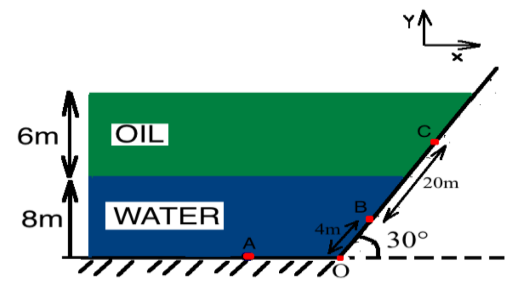
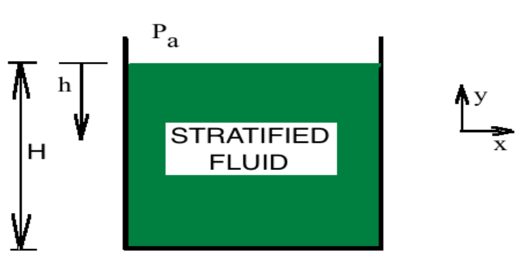
ABE 307
Homework 1
Fall 2017
<u>Assigned: 08/25/2017</u>
<u>Due: 09/01/2017</u>

1. A force of 115.47 kN is applied at a point on a body, at an angle of 60 degree with Y axis. The X component of the shear force on the XZ plane is 86.6 kN. Find the other component of the shear force on XZ plane. Also, find the angle between the force and X axis. Show your work with the body diagram for force components. (10 Points)

2. Two fluids are confined by a gate inclined at an angle of 30° as shown in the figure below. The lower fluid is water and the upper fluid (oil) has a specific gravity of 0.8. The surface of the upper fluid is at a height of 14m and the interface between the fluids is at a height of 8m with respect to the ground level. Point A is located at the ground level, points O, B and C are located along the gate with line segments OB = 4m, and BC = 20m. Calculate the pressures at points A, B, and C. (10 Points)



3. A tank is filled with a 'stratified fluid' (fluid with non-uniform density), the density of which follows a linear relationship: $\rho = \rho_o + kh$, where '***h***' is the distance measured from the liquid surface along the downward direction. If the atmospheric pressure is $\boldsymbol{P_a}$, the height of the tank is ***H***, and the acceleration due to gravity is ***g***, find the pressure at the bottom of the tank. (10 point)



Solution:

1) F = 115.47kN.

Assume the projection of F onto the XZ plane to be F_{xz}, and the projections of Fxz on the X and Z axes to be F_x and F_z respectively.

We have,

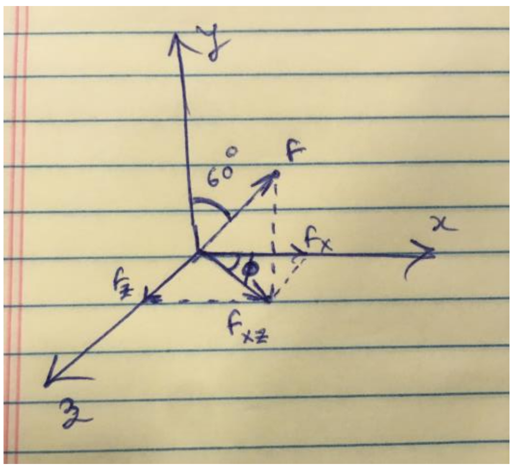
F $\sin(60^o)\cos(\emptyset) = F_x = 86.6$kN

=> $\emptyset = 30^o$

=> $F_z = F_{xz}\sin(30^o)$ = 50.0kN

The angle between F and the X axis θ_x is such that:

$\cos(\theta_x) = \frac{F_x}{F}$ = 86.6/115.47 => $\theta_x = 41.41^o$



2) Let h_B and h_C be depths of points B and C respectively as measured from the surface of oil. From the given information and using geometry, we have:

$$h_B = 14m - (4m.\sin(30^o)) = 12m,$$

$$h_C = 14m - (24m.\sin(30^o)) = 2m.$$

Since point C lies in oil,
$P_C = P_o + \rho_{oil} g h_C,$

But since point B is in water,

$$P_B = P_{int} + \rho_w g(h_B - 6m).$$

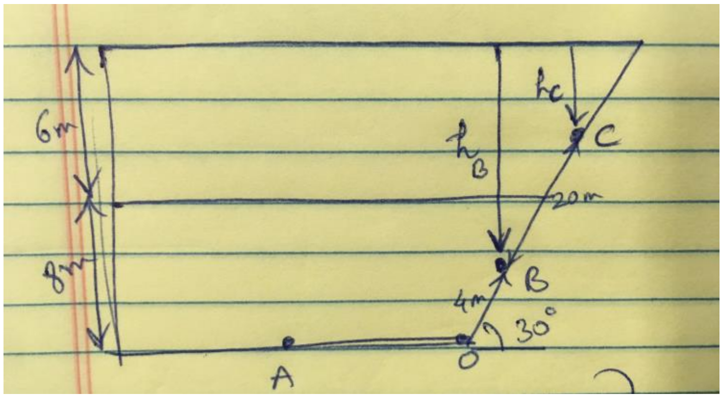
Similarly, $P_A = P_{int} + \rho_w g(8m)$

Where P_{int} is the pressure at the oil-water interface and 6m being the depth of the interface.

Using $P_{int} = P_o + \rho_{oil} g(6m)$,

Po = 1.01x10^{5}Pa, $\rho_w = 1000\ kg/m^3$, and $\rho_{oil} = 0.8 * 1000 \frac{kg}{m^3} = 800 \frac{kg}{m^3}$, and g = 9.8m/s^2,

we have P_A = 2.26x10^{5}Pa, P_B = 2.07x10^{5}Pa, and P_C = 1.17x10^{5}Pa.



3) An infinitesimal increment in static pressure is related to that in depth as:

$$dP = \rho g dh.$$

In the given stratified liquid, we have $\rho = \rho_o + kh$.

⇨ $dP = (\rho_o + kh) g dh$

Assuming P_o to be the atmospheric pressure and integrating both sides with appropriate limits, we have:

$$\int_{P_o}^{P_H} dP = \int_0^H (\rho_o + kh) g dh ,$$

$$P_H = P_o + \rho_o g H + kg \frac{H^2}{2} .$$