6. DENSITY, THERMAL CONDUCTIVITY AND THERMAL DIFFUSIVITY OF BIOLOGICAL MATERIALS

6.1. Introduction

We have just completed looking at those thermophysical properties of foods which determine HOW MUCH ENERGY? is required to heat or to cool foods. These properties are, of course, enthalpy, specific heat and latent heat.

We now turn to those thermophysical properties which determine HOW FAST? a food can be heated or cooled. These ***"heat transfer"*** properties are the thermal conductivity, the thermal diffusivity and, in this context, density and porosity.

Thermal conductivity (symbol λ, units $\frac{W}{mK}$ is a measure of a material's ability to conduct heat under steady state conditions.

Thermal diffusivity (symbol α, units $\frac{m^2}{s}$) is a measure of the rate of propagation of temperature in a material undergoing unsteady state heating or cooling; it is a measure of the material's ability to dissipate temperature gradients within itself. Thermal diffusivity is defined by the equation

$$\alpha = \frac{\lambda}{\rho c} \quad \left(\frac{m^2}{s}\right) \tag{86}$$

where

ρ = substance density, apparent density or bulk density $\left(\frac{kg}{m^3}\right)$ as appropriate

c = specific heat $\left(\frac{J}{kgK}\right)$

ρc = volumetric specific heat $\left(\frac{J}{m^3K}\right)$

Thermal diffusivity is perhaps a more important property than thermal conductivity because, in food processing, heating and cooling are most commonly unsteady state operations. However, it is important to be able to predict the thermal conductivity of foods for two reasons:

(a) In some heat transfer operations, e.g. heating and cooling liquids in continuous heat exchangers, steady state conditions prevail, and a value of λ is required for heat transfer calculations.

(b) Thermal diffusivity can be calculated from equation (86) provided values for λ and ρ and c are available or can themselves be calculated.

We will look now at how to predict the density, thermal conductivity and thermal diffusivity of foods, in that order. It is convenient to consider density (and porosity) first because data for these two properties is a prerequisite for the prediction of conductivity and diffusivity.

Note that density and thermal conductivity do not depend on latent heat effects, as enthalpy and specific heat do, because they are properties of heat **transfer**, and are not any measure of the amount of heat that must be added or removed to effect heating or cooling. Therefore, except in so far as the presence of fat or of ice affects the overall composition of a food, no distinction needs to be made, where these two properties are concerned, between fatty and non-fatty foods, or between frozen and non-frozen foods.

Thermal diffusivity, as it includes specific heat, **does** depend on latent heat effects in the freezing region (because of the phase change water $\leftrightarrow$ ice) and on latent heat effects in fatty foods (because of the phase change solid trigylcerides $\leftrightarrow$ liquid triglycerides). For these cases, alone or in combination, equation (86) must be written

$$\alpha = \frac{\lambda}{\rho \; c_{app}} \quad \left(\frac{m^2}{s} \right) \tag{86a}$$

6.2. Density and porosity of foods above and below freezing

Density, and its various forms, have already been covered in the Introductory handout on *Physical properties of foods and packaging materials.*

Density in general depends on composition and temperature. For a typical food of given **unfrozen** composition and given porosity, the variation of density with temperature is as shown in Fig. 23.

Compare Fig. 23 with Fig. 18. Above freezing, density is almost independent of temperature. As temperature drops below freezing water turns to ice. Because ice is **less dense** than liquid water, the density of the food decreases as the ice fraction increases. When all freezable water has frozen the density again becomes more or less invariant with temperature. The decrease in density on freezing is not, in fact, that much: less than 10%.

At any given temperature, density depends on the composition at that temperature; it must be note that if the temperature is lower than the initial freezing point, ice will be one of the components of the material.

The importance of density and porosity in the thermophysical properties context is that

- Density occurs in the definition of thermal diffusivity, α (equations (86) and (86a), and which form of density to use $(\rho_{s,}\ \rho_{app} \text{ or } \rho_B)$ depends on porosity.
- Density and porosity values are required for working out the composition of a food in terms of the volume fractions, as opposed to the mass fractions, of the food's components; volume fractions are variables in the equations used to predict thermal conductivity (and therefore, indirectly, thermal diffusivity).

The equations needed are given as follows. It is necessary to distinguish between non-porous foods, porous foods, and a bulk collection of particles of a food (porous or non-porous).

6.2.1. Non-porous foods

$$\rho_s = \rho_t = \frac{1}{\sum\left(\frac{x_i}{\rho_i}\right)} \qquad \left(\frac{kg}{m^3}\right) \tag{87}$$

where

ρ_s = substance density $(kg\, m^{-3})$
ρ_t = true density $(kg\, m^{-3})$
ρ_i = density of i th component

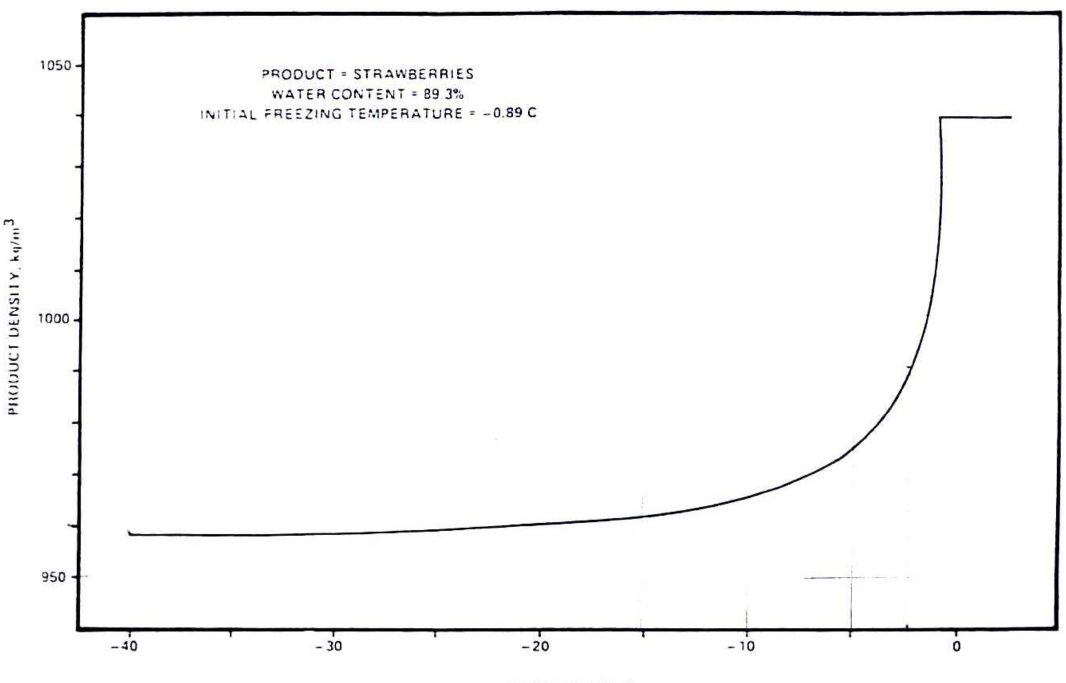


Fig 23. Plot of density against temperature for strawberries, a typical non-fatty food. (From Okos, 1986),

