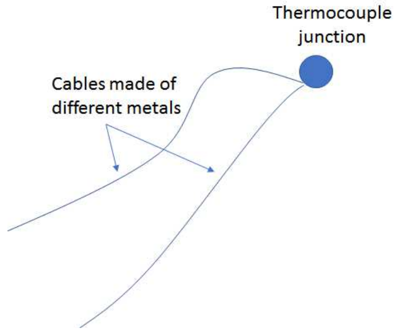
<u>**ABE 30800 - Unstedy State Heat Transfer**</u>

Problem 1



$kJ := 1000 \cdot J$ $\quad k_{therm} := 20 \cdot \frac{W}{m \cdot K}$ $\quad c_{therm} := 0.40 \cdot \frac{kJ}{kg \cdot K}$ $\quad \rho_{therm} := 8500 \cdot \frac{kg}{m^3}$

$h_1 := 400 \cdot \frac{W}{m^2 \cdot K}$ $\quad T_{f1} := 200 \cdot K$ $\quad T_{f\_ini} := 25K$ $\quad t_{const} := 1 \cdot s$

Unfortunately, we cannot calculate the biot number to check if we can use the lumped parameter approach because we do not know the radius of the thermocuple ball. Let's assume that the biot number will be smaller than 0.1

The time constant can be estimated as: t_{const} = mc/hA (see slides and notes for more description on this

$$m_{therm} = \rho_{therm} \cdot V_{therm} \qquad V_{therm} = \frac{4}{3}\pi R_{therm}^3 \qquad A_{therm} = 4\pi R_{therm}^2$$

$$t_{const} = \frac{m_{therm} c_{therm}}{h_1 A_{therm}} = \frac{\frac{4}{3}\pi R_{therm}^3 \rho_{therm} c_{therm}}{h_1 4\pi R_{therm}^2} = \frac{1}{3}\frac{R_{therm}\rho_{therm} c_{therm}}{h_1}$$

$$R_{therm} = \frac{3 t_{const} h_1}{\rho_{therm} \cdot c_{therm}}$$

$R_{therm} := \frac{3 \cdot t_{const} \cdot h_1}{\rho_{therm} \cdot c_{therm}}$ $\quad R_{therm} = 0.353\,mm$ $\quad d_{therm} := 2 \cdot R_{therm}$

$$\boxed{d_{therm} = 7.1 \times 10^{-4}\, m}$$

Let's check now the Biot Number

$Bi_{therm} := \frac{h_1 \cdot R_{therm}}{k_{therm}}$ $\quad Bi_{therm} = 7.059 \times 10^{-3}$

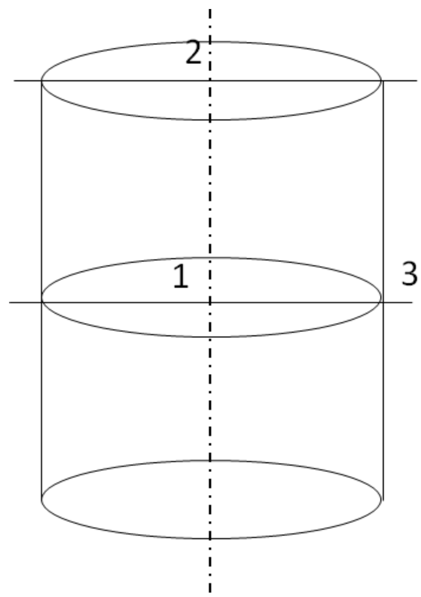
The assumption ws fine, and the Lumped parameter analysis is applicable

(b) Time to reach a temperature of 199C (because time differences are used and to avoid unit problem we use the temperatures in Kelvin)

$$T_{f\_end} := 199 \cdot K \qquad m_{therm} := \frac{4}{3}\pi \cdot R_{therm}^{3} \cdot \rho_{therm} \qquad A_{therm} := 4\pi \cdot R_{therm}^{2}$$

$$t_t := -\left(\frac{m_{therm} \cdot c_{therm}}{h_1 \cdot A_{therm}}\right) \cdot \ln\left(\frac{T_{f\_end} - T_{f1}}{T_{f\_ini} - T_{f1}}\right) \qquad \boxed{t_t = 5.2\,s}$$

