Solution homework 6 - Fall 2017

Question 1

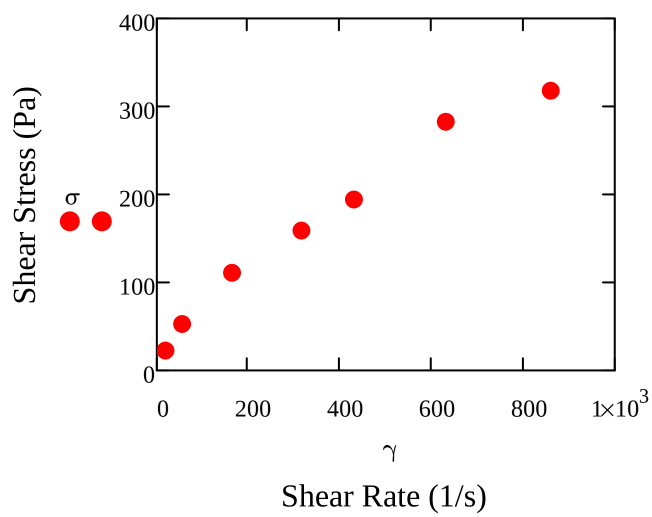
$mN := 10^{-3} \cdot N$ $\qquad \theta := 2 \cdot deg$ $\qquad \theta = 0.035 \cdot rad$

$R_c := 30 \cdot mm$

(a)

$$\Omega_c := \begin{pmatrix} 0.63 \\ 1.9 \\ 5.7 \\ 11 \\ 15 \\ 22 \\ 30 \end{pmatrix} \cdot \frac{rad}{s} \qquad M := \begin{pmatrix} 1.3 \\ 3 \\ 6.3 \\ 9 \\ 11 \\ 16 \\ 18 \end{pmatrix} \cdot mN \cdot m$$

$$\gamma := \frac{\Omega_c}{\theta} \qquad \gamma = \begin{pmatrix} 18 \\ 54.4 \\ 163.3 \\ 315.1 \\ 429.7 \\ 630.3 \\ 859.4 \end{pmatrix} \frac{1}{s} \qquad \sigma := \frac{3 \cdot M}{2\pi \cdot R_c^{\,3}} \qquad \sigma = \begin{pmatrix} 22.989 \\ 53.052 \\ 111.408 \\ 159.155 \\ 194.523 \\ 282.942 \\ 318.31 \end{pmatrix} Pa$$



(b) Determine the rheological parameters of the liquid

We can estimate a power regression to the shear stress versus shear rate data illustrated in the above plot. In MathCad that is done with the genfit function. Variables cannot contain units so below are defined shear rate and shear stress withut unis

$$\gamma_{nu} := \gamma \cdot s \qquad \sigma_{nu} := \sigma \cdot \frac{1}{Pa}$$

$$f(x,k,n) := k \cdot x^{n} \qquad guess := \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$Fit := genfit(\gamma_{nu}, \sigma_{nu}, guess, f)$$

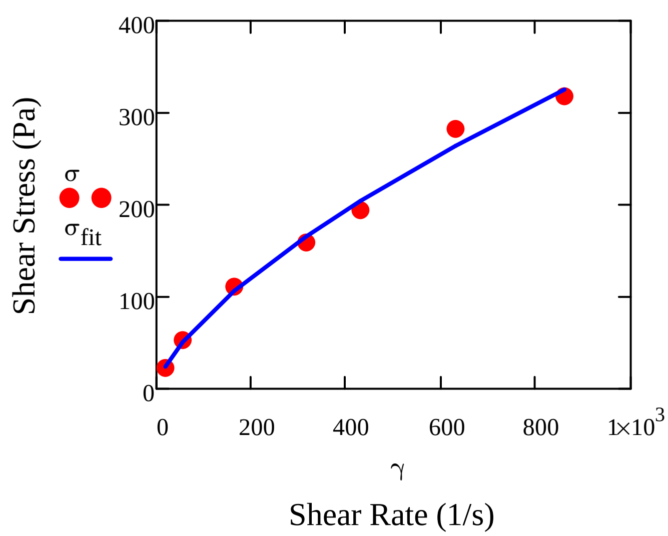
Result of the Fit $\quad Fit = \begin{pmatrix} 3.481 \\ 0.672 \end{pmatrix} \qquad k_{nu} := Fit_0 \qquad n_{nu} := Fit_1$

