ABE 303 – Homework 5
Fall 2017
<u>Solutions</u>

Question 1

The apparent viscosity η_{app} can be determined as the shear stress σ divided by the shear rate $\dot{\gamma}$

$$\eta_{app} = \frac{\sigma}{\dot{\gamma}}$$

By using the Casson model, the above equation becomes'

$$\eta_{app} = \frac{\left(\sqrt{\sigma_o} + K\sqrt{\dot{\gamma}}\right)^2}{\dot{\gamma}} = \frac{\sigma_o + 2K\sqrt{\sigma_o}\sqrt{\dot{\gamma}} + K^2\dot{\gamma}}{\dot{\gamma}} = \frac{\sigma_o}{\dot{\gamma}} + \frac{2K\sqrt{\sigma_o}}{\sqrt{\dot{\gamma}}} + K^2$$

The equation above shows that when the shear rate is small the viscosity of the chocolate is very large whereas when the shear rate is large the apparent viscosity of the chocolate → K^2

Question 2

2. The following data is available for liquid

T (°C)	μ (cP)
5	19
30	6.4
40	5.5
50	4.1
60	3.1

Determine an equation to estimate the viscosity of the material as a function of the temperature.

Assume that the viscosity of the liquid changes with temperature following an Arrhenius relationship and that there is no coagulation of the protein in that range of temperatures

$$\mu = A e^{B/T} \quad (1)$$

A and B are empirical constants, they can be determined from the data

Consider two experimental points, e.g. extreme temperatures

$T_1 = 5+273 = 278 K$ $\quad\quad$ $\mu_1 = 19$ cP

$T_2 = 60+273 = 333 K$ $\quad\quad$ $\mu_2 = 3.1$ cP

$$\left.\begin{aligned} \ln \mu_1 &= \ln A + B/T_1 \\ \ln \mu_2 &= \ln A + B/T_2 \end{aligned}\right\} \Rightarrow \ln \frac{\mu_1}{\mu_2} = B\left[\frac{1}{T_1} - \frac{1}{T_2}\right] \quad (2)$$

$$B = \frac{\ln \mu_1/\mu_2}{\left(\frac{1}{T_1} - \frac{1}{T_2}\right)} \qquad (3)$$

$$\ln A = \ln \mu_1 - \frac{B}{T_1} \qquad (4)$$

From Eq. (3) $B = \dfrac{\ln 19/3.1}{\left(\frac{1}{278} - \frac{1}{333}\right)} = \dfrac{1.81}{0.0006} = 3046.5\ K$

$$B = 3046.5\ K$$

From Eq. (4) $\ln A = \ln 19 - \dfrac{3046.5}{278} = -8.01$

$$A = e^{-8.01} = 3.307 \times 10^{-4}$$

$$\boxed{A = 3.307 \times 10^{-4}\ CP}$$

$$\boxed{\mu = 3.307 \times 10^{-4}\ e^{\frac{3046.5}{T}}}$$

Let's check how well the equation describes the data

Let's consider $T = 40°C = 40 + 273 = 313\ K$

$\mu = 3.307 \times 10^{-4}\ e^{\frac{3046.5}{313}} = 5.6\ cp$ [very close to the experimental data]

Question 3a

$$\boxed{\frac{Q}{\pi R^3} = \frac{1}{\sigma_w^3}\int_0^{\sigma_w} \sigma^2 f(\sigma)\, d\sigma} \quad (12)$$

