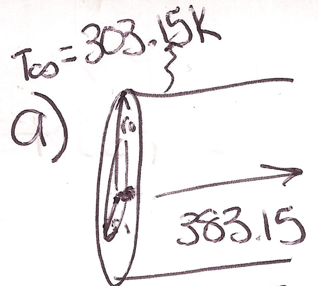
QUESTION ONE

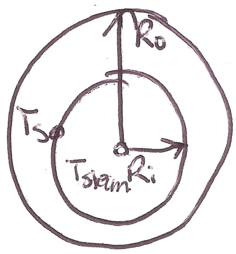
②

a) 

$h_i >> 1$

$R_{conv} = \frac{1}{\infty} = 0$

Assume:
→ s. state
→ 1D (r) one D heat flow in (r)
→ $h_{inside, steam}$ very large ∴ $R_{conv, inside}$ negligible ∴ $T_{steam} = T_s$



$$\left[\frac{\text{watts}}{m}\right] = q' = \frac{q}{L} = \frac{\overbrace{T_{steam} - T_\infty}^{\substack{\text{Driving force} \\ (T_{HOT} - T_{COLD})}}}{\sum R}$$

Uninsulated:

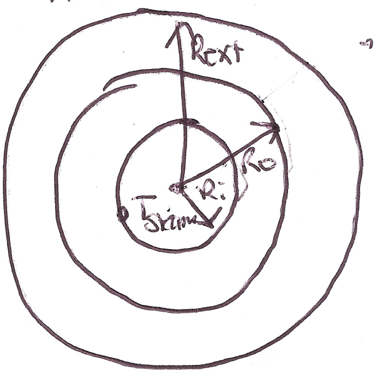
$$R_{cond} = \frac{\ln\left(\frac{R_o}{R_i}\right)}{2\pi K_{tube}} \qquad R_{conv} = \frac{1}{2\pi R_o h_o}$$

→ We do not have L so let's use q'

no L terms in the resistances ∴ q' units $\left[\frac{\text{watts}}{m}\right]$

$$q'_{un} = \frac{T_{steam} - T_\infty}{\left(\frac{\ln\left(\frac{R_o}{R_i}\right)}{2\pi k_{tube}}\right) + \left(\frac{1}{2\pi R_o h_o}\right)}$$

INSULATED What changes? New $R_{cond, insulation}$ and R_{conv} uses a new radius.



$$\underbrace{R_{cond, pipe}}_{\text{same}} + R_{cond, insulation} + R_{conv, new} = \sum R$$

$$R_{cond, insulation} = \left(\frac{\ln\left(\frac{R_{ext}}{R_o}\right)}{2\pi K_{ins}}\right) \qquad R_{conv, new} = \left(\frac{1}{2\pi h_o R_{ext}}\right)$$

$$\left. q'_{INS} = \frac{T_{steam} - T_\infty}{\sum R} \right\} \text{Solve for } q'_{INS}$$

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b) We want to compare q' results using different reference areas for U

~~INSULATED~~, INSIDE AREA

b) $\boxed{U_{ii} = \frac{1}{\sum R A_{ii}}}$

$$\cancel{A_{ii}}\left(\frac{\ln\left(\frac{R_o}{R_i}\right)}{2\pi K_{pipe}} + \frac{\ln\left(\frac{R_{ext}}{R_o}\right)}{\cancel{2\pi} k_{ins}} + \frac{1}{\cancel{2\pi} h_o R_{ext}}\right) \underbrace{(2\pi R_i)}_{A_i}$$

$A_{ii} = 2\pi R_i$
Area inside

$$\frac{R_i \ln\frac{R_o}{R_i}}{K_{pipe}} + \frac{\ln\left(\frac{R_{ext}}{R_o}\right) R_i}{k_{ins}} + \frac{R_i}{h_o R_{ext}} = A_{ii} \sum R$$