$k = 3.481$ Pa $s^{0.672}$

$n = 0.672$

$$\sigma_{fit} := k_{nu} \cdot \gamma_{nu}^{\,n_{nu}} \cdot Pa \qquad \sigma_{fit} = \begin{pmatrix} 24.303 \\ 51.013 \\ 106.699 \\ 165.937 \\ 204.37 \\ 264.327 \\ 325.55 \end{pmatrix} Pa$$

Now the experimental data can be plotted with the fitted model



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Question 2

This problem can be solved using excell and graphically, i.e. plot the data in excel and graphically solve the problem to estimate the shear stress at the wall and the shear rate. to avoid using plots instead this problem was using tools in MathCad that allow us directly interpolate the data used to construct the plot.

Let's first create two vectors, one is the pressure drop called Δp and the volumetric flowrate that wewill call Q_v, in addition we are incorporating units to these variables

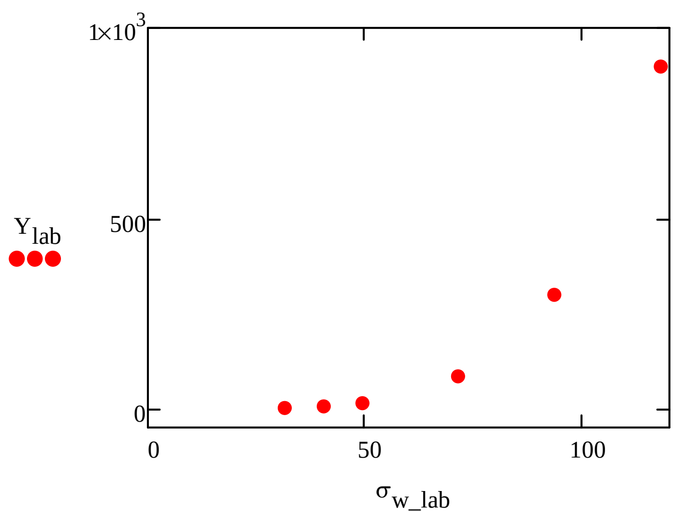
$$\Delta p := \begin{pmatrix} 18.9 \\ 24.3 \\ 29.6 \\ 42.8 \\ 56.1 \\ 70.8 \end{pmatrix} \cdot kPa \qquad Q_v := \begin{pmatrix} 1.81\cdot 10^{-7} \\ 5.89\cdot 10^{-7} \\ 1.47\cdot 10^{-6} \\ 8.34\cdot 10^{-6} \\ 2.94\cdot 10^{-5} \\ 8.83\cdot 10^{-5} \end{pmatrix} \frac{m^3}{s}$$

$$D_{p\_lab} := 0.01\cdot m \qquad L_{lab} := 1.5\cdot m$$

To perform the scale up procedure we have to plot $8u/D_p$ as a functionof σ_w. For convenience we will call those parameters Y_{lab} and σ_{w_lab}, respectively. We need to calculate also the mean velocity that will be called u_{mean}

$$u_{mean} := \frac{Q_v}{\pi\cdot\left(\frac{D_{p\_lab}}{2}\right)^2} \qquad Y_{lab} := \frac{8\cdot u_{mean}}{D_{p\_lab}} \qquad \sigma_{w\_lab} := \frac{\Delta p\cdot\frac{D_{p\_lab}}{2}}{2\cdot L_{lab}}$$

