TUTORIAL QUESTIONS – Exam 1

Question 1

Kiwifruit juice (a non-fatty food), extracted from the fresh fruit at 20°C, is to be cooled to temperature some degrees below its initial freezing point to cause some of the water in the juice to turn into ice. The ice crystals are then to be mechanically separated from the mixtu leaving a concentrated juice (this process is called freeze concentration). The initial mass composition of the juice is:

Water 86.5%

Total solids (SNF) 13.5%

It is desired to concentrate the juice to 60% total solids by weight. All the water in the juice is freezable, i.e. none is bound.

(a) Calculate at what temperature the juice will have to be cooled to achieve the desired concentration. Hint: use 1 kg of unfrozen juice as the basis of the mass balance to estimate that temperature.

(b) Calculate how much heat energy (in kJ/kg) will have to be removed from the fresh jui during the freeze concentration process.

DATA

Initial Freezing point of fresh kiwifruit juice = -1.3C

Specific heat of kiwifruit dry solids = 1.50 kJ/kgK

Specific heat of water = 4.18 kJ/kgK

Specific heat of ice = 2.11 kJ/kgK

Latent heat of fusion of water = 320 kJ/kg

(a) The amount of ice contained in the product can be related to the temperature by the following equation:

$$\frac{x_{ice}}{x'_w} = \left(1 - \frac{\theta_{if}}{\theta}\right)$$

where the mass fraction of ice is x_{ice} and x'_w is the amount of freezable water calculated as:

$$x'_w = x_w - BW$$

Let's assume 1 kg of unfrozen kiwifruit juice, so it contains:

0.865 kg water

0.135 kg solids non-fat

$x_W := 0.865$ $\quad$ $x_{SNF} := 0.135$

So after the juice is frozen and ice is separated the concentration of solids will be 60%. In other words

$$\frac{0.135}{0.135 + x} = 0.60 \text{ solve } \rightarrow 0.09$$

$$x_{wnew} := 0.09$$

The command Solve can be used in MathCad to evaluate the value of x, which in this case is the amount of water in the kiwi juice after the ice is separated, so that the concentration of solids is 0.6 (60%)

The amount of ice that has to be separated is: $x_{ice} := 0.865 - x_{wnew}$

Let's assign all the data (All consistent units are chosen)

$kJ := 1000 \cdot J$ $\quad L_f := 320 \quad x_{ice} = 0.775$

$\theta_1 := 20$ Initial Temperature

$\theta_{if} := -1.3 \quad C_{SNF} := 1.50 \quad C_w := 4.18 \quad C_{ice} := 2.11 \quad x_{BW} := 0$

From Eq.(1) $\theta_2 := \dfrac{\theta_{if}}{1 - \dfrac{x_{ice}}{x_w}}$ $\quad \theta_2 = -12.5$ This temperature is in celcius

(b) Energy Removed

$$\Delta h_{20\to-12.5} = \underbrace{\Delta h_{20\to-1.3}}_{\text{Above Freezing}} + \underbrace{\Delta h_{-1.3\to-12.5}}_{\text{Below Freezing}}$$

Above Freezing Point

$$\Delta h_{\theta 1\_\theta if} := 1.55 \cdot (1 - x_w) \cdot (\theta_1 - \theta_{if}) \ldots$$
$$+ 2.09 \cdot 10^{-3} \cdot (1 - x_w)\left(\theta_1{}^2 - \theta_{if}{}^2\right) \ldots$$
$$+ x_w \cdot C_w \cdot (\theta_1 - \theta_{if})$$

