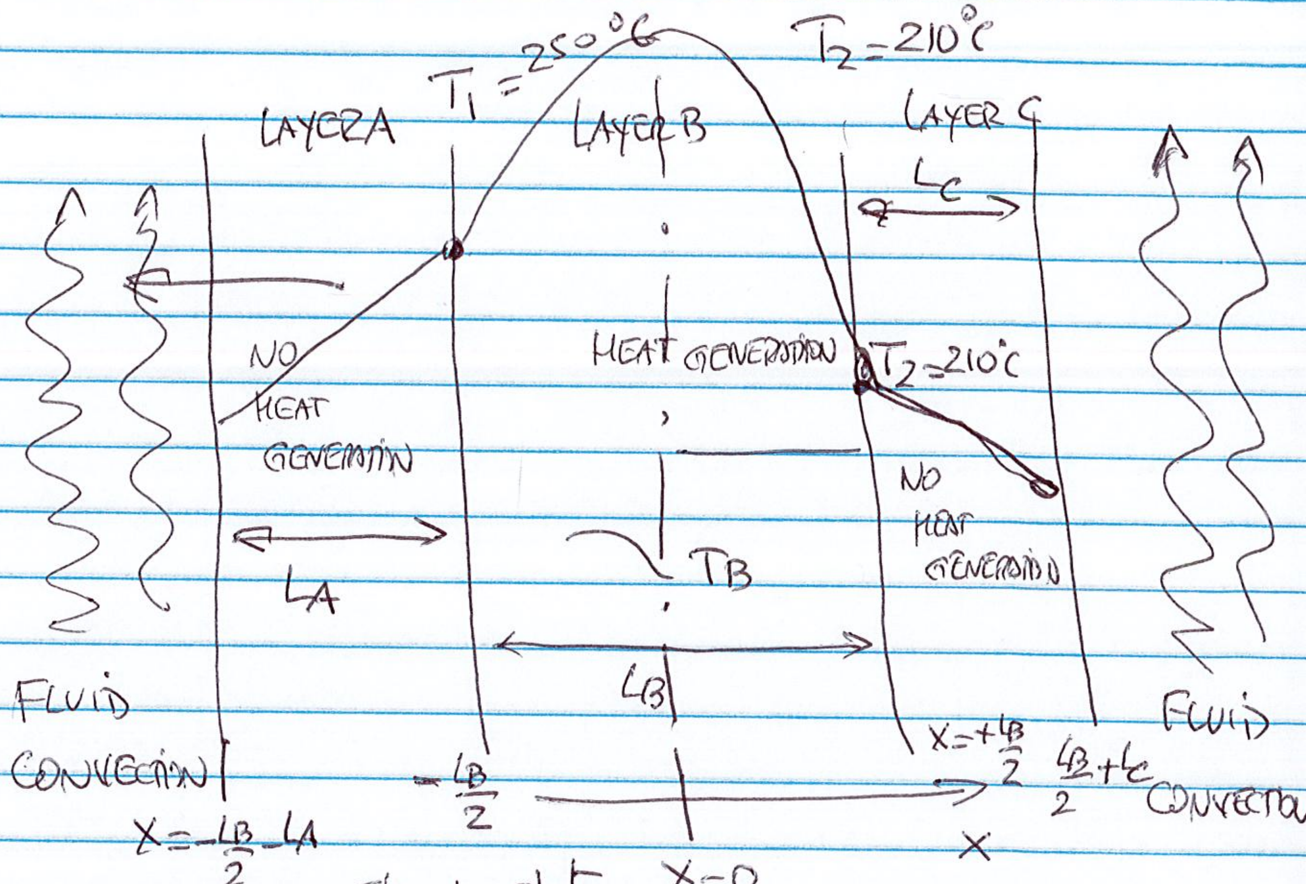
Problem 2 - HOMEWORK 3



- steady state
- Heat transfer in one direction X

Layer B

$$
\begin{cases}
\dfrac{d^2 T_B(X)}{dX^2} = -\dfrac{\dot{q}_B}{K_B} \\
X = +\dfrac{L_B}{2} \quad T = T_2 \\
X = -\dfrac{L_B}{2} \quad T = T_1
\end{cases}
$$

$$\frac{dT_B}{dx} = -\frac{\dot{q}_B}{k_B} x + C_1 \qquad (2)$$

$$\boxed{T_B(x) = -\frac{\dot{q}_B}{2k_B} x^2 + C_1 x + C_2}$$

C_1 & C_2 will be different because the Boundary conditions are different !!

