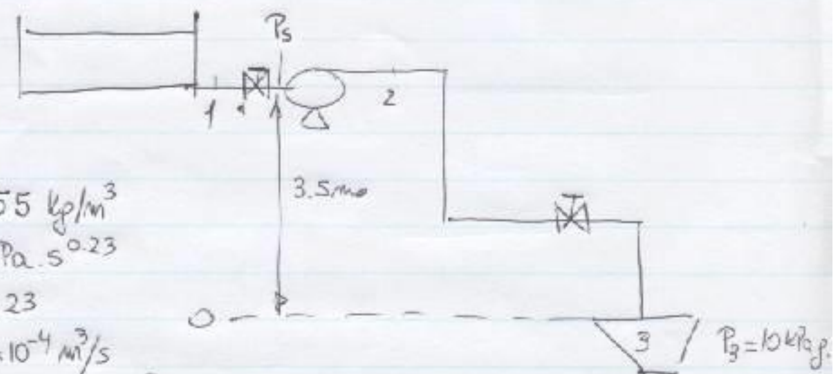
TUTORIAL MECHANICAL ENERGY BALANCE

ABE 303 TUTORIAL
Flow in Pipes

1-1

Problem 1

(a) schematic system



$\rho = 1055\ kg/m^3$

$K = 71\ Pa.s^{0.23}$

$n = 0.23$

$Q = 6 \times 10^{-4}\ m^3/s$

$D = 5cm = 5 \times 10^{-2}\ m$

$L_{eb} = 30D$

$L_{ev} = 36D$

$L = 17\ m$

(b) Required discharge pressure of the pump (P_2)

An energy balance between the locations 2 and 3 gives:

$$\Delta P_{pump} = (P_3 - P_2) + \Delta P_{s\,2\to3} + \Delta P_{K\,2\to3} + \Delta P_{f\,2\to3}$$

Because between 2 and 3 there is no pump $\Delta P_{pump} = 0$
therefore

$$P_2 = P_3 + \Delta P_{s\,2\to3} + \Delta P_{K\,2\to3} + \Delta P_{f\,2\to3} \quad (1)$$

Since the diameter does not change $\Delta P_{K 2\to3} = 0$ 1-2

therefore Eq. (1) becomes:

$$P_2 = P_3 + \Delta P_{S 2\to3} + \Delta P_{f 2\to3} \qquad (2)$$

$$\Delta P_{S 2\to3} = \rho g (h_3 - h_2)$$

and

$$\Delta P_{f 2\to3} = \frac{2LK}{R}\left(\frac{3n+1}{4n}\right)^n \left(\frac{4Q}{\pi R^3}\right)^n$$

In the section 2-3 there are 17 m of straight pipe, 3 90° bends and one butterfly valve so

$$L_e = 17 + 30D + 3 \times 36D$$

$$L_e = 17 + 30 \times 0.05 + 3 \times 36 \times 0.05 = 23.9\,m$$

substituting values into Eq (2)

$$P_2 = 10\,KPa + \frac{2 \times 23.9 \times 71}{0.025} \times \left(\frac{3 \times 0.23 + 1}{4 \times 0.23}\right)^{0.23} \times \left(\frac{4 \times 6 \times 10^{-4}}{\pi \times 0.025^3}\right)^{0.23} * \frac{1\,KPa}{10^3\,Pa}$$

$$+ 1055 \times 9.81 \times (-3.5) \times \frac{1\,KPa}{1000\,Pa}$$

$$P_2 = 10 + 382 - 36.2 \cong 356\,KPa\,g$$

(c) A mechanical energy balance between 1 and 3 gives:

$$\Delta P_{pump} = (P_3 - P_1) + \Delta P_{K 1\to3} + \Delta P_{S 1\to3} + \Delta P_{f 1\to3} \qquad (3)$$