The last term is not used because is negligible at high moistures

$\boxed{\Delta h_{\theta 1\_\theta if} = 81.6}$ Units of kJ/kg

Below Freezing Point

$$\begin{aligned}\Delta h_{\theta 2\_\theta if} := 1.50\cdot(1 - x_w)\cdot(\theta_{if} - \theta_2) \ \ldots & = 286.704 \\ + C_{ice}\cdot x_{BW}\cdot(\theta_{if} - \theta_2) \ \ldots & \\ + C_w\cdot x_w\cdot\theta_{if}\cdot\ln\left(\frac{\theta_{if}}{\theta_2}\right) \ \ldots & \\ + C_{ice}\cdot x_w\cdot(\theta_{if} - \theta_2) \ \ldots & \\ + C_{ice}\cdot x_w\cdot\theta_{if}\cdot\ln\left(\frac{\theta_{if}}{\theta_2}\right) \ \ldots & \\ + -L_f\cdot x_w\cdot\left(\frac{\theta_{if}}{\theta_2} - 1\right) & \end{aligned}$$

$\boxed{\Delta h_{\theta 2\_\theta if} = 286.704}$ Units of kJ/kg

Total Energy/Heat Removed

$$\Delta h_{total} := \Delta h_{\theta 1\_\theta if} + \Delta h_{\theta 2\_\theta if}$$

$\boxed{\Delta h_{total} = 368.3}$ kJ/kg

Question 2

A liquid ice cream mix is to be pasteurized and then cooled in continuous heat exchangers prior to whipping and freezing. Values of the density, specific heat and thermal conductivi the mix are required for use in heat exchanger design calculations. Predict such values usi the data and equations given below.

The mix may be thought of as a random dispersion of milk fat globules in a continuous aqueous phase which itself consists of a random dispersion of non-fat milk solids in a 16.7 sucrose solution. The mix is non-porous.

Hint: volume fractions used at any state of your calculations must be fractions of the part of the mix under consideration at that state.

DATA

Mass composition of mix

Water	60%
Sucrose	12%
Non-fat milk solids	10%
Milk fat	18%

Thermophysical properties

Substance density of 16.7% sucrose solution = 1070 kg/m^3

Substance density of non-fat milk solids = 1380 kg/m^3

Density of milk fat = 930 kg/ m^3

Thermal conductivity of 16.7% sucrose solution = 0.55 W/ m.K

Thermal conductivity of non-fat milk solids = 0.20 W / m.K

Thermal conductivity of milk fat = 0.19 W / m.K

Specific heat of 16.7% sucrose solution = 3.77 kJ / kg.K

Specific heat of non-fat milk solids = 1.70 kJ / kg.K

Specific heat of milk fat = 2.95 kJ / kg.K

$x_{wic} := 0.60$ $\quad x_{sic} := 0.12$ $\quad x_{SNF\_milk} := 0.10$

$x_{suc\_sol} := x_{wic} + x_{sic}$ $\quad x_{suc\_sol} = 0.72$ $\quad x_{fat} := 0.18$

$\rho_{suc\_sol} := 1070 \cdot \frac{kg}{m^3}$ $\quad \rho_{fat} := 930 \cdot \frac{kg}{m^3}$ $\quad \rho_{SNF} := 1380 \cdot \frac{kg}{m^3}$

$c_{suc\_sol} := 3.77 \cdot \frac{kJ}{kg \cdot K}$ $\quad c_{SNF\_milk} := 1.70 \cdot \frac{kJ}{kg \cdot K}$ $\quad c_{fat} := 2.95 \cdot \frac{kJ}{kg \cdot K}$

Since the mixture is non-porous the substance density will be:

