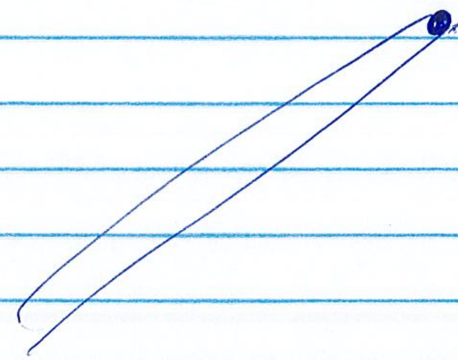
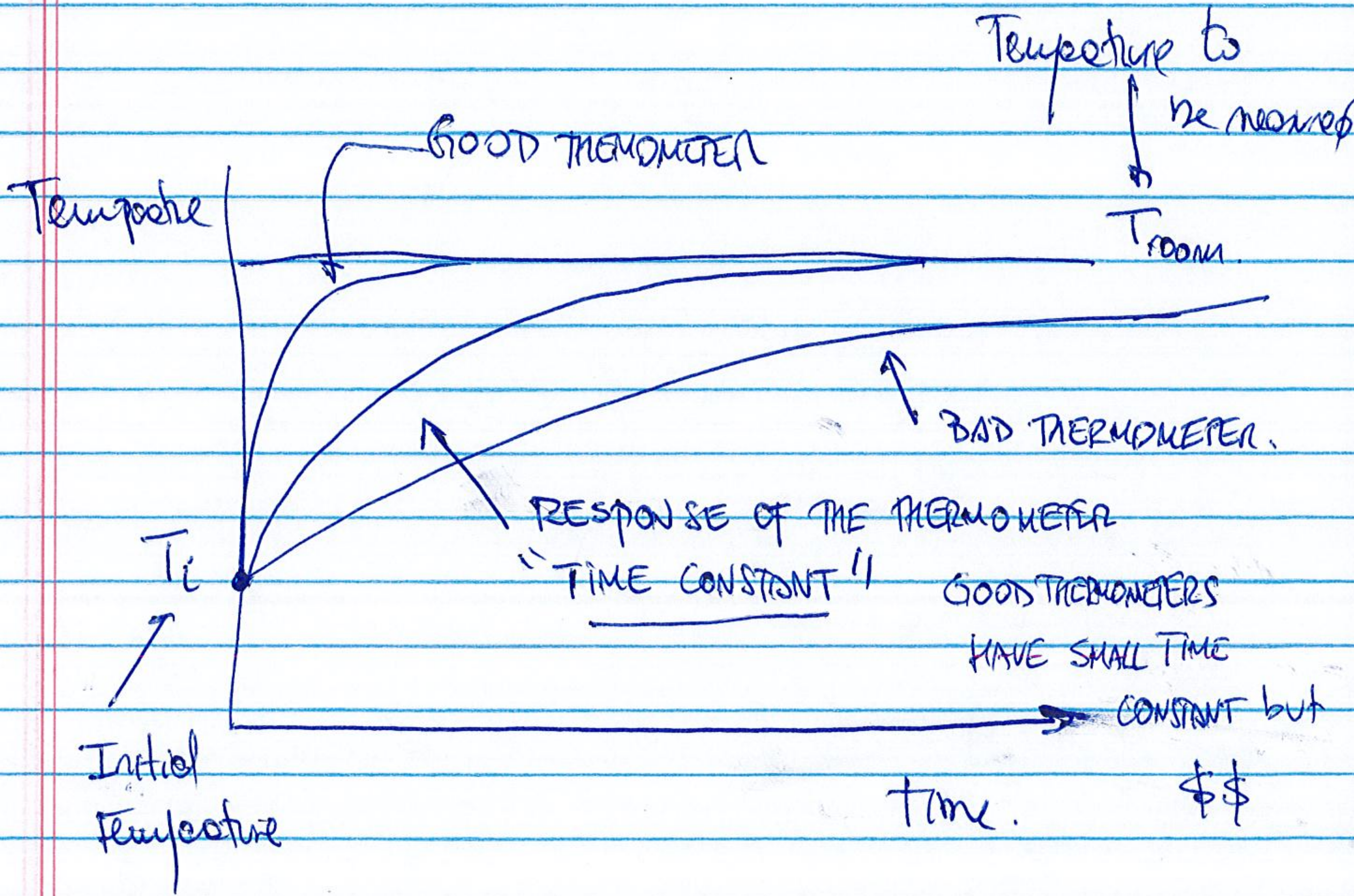
NOTES LECTURE 2-8-18

