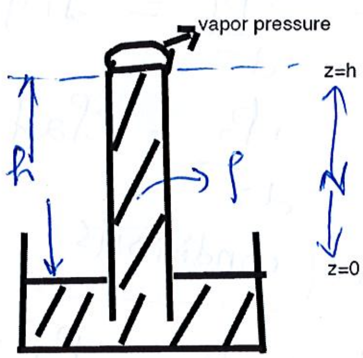
DATE: Aug $30^{th}$, 2017

# FLUID STATICS
## Pressure and Density Measuring Devices

**Barometers: Measuring Absolute Pressure**

A barometer is a device for measuring the absolute pressure of the atmosphere. The one illustrated below consists of a single tube closed at one end and immersed in the barometer fluid (usually mercury). The liquid vaporizes at the closed end and the pressure there is the vapor pressure of the liquid. For mercury, the vapor pressure at room temperature is negligible (around 3 x $10^{-6}$ atm) and therefore may be considered zero.

$$\frac{\partial p}{\partial z} = -\rho g$$




$$P = -\rho g z + C_1$$

h = height of the liquid column in the tube.

at $z=h$, $P \approx P_{vapour}$.

$P_{vapour} \approx 0$

at $z=h$, $P \approx 0$.

$$\Rightarrow \quad 0 = -\rho g h + C_1$$

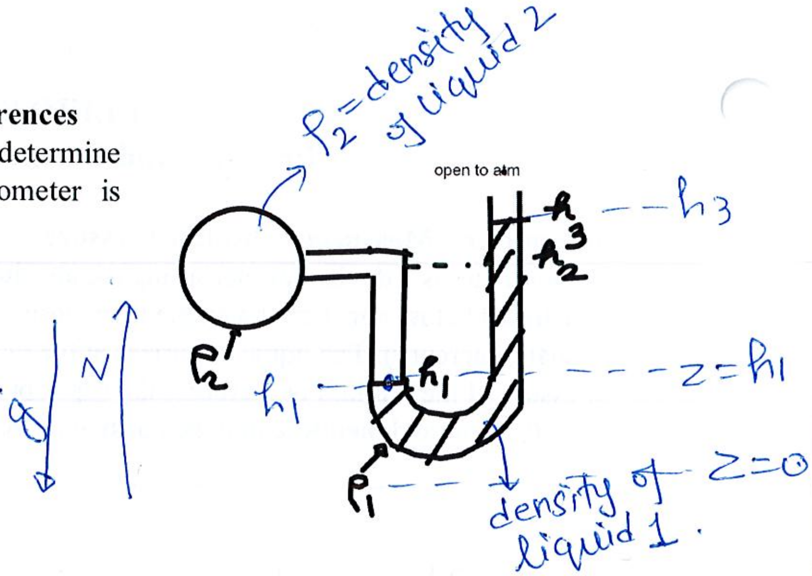
$$\Rightarrow \quad C_1 = \rho g h.$$

$$P = \rho g (h - z).$$

at $z=0$, $P = P_{atm}$ (liquid is open to atmosphere)

$$\boxed{P_{atm} = \rho g h}$$

**Manometers: Measurement of Pressure Differences**
Devices which make use of columns of liquid to determine pressure differences. A simple U tube manometer is illustrated here.




Criteria for choosing liquid
→ liquid should be immiscible.

$$\frac{dp_1}{dz} = -\rho_1 g.$$

$$\frac{dP_2}{dz} = -\rho_2 g$$

Boundary conditions

at $z = h_3$, $P_1 = P_{atm} = P_0$.

at $z = h_1$, $P_1 = P_2$

(Pressure is continous function in static fluid).

$$P_1 = -\rho_1 g z + C_1$$

at $z = h_3$ ; $P_1 = P_0$

$\Rightarrow$ $P_0 = -\rho_1 g h_3 + C_1$

$C_1 = P_0 + \rho_1 g h_3$

$$\boxed{P_1 = \rho_1 g (h_3 - z) + P_0} \quad \text{— (I)}$$

Solving for Pressure variation in liquid 2.

$$\boxed{P_2 = -\rho_2 g z + C_2.} \quad \text{— (II)}$$

at $z = h_1$, $P_1 = P_2$

substitute, $z = h_1$ in eq$^n$ (I).

$$P_1 = \rho_1 g (h_3 - h_1) + P_0.$$

R.H.S

$$\frac{kg}{\cancel{m^3}} \times \frac{m}{s^2} \times \cancel{m} \times \cancel{m^2}$$

$$= kg.\frac{m}{s^2} = \text{Force units.}$$

→ checking the dimension for Example 2 in Problem sheet.

$\cancel{P_2 = -\rho_2 g}$

$$P_2 = -\rho_2 g h_1 + C_2$$

Equating it to $P_1$

$$\rho_1 g(h_3 - h_1) + P_0 = -\rho_2 g h_1 + C_2$$

$$C_2 = \rho_1 g(h_3 - h_1) + \rho_2 g h_1 + P_0$$

$$\cancel{C_2 = \rho_1 g h_3 + g h_1(\rho_2 - \rho_1) + P_0}$$

$$\Rightarrow \cancel{\boxed{P_2 = \rho_1 g h_3 - \rho_2 g h_1 + g h_1(\rho_2 - \rho_1) + P_0}}$$

Put $C_2$ in Equ (11).

$$P_2 = -\rho_2 g z + \rho_1 g(h_3 - h_1) + \rho_2 g h_1 + P_0$$

$$P_2 = \rho_2 g(h_1 - z) + \rho_1 g(h_3 - h_1) + P_0$$

$P_2 =$

Gauge pressure $= P_2 - P_0$ (Pressure in bulb − Pressure atmospheric)

$$P_2 - P_0 = \rho_2 g(h_1 - z) + \rho_1 g(h_3 - h_1) + P_0$$

(psig)