Since the preparation tank is open $P_1 = P_0$ so $P_1 = 0\,KPa\,g$

diameter does not change so $\Delta P_{K 1\to3} = 0$

$\Delta P_{s\,1\to3} = \rho g (h_3 - h_1)$ 1-3

$\Delta P_{s\,1\to3} = 1055 \times 9.81 (-3.5 - 0) = 36{,}223.4\ Pa = 36.2\ kPa$

$\Delta P_{s\,1\to3} = -36.2\ kPa$

$$\Delta P_{f\,1\to3} = \frac{2LK}{R}\left(\frac{3n+1}{4n}\right)^n \left(\frac{4Q}{\pi R^3}\right)^n$$

all the variables are the same except the length L because we need to incorporate the second valve.
the new value of L is :

$L = 23.9 + 36D = 23.9 + 36 \times 0.05 = 25.7\ m$

the pressure loss is now calculated as:

$$\Delta P_{f\,1\to3} = \frac{2 \times 25.7 \times 71}{0.025}\left(\frac{3 \times 0.23 + 1}{4 \times 0.23}\right)^{0.23} \times \left(\frac{4 \times 6 \times 10^{-4}}{\pi \times 0.025^3}\right)^{0.23}$$

$\Delta P_{f\,1\to3} = 410{,}723.7\ Pa$

$\Delta P_{f\,1\to3} = 410.7\ kPa$

substituting values into Eq. (3)

$\Delta P_{pump} = 10\ kPa - 36.2 + 410.7 = 384.5\ kPa$

$\Delta P_{pump} = 384.5\ kPa$

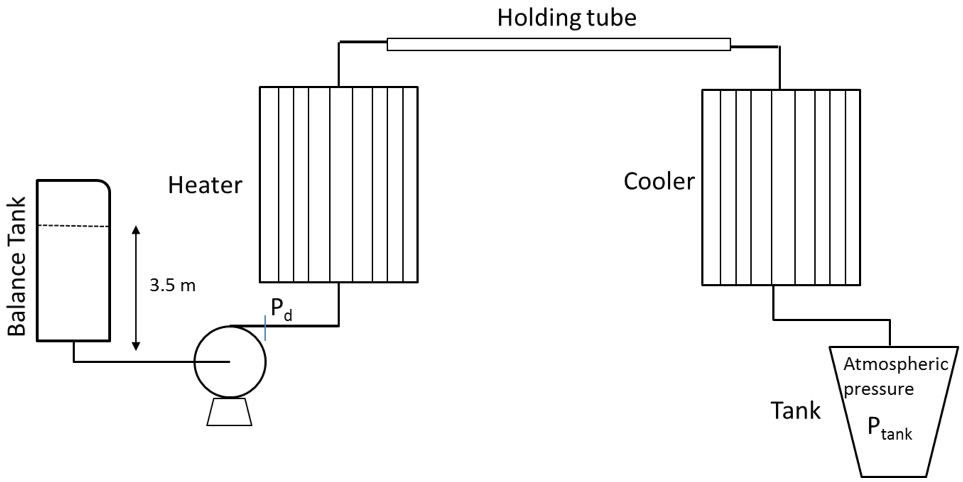
the Power in the pump is :

$Power = \Delta P_{pump} \times Q = 384.5 \times 6 \times 10^{-4}\ kPa \times \frac{m^3}{s}$

$\underline{Power = 0.23\ Kwatts}$

Problem 2

(a)



(b) Calculations were done by using MathCad

Unit Conversions $\quad lt := 10^{-3} \cdot m^3 \qquad hr := 3600 \cdot s$

Data $\quad Q := 4000 \cdot \frac{lt}{hr} \qquad Q = 1.111 \times 10^{-3} \frac{m^3}{s}$

$D_p := 5.5 \cdot cm \qquad D_p = 0.055\,m \qquad D_t := 2 \cdot cm \qquad D_t = 0.02\,m$