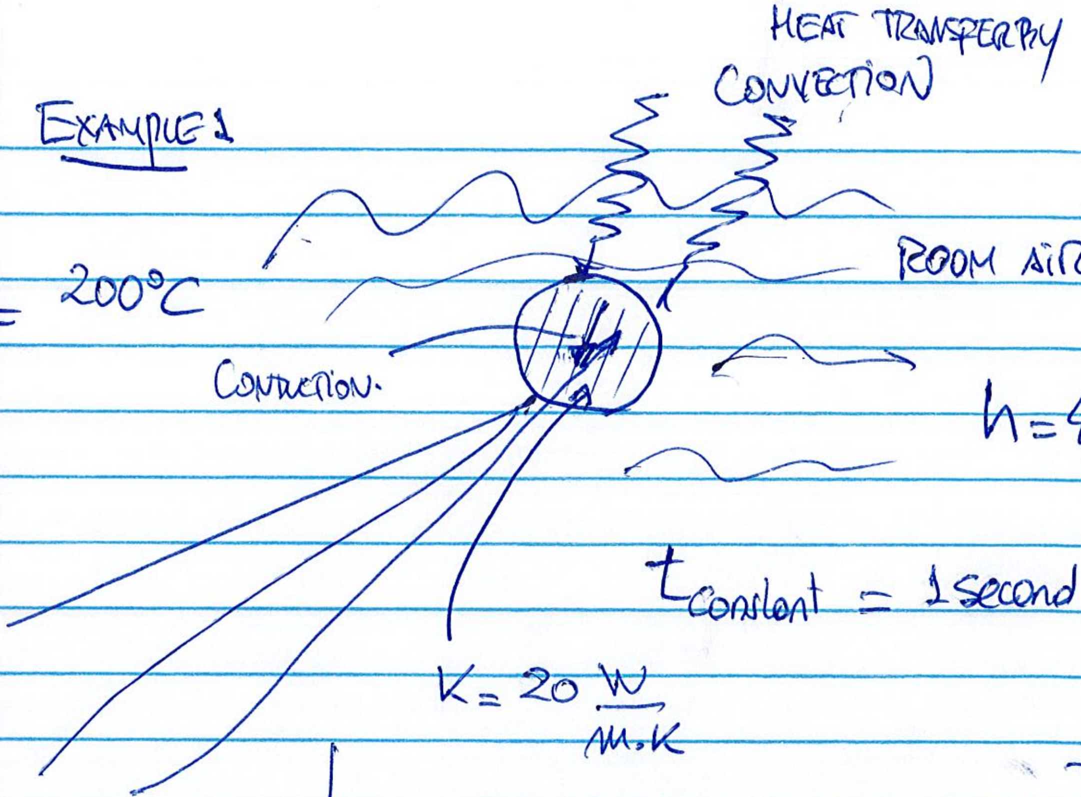




GOOD THERMOMETERS HAVE SMALL TIME CONSTANT but $$

EXAMPLE 1

$T_\infty = 200°C$



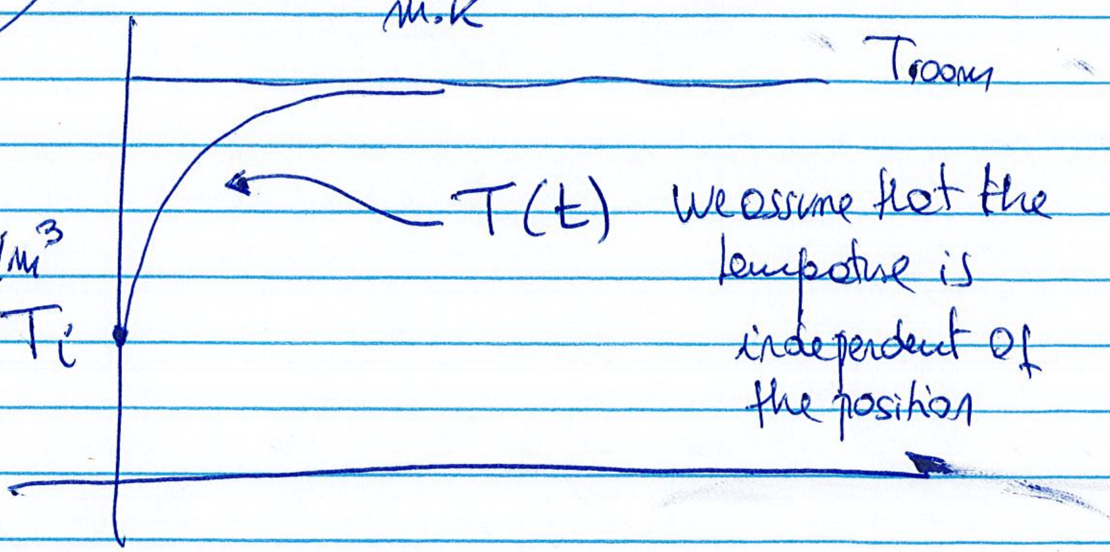
$h = 400 \frac{W}{m^2K}$

$t_{constant} = 1 \text{ second}$

$K = 20 \frac{W}{m.K}$

$C_m = 0.4 \frac{KJ}{KgK}$

$\rho_m = 8500 \; Kg/m^3$



We assume that the temperature is independent of the position

LUMPED PARAMETER ANALYSIS

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = \exp\left(-\frac{UA}{MC}t\right)$$

$$Bi = \frac{\overset{400\, W/m^2K}{h} R}{\underset{20\, \frac{W}{m.K}}{K}} < 0.1$$

Let's assume the Lumped Parameter model can be used [We will need to check that later]

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = \exp\left[-\frac{UA}{mc}t\right]$$

What is U?

U overall heat transfer coefficient.

$$\frac{1}{U} = \cancel{\text{Resistance to conduction}} + \text{Resistance to convection}$$

Overall resistance to heat transfer

$$\frac{1}{h}$$

