Name: ______________________________

ABE 307

Fall 2017

Test 1

Sep 29th , 2017

25 minutes

This test has 2 problems. For derivations, step marking is involved so show all your work clearly and for assumptions write one line of explanation. Without explanation, assumptions will only fetch half points.

Problem	Points	Points Obtained
1	5	
2	20	
Total	25	

1. Choose the correct answer for following. (5 points)

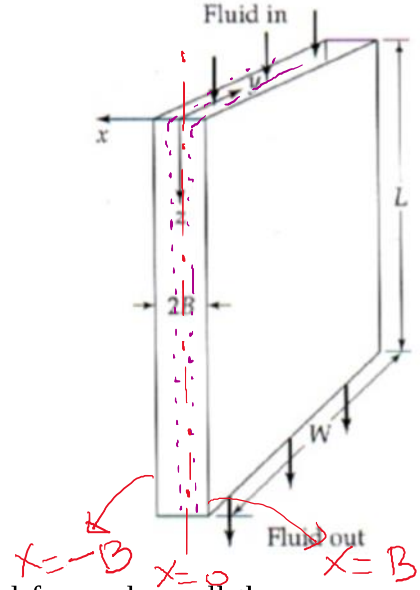
A. Which of the following simplifying assumptions fail in "Thin film" scenario if the width (W) of the plate is not large compared to the film thickness (δ)

a) $\phi_{YZ} = 0$ → if the W is comparable to δ, then we will have $\frac{dv_z}{dy} \neq 0$ and there will be τ_{yz}
b) $V_z|_{z=0}$ is equal to $V_z|_{z=L}$
c) $P = P(x)$
d) None of the above.
e) All of a, b and c

B. Which of the following boundary conditions pertain to a liquid-liquid interface :

a. tangential velocity component and molecular stress components are continuous
b. shear stress is 0
c. relative velocity is 0
d. all of these
e. a and c

2. Consider a fluid flowing through a channel bounded by wall on two sides. Assume steady, laminar flow along the wall. Derive an expression for shear stress variation in the slit and draw a diagram to show the stress profile. (20 Points). Step marking is involved for :



   a. Velocity assumptions for rectilinear flow in this situation. (2)
   b. Clearly showing the co-ordinates and geometry of your shell considered for writing shell momentum balance. (3)
   c. Write the components of the total momentum transfer that will be considered for the total momentum balance along with explanation (2)
   d. Final momentum balance equation in simplified form where all the components (pressure, shear and convective) have been evaluated for the flow situation along with the body forces. (3)
   e. Writing all boundary conditions for this flow situation. (5)
   f. Final expression for shear stress variation in the slit and diagram to show the profile. (5)

(a) $v_x = 0$, $v_z = v_z(x)$, $v_y = 0$

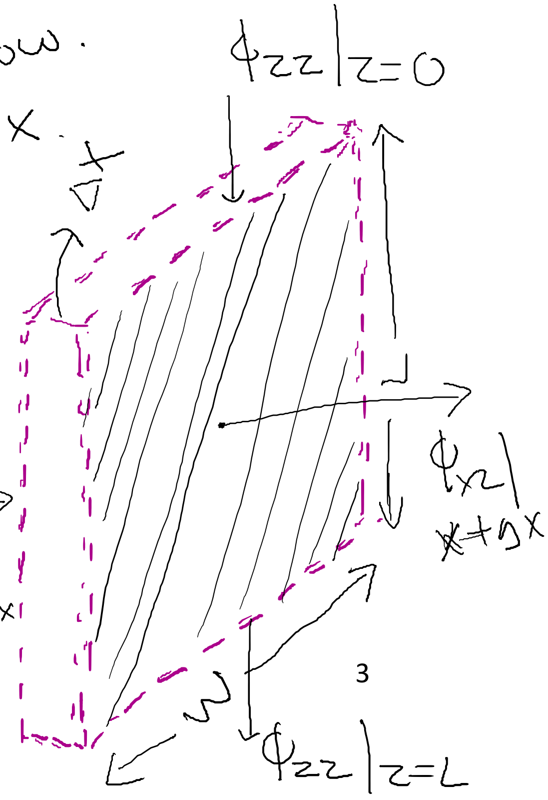
No velocity in x-direction

No velocity in y-direction.

Fully developed, steady flow.

v_z is only function of x.

