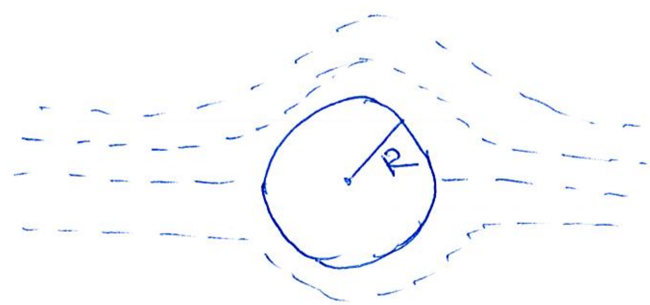
Example 4.3-1

$$\omega(z) = -V_\infty R\left[\frac{z}{R} + \frac{R}{z}\right]$$

$$\omega(z) = \phi(x,y) + i\psi(x,y)$$



① To seperate real & imaginary part

$$z = x + iy$$

$$\omega(z) = -V_\infty R\left[\frac{z}{R} + \frac{R}{z}\right]$$

$$= -V_\infty\left[z + \frac{R^2}{z}\right]$$

$$= -V_\infty\left[(x+iy) + \frac{R^2}{(x+iy)}\right]$$

$$= -V_\infty\left[(x+iy) + \frac{R^2\,(x-iy)}{(x+iy)(x-iy)}\right]$$

multiplying by conjugate

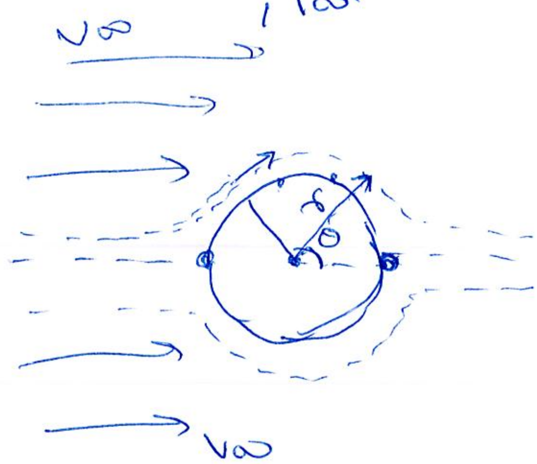
$$= -V_\infty\left[(x+iy) + \frac{R^2\,(x-iy)}{(x^2+y^2)}\right]$$

$$\omega(z) = -V_\infty\left[x + \frac{R^2 x}{(x^2+y^2)} + i\left(y - \frac{R^2 y}{(x^2+y^2)}\right)\right]$$

Velocity potential $\phi(x,y) = -V_\infty\left[x + \frac{R^2 x}{x^2+y^2}\right]$

stream function $\psi(x,y) = -V_\infty\left[y - \frac{R^2 y}{(x^2+y^2)}\right]$.

② Finding the velocity components near the cylinder



$$\frac{dw}{dz} = -V_x + iV_y.$$

$$w = -V_\infty\left(z + \frac{R^2}{z}\right)$$

$$\frac{dw}{dz} = -V_\infty\left[1 - \frac{R^2}{z^2}\right]$$

$$z = re^{i\theta}$$

$$z^2 = r^2 e^{i2\theta}$$

$$e^{i\theta} = \cos\theta + i\sin\theta.$$

$$e^{i2\theta} = \cos 2\theta + i\sin 2\theta$$

$$\frac{dw}{dz} = -V_\infty\left[1 - \frac{R^2}{r^2 e^{i2\theta}}\right].$$

$$= -V_\infty\left[1 - \frac{R^2}{r^2[\cos 2\theta + i\sin 2\theta]}\right]$$

$$= -V_\infty\left[1 - \frac{R^2(\cos 2\theta - i\sin 2\theta)}{r^2\underbrace{(\cos^2 2\theta + \sin^2 2\theta)}_{=1}}\right]$$

$$= -V_\infty\left[1 - \frac{R^2(\cos 2\theta - i\sin 2\theta)}{r^2}\right]$$

$$= -V_\infty\left[1 - \frac{R^2}{r^2}\cos 2\theta\right] - i\left[V_\infty \frac{R^2}{r^2}\sin 2\theta\right]$$

$$v_x = V_\infty\left[1 - \frac{R^2}{r^2}\cos 2\theta\right]$$

$$v_y = -V_\infty\left[\frac{R^2}{r^2}\sin 2\theta\right]$$

on surface of cylinder, $r = R$.

$$v_x = V_\infty\left[1 - \cos 2\theta\right]$$

$$v_y = -V_\infty \sin 2\theta.$$

③ Getting pressure distribution

$$\frac{1}{2}\rho(v_x^2+v_y^2)+P = \text{const.} \qquad \text{(Bernoulli const).}$$

Far from the cylinder

$$\frac{1}{2}\rho(v_\infty^2)+P_\infty = \text{const}$$

On the surface of the cylinder.

$$\frac{1}{2}\rho(v_x^2+v_y^2)+P = \text{const}$$

$$\frac{1}{2}\rho\left[v_\infty^2(1-\cos 2\theta)^2+v_\infty^2\sin^2 2\theta\right]+P$$

$$= \frac{1}{2}\rho v_\infty^2+P_\infty$$

→ $\cos 2\theta = 1-2\sin^2\theta$

→ $\sin 2\theta = 2\sin\theta\cos\theta$

$$v_x^2+v_y^2 = 4v_\infty^2\sin^2\theta\,[\cancel{\sin^2\theta+\cos^2\theta}]^{1}$$

$$= 4v_\infty^2\sin^2\theta$$

$$\frac{1}{2}\rho(4v_\infty^2\sin^2\theta)+P = \frac{1}{2}\rho v_\infty^2+P_\infty.$$

$$P-P_\infty = \frac{1}{2}\rho v_\infty^2[1-4\sin^2\theta]$$