and the volume fraction of the different components ε_i is given by:

$$\varepsilon_i = \frac{\rho_s \times x_i}{\rho_\mathrm{i}} \tag{88}$$

where ε_i = volume fraction of *i* th component

$$\sum \varepsilon_i = 1 \tag{89}$$

6.2.2. Porous foods

$$\varepsilon_{app} = 1 - \frac{\rho_{app}}{\rho s} \tag{90}$$

Therefore,

$$\rho_{app} = \rho_s \ (1 - \varepsilon_{app}) \ \left(\frac{kg}{m^3} \right) \tag{91}$$

For the **solid and liquid components only** of the food (and **not the gas in the pores**).

$$\varepsilon_i = \frac{\rho_{app} \times x_i}{\rho_i} \tag{92}$$

where ε_i = volume fraction of *i* th **solid or liquid** component.

ε_{app} can be calculated using equation (90), provided ρ_{app} and ρ_s are known, or using the equation

$$\varepsilon_{app} = 1 - \sum \ \varepsilon_i \tag{93}$$

where ε_i is calculated using equation (92).

6.2.3 Food particles (porous or non-porous) in bulk

$$\varepsilon_{TP} = 1 - \frac{\rho_B}{\rho_s} \tag{94}$$

where

ε_{TP} = total porosity. It includes the open and closed pores, in the case of a porous food, and the space between particles

ρ_B = bulk density $\left(\frac{kg}{m^3}\right)$

Therefore,

$$\rho_B = \rho_s(1-\varepsilon_{TP}) \quad \left(\frac{kg}{m^3}\right) \tag{95}$$

For the **solid and liquid components only** of the bulk food (and **not the gas in the total pore volume**).

$$\varepsilon_i = \frac{\rho_B \times x_i}{\rho_i} \tag{96}$$

ε_{TP} can be calculated using equation (94), provided ρ_B and ρ_s are known, or using the equation

$$\varepsilon_{TP} = 1 - \sum \varepsilon_i \tag{97}$$

where ε_i is calculated using equation (96).

Equations (87) to (97) can easily be derived from first principles – i.e. the basic definitions of density (= mass per unit volume of food) and porosity (= volume of pores per unit volume of food), and the assumption that the gas in the pores has volume but not mass.

Using these equations requires data for ρ_i, ρ_{app} $(\text{or}\, \varepsilon_{app})$, ρ_B $(\text{or}\, \varepsilon_{TP})$, and x_i. ρ_i values can be calculated using equations (14) to (20) (Table 2), or the values in Table 3 can be used. x_{ice} can be calculated using equation (63):

$$x_{ice} = x_w^{'}\left(1 - \frac{\theta_{if}}{\theta}\right) \tag{63}$$

Other necessary data must be measured experimentally.

6.3. Thermal conductivity of foods

6.3.1. General remarks

The thermal conductivity of a food differs from specific heat, enthalpy and density in that it depends not only on **composition** and **temperature** but also on the **structure** of the food and, in anisotropic foods, on the **direction of heat flow** relative to the way the structural components of the material are arranged spatially.

In this context, *anisotropic means* having different structural arrangements in different directions.

For a given food of given structure and given chemical composition, and a given direction of heat flow, thermal conductivity varies with temperature as shown in Fig. 24.

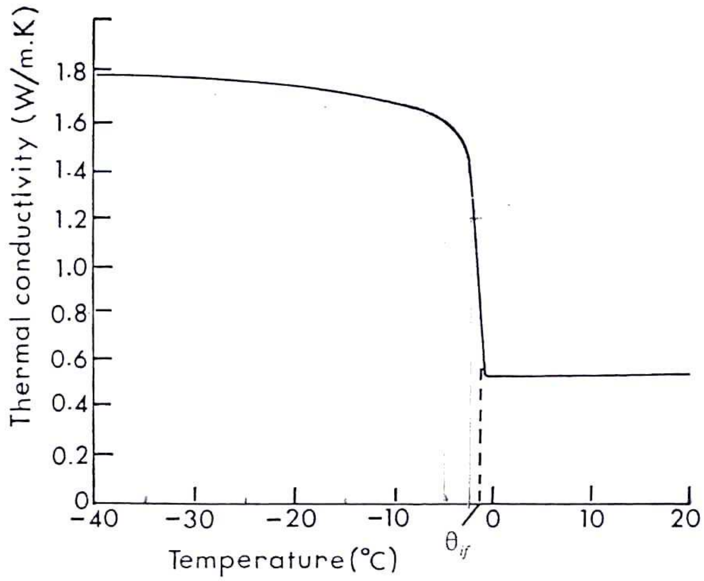


Fig. 24. Plot showing typical variation of thermal conductivity with temperature for a food. (From Lellor, 1978).

COMPARE FIG. 24 WITH FIG. 18. Above freezing, λ is relatively low, and changes little with temperature. But as temperature (on cooling) falls below θ_{if} there is an initial large increase in λ, the rate of increase then falling off as temperature drops to values well below θ_{if}.

There is an obvious similarity between the curves in Fig. 24 and 18. It exists because at $-20°$C for example from Table 3 it is obtained that:

$$\lambda_{water} = 0.533 \frac{W}{mK} \quad , \quad whereas$$
$$\lambda_{ice} = 2.38 \frac{W}{m\,K}$$

The thermal conductivity of ice is nearly four and half times greater than the thermal conductivity of liquid water. It is thus not surprising that thermal conductivity changes with temperature in the region *below freezing* is a very similar way to $x_{ice} / x_w^{'}$.

Phase changes in the fatty part of a fatty food do not have this dramatic effect on λ because the change in the thermal conductivity of the fat resulting from phase changes is usually small.