REMEMBER THIS FOR EXAM AND FUTURE !!

<u>LAYER A</u>

$$\begin{cases} \dfrac{d^2 T_A(x)}{dx^2} = 0 \\ x = -\dfrac{L_B}{2} \quad T = T_1 \\ x = -\dfrac{L_B}{2} - L_A \quad -k_A \dfrac{dT_A}{dx} = h\,[T_{S1} - T_{\infty}) \end{cases}$$

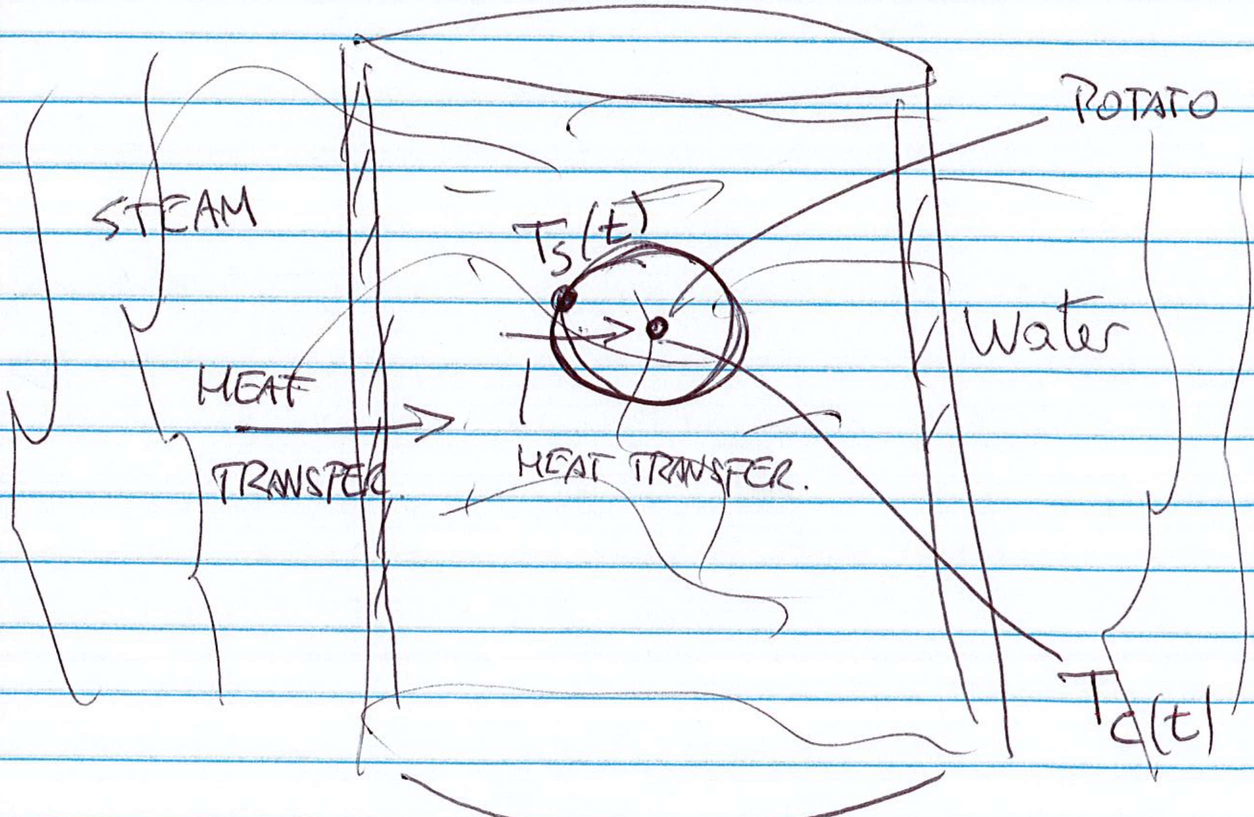
$$\frac{dT_A(x)}{dx} = B_1$$

$$T_A(x) = B_1 x + B_2$$

EXAMPLE 3 – Sterilization of Potato

Question 1 – Is the surface temperature of potato constant? T_s?

Question 2 – why we don't use the concept of resistance inside the potato?



