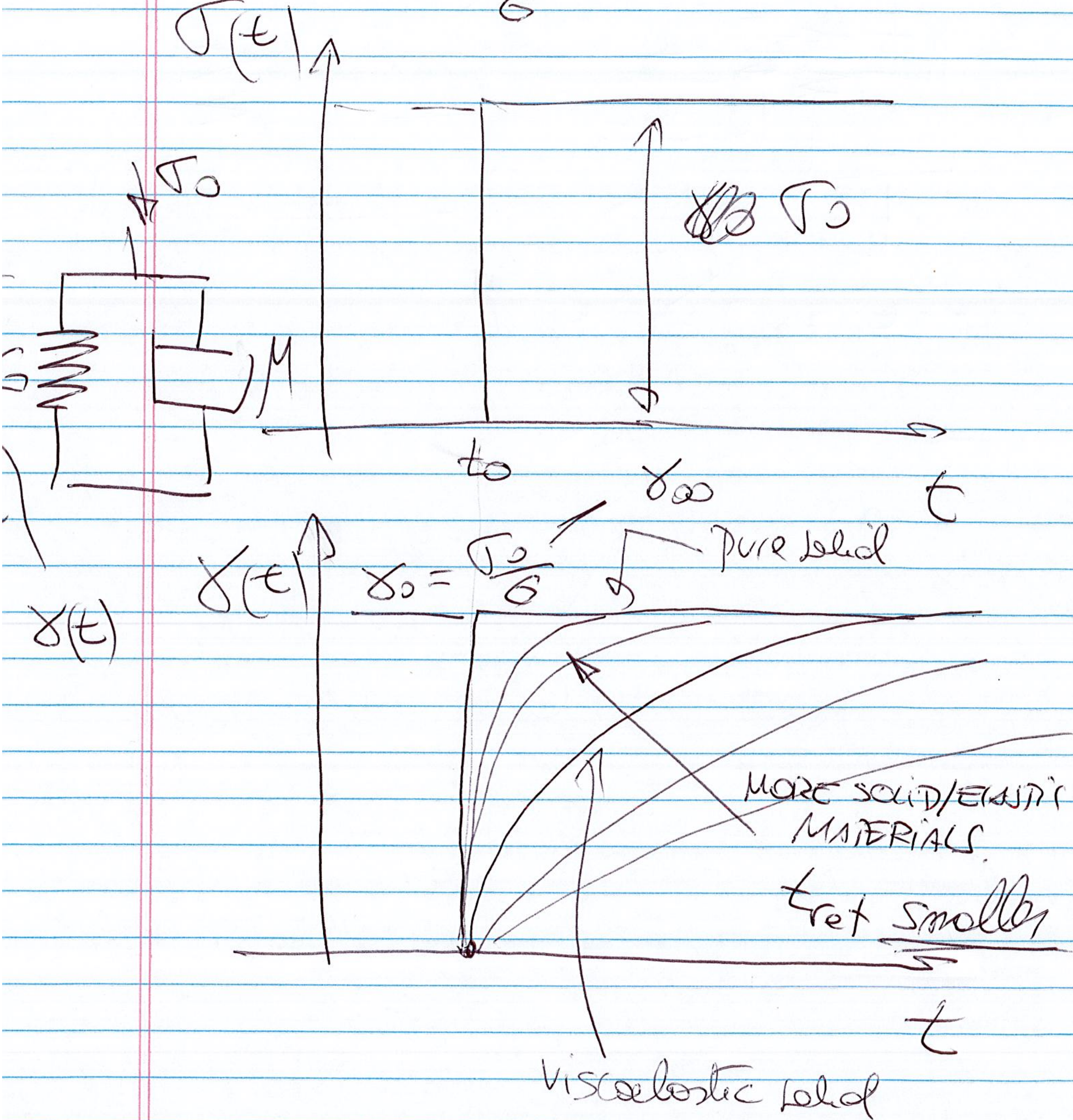
$$0$$

$$\frac{\sigma_0 - G\,\gamma(t)}{\sigma_0} = e^{-\frac{t}{t_{ret}}}$$

RETARDATION TIME

$$1 - \frac{G}{\sigma_0}\,\gamma(t) = e^{-t/t_{ret}}$$

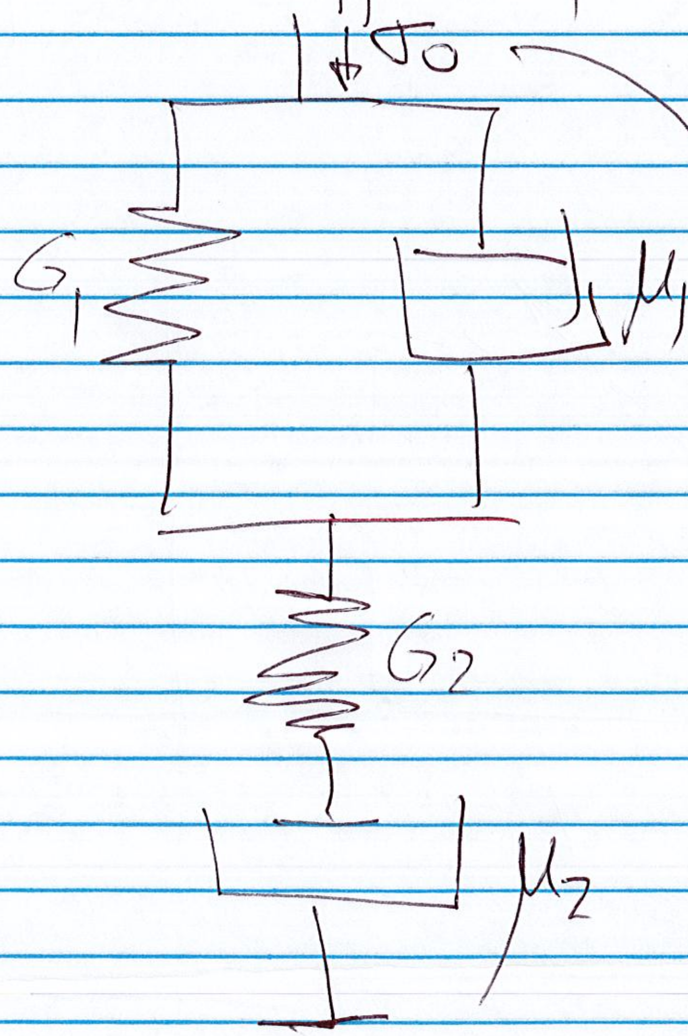
$$\gamma(t) = \frac{\sigma_0}{G}\left[1 - e^{-(t-t_0)/t_{ret}}\right] \qquad (14)$$



$$\gamma(t) = \gamma_\infty \left[1 - e^{-\frac{t-t_0}{t_{ret}}}\right] \quad (15)$$

asymptotic value at $t \to \infty$

what happens if the model is



creep experiment

(16)

