HW-8 SOLUTION:

Problem 1:

(1)

9.8 m/s², 4×10⁻² m

$$f = \frac{4}{3}\frac{gD}{u_\infty^2}\left(\frac{\rho_s - \rho_\ell}{\rho_\ell}\right)$$

$g = 9.8\ m/s^2$, $D = 4\times10^{-2}\ m$

$\rho_s = 2.63\ g/cc = 2630\ kg/m^3$, $\rho_\ell = 1.4\ g/cc = 1400\ kg/m^3$

Rearranging the above eq^n, we have:

$$\log f = \log\left(\frac{\frac{4}{3} g D^3 \rho_\ell(\rho_s-\rho_\ell)}{\mu^2}\right) - 2\log(Re)$$

$$f \cdot Re^2 = \left[\frac{\frac{4}{3} g D^3 \rho_\ell(\rho_s-\rho_\ell)}{\mu^2}\right]$$

$$Re = \sqrt{\frac{(4/3) g D^3 \rho_\ell(\rho_s-\rho_\ell)}{\mu^2 f}}$$

at $f = 0.1$, $Re \simeq 120,000$

$f = 1.0$, $Re \simeq 38,000$

$\Rightarrow$ from the graph of f vs Re,

point of intersection $= (Re, f) : (0.5\times10^5, 0.46)$

$$\Rightarrow u_\infty = \frac{\mu Re}{\rho D} \simeq \underline{\underline{0.9\ m/s}}$$

Problem 2:

$D = 0.5\ cm \quad m = 0.05\ g \quad V_\infty = 0.5\ cm/s \quad \rho = 0.9\ g/cm^2.$

(a) $f_{gravity} = mg = 0.05 \times 981 = 49.05$ dynes

$f_{bouyancy} = \frac{4}{3}\pi R^3 \rho g = \frac{4}{3}\pi (0.25)^3 (0.9)(981)$

$= 57.78$ dynes

$F_{drag} = f_{bou} - f_{grav} = 57.78 - 49.05 = 8.73$ dynes.

(b) $f_d = (\pi R^2)\ (\frac{1}{2}\rho V_\infty^2) f$

$f = \frac{2 f_d}{\pi R^2 \rho V_\infty^2} = \frac{2 \times 8.73}{\pi (0.25)^2 (0.9)(0.5)^2} = 395.35$

(c) $f = \frac{24}{Re} \quad Re = \frac{24}{f}$

$\frac{D V_\infty \rho}{\mu} = \frac{24}{f} \Rightarrow \mu = \frac{f D V_\infty \rho}{24}$

$= \frac{395.35 \times 0.5 \times 0.5 \times 0.9}{24}$

$= 3.71\ g/cm.s.$

Problem 3:

$T = 68°f$

$L = 1320\ ft$

$D = 6 in = 0.5 ft$

$P_0 - P_L = 0.25\ psi = 1158\ lb\, m/ft.s^2.$

$\mu = 6.73 \times 10^{-4}\ \frac{lbm}{ft.s}$

$$Re\sqrt{f} = \frac{D\rho}{\mu}\sqrt{\frac{(P_0-P_L)D}{2L\cdot\rho}} = \frac{0.5\times 62.4}{6.73\times 10^{-4}}\sqrt{\frac{1158\times 0.5}{2\times 1320\times 62.4}}$$

$$= 2738.9 \dashrightarrow 2740$$

$$Re\sqrt{f} = 2740$$

at $f=0.04$ at $f=0.01$

$Re = 13745$ $Re = 27490$

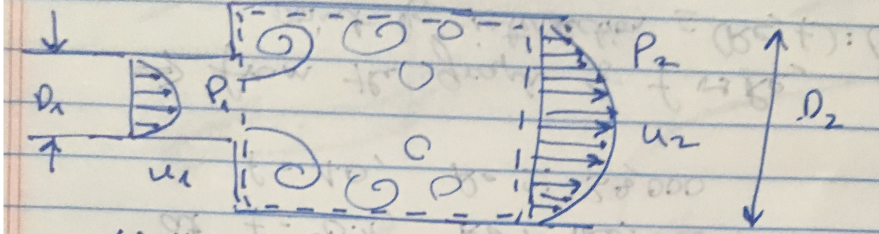
form moody chart $Re = 35000$ approximately

$$V_\infty = \frac{Re\,\mu}{\rho_f D} = \frac{35000\cdot 6.73\times 10^{-4}}{0.5\times 62.4} = 0.745 \text{ ft/s.}$$

$$Q = \pi R^2 V = 0.745 \times \pi \times 0.25 \times 0.25$$

$$= 3930 \text{ gal/hr.}$$

Problem 4:



Writing C.V mass balance for the CV above,

$$\bar{u}_1 D_1^2 = \bar{u}_2 D_2^2 \quad (1)$$

Momentum balance:

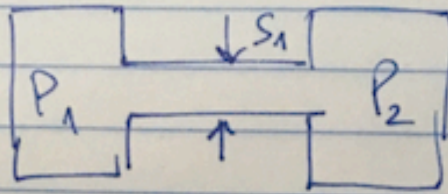
$$-\rho\bar{u}_1^2 D_1^2 + \rho\bar{u}_2^2 D_2^2 = \underbrace{(P_1 - P_2)}_{-\Delta P} D_2^2 \quad (2)$$

Using (1) & (2), $\Delta P = \rho\bar{u}_2^2\left\{\frac{D_2^2}{D_1^2} - 1\right\}$

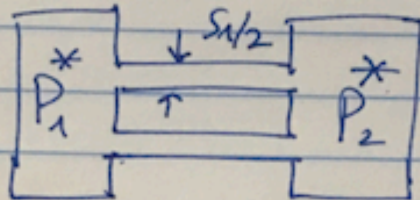
$$\simeq 726 \text{ lb/ft}^2$$

Problem 5:

Case 1



Case 2



$$S_1 = \pi D_1^2 \, , \quad S_2 = S_1/2 = \pi D_2^2$$

$$\Rightarrow D_2 = D_1/\sqrt{2} \quad \text{—} \; (1)$$

$$Q_1 = \bar{u}_1 \pi D_1^2 \, , \quad Q_2 = \underset{\text{(each tube)}}{\frac{Q_1}{2}} = \frac{\bar{u}_2 \pi D_1^2}{2}$$

$$\Rightarrow \bar{u}_1 = \bar{u}_2 \quad \text{—} \; (2)$$

we ~~know~~ know:

$$Re\sqrt{f} = \sqrt{\frac{\Delta P \, D^3 \rho}{2\mu^2 L}}$$

$$Re_2 = \frac{\rho \bar{u}_2 D_2}{\mu} = \frac{Re_1}{\sqrt{2}} \quad \text{—} \; (3)$$

~~(1), (2) and (3)~~ given $f = \dfrac{0.08}{Re^{1/4}}$ — (4)

$$\Rightarrow Re_1 \cdot \sqrt{\frac{0.08}{Re_1^{1/4}}} = \sqrt{\frac{\Delta P_1 \, D_1^3 \rho}{2\mu^2 L}} \quad \text{—} \; (5)$$

$$Re_2 \sqrt{\frac{0.08}{Re_2^{1/4}}} = \sqrt{\frac{\Delta P_2 \, D_2^3 \rho}{2\mu^2 L}} \quad \text{—} \; (6)$$

Using (3), (4), (5), (6) $\Rightarrow \underline{\underline{\Delta P_2 = \Delta P_1 \cdot 2^{5/8}}}$