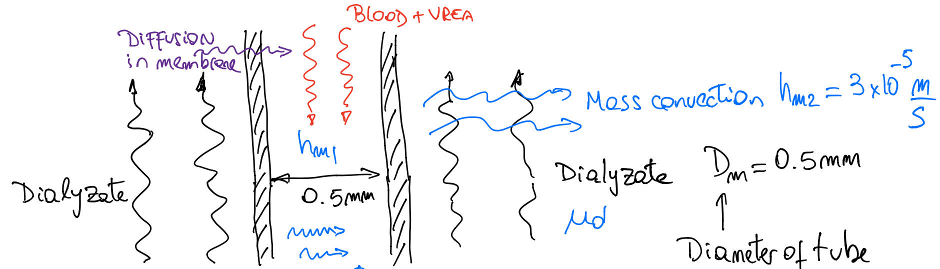
ABE 30800 STEADY STATE MASS TRANSFER

EXAMPLES – SPRING 2018

(1)

Problem 1



$\mu_b = 3\times10^{-3}$ Pa.s viscosity of blood

$\rho_b = 1020\ \frac{kg}{m^3}$ Density of blood

$C_{ub} = 2\times10^{-4}\ \frac{g}{cm^3}$ Concentration of urea in the blood

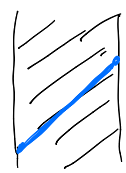
$C_{u,d} = 0\ \frac{g}{cm^3}$ concentration of urea in the dialyzate

$D_{u,m} = 1\times10^{-10}\ \frac{m^2}{s}$ Diffusivity of urea in membrane

$D_{u,b} = 1.5\times10^{-9}\ \frac{m^2}{s}$ Diffusivity of urea in blood

$K^* = 2$

$\bar{u}_b = 2.5$ m/s velocity of blood in the tube



Linear profile because steady state is assumed, along slab geometry because thickness is << radius

(a) Mass transfer coefficient on the blood side

This is an internal flow

$$Re = \frac{D_m \bar{u}_b \rho_b}{\mu_b} = \frac{0.5\times10^{-3}\ m \times 2.5\ m/s \times 1020\ kg/m^3}{3\times10^{-3}\ Pa.s} = 425$$

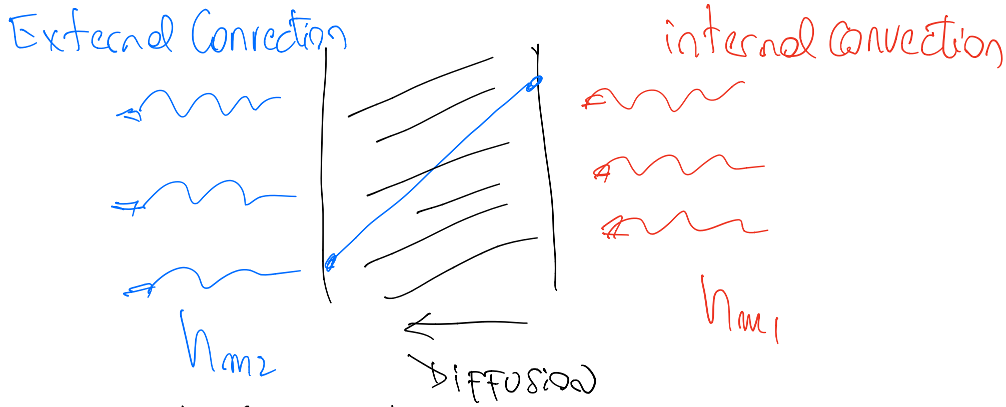
$Re = 425$ LAMINAR FLOW

FOR INTERNAL AND LAMINAR FLOW $Sh = 3.66 = \frac{h_{m1} D_m}{D_{u,b}} \Rightarrow h_{m1} = \frac{3.66\, D_{u,b}}{D_m}$

$$h_{m1} = \frac{3.66\times1.5\times10^{-9}\ m^2/s}{0.5\times10^{-3}\ m} \simeq 1.1\times10^{5}\ m/s$$

$$\boxed{h_{m1} = 1.1\times10^{-5}\ m/s}$$

(b) Overall mass transfer coefficient (2)



By assuming slab geometry (see above)

$$\frac{1}{U_m} = \frac{1}{h_{m1}} + \frac{\Delta L}{K^* D_{u,m}} + \frac{1}{h_{m2}}$$

