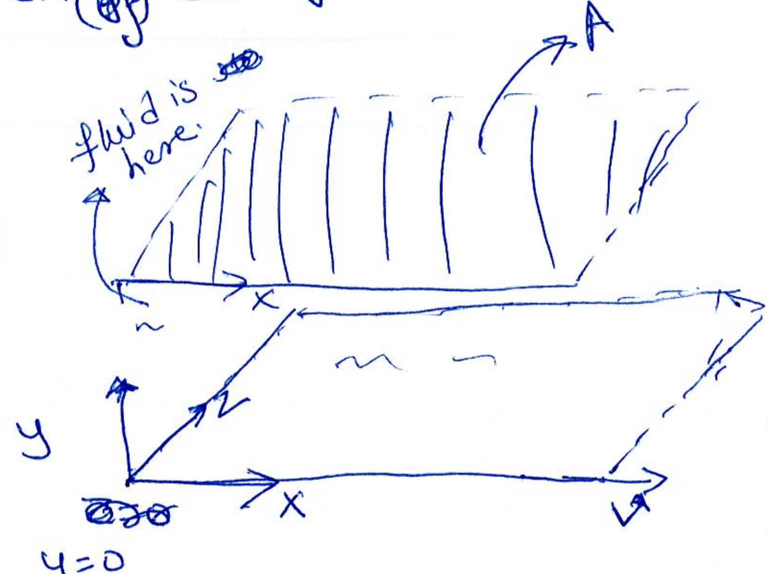
Mechanisms of Momentum Transport.

Sep 8th, 2017.
Friday.

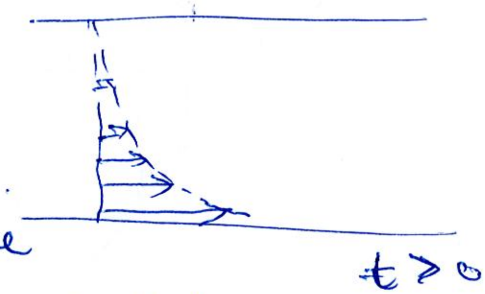
Molecular Momentum Transport → Due to molecular interactions
→ consider viscosity (μ)

Convective Momentum Transport → Due to bulk flow, dependent on (ρ) density.



① Newton's Law of Viscosity.

→ Consider a plate (two parallel) stationary in the beginning
→ Fluid is resting initially.
→ At $t=0$, the lower plate starts moving with velocity $\vec{V}$



$t > 0$

Unsteady state, since velocity of fluid changes with time.

Once steady state profile is reached, we need to apply a constant Force F, to maintain the steady state velocity profile in fluid. This is in order to overcome the shear rate of moving fluid.

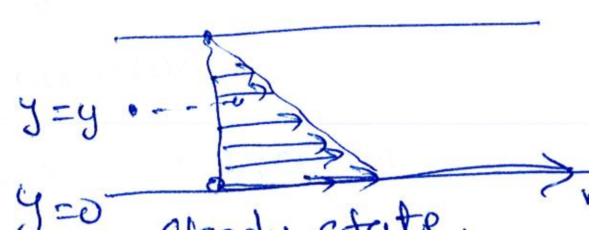
$$\frac{F}{A} \propto \frac{dV_x}{dy}$$

$$\frac{F}{A} = \mu \frac{dV_x}{dy}$$

μ → viscosity.

$$\tau_{yx} = -\mu \frac{dV_x}{dy}$$

→ Applicable in laminar flow or orderly flow.



Steady state.

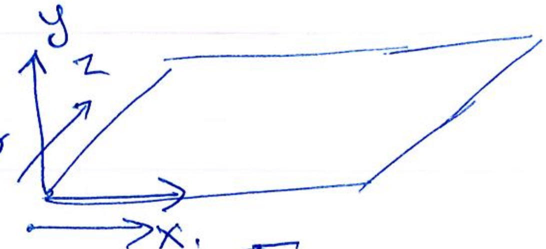
V_x = Velocity of fluid in x direction.

momentum transfer interpretation.

$\tau_{yx} \rightarrow$ transfer of x-momentum in the y-direction.

Shear stress interpretation

$\tau_{yx} \rightarrow$ shear acting in x direction on a plane ~~*~~ for which perpendicular is y direction.



$\tau_{y\underline{x}}$

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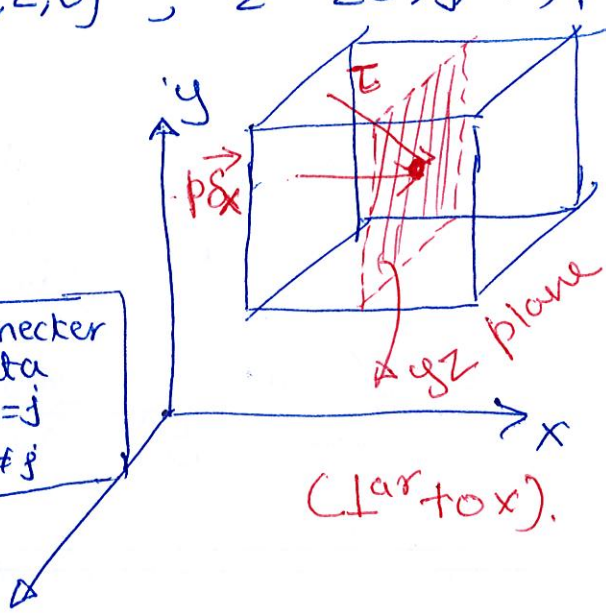
(2) Consider a general flow pattern.

$v_x = v_x(x, y, z, t)$; $v_y = v_y(x, y, z, t)$; $v_z = v_z(x, y, z, t)$.

τ_{ij} = 9 compoments. $i = 1, 2, 3$ (x, y, z); $j = 1, 2, 3$ (x, y, z)

$\boxed{\pi_{ij} = p\vec{\delta}_{ij} + \tau_{ij}}$ (momentum). $\rightarrow$ δ_{ij} is kronecker delta; $\delta_{ij} = 1;\ i = j$; $\delta_{ij} = 0;\ i \neq j$

Molecular stress. Tensor.



τ_{ij} = shear ~~stress tensor~~ stress in j direction on a area ⊥ar to i direction.

τ_{xy} = shear stress in y direction on an area ⊥ar to x direction

= transfer of y-momentum in x direction.

$$\begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix} \longrightarrow \text{Normal stresses. due to viscous forces.}$$

→ off-diagonal ellements. are shear stresses.

τ_{ij} where $i=j$ → normal stresses.

τ_{ij} wher $i \neq j$ → shear stresses.

$\pi_{xx} = p\vec{\delta}_x + \tau_{xx}$ = Normal stress in x-direction.

$\pi_{xy} = \tau_{xy}$.

$$\tau_{ij} = \sum_k \sum_l \mu_{ijkl} \frac{\partial v_k}{\partial y_l} \qquad i, j, k, l = 1, 2, 3.$$

linear combination of all velocity gradients.

For pure rotational flow.

→ $\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$ and $\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)\delta_{ij}$

$$\tau_{ij} = A\left(\frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j}\right) + B\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)\delta_{ij}$$

τ_{ij} → this is termed as viscous stress tensor.