For a power low fluid $\sigma = K\dot{\gamma}^n \Rightarrow \dot{\gamma} = f(\sigma) = \left(\frac{\sigma}{K}\right)^{1/n}$

substituting into Eq.(12)

$$\frac{Q}{\pi R^3} = \frac{1}{\sigma_w^3}\int_0^{\sigma_w} \frac{\sigma^2 \sigma^{1/n}}{K^{1/n}}\, d\sigma = \frac{1}{\sigma_w^3 K^{1/n}}\int_0^{\sigma_w} \sigma^{2+1/n}\, d\sigma$$

$$\frac{Q}{\pi R^3} = \frac{1}{\sigma_w^3 K^{1/n}} \left. \frac{\sigma^{3+1/n}}{3+\frac{1}{n}} \right|_0^{\sigma_w}$$

$$\frac{Q}{\pi R^3} = \frac{1}{\sigma_w^3 K^{1/n}} \frac{\sigma_w^{3+1/n}}{\frac{3n+1}{n}} = \frac{n}{3n+1}\frac{\sigma_w^{1/n}}{K^{1/n}}$$

and

$$\sigma_w = K\left(\frac{3n+1}{n}\frac{Q}{\pi R^3}\right)^n \quad (13)$$

By comparing with $\sigma = K\dot{\gamma}_w^n$

$$\boxed{\dot{\gamma}_w = \frac{3n+1}{n}\frac{Q}{\pi R^3}}$$

Eq. (13)

$$\tau_w = K\left(\frac{3n+1}{4n}\ \frac{4Q}{\pi R^3}\right)^n$$

but $\tau_w = \dfrac{\Delta P \times R}{2L}$

$$\frac{\Delta P \times R}{2L} = K\left(\frac{3n+1}{4n}\right)^n\left(\frac{4Q}{\pi R^3}\right)^n$$

$$\Delta P = \frac{2LK}{R}\left(\frac{3n+1}{4n}\right)^n\left(\frac{4Q}{\pi R^3}\right)^n$$

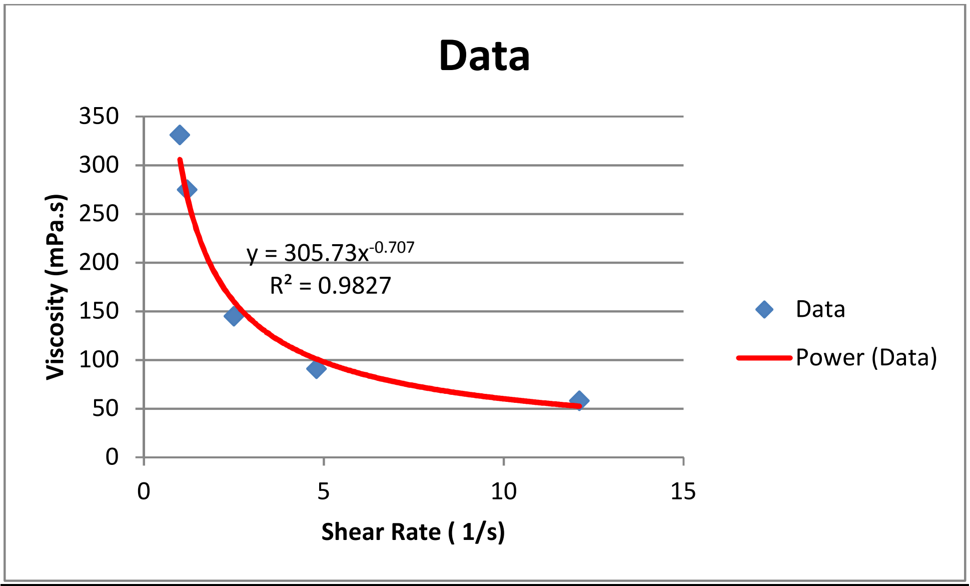
or

$$\Delta P = 2LK\left(\frac{3n+1}{4n}\right)^n\left(\frac{4Q}{\pi R^{3+\frac{1}{n}}}\right)^n$$

or

$$\boxed{\Delta P = 2LK\left(\frac{3n+1}{4n}\right)^n\left(\frac{4Q}{\pi R^{\frac{3n+1}{n}}}\right)^n}$$

Question (3b)



