ABE 307

Homework 5

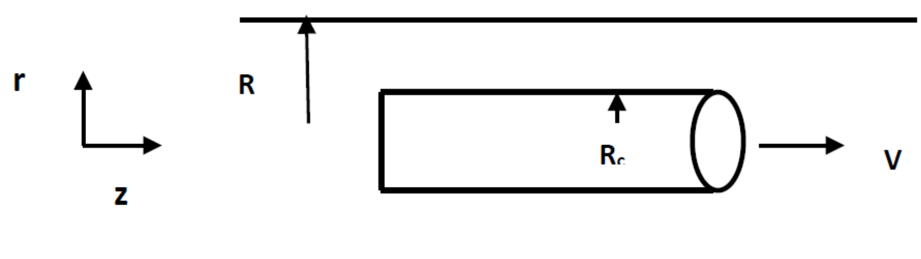
Due Date: Oct 27th 2017

Points : 75 points

Only two problems will be graded, so final total will be 50 points.

1. Consider a catheter of radius R_c placed in a small artery of radius R as shown in the figure below. The catheter moves at a constant speed V. In addition, blood flows through the annular region between R_c and R under a pressure gradient $\Delta p/L$. The capillary can be assumed to be horizontal. We want to determine the effect of the catheter upon the shear stress at r = R. Assume steady, fully developed flow of a Newtonian fluid. (25 Points)

   a. Write the equation of continuity.
   b. Write the equation of motion along the axial direction.
   c. Identify the boundary conditions
   d. Solve for the velocity profile.
   e. Calculate the wall shear stress and plot the wall shear stress for different values of catheter velocity. Discuss the effect.
   f. Use the following values to determine the shear stress acting on the blood vessel surface: R =0.17 cm; R_c = 0.15 cm, V = 10 cm/sec ; μ= 0.03 gm cm^{-1} s^{-1};
   $\Delta p/L$ = 100 dyne/cm^3



2. A liquid of viscosity μ is flowing through a horizontal rectangular slit of height h at steady state with an imposed pressure gradient of $(p_0 - p_L)/L$ along z direction. The width of the slit is much larger than the height so that the flow is one dimensional with $v_x = 0$ $v_y = 0$. The flow is allowed to reach steady state. At time $t = 0$ (which refers to steady state flow at imposed pressure gradient of $(p_0 - p_L)/L$, the pump is shut off so that the imposed pressure gradient is now zero. One would like to obtain the transients of the velocity profile before the system reaches the new steady state. (25 Points)

   a. Write the unsteady state Navier Stokes equation along z .

b. Identify the initial and boundary conditions for the velocity $v_z(y,t)$, y being the direction along which the velocity is varying.
c. Show Identify dimensionless variables and recast the equation of motion in terms of these dimensionless variables.
d. Use the separation of variables to solve for the dimensionless velocity profile.
e. Plot the dimensionless velocity profile as a function of dimensionless distance along y axis at different dimensionless times.

3. Consider a semi-infinite fluid initially moving with a velocity $U(y) = U_0$, as shown in the figure below. The viscosity and density of the fluid are μ, and ρ, respectively. At time t = 0, a stationary plate (*u(y=0) = 0* for *t>0*) comes in contact with the semi-infinite fluid. Use the method of combination of variables (covered in class) to solve for the transient velocity field in the semi-infinite fluid $u(y)$. (25 points)

a. State the assumptions and boundary conditions used.
b. Write down the Navier Stokes equation of the u velocity under the assumptions stated in part a.
c. Use the combination of variables method covered in class to solve for ***u(y).***