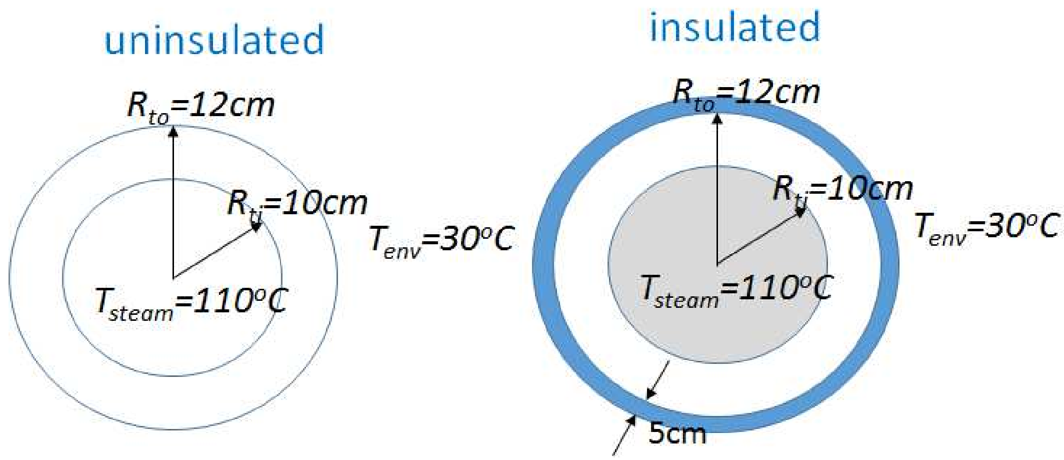
Homework 2 - Solutions

Question 1



(a)

$R_{ti} := 10 \cdot cm$ $\quad$ $R_{to} := 12 \cdot cm$ $\quad$ $e_{ins} := 5 \cdot cm$

$k_{tube} := 400 \frac{W}{m \cdot K}$

h_{steam} is very large so inner temperature is same than the temperature of the steam

$T_{steam} := 110 \cdot K$ $\quad$ $T_{env} := 30 \cdot K$ $\quad$ $k_{ins} := 0.20 \cdot \frac{W}{m \cdot K}$ $\quad$ $h_{out} := 15 \cdot \frac{W}{m^2 \cdot K}$

Uninsulated Tube

$$R_{unins} := \frac{\ln\left(\frac{R_{to}}{R_{ti}}\right)}{2\pi \cdot k_{tube}} + \frac{1}{h_{out} \cdot 2 \cdot \pi \cdot R_{to}} \qquad R_{unins} = 0.088 \cdot \frac{m \cdot K}{W}$$

$$q_{unins\_1p} := \frac{T_{steam} - T_{env}}{R_{unins}} \qquad \boxed{q_{unins\_1p} = 904.037 \cdot \frac{W}{m}}$$

Insulated Tube

$$R_{ext} := R_{to} + e_{ins} \qquad R_{ins} := \frac{\ln\left(\frac{R_{to}}{R_{ti}}\right)}{2\pi \cdot k_{tube}} + \frac{\ln\left(\frac{R_{ext}}{R_{to}}\right)}{2\pi \cdot k_{ins}} + \frac{1}{2\pi\, h_{out} \cdot R_{ext}} \qquad R_{ins} = 0.34 \cdot \frac{m \cdot K}{W}$$

$$q_{ins\_1p} := \frac{T_{steam} - T_{env}}{R_{ins}} \qquad \boxed{q_{ins\_1p} = 235.53 \cdot \frac{W}{m}}$$

(b) Overall heat Transfer Coefficients

Uninsulated Tube

$$U_{i\_unin} := \frac{1}{\frac{R_{ti}}{k_{tube}} \ln\left(\frac{R_{to}}{R_{ti}}\right) + \frac{R_{ti}}{R_{to}} \cdot \frac{1}{h_{out}}}$$

$$\boxed{U_{i\_unin} = 17.985 \cdot \frac{W}{m^2 \cdot K}}$$

$$U_{o\_unin} := \frac{1}{R_{to} \cdot \frac{\ln\left(\frac{R_{to}}{R_{ti}}\right)}{k_{tube}} + \frac{1}{h_{out}}}$$

