Read

Section 4.3 - BSL.

Nov 3rd, 2017.
Friday.

Flow of Inviscid Flow Using Velocity Potential.

considering a situation where effect of viscosity is negligible.
Re >> 1, very high Reynolds number or fast flow.

~~Situations where Potent~~

Flow over a submerged body, the whole flow field can be determined using Potential flow theory and Boundary layer theory.

(1) Potential Flow Theory.

Def :→ Irrotational flow, two dimensional flow (planar flow), Incompressible flow, Viscous forces are neglected, Steady flow.

Equation of motion for Inviscid flow:

$$\rho \frac{D\vec{V}}{Dt} = -\nabla p + \rho \vec{g}$$

(Euler equation)

[Eqn is used to get the velocity and pressure distribution].

Assumptions,

(1) Incompressible flow. ρ = const (density is constant).

$\nabla . \vec{V} = 0$ (from eqn. of continuity).

(2) Irrotational flow

$\nabla \times \vec{V} = 0$ (curl of velocity)

(Two dimensional, steady flow).

$$\rho \frac{D\vec{v}}{Dt} = -\nabla p + \rho \vec{g}$$

$$\rho\left[\frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\nabla\vec{v}\right] = -\nabla p + \rho\vec{g}$$

$\nabla \times \vec{v} = 0$ for irrotational flow.

$$\vec{v}\cdot\nabla\vec{v} = \frac{1}{2}\nabla v^2 - \vec{v}\times\cancel{[\nabla\times\vec{v}]}^{\,0}$$ From appendix.

Eqn reduces to

$$\rho\left[\cancel{\frac{\partial \vec{v}}{\partial t}}^{\,0} + \frac{1}{2}\nabla v^2\right] = -\nabla p + \rho\vec{g}$$

$$\rho\left[\frac{1}{2}\nabla v^2\right] = -\nabla p + \rho\vec{g} \qquad \mathcal{P} = p + \rho g z.$$

$$\rho\int\frac{1}{2}\nabla v^2 = \int -\nabla\mathcal{P}$$

$$\Rightarrow \rho\frac{1}{2}v^2 = -\mathcal{P} + \text{const.}$$

$$\rho\frac{1}{2}v^2 + \mathcal{P} = \text{const.}$$ $\longrightarrow$ Bernoulli eqn for incompressible flow, steady, irrotational flow.

(I) $\leftarrow$ $$\boxed{\rho\left[\frac{1}{2}v_x^2 + \frac{1}{2}v_y^2\right] + \mathcal{P} = \text{const.}}$$

To solve for velocity & pressure profile we need additional equations.

$$\nabla\cdot\vec{v} = 0$$

(II). $$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0. \qquad \text{[2-D continuity eqn]}$$

$$\nabla\times\vec{v} = 0 \qquad \text{(Irrotational).}$$

(III) $$\frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x} = 0.$$

3

Solve. $V = -\cancel{\ } \nabla \phi$ ϕ = velocity potential.

why we picked $V = -\nabla\phi$,

$\boxed{\begin{array}{l}\nabla \times \vec{V} = 0 \\ \nabla \times (-\nabla \phi) = 0.\end{array}}$

because $\nabla \cdot V = 0$

$\Rightarrow \nabla \cdot (-\nabla \phi) = \nabla^2 \phi = 0 \rightarrow$ Laplace eqn.

$\phi(x,y)$

$v_x = \frac{\partial \phi}{\partial x}$

$v_y = \frac{\partial \phi}{\partial y}$.

$\cancel{\psi(x,y)}$. $\psi(x,y)$.

$v_x = \frac{\partial \psi}{\partial y}$

$v_y = \frac{-\partial \psi}{\partial x}$.

Complex ~~vel~~ velocity ~~pot~~

$w(z) = \phi(x,y) + i\psi(x,y)$.

$z = x + iy$

$\left[\nabla^2 \phi = 0 \quad \nabla^2 \psi = 0.\right]$

→ The velocity potential ϕ is ~~as~~ assumed to simplify the solution approach for velocity and pressure distribution. ϕ is picked as $\vec{V} = -\nabla \phi$ so that ~~ϕ~~ ~~$\nabla \times \vec{V}$ is auto~~ $\nabla \times \vec{V} = 0$ is automatically satisfied, i.e. the irrotational flow assumption ~~is~~ is satisfied.

From definition of velocity potential & stream function.

$$\boxed{\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}} \qquad \boxed{\frac{\partial \phi}{\partial y} = \frac{-\partial \psi}{\partial x}}$$

These are known as ~~Cauchy~~ Cauchy-Riemann equations. There allow to come up with the complex velocity function, as the real & imaginary part must ~~be~~ satisfy this condition.

Proof that ϕ & ψ satisfies the Laplace equation.

$$\nabla^2 \phi = 0.$$

Proof:

$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}. \qquad \frac{\partial \phi}{\partial y} = \frac{-\partial \psi}{\partial x}.$$

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{\partial^2 \psi}{\partial x \partial y}. \quad \text{— (a)} \qquad \frac{\partial^2 \phi}{\partial y^2} = \frac{-\partial^2 \psi}{\partial y \partial x}. \quad \text{— (b)}$$

$$a+b \Rightarrow \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \cancel{\frac{\partial^2 \phi}{\partial x \partial y}} - \cancel{\frac{\partial^2 \psi}{\partial y \partial y}}$$

$$\boxed{\nabla^2 \phi = 0} \rightarrow \text{Proved.}$$

Calculus of complex variable.

$$z = U(x,y) + iv(x,y).$$

$$z' = \frac{\partial U}{\partial x} + \frac{i\partial v}{\partial x}$$