HW-7 SOLUTION:

Problem 1:

a) The velocity profile $U_x = \frac{U_\infty y}{\delta}$ satisfies the following boundary conditions: At $y = 0$, $U_x = 0$ (No slip at wall), and at $y = \delta$, $U_x = U_\infty$. However, it doesn't satisfy the condition that at $y = \delta$, $dU_x/dy = 0$. In fact, the derivative of U_x is not continuous at $y = \delta$. Hence, although the given velocity profile is an acceptable boundary layer profile, it can be expected to be less accurate than the velocity profile in the textbook.

b)

Given: $V_x = \frac{V_\infty y}{\delta}$ — (1)

b) V.K.E:

$$\mu \left.\frac{\partial u_x}{\partial y}\right|_{y=0} = \rho \int_0^\infty \frac{d}{dx}\left(V_e u_x - u_x^2\right) dy + \rho \int_0^\infty (V_e - u_x)\frac{dV_e}{dx}\, dy \quad \text{— (3)}$$

$V_e = U_\infty$ — (2)

(1), (2) and (3)

$$\Rightarrow \underbrace{\frac{\mu U_\infty}{\delta}}_{L.H.S} = R.H.S$$

$$RHS = \rho \int_0^\infty \frac{d}{dx}\left(V_\infty u_x - u_x^2\right) dy + \rho \int_0^\infty \cancel{(u_\infty - u_x)\frac{du_\infty}{dx}}\, dy \rightarrow 0$$

$$\frac{du_\infty}{dx} = 0$$

$$\Rightarrow RHS = \rho \int_0^\delta \frac{d}{dx}\left[\frac{V_\infty^2 y}{\delta} - \frac{U_\infty^2 y^2}{\delta^2}\right] dy + \int_\delta^\infty \frac{d}{dx}\underbrace{\left[V_\infty^2 - U_\infty^2\right]}_{=0} dy \rightarrow 0$$

$$\Rightarrow RHS = \rho U_\infty^2 \int_0^\delta \frac{d}{dx}\left[\frac{y}{\delta} - \frac{y^2}{\delta^2}\right] dy$$

$$= \rho U_\infty^2 \frac{d}{dx}\left[\int_0^\delta \left(y/\delta - y^2/\delta^2\right) dy\right]$$

$$= \frac{\rho U_\infty^2}{6}\frac{d(\delta)}{dx}$$

LHS = RHS

LHS = RHS

$$\Rightarrow \quad \frac{\mu u_\infty}{\delta} = \frac{\rho u_\infty^2}{6}\frac{d\delta}{dx}$$

Solving the above ODE, with BC: $\delta(x=0) = 0$,

$$\delta(x) = \sqrt{\frac{12\nu x}{u_\infty}} = 3.46\sqrt{\frac{\nu x}{u_\infty}}$$

c) error in 'δ':

$$\frac{4.64 - 3.46}{4.64} \simeq 25\%$$

Book solution

d)

$$\text{Drag force} = 2\int_0^W\int_0^L \mu \left.\frac{\partial u_x}{\partial y}\right|_{y=0} dx\,dz$$

Top & Bottom

$$= 2\int_0^W\int_0^L \frac{\mu u_\infty}{\delta(x)}\,dx\,dz = 2\int_0^W\int_0^L \frac{\mu u_\infty}{3.46\sqrt{\frac{\nu x}{u_\infty}}}\,dx\,dz$$

$$\simeq 1.156\sqrt{\mu\rho W^2 L u_\infty^3}$$

e)

$$\%\,\text{error} = \frac{1.293 - 1.156}{1.293} \simeq 10.596\%$$

Book solution

Problem 3:

To solve the boundary layer profile, we need information about the free-stream velocity outside of the boundary layer (U_∞), which we can get by using the potential flow theory applied to the region (of negligible viscous effect) outside of the boundary layer.

But to solve for the free-stream velocity using potential flow, we need information about the thickness of the boundary layer (trajectory along which to specify the constant potential φ boundary condition). Thus, the boundary layer solution and potential flow theory need each other and are complimentary methods.

Problem 2: (Use results from the book)

(a) $Re = \frac{x V_\infty}{\mu/\rho} = \frac{(3)^{ft} \times 20 \text{ ft/sec}}{}$

Use μ & ρ for air at 1atm 20°C to get these values.

(b) $\delta(x) = 4.64\sqrt{\frac{\nu x}{V_\infty}}$ $\nu = \frac{\mu}{\rho} =$

$=$

(c) $F_x = 1.293\sqrt{\rho\mu L W^2 V_\infty^3}$

$= 1.293\sqrt{\rho\mu (3)\times(10)\times(20)^3}$

Problem 4:

a)

$$\Delta p = f(V, D, \rho, \mu)$$

$$\Delta p = \rho^a \mu^b V^c D^d.$$

$$ML^{-1}T^{-2} = [ML^{-3}]^a [ML^{-1}T^{-1}]^b [LT^{-1}]^c L^d.$$

$$a+b = 1$$

$$-3a-b+c+d = -1$$

$$-b-c = -2.$$

$$b+c = 2.$$

$$\cancel{b-c = 1+d-3a}$$

$$\Rightarrow \quad a = c-1$$

$$b = 2-c$$

$$d = c-2$$

$$\Delta p = \rho^{c-1}\mu^{2-c} V^c D^{c-2}$$

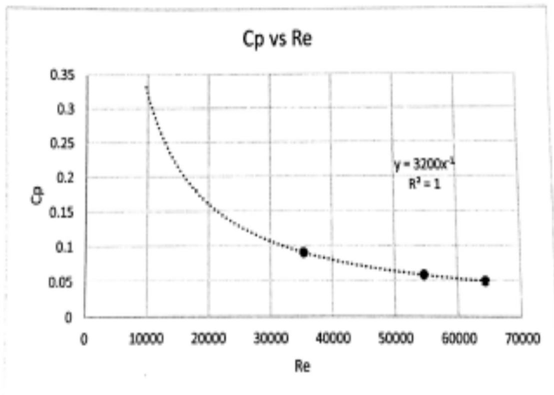
$$= \left(\frac{\rho V d}{\mu}\right)^c \left(\frac{\mu^2}{\rho D^2}\right)$$

$$\Delta p = \left(\frac{\rho V d}{\mu}\right)^{2+d} \frac{\mu^2}{\rho D^2} = \left(\frac{\rho V D}{\mu}\right)^d \rho V^2$$

$$\frac{\Delta p}{\rho V^2} = \left(\frac{\rho V D}{\mu}\right)^d.$$

b)

V	Delta P	Re	Cp
3	192	9649.5	0.331623
11	704	35381.5	0.090443
17	1088	54680.5	0.058522
20	1280	64330	0.049744



From the data given, we see that C_p and *Re* are related as $C_p \: \alpha \: 1/Re$.

c) The limitation of the relation proposed is that it is applicable only in the range of *Re* values presented in the data. If we wish to determine the relationship between Cp and *Re* for *Re = 90000,* we would need data from experiments performed on *Re > 70000*.

Also, since the process employed in the method is purely mathematical (curve fitting), no information about the physics leading to the determined relation can be acquired from it. We would need further experiments or analytical modeling to get insights into the underlying physics in the problem of interest.