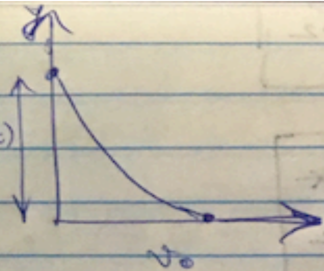
TAKE HOME TEST-3

SOLUTION:

Problem 1:



Given:

$$\frac{d}{dt}\int_0^\infty \rho u\,dy = \tau_{yx}\Big|_{y=0}$$

$$u = u_0\left(1 - \frac{3}{2}\frac{y}{\delta} + \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right)$$

$$\tau_{yx}\Big|_{y=0} = -\mu\frac{\partial u}{\partial y}\Big|_{y=0} = \mu\frac{3u_0}{2\delta} \quad \text{— (1)}$$

$$\frac{d}{dt}\int_0^\infty \rho u\,dy = \rho u_0\frac{d}{dt}\left[\int_0^\delta\left[1 - \frac{3}{2}\frac{y}{\delta} + \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right]dy\right] + \cancel{\rho u_0\frac{d}{dt}\int_\delta^\infty 0\,dy} \to 0$$

$$\Rightarrow \frac{d}{dt}\int_0^\infty \rho u\,dy = \rho u_0\frac{d}{dt}\left[\delta - \frac{3\delta}{4} + \frac{\delta}{6}\right] = \frac{5}{12}\rho u_0\frac{d\delta}{dt} \quad \text{— (2)}$$

$$(1)\ \&\ (2) \Rightarrow \frac{5}{12}\rho u_0\frac{d\delta}{dt} = \frac{3\mu u_0}{2\delta}$$

$$\Rightarrow \int_0^\delta 2\delta\,d\delta = \int_0^t \frac{36\nu}{5}\,dt$$

$$\Rightarrow \delta^2 = \frac{36\nu t}{5}$$

$$\Rightarrow \boxed{\delta = \sqrt{\frac{36\nu t}{5}}}$$

Problem 2:

Given parameters: $d, \mu, \rho_l, \rho_g, g, \sigma, U$

$U = f(d, \mu, g, \rho_l, \rho_g, \sigma)$ $\rightarrow MT^{-2}$ (force per unit length)

LT^{-1} L $ML^{-1}T^{-1}$ LT^{-2} ML^{-3}

we choose ρ_l, μ, d as the primary variables.

$\pi 1$:

$U : (L^{1}T^{-1}).\rho^{a}\mu^{b}d^{c} \equiv \boxed{M^0 L^0 T^0 \equiv L^1 T^{-1}(ML^{-3})^a (ML^{-1}T^{-1})^b L^c}$ — (1)

solving for a, b & c from eq (1),

$\downarrow$ $\downarrow$ $\downarrow$

1 $\;-1$ $\;1$

$\Rightarrow \boxed{\pi 1: \frac{\rho_l U d}{\mu}}$

$\pi 2$:

$g : (LT^{-2})\rho_l^{a}\mu^{b}d^{c} \equiv \boxed{M^0 L^0 T^0 \equiv (LT^{-2})(ML^{-3})^a (ML^{-1}T^{-1})^b L^c}$ — (2)

solving for a, b & c from eq (2),

$\downarrow$ $\downarrow$ $\downarrow$

2 $\;-2$ $\;3$

$\boxed{\pi 2: \frac{g d^3 \rho_l^2}{\mu^2}}$

$\pi 3$:

$\rho_g : (ML^{-3})\rho^{a}\mu^{b}d^{c} \equiv \boxed{M^0 L^0 T^0 \equiv (ML^{-3})(ML^{-3})^a (ML^{-1}T^{-1})^b L^c}$ — (3)

$a = -1, \; b = 0, \; c = 0$

$\boxed{\pi 3: \rho_g / \rho_l}$

$\pi 4$:

$\sigma : (MT^{-2})\rho^{a}\mu^{b}d^{c} \equiv \boxed{M^0 L^0 T^0 \equiv (MT^{-2})(ML^{-3})^a (ML^{-1}T^{-1})^b L^c}$ — (4)

~~a = 1, b = -2, c = 1~~ we have $a = +1, b = -2, c = 1$

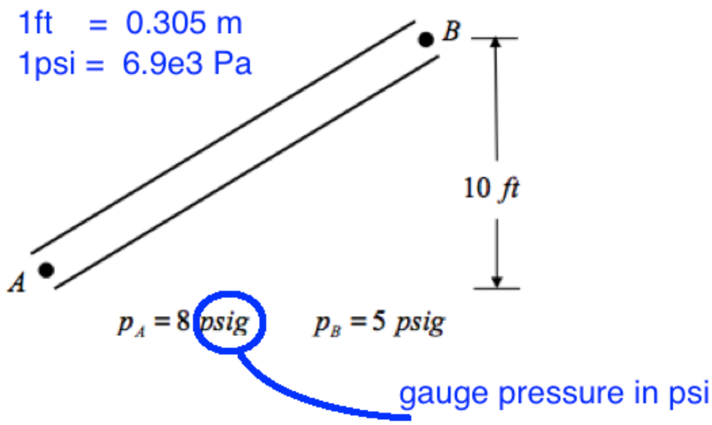
$\boxed{\pi 4 = \frac{\rho_l \sigma d}{\mu^2}}$

$\pi_1 = f(\pi_2, \pi_4, \pi_3)$,

$\Rightarrow \frac{\rho U d}{\mu} = f\left(\frac{g d^3 \rho^2}{\mu^2}, \frac{\sigma d \rho}{\mu^2}, \frac{\rho_g}{\rho}\right)$ (ρ being the liquid density).

Here, π_1, π_2, and π_4 are Reynolds number, Archimedes number and Ohnesorge number respectively.

Problem 3:



Using the Bernoulli's equation between points A and B, assuming the flow to be from A to B, we have:

$$P_A + \frac{1}{2}\rho u_A{}^2 + \rho g h_A = P_B + \frac{1}{2}\rho u_B{}^2 + \rho g h_B + \varphi_d.$$

$'\varphi_d'$ is the pressure loss due to viscous friction at the wall, and is positive if the flow is from A to B.

We know from mass balance that $u_A = u_B$.

Rearranging the above expression, we have:

$$(P_A - P_B) + \rho g(h_A - h_B) = \varphi_d.$$

Plugging in the values of P_A, P_B, h_A, h_B, and assuming the liquid to be water with density *1000kg/m³*, gravity being *9.8m/s²*, (and converting the given values to SI units), we have:

$$\varphi_d = -9.19e3\ Pa.$$

This means that the original assumption of flow from *A to B* is incorrect and that the actual flow direction is from ***B to A***.