$U \approx h$ Because we are assuming the Lumped PARAMETER MODEL.

$$\exp\left[-\frac{hA}{mc}t\right] = \exp\left[-\frac{t}{\frac{mc}{hA}}\right] \quad (4)$$

$$\text{Units } \frac{mc}{hA} = \frac{\cancel{kg} \times \frac{J}{\cancel{kg}\cancel{K}}}{\frac{W}{\cancel{m^2}\cancel{K}} \times \cancel{m^2}} = \frac{J}{W} = \frac{\cancel{J}}{\frac{\cancel{J}}{s}} = s$$

$$t_{constant} = \frac{mc}{hA} = 1s$$

$$m = V_b \rho_m = \frac{4}{3}\pi R^3 \rho_m$$

$$A = 4\pi R^2$$

$$t_{constant} = 1 sec = \frac{\frac{\cancel{4\pi}}{3} R^{\cancel{3}} \rho_m \times c_m}{h \times \cancel{4\pi R^2}}$$

$$1 sec = \frac{\rho_m c_m R}{3h} \Rightarrow R = \frac{3h}{\rho_m c_m}$$

$$R = \frac{3 \times 400 \ W/m^2K}{8500 \frac{kg}{m^3} \times 0.4 \times 1000 \frac{J}{kgK}} = 3.5 \times 10^{-4} m$$

$\boxed{R = 0.35 \ mm}$ $\boxed{D = 0.7 \ mm}$

(5)

$$Bi = \frac{400\ W/m^2K \times 3.5\times10^{-4}\ m}{20\ W/m.K} = 0.007$$

$$Bi = 0.007 < 0.1$$

GOOD ASSUMPTION.

