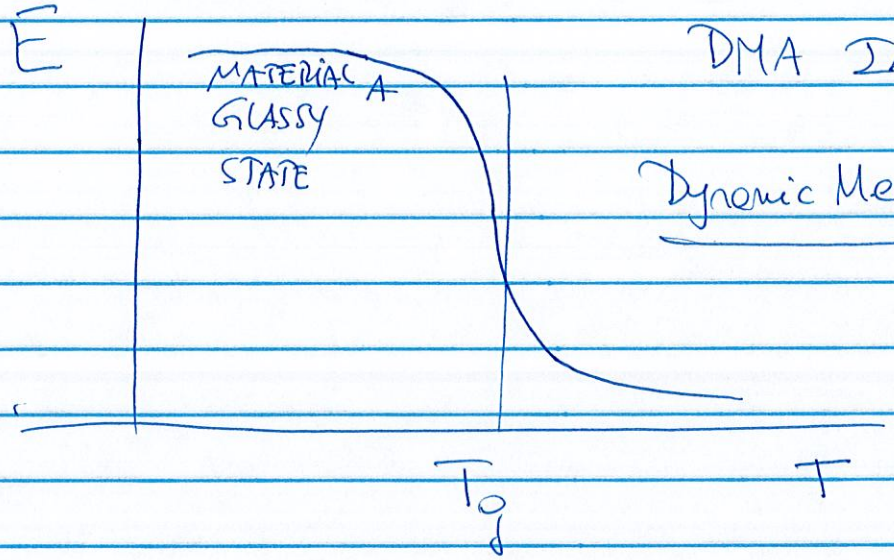
NOTES LECTURE 11/30/2017

MECHANICAL PROPERTY



DMA Instrument

Dynamic Mechanical Analyzer

Controlled strain mode

$$\begin{cases} \gamma(t) = \gamma_0 \sin \omega t & \text{INPUT} \\ \sigma(t) = \sigma_0 \sin(\omega t + \delta) & \text{OUTPUT} \end{cases}$$

$$\Downarrow$$

$$\sigma(t) = \sigma_0 \left[(\sin \omega t \cos \delta) + (\sin \delta \cos \omega t) \right]$$

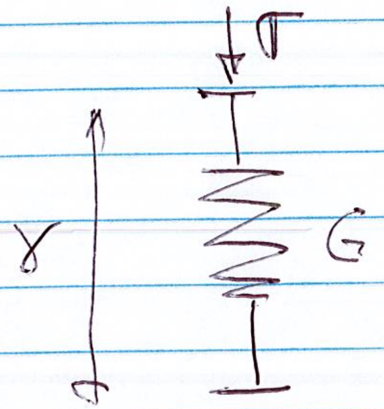
$$\sigma(t) = \underbrace{\sigma_0 \cos \delta}\, \sin \omega t + \underbrace{\sigma_0 \sin \delta}\, \cos \omega t$$

G'' loss modulus

out of phase with input [90°]

IN PHASE WITH INPUT $\Longrightarrow$ G' [Storage modulus

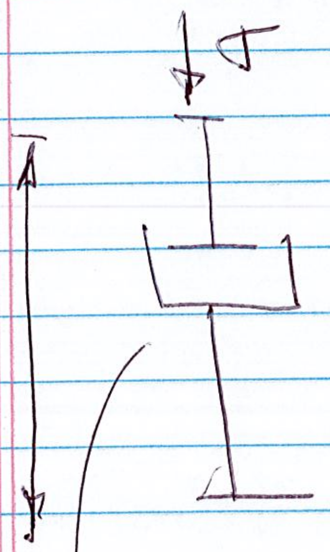
HOOKE LAW DESCRIBES MECHANICAL BEHAVIOR OF A PURELY ELASTIC MATERIAL.