$\rho := 1066 \cdot \frac{kg}{m^3} \qquad Le := 160 \cdot D_p \qquad L_p := 30 \cdot m$

$L_{heater} := 3 \cdot m \qquad L_{cooler} := 3 \cdot m \qquad t_{holding} := 20 \cdot s$

$k := 18.7 \cdot Pa \cdot s^{0.28} \qquad n := 0.28$

Let's calculate the critical Reynolds Number

$$N_{Rec} := \frac{2100 \cdot (4 \cdot n + 2) \cdot (5 \cdot n + 3)}{3 \cdot (1 + 3 \cdot n)^2} \qquad N_{Rec} = 2.838 \times 10^3$$

Let's calculate the Generalized Reynolds number in the pipe, because the material is a power law fluid n'=n and k' = k((3n'+1)/4n')$^{n'}$

$$n' := n \qquad k' := k \cdot \left(\frac{3n' + 1}{4 \cdot n'}\right)^{n'} \qquad k' = 21.489 \frac{kg}{m \cdot s^{1.72}}$$

$$u_p := \frac{Q}{\pi \cdot \frac{D_p^2}{4}} \qquad u_p = 0.468 \frac{m}{s}$$

Because of a problem with units in Mathcad when the power s not an integer, let's get rid of the units but express the variables in metric units.

$\rho_{nu} := 1066 \qquad D_{pnu} := 0.055 \qquad u_{pnu} := 0.468 \qquad k'_{nu} := 21.489$

$$N_{Regen\_pipe} := \frac{D_{pnu}^{n'} \cdot u_{pnu}^{2-n'} \cdot \rho_{nu}}{k'_{nu} \cdot 8^{n'-1}} \qquad N_{Regen\_pipe} = 26.662$$

In the pipe $N_{Regen} < N_{Rec}$ so the flow is laminar in the pipe

Let's calculate now the Generalized Reynolds Number in the tube of the heater and cooler. In order to calculate the flow in the heater/cooler we need to divide the total flow by the number of tubes, so:

$$Q_{h\_c} := \frac{Q}{20} \qquad Q_{h\_c} = 5.556 \times 10^{-5} \frac{m^3}{s} \qquad Q_{hcnu} := 5.556 \cdot 10^{-5}$$

$$u_{h\_c} := \frac{Q_{h\_c}}{\pi \cdot \frac{D_t^{\,2}}{4}} \qquad u_{h\_c} = 0.177 \frac{m}{s} \qquad u_{h\_c\_nu} := 0.177$$

$$N_{Regen\_h\_c} := \frac{D_{pnu}^{\,n'} \cdot u_{h\_c\_nu}^{\,2-n'} \cdot \rho_{nu}}{k'_{nu} \cdot 8^{n'-1}} \qquad N_{Regen\_h\_c} = 5.007$$

In the tubes of the heater/cooler $N_{Regen} < N_{Rec}$ so the flow is laminar

(c) Calculate the length of the holding tube

The maximum velocity in the holding tube is

$$u_{max} := u_p \frac{3 \cdot n + 1}{n + 1} \qquad u_{max} = 0.672 \frac{m}{s}$$

$$L_{H\_T} := u_{max} \cdot t_{holding} \qquad L_{H\_T} = 13.446 m$$

(d) Estimate the discharge pressure (P_d)

A balance of the mechanical energy between the pressure discharge and the filling tank at atmospheric pressure P_0 will be:

$$\Delta P_{pump} = P_d - P_0 + \frac{1}{2\alpha}\rho\left(\bar{u}_o^2 - \bar{u}_d^2\right) + \rho g\left(h_0 - h_d\right) + \Delta p_{f\_d->0}$$