ω	G^*	$\tan\delta$	$G'(\omega)$
—	—	—	X
—	—	—	X
	—	—	X
—			X

$$\tan\delta = \frac{G''}{G'} \Longrightarrow \cancel{G' = \tan\delta\, G''}$$

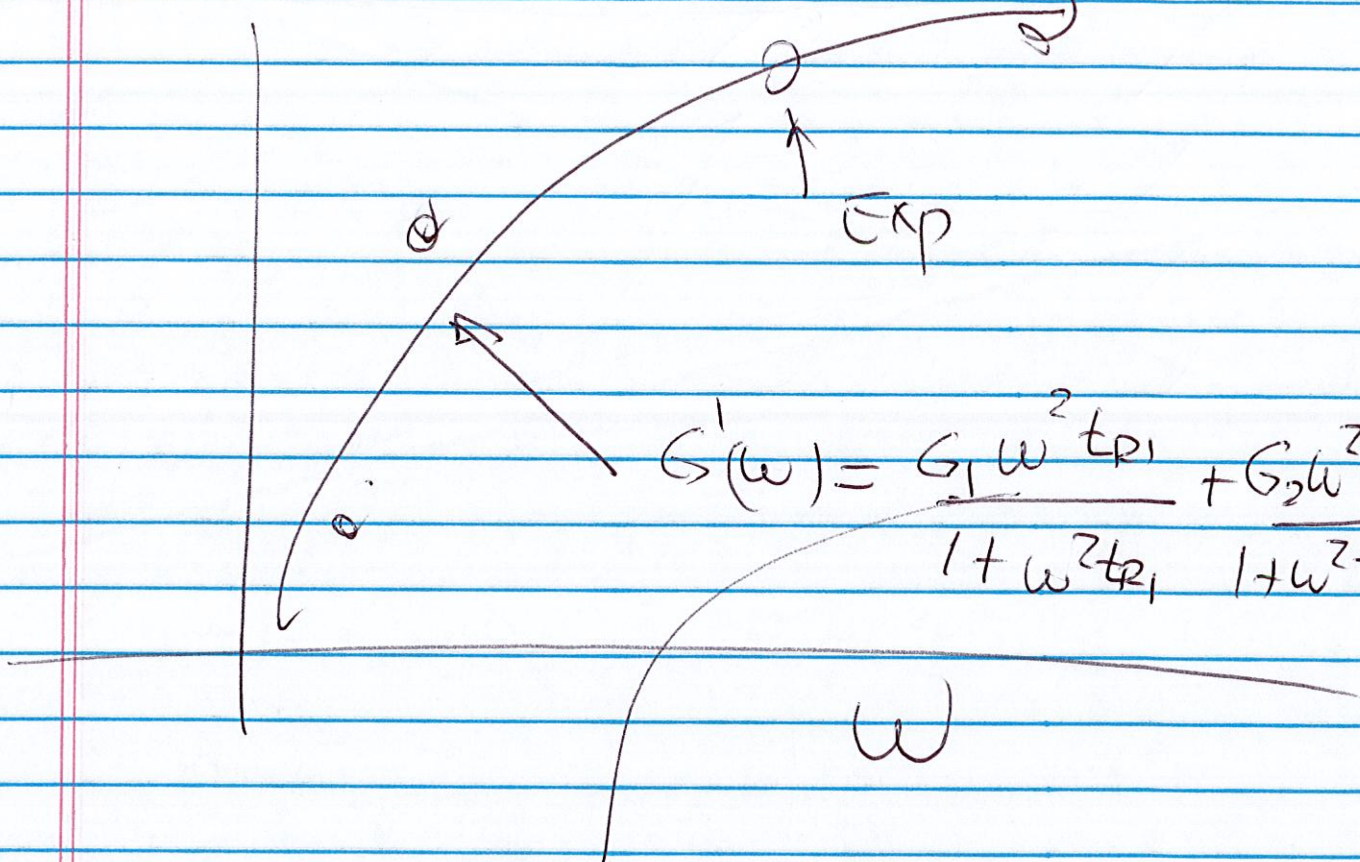
$$\cancel{G^* = \sqrt{G'^2 + G''^2} = \tan^2\delta\, G''}$$

$$\cancel{G' = \frac{G''}{\tan\delta}} \qquad G'' = \tan\delta\, G'$$

$$G^* = \sqrt{G'^2 + \tan^2\delta\, G'^2} = G'\sqrt{1+\tan^2\delta}$$

$$\boxed{G'(\omega) = \frac{G^*(\omega)}{\sqrt{1+\tan^2\delta(\omega)}}}$$

DATA IS CONVERTED TO $G'(\omega)$ vs ω (17)



$$G'(\omega) = \frac{G_1 \omega^2 t_{R1}}{1+\omega^2 t_{R1}} + \frac{G_2 \omega^2 t_{R2}}{1+\omega^2 t_{R2}}$$

ADJUST PARAMETERS
USING SOLVER TO
GET G_1, t_{R1}, G_2, t_{R2}

$$t_{R1} = \frac{\mu_1}{G_1}$$

$$t_{R2} = \frac{\mu_2}{G_2}$$

$\Downarrow$

μ_1 and μ_2

t_{R1} G_1

t_{R2} G_2

Experimental DATA

ω	$G'(\omega)_{exp}$	$G'(\omega)_{model}$	$\left(\frac{G'_{exp} - G'_{mo}}{G'_{exp}}\right)^2$
—	—	X	( )
—	—	X	( )
—	—	X	( )
—	—	X	( )
—	—	X	( )
—	—	X	( )
		X	( )
		SUM	( )

SUM OF SQUARE