$$\boxed{U_{o\_unin} = 14.988 \cdot \frac{W}{m^2 \cdot K}}$$

However, the calculation of the heat flow is the same and independent on the area that is considered, which is demostrated next, using values of the areas per unit of length of the inner and outer surface A_i and A_o, in m^2/m length

$$A_i := 2 \cdot \pi \cdot R_{ti} \qquad q_{i\_unins} := U_{i\_unin} \cdot A_i \cdot \left(T_{steam} - T_{env}\right)$$

$$\boxed{q_{i\_unins} = 904.037 \cdot \frac{W}{m}}$$

$$A_{o\_unins} := 2 \cdot \pi \cdot R_{to} \qquad q_{o\_unins} := U_{o\_unin} \cdot A_{o\_unins} \cdot \left(T_{steam} - T_{env}\right)$$

$$\boxed{q_{o\_unins} = 904.037 \cdot \frac{W}{m}}$$

Insulated Tube

$$U_{i\_ins} := \frac{1}{\frac{R_{ti}}{k_{tube}} \ln\left(\frac{R_{to}}{R_{ti}}\right) + \frac{R_{ti}}{k_{ins}} \cdot \ln\left(\frac{R_{ext}}{R_{to}}\right) + \frac{R_{ti}}{R_{ext}} \cdot \frac{1}{h_{out}}}$$

$$\boxed{U_{i\_ins} = 4.686 \cdot \frac{W}{m^2 \cdot K}}$$

$$q_{i\_ins} := U_{i\_ins} \cdot A_i \cdot \left(T_{steam} - T_{env}\right)$$

$$\boxed{q_{i\_ins} = 235.53 \cdot \frac{W}{m}}$$

$$U_{o\_in} := \frac{1}{R_{ext} \cdot \frac{\ln\left(\frac{R_{to}}{R_{ti}}\right)}{k_{tube}} + R_{ext} \cdot \frac{\ln\left(\frac{R_{ext}}{R_{to}}\right)}{k_{ins}} + \frac{1}{h_{out}}}$$

$$\boxed{U_{o\_in} = 2.756 \cdot \frac{W}{m^2 \cdot K}}$$

$$A_{o\_ins} := 2\pi \cdot R_{ext} \qquad q_{o\_ins} := U_{o\_in} \cdot A_{o\_ins} \cdot \left(T_{steam} - T_{env}\right)$$

$$\boxed{q_{o\_ins} = 235.53 \cdot \frac{W}{m}}$$

Question 2

(a) Included in a a different file

(b)

$d_i := 2\cdot cm$ $\qquad r_i := \frac{d_i}{2}$ $\qquad k_{insul} := 0.17\cdot\frac{W}{m\cdot K}$ $\qquad h := 2\cdot\frac{W}{m^2\cdot K}$

$r_c := \frac{k_{insul}}{h}$ $\qquad r_c = 8.5\cdot cm$ $\qquad e_{crit} := r_c - r_i$ $\qquad e_{crit} = 7.5\cdot cm$

$T_i := 100\cdot K$ $\qquad T_{inf} := 10\cdot K$

$$R_{cond} := \frac{\ln\left(\frac{r_c}{r_i}\right)}{2\cdot\pi\cdot k_{insul}} \qquad R_{cond} = 2.004\cdot\frac{m\cdot K}{W}$$

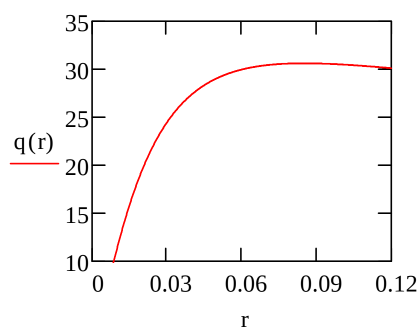
$$R_{conv} := \frac{1}{2\cdot\pi\cdot h\cdot r_c} \qquad R_{conv} = 0.936\cdot\frac{m\cdot K}{W}$$

$$q_{rc} := \frac{T_i - T_{inf}}{R_{cond} + R_{conv}} \qquad \boxed{q_{rc} = 30.615\cdot\frac{W}{m}}$$