$\Delta L = 2.5\,\mu m = 2.5 \times 10^{-6}\, m$ ← membrane thickness

$$\frac{1}{U_m} = \frac{1}{1.1 \times 10^{-5}\, \frac{m}{s}} + \frac{2.5 \times 10^{-6}\, m}{2 \times 1 \times 10^{-10}\, m^2/s} + \frac{1}{3 \times 10^{-5}\, \frac{m}{s}} = 249\,242.4\, \frac{s}{m}$$

$$\boxed{U_m = 4 \times 10^{-6}\, \frac{m}{s}}$$

(c) Number of tubes of 24 cm necessary to remove $5\, \frac{g}{h}$ urea

$$N_{urea} = U_m A \left(C_{u,b} - C_{u,d} \right) \qquad (1)$$

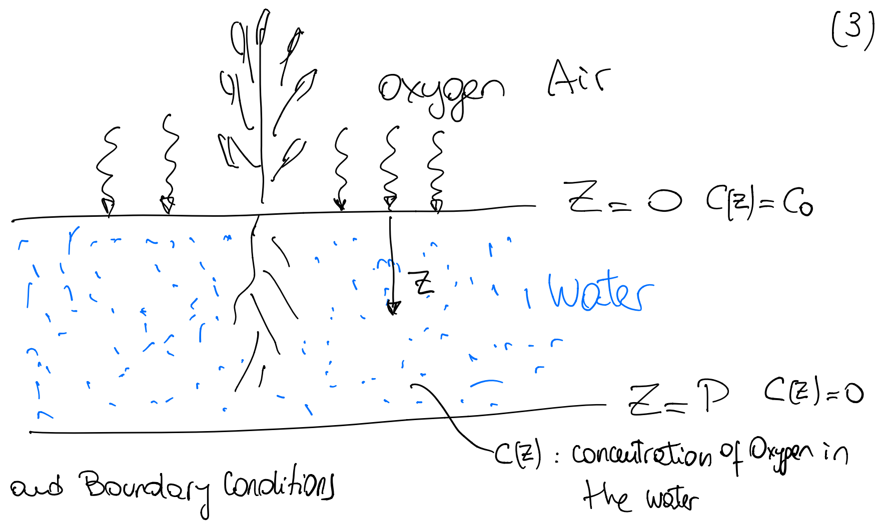
$$A = N_{Tubes}\, \pi D L \qquad (2)$$

$$N_{urea} = U_m N_T \pi D_m L \left(C_{u,b} - C_{u,d} \right) \implies N_T = \frac{N_{urea}}{U_m \pi D_m L \left(C_{u,b} - C_{u,d} \right)}$$

$$N_T = \frac{5\, \cancel{g}/\cancel{h} \times 1\cancel{h}/3600\, \cancel{s}}{4 \times 10^{-6}\, \frac{\cancel{m}}{\cancel{s}} \times \pi \times 0.5 \times 10^{-3}\, \cancel{m} \times 0.24\, \cancel{m} \times 2 \times 10^{-4}\, \frac{\cancel{g}}{\cancel{cm^3}} \times 10^6\, \frac{\cancel{cm^3}}{\cancel{m^3}}} =$$

$$\boxed{N_T \cong 4605 \text{ tubes}}$$

Problem 2



(a) & (b) Equations and Boundary Conditions

Assumptions

- steady state
- No convection
- reaction is Zero order $r_0 = -K$
- diffusion in direction Z

$$\underset{\text{steady state}}{\frac{\partial C(z)}{\partial t}} + \underset{\text{No convection}}{\cancel{u\frac{\partial C(z)}{\partial z}}} = \underset{\text{diffusion}}{D_{o,w}\frac{\partial^2 C(z)}{\partial z^2}} + \underset{\text{reaction}}{r_0}$$

$$\begin{cases} D_{o,w}\dfrac{d^2C(z)}{dz^2} - K = 0 & (1) \\ C(z) = C_0 \quad \text{at } Z = 0 & (1a) \\ C(z) = 0 \quad \text{at } Z = P & (1b) \\ \dfrac{dC(z)}{dz} = 0 \quad \text{at } Z = P & (1c) \end{cases}$$

(c) Determine P

we need to find the concentration of oxygen Profile by solving the differential equation and Boundary conditions given by Equations (1), (1a,b,c)

Integrating once $\quad \dfrac{dC(z)}{dz} = \dfrac{K}{D_{o,w}} z + A \qquad (2)$

Integrating again $\quad C(z) = \dfrac{K}{2D_{o,w}} z^2 + Az + B \qquad (3)$

The integration constants can be obtained from the Boundary Conditions (4)

Using Eq.(2) in Eq.(1c)

$$0 = \frac{K}{D_{o,w}} P + A \Rightarrow A = -\frac{K P}{D_{o,w}} \quad (4)$$

$$C(z) = \frac{K}{2D_{o,w}} z^2 - \frac{K}{D_{o,w}} P z + B \quad (5)$$

using BC [Eq. 1a] $\quad C(z=0) = C_o = B \quad (6)$

Substituting into Eq.(5)

