Date : Sep 11th (Monday)
and Sep 13th (Wednesday)

ABE 307
Method of Shell Momentum Balance

The method of shell momentum balance is applied for laminar, steady rectilinear flows in order to derive velocity profile. Velocity profile can then be utilized to get maximum velocity and the shear rate on the solid surfaces due to the flow of fluid.

Flow Conditions Considered for Application of Shell Momentum Balance

a. **Steady Flow :** Pressure (P), Density (ρ), velocity ($\vec{v}$) components do not change with time.

Note: This does not imply that these variables are constant. It can still change with location.

b. **Incompressible Liquid Flow** ρ is cotl. throughout space and time.

c. **Laminar Flow** Orderly flow with not lot of mixing. Laminar flow is characterized by low reynolds number.

d. **Rectilinear Flow** Flow in one direction or a straight line.

Objective To obtain an expression for spatial variation of velocity and pressure. This allows us to calculate shear forces by fluid on surfaces, important for design considerations.

Concepts Required for Solving Fluid Flow Problem Using Shell Momentum Balance

1. **Viscosity** Since velocity of fluid depends on μ (viscosity) it is an important consideration for deriving expressions for velocity profile.

2. **Momentum Transfer** which direction the momentum is being transferred. consider all components (9 for molecular, 9 for convective).

3. Momentum Balance Expression

Momentum is being transferred as the fluid is flowing under the effect of various forces. write the general momentum balance to develop expression.

4. Boundary Conditions

To solve expression for your specific situation identify the boundaries and specific boundary condition.

General Momentum Balance Expression

Rate of change of momentum = Body forces + ~~surface forces~~. (in case of no external surface force on fluid)

For Steady State.

rate of momentum out − rate of momentum in = Body forces.

$$\left\{ \text{rate of momentum in} \right\} - \left\{ \text{rate of momentum out} \right\} + \text{Body forces} = 0$$

Equ^n (2.1-1) in BSL [This equation breaks down the components to molecular and convective momentum transport components. These components were discussed in chapter 1 (See section 1.7 & 1.2)

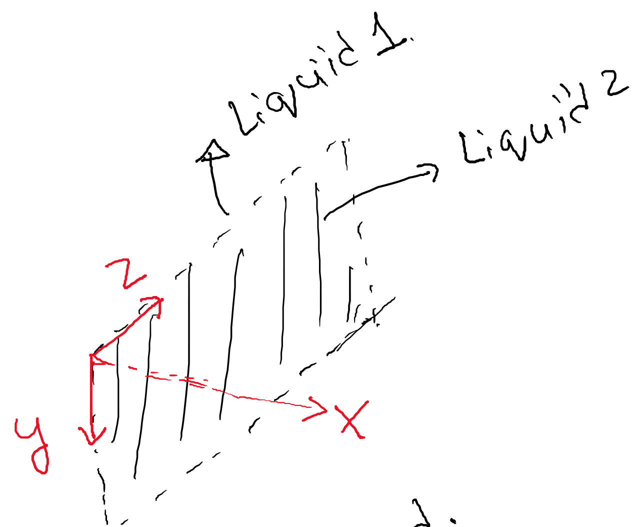
Boundary Conditions for Applying Shell Momentum Balance

For any general fluid flow condition, you can come across three kind of boundary conditions

1. Solid Liquid

At solid-liquid interface "No slip condition", relative velocity is 0.

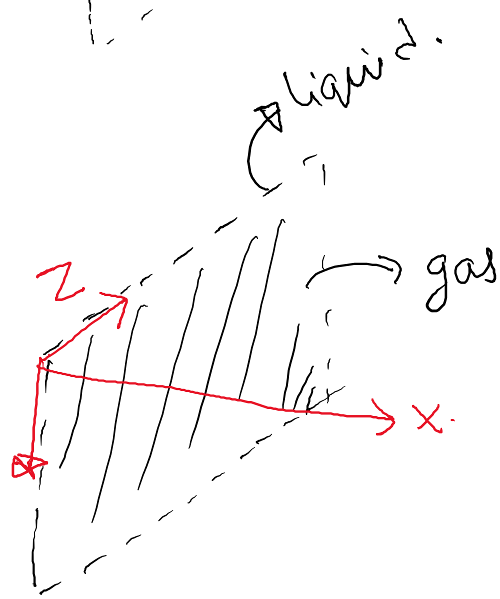
2. Liquid-Liquid



→ continuity assumption.
τ_{xy}, τ_{xz} & $p+\tau_{xx}$ are same at interface.

→ No slip, v_y & v_z are equal

3. Liquid-Gas

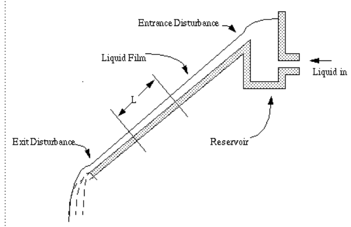


$\mu_{gas} << \mu_{liquid}$.

τ_{xy} & τ_{xz} are zero.

Date : ________________

Steps in Shell Momentum Balance Method



Hint	Step
What is the aim and what kind of flow you are looking at ?	Rectiliner flow → Identify the direction of flow → Find the non-zero velocity component → co-ordinate system.
What do we know about the flow without knowing any details about the objective you are trying to accomplish? Is there a fundamental principle that is applicable ?	Momentum balance principle applies.
Can you get to an equation that helps in relating the variable of interest to co-ordinates of flow ? If yes, how ?	Assume a shell for applying your momentum balance to develop a differential eqn.
Can you solve the equation to describe the flow situation at hand ? Is yes, what do you obtain at this stage ?	Integrate to get momentum flux.
Are you able to get to velocity variable? If yes, what did you do to reach here? If no, what would you need?	Identify the relatioship between momentum flux component and viscosity.
Can you solve for velocity profile? What other information do you need ?	Get velocity differential equation & Integrate using B.C.
Can you calculate maximum velocity, average velocity, mass flow rates and force by fluid on solid surface ?	$v = v(x, y, z)$ velocity as function of x, y & z we can obtain the velocity as function of (r, θ, z) for cylindrical coordinates.

$$p + \overline{\tau} + \rho \vec{v}\vec{v}$$

→ Then its a matter of maxima principle & other general calculation.