Date : Sep 25th, 2017
Monday

Equation of Change
ABE 307

Associated Readings : Chapter 3 Introduction, Section 3.1 and 3.2

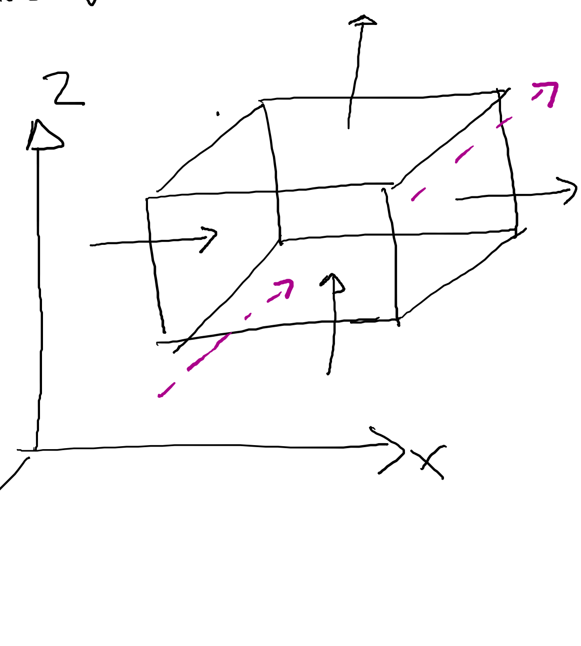
Generalized Equations to Start Fluid Flow problem. Applicable on any type of fluid flow.

Equation of Continuity : General mass balance equation

Equation of Motion : General momentum balance equation

1. Equation of Continuity :

Consider a differential element of size $\Delta x \Delta y \Delta z$



$$\text{rate of mass increase} = \left\{\begin{matrix}\text{rate of}\\ \text{mass in}\end{matrix}\right\} - \left\{\begin{matrix}\text{rate of}\\ \text{mass out}\end{matrix}\right\}$$

$$(\Delta x \Delta y \Delta z)\frac{\partial \rho}{\partial t} = \left\{\rho v_x|_x - \rho v_x|_{x+\Delta x}\right\}\Delta y \Delta z + \left\{\rho v_y|_y - \rho v_y|_{y+\Delta y}\right\}\Delta x \Delta z + \left\{\rho v_z|_z - \rho v_z|_{z+\Delta z}\right\}\Delta x \Delta y$$

Divide by $(\Delta x)(\Delta y)(\Delta z)$ and take limits
$\Delta x \to 0$, $\Delta y \to 0$, $\Delta z \to 0$

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial x}(\rho v_x) - \frac{\partial}{\partial y}(\rho v_y) - \frac{\partial}{\partial z}(\rho v_z)$$

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$$\frac{\partial \rho}{\partial t} = -\left[\frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z)\right]$$

$$\boxed{\frac{\partial \rho}{\partial t} = -\left[\nabla \cdot (\rho \vec{V})\right]} \rightarrow \text{Generalized Eqn of continuity.}$$

∇ = del operator

$$\nabla = \vec{i}\,\frac{\partial}{\partial x} + \vec{j}\,\frac{\partial}{\partial y} + \vec{k}\,\frac{\partial}{\partial z}$$