Let's plot the data



As discussed once the plot is constructed we can get the information directly from the graph or by interpolation the data. Another option would be to fit the experimental data by an equation and get the parameters of the equation, A good fitting approach is the power-law model given by the equation $Y_{lab} = aX^b$ where X is σ_{w_lab}

The following is taken from the help in MathCad Help to fit a power law equation to the data in the plot. See also Question 1:

genfit(vx,vy,vg.F) return a vector containing the parameters that make the F of X. vx is the x-data and vy is the y-data, whereas vg is the vector with solutions. As in question 1 we need to get rid of the units

$$\Delta p_{no\_untis} := \frac{\Delta p}{Pa} \qquad Q_{v\_no\_units} := \frac{Q_v}{\frac{m^3}{s}} \qquad D_{p\_lab\_no\_units} := \frac{D_{p\_lab}}{cm}$$

$$L_{lab\_no\_units} := \frac{L_{lab}}{m} \qquad u_{mean\_no\_units} := \frac{Q_{v\_no\_units}}{\pi \cdot \left(\frac{D_{p\_lab\_no\_units}}{2}\right)^2}$$

$$Y_{lab\_no\_units} := \frac{Y_{lab}}{\frac{1}{s}} \qquad \sigma_{w\_no\_units} := \frac{\sigma_{w\_lab}}{Pa}$$

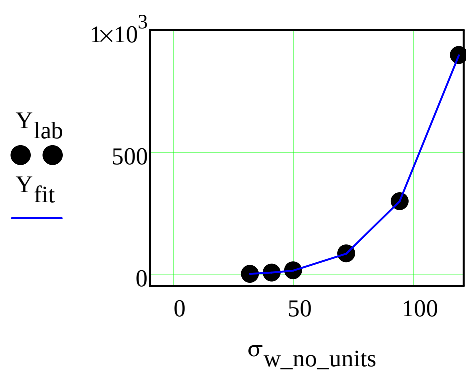
$$guess := \begin{pmatrix} 11 \\ 10 \end{pmatrix} \qquad f_{question\_2}(x,a,b) := a \cdot x^b$$

$$Y_{model\_fit} := genfit\left(\sigma_{w\_no\_units}, Y_{lab\_no\_units}, guess, f_{question\_2}\right)$$

$$Y_{model\_fit} = \begin{pmatrix} 1.536 \times 10^{-7} \\ 4.714 \end{pmatrix}$$:--- a value <--- b value

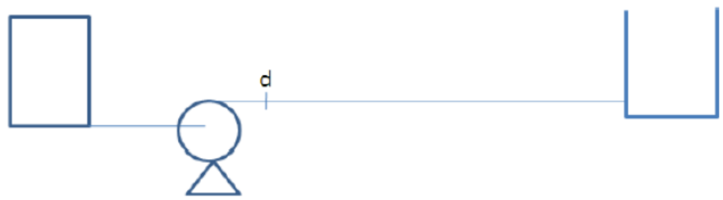
$$Y_{fit} := 1.536 \cdot 10^{-7} \cdot \sigma_{w\_no\_units}^{\ 4.714}$$

Let's plot the experimental data and the model to check the model accuracy



The fit of the experimentl is very good as observed in the above graph

The factory situation is given in the figure below:



The static pressure drop, the kinetic pressure drop and the pressure drop in the pump, all can be considered to be 0. The pressure drop in the pump is zero because there is no pump betwween the pump discharge (d) and the exit. The balance of energy will be p_d-p_o=Δp_f where p_o is the atmospheric pressure, therefore p_{d_gauge}. Δp_f can be calculated from the factory situation where the dimensions of the factory piping system are known, and we can use the rheological model given by Y_{fit}

$$D_{factory} := 6\cdot cm \qquad L_{factory} := 65\cdot m \qquad Q_{factory} := 70\cdot\frac{l}{hr}$$