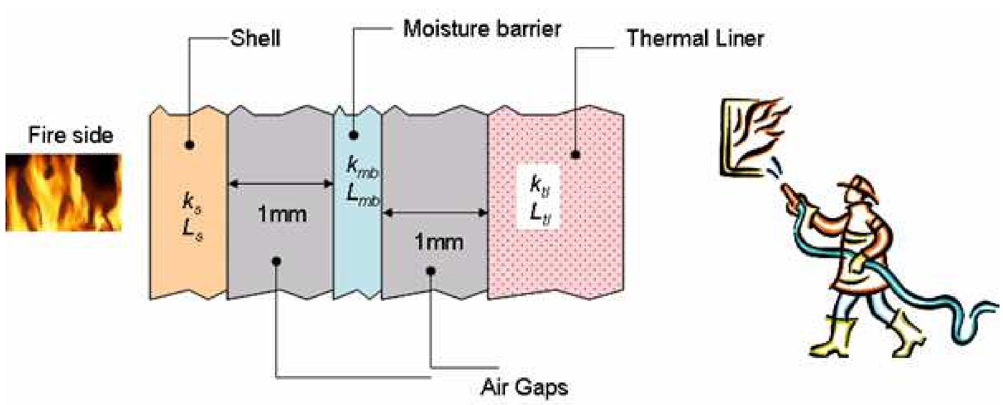
Plot

$$R_{cd}(r) := \frac{\ln\left(\frac{r}{r_i}\right)}{2\cdot\pi\cdot k_{insul}} \qquad R_{cv}(r) := \frac{1}{2\cdot\pi\cdot h\cdot r}$$

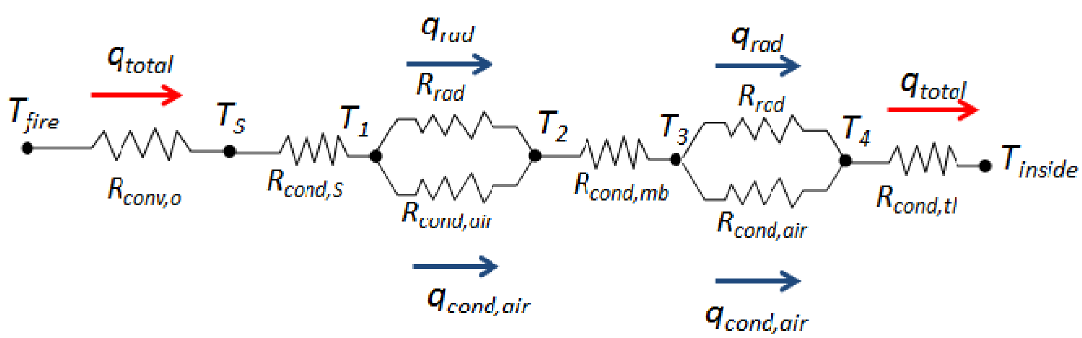
$$q(r) := \frac{T_i - T_{inf}}{R_{cd}(r) + R_{cv}(r)}$$



Question 3



(a)



(b) All the resistance calculations are straight forward except in the air gaps. It was demonstrated that for resistances in parallel, for example between the temperatures T_3 and T_4 the "equivalent" thermal resistance can be calculated as:

$$R_{equivalent} = \frac{1}{\dfrac{1}{R_{rad}} + \dfrac{1}{R_{still\ air}}}$$

Data

Shell Layer

$L_{shell} := 1\text{mm}$

$k_{shell} := 0.06 \cdot \frac{\text{W}}{\text{m} \cdot \text{K}}$

Moisture Barrier

$L_{mb} := 0.7 \cdot \text{mm}$

$k_{mb} := 0.015 \cdot \frac{\text{W}}{\text{m} \cdot \text{K}}$

Thermal Liner

$L_{tl} := 4\text{mm}$

$k_{tl} := 0.045 \cdot \frac{\text{W}}{\text{m} \cdot \text{K}}$

Let's calculate all the resistances which will be evaluated by unit of area (mainly because the area is not known). Since we do not know the convection coefficient outside (fire side), we can assume that is very large so the thermal resistance is negligible.

