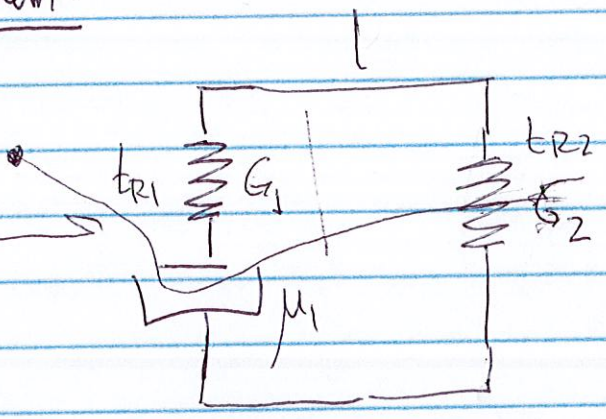
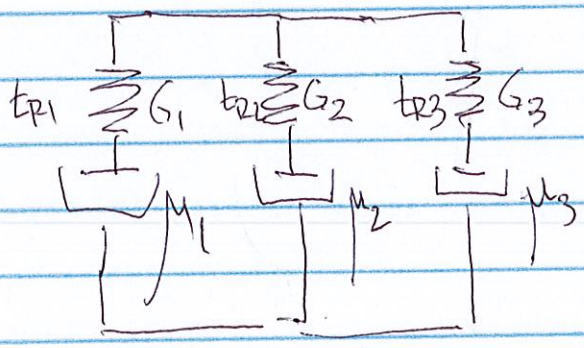
Problem 1



$$t_{R1} = \frac{\mu_1}{G_1}$$

$$t_{R2} = \frac{\mu_2}{G_2}$$

Generalized Maxwell model (3 elements)



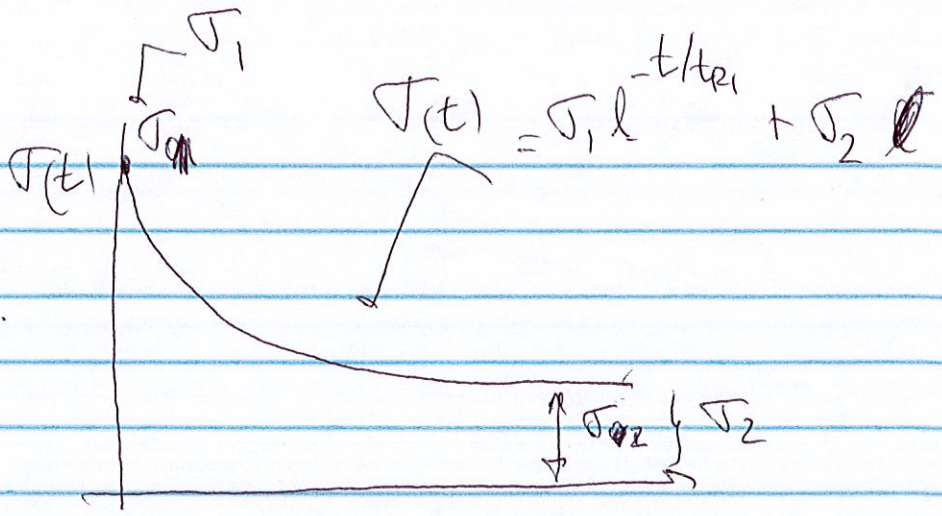
Problem

(a) $\sigma(t) = \sigma_{01} e^{-t/t_{R1}} + \sigma_{02} e^{-t/t_{R2}}$

t_{R2} is very large $t_{R2} \to \infty$ [Because there is a elastic element in branch 2]