(b) Consider the shell at the center of the slit.



Notice that the coordinate $\phi_{xz}|_{x=x}$ section selected here is $x=0$ at center.

(c) $\phi_{zz}|_{z=0} \rightarrow$ z momentum in z direction entering the shell at $z=0$

$\phi_{zz}|_{z=L} \rightarrow$ z momentum in z-direction leaving the shell at $z=L$

$\phi_{xz}|_{x=x} \rightarrow$ z momentum in x-direction entering the shell at $x=x$

$\phi_{xz}|_{x=x+\Delta x} \rightarrow$ z momentum in x-direction leaving the shell at $x=x+\Delta x$

$\phi_{yz}=0$, $B \lll W$, we assume no velocity gradient in y diretion $\frac{dV_z}{dy}=0$ (so viscous component is zero)

& $v_y=0$ (so convective component is also zero)

(d) $$LW\left(\phi_{xz}|_{x} - \phi_{xz}|_{x+\Delta x}\right) + W\Delta x\left(\phi_{zz}|_{z=0} - \phi_{zz}|_{z=L}\right) + LW\Delta x(\rho g) = 0$$

$$\phi_{xz} = \tau_{xz} + \rho \cancel{v_x v_z} \to 0 \quad v_x = 0$$

$$\phi_{zz} = P + \cancel{\tau_{zz}} + \rho v_z v_z$$

$$0 \quad \tau_{zz} = -\mu \frac{\partial v_z}{\partial z}$$

v_z is only a function of x.

So, the final momentum balance with equations evaluated becomes —

$$LW\left[\tau_{xz}\big|_x - \tau_{xz}\big|_{x+\Delta x}\right]$$

$$+ \; W\Delta x\left[P + \cancel{\rho v_z v_z}\big|_{z=0} - \cancel{(P + \rho v_z v_z)}\big|_{z=L}\right]$$

$$+ \rho g \,(LW\Delta x) = 0$$

Again from assumption of fully developed flow v_z is not a function of z, therefore $v_z|_{z=0}$ & $v_z|_{z=L}$ are same

$$LW\left(\tau_{xz}|_x - \tau_{xz}|_{x+\Delta x}\right) + W\Delta x\left(P|_{z=0} - P|_{z=L}\right) + \rho g LW\Delta x = 0$$

(e) at $x = B$, $v_z = 0$ (solid-liquid interface)

$x = -B$, $v_z = 0$ (solid-liquid interface)

we do not know anything for shear stress directly.

(f) Using equation from (d), dividing throughout by $LW\Delta x$

$$\lim_{\Delta x \to 0} \frac{\left(\tau_{xz}|_x - \tau_{xz}|_{x+\Delta x}\right)}{\Delta x} + \frac{\left(P|_{z=0} - P|_{z=L}\right)}{L} + \rho g = 0$$

$$-\frac{d\tau_{xz}}{dx} + \frac{\left(\mathcal{P}_0 - \mathcal{P}_L\right)}{L} = 0$$

$$\boxed{\begin{aligned}\mathcal{P}_0 &= P|_{z=0}\\ \mathcal{P}_L &= P|_{z=L} - \rho g L\end{aligned}}$$

→ modified pressure

→ This is just a constant, so even if we do not use this exact formulation treat this as constant.

$$\frac{d\tau_{xz}}{dx} = \frac{P_0 - P_L}{L}$$

$$\tau_{xz} = \left(\frac{P_0 - P_L}{L}\right) x + C_1$$

Since, we do not know anything about shear stress boundary condition; we will have to use velocity conditions.

$$\tau_{xz} = -\mu \frac{dv_z}{dx}$$

$$-\mu \frac{dv_z}{dx} = \left(\frac{P_0 - P_L}{L}\right) x + C_1$$

$$v_z = -\frac{(P_0 - P_L)}{2\mu L} x^2 - \frac{C_1}{\mu} x + C_2$$

using
$v_z = 0$ at $x = B$
$v_z = 0$ at $x = -B$

$$C_1 = 0, \quad C_2 = \frac{P_0 - P_L}{2\mu L} B^2$$

$$\therefore \boxed{\tau_{xz} = \left(\frac{P_0 - P_L}{L}\right) x}$$

This straight line shows the variation of shear stress from $x=0$, to $x=-B$.