But between d and 0 there is no pump, also from the figure $h_d = h_0$ and if the diameter of the pipe does not change at the discharge and the entrance to the filling tank the corresponding medium velocities are the same. Thus, the mechanical balance simplifies to:

$$P_d - P_0 = \Delta P_{f\_d->0}$$

Since P_0 is subtracted from P_d the result will be in gauge pressure. In addition:

$$\Delta P_{f\_d->0} = \Delta P_{f,heater} + \Delta P_{pipeline} + \Delta P_{H\_T} + \Delta P_{f,cooler}$$

$$\Delta P_{f\_heater} := \frac{2knu \cdot L_{heater\_nu}}{\frac{D_{tnu}}{2}} \cdot \left(\frac{3 \cdot n + 1}{4 \cdot n}\right)^n \cdot \left[\frac{4 \cdot Q_{hcnu}}{\pi \cdot \left(\frac{D_{tnu}}{2}\right)^3}\right]^n$$

$\Delta P_{f\_heater} = 4.249 \times 10^4$ By dividing this value by 1000 the result is in kPa

$$\Delta P_{f\_pipeline} := \frac{2knu \cdot \left(L_{pipe\_nu} + Le_{nu}\right)}{\frac{D_{pnu}}{2}} \cdot \left(\frac{3 \cdot n + 1}{4 \cdot n}\right)^n \cdot \left[\frac{4 \cdot Q_{nu}}{\pi \cdot \left(\frac{D_{pnu}}{2}\right)^3}\right]^n$$

$\Delta P_{f\_pipeline} = 1.976 \times 10^5$ By dividing this value by 1000 the result is in kPa

$$\Delta P_{f\_H\_T} := \frac{2knu \cdot L_{H\_Tnu}}{\frac{D_{pnu}}{2}} \cdot \left(\frac{3 \cdot n + 1}{4 \cdot n}\right)^n \cdot \left[\frac{4 \cdot Q_{nu}}{\pi \cdot \left(\frac{D_{pnu}}{2}\right)^3}\right]^n$$

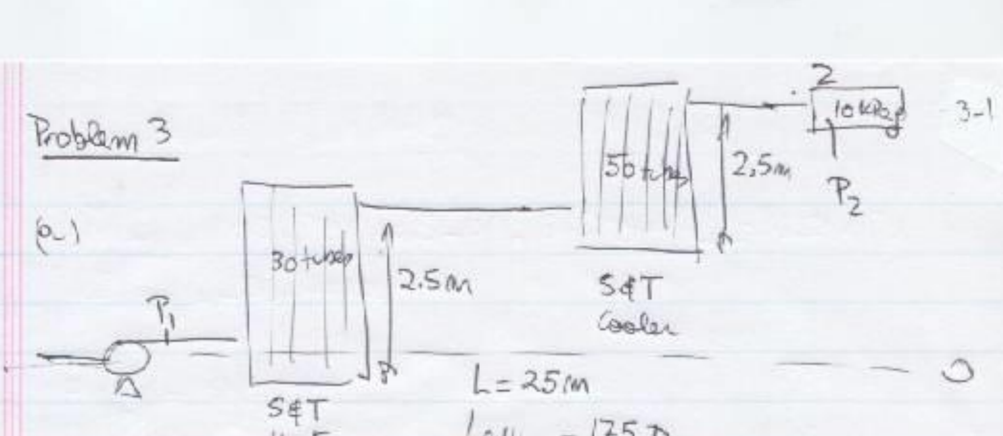
$\Delta P_{f\_H\_T} = 6.849 \times 10^4$

$$P_{d\_gauge} := \frac{\left(2 \cdot \Delta P_{f\_heater} + \Delta P_{f\_pipeline} + \Delta P_{f\_H\_T}\right)}{1000}$$

$P_{d\_gauge} = 351.107$ kPa

Problem 3

(a)



