**ABE 307**

**Method of Shell Momentum Balance**

The method of shell momentum balance is applied for laminar, steady rectilinear flows in order to derive velocity profile. Velocity profile can then be utilized to get maximum velocity and the shear rate on the solid surfaces due to the flow of fluid.

**Flow Conditions Considered for Application of Shell Momentum Balance**

1. Steady Flow: pressure, density, velocity components do not change with time
   1. Note: does not mean they are constant. They may change with location.
2. Incompressible Liquid Flow: ρ is constant through time and space
3. Laminar flow: orderly flow with not a lot of mixing.
   1. Laminar flow is characterized by low Reynold’s Number.
4. Rectilinear Flow: flow in one direction or a straight line

Objective: to obtain an expression for spatial variation of velocity and pressure. This allows us to calculate shear forces by fluid on surfaces, important for design.

**Concepts Required for Solving Fluid Flow Problem Using Shell Momentum Balance**

1. Viscosity: Since velocity of fluid depends on μ (viscosity), it is an important consideration for deriving expressions for velocity profile
2. Momentum Transfer: which direction is the momentum being transferred? Consider all components (q for molecular, q for convective).
3. Momentum Balance Expression: momentum is being transferred as the fluid is flowing under the effect of various forces. Write the general momentum balance to develop expressions.
4. Boundary Conditions: to solve expression for your specific situation identify the boundaries and specific boundary conditions.

**General Momentum Balance Expression**

Rate of change of momentum = body forces + surface forces

* For static fluid, rate of change of momentum = 0
* For steady state, rate of change of momentum = 0
* For unsteady state, rate of change of momentum is nonzero.

Rate of momentum out - rate of momentum in = body forces

{Rate of momentum in} - {rate of momentum out} + body forces = 0

**Boundary Conditions for Applying Shell Momentum Balance**

For any general fluid flow condition, you can come across three kinds of boundary conditions

1. Solid-liquid: at solid-liquid interface, “no-slip condition”, relative velocity is zero.
2. Liquid-liquid: continuity assumption: 𝜏xy, 𝜏yx and p + 𝜏xx are same at interface. “No slip condition”: vy and vz are equal.
3. Liquid-gas: μgas << μliquid --> 𝜏xy and 𝜏xz are zero.

**Steps in Shell Momentum Balance Method**

1. What is aim and what kind of flow are you looking at?
   1. Rectilinear flow:
      1. Identify direction of flow
      2. Find the nonzero velocity component
      3. Write coordinate system
2. What do we know about the flow without knowing any details about the objective you are trying to accomplish? Is there a fundamental principle that is applicable?
   1. Momentum balance principle applies
3. Can you get to an equation that helps in relating the variable of interest to coordinates of flow? If yes, how?
   1. Assume a shell for applying your momentum balance.
4. Can you solve the equation to describe the flow situation at hand? If yes, what do you obtain at this stage?
   1. Integrate to get a momentum flux.
5. Are you able to get a velocity variable? If yes, what did you do to reach here? If no, what would you need?
   1. Identify the relationship between momentum flux component and viscosity.
6. Can you solve for velocity profile? What other information do you need?
   1. Get velocity differential equation and integrate using boundary conditions.
7. Can you calculate maximum velocity, mass flow rates, and force by fluid on solid surface.
   1. V = v(x, y, z)