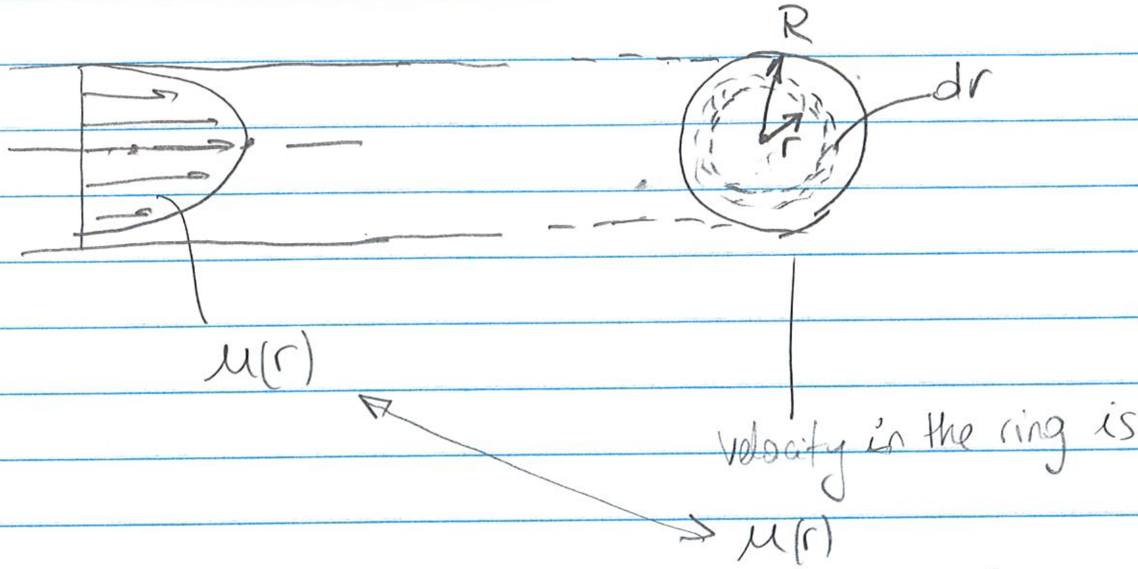
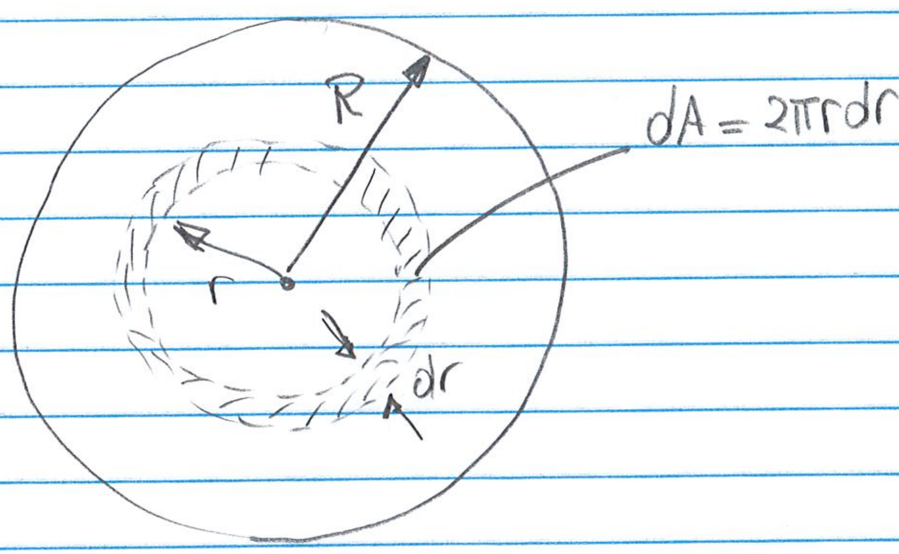