$$T_{potato,\,initial} = 30°C$$

$$\frac{1}{U} \approx \frac{1}{h_{steam}} + \frac{Thickness_{Can}}{K_{can.}} + \frac{1}{h_{water}}$$

data given.

Thermal Resistance of steam

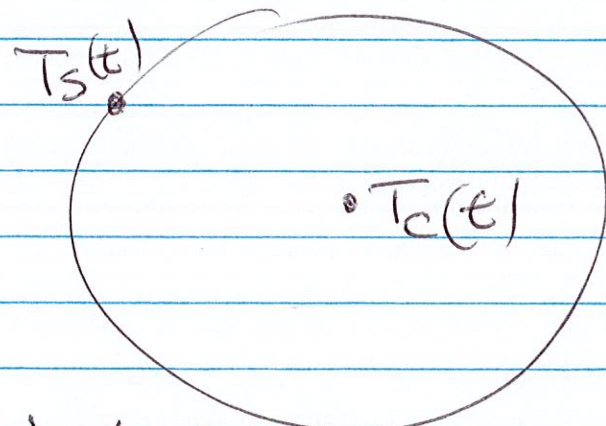
$\underbrace{T_s(t) \text{ and } T_c(t)}_{\text{Function of } t}$ (4)

are they equal?

They are not equal.!!

Because $B_i > 0.1$

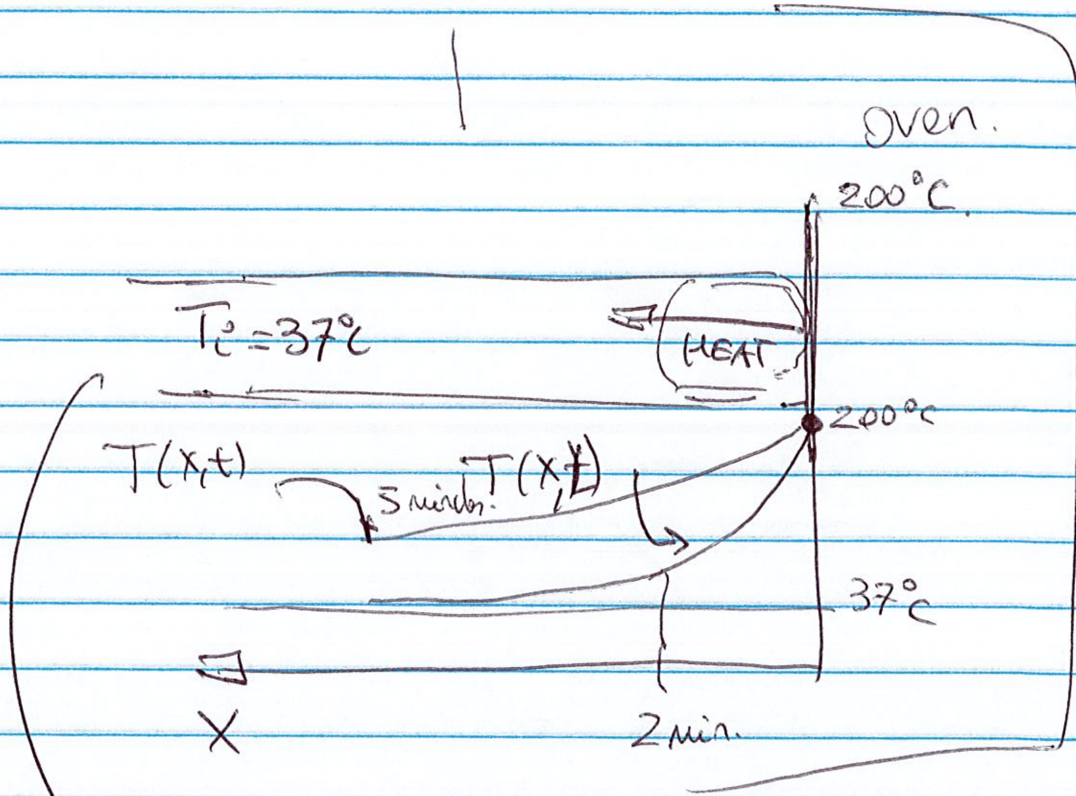
Why we cannot use the concept of resistance to analyze the potato?