We want to know time to reach 199°C

199

200°C

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = \exp\left(-\frac{hA}{mc}t\right)$$

25°C

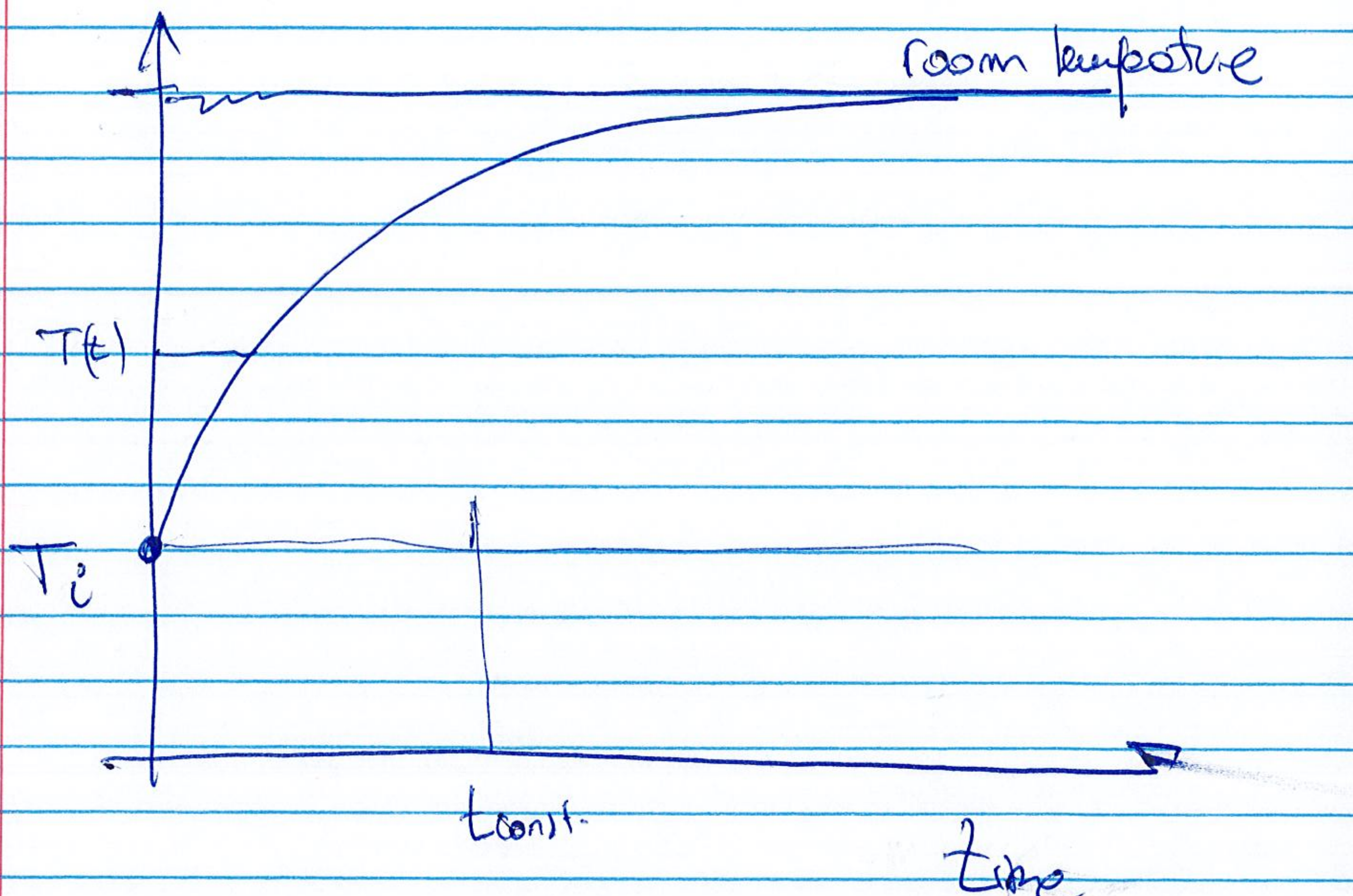
200

$$\ln\frac{199-200}{25-200} = -\frac{hA}{mc}t$$

$t_{const} = 1 sec.$

$$t = -\frac{mc}{hA}\ln\frac{199-200}{25-200} \simeq 5s$$

