ABE 307
Homework 4 (60 Points)
Fall 2017
Assigned: 10/11/2017
Due: 10/18/2017

1. Explain the difference between Lagrangian and Eulerian frames based on the example on radiation concentration shown in the video posted. Why do we need Lagrangian reference? (5 points)

2. What are the three different type of differentials you will encounter in fluid mechanics? Which frame of reference is most commonly used to make observations? (3)

3. Is Steady state an Eulerian or Lagrangian concept? Explain your answer. (3 point)

4. Does D/Dt = 0 mean steady flow? Explain? (4 points)

5. The viscosity of 60% by weight sucrose solution is to be measured using a concentric cylinder viscometer as depicted by Fig. 3.6-1 of BSL. The instrument has an inner cylinder 4 cm in diameter surrounded by a rotating outer concentric cylinder 4.5 cm in diameter. The length of the two cylinders is 4 cm. The measured torque at the rotational speed of 10 rpm is 5.0×10^{-5} N.m. Calculate the viscosity of sucrose solution. (10 points)

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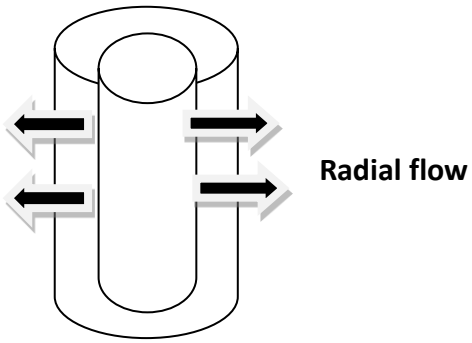
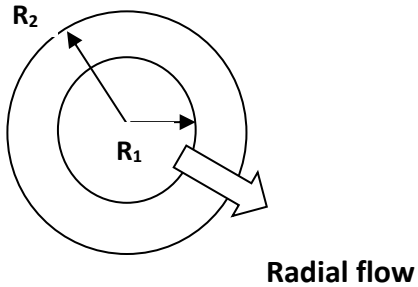
6. Hollow fiber membranes are used for filtration. They consist of a bundle of two concentric cylinders (hollow fibers) of extremely small diameter inner cylinder. These filters are extremely compact. The inner cylinder is porous and acts as a filer. In a real system, the liquid flow through a bundle of inner tubes inside a relatively large outer tube. Because of high pressure of the liquid, the liquid also flows radially through the porous inner tube into the outer tube. We can simplify the problem by neglecting the axial flow of liquid through the inner tube and by considering only the radial flow from the inner tube of radius R_1 into the outer tube of radius R_2 (see figure below). (25 points)





a) List all the assumptions in your analysis.
b) Write the equation of continuity and solve for v_r.
c) Write the equation of motion in the radial direction and obtain an expression for the pressure gradient along the radial direction.
d) Integrate the pressure gradient to obtain the following expression for the pressure

$$p(r) - p(R_1) = -\frac{1}{2}\rho\left(v_r\big|_{r=R_1}\right)^2\left[1-\left(\frac{R_1}{r}\right)^2\right]$$

e) Use the above expression to calculate the mass flow rate per unit length of hollow fiber membrane of inner and outer radii of 1 mm and 5 mm respectively when the pressure difference is 15 Pa. What will be the product flow rate for a hollow fiber cartridge of length 1m consisting of 100 hollow fibers. The density of the fluid is 1000 kg m^{-3}.

7. Recall the rotational viscometer problem in class. Using the same geometrical set up for the rotational viscometer, derive the velocity distribution equation for Searle viscometer (i.e. when the outer cylinder is fixed and the inner cylinder is rotated with an angular velocity Ω_i (10)