ABE 30300 - Fall 2017
HOMEWORK 2

Problem 1

The food is above of the freezing point so we can use the equation of the enthalpy for conditions above the freezing point. Let's calculate the composition of the entire food taking into account the composition of each food commponent. Let's start to enter the data

$m_{pot} := 250$ $\quad m_{peas} := 180$ $\quad m_{pork} := 150$ $\quad m_{but} := 30$

$m_{food} := m_{pot} + m_{peas} + m_{pork} + m_{but}$ $\quad m_{food} = 610$

Composition of each food ingredient (fat 1 = pork and fat 2 = butter fat)

Potato $\quad x_{wp} := 0.80$ $\quad x_{f1p} := 0$ $\quad x_{SNFp} := 0.20$

$x_{f2p} := 0$

Peas $\quad x_{wpeas} := 0.75$ $\quad x_{f1peas} := 0$ $\quad x_{SNFpeas} := 0.25$

$x_{f2peas} := 0$

Pork $\quad x_{wpork} := 0.45$ $\quad x_{f1pork} := 0.29$ $\quad x_{SNFpork} := 0.26$

$x_{f2pork} := 0$

Butter $\quad x_{wb} := 0.12$ $\quad x_{f1b} := 0$ $\quad x_{SNFb} := 0$

$x_{f2b} := 0.89$

Composition of in the whole food

Water

$$x_{w\_food} := \frac{m_{pot} \cdot x_{wp} + m_{peas} \cdot x_{wpeas} + m_{pork} \cdot x_{wpork} + m_{but} \cdot x_{wb}}{m_{food}}$$

$\boxed{x_{w\_food} = 0.67}$

Fat 1

$$x_{f1\_food} := \frac{m_{pot} \cdot x_{f1p} + m_{peas} \cdot x_{f1peas} + m_{pork} \cdot x_{f1pork} + m_{but} \cdot x_{f1b}}{m_{food}}$$

$\boxed{x_{f1\_food} = 0.07}$

Fat 2

$$x_{f2\_food} := \frac{m_{pot} \cdot x_{f2p} + m_{peas} \cdot x_{f2peas} + m_{pork} \cdot x_{f2pork} + m_{but} \cdot x_{f2b}}{m_{food}}$$

$\boxed{x_{f2\_food} = 0.04}$

Solids Non Fat

$$x_{SNF\_food} := \frac{m_{pot} \cdot x_{SNFp} + m_{peas} \cdot x_{SNFpeas} + m_{pork} \cdot x_{SNFpork} + m_{but} \cdot x_{SNFb}}{m_{food}}$$

$$\boxed{x_{SNF\_food} = 0.22}$$

Selection of temperatures, the convention given in class is used assigning the high temperature to θ_2 and the lower temperature as θ_1. Other conventions will not change the numerical value of the calculated enthalpy but it will change its sign. With the convention used all enthalpy are positive, but you have to remember when heat (enthalphy) is supplied to the system the convention commonly used indicates that the eheat (enthalpy) is positive because increases the energy of the system. Otherwise, it is considered the enthalpy is considered negative.

$\theta_2 := 80$ $\qquad$ $\theta_1 := -1.0$

Calculation of Enthapy Change - Above of freezing point

$x_{fat\_food} := x_{f1\_food} + x_{f2\_food}$ $\qquad$ $x_{non\_fat\_food} := 1 - (x_{f1\_food} + x_{f2\_food})$

$x_{non\_fat\_food} = 0.88$

$$x_{w\_non\_fat\_food} := \frac{x_{w\_food}}{x_{non\_fat\_food}} \qquad C_w := 4.2$$

$$\Delta h_{SNF\_food} := 1.55 \cdot (1 - x_{w\_non\_fat\_food}) \cdot (\theta_2 - \theta_1) + 2.09 \cdot 10^{-3} \cdot (\theta_2{}^2 - \theta_1{}^2) \ldots$$
$$+ x_{w\_non\_fat\_food} \cdot C_w \cdot (\theta_2 - \theta_1) - 0.376 \cdot \exp\left(-43 x_{w\_non\_fat\_food}{}^{2.3}\right) \cdot (\theta_2 - \theta_1)$$

$\Delta h_{SNF\_food} = 300.4$ $\qquad$ Units are in kJ/kg

Empirical enthalpy data for pork (1) and butter fat (2), temperature in Celsius and Enthalpy is kJ/kg

$$Temp := \begin{pmatrix} -5 \\ 0 \\ 10 \\ 30 \\ 60 \\ 90 \end{pmatrix} \qquad H_{fat1} := \begin{pmatrix} 77.11 \\ 86.15 \\ 112.9 \\ 182.6 \\ 301.0 \\ 361.5 \end{pmatrix} \qquad H_{fat2} := \begin{pmatrix} 86.27 \\ 101.7 \\ 141.4 \\ 223.9 \\ 294.2 \\ 355.3 \end{pmatrix}$$

Mathcad allows for the interpolation of data. The command can be obtained from MathCad help and is used as linterp(vx,vy,x) which returns a linearly interpolated value at x for data vectors vx and vy which for this problem are Temp, H_{fat1} and H_{fat2}. Thus to calculate the enthalpy for fats 1 and 2 at temperatures 80C and -1.5C we can write:

Fat 1

$h_{fat1\_\theta2} := \mathrm{linterp}(Temp, H_{fat1}, \theta_2)$ $h_{fat1\_\theta1} := \mathrm{linterp}(Temp, H_{fat1}, \theta_1)$