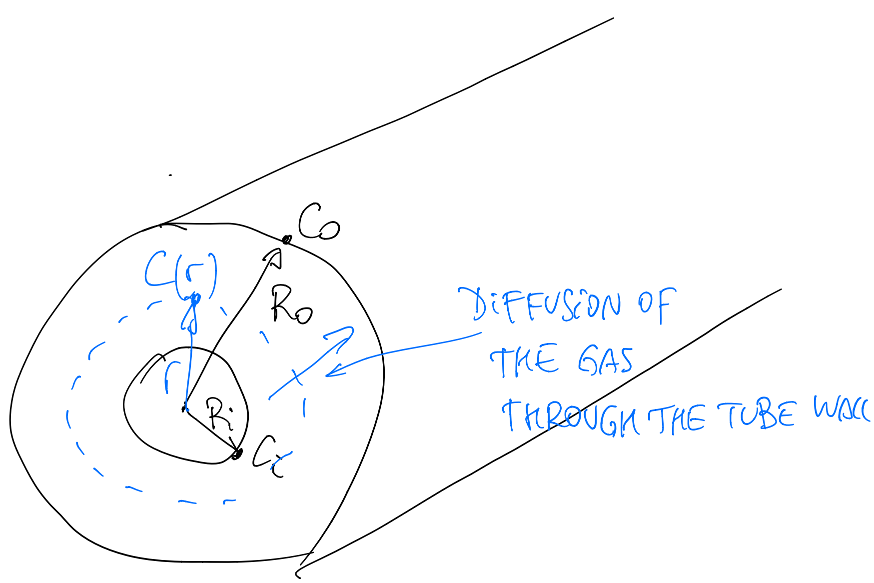
$$C(z) = C_o + \frac{KP}{D_{o,w}} \left[\frac{z^2}{2P} - z\right] \quad (7)$$

Applying Boundary Condition (Eq. 1b) in Equation (7) we can get

$$C(z=P) = 0 = C_o + \frac{KP}{D_{o,w}} \left[\frac{P^{\cancel{2}}}{2\cancel{P}} - P\right] \Longrightarrow \frac{KP^2}{2D_{o,w}} = C_o \Rightarrow P = \sqrt{\frac{2D_{o,w} C_o}{K}}$$

$$\boxed{P = \sqrt{\frac{2 D_{o,w} C_o}{K}}}$$

Problem 3



By assuming cylindrical geometry, steady state, no convection, radial diffusion and no chemical reaction the mass transfer equation can be simplified

(5)

$$\frac{\partial C(\cancel{\varphi},t)}{\cancel{\partial t}} = D\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C(r,t)}{\partial r}\right) + \cancel{\frac{1}{r^2}\frac{\partial^2 C(r,t)}{\partial \varphi^2}} + \cancel{\frac{\partial^2 C(r,t)}{\partial z^2}}\right] + \cancel{r} \quad (1)$$

The simplified equation and boundary conditions are :

$$\begin{cases} \frac{1}{r}\frac{d}{dr}\left(r\frac{dC(r)}{dr}\right) = 0 & (2) \\ C(r=R_i) = C_i \text{ at } r = R_i & (2a) \\ C(r=R_o) = C_o \text{ at } r = R_o & (2b) \end{cases}$$

By integrating Eq.(2) twice

$$r\frac{dC(r)}{dr} = A \Longrightarrow \frac{dC(r)}{dr} = \frac{A}{r} \quad (3)$$

$$C(r) = A\ln r + B \quad (4)$$

By using the Boundary Conditions

$$C(r=R_i) = A\ln R_i + B = C_i \quad (5)$$

$$C(r=R_o) = A\ln R_o + B = C_o \quad (6)$$

From Eqs (5) and (6) $\quad C_o - C_i = A\ln\frac{R_o}{R_i} \Longrightarrow A = \frac{C_o - C_i}{\ln R_o/R_i} \quad (7)$

From Eq.(5) $\quad B = C_i - \frac{C_o - C_i}{\ln R_o/R_i}\ln R_i \quad (8)$

Substituting into Eq.(4)

$$C(r) = C_i - \frac{C_o - C_i}{\ln R_o/R_i}\ln R_i + \frac{C_o - C_i}{\ln R_o/R_i}\ln r$$

$$\boxed{C(r) = C_i + \frac{C_o - C_i}{\ln R_o/R_i}\ln r/R_i} \quad (9)$$

The Diffusion flow of a component A is :

$$N_A = -D_A A\left.\frac{dC_A}{dr}\right|_{r=R_i} = -D_A 2\pi R_i L\left.\frac{dC_A}{dr}\right|_{r=R_i} \quad (10)$$

From Eq.(9) (6)

$$\frac{dC(r)}{dr}\bigg|_{r=R_i} = \frac{C_o - C_i}{\ln R_o/R_i}\frac{1}{R_i} \quad \text{and} \quad -\frac{dC(r)}{dr}\bigg|_{r=R_i} = \frac{C_i - C_o}{\ln R_o/R_i}\frac{1}{R_i} \qquad (11)$$