There are many theoretical equations available for predicting the thermal conductivity of heterogeneous materials such as foods. A number of the more straightforward of these (some are very complex) will be identified and discussed here with the aim of showing how they can be used by the engineer to calculate sensible estimates of the thermal conductivities of foods. Derivations of the equations will not be given.

Before looking at these equations, it is important to note that in thermal conductivity prediction it is the **volume** fractions of the components of the food, and not the mass fractions, that are important. The volumetric composition of a food **including pores** together with the food's structure, determines what heat "sees" as it flows through the food.

Thus for porous foods, the apparent porosity (the volume fraction of all the pores) is important, while for foods in bulk the total porosity (the total volume fraction of all pores plus the interparticle gas space) is important.

The calculation of volume fractions was covered in the last section (Section 6.2).

6.3.2. Equations for predicting the thermal conductivity of foods

<u>There are four useful general equations or models</u>. These will be introduced in turn.

The predicted minimum thermal conductivity of a given food is given by the so-called series model:

$$\lambda_{ser} = \frac{1}{\sum\left(\frac{\varepsilon_i}{\lambda_i}\right)} \quad \left(\frac{W}{m\,K}\right) \tag{98}$$

where

λ_{ser} = minimum thermal conductivity $\left(\frac{W}{m\,K}\right)$

λ_i = thermal conductivity of *i*- th component $\left(\frac{W}{m\,K}\right)$

For this model, it is assumed that the components of the food are arranged as parallel layers perpendicular to the direction of heat flow (ϕ_{ser} in Fig. 25(a)). The food's thermal conductivity is limited by the component with the lowest λ_i.

The maximum predicted thermal conductivity is given by the so-called parallel model:

$$\lambda_{par} = \sum(\lambda_i \varepsilon_i) \quad \left(\frac{W}{m\,K}\right) \tag{99}$$

where λ_{par} = maximum thermal conductivity $\left(\frac{W}{mK}\right)$

For this model it is assumed that the components of the food are parallel both with each other and with the direction of heat flow (ϕ_{par} in Fig. 25 (a) and (b)). The components can be layers (Fig. 25 (a)), or fibers plus a continuous phase that contains them (Fig. 25 (b)).

The true thermal conductivity of a heterogeneous food, unless exactly one of these structure-heat flow combinations exist, will lie somewhere between the limits given by equations (98) and (99); in many foods, the real pathway for heat conduction is likely to have both series and parallel elements.

The third possible conceptual structure for a food (Fig. 25(c)) is one in which the food's components are distributed randomly, so that the structure is the same in any direction (isotropic). Here, the direction of heat flow is of no importance, and thermal conductivity can be predicted using the so-called random model:

$$\lambda_{ran} = \lambda_1^{\varepsilon_1} \cdot \lambda_2^{\varepsilon_2} \cdot \lambda_3^{\varepsilon_3} \cdots\cdots \lambda_n^{\varepsilon_n} \quad \left(\frac{W}{m\,K}\right) \tag{100}$$

where λ_{ran} = predicted thermal conductivity $\left(\frac{W}{mK}\right)$

For a food or given composition and porosity, equation (100) will predict a value lying somewhere between those predicted by equations (98) and (99).

The fourth, and last, general model is the so-called Effective Medium Theory (EMT) equation:

$$\sum\left[\varepsilon_i\left(\frac{\lambda_{EMT} - \lambda_i}{\lambda_i + 2\lambda_{EMT}}\right)\right] = 0 \tag{101}$$

where λ_{EMT} = predicted thermal conductivity $\left(\frac{W}{m\,K}\right)$.

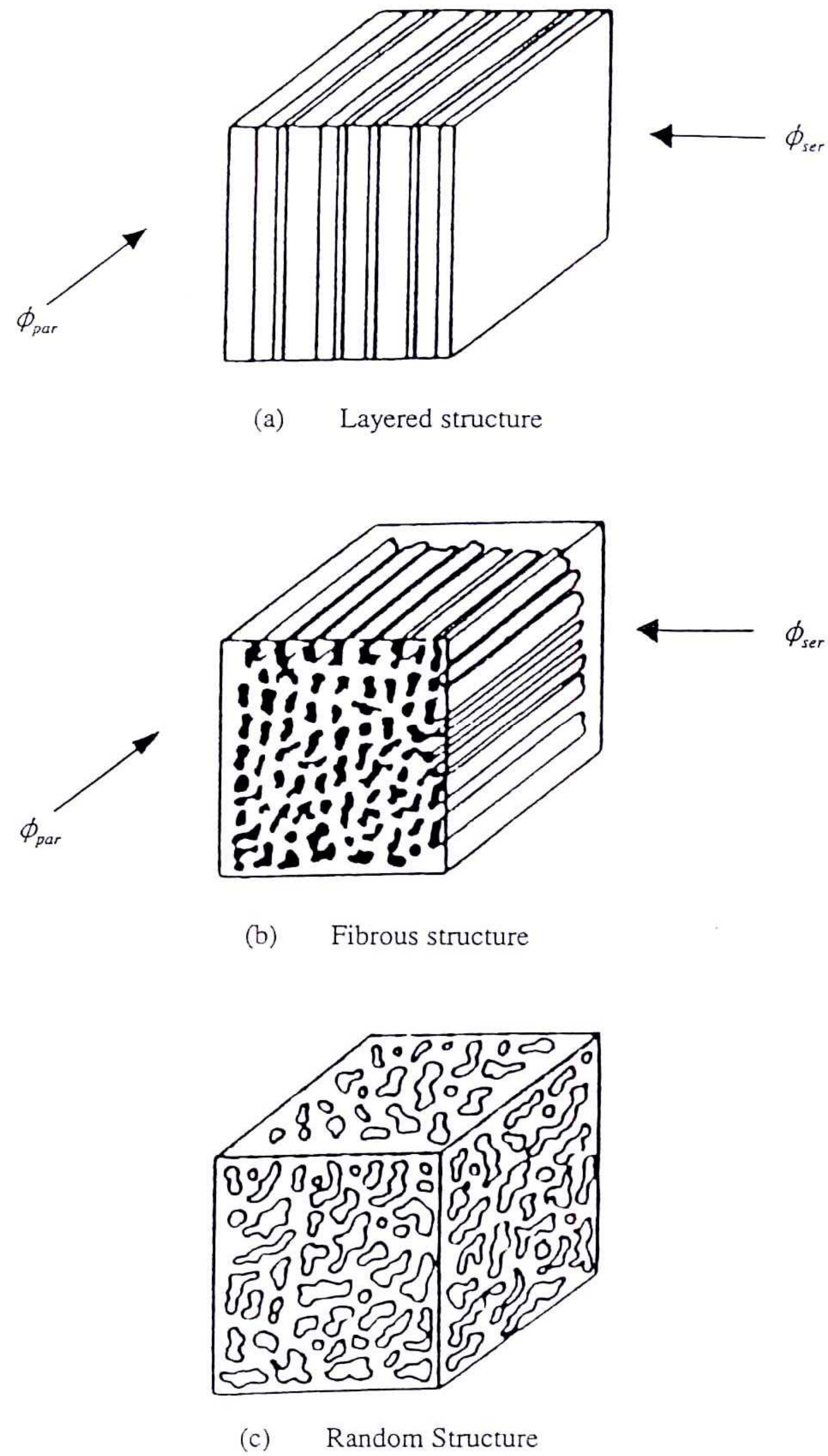


Fig. 25. Conceptual structures of two component foods. $\phi = heat\ \ flow\,(W)$