From the chart we can see that K=305.7 mPa.s$^{0.3}$ and n-1 = -0.70 → n = 0.3

The apparent viscosity can be calculated as: $\mu_{app} = K\,\dot{\gamma}^{0.3-1} = K\dot{\gamma}^{-0.7}$

or

$$\mu_{app} = 305.7\ \ mPa.s^{0.3}\ 20^{-0.7}\ \frac{1}{s^{-0.7}} = 37.5\,mPa.s$$

Question 4

A shear sensitive non-Newtonian protein solution having a density of 1041 kg/m^3 is flowing through 14.9 meters of tubing having an inside diameter of 5.24 cm. Capillary viscometer measurements have shown that the solution is time-independent and that, for the flow conditions in the processing line the following rheological parameters apply n = 0.4 and k = 15.2 Pa.s$^{0.4}$. The mass flow rate is 590 kg/h.

(a) What is the type of flow behavior being exhibited by the fluid? Explain your answer, i.e., why do you think the protein solution is exhibiting that behavior?

The fluid is shear thinning because the value of n < 1. Probably due to shear flow the orientation of the protein in the solution is enhanced and the viscosity of the solution, which is the resistance of the material to flow, decreases. There is no information on the rheological data of the presence of a yield stress, so it is not a plastic fluid.

(b) Calculate the pressure drop due to viscous friction in the pipeline assuming laminar flow.

(c) An increase in the throughput requires increasing the mass flowrate 50%. An existing spare pump can deliver the new required flowrate but at a maximum permissible discharge pressure of 50kPa (gauge). Determine whether that pump will do the job or it would be necessary to acquire one with a higher maximum permissible discharge pressure.

(d) The maximum shear rate that can be applied without causing the protein to denature is 2001/s. Find if the new flow given in (c) will cause shear denaturation on the protein

(b)

$K = 15.2\ Pa\ s^{0.4}$ $\quad n = 0.4$ $\quad Q = \dfrac{590\ kg/h}{1041\ \frac{kg}{m^3} \times 3600\ sec}$

$L = 14.9\ m$

$D = 5.24 \times 10^{-2}\ m$

$R = 2.62 \times 10^{-2}\ m$ $\quad Q = 1.574 \times 10^{-4}\ \dfrac{m^3}{s}$

$$\Delta P_f = \frac{2 \times 14.9 \times 15.2}{2.62 \times 10^{-2}} \left(\frac{3 \times 0.4 + 1}{4 \times 0.4}\right)^{0.4} \left(\frac{4 \times 1.574 \times 10^{-4}}{\pi \times (2.62 \times 10^{-2})^3}\right)^{0.4}$$

$$\Delta P_f = 1.729 \times 10^4 \times 1.14 \times 2.623 = 51{,}700.9\ Pa$$

$$\underline{\Delta P_f \cong 51.7\ KPa}$$

(c) If we assume that the inlet and outlet of the pipe are at the same height, the same pipe diameter is used and overall pressures are at atmospheric pressures;

$$Pd \cong \Delta P_f = 51.7\ KPa$$

So an increase in the flow by 50% will increase the discharge pressure and the spare pump will not work

(d) For a power law fluid the maximum shear rate is at the wall and given as:

$$\dot{\gamma}_w = \frac{3n+1}{4n} \frac{4Q}{\pi R^3}$$

or

$$Q = \frac{4n}{3n+1} \dot{\gamma}_w \pi R^3 = \frac{4 \times 0.4}{3 \times 0.4 + 1} \times 200 \times \pi \times (2.62 \times 10^{-2})^3\ \frac{m^3}{s}$$

$$\underline{Q = 8.218 \times 10^{-3}\ \frac{m^3}{s}} \quad \text{or} \quad Q = 8.218 \times 10^{-3}\ \frac{m^3}{s} \times 1041\ \frac{kg}{m^3} \times \frac{3600\ s}{1h}$$

$$\underline{Q = 30{,}798.8\ kg/h}\ \text{Big throughput.}$$