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| **Flow Type** | **Solution** |
| Rectilinear Flow, Steady State, Laminar | Shell Momentum Balance |
| Rectilinear, One Directional Flow, Steady State, Laminar, ρ and μ constant | Navier Stokes’ Equation |
| Unsteady State Flow: v = v(t,x,y,z), ρ and μ constant | Navier Stokes’ Equation   1. Semi-Infinite Fluid: Combination of Variables 2. Bounded Fluid: Separation of Variables |
| Boundary Layer Flow: Fluid flow near solid surface | Von-Karmen Integral Equation |
| Potential Flow: Flow far from solid surface | Using velocity potential |

**Unsteady State: Laminar Flow near a Wall Suddenly Set in Motion (Semi-Infinite Body of Fluid)**

* Semi-infinite fluid - only bounded at lower end
* Plate set to motion at t = 0 with v0 velocity
* Fluid moves due to momentum transfer
* ρ and μ continuous
* No pressure or gravitational force in the direction of motion

**Step 1:** Start with Navier-Stokes Equation

∂vx/∂t = γ ∂2vx/∂t2

γ = μ/ρ

**Step 2:** Initial Conditions

t = 0, vx = 0

Boundary Conditions

t > 0, vx = v0 @ y = 0

t > 0, vx = 0 @ y --> ∞

**Step 3:** Create a Dimensionless Variable

Φ = vx/v0

**Step 4:** Put Original Equation in Dimensionless Variable

∂vx/∂t = v0γ ∂2vx/∂y2 ==> ∂Φ/∂t \* ∂vx/∂Φ

v0∂Φ/∂t = v0 γ ∂2Φ/∂y2

∂Φ/∂t = γ ∂2Φ/∂y2

Φ = Φ(y,t)

∂/∂y \* ∂vx/∂y

= ∂/∂y [∂vx/∂Φ \* ∂Φ/∂y]

= ∂/∂y [v0 \* ∂Φ/∂y]

= v0 \* ∂2Φ/∂y2

**Step 5:** Change Boundary Conditions to Dimensionless Variable

vx = v0 @ y = 0, t > 0 --> Φ = 1 @ y = 0, t > 0

vx = v0 @ y --> ∞, t > 0 --> Φ = 0 @ y --> ∞, t > 0

New variables: Φ, y, t, γ

η = y/sqrt(4γt)

We want to convert equation to ordinary differential equation, hence choose the combination of variables method.

**Step 6:** Convert partial differential equation to ordinary differential equation

∂Φ/∂t = γ ∂2Φ/∂y2

∂Φ/∂t = ∂Φ/∂η \* ∂η/∂t

∂Φ/∂t = -1/2 \* η/t \* ∂Φ/∂η

∂2Φ/∂y2 = ∂/∂y \* ∂Φ/∂y

∂2Φ/∂y2 = ∂/∂y \* ∂Φ/∂η \*∂η/∂y

∂2Φ/∂y2 = 1/4γt \* ∂2Φ/∂η2

Original Equation:

∂Φ/∂t = γ ∂2Φ/∂y2

Modified Equation:

-1/2 \* η/t \* ∂Φ/∂η = 1/4γt \* ∂2Φ/∂η2

-η \* ∂Φ/∂η = 1/2 \* ∂2Φ/∂η2

0 = 2η \* ∂Φ/∂η + ∂2Φ/∂η2

**Step 7:** Convert B.C. in terms of Φ, η

Φ = 1, at y = 0, η = 0

Φ = 0, at y => ∞, η => ∞

**Step 8:** Solve

d2Φ/dη2 + 2η dΦ/dη = 0

Ψ = dΦ/dη

dΦ/dη + 2ηψ = 0

dΦ/dη = -2ηψ

lnψ = -η2-C0

ψ = e-η2-C0

dΦ/dη = e-η2-C0

Φ = ∫0ηC1e-η2

At Φ = 1, η = 0

1 = ∫00C + C2

C2 = 1

At Φ = 0, η => ∞

0 = C1 ∫0∞e-ηbar2dη+ 1 (-η2 is now -ηbar2)

C1 = -1/ ∫0∞e-ηbar2dη

Φ = ∫0ηe-η2dη / ∫0∞e-ηbar2dη



Boundary Layer Thickness: distance from the plate till which we can expect the effect of moving plate by momentum transfer.

2.0 = δ/sqrt(4γt)

δ = 4sqrt(γt)