ABE 307
Date: Nov 13th, 2017.

Friction Factors

Friction factor is a generalized variable that is useful for calculation of force exerted by fluid on the system for any kind of system. We will study the friction factor for two most common system encountered in engineering problems.

a) Flow through channels/pipes such as pumping of fluids through pipes, extrusion, or blood flow through artery etc. The main goal in these cases is to get pressure drop with volumetric flow.

b) Flow around submerged objects such as fluid flow around heat exchanger tube, flow around particles in a mixing tank. The main goal in these cases is to find relationship between fluid velocity and drag exerted by the fluid on the submerged object.

Defining Friction Factor

Forces exerted by the fluid on surfaces.

$$F_{fluid \rightarrow surface} = F_K + F_S$$

F_K = Kinetic force

F_S = Static force.

We are interseted in kinetic forces.

$$F_K \propto AK$$

$$F_K = fAK$$

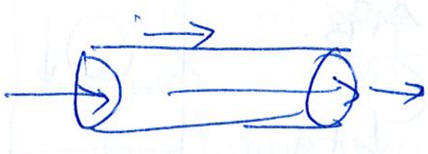
A = characteristic area

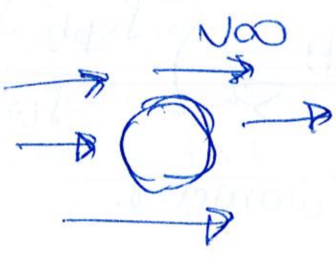
K = Kinetic energy per unit volume.

f = friction factor (dimensionless variables).

(a) For fluid flowing in pipes/conduits (channel) F_K is in the direction of average velocity.

(b) For submerged objects, the force is in direction of approach velocity

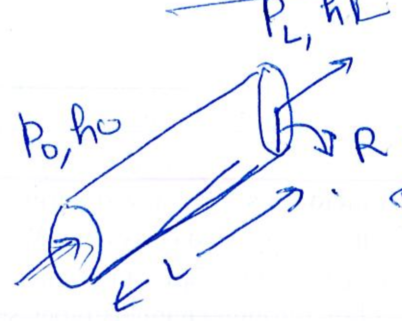




Applicable for both laminar & turbulent flow.

a. Friction Factor for Flow in Conduits

$$F_K = AKf \quad \text{— (I)}$$



$\mathcal{P} = P + \rho g h$

$$A = 2\pi R L$$

$$K = \frac{1}{2}\rho \langle v \rangle^2.$$

$\langle v \rangle \rightarrow$ average velocity

$$F_K = \left[(p_0 + \rho g h_0) - (p_L + \rho g h_L) \right] \pi R^2. \quad \text{— (II)}$$

$$AKf = (\mathcal{P}_0 - \mathcal{P}_L)\pi R^2 \qquad \mathcal{P} = \text{modified pressure.}$$

$$f = \frac{(\mathcal{P}_0 - \mathcal{P}_0)\pi R^2}{(2\pi R L)\left(\frac{1}{2}\rho \langle v \rangle^2\right)} = \frac{1}{4}\left(\frac{D}{L}\right)\left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{\frac{1}{2}\rho\langle v \rangle^2}\right)$$

= Fanning friction factor (Used in Moody Charts).

b. Friction Factor for Flow Around Submerged Objects

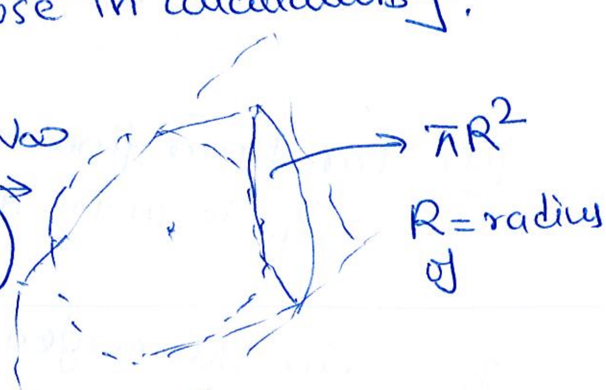
$$F_K = AKf \quad \text{— (I)} \rightarrow$$ [This is not a physical law. It is an empirical relationship to come up with dimensionless charts for use in calculations].

