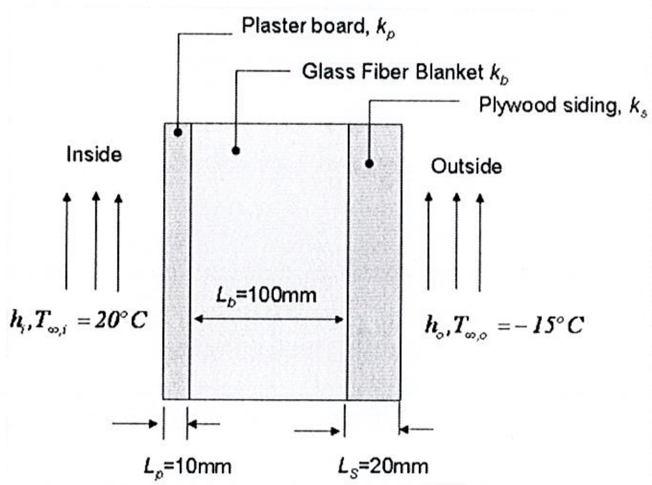
ABE 30800 - Heat and Mass Transfer

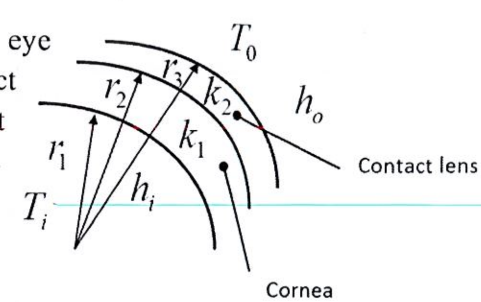
Steady State Conduction Examples

Example 1. A house has a composite wall of plasterboard, fiberglass insulation and plywood siding, as indicated in the figure below. On a cold winter day the convection heat transfer coefficients outside and inside the house are h_o=150W/m^2.K and h_i=20W/m^2.K. The total wall surface area is 400m^2. Values of thermal conductivity for the plasterboard, the glass fiber blanket and the plywood siding are 0.1 W/m.K, 0.04W/m.K and 0.15 W/m.K, respectively.

(a) Determine the total heat loss through the wall.
(b) For design purposes, you will like to know the temperature of the material at the boundary between the glass fiber blanket and the plywood siding but it is becoming difficult to inset a thermometer so **you would need to estimate that temperature**
(c) It is recommended to reduce the heat loss through the wall by 30% and since the glass fiber blanket is the best insulator, you will need to estimate the new thickness of this material of glass fiber blanket to achieve that reduction in the heat flow.



Example 2. The heat transferred from the anterior chamber of the eye through the cornea varies considerably depending on whether a contact lens is worn. Assume the eye as a spherical surface and assume the heat transfer to be at steady state and that the convection coefficient outside ho is not changed with and without the contact lens in place. The cornea and the lens cover 1/3 of the spherical surface area. A schematic is given below:



jjjjkjkValues of the parameters in the figure above are:

r_1 =10.2mm T_o = 21°C

r_2 = 12.7mm k_1 = 0.35 W/m.K

r_3 = 16.5mm $\quad$ k_2 = 0.80 W/m.K

T_i = 37°C $\quad$ h_i = 12 W/m^2.K $\quad$ h_o = 6 W/m^2.K

Determine the heat loss from the anterior chamber with and without the contact lens in place. Assume steady state, eye is 1/3 sphere.

Example 3. A stainless-steel tube is used to transport a chilled pharmaceutical fluid. The tube has an inner diameter of 36mm and a wall thickness of 2mm. The pharmaceutical fluid and ambient air are at temperatures of 6°C and 23°C respectively. The corresponding inner and outer convection coefficients are 400 W/m^2.K and 6W/m^2.K respectively.

(a) What is the heat gain per unit tube length? Assume that the thermal conductivity of the tube is 15 W/m.K

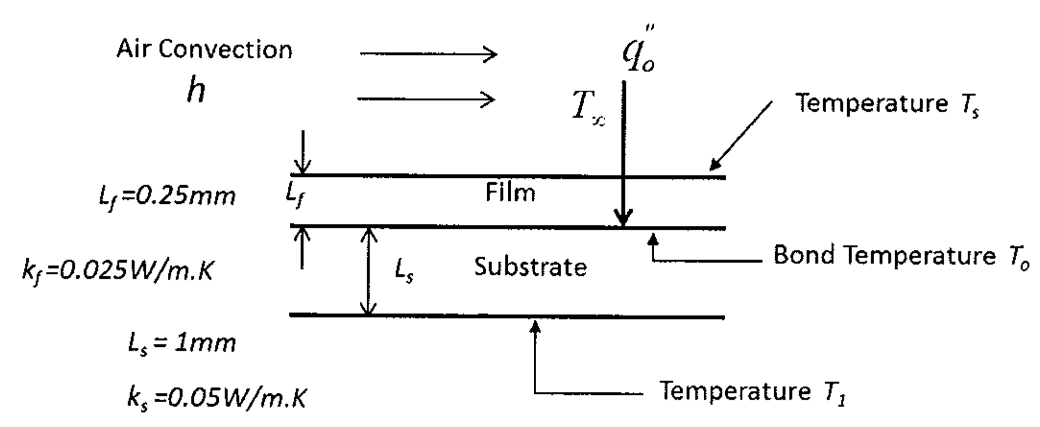
(b) What is the heat gain per unit length if a 10mm-thick layer of calcium silicate insulation with a thermal conductivity of 0.050W/m.K is applied to the tube?

