DATE: Aug 28th, 2017 Monday

FLUID STATICS

Application of the Linear Momentum Principle

Developing Relationship for Pressure Variation in a Static Fluid

General momentum principle application on a static fluid will be utilized to develop a relationship that describes the pressure variation in the fluid. The steps are as described below.

Consider a large reservoir of fluid kept stationary. The distance of the surface of the fluid from the bottom of tank is L. Assume the height of tank in z direction. z = 0 is the bottom of tank and z = L is the surface of liquid. The goal is to find a general expression for pressure at any depth "h" from the surface of liquid in tank.

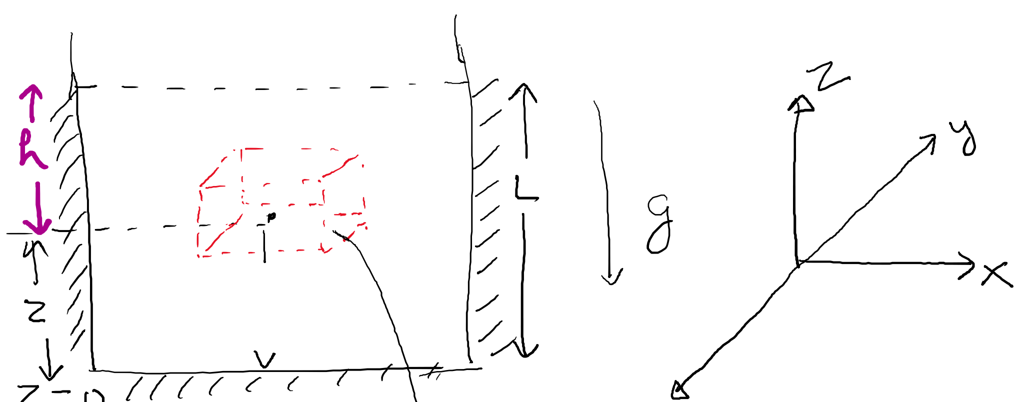
Step 1. Consider a differential element of fluid at z in the fluid with dimensions Δx, Δy and Δz. Write the momentum balance equation for this element.

Step 2: Divide by ΔxΔyΔz and take limits Δx → 0, Δy → 0 and Δz → 0

Step 3: Equate specific terms to 0

Step 4: Integrate the pressure variation equation with the proper boundary conditions.

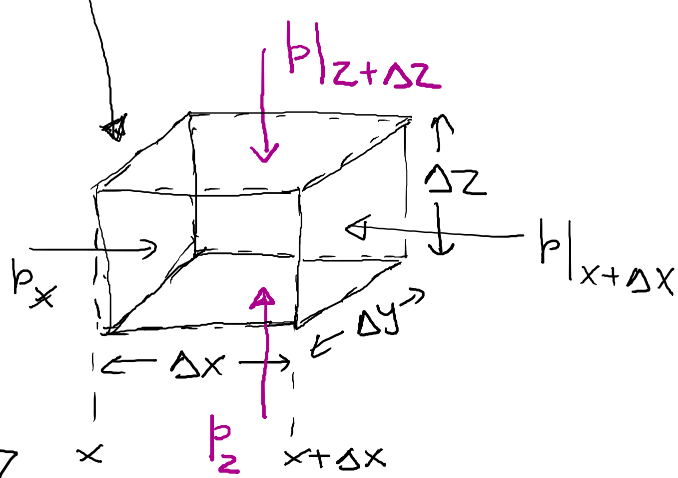
$\boxed{h = L - z}$



Step 1

General momentum principle for static fluid

$0 =$ Sum of body forces + sum of surface forces



$$= \rho \vec{g} \Delta V + \vec{i} p_x \Delta y \Delta z - \vec{i} p_{x+\Delta x} \Delta y \Delta z + \vec{j} p_y \Delta x \Delta z$$

$$-\vec{j}\, p_{y+\Delta y}\, \Delta x \Delta z + \vec{K}\, p_z \Delta x \Delta y$$

$$-\vec{K}\, p_{z+\Delta z}\, \Delta x \Delta y$$

$$0 = -\rho \vec{K} g \Delta x \Delta y \Delta z + \vec{i}\, p_x \Delta y \Delta z$$

$$-\vec{i}\, p_{x+\Delta x}\, \Delta y \Delta z + \vec{J}\, p_y \Delta x \Delta z$$

$$-\vec{j}\, p_{y+\Delta y}\, \Delta x \Delta z + \vec{K}\, p_z \Delta x \Delta y$$

Step 2

$$-\vec{K}\, p_{z+\Delta z}\, \Delta x \Delta y$$

$$0 = -\rho \vec{K} g + \vec{i}\, \frac{(p_x - p_{x+\Delta x})}{\Delta x}$$

$$+ \vec{j}\, \frac{\left(p_y - p_{y+\Delta y}\right)}{\Delta y} + \vec{K}\, \frac{\left(p_z - p_{z+\Delta z}\right)}{\Delta z}$$

Taking limits, $\Delta x \to 0$, $\Delta y \to 0$ & $\Delta z \to 0$

$$0 = -\rho \vec{K} g - \left[\vec{i}\, \frac{\partial p}{\partial x} + \vec{j}\, \frac{\partial p}{\partial y} + \vec{k}\, \frac{\partial p}{\partial z}\right]$$

step 3

Individual components should be zero

$\boxed{\frac{\partial p}{\partial x} = 0}$; $\boxed{\frac{\partial p}{\partial y} = 0}$ and $\boxed{\frac{\partial p}{\partial z} + \rho g = 0}$

$$\frac{\partial p}{\partial z} + \rho g = 0$$

Integrate between limits to get pressure variation in z direction

Step 4

Identify boundary conditions for limits on your differential equations.

$z = L, \; p = P_{atm}$ [from continous property of Pressure variation]

At interface pressure changes as a continous function.

$$\frac{\partial p}{\partial z} = -\rho g$$

$$p = -\rho g z + c_1 \quad \text{—— (1)}$$

Since, at $z = L$, $p = P_{atm}$

$$P_{atm} = -\rho g L + c_1$$

$$c_1 = P_{atm} + \rho g L$$

Substituting in the equ^n (1)

$$p = -\rho g z + P_{atm} + \rho g L$$

$$p = P_{atm} + \rho g (L - z) \Rightarrow \boxed{p = P_{atm} + \rho g h}$$