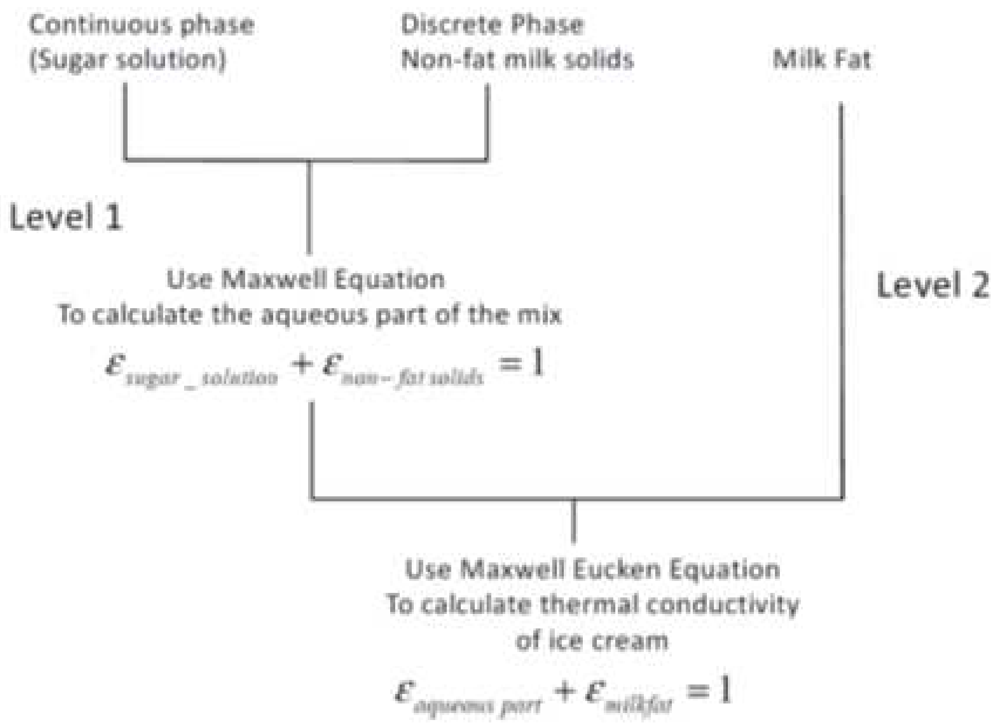
$$\rho_s := \frac{1}{\frac{x_{suc\_sol}}{\rho_{suc\_sol}} + \frac{x_{SNF\_milk}}{\rho_{SNF}} + \frac{x_{fat}}{\rho_{fat}}} \qquad \rho_s = 1065.1 \frac{kg}{m^3}$$

And the specific heat (above the freezing point) is calculated as:

$$c_{ice\_cream\_mix} := x_{suc\_sol} \cdot c_{suc\_sol} + x_{SNF\_milk} \cdot c_{SNF\_milk} + x_{fat} \cdot c_{fat}$$

$$c_{ice\_cream\_mix} = 3.415 \cdot \frac{kJ}{kg \cdot K}$$

A hierarchy of calculations is required. For calculations at a given level, put attention in the calculated volume fractions and make sure that these fractions of the part of the material being considered at that level, and not fractions of the total material. Except at the finakl level where **<u>all the material is being considered</u>**



Let's calculate the level 1 in which the sugar solution is the continuous phase and the non-fat milk solids is the discrete phase. In the system where the fat is not included, tl composition of sucrose in a solution that does not include fat would be:

$$x_{suc\_sol\_level1} := \frac{x_{suc\_sol}}{x_{suc\_sol} + x_{SNF\_milk}} \qquad \boxed{x_{suc\_sol\_level1} = 0.878}$$

Let's calculate now the concentration of solid non fat milk solids, also for the level 1

$$x_{SNF\_milk\_level1} := \frac{x_{SNF\_milk}}{x_{suc\_sol} + x_{SNF\_milk}} \qquad \boxed{x_{SNF\_milk\_level1} = 0.122}$$

We can also check that the sum of those mass fractions is equal to 1 as they are the only two components in the system (level 1). The volume fraction can be then calculate if we know the substance density of the system at the level 1.

$$\rho_{s\_level\_1} := \frac{1}{\dfrac{x_{suc\_sol\_level1}}{\rho_{suc\_sol}} + \dfrac{x_{SNF\_milk\_level1}}{\rho_{SNF}}}$$

$$\boxed{\rho_{s\_level\_1} = 1100.1 \frac{kg}{m^3}}$$

$$\varepsilon_{SNF\_milk} := \frac{x_{SNF\_milk\_level1} \cdot \rho_{s\_level\_1}}{\rho_{SNF}} \qquad \boxed{\varepsilon_{SNF\_milk} = 0.097}$$