Equation (101) requires no prior assumptions to be made about the structure of the food or about the direction of heat flow.

Equation (101), also, will predict a value lying somewhere between those predicted by equations (98) and (99), but it will be different from that predicted by equation (100).

As these four equations will all predict different values of a food's thermal conductivity, which one should be used? Equations (98) and (99) are useful because at least they will predict minimum (i.e. conservatively low) and maximum values, respectively. However, a more exact value can often be calculated by:

- carefully deciding what physical structure the food actually has,
- identifying the direction of heat flow relative to this structure, and then,
- carrying out a **hierarchy of calculations** in which, at each level in the hierarchy, only a part of the food, consisting of **two** components, is considered. The thermal conductivity of this two-component part is calculated using an appropriate equation from among the five given below (three of which are equations (98), (99) and (101) written for two components).

The necessary equations are now presented, and then the hierarchy of calculations explained. Each of the following equations is applicable to a two component (two phase) material comprising one phase (the dispersed or discontinuous phase, subscript *d*) dispersed in another, continuous, phase (subscript *c*).

Layered structure (Fig. 25(a))

From equation (98),

$$\lambda_{ser} = \frac{1}{\dfrac{\varepsilon_d}{\lambda_d} + \dfrac{\varepsilon_c}{\lambda_c}} \qquad (102)$$

From equation (99),

$$\lambda_{par} = \lambda_d \ \varepsilon_d + \lambda_c \ \varepsilon_c \qquad (103)$$

Fibrous structure (Fig. 25 (b))

Introducing a new, fifth, equation:

$$\lambda_{ser} = \lambda_c \left(\frac{1-\xi}{1-\xi\left(1-\varepsilon_d^{1/2}\right)} \right) \qquad (104)$$

where

$$\xi = \varepsilon_d^{1/2}\left(1-\frac{\lambda_d}{\lambda_c}\right) \tag{105}$$

ε_d is the volume fraction of the fibers.

From equation (99),

$$\lambda_{par} = \lambda_d\varepsilon_d + \lambda_c\varepsilon_c \tag{103}$$

Random structure (Fig. 25c))

Equation (100) could be used, in this form:

$$\lambda_{ran} = \lambda_d^{\varepsilon_d} \cdot \lambda_c^{\varepsilon_c} \tag{100(a)}$$

However, the following equation is preferable because it has proved useful for foods:

$$\lambda_{M-E} = \lambda_c\left[\frac{2\lambda_c + \lambda_d - 2\varepsilon_d\left(\lambda_c - \lambda_d\right)}{2\lambda_c + \lambda_d + \varepsilon_d\left(\lambda_c - \lambda_d\right)}\right] \tag{106}$$

Equation (106) is known as the Maxwell-Eucken equation. The equation was originally developed for predicting the thermal conductivity of any two component system consisting of relatively high thermal conductivity spheres randomly dispersed in a relatively low thermal conductivity continuous phase. <u>It is strictly applicable only when the volume fraction of the dispersed phase is low</u>.

The Maxwell-Eucken equation has been found to be particularly successful for predicting the thermal conductivity of frozen foods; a frozen food can be considered grossly as a dispersion of relatively high thermal conductivity ice crystals dispersed in a relatively low thermal conductivity continuous phase comprising all the other components of the food.

Any structure and any heat flow direction

The EMT equation (equation (101), for two components, can be written as:

$$\varepsilon_d\left(\frac{\lambda_{EMT} - \lambda_d}{\lambda_d + 2\lambda_{EMT}}\right) + \varepsilon_c\left(\frac{\lambda_{EMT} - \lambda_c}{\lambda_c + 2\lambda_{EMT}}\right) = 0 \tag{107}$$

λ_{EMT} can be obtained from (107) to give:

$$\lambda_{EMT} = \lambda_c \left(\kappa + \left(\kappa^2 + \frac{\lambda_d}{2\lambda_c} \right)^{1/2} \right) \tag{108}$$

where

$$\kappa = \frac{3\varepsilon_c - 1 + \left(\lambda_d / \lambda_c\right)\left(2 - 3\ {}_c\right)}{4} \tag{109}$$

6.3.3. Hierarchy of calculations for predicting thermal conductivity

Before applying the hierarchy of calculations method for predicting the thermal conductivity of a food, you must first (as suggested above)

- decide what conceptual structure best represents the real physical structure of the food, and
- specify or identify the direction of heat flow relative to this structure. (this step is not, of course, necessary if the structure is assumed to be random).

The method is best explained by going through an example – the prediction of the thermal conductivity of a fibrous vegetable tissue (celery tissue) in which heat is flowing **perpendicular to** the fibers.

Example 6. Thermal conductivity of celery at $75°C$

Most vegetable tissue contains intercellular air, and celery is no exception; celery tissue is a **porous** material.

As celery is a non-fatty food, its composition from the point of view of predicting thermal conductivity is:

Water
SNF
Air

In terms of mass fractions,

$$x_{SNF} + x_w = 1$$

In terms of **volume** fractions, which are what is important here,

$$\varepsilon_{SNF} + \varepsilon_w + \varepsilon_{air} = 1$$

Assume that, in simple terms, celery consists of air bubbles (i.e. pores) dispersed in a continuous phase **which itself consists of** parallel SNF fibers (dispersed phase) immersed in water (continuous phase).

The thermal conductivity of celery (for heat flow perpendicular to the SNF (fibers)) can be calculated by means of the following two-level hierarchy of calculations.

