HW-6 SOLUTION:

Problem 1:

$$u_x = \frac{\partial \psi}{\partial y} \qquad u_y = -\frac{\partial \psi}{\partial x}$$

a) Given : $u_x = 25y$ (ft/s).

$$\Rightarrow \int_{\psi_0}^{\psi} d\psi = \int_{0}^{y} 25y\,dy$$

$$\Rightarrow \boxed{\psi = \frac{25y^2}{2} + \psi_0} \rightarrow \text{constant}$$

b) $Q(y) = \psi(y) - \psi(0)$

$$Q(y) = \frac{1}{2} Q(4\,\text{ft})$$

$$\Rightarrow 12.5y^2 = \frac{1}{2} \times 12.5 \times 4^2$$

$$\Rightarrow y = \frac{4}{\sqrt{2}} = 2\sqrt{2} \simeq \boxed{2.828\,\text{ft}}$$

Problem 2:

a) $w = -V_0 z^2$

$z = x + iy$

$w = -V_0 (x+iy)^2.$

$= -V_0 (x^2 - y^2) - i(2xy\, V_0)$

$w = \phi(x,y) + i\psi(x,y)$

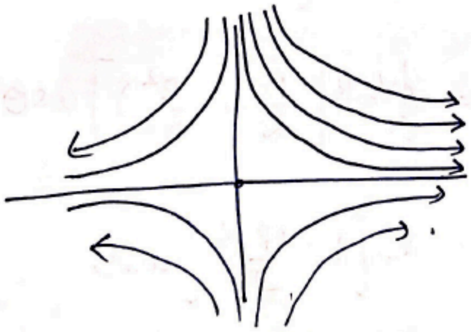
$\phi(x,y) = -V_0(x^2 - y^2)$

$\psi(x,y) = V_0(-2xy)$

$\hookrightarrow$ complex component.

It describes hyperbola. with the streamlines as shown

It describes flow near a plane stagnation point



b.) $\frac{dw}{dz} = -V_x + iV_y$

$\frac{dw}{dz} = \frac{d(-V_0 z^2)}{dz} = -2V_0 z$

$= -2V_0(x+iy)$

$= -2V_0 x + i(-2V_0 y)$

$\Rightarrow V_x = 2V_0 x$

$V_y = -2V_0 y$

c) The sign of V_o indicates the direction of the jet and the magnitude of V_o indicates the magnitude of the flow and can be used to compute the stagnation pressure at the wall (using Bernoulli's principle).

Problem 3:

a)

$$\phi = -V_\infty R\left[\left(\frac{r}{R}\right) + \frac{1}{2}\left(\frac{R}{r}\right)^2\right]\cos\theta$$

$$V_r = -\frac{\partial\phi}{\partial r} = V_\infty R\left[\frac{1}{R} - \left(\frac{R^2}{r^3}\right)\right]\cos\theta$$

$$= V_\infty\left[1 - \left(\frac{R}{r}\right)^3\right]\cos\theta$$

$$V_\theta = -\frac{1}{r}\frac{\partial\phi}{\partial\theta} = \frac{\partial}{\partial\theta} V_\infty R\left(\frac{1}{R} + \frac{1}{2}\frac{R^2}{r^3}\right)\cos\theta$$

$$= -V_\infty\left(1 + \frac{1}{2}\left(\frac{R}{r}\right)^3\right)\sin\theta.$$

b.)

$$\frac{1}{2}\rho(V_r^2 + V_\theta^2) + P = P_\infty + \frac{1}{2}\rho V_\infty^2$$

$$\frac{1}{2}\rho V_\infty^2\left[\left(1-\left(\frac{R}{r}\right)^3\right)^2\cos^2\theta + \left[1 + \frac{1}{2}\left(\frac{R}{r}\right)^3\right]^2\sin^2\theta\right] + P = P_\infty + \frac{1}{2}\rho V_\infty^2$$

$$P - P_\infty = \frac{1}{2}\rho V_\infty^2\left[1 - \left[\cos^2\theta\left(1 - 2\left(\frac{R}{r}\right)^3 + \left(\frac{R}{r}\right)^3\right) + \sin^2\theta\left(1 + \left(\frac{R}{r}\right)^3 + \frac{1}{4}\left(\frac{R}{r}\right)^6\right)\right]\right]$$

$\therefore$ at $r = R$.

$$P - P_\infty = \frac{1}{2}\rho V_\infty^2\left[1 - \left(1 + \frac{1}{2}\right)^2\sin^2\theta\right]$$

$$= \frac{1}{2}\rho V_\infty^2\left(1 - \frac{9}{4}\sin^2\theta\right).$$