Example 4. In a manufacturing process, a transparent film is being bonded to a substrate as shown in the sketch below. To cure the bond at a temperature T_o, a radiant source is used to provide a heat flux $q_o''\left(W/m^2\right)$, all of which is absorbed at the bonded surface. The back of the substrate is maintained at T_l while the free surface of the film is exposed to air at T_∞ and a convection heat transfer coefficient h.

(a) Show the thermal circuit representing the steady state heat transfer

(b) Assume the following conditions T_∞ = 20°C, h=50W/m^2.K and T_l=30°C. Calculate the heat flow $q_o''\left(W/m^2\right)$ that is required to maintain the bonded surface at T_o = 60°C.

(c) If the film is not transparent and all of the radiant heat flux is absorbed at it upper surface, determine the heat flux required to achieve bonding.



Example 5. Compare the heat loss (in Watts) from two different windows in a room. One window gas a single sheet of glass 10mm thick whereas the other has two sheets of glass, 5mm each but separated by an air gap of 5mm. The second window is called a thermopane. The windows sizes are 1mx1m. The air temperature in the room is 72°F and the outside temperature is 18°F. The thermal conductivity of the glass is 0.9 W/m.K and the average thermal conductivity at of air is 0.03 W/m.K. The convection coefficient inside the room is h_i=10 $W/m^2.K$ and outside due to a strong wind is h_o = 200 W/m.K.

(a) Compare the heat losses between these two windows designs, if the air in the gap is stagnant, i.e. heat though the air gap is only transmitted by conduction
(b) Consider that in the air gap you also have some radiation and that the radiation coefficient is $h_r = \sigma(T_1 + T_2)(T_1^2 + T_2^2)$, using the main equation used for radiation demonstrate the validity of the expression to estimate h_r.
(c) Assuming that $h_r \approx 4\sigma T_{avg}^3$, estimate and compare how the resistance to heat transfer changes when radiation is also considered in addition to conduction alone. Therefore, you have to estimate the differences in the resistance to the heat transfer considering and without considering radiation in the air gap, and what is the effect in the heat flow?

Example 6 (include convection too). The wind chill, which is experienced on a cold windy day, is related to increased heat transfer from exposed human skin to the surrounding atmosphere. Consider a layer of fatty tissue that is 3mm thick and whose interior surface is maintained at 36°C. On a calm day the convection heat transfer coefficient at the outer surface is 25 $W/m^2.K$, but with a 20 miles/h wind it reaches 65 $W/m^2.K$. In both cases the ambient temperature is -15°C.

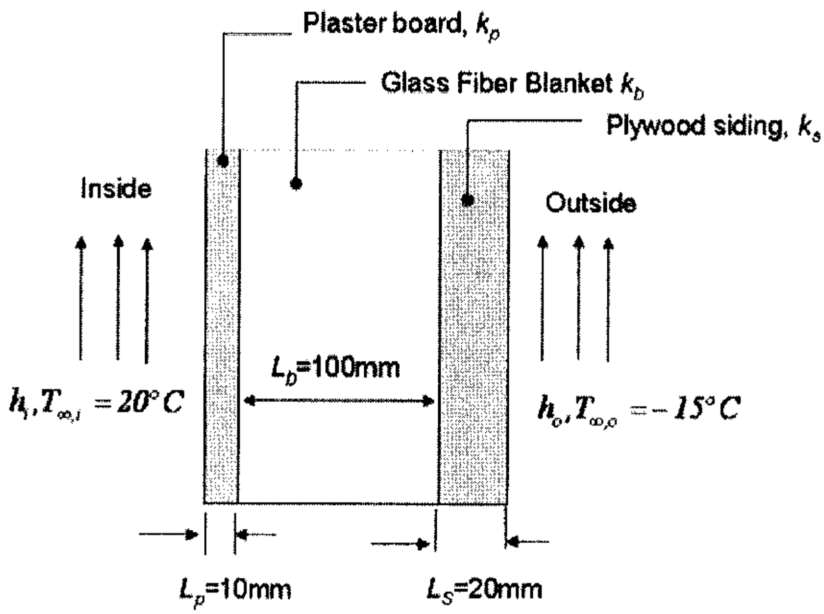
(a) What is the ratio of the heat loss per unit area from the skin for the calm day to that for the windy day?