$$Q_{fac\_no\_units} := \frac{Q_{factory}}{\frac{m^3}{s}} \qquad u_{mean\_fac} := \frac{Q_{factory}}{\pi\cdot\left(\frac{D_{factory}}{2}\right)^2} \qquad Y_{factory} := \frac{8\cdot u_{mean\_fac}}{D_{factory}}$$

$$Y_{fac\_no\_units} := \frac{Y_{factory}}{\frac{1}{s}} \qquad \sigma_{w\_fac\_no\_units} := \left(\frac{Y_{fac\_no\_units}}{1.536\cdot 10^{-7}}\right)^{\frac{1}{4.714}}$$

$$\boxed{\sigma_{w\_fac\_no\_units} = 27.4} \qquad \sigma_{w\_fac} := \sigma_{w\_fac\_no\_units}\cdot Pa$$

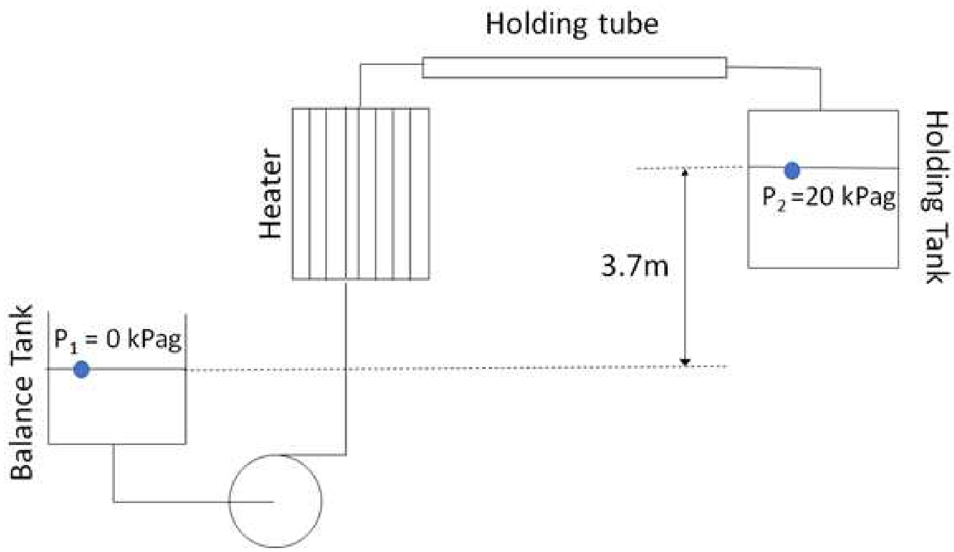
With that value we can calculate the pressure loss in the pipe

$$\Delta p_{f\_factory} := \frac{\sigma_{w\_fac}\cdot 2\cdot L_{factory}}{\frac{D_{factory}}{2}} \qquad \Delta p_{f\_factory} = 118.64\cdot kPa$$

$$p_d := \Delta p_{f\_factory} \qquad \boxed{p_d = 118.64\cdot kPa}$$

Question 3

(a) Draw a diagram of the system



(b)

$$\Delta p_{pump} = (p_2 - p_1) + \Delta p_{k1\text{-}2} + \Delta p_{s1\text{-}2} + \Delta p_{f1\text{-}2}$$

Since 1 and 2 are over the surfaces, and they are not moving the fluid velocities at 1 and 2 are zero and $\Delta p_{k1\text{-}2}=0$. If we consider the level zero at the level 1, then $h_1 = 0$ and $h_2 = 3.7$m

$\rho_{fluid} := 995\cdot\frac{kg}{m^3}$ $\quad Q := 4000\cdot\frac{l}{hr}$ $\quad \eta_{pump} := 0.8$ $\quad \eta_{motor} := 0.75$