$\lambda_{\mathbf{w}}$ $\lambda_{\mathbf{SNF}}$ $\lambda_{\mathbf{air}}$

Level 1:

Use equation (104) with

$\lambda_d = \lambda_{SNF}$

$\lambda_c = \lambda_w$

$\epsilon_d = \epsilon_{SNF}$

ϵ_d must be calculated such that:

$\epsilon_d + \epsilon_c = 1$

$\lambda_{\mathbf{SNF\text{-}in\text{-}water}}$

Level 2:

Use equation (106) or (108) with

$\lambda_d = \lambda_{air}$

$\lambda_c = \lambda_{SNF\text{-}in\text{-}water}$

Volume fractions must be calculated such that

$\epsilon_d + \epsilon_o = 1$

$\lambda_{\mathbf{celery}}$

(for series heat flow)

Note that the levels from level 1 to the penultimate level (level 1 actually is itself the penultimate level in this example) each considers a two-component part of the total food. The last level (level 2 in this example) considers the total food.

Note that for the thermal conductivity calculation at each level, volume fractions must be calculated such that their sum is equal to 1, i.e. the volume fractions must be fractions of that part of the food (e.g. water plus SNF) being considered at that level NOT fractions of the total food (except, of course, at the last level where the total food itself is being considered).

Thus, in level 1 for celery, the water plus SNF part (which is non-porous) is considered as a material in its own right, and volume fractions must be calculated as

$$\varepsilon_w = \frac{\rho_s \times x_w}{\rho_w} \tag{88}$$

$$\varepsilon_{SNF} = \frac{\rho_s \times x_{SNF}}{\rho_{SNF}} \tag{88}$$

where

$$\rho_s = \frac{1}{\dfrac{x_w}{\rho_w} + \dfrac{x_{SNF}}{\rho_{SNF}}} \tag{87}$$

Using the following data (physical property values from Table 3), we can put numbers in to see how the calculations work:

ρ_w	=	$974.6\,kg\ m^{-3}$
ρ_{SNF}	=	$\rho_{F1} = 1284\,kg\,m^{-3}$
λ_w	=	$\lambda_c = 0.666\,W\,m^{-1}K^{-1}$
λ_{SNF}	=	$\lambda_{Fi} = \lambda_d = 0.259\,W\ m^{-1}K^{-1}$
λ_{air}	=	$0.025\,W\ m^{-1}K^{-1}$
x_w	=	0.8
x_{SNF}	=	0.2
ε_{app}	=	$\varepsilon_{air} = 0.18$ (assume this value)

Level 1 calculations

$$\rho_s = \frac{1}{\frac{0.8}{974.6}+\frac{0.2}{1284}} = 1024\ kg\ m^{-3} \tag{87}$$

$$\varepsilon_w = \frac{1024\times 0.8}{974.6} = 0.84 \tag{88}$$

$$\varepsilon_{SNF} = \frac{1024\times 0.2}{1284} = 0.16\ (=1-0.84) \tag{88}$$

$$\xi = (0.16)^{1/2}(1-0.259/0.666) = 0.244 \tag{105}$$

$$\begin{aligned}\lambda_{SNF-in-water} &= 0.666\left[\frac{1-0.244}{1-0.244\left(1-(0.16)^{1/2}\right)}\right] \\ &= 0.569\end{aligned} \tag{104}$$

Level 2 calculations

As level 2 is the last level (in this example), the volume fractions are fractions of the total food (celery).

$$\begin{aligned}\rho_{app} &= \rho_s(1-\varepsilon_{app}) \\ &= 1024(1-0.18) \\ &= 839.7\ kg\ m^{-3}\end{aligned} \tag{91}$$

$$\begin{aligned}\varepsilon_{SNF-in-water} &= \frac{\rho_{app}\times x_{SNF-in-water}}{\rho_{SNF-in-water}} \\ &= \frac{839.7\times 1}{1024} \\ &= 0.82\end{aligned} \tag{90}$$

Obviously, since at this stage there are only two components, one of which is air, $\varepsilon_{SNF-in-water}$ could have been calculated directly as:

$$\begin{aligned}\varepsilon_{SNF-in-water} &= 1-\varepsilon_{air} \\ &= 1-0.18 \\ &= 0.82\end{aligned}$$

Conversely, if we knew ρ_{app} but not ε_{air}, we could calculate ε_{air} as:

$$\varepsilon_{app} = 1-0.82 = 0.18$$

The final answer, the thermal conductivity of celery with series heat flow, can now be calculated using either equation (106) or equations (108) and (109).

Using equation (106), with $\lambda_c = \lambda_{SNF-in-water}$,

$$\begin{aligned}\lambda_{celery} &= \lambda_c\left[\frac{2\lambda_c+\lambda_{air}-2\varepsilon_{air}(\lambda_c-\lambda_{air})}{2\lambda_c+\lambda_{air}+\varepsilon_{air}(\lambda_c-\lambda_{air})}\right] \\ &= 0.589\left[\frac{(2\times0.589)+0.025-(2\times0.18)(0.589-0.025)}{(2\times0.589)+0.025+0.18(0.589-0.025)}\right] \qquad (106) \\ &= 0.451\,W\ m^{-1}K^{-1}\end{aligned}$$

Using the EMT model (with subscript *d* meaning air, and subscript *c* meaning SNF-*in-water),*

$$\begin{aligned}\kappa &= \frac{(3\times0.82)-1+(0.025/0.589)\,(2-3(3\times0.82))}{4} \qquad (109) \\ &= 0.360\end{aligned}$$

$$\begin{aligned}\lambda_{celery} &= 0.589\left(0.360+\left((0.360)^2+(0.025/2\times0.589)\right)^{1/2}\right) \qquad (108) \\ &= 0.430\,W\ m^{-1}K^{-1}\end{aligned}$$

For comparison, the minimum and maximum predicted values of the thermal conductivity of celery can be calculated using equations (98) and (99). But first, the correct volume fractions must be determined:

$$\varepsilon_{air} = 0.18$$

$$\begin{aligned}\varepsilon_w &= \varepsilon_{w,\,SNF-in-water} \times \varepsilon_{SNF-in-water} \\ &= 0.84 \times 0.82 \\ &= 0.69\end{aligned}$$

$$\begin{aligned}\varepsilon_{SNF} &= \varepsilon_{SNF,\,SNF-in-water} \times \varepsilon_{SNF-in-water} \\ &= 0.16 \times 0.82 \\ &= 0.13\end{aligned}$$

Note that $(0.18 + 0.69 + 0.13) = 1$

$$\begin{aligned}\lambda_{\min} = \lambda_{ser} &= \frac{1}{\dfrac{0.18}{0.025} + \dfrac{0.69}{0.666} + \dfrac{0.13}{0.259}} \\ &= 0.114\,W\ m^{-1}K^{-1}\end{aligned} \tag{98}$$

Here, the thermal conductivity is dominated by $\lambda_{air,}$ which is the lowest λ_i.

