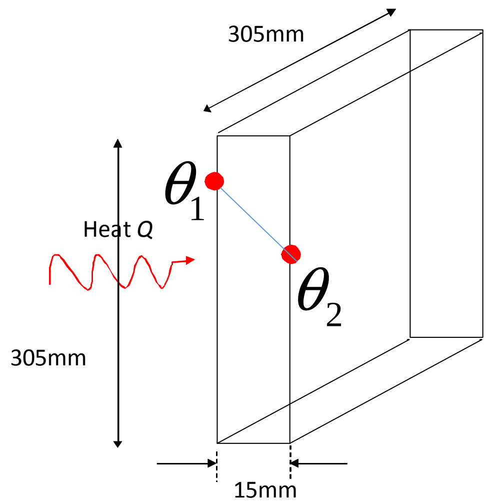
Homework 3 - Fall 2017
Solutions

Problem 1



The 1D heat transfer through a wall can be calculated by the Fourier Law that is written below:

$$Q = -kA_p d\theta/dx$$

where A_p is the area of the system perpendicular to the heat flow (i.e. the are 305mmx305mm in the figure, $d\theta/dx$ is the temperature (θ) in the direction of the heat flow (i.e. in this case). k is the thermal conductivity of of the material. The negative sign is because heat transfer is in the direction of the decreasing temperatures so $d\theta/dx$ is negative, so the negative is to have a positive heat flow. As explained in class it is mpore convenient to estimate the heat flow from the temperatures on the sides of the plate θ_1 and θ_2 ($\Delta\theta = \theta_1 - \theta_2$). By assuming steady state the expression to estimate the heat is (see derivation done in lectures):

$$Q = kA_p (\theta_1-\theta_2)/d = kA_p \Delta\theta/d$$

so

$$k = Q * d / (Ap*\Delta\theta)$$

So let's enter the data to solve the problem:

$\Delta\theta := 4 \cdot K$

In order to have consistent units, the units of temperature use are in Kelvin (K). Working with temperature units could be problematic unless we can keep the units. Since is a temperature difference I can use units in kelvins or Celcius and will be the same

$d := 15mm \qquad L_p := 305mm \qquad W_p := 305 \cdot mm \qquad A_p := L_p \cdot W_p$

$A_p = 0.093\,m^2 \qquad Q := 10 \cdot W$

$$k := \frac{Q \cdot d}{A_p \cdot \Delta\theta} \qquad \boxed{k = 0.403 \cdot \frac{W}{m \cdot K}}$$

Problem 2

Data

$kJ := 1000 \cdot J \qquad L_b := 100 \cdot mm \qquad W_b := 100 \cdot mm \qquad H_b := 150 \cdot mm$

$\rho_{s\_cheese} := 1055 \cdot \frac{kg}{m^3} \qquad c_{cheese} := 3.41 \cdot \frac{kJ}{kg \cdot K}$

$k_{cheese} := 0.306 \cdot \frac{W}{m \cdot K} \qquad k_{air} := 0.026 \cdot \frac{W}{m \cdot K}$ (at 25^{o}C taken from internet)

$M_{cheese} := 0.6 \cdot kg \qquad V_{box} := L_b \cdot W_b \cdot H_b \qquad V_{box} = 1.5 \times 10^{-3} \cdot m^3$

$$\rho_{Bulk} := \frac{M_{cheese}}{V_{box}} \qquad \boxed{\rho_{Bulk} = 400 \frac{kg}{m^3}}$$

$$\varepsilon_{Bulk} := 1 - \frac{\rho_{Bulk}}{\rho_{s\_cheese}} \qquad \boxed{\varepsilon_{Bulk} = 0.621}$$

(a) Let's consider that the box with grated cheese is a composite system in which the dispersed phase (d) is the grated cheese whereas the air is the continuous phase. Thus the volume fraction of air is equal than the bulk porosity of the system. The discrete phase is the grated cheese and its volume fraction can that can be calculated as $1 - \varepsilon_B$

$$\varepsilon_d := 1 - \varepsilon_{Bulk}$$

$$k_d := k_{cheese} \qquad\qquad k_c := k_{air}$$

Let's use the Maxwell-Eucken Equation

$$k_{cheese\_box} := \frac{k_c \cdot \left[2 \cdot k_c + k_d - 2 \cdot \varepsilon_d \cdot (k_c - k_d)\right]}{2 \cdot k_c + k_d + \varepsilon_d \cdot (k_c - k_d)}$$

$$\boxed{k_{cheese\_box} = 0.059 \cdot \frac{W}{m \cdot K}}$$

Because the weight of the air is negligible the heat specific of the box with the cheese can be approximated to that of the cheese

$$\alpha_{cheese\_box} := \frac{k_{cheese\_box}}{\rho_{Bulk} \cdot c_{cheese}}$$

$$\boxed{\alpha_{cheese\_box} = 4.317 \times 10^{-8} \frac{m^2}{s}}$$

(b) If the new bulk density is 0.50 what is the new packing weight?

$$\varepsilon_{Bulk\_new} := 0.5 \qquad \varepsilon_{d\_ch\_new} := 1 - \varepsilon_{Bulk\_new}$$

$$\varepsilon_{d\_ch\_new} = 1 - \rho_B/\rho_{s\_cheese} \;\text{-->}\; \rho_B = \rho_{s\_cheese}(1 - \varepsilon_{d\_ch\_new})$$

$$\rho_{B\_new} := \rho_{s\_cheese} \cdot (1 - \varepsilon_{d\_ch\_new}) \qquad\qquad \rho_{B\_new} = 527.5 \frac{kg}{m^3}$$

and the new weight (or mass) is calculated as:

$$M_{new} := \rho_{B\_new} \cdot V_{box} \qquad \boxed{M_{new} = 0.791\,kg}$$

(c) To calculate the new thermal diffusivity for a bulk density of 0.50. The thermal conductivity has to be recalculated.

$$k_{ch\_bx\_nw} := \frac{k_c \cdot \left[2 \cdot k_c + k_d - 2 \cdot \varepsilon_{d\_ch\_new} \cdot \left(k_c - k_d\right)\right]}{2 \cdot k_c + k_d + \varepsilon_{d\_ch\_new} \cdot \left(k_c - k_d\right)}$$

$$\boxed{k_{ch\_bx\_nw} = 0.076 \cdot \frac{W}{m \cdot K}}$$

$$\alpha_{cheese\_new} := \frac{k_{ch\_bx\_nw}}{\rho_{B\_new} \cdot c_{cheese}}$$

$$\boxed{\alpha_{cheese\_new} = 4.23 \times 10^{-8} \frac{m^2}{s}}$$

This new value did not change much respect to the one calculated in (a). The bulk density increase when the bulk porosity decreases so one would expect that thermal diffusivity would decrease. However when the bulk density increases the thermal conductivity of the system increases so that compensates the increase in the bulk density.

Problem 3

1. Addition of sodium chloride

Water activity decreases because I am adding small molecular size material

2. Addition of native starch (granules)

No much happens, startch has a very large molecular weight

3. Heating the food with starch and cooling again to the original temperature without loss of water

It will reduce a_w, water gets immbilized due to gelatinization

4. Enzymatic hydrolysis of the protein present

Molecular wright/size of the protein decreases with the hydrolisis, so depending on the extent of the hydrolysis a_w will decrease in more or less extent

5. Freezing part of the water

Some water will become immobilized so a_w will decrease