$\nabla \cdot (\rho \vec{V}) \rightarrow$ divergence of $\vec{\rho V}$

2. Equation of Motion

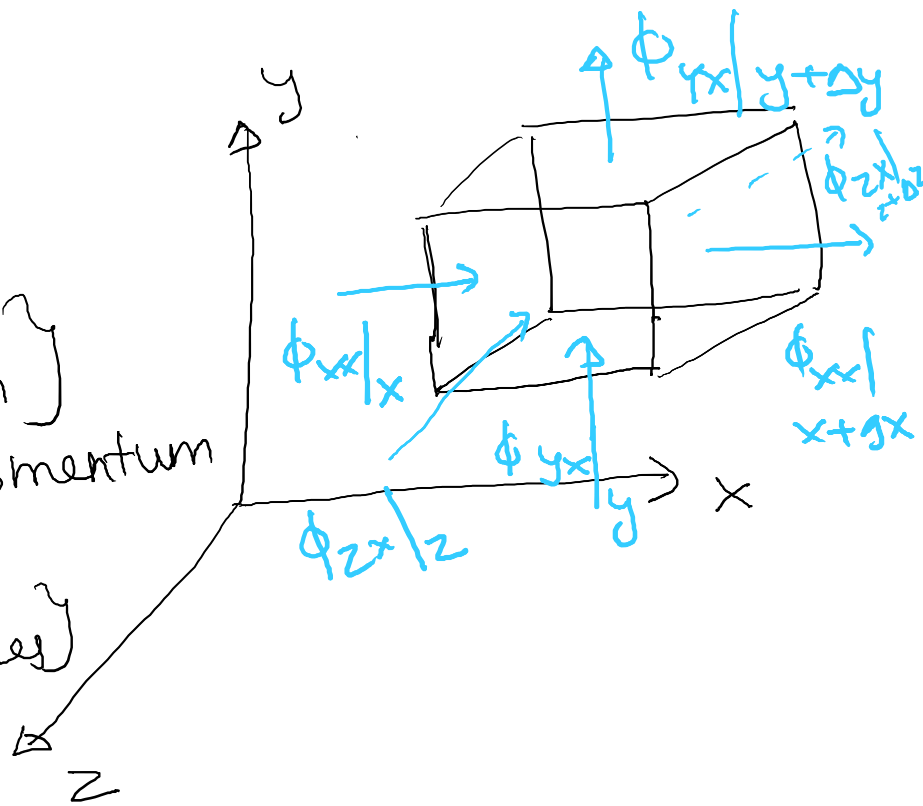
Applying the momentum balance in a very general situation.

rate of increase of momentum

= {rate of momentum in}

− {rate of momentum out}

+ {Body forces}



X-momentum transfer equation.

$$\frac{\partial}{\partial t}(\rho v_x)(\Delta x \Delta y \Delta z) = \left\{ \phi_{xx}|_x - \phi_{xx}|_{x+\Delta x} \right\} \Delta y \Delta z$$

$$+ \left\{ \phi_{yx}|_y - \phi_{yx}|_{y+\Delta y} \right\} \Delta x \Delta z$$

$$+ \left\{ \phi_{zx}|_z - \phi_{zx}|_{z+\Delta z} \right\} \Delta y \Delta x$$

$$+ (\Delta x \Delta y \Delta z) \rho g_x$$

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Divide by $(\Delta x \Delta y \Delta z)$
& limits $\Delta x, \Delta y, \Delta z \to 0$

$$\frac{\partial}{\partial t}(\rho v_x) = -\frac{\partial}{\partial x}(\phi_{xx}) - \frac{\partial}{\partial y}(\phi_{yx}) - \frac{\partial}{\partial z}(\phi_{zx}) + \rho g_x$$

$$\frac{\partial}{\partial t}(\rho v_y) = -\frac{\partial}{\partial x}(\phi_{xy}) - \frac{\partial}{\partial y}(\phi_{yy}) - \frac{\partial}{\partial z}(\phi_{zy}) + \rho g_y$$

y-momentum balance

z-component balance

$$\frac{\partial}{\partial t}(\rho v_z) = -\frac{\partial}{\partial x}(\phi_{xz}) - \frac{\partial}{\partial y}(\phi_{yz}) - \frac{\partial}{\partial z}(\phi_{zz}) + \rho g_z$$

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here $i = x, y, z$

$$\frac{\partial}{\partial t}(\rho v_i) = -[\nabla \cdot \phi]_i + \rho g_i$$

$$\boxed{\frac{\partial}{\partial t}(\rho \vec{v}) = -[\nabla \cdot \phi] + \rho \vec{g}}$$

General Equation of motion.

Now, introduce all the components for momentum transport

$$\phi = P + \tau + \rho \vec{v}\vec{v}$$

$$\frac{\partial}{\partial t}(\rho \vec{v}) = \left(-[\nabla P] - [\nabla \cdot \vec{\tau}]\right) \left(-[\nabla \cdot \rho \vec{v}\vec{v}]\right) + \rho \vec{g} \quad \text{—I}$$

rate of momentum transport due to molecular

rate of change of momentum per unit volume

rate of momentum transport due to convective mechanism.

Equation I is the most general momentum balance equation called as Equ^n. of motion. or cauchy momentum balance.