$\sigma(t) = \sigma_{01} e^{-t/t_{R1}} + \sigma_{02} e^{0}$

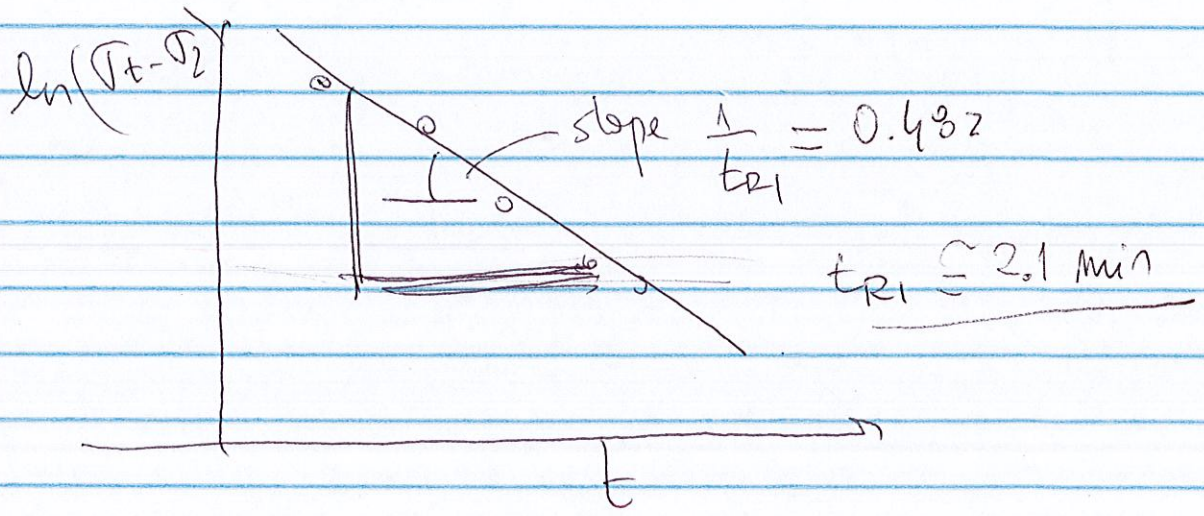
$e^{0} = 1$

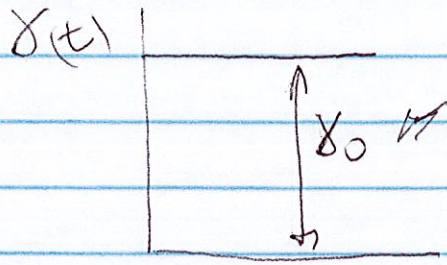


$$\sigma(t) = \sigma_1 e^{-t/t_{R1}} + \sigma_2 \qquad (2)$$

$$\sigma(t) - \sigma_2 = \sigma_1 e^{-t/t_{R1}}$$

$$\ln(\sigma(t) - \sigma_2) = \ln \sigma_1 - \frac{t}{t_{R1}}$$

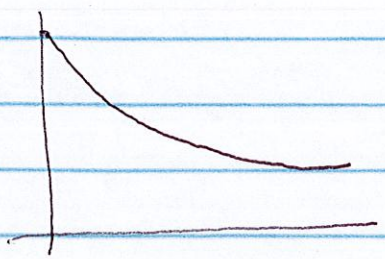




Not given, so assume $\gamma_0 = 1$

$$t_{R1} = \frac{\mu_1}{G_1}$$

$$\mu_1 = t_{R1} \times G_1$$



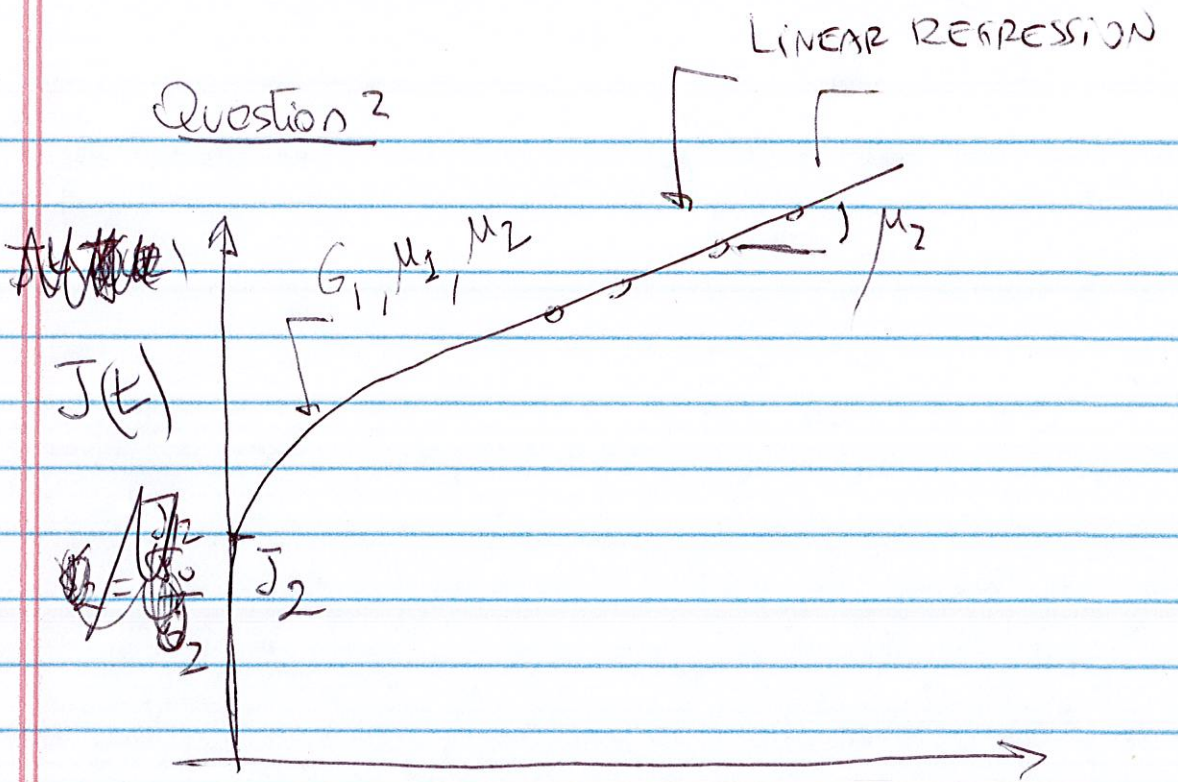