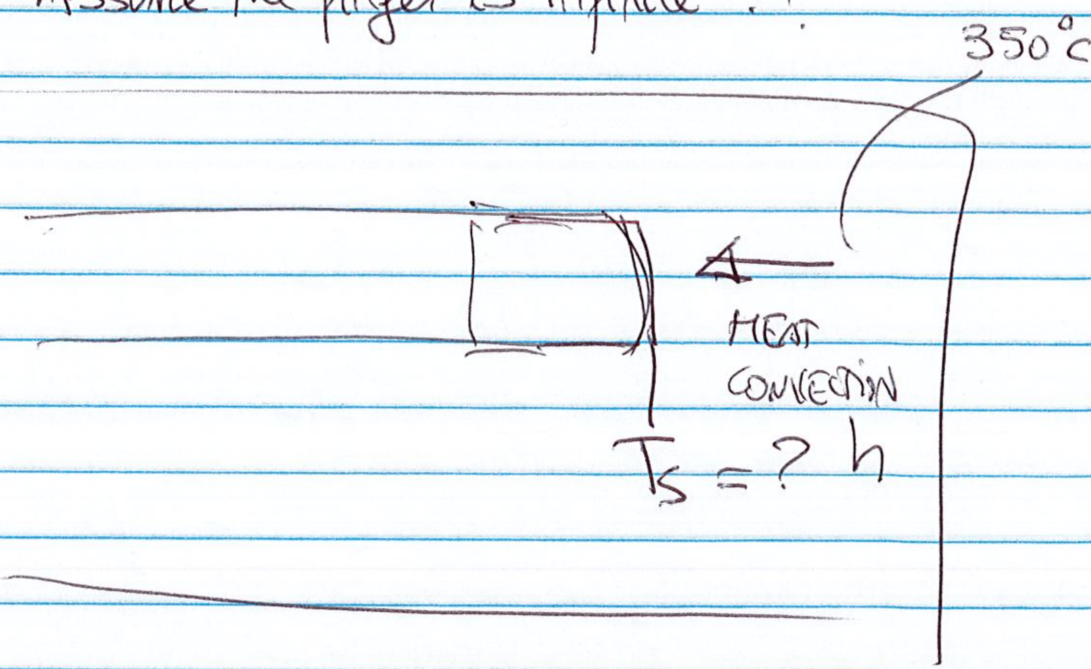
$$q \neq \frac{T_s(t) - T_c(t)}{R_{potato}} \quad ??$$

Because $T_s(t)$ and $T_c(t)$ are changing time and the heat transfer is under unsteady state.

