Example 7.5.1 Power requirement for Pipeline Flow.

Q→ what is the ~~power~~ required power output from the pump at steady state in the system shown in Fig. 7.5-1

ID = 4 inch

R = 2 inch.

$$w = \rho \langle v \rangle \pi R^2$$

$$\langle v \rangle = \frac{w}{\rho \pi R^2} = \frac{(w/\rho)}{\pi R^2}$$

$\frac{w}{\rho}$ = Volumetric flowrate.

= 12 ft^3/min

= $\frac{12}{60}$ ft^3/sec.

$$\langle v \rangle = \frac{12/60}{3.14 \times (1/6)^2} = ft/sec$$

$$= 2.30 \, ft/sec.$$

1 ft = 12 inch.

1 inch = $\frac{1}{12}$ ft.

R = 2 inch = $\frac{1}{12} \times 2 = \frac{1}{6}$ ft

$$Re = \frac{\rho \langle v \rangle D}{\mu} = \frac{62.4 \times 2.30 \times 4}{12 \times 1.0(\;)}$$

$$= \frac{62.4 \times 2.30 \times 4}{12 \times 6.72 \times 10^{-4}}$$

$$Re = 7.11 \times 10^4$$

$\mu = 6.72 \times 10^{-4}$. in ft unit = 1 centipoise.

~~Contribution~~

$$\frac{1}{2}(v_2^2 - v_1^2) + g(h_2 - h_1) + \int_1^2 \frac{1}{\rho} dP = \hat{W}_m - \sum_i \left(\frac{1}{2} v^2 \frac{L}{R_h} f\right)_i - \sum_i \left(\frac{1}{2} v^2 e_v\right)_i$$

For straight sections.

$D_{eq} = 4R_h$.

$$\sum_i \left(\frac{1}{2} v^2 \frac{L f}{R_h}\right)_i = \frac{\frac{1}{2} v^2}{\frac{D_{eq}}{4}} \sum_i Lf$$

$$= \frac{2v^2}{D_{eq}} \sum_i Lf$$

$$= \frac{2v^2 \times 0.0049}{D_{eq}} \sum_i L_i$$

at Re = 7.11×10^4

f = 0.0049

For smooth circular pipe

From friction factor chart.

$$= \frac{2 \times (2.30)^2 \times 0.0049}{(1/3)} [5 + 300 + 100 + 120 + 20]$$

$$= 85 \ ft^2/sec^2$$

contribution from other fittings, contraction & expansion :-

$$\sum_i \left(\tfrac{1}{2} v^2 e_v\right)_i = \tfrac{1}{2} v^2 [\underbrace{0.45}_{\text{sudden contraction}} + \underbrace{3(\tfrac{1}{2})}_{3 \text{ elbows}} + \underbrace{1}_{\substack{\text{sudden expansion} \\ \beta = 0}}]^{e_v} \quad (\beta = 0$$

$$= 8 \ ft^2/sec^2$$

$$\tfrac{1}{2}(\cancel{v_2^2 - v_1^2})^{0} + g(h_2 - h_1) + \int_1^2 \frac{1}{\rho} dP = \hat{w}_m - \sum_i \left(\tfrac{1}{2} v^2 \frac{L}{R_h} f\right)_i - \sum_i \left(\tfrac{1}{2} v^2 e_v\right)_i$$

$g = 32.2 \ ft/sec^2$

$$(32.2)[105 - 20] + 0 = \hat{w}_m - 85 - 8$$

$$\hat{w}_m = 2740 + 85 + 8 \approx 2830 \ ft^2/sec^2$$

Per unit mass of fluid flow. or

$$W_m = w \hat{W}_m$$

$$w = (1/60) \frac{ft^3}{sec} \times 62.41 \frac{lbm}{ft^3}$$

$$w = 12.5 \frac{lbm}{sec}$$

work per unit mass.

$$= \frac{2830 \ ft^2/sec^2}{32.2 \ ft/sec^2}$$

$$\hat{w}_m = 88 \ ft \frac{lbf}{lbm}$$

$g = 32.3 \ ft/sec^2$

$$w_m = 88 \left(ft \cdot \frac{lbf}{lbm}\right) \times 12.5 \frac{lbm}{sec}$$

$$= \frac{ft \cdot lbfm}{sec}$$

$$1\ lbf = 32.2\ lbm \frac{ft}{sec^2}$$

$$\hat{W}_m = 2830\ ft^2/sec^2$$

$$= 2830\ ft \cdot ft/sec^2$$

$$= \frac{2830\ ft \cdot ft/sec^2}{32.2\ lbm \cdot ft/sec^2}$$

$$\hat{W}_m = 2830 \frac{ft^2}{sec^2}$$

$$1\ lbf = 32.2\ lbm \frac{ft}{sec^2}$$

$$\hat{W}_m = 2830\ ft \cdot \frac{ft}{sec^2}$$

$$= 2830\ ft \cdot \frac{1\ lbf}{32.2\ lbm}$$

$$\hat{W}_m = 88\ ft \cdot \frac{lbf}{lbm} = 88 \quad \frac{(F.d) \to \text{work done}}{(m) \quad \text{mass}}$$

$$F = m.a$$

$$32.2\ lbm \frac{ft}{sec^2} = 1\ lbf$$

$$1\ lbm \frac{ft}{sec^2} = \frac{1}{32.2} lbf$$

work done

$= F.d$

$= m\ a\ .\ d$

$$= \frac{m \cdot \frac{ft}{sec^2} \cdot ft}{lbm}$$

$$32.2\ lbm \frac{ft}{sec^2} = 1\ lbf$$

$$1\ lbm \frac{ft}{sec^2} = \frac{1}{32.2} lbf$$

$$1 \cdot \frac{ft}{sec^2} = \frac{1\ lbf}{32.2\ lbm}$$

$$W_m = w\ \hat{W}_m$$

$$= \left(\left(12/60\right) \times 62.4 \frac{lbm}{sec}\right) \times 88 \times ft \cdot \frac{lbf}{lbm}$$

$$= 1100\ ft \cdot \frac{lbf}{sec}$$

$$= 2\ hp = 1.5\ KW$$

$$1\ ft \cdot \frac{lbf}{sec} = 1.818 \times 10^{-3}\ hp$$