Resistances

Shell Layer

$$R_{shell} := \frac{L_{shell}}{k_{shell}}$$

$$\boxed{R_{shell} = 0.017 \cdot \frac{m^2 \cdot K}{W}}$$

Moisture Barrier

$$R_{mb} := \frac{L_{mb}}{k_{mb}}$$

$$\boxed{R_{mb} = 0.047 \cdot \frac{m^2 \cdot K}{W}}$$

Thermal Liner

$$R_{tl} := \frac{L_{tl}}{k_{tl}}$$

$$\boxed{R_{tl} = 0.089 \cdot \frac{m^2 \cdot K}{W}}$$

To estimate the resistance in the air gap it is necessary to estimate the resistance due to radiation and conduction in the air. Let's assume that the conductivity of air is k_{air}=0.04 W/m.K taken at approximately 470K. In Appendix Table C8 from the textbook.

$$k_{air} := 0.04 \cdot \frac{W}{m \cdot K} \qquad L_{air} := 1mm$$

$$R_{cond\_air} := \frac{L_{air}}{k_{air}} \qquad \boxed{R_{cond\_air} = 0.025 \cdot \frac{m^2 \cdot K}{W}}$$

Now we have to calculate the resistance due to radiation

$$\sigma := 5.676 \cdot 10^{-8} \cdot \frac{W}{m^2 \cdot K^4} \qquad T_{avg} := (470 + 273)K \qquad h_{rad} := 4 \cdot \sigma \cdot T_{avg}^{\ 3}$$

$$R_{radiation} := \frac{1}{h_{rad}} \qquad \boxed{R_{radiation} = 0.011 \cdot \frac{m^2 \cdot K}{W}}$$

$$R_{air\_gap} := \frac{1}{\frac{1}{R_{radiation}} + \frac{1}{R_{cond\_air}}} \qquad \boxed{R_{air\_gap} = 7.512 \times 10^{-3} \cdot \frac{m^2 \cdot K}{W}}$$

From the calculated resistances the thermal liner is what produces the highest thermal resistance.

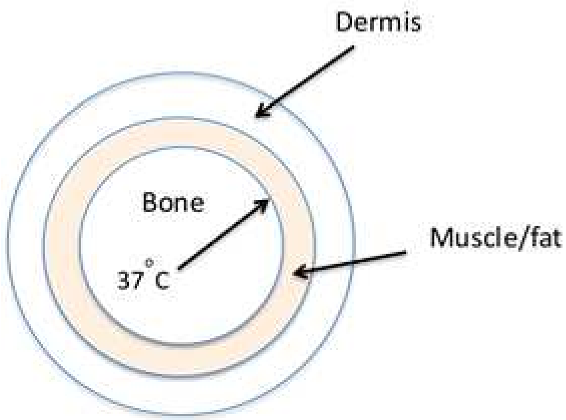
$$q_{fire} := 2500 \cdot \frac{W}{m^2} \qquad T_{inside} := (50 + 273) \cdot K$$

$$R_{total} := R_{shell} + R_{mb} + 2 \cdot R_{air\_gap} + R_{tl} \qquad \boxed{R_{total} = 0.167 \cdot \frac{m^2 \cdot K}{W}}$$

$$T_s := T_{inside} + q_{fire} \cdot R_{total} \qquad \boxed{T_s = 741.114K}$$

So temperature outside of the coat will be about 741.1K or 468.1C

Question 4



Data

$D_{bone} := 25 \cdot mm$ $\quad r_{bone} := \frac{D_{bone}}{2}$ $\quad e_{muscle} := 2.1 \cdot mm$ $\quad e_{dermis} := 0.4 \cdot mm$

$k_{m\_fat} := 0.3 \cdot \frac{W}{m \cdot K}$ $\quad k_{dermis} := 0.1 \cdot \frac{W}{m \cdot K}$ $\quad k_{glove} := 0.02 \cdot \frac{W}{m \cdot K}$

$h_{amb} := 100 \cdot \frac{W}{m^2 \cdot K}$ $\quad e_{glove} := 1.4 \cdot mm$

$r_{m\_fat} := r_{bone} + e_{muscle}$ $\quad r_{dermis} := r_{m\_fat} + e_{dermis}$

(a) The heat loss per unit length q_{prime} can be calculated from the resistances in the different layers of the finger, so the resistances need to be estimated