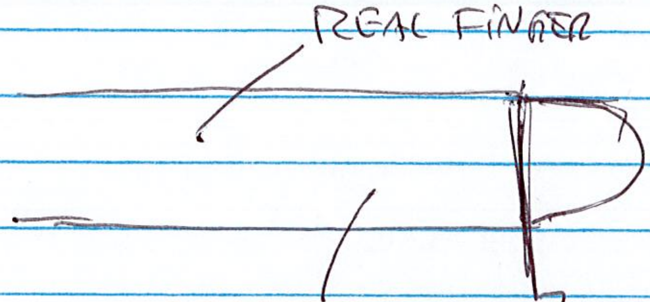
EXAMPLE 5



Assume the finger is infinite !!

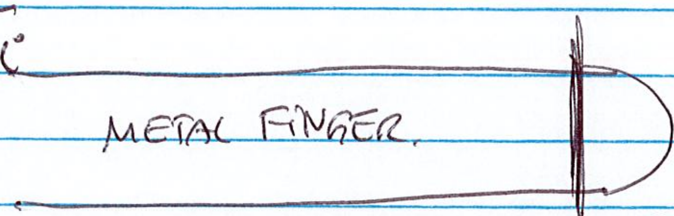


Is ok to assume that finger is infinite? (6)



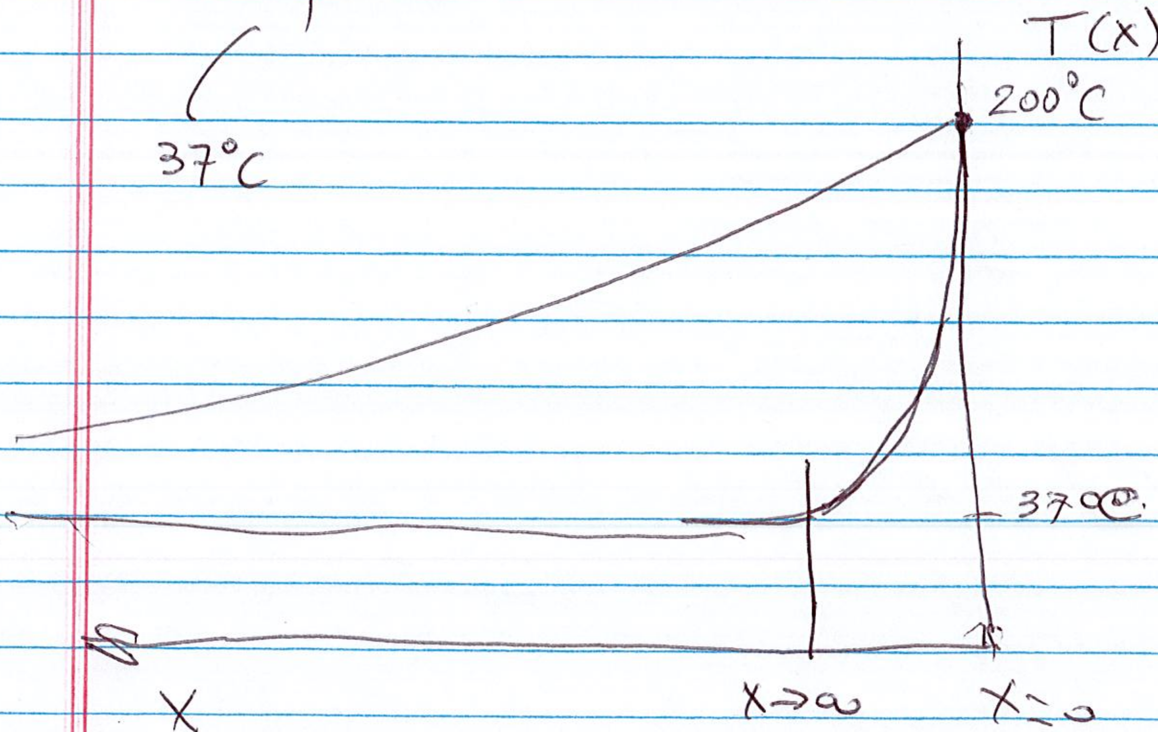
$$\begin{cases} \dfrac{\partial T(x,t)}{\partial t} = \alpha \dfrac{\partial^2 T(x,t)}{\partial x^2} \\ \text{at } x=0 \quad T=T_s \\ \text{at } x\to\infty \quad T=T_i \end{cases}$$