This is the dispersed phase in the level 1, so

$$\varepsilon_{d\_l1} := \varepsilon_{SNF\_milk}$$

The volume fraction of continuous phase in the level 1 is the volume fraction of the sucrose solution that can be calculated as $1-\varepsilon_{d\_level1}$, or by the definition of the volume fraction

$$\varepsilon_{suc\_sol\_level\_1} := \frac{x_{suc\_sol\_level1} \cdot \rho_{s\_level\_1}}{\rho_{suc\_sol}}$$ $$\varepsilon_{suc\_sol\_level\_1} = 0.903$$

or $$\varepsilon_{c\_l1} := 1 - \varepsilon_{SNF\_milk}$$ $$\varepsilon_{c\_l1} = 0.903$$

As illustrated above, and as expected, the two values agree. Now for example we can us the Maxwell or other Equation (for example the EMT to estimate the thermal conductivity at the level 1. Let's use the Maxwell-Eucken equation, but first let'd define the thermal conductivity of the system:

$$k_{suc\_sol} := 0.55 \cdot \frac{W}{m \cdot K}$$

This is also the thermal conductivity of the continuous phase, which we can call k_{c_l1}

$$k_{c\_l1} := k_{suc\_sol}$$

$$k_{SNF\_milk} := 0.20 \cdot \frac{W}{m \cdot K}$$

This is also the thermal conductivity of the discrete phase, which we can call kd $_{\_l1}$

$$k_{d\_l1} := k_{SNF\_milk}$$

So, the thermal conductivity of the system **at level 1** will be

$$k_{l1} := \frac{k_{c\_l1} \cdot \left[2 \cdot k_{c\_l1} + k_{d\_l1} - 2 \cdot \varepsilon_{d\_l1} \cdot \left(k_{c\_l1} - k_{d\_l1}\right)\right]}{2 \cdot k_{c\_l1} + k_{d\_l1} + \varepsilon_{d\_l1} \cdot \left(k_{c\_l1} - k_{d\_l1}\right)}$$ $$k_{l1} = 0.508 \cdot \frac{W}{m \cdot K}$$

Now, we move to the level 2. In this clase the continuous phase will be the sucrose solution and the discrete phase will be the fat. Now we have to calculate the volume fractions of each component. Since the discrete phase is fat

$$\varepsilon_{fat} := \frac{\rho_s \cdot x_{fat}}{\rho_{fat}}$$

The density is the density of the whole mix that was calculated before:

And $$\varepsilon_d := \varepsilon_{fat}$$ $$\varepsilon_d = 0.206$$

The continuous phase for the calculation at the level 2 will be ε_c-1, or calculated by the equation for volume fraction as:

$$\varepsilon_{c\_l2} := \frac{\rho_s \cdot (1 - x_{fat})}{\rho_{s\_level\_1}} \qquad \boxed{\varepsilon_{c\_l2} = 0.794}$$

or $$\varepsilon_c := 1 - \varepsilon_d \qquad \boxed{\varepsilon_c = 0.794}$$

So the two values are in agreement, it is checking because this values is not used in the Maxwell Eucken equation, for using this equation we need to use the thermal conductivity of fat, which will be the dispersed phase whereas the thermal conductivity calculated at level 1 will be the conituous phase:

$$k_{fat} := 0.19 \cdot \frac{W}{m \cdot K} \qquad k_d := k_{fat}$$

Let's apply the Maxwell Eucken equation now:

$$k_{ice\_cream} := \frac{k_{l1} \cdot \left[2 \cdot k_{l1} + k_d - 2 \cdot \varepsilon_d \cdot (k_{l1} - k_d)\right]}{2 \cdot k_{l1} + k_d + \varepsilon_d \cdot (k_{l1} - k_d)}$$

$$\boxed{k_{ice\_cream} = 0.429 \cdot \frac{W}{m \cdot K}}$$

And

$$\alpha_{ice\_cream} := \frac{k_{ice\_cream}}{\rho_s \cdot c_{ice\_cream\_mix}}$$

$$\boxed{\alpha_{ice\_cream} = 1.18 \times 10^{-7} \frac{m^2}{s}}$$

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