What is time constant ?
Of a thermometer

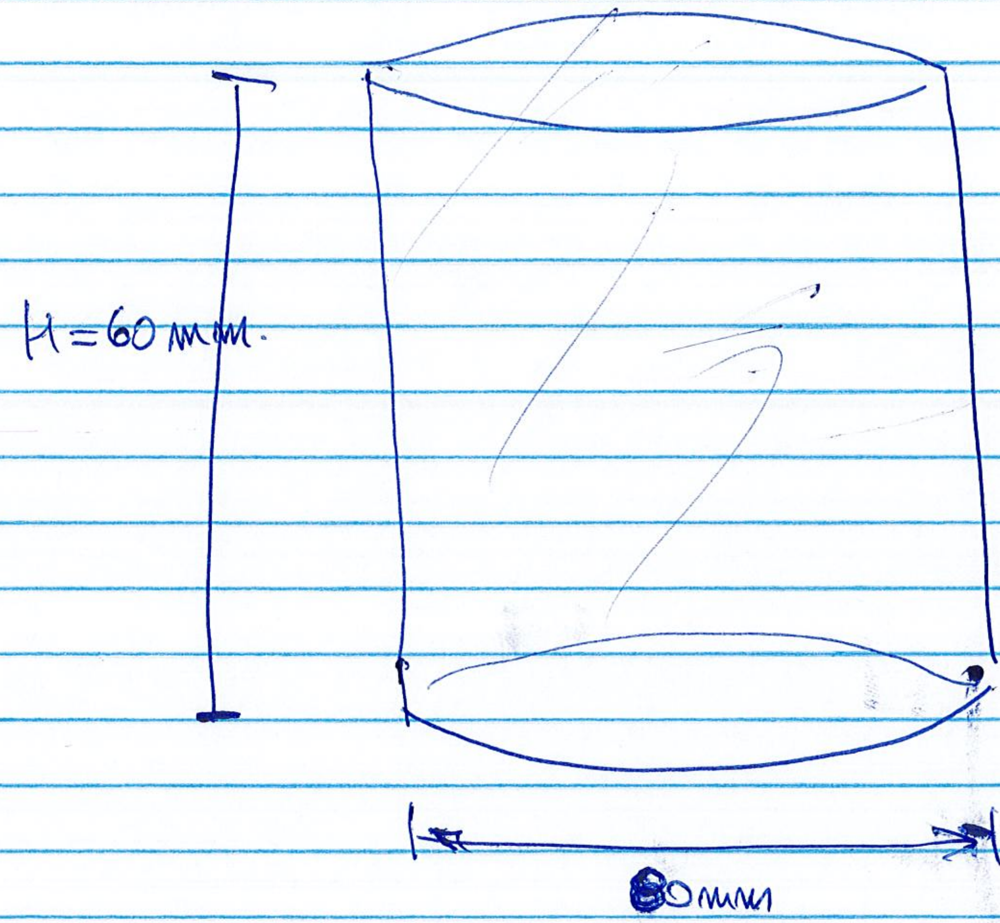


$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-\frac{t}{\frac{mc}{hA}}} = e^{-\frac{t}{t_{const.}}}$$

if $t = t_{const.}$

$$\frac{T_\infty - T(t)}{T_\infty - T_i} = \frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-1} = 0.37$$