$A_o = 2\pi R_o$
Area outside, ~~ins~~ uninsulated

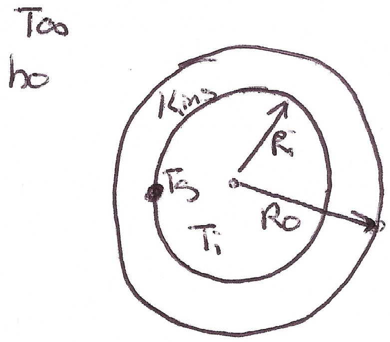
$$\boxed{U_{ii} = \frac{1}{\frac{R_i \ln\left(\frac{R_o}{R_i}\right)}{K_{pipe}} + \frac{\ln\left(\frac{R_o}{R_i}\right) R_i}{K_{ins}} + \frac{R_i}{h_o R_{ext}}}}$$

Use $q' = UA_i(T_s$
for all four scenarios:
INS, INSIDE A
INS, OUTSIDE A
UNINS, INSIDE A
UNINS, OUTSIDE A

$$q'_{INS} = UA_i(T_{steam} - T_\infty) = U_o A_o (T_{steam} - T_\infty)$$ ← This will be true

QUESTION TWO

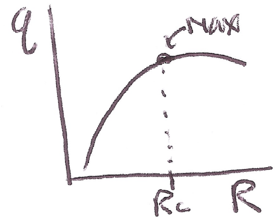
a)



$T_s = T_i$

$$\frac{q}{L} = q' = \frac{T_i - T_\infty}{R_{cond,ins} + R_{conv}}$$

$$\left. q' = \frac{T_i - T_\infty}{\frac{\ln\left(\frac{R}{R_i}\right)}{2\pi K_{insulation}} + \frac{1}{2\pi h_o R}} \right\} \text{Graph c)}$$



$q = f(r)$

We want q max, when $\frac{dq'}{dr} = 0$

□ This is where q' stops increasing and starts decreasing I.e. where the value of R is $R_{critical}$

$$\frac{dq'}{dr} = 0 = \frac{d}{dr}\left(\frac{1}{\frac{\ln\left(\frac{R}{R_i}\right)}{K} + \frac{1}{h_o R}}\right)$$

Chain Rule:

$$u = \frac{\ln\left(\frac{R}{R_i}\right)}{k} + \frac{1}{hR}$$

$$\frac{du}{dr} = \frac{1}{KR} - \frac{1}{hR^2}$$

$$0 = \frac{d}{du}\left(\frac{1}{u}\right)\frac{du}{dr}$$

$$0 = \frac{\cancel{-1}}{\cancel{u^2}}\left(\frac{1}{KR} - \frac{1}{hR^2}\right)$$

$$\frac{d}{du}\left(\frac{1}{u}\right) = \frac{-1}{u^2}$$

$$0 = \frac{1}{KR} - \frac{1}{hR^2}$$

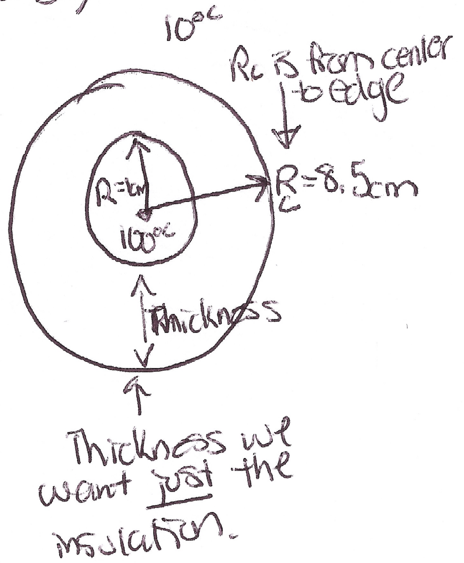
$$0 = \frac{Rh - K}{\cancel{KhR^3}}$$

$$R_c = \frac{K_{insulation}}{h_{outside}}$$

↑ $R_{critical}$

□ Note: I simplified the expression Remaining $2\pi, (T_i - T_\infty)$ because zero is on the left. Same method to get rid of $\left(\frac{-1}{u^2}\right)$, I didn't bother writing u^2 back in because we can multiply it by both sides.