$h_1 := 0\cdot m$ $\quad h_2 := 3.7\cdot m$

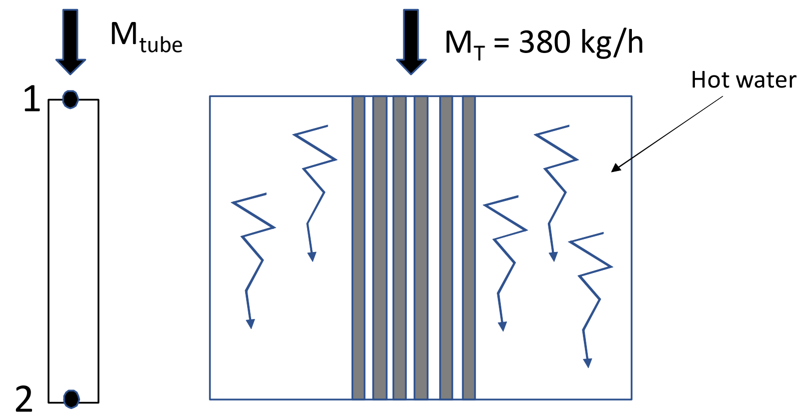
$\Delta p_{s1\_2} := \rho_{fluid}\cdot g\cdot(h_2 - h_1)$ $\quad \Delta p_{s1\_2} = 36.103\cdot kPa$

$p_1 := 0\cdot kPa$ $\quad p_2 := 20\cdot kPa$ $\quad \Delta p_{f1\_2} := 1250\cdot kPa$

$\Delta p_{pump} := (p_2 - p_1) + \Delta p_{s1\_2} + \Delta p_{f1\_2}$ $\quad \Delta p_{pump} = 1306.1\cdot kPa$

$$Power := \frac{\Delta p_{pump}\cdot Q}{\eta_{pump}\cdot\eta_{motor}}$$ $\quad \boxed{Power = 2.4\cdot kW}$

Question 4



$D_{tube} := 4 \cdot cm$ $\qquad \rho_f := 870 \cdot \frac{kg}{m^3}$ $\qquad M_T := 380 \cdot \frac{kg}{hr}$

$n := 0.32$ $\qquad k := 90 \cdot Pa \cdot s^{0.32}$ $\qquad Q_T := \frac{M_T}{\rho_f}$

Let's consider a balance of mechanical energy between poits 1 and 2 for a tube in the diagram shown above

$\Delta p_{pump} = (p_2 - p_1) + \rho_f\, g\, (h_2 - h_1) + 1/2\, \rho_f\, (u_2 - u_1)/\alpha + \Delta p_{f1\text{-}2}$

Because the pipe is open to the atmosphere p_1 and p_2 are equals to the atmospheric pressure and $p_2 - p_1 = 0$. Also $h_2 - h_1 = -L$, where L is the length of the pipe. The diameters of the tube at 1 and 2 are equals there the kinetic change is zero, as well as Δp_{pump} because there is no pump betweem 1 and 2. Therefore if you use the mechanical balance between 1 and 2 we get $\quad -\rho_f\, g\, L + \Delta p_{f1\text{-}2} = 0 \quad$ ---> $\quad \rho_f\, g\, L = 2Lk/R\ .\ ((3n+1)/4n)^n\ .\ ((4Q_{tube})/\pi R^3_{tube})^n$

from the above equaion we can calculate Q_{tube}

$$Q_{tube} := \left[\frac{\rho_f \cdot g \cdot \frac{D_{tube}}{2}}{2 \cdot k} \cdot \left(\frac{4 \cdot n}{3n + 1}\right)^n\right]^{\frac{1}{0.32}} \cdot \pi \cdot \frac{\left(\frac{D_{tube}}{2}\right)^3}{4}$$

$$Q_{tube} = 3.472 \times 10^{-6}\, \frac{m^3}{s}$$

$$N_{tubes} := \frac{Q_T}{Q_{tube}} \qquad \boxed{N_{tubes} = 35}$$