$$\sigma(t) = G\,\gamma(t)$$

FORCE DEFORMATION

NEWTON LAW DESCRIBES THE MECHANICAL BEHAVIOR OF A PURELY LIQUID MATERIAL



$$\sigma(t) = \mu \frac{d\gamma}{dt} = \mu\,\dot{\gamma}(t)$$

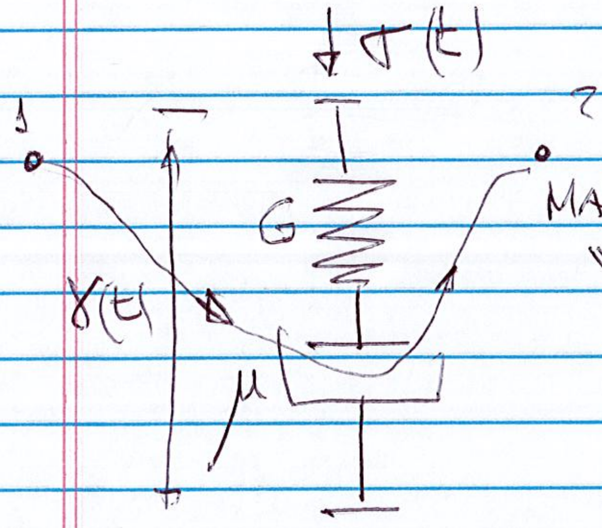
VISCOSITY

$\frac{d\gamma}{dt}$

DASHPOT FORCE FLOW

THE DERIVATIVE OF γ RESPECT TO TIME INDICATES A FLOW

How we can combine these simple ideal "~~Mena~~ Mechanical Analogues" to describe viscoelastic behavior?

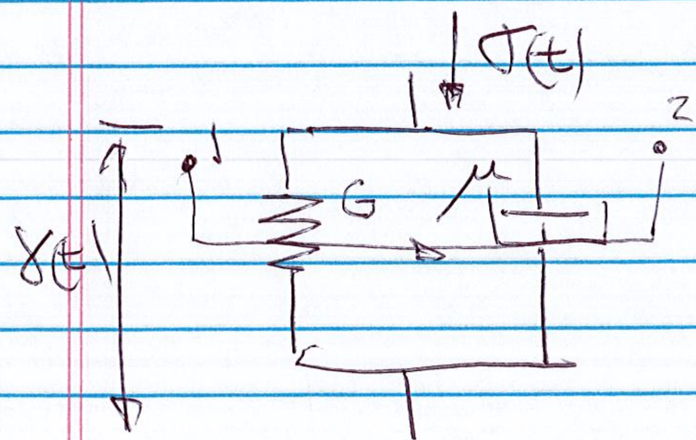


MAXWELL ELEMENT

"LIQUID VISCOELASTIC MATERIAL"

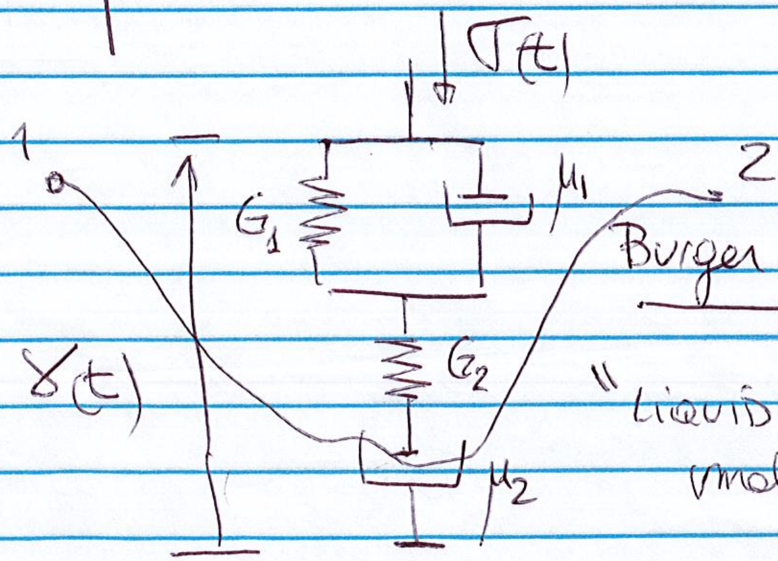
$t_R = \mu / G$ [RELAXATION TIME]

$t_R = \mu / G$ [RETARDATION TIME]



Kelvin-Voigt ELEMENT

"SOLID VISCOELASTIC MATERIAL"



Burger Model.