2b)



$$R_c = \frac{k}{h_o} = \frac{0.17}{2} = 8.5\text{cm}$$

Insulation Thickness = $R_c - R_{tube} = 8.5 - 1 = 7.5$

$$q' = \frac{T_i - T_\infty}{R_{cond,ins} + R_{conv}}$$

$$q' = \frac{T_i - T_\infty}{\left(\frac{\ln\left(\frac{R_{ins}}{R_i}\right)}{2\pi K_{ins}}\right) + \left(\frac{1}{2\pi R_{insulation} h_o}\right)}$$

$R_{insulation} = R_c$
I wrote R_{ins} meaning the critical insulation thickness we just solved for.

d) Sphere Derive R_c:

$$q' = \frac{T_i - T_\infty}{\left(\frac{R - R_i}{4\pi K_{ins} R R_i}\right) + \left(\frac{1}{4\pi R^2 h_o}\right)}$$

Chain Rule

$$u = \frac{R - R_i}{K_{ins} R R_i} + \frac{1}{R^2 h_o}$$

$$\frac{du}{dr} = \text{ try this one}$$

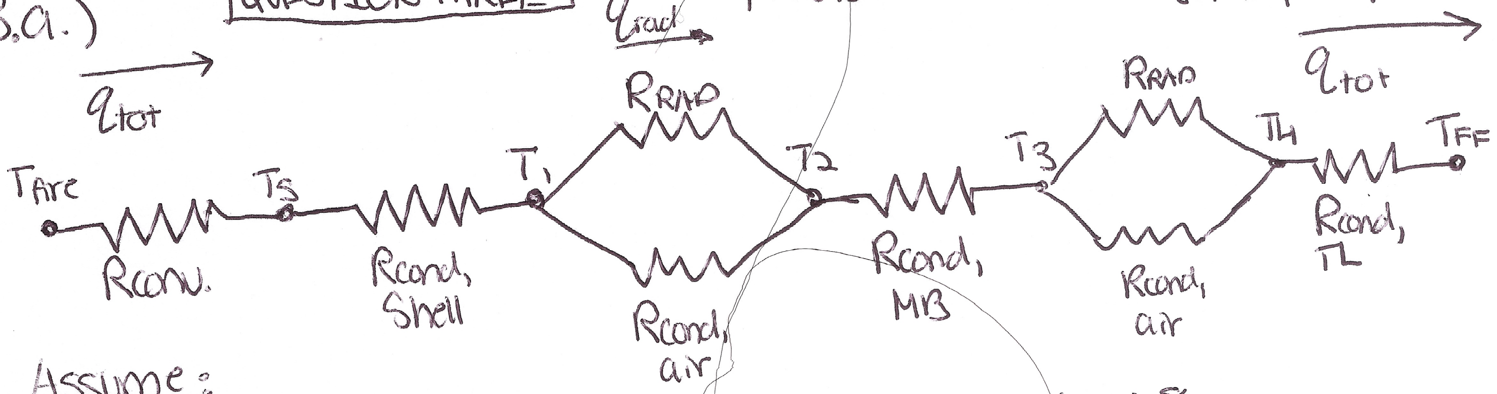
$$\frac{dq'}{dr} = 0 = \frac{d}{dr}\left(\frac{1}{\left(\frac{R - R_i}{K_{ins} R R_i}\right) + \left(\frac{1}{R^2 h_o}\right)}\right)$$

Same method, math is just chain rule and simplify.

I will set up this one, similar to window problem last week (examples).

QUESTION THREE

3.a.)