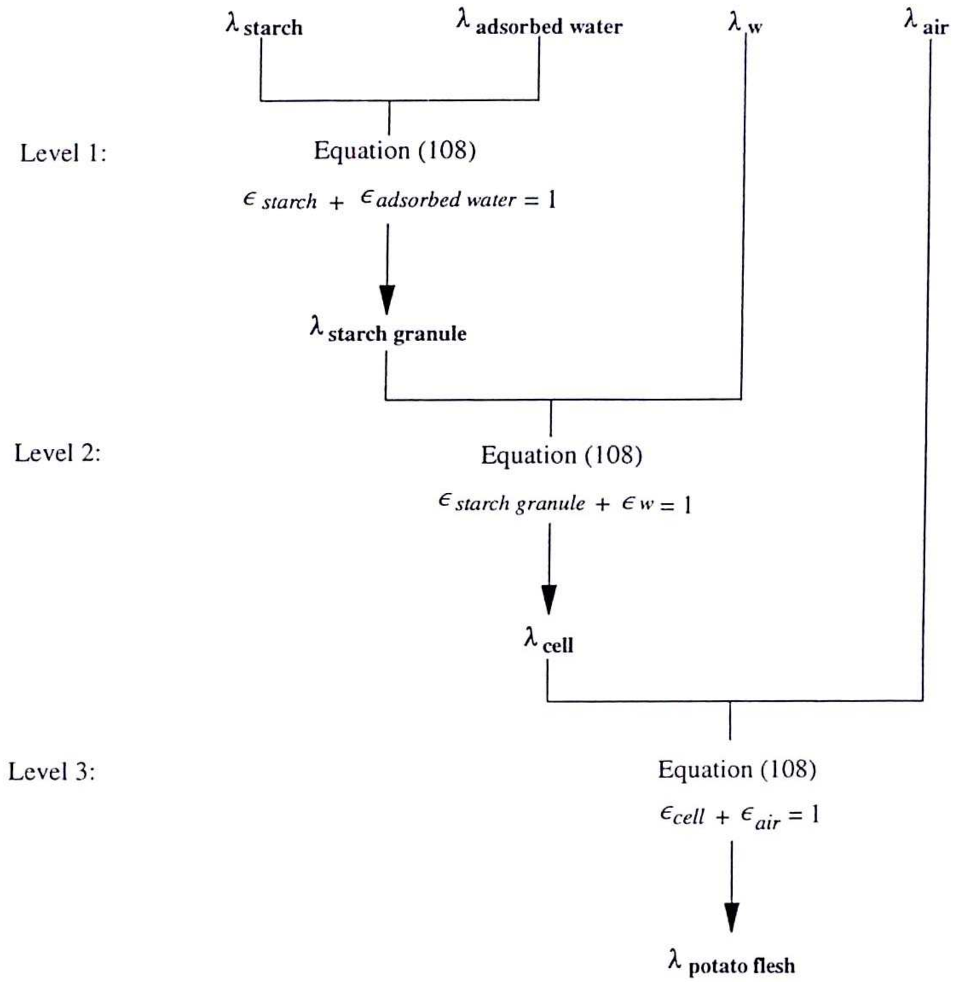
$$\begin{aligned}\lambda_{\max} = \lambda_{par} &= (0.18 \times 0.025) + (0.69 \times 0.666) + (0.13 \times 0.259) \\ &= 0.497\,W\ m^{-1}K^{-1}\end{aligned} \tag{99}$$

Note that the values given by the hierarchy of calculations, using the random and EMT models at level 2, lie between these minimum and maximum limits, as would be expected.

Comment: Equations (104) and (106) are the only equations from among those used in hierarchies of calculations (equations (102) – (109)) for which a decision has to be made as to which of the two phases is dispersed and which continuous. If this is not obvious from a careful consideration of the likely physical structure of the food, then it is best to assume that the phase with the larger volume fraction is the continuous phase, and the phase with the smaller volume fraction the dispersed phase.

The hierarchy of calculations has more levels and/or more calculation steps the more components the food contains. For example, the thermal conductivity of potato flesh, a four component material, can be successful predicted using a hierarchy of calculations with three levels, with the EMT equation (equation (108)) being employed at each level.

Potato flesh can be thought of as a continuous matrix of cells containing intercellular air as a dispersed phase. Each cell consists of a dispersion of starch granules in water. Each starch granule consists of starch saturated with absorbed water. The necessary hierarchy of calculations is as follows:



The volume fractions for use in these calculations are calculated using experimental physical and physico-chemical data reported in the literature. A comparison between predicted and experimental thermal conductivities for potato flesh is shown in Fig. 26; the predicted results are consistently lower than the experimental, but only by a maximum of 7.5% (at $X / X_o = 1$). The prediction method can therefore be said to work well.