So flow in the ring is small [differential]

$$dQ = \mu(r)\,\underbrace{dA}_{\text{differential area of the ring}} \qquad (1)$$

$$dA = 2\pi r\,dr \qquad (2)$$



Substituting Eq.(2) into Eq.(1)

$$dQ = u(r)\, 2\pi r\, dr$$

TOTAL VOLUMETRIC FLOW Q IS CALCULATED AS:

$$Q = \int dQ = \int_0^R u(r)\, 2\pi r\, dr$$

$$\boxed{Q = 2\pi \int_0^R u(r)\, r\, dr}$$

But $u(r)$ is the velocity profile, which is a function of the fluid rheology, so in order to incorporate the RHEOLOGY OF THE MATERIAL IN THE CALCULATION OF FLOW WE NEED TO RELATE THE RHEOLOGY OF THE MATERIAL WITH $u(r)$

Let's look how the rheology is related to $u(r)$

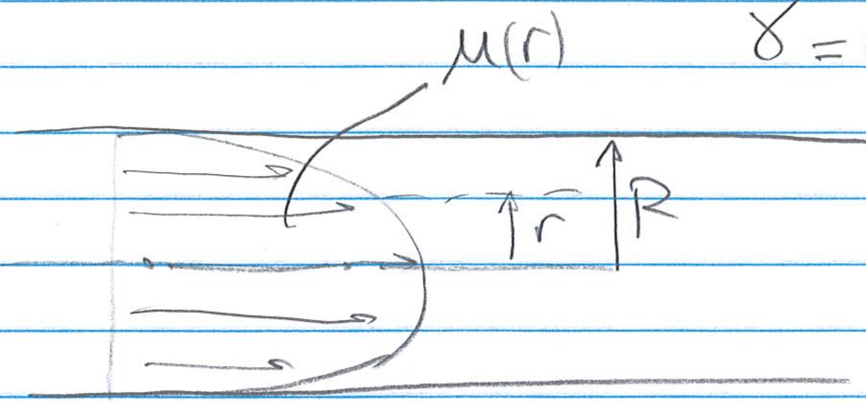
velocity profile.

For a Newtonian material

$$\sigma = \mu \dot{\gamma}$$

SHEAR STRESS

SHEAR RATE



$$\dot{\gamma} = -\frac{du(r)}{dr}$$

$$Q = 2\pi \int_0^R u(r)\, r\, dr$$

INTEGRATION BY PARTS

$u(r).v(r) \leftarrow$ Defined

$$d\left[u(r).v(r)\right] = u(r)\,\underline{dv(r)} + v(r)\,\underline{du}(r)$$

$$\int_0^R d(u(r).v(r)) = \int_0^R u(r)\,dv(r) + \int_0^R v(r)\,du(r)$$

$$u(r)\,v(r)\Big|_0^R = \int_0^R u(r)\,dv(r) + \int_0^R v(r)\,du(r) \quad (3)$$

Now we can define $u(r)$ and $v(r)$

$$Q = 2\pi \int_0^R \underbrace{\mu(r)}_{u(r)}\,\underbrace{r\,dr}_{dv(r)}$$

$$u(r) \equiv \mu(r) \quad (4)$$

$$dv(r) = r\,dr \Rightarrow v(r) = \frac{r^2}{2} \quad (5)$$

Substituting Eqs (4) and (5) in Eq.(3)

$$\mu(r)\frac{r^2}{2}\Big|_0^R = \int_0^R \mu(r)\,r\,dr + \int_0^R \frac{r^2}{2}\underbrace{\frac{d\mu(r)}{dr}}_{-\dot{\gamma}(r)}dr \quad (5)$$

$$\Downarrow$$

$$0 = \int_0^R \mu(r)\,r\,dr - \int_0^R \frac{r^2}{2}\dot{\gamma}(r)\,dr$$

$$\int_0^R \mu(r)\,r\,dr = \int_0^R \frac{r^2}{2}\dot{\gamma}(r)\,dr$$

$$Q = \cancel{2}\pi\int_0^R \frac{r^2}{\cancel{2}}\dot{\gamma}(r)\,dr$$

$$\sigma(r) = \frac{\Delta P\, r}{2L} \text{ AND } \sigma_w = \frac{\Delta P R}{2L}$$

$$\frac{\sigma(r)}{\sigma_w} = \frac{\frac{\cancel{\Delta P}\,r}{\cancel{2L}}}{\frac{\cancel{\Delta P}\,R}{\cancel{2L}}} = \frac{r}{R}$$