Problem 2



$$\rho_{cyl} := 7900 \cdot \frac{kg}{m^3} \qquad c_{cyl} := 526 \cdot \frac{J}{kg \cdot K} \qquad h_2 := 500 \cdot \frac{W}{m^2 \cdot K}$$

$$T_{cyl\_ini} := 327 \cdot K \qquad T_{f\_cyl} := 27 \cdot K \qquad k_{cyl} := 17.4 \cdot \frac{W}{m \cdot K}$$

$$\alpha_{cyl} := \frac{k_{cyl}}{\rho_{cyl} \cdot c_{cyl}} \qquad \alpha_{cyl} = 4.187 \times 10^{-6}\,\frac{m^2}{s} \qquad t_q := 3 \cdot min$$

$$D_{cyl} := 80 \cdot mm \qquad R_{cyl} := \frac{D_{cyl}}{2} \qquad H_{cyl} := 60 \cdot mm \qquad L_{half} := \frac{H_{cyl}}{2}$$

(1) Temperature in location (center of the infinite cylinder and the slab slicing the cylinder

$$r_{1\_cyl} := 0 \qquad n_{1\_cyl} := \frac{r_{1\_cyl}}{R_{cyl}} \qquad m_{1\_cyl} := \frac{k_{cyl}}{h_2 \cdot R_{cyl}} \qquad m_{1\_cyl} = 0.87$$

$$Fo_{1\_cyl} := \frac{\alpha_{cyl} \cdot t_q}{R_{cyl}^{2}} \qquad Fo_{1\_cyl} = 0.471$$

Since Fo > 0.2 we can use either charts or the tables. Let's use both for practice

Using Tables

Infinite Cylinder

$$Bi_{1\_cyl} := \frac{1}{m_{1\_cyl}} \qquad Bi_{1\_cyl} = 1.15$$

From tables for an infinite cylinder and interpolation $\gamma_1 = 1.31$ and $C_1 = 1.23$

$$C_{11\_cyl} := 1.23 \qquad \gamma_{11\_cyl} := 1.31$$

$$Y_{1\_inf\_cyl} := C_{11\_cyl} \cdot \exp\left(-\gamma_{11\_cyl}^{2} \cdot Fo_{1\_cyl}\right) \cdot J0\left(\gamma_{11\_cyl} \cdot \frac{r_{1\_cyl}}{R_{cyl}}\right)$$

$Y_{1\_inf\_cyl} = 0.548$ The chart gives a value of 0.55

Infinite Slab

$$x_{1\_slab} := 0 \qquad n_{1\_slab} := \frac{x_{1\_slab}}{L_{half}}$$

$$Fo_{1\_slab} := \frac{\alpha_{cyl} \cdot t_q}{L_{half}^{2}} \qquad Fo_{1\_slab} = 0.837$$

> 0.2 so we can use the approximate equations

$$m_{1\_slab} := \frac{k_{cyl}}{h_2 \cdot L_{half}} \qquad Bi_{slab} := \frac{1}{m_{1\_slab}} \qquad Bi_{slab} = 0.862$$

From tables for an infinite slab and Biot 0.862 and interpolation $\gamma_1 = 0.792$ and $C_1 = 1.106$

$$C_{11\_slab} := 1.106 \qquad \gamma_{11\_slab} := 0.792$$

$$Y_{1\_inf\_slab} := C_{11\_slab} \cdot \exp\left(-\gamma_{11\_slab}^{2} \cdot Fo_{1\_slab}\right) \cdot \cos\left(\gamma_{11\_slab} \cdot n_{1\_slab}\right)$$

$Y_{1\_inf\_slab} = 0.654$ The chart gives a value of 0.65

$$Y_{1\_finite\_cyl} := Y_{1\_inf\_cyl} \cdot Y_{1\_inf\_slab} \qquad Y_{1\_finite\_cyl} = 0.358$$

Temperature in Celcius

$$T_{1\_finite\_cyl} := T_{f\_cyl} + Y_{1\_finite\_cyl} \cdot \left(T_{cyl\_ini} - T_{f\_cyl}\right) \qquad \boxed{T_{1\_finite\_cyl} = 134.5\,K}$$