(b) What will be the skin outer surface temperature for the calm day?

(c) And what for the windy day?

Examples - Steady State heat Transfer Spring 2018

Example 1



<u>Data</u>

$$h_i := 20 \cdot \frac{W}{m^2 \cdot K} \qquad h_o := 150 \cdot \frac{W}{m^2 \cdot K} \qquad A_{area} := 400 \cdot m^2 \qquad T_{inf\_o} := -15 \cdot K$$

$$L_p := 10 \cdot mm \qquad L_b := 100 \cdot mm \qquad L_s := 20 \cdot mm \qquad T_{inf\_i} := 20 \cdot K$$

$$k_p := 0.1 \cdot \frac{W}{m \cdot K} \qquad k_b := 0.04 \cdot \frac{W}{m \cdot K} \qquad k_s := 0.15 \cdot \frac{W}{m \cdot K}$$

(a)

$$\Sigma Res := \frac{1}{h_i \cdot A_{area}} + \frac{L_p}{k_p \cdot A_{area}} + \frac{L_b}{k_b \cdot A_{area}} + \frac{L_s}{k_s \cdot A_{area}} + \frac{1}{h_o \cdot A_{area}}$$

$$\Sigma Res = 0.007 \cdot \frac{K}{W}$$

$$\boxed{q := \frac{T_{inf\_i} - T_{inf\_o}}{\Sigma Res} = 5017.9\,W}$$

(b) Estimate the temperature at the boundary between the glass fiber blanket and the plywood siding. It will be called T_{bound} and the resistances bewtween the interior of the house will include the convection inside, conduction through the plaster board a the glass fiber blanket

$$\Sigma Res\_new := \frac{1}{h_i \cdot A_{area}} + \frac{L_p}{k_p \cdot A_{area}} + \frac{L_b}{k_b \cdot A_{area}}$$

$$\Sigma Res\_new = 0.007 \cdot \frac{K}{W}$$

The heat flow remains the same but the resistance changes, from the equation considering the driving temperature difference T_{inf_i}-T_{bound} we can estimate T_{bound}

$q = (T_{inf\_i} - T_{bound}) / \Sigma Res\_new$ -- > $T_{bound} := T_{inf\_i} - q \cdot \Sigma Res\_new$

$\boxed{T_{bound} = -13.2\,K}$ or -13.2C

(c) All the materials thickness will be the same except the fiber glass blanket to reduce the heat flow by 30%

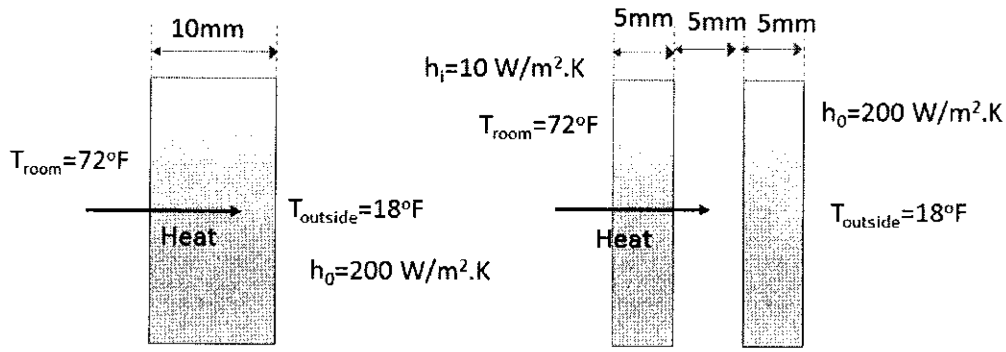
$$q_{new} := 0.30 \cdot q$$

$$R_{ci} := \frac{1}{h_i \cdot A_{area}} \qquad R_{pb} := \frac{L_p}{k_p \cdot A_{area}} \qquad R_{ps} := \frac{L_s}{k_s \cdot A_{area}} \qquad R_{co} := \frac{1}{h_o \cdot A_{area}}$$

$$R_{gb} := \frac{T_{inf\_i} - T_{inf\_o}}{q_{new}} - \left(R_{ci} + R_{pb} + R_{ps} + R_{co}\right) \qquad R_{gb} = 0.023 \cdot \frac{K}{W}$$

$$L_{bnew} := R_{gb} \cdot k_b \cdot A_{area} \qquad \boxed{L_{bnew} = 360.4 \cdot mm}$$