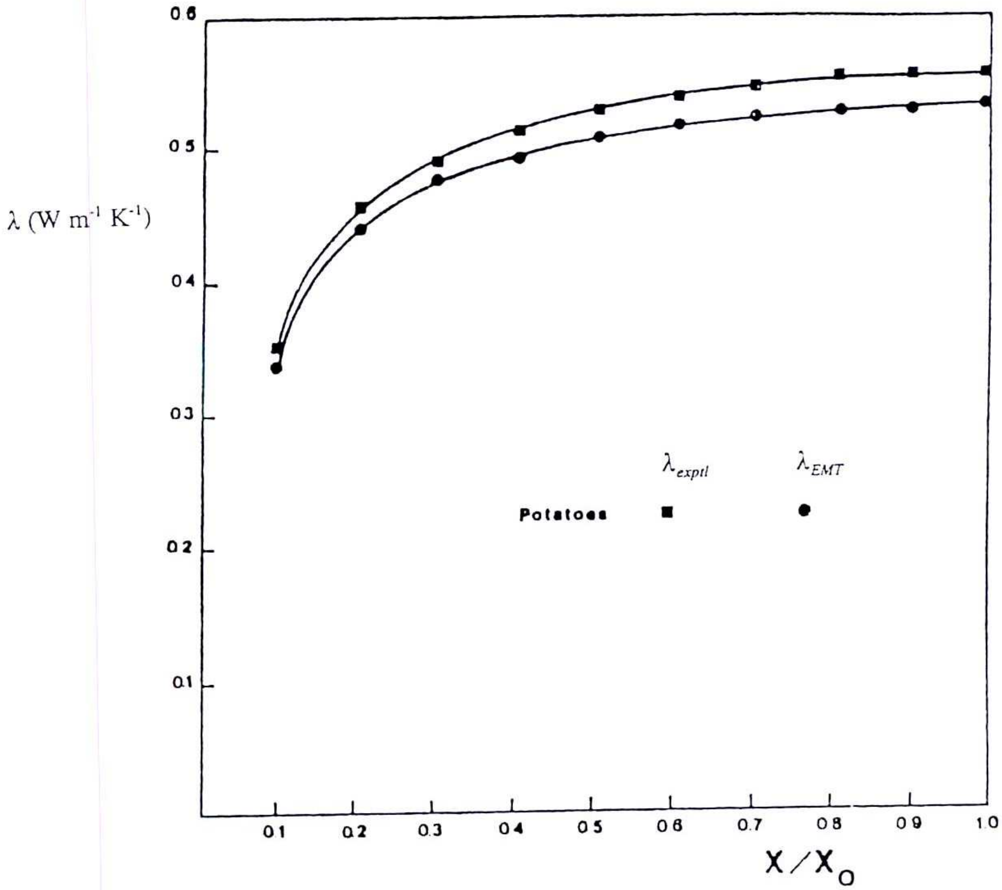


Fig. 26. Comparison between experimental thermal conductivities of potato, and thermal conductivities predicted using the EMT equation (equation (108)) in a hierarchy of calculations. ($X / X_o =$ kg water per kg SNF). (From Mattea *et al*, 1986).

The prediction of the thermal conductivity of a food containing ice and/or fat is carried out using exactly the same sort of hierarchy of calculations. The ice and the fat are merely two further components of the food.

For example, the thermal conductivity of a partly frozen hamburger mix (composed of SNF, water, fat, ice and air) might be predicted by considering the mix to be, structurally, a dispersion of air bubbles (i.e. pores) in a continuous phase which itself consists of *SNF plus fat* phase which in turn might be considered as being SNF dispersed in a continuous fat phase, and the *ice plus brine* phase as being ice crystals dispersed in brine.

Great care must be taken when doing these hierarchies of calculations for densities, and consequently volume fractions, are correctly calculated. Be careful to note whether or not the food is porous, and make sure that you have a clear picture in your mind of the physical structure of the food.

6.3.4. Effective thermal conductivity of a bulk collection of food particles.

The effective thermal conductivity of food particles in bulk (be they, for example, milk powder particles or apples) can be calculated using the hierarchical approach already described. The only difference is that volume fractions must be calculated using **bulk density** and **total porosity** data instead of apparent density and apparent porosity data (as described in Section 6.2.3).

6.3.5. Note on the effect of porosity on thermal conductivity

The hierarchy approach to predicting thermal conductivity, when applied to porous foods, assumes that the gas in pores merely conducts heat as if it was a low thermal conductivity opaque solid. In reality, heat conduction in porous materials is **enhanced** by convection and radiation in the pores and inter-particle spaces in porous and bulk foods. This means that the true effective thermal conductivity of such a food will be larger than the value predicted by the hierarchy method. The predicted value will thus be a conservatively low one.

Pores can contain gases other than air at pressures other than atmospheric pressure, and, in food, will nearly always contain water vapor as well. **However, for the purpose of predicting reasonable estimates of thermal conductivity, it can usually be assumed that pores and gas spaces contain only air at atmospheric pressure**. Thermal conductivity values for air at different temperatures, at atmospheric pressure, are given in Table 3.

6.3.6. Empirical data on the thermal conductivity of foods

If high quality empirical data on the thermal conductivity of a particular food are available (but this is often not the case), they might well be more accurate than values predicted by a hierarchies of calculations. However, they will usually be valid only for a fixed composition or limited ranges of compositions.

Very accurate empirical equations for predicting the thermal conductivity of a range of foods, both above and below freezing, have been published (Willix and Amos, 1995). These equations are based on precise experimental measurements. Typical values (at $20°C$) calculated using the appropriate equations are:

Mozzarella cheese	0.359 $\frac{W}{mK}$
Unsalted butter	0.203

Apples	0.438
Lean pork	0.474
Beef mince	0.382
Snapper fillets	0.494

There exists in the food literature partly empirical-partly theoretical equations specially developed for predicting the thermal conductivities of specific foods or classes of foods, e.g. a modified version of the Maxwell-Eucken equation specifically applicable to partly frozen meat, and equations developed for porous foods. These equations are likely to give more exact values than the general hierarchy approach taken here. It is possibly preferable to use them if the data they need is available. However, the advantages of the hierarchy approach is its generality – its applicability to **any** food.

6.4. Thermal diffusivity of foods

Remember that thermal diffusivity is defined by equation (86) or equation (86(a)):

$$\alpha = \frac{\lambda}{\rho\, c}\left(\frac{m^2}{s}\right) \tag{86}$$

$$\alpha = \frac{\lambda}{\rho\, c_{app}}\left(\frac{m^2}{s}\right) \tag{86a}$$

Note that in these two equations must be defined as ρ_s or ρ_{app} or ρ_B depending on whether the food is non-porous, porous or in bulk.

Factors that affect the values of λ, ρ and c (or c_{app}) obviously will affect the value of α. α, thus, depends on composition, temperature, structure and, unless the structure is random, on the direction of heat flow as well.

The variation of thermal diffusivity with temperature for a food with a given unfrozen composition, structure and direction of heat flow is shown in Fig. 27.