(2) Temperature in one of the bases of the cylinder (point 2)
It is calculated in the center of the infinite cylinder (calculated before) and the surface of the slab

$$x_{2\_slab} := L_{half} \qquad n_{2\_slab} := \frac{x_{2\_slab}}{L_{half}} \qquad check := \cos(\pi) \qquad cl$$

All other parameters are the same

$C_{12\_slab} := C_{11\_slab}$ $\gamma_{12\_slab} := \gamma_{11\_slab}$ $Y_{2\_inf\_cyl} := Y_{1\_inf\_cyl}$

$$Y_{2\_inf\_slab} := C_{12\_slab} \cdot \exp\left(-\gamma_{12\_slab}^{2} \cdot Fo_{1\_slab}\right) \cdot \cos\left(\gamma_{12\_slab} \cdot n_{2\_slab}\right)$$

$Y_{2\_inf\_slab} = 0.459$ The chart gives a value of 0.43

$$Y_{2\_finite\_cyl} := Y_{2\_inf\_cyl} \cdot Y_{2\_inf\_slab}$$

$$T_{2\_finite\_cyl} := T_{f\_cyl} + Y_{2\_finite\_cyl} \cdot \left(T_{cyl\_ini} - T_{f\_cyl}\right)$$ $\boxed{T_{2\_finite\_cyl} = 102.5\,K}$

(3) Temperature in the side of the cylinder (point 3)

It is calculated in side of the infinite cylinder and the surface of the slab (calculated before)

$r_{3\_cyl} := R_{cyl}$ $n_{3cyl} := \dfrac{r_{3\_cyl}}{R_{cyl}}$

All the other parameters remain the same

$C_{13\_cyl} := C_{11\_cyl}$ $\gamma_{13\_cyl} := \gamma_{11\_cyl}$

$C_{13\_slab} := C_{11\_slab}$ $\gamma_{13\_slab} := \gamma_{11\_slab}$

$$Y_{3\_inf\_slab} := Y_{2\_inf\_slab}$$

$$Y_{3\_inf\_cyl} := C_{13\_cyl} \cdot \exp\left(-\gamma_{13\_cyl}^{2} \cdot Fo_{1\_cyl}\right) \cdot J0\left(\gamma_{13\_cyl} \cdot n_{3cyl}\right)$$

$$Y_{3\_finite\_cyl} := Y_{3\_inf\_cyl} \cdot Y_{3\_inf\_slab}$$

$$T_{3\_finite\_cyl} := T_{f\_cyl} + Y_{3\_finite\_cyl} \cdot \left(T_{cyl\_ini} - T_{f\_cyl}\right)$$ $\boxed{T_{3\_finite\_cyl} = 73.4\,K}$

<u>Problem 3</u>

Let's assume that the potato has a spherical geometry and that the coldest oint is the center of the potato

$h_3 := 20 \cdot \dfrac{W}{m^2 \cdot K}$ $k_{pot} := 0.4 \cdot \dfrac{W}{m \cdot K}$ $\alpha_{pot} := 1.5 \cdot 10^{-7} \cdot \dfrac{m^2}{s}$

$D_{pot} := 4 \cdot cm$ $R_{pot} := \dfrac{D_{pot}}{2}$ $T_{steam} := 121 \cdot K$

$T_{pot\_ini} := 30 \cdot K$ $T_{center} := 115 \cdot K$

$Bi_{pot} := \frac{h_3 \cdot R_{pot}}{k_{pot}}$ $\qquad Bi_{pot} = 1$ We cannot use the Lumped Parameter equation

from charts for an spherical geometry

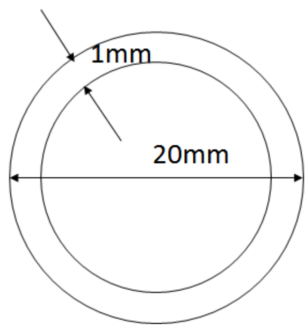
$r_{3sph} = 0$ --> $n_{3sphe} = 0$

m = 1

Y = (115-121)/(30-121)

$F_{o\_sp} := 1.2$ $\qquad t_{heating} := \frac{F_{o\_sp} \cdot R_{pot}^2}{\alpha_{pot}}$ $\qquad \boxed{t_{heating} = 53.3\,min}$

