Fall 2015
ABE 307
Final Exam Problem

1. A pipe needs to be designed for carrying water at a specified mass flow rate. The known parameters for the pipe to be designed is pressure drop (P_0-P_L), length (L) mass flow rate (w) and fluid properties (μ, ρ). **(12 Points)**

   a. Derive a relationship between friction factor (f) and the known quantities :$(P_0 - P_L)$, mass flow rate (w), Length (L) and Reynolds nr (Re).
   [Hint: Use the method of eliminating unknowns to relate the known variables to friction factor] **(6 Points)**

   Mass flow rate: $w = \frac{\pi D^2 \langle v_z \rangle \rho}{4}$

   Reynolds number: $Re = \frac{\rho \langle v_z \rangle D}{\mu}$

   Friction factor: $f = \frac{(P_0 - P_L) D}{2 \rho \langle v_z \rangle^2 L}$

D L

$$w = \frac{\pi D^2 \langle v_z \rangle \rho}{4}$$

$$\Leftrightarrow \langle v_z \rangle = \frac{4w}{\pi \rho D^2}$$

$$Re = \frac{\rho \langle v_z \rangle D}{\mu} = \frac{\rho D}{\mu} \times \frac{4w}{\pi \rho D^2} = \frac{4w}{\pi \mu D}$$

$$D = \frac{4w}{\pi \mu Re}$$

$$\langle v_z \rangle = \frac{4w}{\pi \rho \left[\frac{4w}{\pi \mu Re}\right]^2} = \frac{\pi \mu^2 Re^2}{4w\rho}$$

$$f = \frac{(P_0 - P_L)}{2\rho L} \left(\frac{4w}{\pi \mu Re}\right) \frac{1}{\left(\frac{\pi^2 \mu^4 Re^4}{16 w^2 \rho^2}\right)}$$

$$f = \frac{(P_0 - P_L) 32 w^3 \rho}{Re^5 \mu^5 \pi^3 L}$$

$$\log f = -5 \log Re + \log \left[\frac{(P_0 - P_L) 32 w^3 \rho}{\mu^5 \pi^3 L}\right]$$

b. Calculate the diameter (D) of the pipe for the following parameters. Assume that the pipe is hydraulically smooth. The friction factor chart for pipe is provided. **(6 Points)**

Mass flow rate w = 5 X 10^3 lbm/hr
Density (ρ) = 62.3 lbm/(ft^3)
L = 1000 ft
P_0-P_L = 2.78 x (10^4) lbm/(ft-sec^2)
Viscosity (μ) = 6.93 x (10^{-4}) lbm/ft-sec

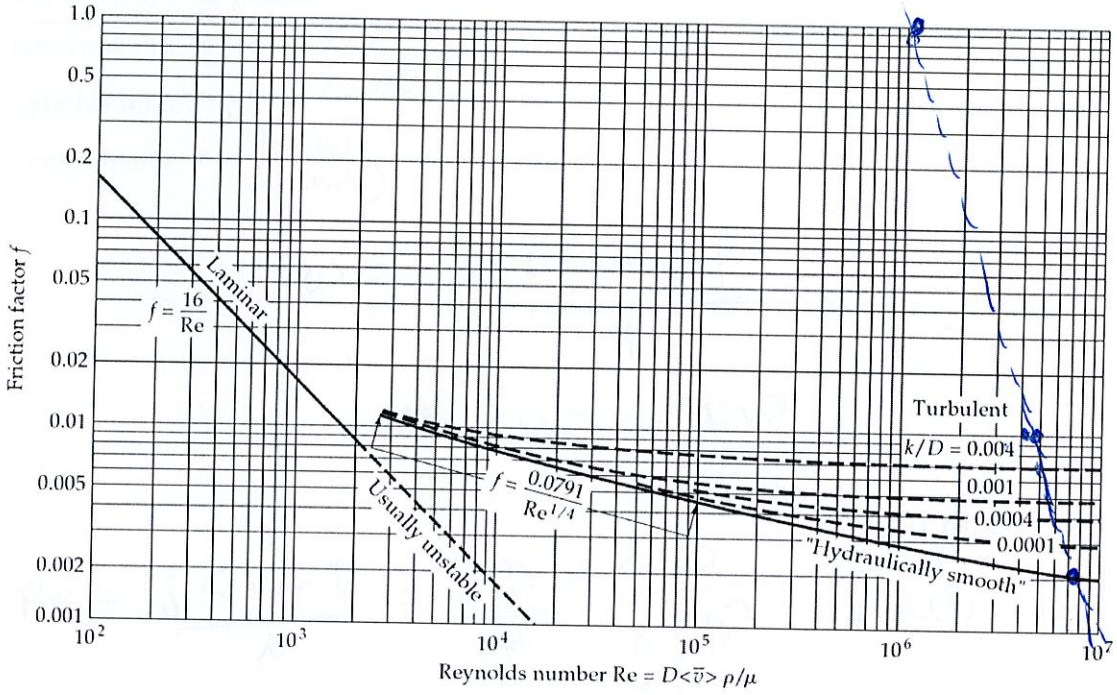

Fig. 6.2-2. Friction factor for tube flow (see definition of f in Eqs. 6.1-2 and 6.1-3). [Curves of L. F. Moody, *Trans. ASME*, **66**, 671–684 (1944) as presented in W. L. McCabe and J. C. Smith, *Unit Operations of Chemical Engineering*, McGraw-Hill, New York (1954).]

This is a log-log graph (Base 10)

$$\log f = -5 \log Re + \log\left[\frac{(P_0 - P_L)\, 32\, \omega^3\, \rho}{\mu^5\, \pi^3\, L}\right]$$

$$\log f = -5 \log Re + 30.146.$$