$\alpha = 2{,}5\times10^{-7}\ m^2/s \qquad \dfrac{k}{\rho c}$



Initial temperature

$\alpha = 2{,}5\times10^{-2}\ m^2/s \qquad \dfrac{k}{\rho c}$



solution is the error funtion.

$$\frac{T(x,t)-T_i}{T_s-T_i} \cancel{T(x,t)} = 1 - erf\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

$$\frac{x}{2\sqrt{\alpha t}} \geq 2 \qquad erf(\sim 2) \longrightarrow 1$$

when $\frac{x}{2\sqrt{\alpha t}}$ $\quad \frac{T(x,t)-T_i}{T_s-T_i} \longrightarrow 0 \qquad T(x,t) \longrightarrow T_i$

$$\frac{x}{2\sqrt{\alpha t}} \geq 2$$

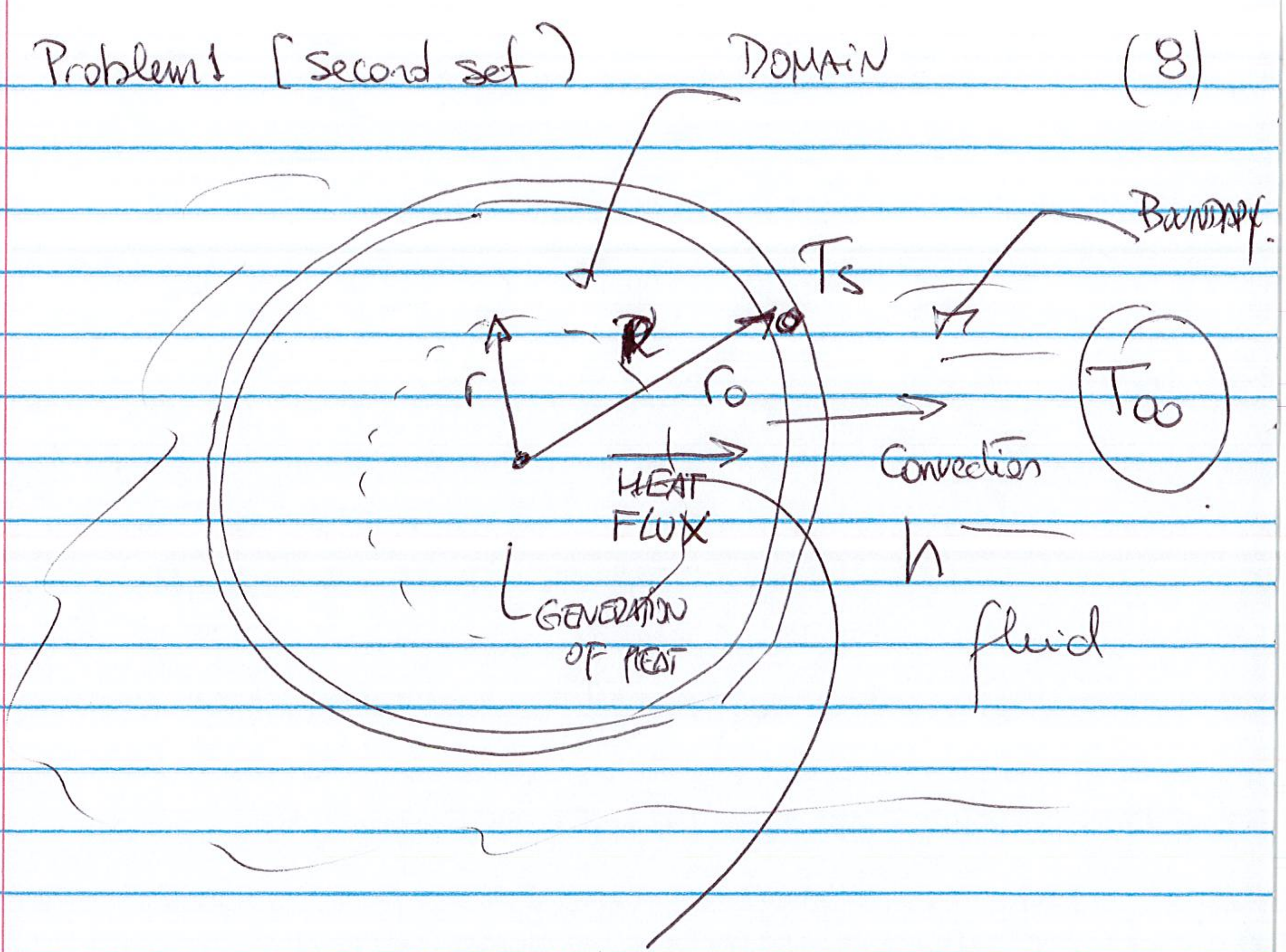
$$x \geq 4\sqrt{\alpha t}$$

x is considered infinite

$$x \geq 4\sqrt{2.5\times10^{-7}\,\frac{m^2}{\cancel{s}} \times 2\cancel{s}} =$$

$$x \geq 4 \times 7\times10^{-4} = 28\times10^{-4}\,m = 0.28\,mm$$

Problem 1 [second set)



We need to determine $T(r)$

$$0 = \frac{K}{\cancel{\rho c}} \frac{1}{r^2} \frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) + \frac{\dot{q}}{\cancel{\rho c}}$$

$$\left\{\begin{array}{l} \dfrac{1}{r^2} \dfrac{d}{dr}\left[r^2 \dfrac{dT}{dr}\right] = -\dfrac{Q_0}{K}\left[1-\left(\dfrac{r}{r_0}\right)^2\right] \\[2ex] \text{at } r=r_0 \quad -K \left.\dfrac{dT}{dr}\right|_{r=r_0} = h\left[T_s - T_\infty\right] \\[2ex] \text{at } r=0 \quad \dfrac{dT}{dr} = 0 \end{array}\right.$$

