Writing Equations: (Review class).

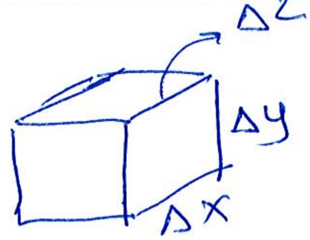
(1) Flow rate across a surface is written in terms of

ⓐ → surface integral.

(2)

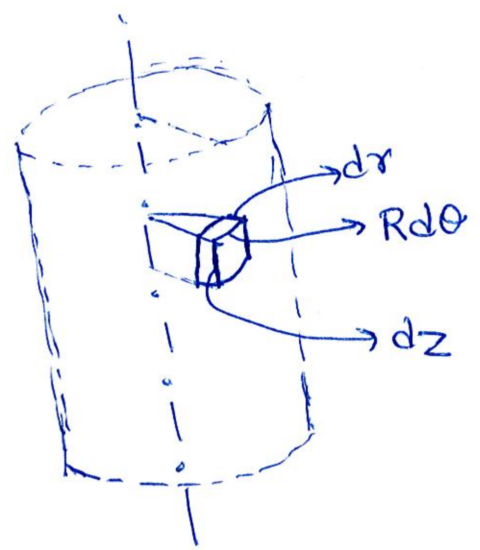
(a) Differential volume element in cartesian coordinate.

$$\int_0^H \int_0^W \int_0^L \Delta x \, \Delta y \, \Delta z.$$



ⓑ Differential volume element in cylindrical coordinate

$$\int_0^L \int_0^R \int_0^{2\pi} R \, d\theta \cdot dr \cdot dz$$



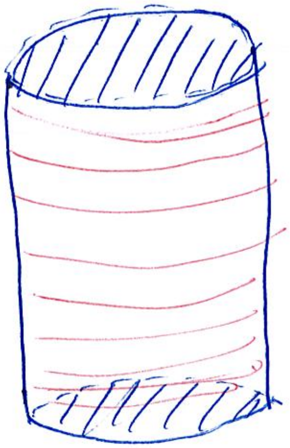
(c) Differential surface area in cartesian coordinates.

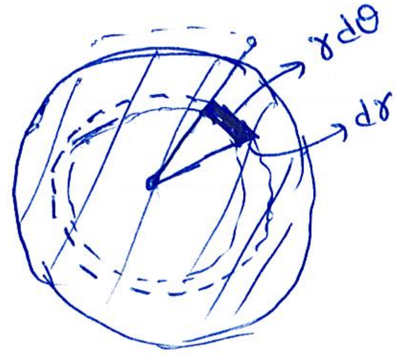
$\Delta x \, \Delta y$
or $\Delta y \, \Delta z$
or $\Delta z \, \Delta x$

} you should pay attention to your choice of coordinate for the surface.

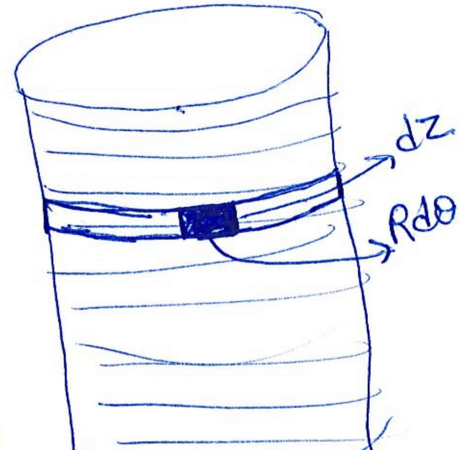
2d Differential surface element in cylindrical coordinates.

For this problem, we can look at two surfaces.
One is the top surface of the cylinder and other is the curved surface.





Top surface
(or bottom surface).



Curved surface.

$$\int_0^R \int_0^{2\pi} r\,d\theta\,dr$$

$$\Downarrow$$

$$2\pi \int_0^R r\cdot dr$$

$$= 2\pi \left.\frac{r^2}{2}\right|_0^R$$

$$= \pi R^2$$

$$\int_0^L \int_0^{2\pi} R\cdot d\theta\cdot dz$$

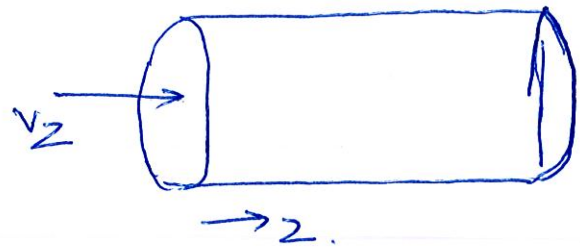
$$= 2\pi \int_0^L R\cdot dz$$

$$= 2\pi R L.$$

When you have to calculate volumetric or mass flow rate for a cylindrical geometry, make sure to use the correct surface integral for calculation of volumetric/mass flow rate. Find out the surface through which the fluid is flowing.

(3)

(a) $v_z = v_z(r, \theta)$.