$$\frac{\sigma(r)}{\sigma_w} = \frac{r}{R}$$

so
$$r^2 = \frac{\sigma^2}{\sigma_w^2} R^2$$

$$dr = \frac{d\sigma}{\sigma_w} R$$

$$Q = \pi \int_0^R r^2 \, \dot{\gamma}(r) \, dr$$

$$Q = \pi \int_0^R \frac{\sigma^2 R^2}{\sigma_w^2} \frac{d\sigma}{\sigma_w} R \, \dot{\gamma}(r)$$

$$Q = \frac{\pi R^3}{\sigma_w^3} \int_0^R \sigma^2 \, d\sigma \, \dot{\gamma}(r)$$

RHEOLOGY

$$Q = \frac{\pi R^3}{\tau_w^3} \int_0^R \tau^2 f(\tau)\, d\tau \quad (7)$$

$\dot{\gamma}$

$$\tau = \mu \dot{\gamma} \Rightarrow \dot{\gamma} = \frac{\tau}{\mu} = f(\tau)$$

DONE ON 10/24/2017

We got THE HAGEN - POISEUILLE

EQUATION FOR PRESSURE LOSS

LECTURE ON 10/26/2017

$$\boxed{Q = \frac{\pi R^3}{\sigma_w^3}\int_0^{\sigma_w} f(\sigma)\,\sigma^2\, d\sigma} \quad (1)$$

How could we calculate the pressure loss for a power-law fluid.

We need to define the Rheological model describing the fluid

$$\sigma = K\dot{\gamma}^n \Longrightarrow \dot{\gamma} = \left(\frac{\sigma}{K}\right)^{1/n} = f(\sigma)$$

What is $f(\sigma)$ in Eq. (1) ?

$$f(\sigma) \equiv \dot{\gamma}$$

$$Q = \frac{\pi R^3}{\sigma_w^3}\int_0^{\sigma_w} \sigma^2 \left(\frac{\sigma}{K}\right)^{1/n} d\sigma$$

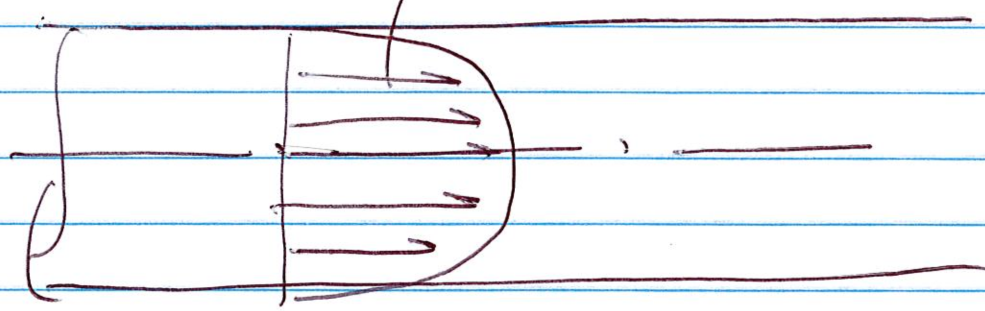
$$Q = \frac{\pi R^3}{\sigma_w^3 K}\int_0^{\sigma_w} \sigma^{2+\frac{1}{n}}\, d\sigma$$

Power Law MODEL [Rheological Model] (9)

$$\tau_{(r)} = K \dot{\gamma}^{n}_{(r)}$$ CALCULATED EVERY "r" in the pipe

$$\tau_{w} = K \dot{\gamma}^{n}_{w}$$ CALCULATED AT THE WALL.