$\dot{q} = Q_0\left[1-\left(\frac{r}{r_0}\right)^2\right]$

$$\frac{d}{dr}\left[r^2\frac{dT}{dr}\right] = -\frac{Q_0}{k}\left[r^2 - \frac{r^4}{r_0^2}\right]$$

integrating once

$$r^2\frac{dT}{dr} = -\frac{Q_0}{k}\left[\frac{r^3}{3} - \frac{r^5}{5r_0^2}\right] + C_1$$

$$\frac{dT}{dr} = -\frac{Q_0}{k}\left[\frac{r}{3} - \frac{r^3}{5r_0^2}\right] + \cancel{\frac{C_1}{r^2}}$$

at $r=0$ $\quad \frac{dT}{dr} \to 0 \quad$ so $C_1 = 0$

$$\frac{dT}{dr} = -\frac{Q_0}{k}\left[\frac{r}{3} - \frac{r^3}{5r_0^2}\right]$$

$$T(r) = -\frac{Q_0}{k}\left[\frac{r^2}{6} - \frac{r^4}{20r_0^2}\right] + C_2$$

$$+\cancel{k}\frac{Q_0}{\cancel{k}}\left[\frac{r_0}{3} - \frac{r_0^{\cancel{3}}}{5\cancel{r_0^2}}\right] = h\left(T_s - T_\infty\right)$$

$$T_s = T(r=r_0) = -\frac{Q_0}{k}\left[\frac{r_0^2}{6} - \frac{r_0^2}{20}\right] + C_2$$

(10)

$$\cancel{k}\frac{Q_0}{\cancel{k}}\left[\frac{r_0}{3}-\frac{r_0}{5}\right]=h\left[-\frac{Q_0}{k}\left(\frac{r_0^2}{6}-\frac{r_0^2}{20}\right)+C_2-T_\infty\right]$$

$$\Downarrow$$

$$C_2$$