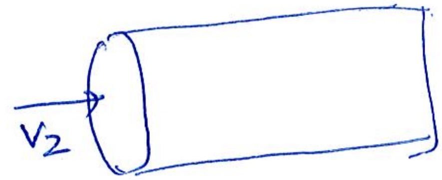
Volumetric flow rate

$$V = \int_0^{2\pi}\int_0^{R} v_z \, r \cdot d\theta \cdot dr$$

Since v_z is varying over θ & r, and flowing through the flat surface, we use surface integral for fluid flowing through flat surface.

(if you have the functional form of v_z as function of r & θ, you can substitute in equation above and calculate the volumetric flow rate.

(b) $V_z = \text{const.} = A.$ (m/s.)



$$V = \int_0^{2\pi}\int_0^{R} V_z \, r \, d\theta \cdot dr$$

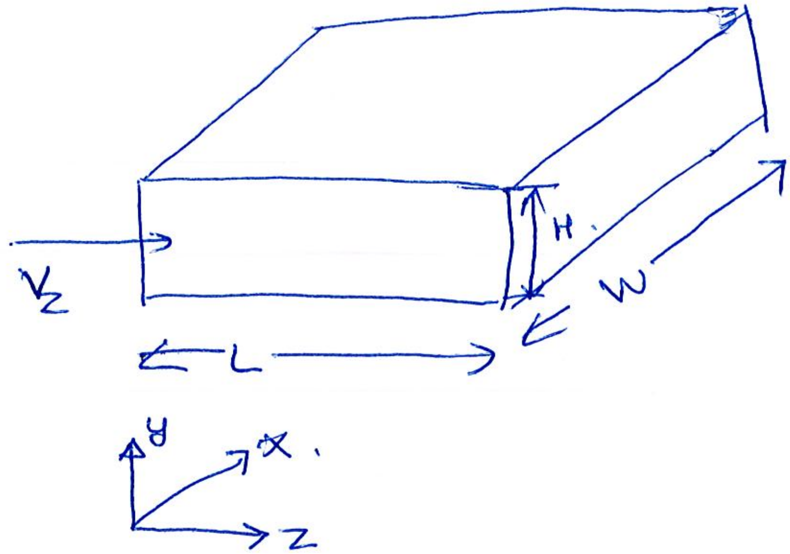
$$= A \int_0^{2\pi}\int_0^{R} r \cdot dr \cdot d\theta$$

$$= A \, 2\pi \times \frac{r^2}{2}\Big|_0^R = A\left(\pi R^2\right)$$

A is the const. velocity. Here, we could directly tell that volumetric flow rate is $A\pi R^2$ (we get the same result from integral).

3c

$$\text{Vol.} = \int_0^H \int_0^W V_z \, dx \cdot dy$$



Here ~~o~~ again pay attention to which is the surface through which the fluid is crossing.

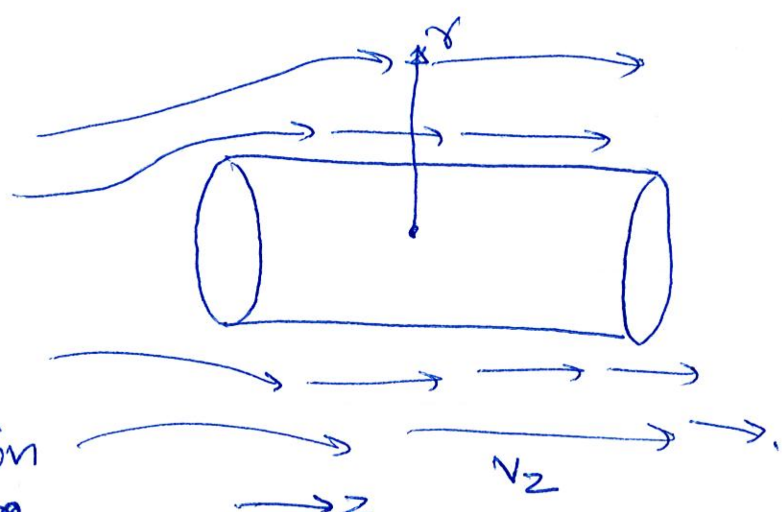
$V_z = V_z(x, y)$

V_z is function of x & y.

(4) (a).

Fluid flowing over a cylinder.

$V_z = V_z(r)$



As you go far from cylider in radial direction the velocity varies ~~with~~ in that direction. ie. your

$\frac{dV_z}{dr}$ is causing the shear stress.

The surface on ~~with~~ which it is acting is the cylinder surface area

$$\int_0^L \int_0^{2\pi} \tau_{rz} \cdot R \, d\theta \, dz$$

L = Length of cylinder

R = Radius of cylinder.

4b → It is same as above, except that fluid is now flowing inside the pipe.

$\boxed{4c}$ For multiple ~~dimensio~~ dimensions you will use the generalized form of τ and use that to get total shear stress. ~~others~~