Example 5



Data

- Window 1

$$k_{glass} := 0.9 \cdot \frac{W}{m \cdot K} \qquad x_1 := 10 \cdot mm \qquad h_{in} := 10 \cdot \frac{W}{m^2 \cdot K} \qquad h_{out} := 200 \cdot \frac{W}{m^2 \cdot K}$$

(a) **Units for the data temperatures are in Farenheit**

$$T_{room} := 72 \qquad T_{out} := 18$$

Conversion of Temperatures from Farenheit to Kelvins

$$T_{room\_K} := \frac{T_{room} - 32}{1.8} \cdot K \qquad T_{out\_K} := \frac{T_{out} - 32}{18} \cdot K$$

- Windows 2

$$k_{air} := 0.03 \cdot \frac{W}{m \cdot K} \qquad x_2 := 5 \cdot mm$$

- Heat Transfer for Window 1 - Calculated as Flux, that is heat flow per unit area

$$R_{cv\_room} := \frac{1}{h_{in}} \qquad R_{cd\_glass} := \frac{x_1}{k_{glass}} \qquad R_{cv\_out} := \frac{1}{h_{out}}$$

$$q_1 := \frac{T_{room\_K} - T_{out\_K}}{R_{cv\_room} + R_{cd\_glass} + R_{cv\_out}} \qquad \boxed{q_1 = 198.09 \cdot \frac{W}{m^2}}$$

- Heat Transfer for Window 2 - Calculated as Flux, that is heat flow per unit area without considering radiation and only conduction through the air gap

$$R_{cd\_glass1} := \frac{x_1}{k_{glass}} \qquad R_{cd\_air\_gap} := \frac{x_1}{k_{air}} \qquad R_{cd\_glass2} := \frac{x_1}{k_{glass}}$$

$$q_2 := \frac{T_{room\_K} - T_{out\_K}}{R_{cv\_room} + R_{cd\_glass1} + R_{cd\_air\_gap} + R_{cd\_glass2} + R_{cv\_out}}$$

$$\boxed{q_2 = 49.94 \cdot \frac{W}{m^2}}$$

(b)

The Equation for Radiation is:

$$q = \sigma A\left(T_1^4 - T_2^4\right)$$

where is the Stefan-Boltzmann constant and A the area.; temperatures T1 and T2 are absolute temperatures (in degrees Kelvin or Rankine). By using some algebra the equation can be modified as follows:

$$\frac{q}{A} = q'' = \sigma\left(T_1^4 - T_2^4\right) = \sigma\left(T_1^2 - T_2^2\right)\left(T_1^2 + T_2^2\right) = \sigma(T_1 + T_2)\left(T_1^2 + T_2^2\right)(T_1 - T_2) = h_r(T_1 - T_2)$$

and:

$$h_r = \sigma(T_1 + T_2)\left(T_1^2 + T_2^2\right)$$

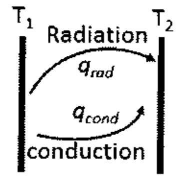
When we consider conduction and radiation through the air gap we have the following:

(c)

$$\frac{q_{rad}}{A} = q''_{rad} = \frac{T_1 - T_2}{R_{rad}} = \frac{T_1 - T_2}{\frac{1}{h_r}}$$

$$\frac{q_{cond}}{A} = q''_{rad} = \frac{T_1 - T_2}{R_{cond}} = \frac{T_1 - T_2}{\frac{x_2}{k_{air}}}$$

When we consider conduction and radiation through the air gap we have the following:



If we use the concept of resistance

$$\frac{q_{rad}}{A} = q"_{rad} = \frac{T_1 - T_2}{R_{rad}} = \frac{T_1 - T_2}{\frac{1}{h_r}}$$

And

$$\frac{q_{cond}}{A} = q"_{rad} = \frac{T_1 - T_2}{R_{cond}} = \frac{T_1 - T_2}{\frac{x_2}{k_{air}}}$$

$$q" = q"_{rad} + q"_{cond} = (T_1 - T_2)\left(\frac{1}{\frac{1}{h_r}} + \frac{1}{\frac{x_2}{k_{air}}}\right) = (T_1 - T_2)\left(h_r + \frac{k_{air}}{x_2}\right) = \frac{(T_1 - T_2)}{\frac{1}{\left(h_r + \frac{kair}{x_2}\right)}} = \frac{(T_1 - T_2)}{R_{eq}}$$

$$R_{eq} = \frac{1}{h_r + \frac{k_{air}}{x_2}}$$

So now rather that having a resistance due to conduction in the air gap will be conduction plus radiation

First let's calculate h_r $\qquad \sigma := 5.678 \cdot 10^{-8} \cdot \frac{W}{m^2 \cdot K^4}$

$T_{1\_K} := T_{room\_K} + 273 \cdot K \qquad T_{2\_K} := T_{out\_K} + 273 \cdot K$