So

$$\sigma_1 = G_1 \gamma_0 = G_1$$

$$\sigma_2 = G_2 \gamma_0 = G_2$$

Question 2

(3)



$$\gamma(t) = \gamma_2 + \gamma_1\left(1 - e^{-\frac{t}{t_{ret.}}}\right) + \frac{t}{\mu_2}$$

$$J(t) = J_2 + J_1\left(1 - e^{-t/t_{ret}}\right) + \frac{t}{\mu_2}$$

FROM LINEAR REGRESSION

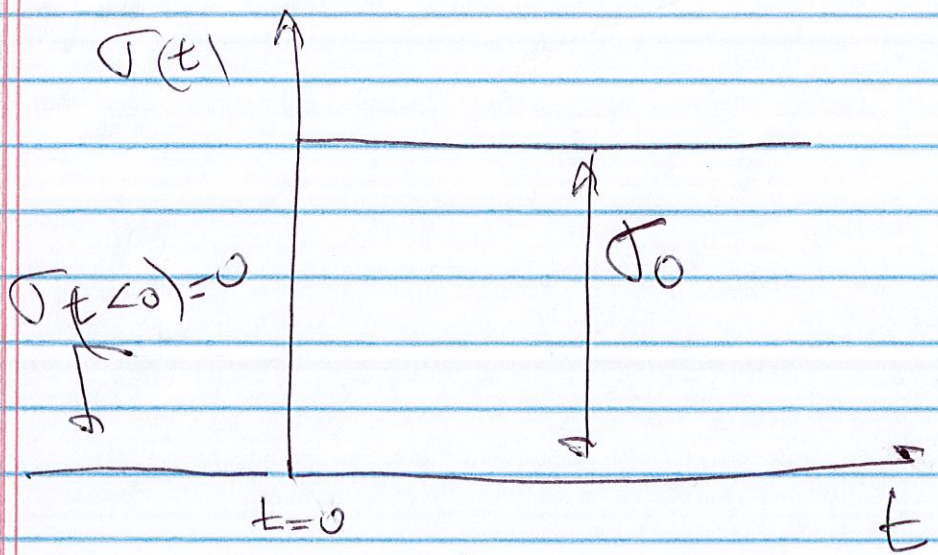
$$y = 0.3x + 30.44$$

slope

$$\text{slope} = 0.3 \times 10^{-5} = \frac{1}{\mu_2} \Rightarrow \mu_2 = \frac{1}{0.3 \times 10^{-5}}$$

$$G_2 = \frac{1}{J_2}$$

Why the initial intercept is J_2 ?

