Homework 1 - Fall 2017

Problem 1

$$\varepsilon_B = 1 - \frac{\rho_B}{\rho_{part}}$$

$$\varepsilon_B = \frac{V_{op} + V_{interparticle}}{V_s + V_{interparticle} + V_{cp}}$$

$$\rho_B = \frac{m_s}{V_s + V_{interparticle} + V_{cp}}$$

$$\rho_{part} = \frac{m_s}{V_s + V_{cp}}$$

$$\varepsilon_B = 1 - \frac{\dfrac{m_s}{V_s + V_{cp} + V_{op} + V_{interparticle}}}{\dfrac{m_s}{V_s + V_{cp}}} = 1 - \frac{V_s + V_{cp}}{V_s + V_{cp} + V_{op} + V_{interparticle}}$$

$$\varepsilon_B = \frac{V_s + V_{cp} + V_{op} + V_{interparticle} - V_s - V_{cp}}{V_s + V_{cp} + V_{op} + V_{interparticle}} = \frac{V_{op} + V_{interparticle}}{V_s + V_{cp} + V_{op} + V_{interparticle}}$$

$$\varepsilon_{cp} = 1 - \frac{\rho_{part}}{\rho_s}$$

$$\varepsilon_{cp} = \frac{V_{cp}}{V_s + V_{cp} + V_{op}}$$

$$\rho_{part} = \frac{m_s}{V_s + V_{cp}}$$

$$\rho_s = \frac{m_s}{V_s}$$

$$\varepsilon_{cp} = 1 - \frac{\dfrac{m_s}{V_s + V_{cp}}}{\dfrac{m_s}{V_s}} = 1 - \frac{V_s}{V_s + V_{cp}}$$

$$\varepsilon_B = \frac{V_s + V_{cp} - V_s}{V_s + V_{cp}} = \frac{V_{cp}}{V_s + V_{cp}}$$

Problem 2

From USDA Database

Water: 91.2%	$x_w := 0.912$
Protein: 2.8%	$x_p := 0.028$
Fat: 0.39%	$x_{fat} := 0.00039$
Carbohydrate: 4.0% Including sugars	$x_{carb} := 0.04$
Fibers: 2.0%	$x_{fiber} := 0.020$

$x_{total} := x_w + x_p + x_{fat} + x_{carb} + x_{fiber}$ $\qquad x_{total} = 1$

$\theta_s := 20$

$\rho_w := \left(9.9718 \cdot 10^2 + 3.1439 \cdot 10^{-3} \theta_s - 3.75744 \cdot 10^{-3} \cdot \theta_s{}^2\right) \cdot \frac{kg}{m^3}$ $\qquad \rho_w = 995.7 \frac{kg}{m^3}$

$\rho_p := \left(1.3299 \cdot 10^3 - 5.184 \cdot 10^{-1} \cdot \theta_s\right) \cdot \frac{kg}{m^3}$ $\qquad \rho_p = 1319.5 \frac{kg}{m^3}$

$\rho_{fat} := \left(9.2559 \cdot 10^2 - 4.1757 \cdot 10^{-1} \cdot \theta_s\right) \cdot \frac{kg}{m^3}$

$\rho_{carb} := \left(1.5991 \cdot 10^3 - 3.1046 \cdot 10^{-1} \cdot \theta_s\right) \cdot \frac{kg}{m^3}$

$\rho_{fiber} := \left(1.3115 \cdot 10^3 - 3.6589 \cdot 10^{-1} \cdot \theta_s\right) \cdot \frac{kg}{m^3}$

$$\rho_{true} := \frac{1}{\frac{x_w}{\rho_w} + \frac{x_p}{\rho_p} + \frac{x_{fat}}{\rho_{fat}} + \frac{x_{carb}}{\rho_{carb}} + \frac{x_{fiber}}{\rho_{fiber}}} \qquad \boxed{\rho_{true} = 1022.5 \frac{kg}{m^3}}$$

Problem 3

$x_{bw} := 0.80$ Berry fruit moisture content 80%

$x_{bc} := 1 - x_{bw}$

$\rho_{bapp} := 605 \cdot \frac{kg}{m^3}$ $\rho_{bBulk} := 500 \cdot \frac{kg}{m^3}$

From the apparent density, we can calculate the apparent porosity if we know the substance density, which can be calculated from the composition of the berries. In order to use the densities of the components calculated in problem 2, we assume a tempreture of 20ºC

$$\rho_{bs} := \frac{1}{\frac{x_{bw}}{\rho_w} + \frac{x_{bc}}{\rho_{carb}}} \qquad \rho_{bs} = 1076.4 \frac{kg}{m^3}$$

And the apparent porosity is:

$$\varepsilon_{app} := 1 - \frac{\rho_{bapp}}{\rho_{bs}} \qquad \varepsilon_{app} = 0.4$$

And the total porosity is:

$$\varepsilon_T := 1 - \frac{\rho_{bBulk}}{\rho_{bs}} \qquad \boxed{\varepsilon_T = 0.5}$$

Problem 4

Let's assume temperature of 20ºC to use the values of densities calculated in problem 2

