**ABE 307**
**Date:** Nov 8th, 2017. Wed.

## Dimensional Analysis

**Why is dimensional analysis needed?**

- To be able to validate some physical laws experimentally.
- For scale up, when you want to maintain the same flow field that you observe in laboratory to a large (plant scale) system. In this case, you will need some rules that you can obtain from dimensional analysis.
- To be able to derive relationship for calculations experimentally.

**What is dimensional analysis?**

- The physical dimensions of both sides of an equation are evaluated and equated.

**Law of Dimensional Homogeneity**

- Every term in an equation that describe a real physical process must be dimensionally the same when it is expressed in terms of fundamental dimensions.

**Fundamental Dimensions**

- Unit of measurement is unimportant.
- 6 Fundamental dimensions

Mass — Temperature
Length — Electrical intensity or current
Time — Luminous Intensity.

**Example: Energy and Work**

Work done = Change in K.E & P.E.

$$F \cdot d. = \frac{1}{2} m v^2 + mgh$$

| Term | Units | Dimensions |
|---|---|---|
| $F \cdot d$ | $Kg \cdot (m/s^2) \cdot m$ | $[M][LT^{-2}][L] = [M][L]^2[T]^{-2}$ |
| $\frac{1}{2} m v^2$ | $kg \cdot (m/s)^2$ | $[M][L]^2[T]^{-2}$. |
| $mgh$ | $kg \cdot (m/s^2) \cdot m$ | $[M][L]^2[T]^{-2}$. |

**Example:**

Deduce the basic dimensions of dynamic viscosity. $(\mu)$.

$$\tau = -\mu \left(\frac{dv_x}{dy}\right).$$

$$[\tau] = \frac{F}{A} = \frac{kg.\,m/s^2}{m^2} = \frac{kg}{m\,s^2} = [M][L]^{-1}[T]^{-2}.$$

$$[v] = m/s = [L][T]^{-1} \Rightarrow [M][L]^{-1}[T]^{-2} = [\mu]\frac{[L][T]^{-1}}{[L]}$$

$$[y] = m = [L] \qquad [\mu] = ML^{-1}T^{-1}$$

**Example: Application of Dimensional Analysis in Deriving Functional Dependency**

The frequency (f) of oscillation of a simple pendulum is dependent on l (length) and gravity (g). Derive the functional relationship between f, l and g.

$$f \propto l. \Rightarrow f \propto lg. \qquad c = \text{constant}$$

$$f \propto g \qquad f = c\,l^a g^b.$$

$$[f] = 1/s = T^{-1} \qquad M^0L^0T^{-1} = L^a(LT^{-2})^b$$

$$[l] = L \qquad M^0L^0T^{-1} = L^{a+b}\,T^{-2b}\,M^0$$

$$[g] = LT^{-2}.$$

$$-2b = -1 \Rightarrow b = 1/2.$$

$$a+b = 0 \Rightarrow a = -b = -1/2.$$

$$f = c\,l^{-1/2} g^{1/2}$$

$$\boxed{f = c\sqrt{g/l}.}$$