$t_{R1} = \mu_1 / G_1$

$t_{R2} = \mu_2 / G_2$

"Liquid viscoelastic material"

How Maxwell got the equation to model a Maxwell element? (4)

~~Ans~~ Answering Maxwell questions

1. $\sigma(t)$ is the stress applied

2. yes

3. • In the spring $\sigma(t) = G\,\gamma_s(t)$ ← HOOKE LAW

$$\gamma_s(t) = \frac{\sigma(t)}{G}$$

• In the dashpot.

$$\sigma(t) = \cancel{\mu\, d\gamma_d}\ \mu \frac{d\gamma_d(t)}{dt}$$

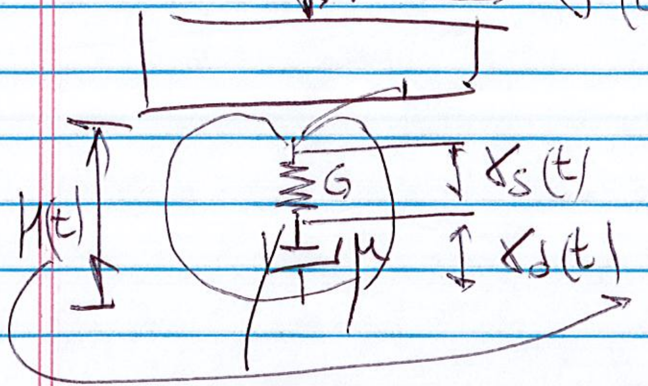
NEWTON LAW

$$\frac{d\gamma_d(t)}{dt} = \dot{\gamma}_d(t) = \frac{\sigma(t)}{\mu}$$

We cannot answered completely the questions.

How we combine the two elements?

$\downarrow F \rightarrow \sigma(t)$



EXPERIMENT GIVES $\sigma(t)$ & $\gamma(t)$

$\gamma(t)$ { MEASURED PROPERTY

MECHANICAL PROPERTIES OF THE APPLE

$$\frac{d\gamma(t)}{dt} \quad \gamma(t) = \gamma_s(t) + \gamma_d(t)$$

$$\frac{\sigma(t)}{G}$$

$$\dot{\gamma}_d(t) = \frac{\sigma(t)}{\mu}$$

$\gamma_0 \sin \omega t$

$$\frac{d\gamma(t)}{dt} = \frac{d\gamma_s(t)}{dt} + \frac{d\gamma_d(t)}{dt}$$

$$\frac{1}{G}\frac{d\sigma(t)}{dt} + \frac{1}{\mu}\sigma(t)$$

$$\gamma_s(t) = \frac{\sigma(t)}{G}$$

$$\frac{d\gamma_s(t)}{dt} = \frac{1}{G}\frac{d\sigma(t)}{dt}$$

$$\frac{d\gamma(t)}{dt} = \frac{1}{G}\frac{d\sigma(t)}{dt} + \frac{1}{\mu}\sigma(t) \quad (6)$$

Multiply by μ the above equation

$$\mu\frac{d\gamma(t)}{dt} = \underbrace{\frac{\mu}{G}}_{t_R}\frac{d\sigma(t)}{dt} + \sigma(t)$$

$\gamma(t) = \gamma_0 \sin \omega t$

$$\boxed{\mu\frac{d\gamma(t)}{dt} = t_R\frac{d\sigma(t)}{dt} + \sigma(t)}$$

For a liquid t_R is small.

$$\sigma(t) = \mu\frac{d\gamma(t)}{dt} \quad [\text{LIQUID}$$

FOR AN ELASTIC t_R IS LARGE

$$\frac{\mu}{G} \rightarrow t_R\frac{d\sigma(t)}{dt} = \mu\frac{d\gamma(t)}{dt} \Longrightarrow \sigma(t) = G\gamma(t)$$