$A \rightarrow$ Projected area $= \pi R^2$.

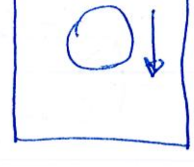
$K = \frac{1}{2}\rho \langle v_\infty \rangle^2$.

$$\text{(II)} \rightarrow F_K = (\text{gravity forces}) - (\text{Buoyancy forces}),$$

$$= \frac{4}{3}\pi R^3 \rho_{sph} g - \left(\frac{4}{3}\pi R^3 \rho_{liquid} g\right)$$



$$f = \frac{4}{3}\frac{gD}{\langle v_\infty \rangle^2}\left[\frac{\rho_{sph} - \rho_{liquid}}{\rho_{liquid}}\right]$$



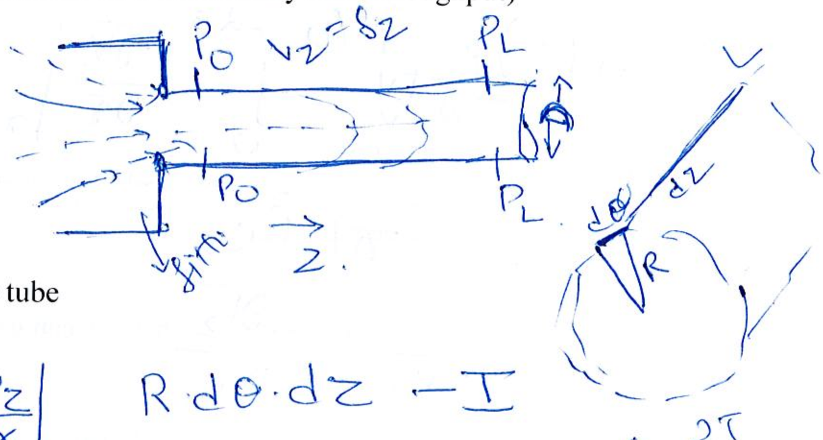
$D \rightarrow$ diameter

Developing Friction Factor Charts for Calculations

Calculation of friction factor for all types of system will become time consuming, so we want an easier and quicker way to calculate these factors for a real system. Thus, we combine dimensional analysis with definition of friction factor to relate the friction factor to one of the dimensionless variables that defines the flow regime for a system.

a. Friction Factor Charts for Flow in Tubes

- Either laminar or Turbulent flow
- Steadily driven turbulent flow (ie turbulent flow with steady total throughput)



Force in the z-direction on the inner wall of the tube

$$F_K = \int_0^L \int_0^{2\pi} -\mu \cdot \frac{\partial v_z}{\partial r}\bigg|_{r=R} R \cdot d\theta \cdot dz \quad \text{— I}$$

$$F_K = \frac{1}{2}\rho \langle v_z \rangle^2 (2\pi R L) f . \quad \text{— II}$$

Note: For Turbulent flow this force can be a function of time, due to fluctuations and the random velocity profile.

$$f(t) = \frac{\int_0^L \int_0^{2\pi} -\mu \cdot \frac{\partial v_z}{\partial r}\Big|_{r=R} R \cdot d\theta \cdot dz}{\frac{1}{2}\rho \langle v_z \rangle^2 (2\pi R L)} .$$

time

In turbulent flow, $\frac{\partial v_z}{\partial r}$ will be a function of time.

To be able to use dimensionless numbers, we will change the friction factor in dimensionless form.

— Since, the empirical charts are developed using model (or scaled down versions of real systems) we want these charts to be developed in non-dimensionless form for results to hold.

