Eulerian vs. Lagrangian

Sep 27th, 2017
Monday.

Lagrangian → moving fluids tracking
→ Lagrangian displacement
→ Lagrangian velocity
→ Difficult to measure.

Eulerian → observing fixed points in space
→ easy to measure in lab coordinates.

Section 3.5 in Book

Types of derivatives

(1) Partial time derivative

$$\left.\frac{\partial f}{\partial t}\right|_{x,y,z}$$

observing the variation of a function at fixed coordinates.

(2) Total time derivative

$$\frac{d}{dt}$$

$$\frac{dc}{dt} = \left(\frac{\partial c}{\partial t}\right)_{x,y,z} + \left(\frac{dx}{dt}\right)\cdot\left.\frac{\partial c}{\partial x}\right|_{y,z} + \left(\frac{dy}{dt}\right)\cdot\left.\frac{\partial c}{\partial y}\right|_{x,z} + \left.\left(\frac{dz}{dt}\right)\left(\frac{\partial c}{\partial z}\right)\right|_{x,y}$$

(3) Substantial or Material Derivative

$$\frac{Dc}{Dt} = \frac{\partial c}{\partial t} + v_x\frac{\partial c}{\partial x} + v_y\frac{\partial c}{\partial y} + v_z\frac{\partial c}{\partial z}$$

$$\frac{Dc}{Dt} = \frac{\partial c}{\partial t} + (v \cdot \nabla c)$$

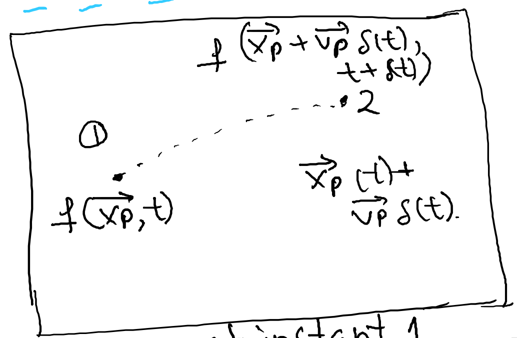
Relationship between Lagrangian & Eulerian

Material Derivative → From the first principles of derivative

$$\frac{Df}{Dt} = \lim_{\delta t\to 0} \frac{f(\vec{x}_p+\vec{v}_p\,\delta(t),\, t+\delta(t)) - f(\vec{x}_p,t)}{\delta t}$$

↓

Change in a property that is observed by the moving particle.



Particle at instant 1 moving with a velocity $\vec{v}_p$, traverses distance $\vec{v}_p\,\delta(t)$ and reaches point 2

Taylor Series Expansion

$$f(x,y) = f(a,b) + (x-a)f_x(a,b) + (y-b)f_y(a,b) + \cancel{\frac{(x-a)^2}{2!}f_{xx}(a,b)} \to 0$$
$$+ \cancel{\frac{(y-b)^2}{2!}f_{yy}(a,b)} \to 0$$

f_x: Partial derivative w.r.t x

f_y: Partial derivative w.r.t y.

$$f(\vec{x}_p+\vec{v}_p\,\delta t,\, t+\delta(t)) = f(\vec{x}_p,t) + \delta(t)\frac{\partial}{\partial t}f(\vec{x}_p,t) + \vec{v}_p\,\delta(t)\,\nabla f(\vec{x}_p,t) + \ldots \to 0$$

(higher order terms go to 0)

$$\lim_{\delta t\to 0}\frac{f(\vec{x}_p+\vec{v}_p\,\delta t,\, t+\delta t) - f(\vec{x}_p,t)}{\delta(t)} = \frac{\partial}{\partial t}f(\vec{x}_p,t) + \vec{v}_p\,\nabla f(\vec{x}_p,t)$$

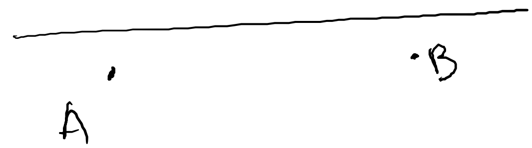
$$\boxed{\frac{Df}{Dt} = \underbrace{\frac{\partial f}{\partial t} + \vec{v}\cdot\nabla f}_{\text{Eulerian expression}}}$$

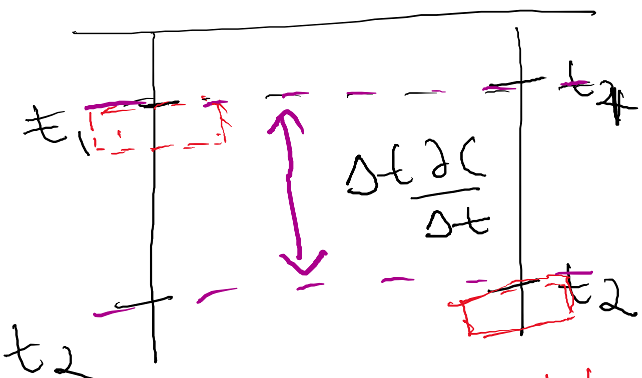
$\frac{Df}{Dt}$: Lagrangian or Material or Substantial Derivative

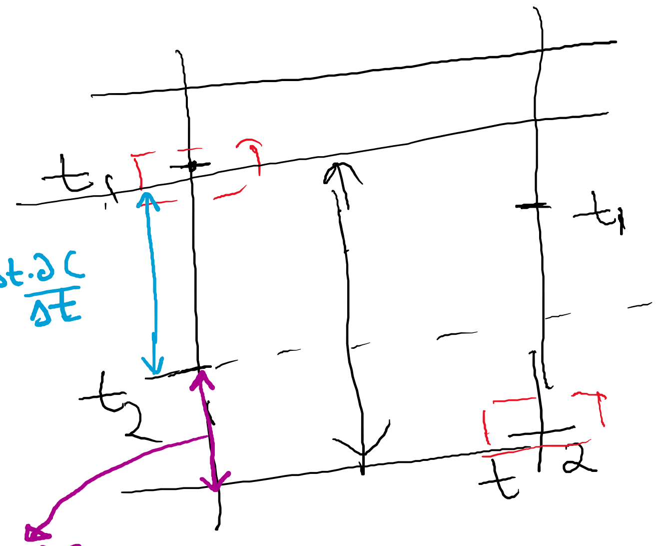
Refer to the video.

Same concentration at A&B at t_1, only decaying with time

Different concentration at A & B at time t_1







Red is indicating particle moving from A & B from t_1 to t_2 and observing what the counter is measuring.

$$\Delta x \cdot \frac{\partial C}{\Delta x}$$

$$\Delta x = \vec{v_p}\, \Delta t$$

$$\Rightarrow \vec{v_p}\, \Delta t \frac{\partial C}{\partial x}$$