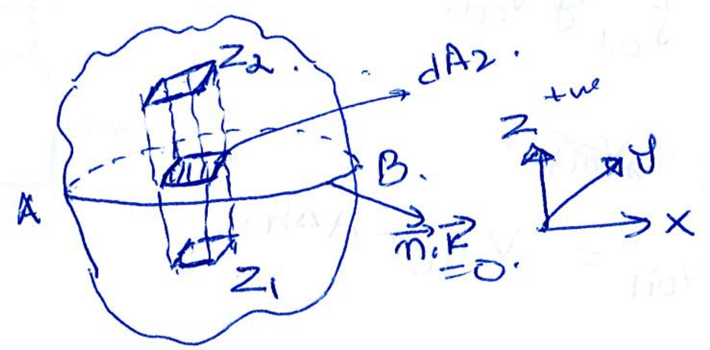
Hydrometers: Determine the ratio of density of two liquids. Based on the principle of Buoyancy force.

Specific Gravity: $\gamma = \dfrac{\rho}{\rho_{H2O}}$

1. Buoyancy: Consider a body of arbitrary shape immersed in a liquid as shown below. Archemedes Principle states that "a body immersed in a liquid is buoyed up by a force equal to the weight of the displaced fluid".



The solid body can be separated into two regions by a curve along which

$$\vec{n}\cdot\vec{k} = 0.$$

The curve then lies on a horizontal plane AB. Take a volume element inside the body as shown above. We can calculate the surface forces acting on the volume element. The force dF acting on the top and the bottom surfaces of the volume element is given by :

$$d\vec{F} = -\vec{n}\,p(z_2)dA_2 + \vec{n}\,p(z_1)\,dA_1$$

$$\vec{F} = \int d\vec{F} = -\int_{A_2} \vec{n}\,p(z_2)\,dA_2 + \int_{A_1} \vec{n}\,p(z_1)\,dA_1.$$

Therefore,

$$\vec{k}\cdot\vec{F} = -\int_{A_2} \vec{n}\cdot\vec{k}\,p(z_2)dA_2 + \int_{A_1} \vec{n}\cdot\vec{k}\,p(z_1)dA_1.$$

$$= -\int_{A_2} p(z_2)\,dA_2 + \int_{A_2} p(z_1)dA_1$$

$$F_z = \int_{A_2} p(z_1) - p(z_2)\,dA_2.$$

V_s = solid volume (Archimedes Principle).

$$p(z_1) - p(z_2) = \rho g(z_2 - z_1) \Rightarrow F_2 = \rho g \int_{A_2} (z_2 - z_1)\,dA_2 = \rho g V_s.$$

The vertical component of the force can then be calculated as,

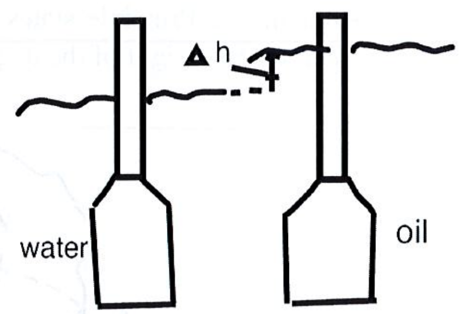
2. Hydrometers

$$Mg = \rho g V_{H_2O}$$

$$Mg = \rho_{oil}\, g\, V_{oil}.$$

$$\cancel{V_{H_2O} = V_{oil} +}$$

$$V_{oil} = V_{H_2O} + A\Delta h.$$



$$\frac{\rho_{oil}}{\rho_{H_2O}} = \sigma_{oil} = \frac{V_{H_2O}}{V_{oil}}.$$

$$\frac{V_{oil}}{V_{H_2O}} = 1 + \frac{A\Delta h}{V_{H_2O}}.$$

$$\boxed{\sigma_{oil} = \frac{1}{1 + \frac{A\Delta h}{V_{H_2O}}}}$$

Example problem on buoyancy!