Assume:
- h_o outside very large
- SS
- 1D - heat flow 1

b)

□ $R_{gap} = \dfrac{1}{\frac{1}{R_{RAD}} + \frac{1}{R_{air,cond}}}$

q_{cond}

Table C.8

K_{air} at 470^{oC} =

□ $R_{RAD} = \dfrac{1}{h_{RAD}}$

$h_{rad} = 4(5.676e^{-8})(\underbrace{470+273.15}_{\text{MUST use K.}})^3$

□ $R_{air,cond} = \dfrac{L_{gap}}{K_{air}}$

Steady state

$q''_{fire} = q''_{total}$

$T_{skin} = 50^{oC} + 273.15$

c) $q'' = 2500 \frac{W}{m^2} = \dfrac{T_s - T_{skin}}{\sum R}$

→ Notice units we do not have an area so use flux

found all of these in b)

$\left.\begin{array}{l} \square\ R_{cond, Shell} \\ \square\ R_{cond, MB} \\ \square\ R_{cond, TL} \\ \square\ 2 \cdot R_{gap} \end{array}\right\} \sum R$

$R_{conv, outside} = 0$

b) Compare R's, which offers best protection? why? Which offers least protection? why?

QUESTION FOUR

(4)

Similar to previous questions, solving for some radius where there is frostbite ie where $T = -1^{\circ C}$

① $$q_{no,\,glove} = \frac{T_B - T_\infty}{\Sigma R}$$

Same, steady state

② $$q_{no,\,glove} = \frac{T_B - T_{FB}}{R_{cond,M} + R_{cond,D}}$$ ← Use Kelvin!

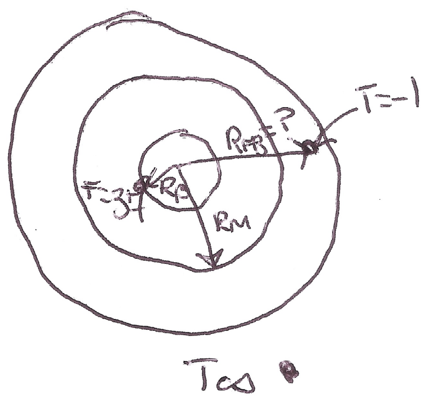
$$q_{no\ glove} = \frac{T_B - T_{FB}}{R_{cond,M} + \frac{\ln\left(\frac{R_{FB}}{R_M}\right)}{2\pi K_D}}$$

← Solve for (R_{FB})

← R_M

$$R_{cond,M} = \frac{\ln\left(\frac{R_M}{R_B}\right)}{2\pi K_M}$$

□ Solve for R_{FB}



Note: You could also use

$$q_{no,\,glove} = \frac{T_{FB} - T_\infty}{R_{conv} + \frac{\ln\left(\frac{R_D}{R_{FB}}\right)}{2\pi K_D}}$$

← outside dermis layer

← radius at $T_{FB} = -1$

because steady state

⑤ Slide 15 L3

QUESTION FIVE

$$q_{fin,infinite} = \sqrt{K_{fin} h_{fin,air} A_c P}\ \theta_b$$

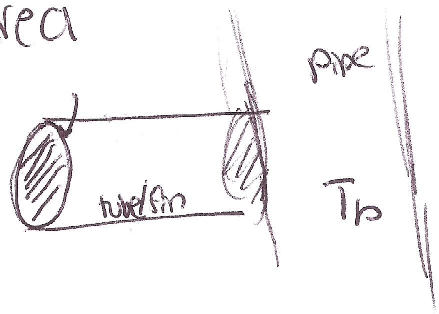
a)

A_c = cross sectional area

$$A_c = \frac{\pi D_{tube}^2}{4} \text{ or } \pi R^2$$

P = perimeter $= 2\pi R$ or πD

$$\theta_b = (T_b - T_\infty)$$



b) Finite fin of 25mm → 0.025m

$q_{no\ fin} = h A_c (T_b - T_\infty)$, Slide 15 Right Side

$$q_{no\ fin} \times \frac{q_{fin,finite}}{q_{non\ fin}} = q_{fin\ finite}$$