EXAMPLE 7

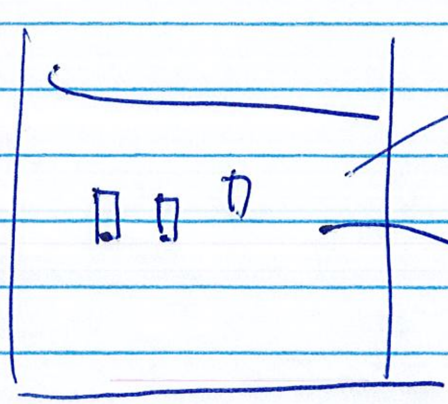


$\rho_c = 7900\ kg/m^3$

$C = 526\ \frac{J}{kg\,K}$

$K = 17.4\ \frac{W}{m.K}$

$T_c = 327^\circ C$



$h = 500\ W/m^2K.$

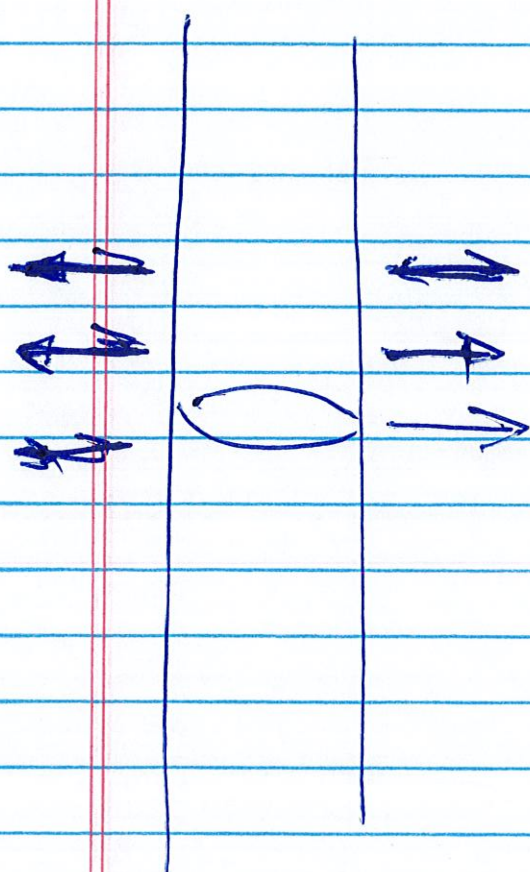
oil BATH OF $27^\circ C$

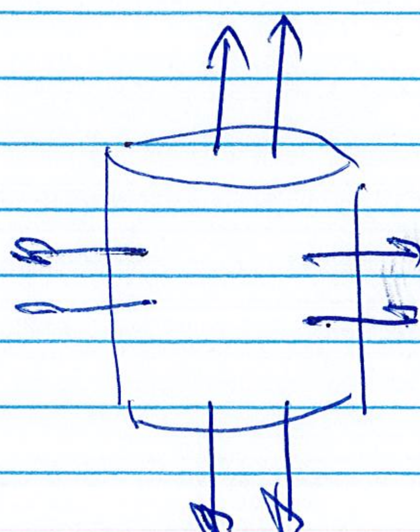
$$Bi = \frac{hR}{k} = \frac{500 \overset{W/m^2K}{} \times 40 \times 10^{-3}\, m}{17.4 \frac{W}{m.K}} = \frac{1}{0.87} \quad (8)$$