$T_{avg} := \frac{T_{1\_K} + T_{2\_K}}{2} \qquad h_r := 4\sigma \cdot T_{avg}^{\;3} \qquad h_r = 5.187 \frac{1}{K} \cdot \frac{W}{m^2}$

$T_{avg} = 283.722\,K$

$$R_{eq} := \frac{1}{h_r + \frac{k_{air}}{x_2}}$$

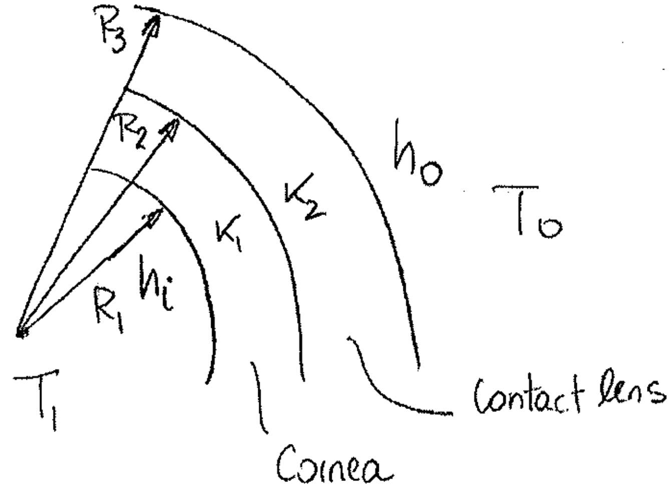
So now the new heat flux including radition is:

$$q_3 := \frac{T_{room\_K} - T_{out\_K}}{R_{cv\_room} + R_{cd\_glass1} + R_{eq} + R_{cd\_glass2} + R_{cv\_out}}$$

$$\boxed{q_3 = 106.182 \cdot \frac{W}{m^2}}$$

The heat flux is larger than the once calculated considering conduction alone

EXAMPLE 2



$R_1 = 10.2\,mm$ $\quad T_o = 21°C$

$R_2 = 12.7\,mm$ $\quad K_1 = 0.35\,W/m.K$

$R_3 = 16.5\,mm$ $\quad K_2 = 0.80\,W/m.K$

$T_i = 37°C$ $\quad h_i = 12\,\frac{W}{m^2K}$

$h_o = 6\,\frac{W}{m^2K}$

Assume eye = 1/3 sphere.

(a) Heat loss with contact lens

$$q_{wc} = \frac{T_i - T_o}{\Sigma Res} \quad (1)$$

$$\Sigma Res = R_{conv,in} + R_{cond,1} + R_{cond,2} + R_{conv,out}$$

$$R_{conv,i} = \frac{1}{4\pi R_1^2 h_i} = \frac{1}{4\pi \times (10.2\times10^{-3})^2 \times 12} = 63.74\ \frac{K}{W}$$

$$R_{cond,1} = \frac{R_2 - R_1}{4\pi K_1 R_1 R_2} = \frac{(12.7-10.2)\times10^{-3}}{4\times\pi\times0.35\times12.7\times10.2\times10^{-6}} = 4.39\ \frac{K}{W}$$

$$R_{cond,2} = \frac{R_3 - R_2}{4\pi k_2 R_2 R_3} = \frac{(16.5 - 12.7)\times 10^{-3}}{4\pi \times 0.80 \times 16.5 \times 12.7 \times 10^{-6}} = 1.804\ \frac{K}{W} \quad (2)$$

$$R_{conv,out} = \frac{1}{4\pi R_3^2 h_o} = \frac{1}{4\pi \times (16.5\times 10^{-3})^2 \times 6} = 48.72\ \frac{K}{W}$$

Substituting into Eq.(1)

$$q_{wc} = \frac{37-21 \quad K}{63.74 + 4.39 + 1.804 + 48.72 \quad \frac{K}{W}} = 0.134\ W$$

$$q_{wc,eye} = \frac{0.134 W}{3} \cong 50 mWatts$$

$$\boxed{q_{wc,eye} \cong 50 mW}$$

WITHOUT CONTACT LESS

$$\sum Res = R_{conv,i} + R_{cond,1} + R_{conv,out}$$

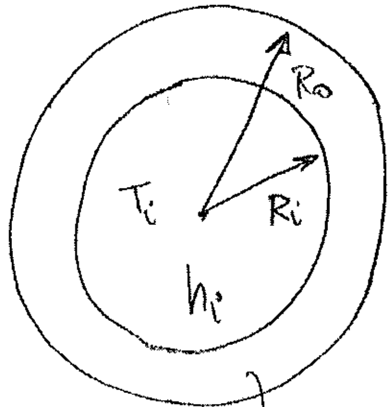
$$R_{conv,out} = \frac{1}{4\pi R_2^2 h_o} = \frac{1}{4\pi \times (12.7\times 10^{-3})^2 \times 6} = 82.23\ \frac{K}{W}$$