$L = 25\,m$

$L_{fitting} = 175\,D$

$D = 0.06\,m$

$Q = 2500\ l/h = \dfrac{2{,}500 \times 10^{-3}\ m^3}{3600\ s}$

$Q \cong 7 \times 10^{-4}\ m^3/s$

$\rho = 1120\ kg/m^3$

Heat Exchangers

Tube ID $= 0.015\ m$

tube length $= 2.5\ m$

(b) A mechanical balance between 1 and 2 gives:

$$\Delta P_{pump} = (P_2 - P_1) + \Delta P_{k\,1\to2} + \Delta P_{s\,1\to2} + \Delta P_{f,1\to2} + \Delta P_{heater} + \Delta P_{f\,cooler} \quad (1)$$

between 1 and 2 there is no pump so $\Delta P_{pump} = 0$, also there is no change in diameter $\Delta P_{k\,1\to2} = 0$

From Equation (1)

$$P_1 = P_2 + \Delta P_{s\,1\to2} + \Delta P_{f\,1\to2} + \Delta P_{f\,heater} + \Delta P_{f\,cooler} \quad (2)$$

$$\Delta P_{s\,1\to2} = \rho g (h_2 - h_1) = 1120 \times 9.81\,(5 - 0) = 54{,}936\ Pa$$

$$\Delta P_{s\,1\to2} = 54.93\ kPa \approx 55\ kPa$$

$\underline{\Delta P_{f\,1\to 2}}$

we know the curve $\frac{8\bar{u}}{D}$ versus τ_w, so let's calculate $\bar{u}$

$$\bar{u} = \frac{Q}{\pi R^2} = \frac{7\times10^{-4}}{\pi\times 0.03^2} = 0.25\ \frac{m}{s}$$

and $$\frac{8\bar{u}}{D} = \frac{8\times 0.25\ m/s}{0.06\ m} \cong 33\ 1/s$$

From a plot $\frac{8\bar{u}}{D}$ versus $\tau_w \Longrightarrow \tau_w \cong 230\ Pa$

$$L = 25 + 175D = 25 + 175\times 0.06 = 35.5\ m$$

So to calculate ΔP_f we can use

$$\tau_w = \frac{\Delta P_f \times R}{2L} \Longrightarrow \Delta P_f = \frac{2L\times\tau_w}{R}$$

$$\Delta P_f = \frac{2\times 35.5\times 230}{0.03} = 544{,}333.3\ Pa$$

$$\underline{\Delta P_{f\,1\to2} = 544.3\ kPa}$$

$\underline{\Delta P_{heater}}$

Flow in a tube $= q_h = \frac{Q}{30} = \frac{7\times10^{-4}}{30}\ \frac{m^3}{s} = 2.33\times10^{-5}\ m^3/s$

mean velocity $$\bar{u}_h = \frac{q_h}{\pi R^2} = \frac{2.33\times10^{-5}\ m^3/s}{\pi\times\left(\frac{0.015}{2}\right)^2} = 0.13\ m/s$$

$$\frac{8\bar{u}_h}{D} = \frac{8\times 0.13}{0.015} = 70.3\ 1/s$$

From the plot $\frac{8\bar{u}_h}{D} = 70.3\ 1/s \longrightarrow \tau_w = 315\ Pa$

the pressure loss can be calculated as: 3-3

$$\tau_w = \frac{\Delta P_{heater} \times R}{2L} \Rightarrow \Delta P_{heater} = \frac{2L\,\tau_w}{R}$$

$$\Delta P_h = \frac{2 \times 2.5 \times 315}{\frac{0.015}{2}} = 210{,}000\ Pa = 210\ KPa$$