Define the following dimensionless quantities

$$\bar{r} = \frac{r}{D}, \quad \bar{v} = \frac{v_z}{\langle v_z \rangle}; \quad \bar{t} = \frac{t}{D/\langle v_z \rangle}$$

$$\bar{P} = \frac{P - P_0}{\rho \langle v_z \rangle^2}, \quad Re = \frac{\rho \langle v_z \rangle D}{\mu}, \quad \bar{z} = \frac{z}{D}.$$

Friction Factor in Dimensionless Equation Form

$$f(\bar{t}) = \frac{\rho D \int_0^{L/D} \int_0^{2\pi} \left.\frac{\partial \bar{v}_z}{\partial \bar{r}}\right|_{\bar{r}=1/2} d\theta \, d\bar{z}}{\pi L Re}$$

Now, if you want to calculate $\frac{\partial \bar{v}_z}{\partial \bar{r}}$, then we can use the dimensionless form of Equation of motion to get $\bar{v}_z$ (3.7)

Boundary conditions in Dimensionless Form

(1) $r = R, \; v_z = 0, \quad \bar{r} = 1/2, \; \bar{v}_z = 0.$

(2) $z = 0, \quad \bar{v}_z = \delta_z \rightarrow$ uniform velocity at inlet

Finally, when the equation for $f(\bar{t})$ is integrated, it will only depend on Re, L/D and $\bar{t}$

$$f(Re, L/D, \bar{t})$$

Next, when we time average the friction factor

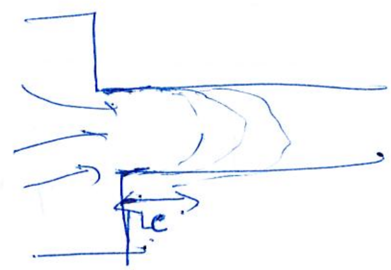
$$\int_0^t f(Re, L/D, \bar{t}) = f(Re, L/D)$$

$\hookrightarrow$ Moody friction factor charts.

Thus, measured friction factor only depends on Re and L/D. or Length to diameter ratio of the pipe.
After the fluid enters the pipe, till a certain length the velocity profile develops and after that the flow is fully developed and it will not depend on z. This entrance length is what is different in laminar and turbulent flow.

In case of Laminar flow, the entrance length is

$$L_e = 0.03 D\, Re.$$



In case of Turbulent flow, the entrance length is

$$L_e \approx 60 D.$$

Beyond the entrance length, the f is not dependent on L/D.

Readings: Please read Sections 6.1, 6.2, and 6.3

Examples from Book

Example 6.2.1: Calculation of Pressure Drop Required for a Given Flow Rate
What pressure gradient is required for fluid of density $\rho = 0.935$ gm/cc and viscosity $\mu=1.95$ cp to flow in a horizontal, smooth, circular tube of inside diameter D = 3 cm at a mass flow rate of 1028 g/s.

$w = 1028$ g/s, $\mu = 1.95$

$$f = (P_0 - P_L) \frac{1}{4} \frac{D}{L} \times \frac{1}{(\frac{1}{2}\rho \langle v_z \rangle^2)}$$

$$Re = \frac{\rho \langle v_z \rangle D}{\mu}$$

$$w = \rho \langle v_z \rangle \pi R^2$$

$$\langle v_z \rangle = \frac{w}{\rho \pi R^2}$$

$$Re = \frac{\cancel{\rho}\, w\, D}{\cancel{\rho}\, \pi R^2 \mu}$$

$$Re = \frac{wD}{\pi R^2 \mu}$$

$$Re = \frac{2w}{\pi R \mu}$$

$$Re = 2.24 \times 10^4$$

$f = 0.006$ (from chart).