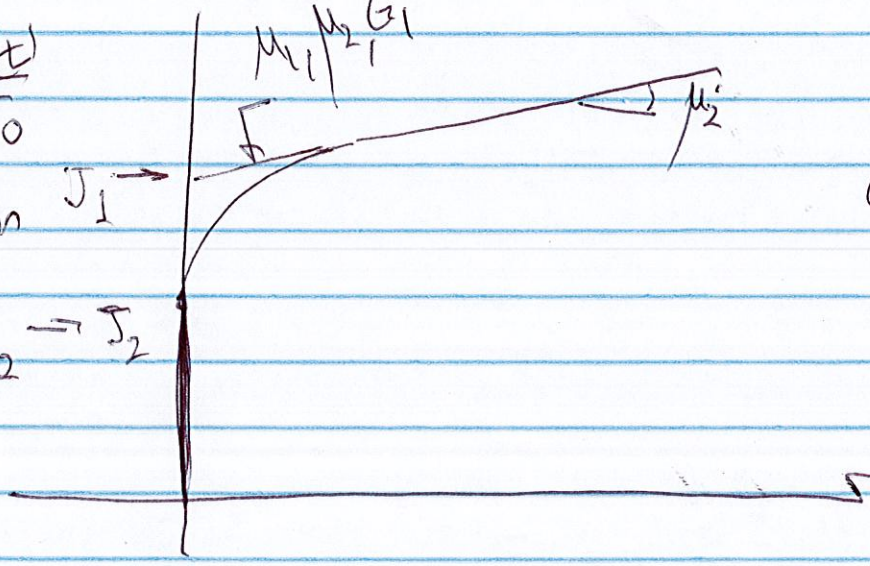


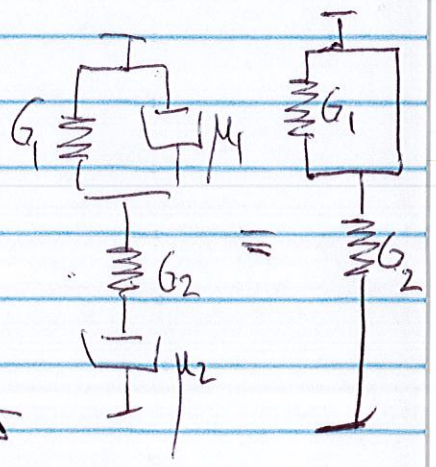
$$J(t) = \frac{\gamma(t)}{\sigma_0}$$

calculated by extrapolation

J_1

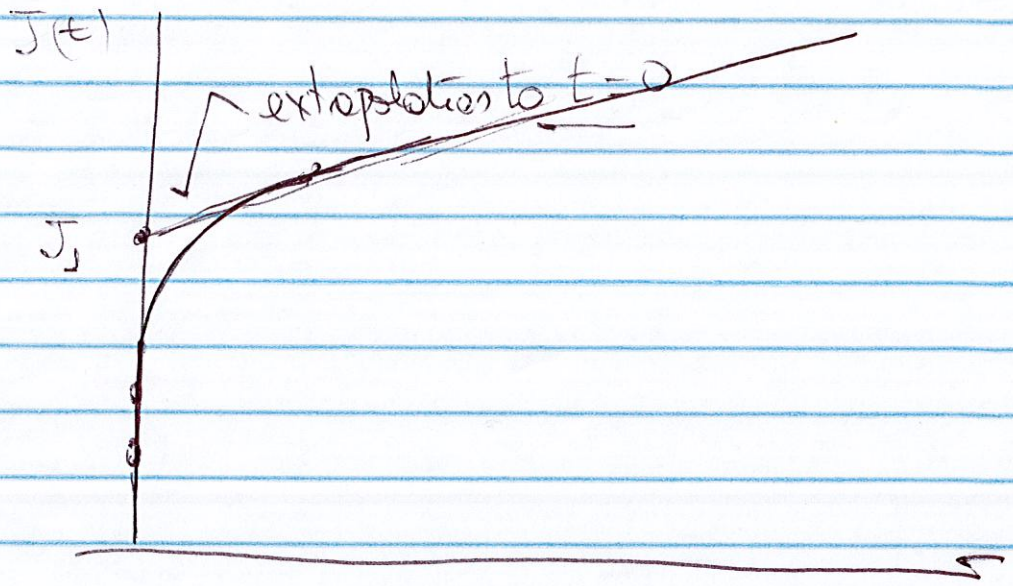
$\frac{\sigma_0}{\gamma_2} = G_2 \rightarrow J_2$





$$\rightarrow J(t) = J_2 + J_1\left(1 - e^{-t/t_{ret}}\right) + \frac{t}{\mu_2}$$

$$\left[J(t) - J_2 - \frac{t}{\mu_2}\right] = J_1\left(1 - e^{-t/t_{ret}}\right)$$



$$J(t) - J_2 = J_1\left(1 - \ell^{-t/t_{ret}}\right) + \frac{t}{\mu_2}$$

$$\frac{J(t) - J_2}{J_1} = 1 - \ell^{-t/t_{ret}}$$

does not count.

$$\ell^{-t/t_{ret}} = 1 - \frac{J(t) - J_2}{J_1}$$

$$-\frac{t}{t_{ret}} = \ln\left[1 - \frac{J(t) - J_2}{J_1}\right]$$

Extrapolation of the linear part to $t=0$ (5)

$$J(t=0) = J_2 + J_1 [1 - 0] + 0$$

$\frac{2\,rad}{s} = \omega$

Problem 3 $\quad \sigma(t) = 300 \sin(2t)$ ← INPUT (6)

$\gamma(t) = 0.5 \sin(2t + 1.1)$ OUTPUT

$\delta = 1.1$ radians. $\quad \delta = \frac{\pi}{2} = \frac{3.14}{2} = 1.57$ rad.

↑ more a liquid

↑ PURE LIQUID

$\sigma_0 = 300$ Pa ← Assuming Pascal.

$\gamma_0 = 0.5$ [NO UNITS BECAUSE STRAIN]

$\delta = 1.1$ radians

$G' = \frac{\sigma_0}{\gamma_0} \cos\delta =$

$G'' = \frac{\sigma_0}{\gamma_0} \sin\delta$

$G^* = \sqrt{G'^2 + G''^2}$

What is the frequency?