$$q_{wout,c} = \frac{37-21\ K}{63.74 + 4.39 + 82.23\ \frac{K}{W}} = 0.106\ W$$

$$q_{out,c\,eye} = \frac{0.106 W}{3} = 0.0355 W = 35.5 mW$$

$$\boxed{q_{out,c\,eye} = 35.5 mW}$$

EXAMPLE 3



$R_i = \frac{36\,mm}{2} = 18\,mm$

$R_o = 18 + 2 = 20\,mm$

$h_o = 6 \frac{W}{m^2 K}$

$h_i = 400 \frac{W}{m^2 K}$

$K_s = 15 \frac{W}{m.K}$ $\quad T_\infty = 23°C$

(a) Heat Gain per unit of tube length

$$q' = \frac{T_\infty - T_i}{\Sigma Res}$$

$$\Sigma Res = R_{conv,out} + R_{cond,pipe} + R_{conv,in}$$

$$R_{conv,out} = \frac{1}{2\pi R_o h_o} = 1.326 \frac{m K}{W}$$

$$R_{conv,in} = \frac{1}{2\pi R_i h_i} = 0.022 \frac{m K}{W}$$

$$R_{cond,pipe} = \frac{\ln R_o/R_i}{2\pi K_s} = 0.00112$$

big resistance to heat transfer is the convection out

$$q' = \frac{(23-6)\,K}{1.326 \frac{m.K}{W} + 0.022 \frac{m.K}{W} + 0.00112 \frac{m.K}{W}} = 12.6 \frac{W}{m}$$

$$\boxed{q' = 12.6 \frac{W}{m}}$$

(b) Heat gain per unit length if a 10 mm - thick layer of calcium silicate insulation ($K_{ins} = 0.05 \frac{W}{m.K}$) is applied to the tube (4)

$R_{ext} = R_o + 10\,mm = 30\,mm$

So

$$R_{conv,out} = \frac{1}{2\pi R_{ext} h_o} = \frac{1}{2\pi \times 30\times10^{-3}\,m \times 6\,\frac{W}{m^2K}} = 0.884\,\frac{mK}{W}$$

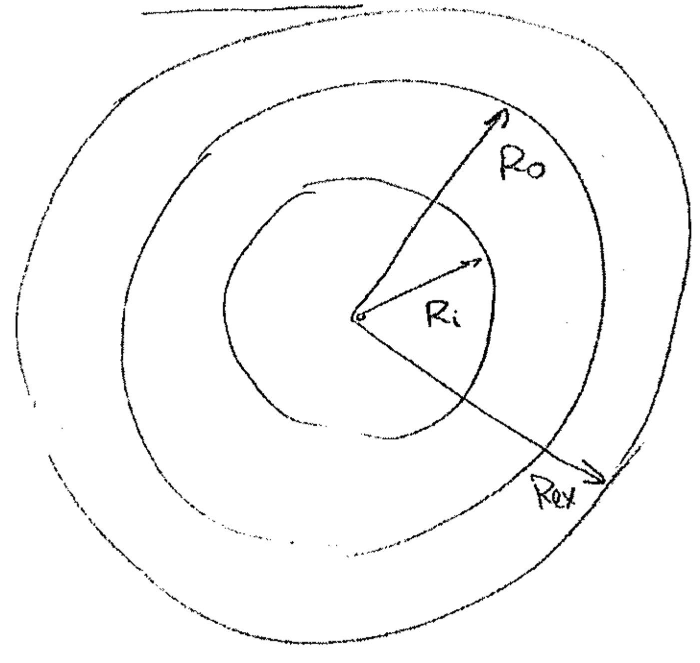
and

$$R_{cond,ins} = \frac{\ln(R_{ext}/R_o)}{2\pi K_{ins}} = \frac{\ln(30/20)}{2\pi\, 0.05\, W/mK} = 1.29\,\frac{mK}{W}$$

$$\sum Res = 0.022 + 0.00112 + 1.29 + 0.884 = 2.20\,\frac{mK}{W}$$

$$q' = \frac{(23-6)\,\cancel{K}}{2.20\,\frac{m\cancel{K}}{W}} = 7.72\,\frac{W}{m} \qquad \boxed{q' = 7.72\,\frac{W}{m}}$$