Substituting Eq.(11) onto Eq.(10)

$$N_A\Big|_{r=R_i} = -D_A\, 2\pi \cancel{R_i} L\, \frac{C_o - C_i}{\ln R_o/R_i}\cdot\frac{1}{\cancel{R_i}} = D_A\, 2\pi L\,\frac{C_i - C_o}{\ln R_o/R_i} = \frac{C_i - C_o}{\frac{\ln R_o/R_i}{2\pi L D_A}}$$

$$N_A\Big|_{r=R_i} = \frac{C_i - C_o}{\underbrace{\frac{\ln R_o/R_i}{2\pi L D_A}}_{Res,diff}} \qquad Res,_{diff} = \frac{\ln R_o/R_i}{2\pi L D_A} \quad (12)$$

(3) Expand to include convection inside and outside with convection coefficients h_{m1} and h_{m2}

Let's assume the concentration of the gas is C_{gi} and the concentration of the gas at the inner surface but in the gas phase is $C_{in,f}$. Convection at the inner surface can be calculated as:

$$N_A\Big|_{r=R_i} = h_{m1}\, 2\pi R_i L\,(C_{gi} - C_{in,f}) = \frac{C_{gi} - C_{in,f}}{\underbrace{\frac{1}{2\pi R_i L h_{m1}}}_{Res,conv1}} \qquad (13)$$

$$Res,_{conv1} = \frac{1}{2\pi R_i L h_{m1}} \qquad (14)$$

If the concentration of gas outside is C_{go} and the concentration of gas at the outer surface but in the gas phase is $C_{out,f}$ the mass convection is:

$$N_A\Big|_{r=R_o} = h_{m2}\, 2\pi R_o L\,(C_{out,f} - C_{go}) \qquad (15)$$

and $N_A|_{r=R_o} = \dfrac{C_{out,f} - C_{so}}{\underbrace{\dfrac{1}{2\pi R_o L h_{m2}}}_{Res_{convo}}}$ (7)

$$Res_{convo} = \frac{1}{2\pi R_o L h_{m2}} \quad (16)$$

By considering that $C_{is} = K^* C_{in,f}$ and $C_{os} = K^* C_{out,f}$ can be shown that for <u>steady state</u>

$$N_A = \frac{C_{is} - C_{os}}{\underbrace{\dfrac{1}{2\pi R_i L h_{m1}} + \dfrac{\ln R_o/R_i}{2\pi L D_A K^*} + \dfrac{1}{2\pi R_o L h_{m2}}}_{\text{TOTAL MASS RESISTANCE}}}$$

$$\text{TOTAL MASS RESISTANCE} = \frac{1}{2\pi R_i L h_{m1}} + \frac{\ln R_o/R_i}{2\pi L K^* D_A} + \frac{1}{2\pi R_o L h_{m2}} \quad (17)$$

$$\boxed{U_m = \frac{1}{\text{TOTAL MASS RESISTANCE}}}$$

Problem 4

(1) Equation for the drug concentration as a function of position (x)

Solution of a problem considering diffusion and a first order reaction under conditions of steady state and not convection are:

$$C(x) = C_o e^{-\sqrt{\frac{K}{D}}x} \quad (1)$$

$$\sqrt{\frac{K}{D}} = \sqrt{\frac{0.6\ 1/\cancel{h} \times 1\cancel{h}/3600s}{40 \times 10^{-11}\ m^2/s}} \simeq 645.5\ \frac{1}{m}$$

$$C(x) = C_o e^{-645.5x} \quad (2)$$

(2) if $C(x=x_o) = 0.10 C_o$

$$C(x_o) = 0.10 \cancel{C_o} = \cancel{C_o} e^{-645.5 x_o}$$

$$X_0 = -\frac{\ln 0.10}{645.5} \cong 3.6\times10^{-3}\,m$$ (8)

$$\boxed{X_0 = 3.6\,mm}$$

(3) $C_{avg} = 8\mu g/g$

$$C_{avg} = \frac{1}{X_0}\int_{X=0}^{X=X_0} C(x)\,dX = \frac{1}{X_0}\frac{C_0}{-645.5}\,\ell^{-645.5}\Big|_{X=0}^{X=X_0}$$

$$C_{avg} = \frac{1}{3.6\times10^{-3}}\frac{C_0}{645.5}\left[1-\ell^{-645.5\times3.6\times10^{-3}}\right]$$

$$8\mu g/g = C_{avg} = \frac{C_0}{2.32}\left[1-\ell^{-2.32}\right] = 0.39\,C_0$$

$$C_0 = \frac{8\mu g/g}{0.39} = 20.6\,\mu g/g \qquad \boxed{C_0 = 20.6\,\mu g/g}$$