Pressure gradient $= \frac{\Delta P}{L} = \frac{P_0 - P_L}{L}$

Example 6.2.2 : Calculation of flow rate for a given Pressure Drop
Determine the flow rate, in lbm/hr, of water at 68 deg F through a 1000-ft length of horizontal 8-in schedule 40 steel pipe (internal diameter 7.981in) under a pressure difference of 3.00 psi. Assume $k/D = 2.3 \times 10^{-4}$

$w \rightarrow$?

w ?, L, D, ΔP.

$$f = \Delta P \frac{1}{4} \frac{D}{L} \left(\frac{1}{\frac{1}{2} \rho \langle v_z \rangle^2} \right).$$

$\Delta P = 3.00$ psi
$L = 1000$ ft

$$Re = \frac{\rho \langle v_z \rangle D}{\mu}$$

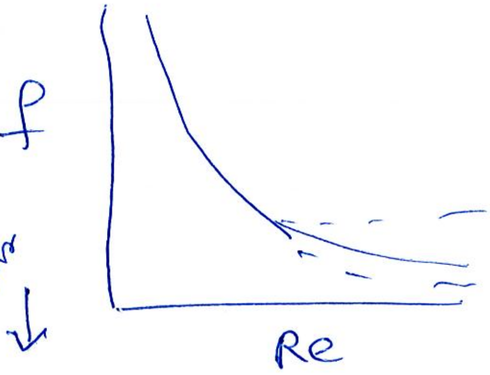
D,
ρ, μ.

f
Re

Method A → Recreating a new graph to use with the known parameters.

$$Re\sqrt{f} = \frac{\rho\langle v_z\rangle D}{\mu}\sqrt{\frac{\Delta P D}{2L\rho\langle \bar{v}_z\rangle^2}}$$

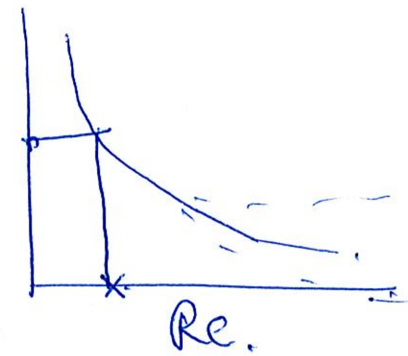
$$= \frac{\rho D}{\mu}\sqrt{\frac{\cancel{\langle \bar{v}_z\rangle^2}\,\Delta P D}{2L\rho\,\cancel{\langle \bar{v}_z\rangle^2}}}$$



$$Re\sqrt{f} = \frac{D\rho}{\mu}\sqrt{\frac{\Delta P D}{2L\rho}}$$

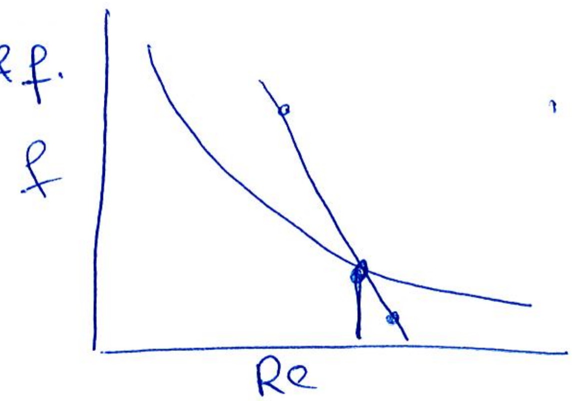
calculated specific to your problem

Develop
New chart for $Re\sqrt{f}$ vs Re from our original moody chart.



$$Re = \frac{\rho\langle v_z\rangle D}{\mu}$$

Method B → Develop a new equation for relationship between Re & f.



$$f = \frac{(Re\sqrt{f})^2}{Re^2}$$

$$\log f = -2\log Re + 2\log(Re\sqrt{f})$$

$$Re = \frac{\rho\langle v_z\rangle D}{\mu}$$

$$w = \cancel{\rho}\rho\langle v_z\rangle \pi R^2$$