Vx (y, ±w/2) = 0 because no cross flow at ends and flows only within channel

[insert written notes from 10/23/17]

0 = ΔP/L + μ(∂2vx/∂y2 + ∂2vx/∂z2)

vx(y) for w >> h

vx = vx(y) + Φ(y,z)

Substitute velocity profile in original equation of motion.

0 = ΔP/L + μ(∂2vx/∂y2 + ∂2Φ/∂y2 + ∂2Φ/∂z2)

For w >> h, Φ = 0

0 = ΔP/L + μ(∂2vx/∂y2)

∂2Φ/∂y2 + ∂2Φ/∂z2 = 0

Write new boundary conditions for modified equations of motion

Y = ±h/2: vx(y) = 0, Φ = 0

Y = 0: ∂vx/∂y = 0 ⇒ dvx/dy = 0, ∂Φ/∂y = 0

Z = ±w/2: vx = 0, vx(y) = -Φ

Z = 0: ∂vx/∂y = 0, ∂Φ/∂z = 0

Solve for vx(y)

0 = ΔP/L + μ(∂2vx/∂y2)

-ΔP/L = μ(∂2vx/∂y2)

dvx/dy = -ΔPy/μL + C0

C0 = 0

Vx = -ΔPy2/2μL + C1

0 = -ΔPy2/2μL + C1

C1 = ΔP(h2/8μL - y2/2μL)

Vx = ΔP/2μL \* [h2/4 - y2]

Solve for Φ(y,z)

Φ(y,z) = Y(y)Z(z)

∂2Φ/∂y2 + ∂2Φ/∂z2 = 0

Z(z)∂2Y/∂y2 + Y(y)∂2Z/∂z2 = 0

1/Y(y) \* ∂2Y/∂y2 + 1/Z(z) \* ∂2Z/∂z2 = 0

1/Y(y) \* ∂2Y/∂y2 = - 1/Z(z) \* ∂2Z/∂z2 = λ2

1/Y(y) \* ∂2Y/∂y2 = λ2

1/Z(z) \* ∂2Z/∂z2 = -λ2

Write new boundary conditions for Φ

Y = 0, ∂Φ/∂y = 0 ⇒ ∂Y/∂y = 0

Y = ±h/2, Φ = 0 ⇒ Y = 0

Z = 0, ∂Φ/∂z = 0 ⇒ ∂Z/∂z = 0

Y(y) = A1sinλy + A2cosλy

Z(z) = B1sinhλz + B2coshλz

Y = 0, ∂Y/∂y = 0

∂Y/∂y|y=0 = A1sinλy + A2cosλy

0 = A1sin(0) + A2cos(0)

0 = A1sin(0)

A1 = 0

Y(y) = A2cosλy

Y = ±h/2, Y = 0

0 = A2cos(λ\*±h/2)

cos(λ\*±h/2) = 0

λh/2 = (2n + 1) \* π/2

Λn = (2n + 1) \* π/h

Z=0, ∂Z/∂z = 0

Z(z) = B1sinhλz + B2coshλz

∂Z/∂z = B1λcoshλz + B2λsinhλz

B1 = 0

Z(z) = B2coshλz