$\mu(r) \longrightarrow \bar{\mu}$ MEAN VELOCITY



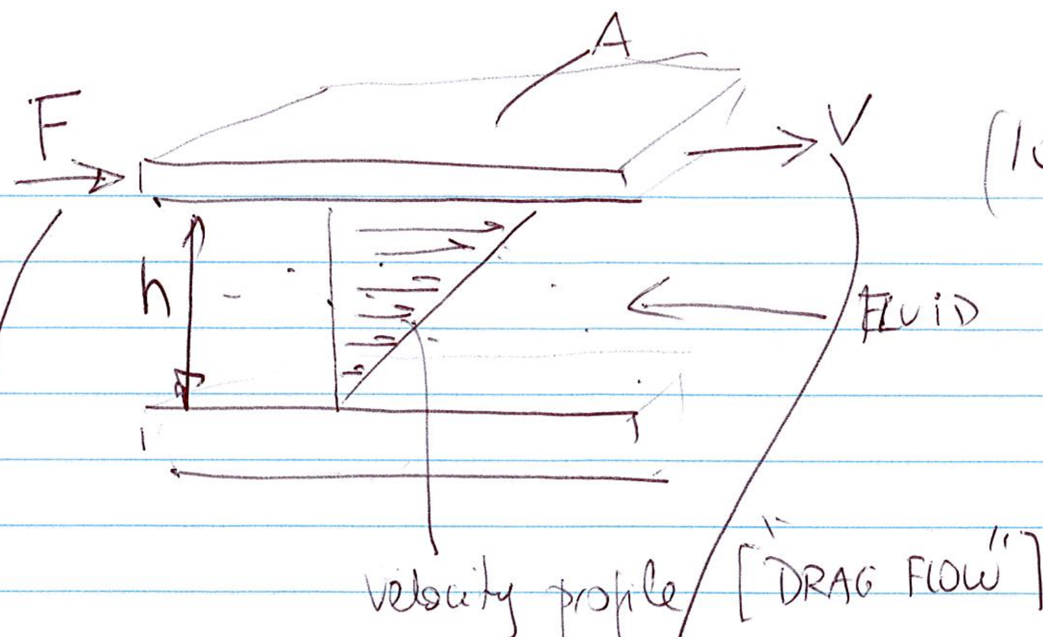
$$Q \cancel{=} \mu(r) \cdot A$$

$$Q = \bar{\mu} \cdot A$$

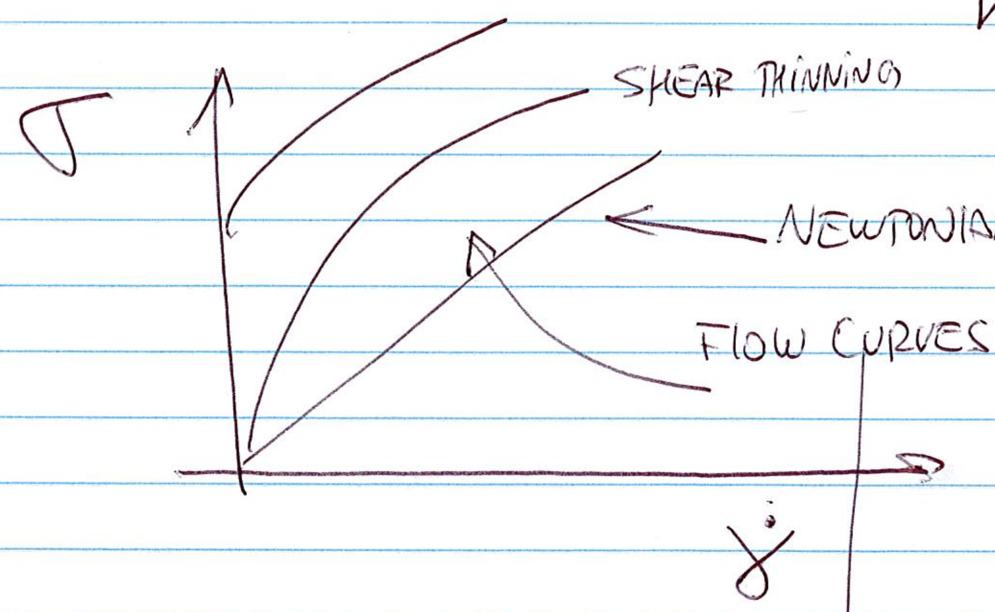
MEAN VELOCITY

Volumetric Flow rate m^3/s

NEWTON EXPERIMEN



$$\sigma = \frac{F}{A} \; [Pa] \qquad \dot{\gamma} = \frac{V}{h} \; [1/s]$$



RHEOLOGICAL MODEL

CONTINUITY EQUATION

$\nabla \cdot \underline{u} = 0$ MICRO-SCALE

INCOMPRESSIBLE FLUID

INTEGRATE

$Q = \text{constant}$ ✓ MACROSCALE