$\boxed{h_{fat1\_\theta2} = 341.33}$ $\boxed{h_{fat1\_\theta1} = 84.34}$

$\Delta h_{fat1\_food} := h_{fat1\_\theta2} - h_{fat1\_\theta1}$

Fat 2

$h_{fat2\_\theta2} := \mathrm{linterp}(Temp, H_{fat2}, \theta_2)$ $h_{fat2\_\theta1} := \mathrm{linterp}(Temp, H_{fat2}, \theta_1)$

$\boxed{h_{fat2\_\theta2} = 334.93}$ $\boxed{h_{fat2\_\theta1} = 98.61}$

$\Delta h_{fat2\_food} := h_{fat2\_\theta2} - h_{fat2\_\theta1}$

$\Delta h_{food} := x_{non\_fat\_food} \cdot \Delta h_{SNF\_food} + x_{f1\_food} \cdot \Delta h_{fat1\_food} + x_{f2\_food} \cdot \Delta h_{fat2\_food}$

$\boxed{\Delta h_{food} = 294.5}$ Units are in kJ/kg

$Meals := 900$ Meals per day

$Meal\_weight := 0.6$ Kg 1 MJ = 10^3 kJ

$Energy := \Delta h_{food} \cdot Meals \cdot Meal\_weight \cdot 10^{-3}$

$\boxed{Energy = 159.03}$ Units are in MJ/day

Question 2

$c_w := 4.2$ $c_{ice} := 2.1$ $c_{SNF} := 1.9$ $x_w := 0.82$

$\theta_{if} := -1$ $\theta_{2b} := 25$ $\theta_{1b} := -2.5$ $L_w := 331$ $x_{SNF} := 0.18$

Linear interpolation in the table, allow to calculate the total concentration of salt to be able to get a freezing point of -3.5^{o}C; the value is 5.8% salt. So the kg of salt (x) in 1 kg uf unsalted fish (i.e. before salt is added

$$\frac{x}{0.8 + x} \cdot 100 = 5.8 \text{ solve } \rightarrow 0.049256900212314225053$$

$x := 0.049$ This is the total amount of salt, so salt to be added by kg of unsalted fish is

$Salt\_to\_add := x - 0.015$ $Salt\_to\_add = 0.03$

By assuming 1 kg of unsalted fish the new composition of the fish after salt is added is

$m_{water} := 0.82$ $m_{prot} := 0.165$ $m_{salt} := 0.049$

So mass fractions can be calculated as:

$$x_{wn} := \frac{m_{water}}{m_{water} + m_{prot} + m_{salt}} \qquad x_{SNFn} := \frac{m_{prot} + m_{salt}}{m_{water} + m_{prot} + m_{salt}}$$

$$x_{wn} = 0.79 \qquad x_{SNFn} = 0.21 \qquad x_{BW} := 0$$

__Unsalted Biomaterial__

Below freezing point

$$\Delta h_{\theta 1b\_\theta if} := c_{SNF} \cdot (1 - x_w) \cdot (\theta_{if} - \theta_{1b}) + c_{ice} \cdot x_{BW} \cdot (\theta_{if} - \theta_{1b}) + c_w \cdot x_w \cdot \theta_{if} \cdot \ln\left(\frac{\theta_{if}}{\theta_{1b}}\right) \dots$$
$$+ c_{ice} \cdot x_w \cdot (\theta_{if} - \theta_{1b}) - c_{ice} \cdot x_w \cdot \theta_{if} \cdot \ln\left(\frac{\theta_{if}}{\theta_{1b}}\right) - L_w \cdot x_w \cdot \left(\frac{\theta_{if}}{\theta_{1b}} - 1\right)$$

$$\boxed{\Delta h_{\theta 1b\_\theta if} = 167.5}$$ ALL UNITS IN kJ/kg

Above freezing point

$$\Delta h_{\theta if\_\theta 2b} := 1.55(1 - x_w) \cdot (\theta_{2b} - \theta_{if}) + 2.09 \cdot 10^{-3} \cdot (1 - x_w) \cdot \left(\theta_{2b}{}^2 - \theta_{if}{}^2\right) \dots$$
$$+ x_w \cdot c_w \cdot (\theta_{2b} - \theta_{if}) - 0.376 \cdot \exp\left(-43 \cdot x_w{}^{2.3}\right) \cdot (\theta_{2b} - \theta_{if})$$

$$\boxed{\Delta h_{\theta if\_\theta 2b} = 97.0}$$ ALL UNITS IN kJ/kg

$$\Delta h_{unsalted\_biom} := \Delta h_{\theta 1b\_\theta if} + \Delta h_{\theta if\_\theta 2b}$$

$$\boxed{\Delta h_{unsalted\_biom} = 264.6}$$ ALL UNITS IN kJ/kg

Salted Biomaterial - The material is all above freezing point because the salt lowers the freezing point to -3.5C

$$\Delta h_{\theta 1b\_\theta 2b} := 1.55(1 - x_{wn}) \cdot (\theta_{2b} - \theta_{1b}) + 2.09 \cdot 10^{-3} \cdot (1 - x_{wn}) \cdot \left(\theta_{2b}{}^2 - \theta_{1b}{}^2\right) \dots$$
$$+ x_{wn} \cdot c_w \cdot (\theta_{2b} - \theta_{1b}) - 0.376 \cdot \exp\left(-43 \cdot x_{wn}{}^{2.3}\right) \cdot (\theta_{2b} - \theta_{1b})$$

$$\Delta h_{\theta 1b\_\theta 2b} = 100.69 \qquad \Delta h_{salted} := \Delta h_{\theta 1b\_\theta 2b}$$

$$\boxed{\Delta h_{salted} = 100.7}$$ ALL UNITS IN kJ/kg

A significant less amount of heat needs to be removed when salt is added to the biomaterial