Protein : 18%	$x_{pr} := 0.18$	$\rho_p = 1319.5 \frac{kg}{m^3}$
Carbohydrate : 4.0%	$x_{ca} := 0.04$	$\rho_{carb} = 1592.9 \frac{kg}{m^3}$
Ash : 0.5%	$x_{As} := 0.005$	$\rho_{As} := 2408 \cdot \frac{kg}{m^3}$
Fat: 2.5%	$x_{bmfat} := 0.025$	$\rho_{fat} = 917.2 \frac{kg}{m^3}$
Water : 75%	$x_{bmw} := 0.75$	$\rho_w = 995.7 \frac{kg}{m^3}$

Checking $\quad x_{pr} + x_{ca} + x_{As} + x_{bmfat} + x_{bmw} = 1$

(a) Let's calculate the composition in dry bases

$$X = \frac{x}{1-x}$$

x : Mass fraction in wet basis

X : Mass fraction in dry basis

$$X_{pr} := \frac{x_{pr}}{1-x_{pr}} \qquad \boxed{X_{pr} = 0.22}$$

$$X_{ca} := \frac{x_{ca}}{1-x_{ca}} \qquad \boxed{X_{ca} = 0.042}$$

$$X_{As} := \frac{x_{As}}{1-x_{As}} \qquad \boxed{X_{As} = 0.005}$$

$$X_{bmfat} := \frac{x_{bmfat}}{1-x_{bmfat}} \qquad \boxed{X_{bmfat} = 0.026}$$

$$X_{bmw} := \frac{x_{bmw}}{1-x_{bmw}} \qquad \boxed{X_{bmw} = 3}$$

Lets assume 100 grams fresh biomaterial

$m_{pr} := 28 \cdot gm \qquad m_{ca} := 4 \cdot gm \qquad m_{As} := 0.5 \cdot gm$

$m_{fat} := 2.5 \cdot gm \qquad m_{w} := 75 \cdot gm$

$m_{solids} := m_{pr} + m_{ca} + m_{As} + m_{fat} \qquad m_{solids} = 35 \cdot gm$

The moisture content of the dried biomaterial is 40% so the amount of water in the dried biomaterial, here designated as X, can be calculated fro the following equation:

$$\frac{X}{m_{solids} + X} = 0.40 \text{ solve}, X \rightarrow 23.333333333333333333 \cdot gm$$

so, let 's use only one decimal figure $\qquad X := 23.3 \cdot gm$

We can calculate the volume of the biomaterial from its density

$$\rho_{bm} := \frac{1}{\frac{x_{pr}}{\rho_p} + \frac{x_{ca}}{\rho_{carb}} + \frac{x_{As}}{\rho_{As}} + \frac{x_{bmfat}}{\rho_{fat}} + \frac{x_{bmw}}{\rho_w}}$$

$$\rho_{bm} = 1059.2\,\frac{kg}{m^3}$$

$$\rho_{bm} = \frac{m_{solids} + m_w}{V_{bm}} \text{ solve}, V_{bm} \rightarrow \frac{0.10384708997713889028 \cdot gm \cdot m^3}{kg}$$

So $V_{bm} := 0.104 \cdot 10^{-3} \cdot m^3$

If the volume of the biomaterial is conserved (i.e. there is no shrinkage), we can calculate the apparent density of the material as:

$$\rho_{app\_driedbm} := \frac{m_{solids} + X}{V_{bm}}$$

$$\boxed{\rho_{app\_driedbm} = 560.6\,\frac{kg}{m^3}}$$

The apparent porosity of the biomaterial can be calculated from its apparent and substance densities, the latter calculated by estimating the new composition of the biomateriali. The composition is calculated with the new amount of water in the biomaterial (X) but assuming that the solid components do not change. Thus

$m_{prdbm} := 18 \cdot gm$ $\quad m_{cadbm} := 4 \cdot gm$ $\quad m_{fatdbm} := 2.5 \cdot gm$

$m_{Asdbm} := 0.5 \cdot gm$ $\quad m_{wdbm} := X$

$$M_{dc} := m_{prdbm} + m_{cadbm} + m_{Asdbm} + m_{fatdbm} + m_{wdbm}$$

$$M_{dc} = 48.3 \cdot gm$$

And new concentrations of the dried biomaterial are:

$$x_{prdbm} := \frac{m_{prdbm}}{M_{dc}} \qquad x_{Asdbm} := \frac{m_{Asdbm}}{M_{dc}} \qquad x_{wdbm} := \frac{m_{wdbm}}{M_{dc}}$$

$$x_{fatdbm} := \frac{m_{fatdbm}}{M_{dc}} \qquad x_{cadbm} := \frac{m_{cadbm}}{M_{dc}}$$

Checking $x_{prdbm} + x_{cadbm} + x_{Asdbm} + x_{fatdbm} + x_{wdbm} = 1$

$$\rho_{dbm} := \frac{1}{\frac{x_{prdbm}}{\rho_p} + \frac{x_{cadbm}}{\rho_{carb}} + \frac{x_{Asdbm}}{\rho_{As}} + \frac{x_{fatdbm}}{\rho_{fat}} + \frac{x_{wdbm}}{\rho_w}}$$

$$\rho_{dbm} = 1136.9\,\frac{kg}{m^3}$$

And the apparent porosity of the biomaterial can be calculated as:

$$\varepsilon_{app\_dbm} := 1 - \frac{\rho_{app\_driedbm}}{\rho_{dbm}} \qquad \boxed{\varepsilon_{app\_dbm} = 0.51}$$