$\underline{\Delta P_{cooler}}$

Flow in a tube $= \frac{Q}{50} = q_H = \frac{7\times10^{-4}\ m^3/s}{50} = 1.40\times10^{-5}\ m^3/s$

the mean velocity $\bar{u}_c$ is: $\bar{u}_c = \frac{q_H}{\pi R^2} = \frac{1.40\times10^{-5}\ m^3/s}{\pi\times\left(\frac{0.015}{2}\right)^2} = 0.08\ \frac{m}{s}$

and

$$\frac{8\bar{u}_c}{D} = \frac{8\times0.08}{0.015} = 42.7\ 1/s$$

From the plot $\frac{8\bar{u}_c}{D}$ versus $\tau_w \Longrightarrow \tau_w \approx 250\ Pa$

and

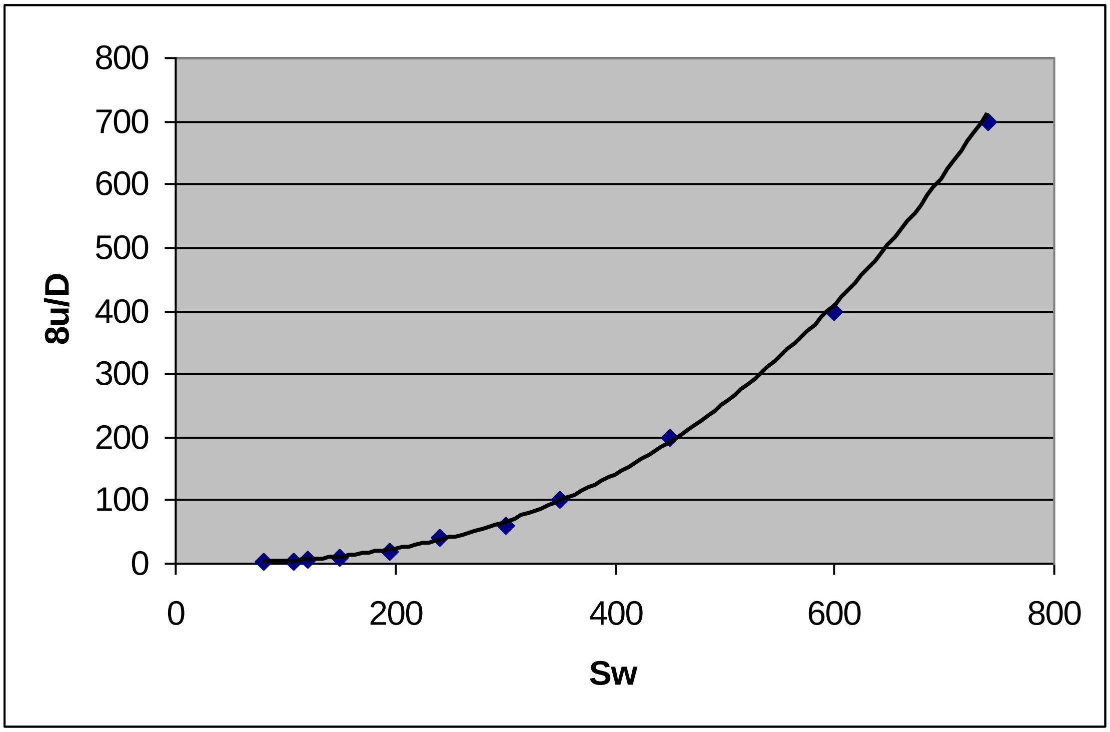
$$\Delta P_{cooler} = \frac{\tau_w \times 2L}{R} = \frac{250\times2\times2.5}{\frac{0.015}{2}} = 166{,}666.7\ Pa$$