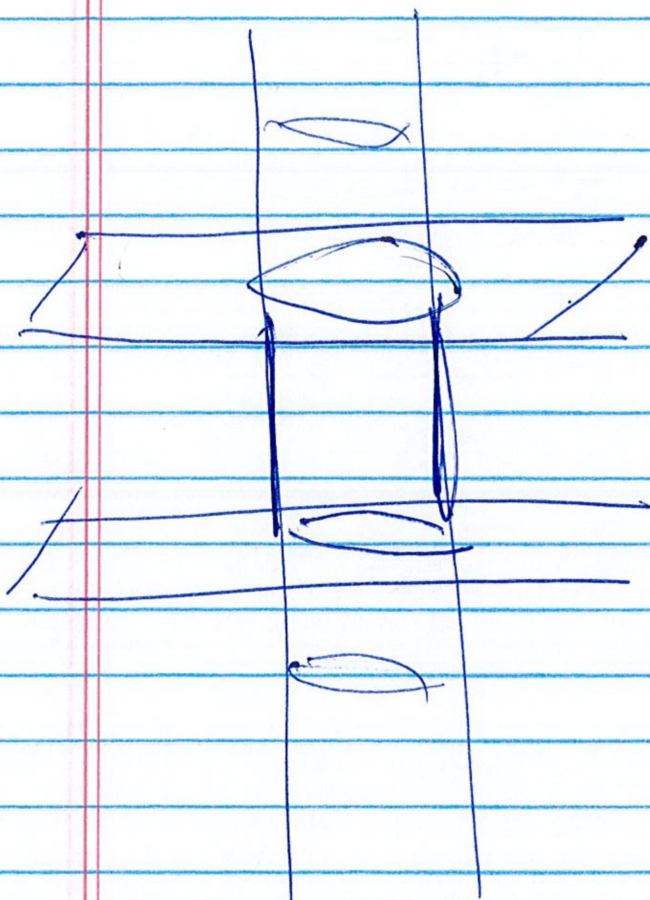
$$\boxed{Bi = 1.1}$$

We cannot use LUMPED PARAMETER MODEL

WE HAVE TO USE CHARTS BUT THEY ARE FOR INFINITE LONG CYLINDERS



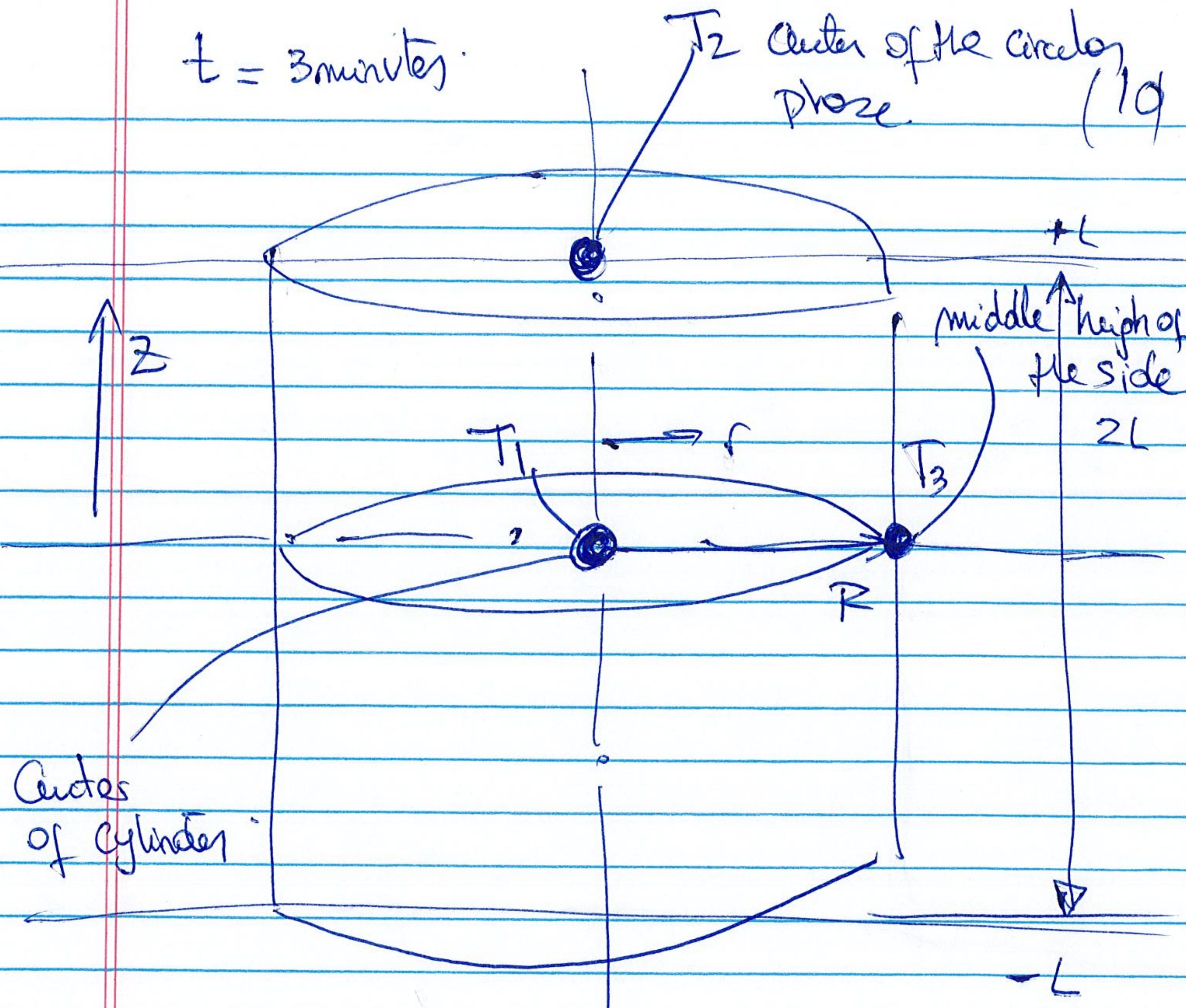




solution

$$Y = Y_c \times Y_{slab}.$$

$t = 3$ minutes.



$$\frac{T(r,z,t) - T_s}{T_i - T_s} = \underbrace{\frac{T(r,t) - T_s}{T_i - T_s}}_{\text{CHART FOR INFINITE CYLINDER.}} \times \underbrace{\frac{T(z,t) - T_s}{T_i - T_s}}_{\text{CHART FOR INFINITE SLAB}}$$