with Isolation



$R_i = 18\,mm$
$R_o = 20\,mm$
$R_{ext} = 30\,mm$

ABE 30800

STEADY STATE CONDUCTION EXAMPLES

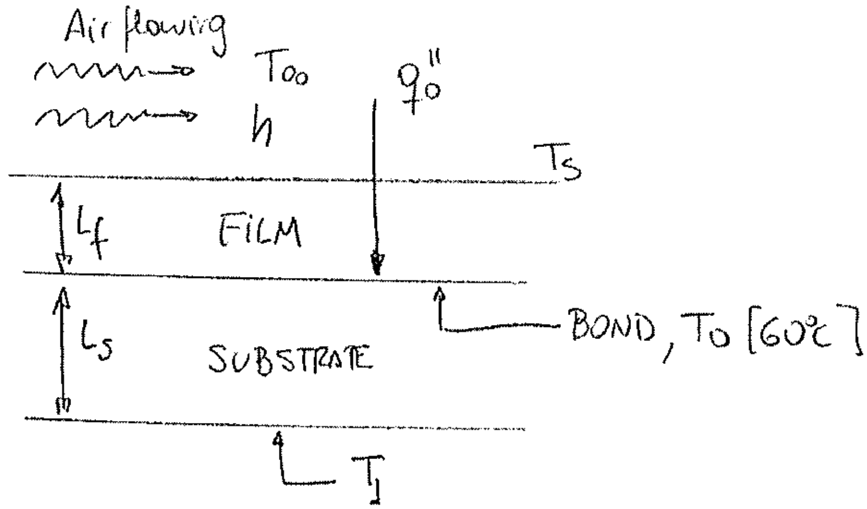
Example 4

$L_f = 0.25\,mm$

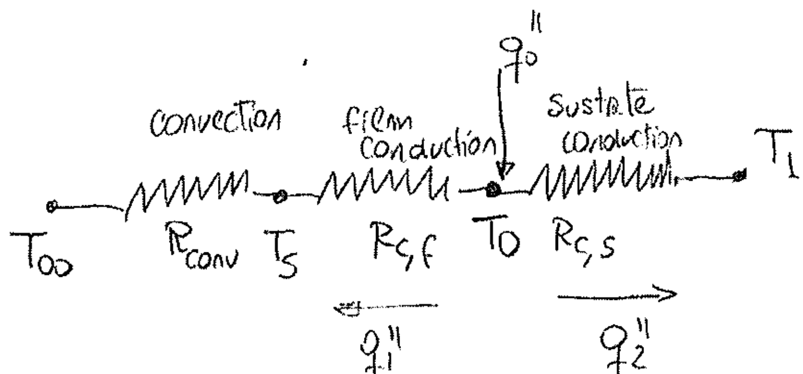
$k_f = 0.025\ \frac{W}{m.K}$

$L_s = 1.0\,mm$

$k_s = 0.05\ W/m.K$



(a) Thermal "circuit" representing the steady-state heat transfer situation for the case the film is transparent



(b) $T_o = 60°C$, $T_\infty = 20°C$, $h = 50\ \frac{W}{m^2K}$ $T_1 = 30°C$

See diagram above

$$\underline{q_o'' = q_1'' + q_2''}$$

Heat Flux q_1''

$$q_1'' = \frac{T_o - T_\infty}{\Sigma Res_1}$$

$$\Sigma Ress = R_{c,f} + R_{conv}$$

$$R_{c,f} = \frac{L_f}{k_f} = \frac{0.25 \times 10^{-3}\, m}{0.025\, \frac{W}{m.K}} = 0.01\, \frac{m^2 K}{W}$$

$$R_{conv} = \frac{1}{h} = \frac{1}{50\, \frac{W}{m^2 K}} = 0.02\, \frac{m^2 K}{W}$$

so

$$q_1'' = \frac{60 - 20\, \cancel{K}}{(0.01 + 0.02)\, \frac{m^2 \cancel{K}}{W}} = 1{,}333\, \frac{W}{m^2} = 1.33\, \frac{kW}{m^2}$$

<u>Heat Flux</u> q_2''

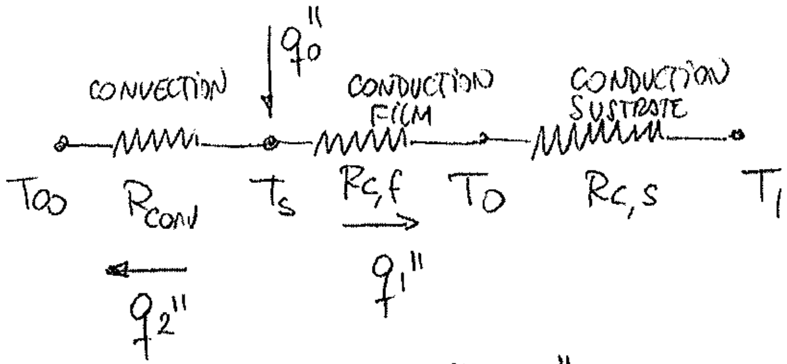
$$q_2'' = \frac{T_0 - T_1}{R_{c,s}}$$

$$R_{c,s} = \frac{L_s}{k_s} = \frac{1 \times 10^{-3}\, m}{0.05\, \frac{W}{m.K}} = 0.02\, \frac{m^2 K}{W}$$