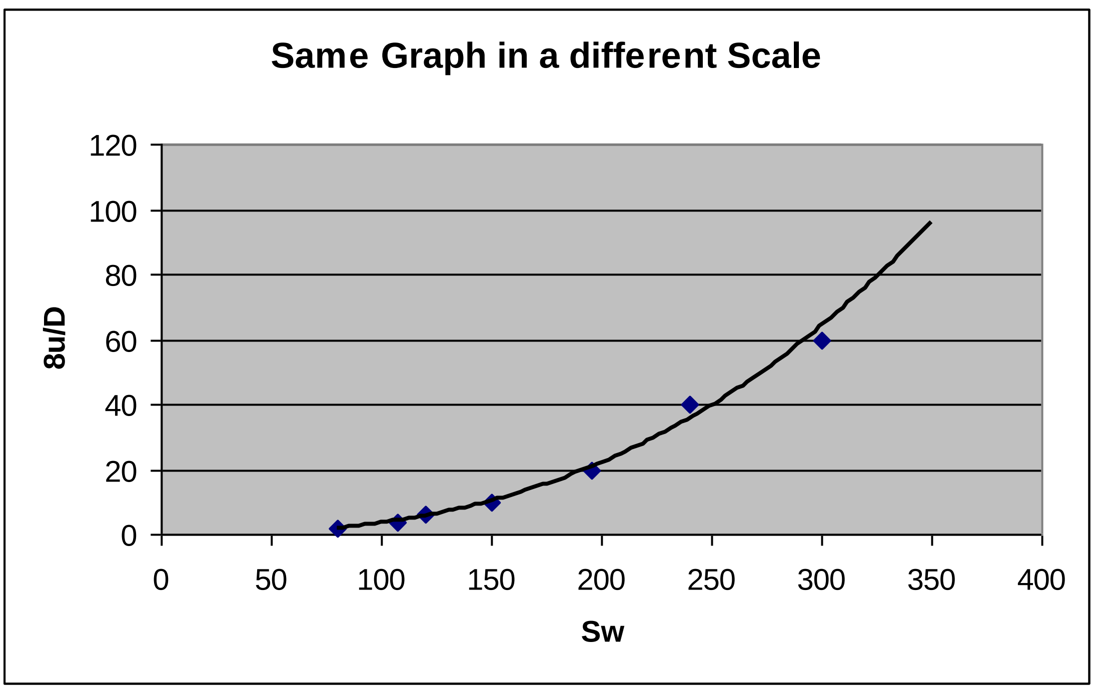
$$\Delta P_{cooler} = \underline{166.7\ KPa}$$

substituting values into Eq.(2)