Without Gloves

$$R_{muscle} := \frac{1}{2\pi \cdot k_{m\_fat}} \cdot \ln\left(\frac{r_{m\_fat}}{r_{bone}}\right) \qquad \boxed{R_{muscle} = 0.082 \cdot \frac{m \cdot K}{W}}$$

$$R_{dermis} := \frac{1}{2\pi \cdot k_{dermis}} \cdot \ln\left(\frac{r_{dermis}}{r_{m\_fat}}\right) \qquad \boxed{R_{dermis} = 0.043 \cdot \frac{m \cdot K}{W}}$$

$$R_{conv\_} := \frac{1}{2\pi \cdot h_{amb} \cdot r_{dermis}} \qquad \boxed{R_{conv\_} = 0.106 \cdot \frac{m \cdot K}{W}}$$

With Gloves

If we add the glove there will be a change in the heat flux per unit length because we need to add the Resistance of the Glove. The radius after the glove is added will be:

$$r_{glove} := r_{dermis} + e_{glove}$$

$$R_{glove} := \frac{1}{2\pi \cdot k_{glove}} \cdot \ln\left(\frac{r_{glove}}{r_{dermis}}\right) \qquad \boxed{R_{glove} = 0.71 \cdot \frac{m \cdot K}{W}}$$

Since the external radius changes due to the glove, convection will change and the new Resistance to Convection will be:

$$R_{new\_conv} := \frac{1}{2\pi \cdot h_{amb} \cdot r_{glove}} \qquad \boxed{R_{new\_conv} = 0.097 \cdot \frac{m \cdot K}{W}}$$

The major resistance is due to the glove. We can calculate (no asked but needed it) the heat fluxes per unit length when you have and when you don't have gloves. I need to enter temperature pleases note that since I will use temperature differences I can use tempernartures in degrees Kelvin

$$T_{ib} := 37 \cdot K \qquad T_{out} := -25 \cdot K$$

Without Gloves

$$q_{prime\_wout\_gloves} := \frac{T_{ib} - T_{out}}{R_{muscle} + R_{dermis} + R_{conv\_}}$$

$$\boxed{q_{prime\_wout\_gloves} = 267.8 \cdot \frac{W}{m}}$$

With Gloves

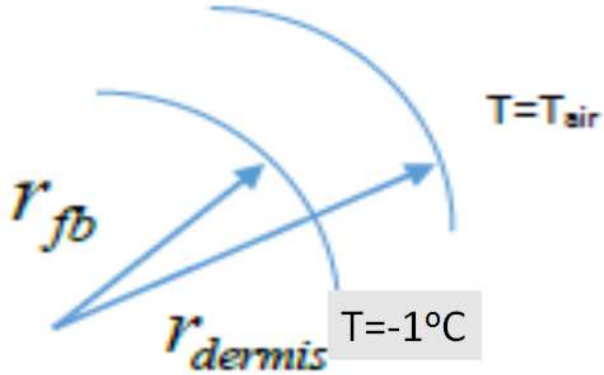
$$q_{prime\_with\_gloves} := \frac{T_{ib} - T_{out}}{R_{muscle} + R_{dermis} + R_{glove} + R_{new\_conv}}$$

$$\boxed{q_{prime\_with\_gloves} = 66.5 \cdot \frac{W}{m}}$$

An important decrease in the heat flow per unit length

Let's assume the frostbite is within the dermis layer and can be located by the radius r_{fb} where the temperature is reaches -1C

$$T_{fb} := -1 \cdot K$$



$$r_{fb\_nogv} := \frac{r_{dermis}}{\exp\left[\frac{2\pi\cdot(T_{fb}-T_{out})\cdot k_{dermis}}{q_{prime\_wout\_gloves}} - \frac{k_{dermis}}{h_{amb}\cdot r_{dermis}}\right]}$$

$$r_{fb\_nogv} = 15.156\cdot mm$$

$Damaged\_dermis := r_{dermis} - r_{fb\_nogv}$ $\qquad Damaged\_dermis = -0.156\cdot mm$

The value is negative, it may indicate that all the dermis is affected and the frostbite can be even move to the muscle/fat zone. Equation to calculate the frosbite distance including that zone can be calculated using the following (it can be derived by assuming the frostbite is with the muscle/fat zone zone), which we will define as new frost bite.