$$q_2'' = \frac{(60 - 30)\, \cancel{K}}{0.02\, \frac{m^2 \cancel{K}}{W}} = 1{,}500\, \frac{W}{m^2} = 1.5\, \frac{kW}{m^2}$$

$$\boxed{q_0'' = 1.33 + 1.5 = 1.83\, \frac{kW}{m^2}}$$

(C) If the film is not transparent, the radiation q_0'' only heat up the surface of the system. the "thermal circuit" changes to : (7)



still $\quad q_0'' = q_1'' + q_2''$

but Now resistances have changed

– Heat flux q_1''

$$q_1'' = \frac{T_s - T_1}{R_{c,f} + R_{c,s}} \qquad (1)$$

but also q_1'' can be calculated as [T_s is NOT KNOWN]

$$q_1'' = \frac{T_0 - T_1}{R_{c,s}} = \frac{60 - 30\,K}{0.02\,\frac{m^2 K}{W}} = 1500\,\frac{W}{m^2}$$

From Equation (1)

$$T_s = T_1 + (R_{cf} + R_{c,s})\,q_1'' = 30 + (0.01 + 0.02)\,1500$$

$$\underline{T_s = 75°C}$$

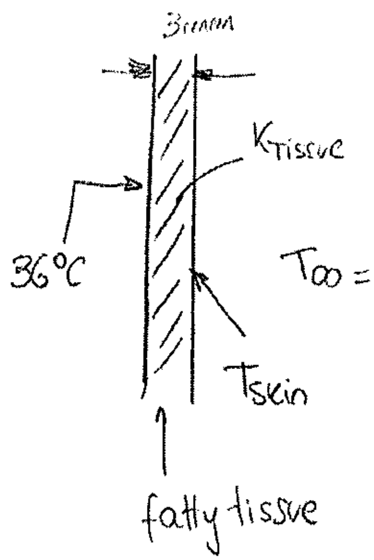
- Heat Flux q_2''

$$q_2'' = \frac{T_s - T_o}{R_{conv}} = \frac{75-20\,K}{0.02\,\frac{m^2 K}{W}} = 2{,}750\,\frac{W}{m^2}$$

$$q_o'' = 1{,}500 + 2{,}750\,\frac{W}{m^2} = 4{,}250\,\frac{W}{m^2} = 4.25\,\frac{kW}{m^2}$$

$$\boxed{q_o'' = 4.25\,\frac{kW}{m^2}}$$

EXAMPLE 6



$h_c = 25\,\frac{W}{m^2K}$ [calm day]

$h_w = 65\,\frac{W}{m^2K}$ [windy day]

$T_\infty = -15°C$ $\quad K_{tissue} = 0.2\,\frac{W}{m.K}$

(a) Ratio of heat loss per unit area between the calm and windy day

$$q_c'' = \frac{36-(-15°)}{R_{tissue} + R_{conv}}$$

$$R_{tissue} = \frac{L_{tissue}}{K_{tissue}} = \frac{3\times10^{-3}\,m}{0.2\,\frac{W}{m.K}} = 0.015\,\frac{m^2K}{W}$$

$$R_{conv,c} = \frac{1}{h_c} = \frac{1}{25} = 0.040\,\frac{m^2K}{W}$$

(5)

$$q_c'' = \frac{36-(-15)\,K}{0.015+0.040\ \frac{K\,m^2}{W}} = 927.3\ \frac{W}{m^2}$$

For the windy day

$$R_{conv,w} = \frac{1}{h_w} = \frac{1}{65\ \frac{W}{m^2K}} = 0.0154\ \frac{m^2K}{W}$$

$$q_w'' = \frac{36-(-15)\ K}{0.015+0.0154} = 1,677.6\ \frac{W}{m^2}$$

$$\frac{q_c''}{q_w''} = \frac{927.3}{1,677.6} = 0.553$$

$$\boxed{\frac{q_c''}{q_w''} = 0.553}$$

(b) skin outer surface temperatures

calm day $q_c'' = \frac{T_{skin} - T_\infty}{R_{conv,c}} = 927.3$

$$T_{skin} = T_\infty + 927.3 \times R_{conv,c} = -15 + 927.3 \times 0.04$$

$$\boxed{T_{skin} = 22.1°C}$$

windy day $q_w'' = \frac{T_{skin} - T_\infty}{R_{conv,w}} = 1,677.6$

$$T_{skin} = -15 + 1677.6 \times 0.0154 = 10.8°C \qquad \boxed{T_{skin} = 10.8°C}$$