$P_1 = 10\ KPag + 55\ KPa + 544.3 + 210 + 166.7$

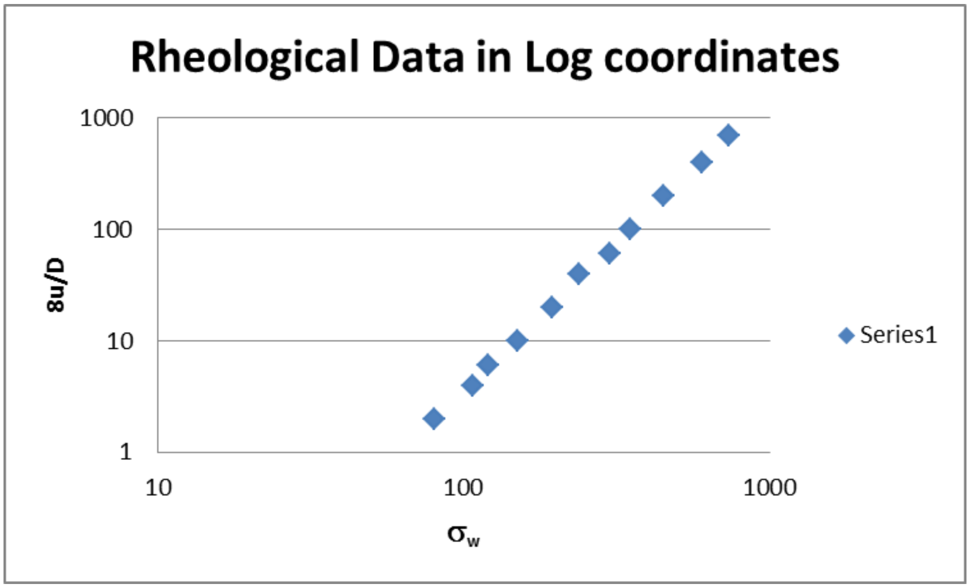
$\underline{P_1 = 986\ KPag}$



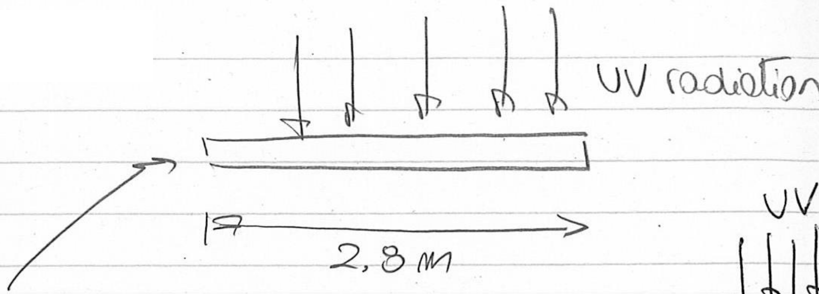


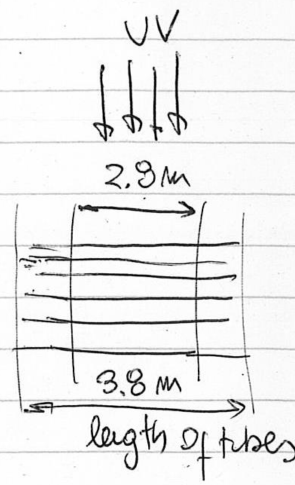
(c) To find the rheological behavior of the material we could plot the data in logarithmic coordinates and we will see that we obtain a straight line so the material is a power law rheological material and the slope of the graph is equal to 1/n where n is the flow index.

A linear regression on below data gives a value of $1/n = 2.66$ and $n = 0.38$ so the material is shear thinning, for a Newtonian $n = 1$.



Problem 4





Let's Consider one tube in the bank

$K = 21\ Pa.s^{0.2}$

$n = 0.2$

(i) Tubes required

$$L = u_{max} \times t \Rightarrow u_{max} = \frac{L}{t} = \frac{2.8\ m}{7\ s} = 0.4\ \frac{m}{s}$$

$$u_{max} = \bar{u}\ \frac{3n+1}{n+1} \Rightarrow \bar{u} = u_{max}\ \frac{n+1}{3n+1}$$

$$\bar{u} = 0.4\ \frac{m}{s} \times \frac{0.2+1}{3\times 0.2+1} = 0.30\ \frac{m}{s}$$

$$\bar{u} = \frac{Q_{Tube}}{\pi R^2} \Rightarrow Q_{Tube} = \bar{u} \times \pi R^2$$

$$Q_{Tube} = 0.30\ m/s \times \pi \times (4.5\times 10^{-3})^2\ m^2$$

$$Q_{Tube} = 1.909\times 10^{-5}\ m^3/s$$

$$N_{Tubes} = \frac{Q_{TOTAL}}{Q_{TUBE}} = \frac{9000 \times 10^{-3}\ m^3}{3600\,s \times 1.909 \times 10^{-5}\ m^3/s}$$

$N_{Tubes} \cong 131$ Tubes.

(ii)

$$\Delta P_f = \frac{2LK}{R}\left(\frac{3n+1}{4n}\right)^n\left(\frac{4Q}{\pi R^3}\right)^n$$

$$\Delta P_f = \frac{2 \times 3.8\,m \times 21\,Pa.s}{4.5 \times 10^{-3}\,m}\left(\frac{3 \times 0.2 + 1}{4 \times 0.2}\right)^{0.2}\left(\frac{4 \times 1.909 \times 10^{-5}}{\pi \times (4.5 \times 10^{-3})^3}\right)^{0.2}$$

$\Delta P_f = 124,521$ Pa

$$\boxed{\Delta P_f = 124.52\ KPa}$$