Problem 4



$D_{sball} := 20 \cdot mm$

$R_{sball} := \frac{D_{sball}}{2}$

$T_{bath} := 1300 \cdot K$

$T_{sball\_ini} := 300 \cdot K$

$T_{curing} := 1000 \cdot K$

$h_4 := 5000 \cdot \frac{W}{m^2 \cdot K}$

$c_{sball} := 500 \cdot \frac{J}{kg \cdot K}$ $\qquad \rho_{sball} := 7800 \cdot \frac{kg}{m^3}$ $\qquad k_{sball} := 50 \cdot \frac{W}{m \cdot K}$

Let's try to see if the Lumped parameter Analysis can be used, and let's calculate the Bi Number

$Bi_{sball} := \frac{h_4 \cdot R_{sball}}{k_{sball}}$ $\qquad Bi_{sball} = 1$ We cannot use the Lumped Parameter Analysis

This is a typical problem where the thermal conductivity of the solid is very high so thermal resistance to conduction in the solid is very small and one would think that the lumped parameter analysis will be suitable. However, the heat transfer outside by convection is very high (thermal resistance even smaller than the thermal resistance in the solid) and all the energy transferred by convection could not conducted without an significant temperature gradient, so the lumped parameter analysis will not work. In that sense the Biot number compares these two resistances.

$Y_{sball} := \frac{T_{curing} - T_{bath}}{T_{sball\_ini} - T_{bath}}$ $\qquad Y_{sball} = 0.3$

$m_{sball} := \frac{1}{Bi_{sball}}$ $\qquad m_{sball} = 1$ $\qquad x_{sball} := 9 \cdot mm$ $\qquad n_{sball} := \frac{x_{sball}}{R_{sball}}$ $\qquad n_{sball} = 0.9$

Using charts for spherical geometries $\boxed{Fo_{curing} := 0.44}$

$$t_{curing} := \frac{Fo_{curing} \cdot \rho_{sball} \cdot c_{sball} \cdot R_{sball}^{2}}{k_{sball}} \qquad \boxed{t_{curing} = 3.4\,s}$$

We can verify this result by using the approximate solution for a sphere, obviosuly these solutions work for Fo > 0.2, which apparasto be this case

For Bi = 1 from tables for spherical geometry we get:

$$C_{1ball} := 1.2732 \qquad \gamma_{1ball} := 1.5708$$

$$Fo\_curing\_Eq := \frac{-1}{\gamma_{1ball}^{2}} \cdot \ln\left(\frac{\gamma_{1ball} \cdot n_{sball}}{\sin\left(\gamma_{1ball} \cdot n_{sball}\right)} \cdot \frac{Y_{sball}}{C_{1ball}}\right)$$

$\boxed{Fo\_curing\_Eq = 0.44}$ Same value

Problem 5

$$T_{finger\_ini} := 33 \cdot K \qquad T_{damage} := 62 \cdot K$$

$$t_{contact} := 2 \cdot s \qquad T_{surface} := 200 \cdot K \qquad \alpha_{finger} := 2.5 \cdot 10^{-7} \cdot \frac{m^{2}}{s}$$

Remember that although the temperatures should be in degrees Celcius we are using temperatures in Kelvins to avoid problems with the units in MathCad. As long we deal with temperature differences there is not problem and no corrections are needed beyong remember that if a temperature is the result, that temperature need to be considered in degree Celcius. Lets' consider the solution for an semi-infinite domain, which is fine because the thermal diffusivity ofthe finger is very small.

$$Y_{finger} := \frac{T_{damage} - T_{finger\_ini}}{T_{surface} - T_{finger\_ini}} \qquad Y_{finger} = 0.174$$

$$\mathrm{erf}(\phi) = 1 - Y_{finger} \ \text{solve} \ \rightarrow 0.96206139227182396223$$

$$\phi_{d} := 0.962 \qquad x_{damaged} := \phi_{d} \cdot 2\sqrt{\alpha_{finger} \cdot t_{contact}}$$

$$\boxed{x_{damaged} = 1.36\,mm}$$

Checking

$$x_{condition} := 4\sqrt{\alpha_{finger} \cdot t_{contact}} \qquad x_{condition} = 2.828\,mm$$

it is right to assume a semi-infite domain neck = −1