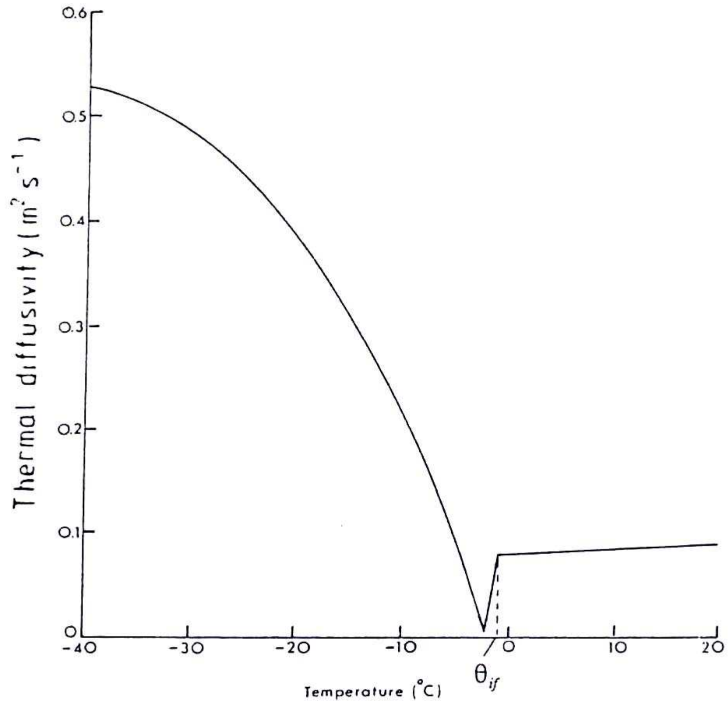


Fig. 27. Typical variation of thermal diffusivity with temperature for a food. (From Mellor, 1978).

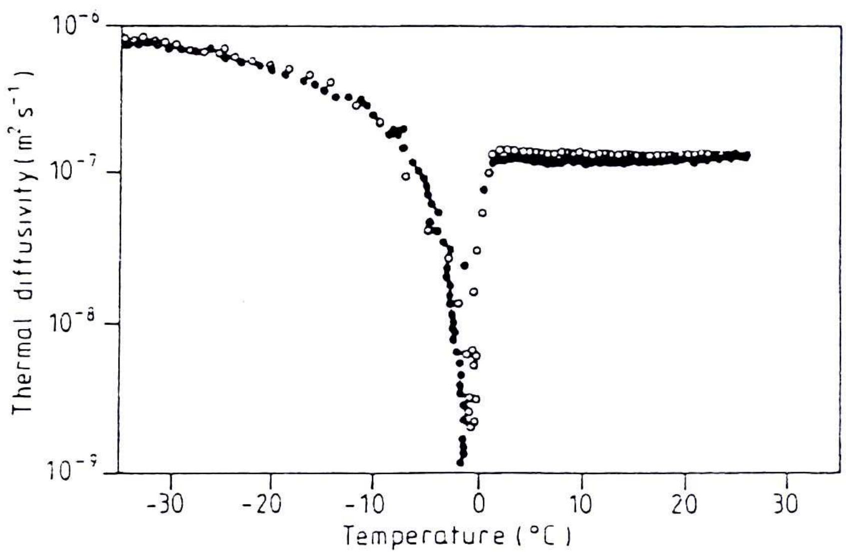


Fig. 28 Plot of thermal diffusivity against temperature for cod fillets (with heat flow perpendicular to the muscle fibers). 0, freezing; •, thawing. (From Nesvadbha, 1982).

COMPARE THE PLOT IN FIG. 27 WITH THOSE IN FIGS. 8, 18, 23 AND 24. Above θ_{if}, α is almost independent of temperature, as are λ, ρ and c_{app}. Just below θ_{if}, on cooling, there is a sharp drop in α. This is caused by the huge increase in c_{app} at this point (Fig. 8); this more than compensates for the increase in λ (Fig. 24) and the decrease in ρ (Fig. 23) – changes which tend to cause an increase in λ (equations (86) and (86(a))). The root cause of these changes is, of course, the appearance of ice at θ_{if}, and the high rate of increase in ice content just below θ_{if} (Fig. 18).

As temperature falls further, c_{app} and ρ both decrease (Figs. 8 and 23) while λ Increases (Fig. 24). The net effect now (refer to equation (86) and (86(a)) is a steady increase in α with falling temperature.

This patter is shown by the experimentally measured thermal diffusivity data for cod fillets plotted in Fig. 28. Heat flow was perpendicular to the muscle fibers.

The thermal diffusivity of a frozen food is nine or ten times higher than that of the unfrozen food (Figs. 27 and 28).

6.4.1. Prediction of thermal diffusivity

Equation (86) or (86a) can be used to predict thermal diffusivity if values of thermal conductivity, density and specific heat are available. These values can, of course, themselves be predicted using the methods described in this document.

Alternatively, an estimate of the thermal diffusivity of a food can be calculated using the equation.

$$\alpha = \sum(\alpha_i \varepsilon_i) \quad \left(\frac{m^2}{s}\right) \tag{110}$$

where α_i = thermal diffusivity of *i*th component $\left(\frac{m^2}{s}\right)$.

Values of α_i are given in Table 3.

Prediction of thermal diffusivity using equation (86) or (86(a)) is the better option, and is recommended.

Note that the thermal diffusivities of foods are very small numbers when the SI units $m^2 s^{-1}$ are used. Typical experimentally measured values (and the temperatures for which they are valid) are as follows:

Bulk wheat $(0-30°C)$	$9.1\times10^{-8}\, m^2 s^{-1}$
Lard $(0-25°C)$	6.1×10^{-8}

Carrot $(20°C)$	1.4×10^{-7}
Avocado flesh $(3-24°C)$	1.05×10^{-7}
Banana flesh $(20°C)$	1.37×10^{-7}
Ground beef $(-35 \text{ to} -24°C)$	1.0×10^{-6}
Egg white $(0-50°C)$	1.46×10^{-7}
Cod $(5°C)$	1.22×10^{-7}
Mayonnaise $(0-50°)$	1.07×10^{-7}

7. THERMOPHYSICAL PROPERTIES OF PACKAGING MATERIALS

These can be of importance in the heating and cooling of **packaged** foods.

The thermal conductivity of the packaging material is the property of major importance because its value can greatly influence the **rate** of heat transfer to or from the package. For example, frozen food is often frozen **after** it has been packaged, and the thermal conductivity of the packaging material (e.g. cardboard), which is usually relatively low, can be the limiting factor with respect to the rate at which heat can be transferred out of the food in order to freeze it. A knowledge of the packaging material's conductivity is therefore necessary for designing food freezing systems.

The thermal conductivities of packaging materials cannot generally be predicted; they must be **measured**.

The specific heats and enthalpies of packaging materials are of little importance. This is because:

- these properties have low values compared to foods, and
- the fraction of the total mass of a packaged food (mass of package plus mass of food) represented by the packaging material is usually very small.

For both these reasons, the thermal energy that must be supplied to, or removed from, the packaging material during the heating or cooling of a packaged food is usually negligible compared to the thermal energy that must be supplied to or removed from the food itself. (But note that the absorption of moisture by a packaging material, e.g. a paper-based one, can result in the material's specific heat and thus its thermal energy demand, increasing to significantly high values).

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