$$r_{fb\_nogv\_new} := \frac{r_{m\_fat}}{\exp\left[\frac{2\pi\cdot(T_{fb}-T_{out})\cdot k_{m\_fat}}{q_{prime\_wout\_gloves}} - \frac{k_{m\_fat}}{k_{dermis}}\cdot\ln\left(\frac{r_{dermis}}{r_{m\_fat}}\right) - \frac{k_{m\_fat}}{h_{amb}\cdot r_{dermis}}\right]}$$

$r_{fb\_nogv\_new} = 16.333\,mm$ $\qquad NewDamaged\_dermis := r_{dermis} - r_{fb\_nogv\_new}$

$NewDamaged\_dermis = -1.333\,mm$ So the frostbite is very severe and we can have temperatures of -1C in the bone

When gloves are weared,you replace in the above equation (the heat flux using gloves)

$$r_{fb\_withgv} := \frac{r_{dermis}}{\exp\left[\frac{2\pi\cdot(T_{fb}-T_{out})\cdot k_{dermis}}{q_{prime\_with\_gloves}} - \frac{k_{dermis}}{h_{amb}\cdot r_{dermis}}\right]}$$

$r_{fb\_withgv} = 12.78\,mm$ $\qquad DamagedDermis_{with\_globe} := r_{dermis} - r_{fb\_withgv}$

$DamagedDermis_{with\_globe} = 2.22\,mm$ Better wear gloves when temperature outside is -25C or -13F

Question 5

(a)

$D_{fin} := 2.5mm$ $\quad A_c := \pi\cdot\frac{D_{fin}^2}{4}$ $\quad P := \pi\cdot D_{fin}$ $\quad k_{fin} := 400\cdot\frac{W}{m\cdot K}$

$T_{fin} := 95\cdot K$ $\quad T_{amb\_air} := 25\cdot K$ $\quad h_{fin\_air} := 10\frac{W}{m^2\cdot K}$

$$q_{fin} := \sqrt{h_{fin\_air}\cdot k_{fin}\cdot P\cdot A_c}\cdot\left(T_{fin} - T_{amb\_air}\right) \qquad \boxed{q_{fin} = 0.87\,W}$$

(b) For a 25mm length let's use the equation below given in the problem, but first we need to calculate the heat with no fin which is:

$$q_{non\_fin} := A_c\cdot h_{fin\_air}\cdot\left(T_{fin} - T_{amb\_air}\right)$$

$m_p := \sqrt{\frac{h_{fin\_air}\cdot P}{k_{fin}\cdot A_c}}$ $\quad m_p = 6.325\frac{1}{m}$ $\quad L_{fin} := 25\cdot mm$

$$q_{finite\_fin} := \sqrt{\frac{k_{fin}\cdot P}{h_{fin\_air}\cdot A_c}}\cdot\frac{\sinh\left(m_p\cdot L_{fin}\right) + \frac{h_{fin\_air}}{m_p\cdot k_{fin}}\cdot\cosh\left(m_p\cdot L_{fin}\right)}{\cosh\left(m_p\cdot L_{fin}\right) + \frac{h_{fin\_air}}{m_p\cdot k_{fin}}\sinh\left(m_p\cdot L_{fin}\right)}\cdot q_{non\_fin}$$

$\boxed{q_{finite\_fin} = 0.14\,W}$ Very different value

(c) For the two solutions be within 5%

sinh m_pL + h/m_pk * cosh m_pL/(cosh m_pL +(h/m_pk) sinh (m_pL) = 0.95

The root tool can be used from where a hypothetical lenght L_{hyp} where the infinite analytical solution (see Equation given in Lectures) can be used within a 5% error can obtained, let's define the function from which the root L can be obtained

$L_f := 6\cdot mm$

$$f\left(L_f\right) := \frac{\sinh\left(m_p\cdot L_f\right) + \frac{h_{fin\_air}}{m_p\cdot k_{fin}}\cdot\cosh\left(m_p\cdot L_f\right)}{\cosh\left(m_p\cdot L_f\right) + \frac{h_{fin\_air}}{m_p\cdot k_{fin}}\sinh\left(m_p\cdot L_f\right)} - 0.95$$

$$L_{hyp} := \mathrm{root}\left(f\left(L_f\right), L_f\right) \qquad \boxed{L_{hyp} = 28.9\cdot cm}$$