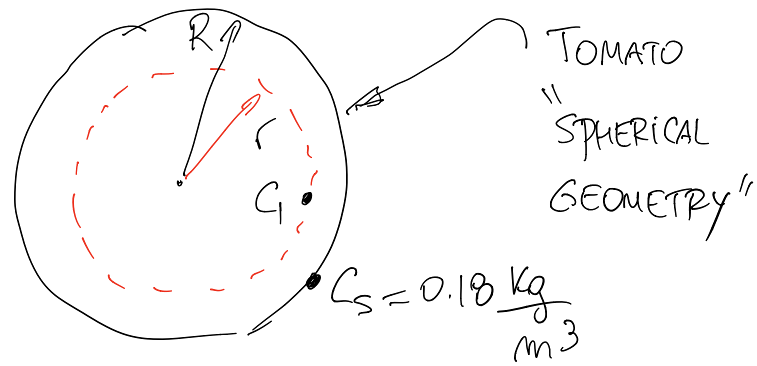
SOLUTIONS HOMEWORK 7
ABE 30800 – SPRING 2018

Problem 1

$C_i = 0.02 \frac{kg}{m^3}$

$D_{CO_2} = D = 2.3 \times 10^{-8}\, m^2/s$

$D_t = 4\ cm$



at $\frac{r}{R} = 0.8$ $\quad C_{CO_2} \equiv C_1 = 0.17 \frac{kg}{m^3}$

(1) Calculate the concentration of CO_2 at the center of the tomato at that time

Let's assume spherical geometry and radial diffusion

We can use the Heisler charts for a sphere

$$Y = \frac{C_1 - C_\infty}{C_i - C_\infty} = \frac{C_1 - C_s}{C_i - C_s} = \frac{0.17 - 0.18}{0.02 - 0.18} = 0.0625$$

$$\left.\begin{array}{l} Y = 0.0625 \\ n = \frac{r}{R} = 0.8 \\ m = \frac{1}{Bi} \to 0 \end{array}\right\} \Longrightarrow Fo = \frac{Dt}{R^2} = 0.2 \Longrightarrow t = \frac{0.2 R^2}{D} = \frac{0.2 \times (2 \times 10^{-2})^2\, m^2}{2.3 \times 10^{-8}\, m^2/s}$$

$$\boxed{t = 3478.3\ s}$$

For that time (i.e. $Fo = 0.2$) we can calculate the concentration at the center using Heisler charts for spheres

$$\left.\begin{array}{l} Fo = 0.2 \\ r/R = 0 \ \text{(center)} \\ m = \frac{1}{Bi} = 0 \end{array}\right\} \Longrightarrow Y = 0.2 = \frac{C_c - C_s}{C_i - C_s} \Longrightarrow C_c = C_s + 0.2(C_i - C_s)$$

$$C_c = 0.18 - 0.2(0.18 - 0.02) = 0.15\ kg/m^3$$

$$\boxed{C_c = 0.15\ kg/m^3}$$

We can solve the problem also using tables which are fine for $F_o \geq 0.2$ (2)

From Tables for $Bi \to \infty$ for a sphere $\gamma_1 = 3.1415$ and $C_1 = 2$

So
$$\frac{C_c - C_s}{C_i - C_s} = C_1 e^{-\gamma_1^2 F_o} \cdot \frac{1}{\gamma_1} \cdot \frac{\sin(\gamma_1 r/R)}{r/R} \qquad (1)$$

For $\frac{r}{R} = 0$ (center) we have $\frac{\sin(\gamma_1 r/R)}{r/R} = \frac{0}{0}$ but in reality we need

to find out $\lim\limits_{r/R \to 0} \frac{\sin(\gamma_1 r/R)}{r/R} \to \gamma_1$

Therefore Eq. (1) for $r/R \to 0$ yields:

$$\frac{C_c - C_s}{C_i - C_s} = C_1 e^{-\gamma_1^2 F_o} \frac{1}{\cancel{\gamma_1}} \cdot \cancel{\gamma_1} = C_1 e^{-\gamma_1^2 F_o} = 2 e^{-3.1415^2 \times 0.2} = 0.28$$

$$C_c = C_s - 0.28(C_s - C_i) = 0.18 - 0.28(0.18 - 0.02) = 0.14$$

$\boxed{C_c = 0.14 \text{ kg/m}^3}$ Very close value at that obtained using the Heisler charts

(2) if a concentration of 0.16 kg/m³ CO_2 is required at the center, what additional time is necessary

$$\left.\begin{aligned} Y &= \frac{0.16 - 0.18}{0.02 - 0.18} = 0.13 \\ n &= r/R = 0 \\ m &= 0 \end{aligned}\right\} \quad \begin{aligned} &F_o \cong 0.6 \\ &F_o = \frac{Dt}{R^2} \Rightarrow t = \frac{0.6 \times (2 \times 10^{-2})^2 m^2}{2.3 \times 10^{-8} m^2/s} = 10{,}434.8 s \end{aligned}$$

So $t = 3478.3 s \times \frac{1h}{3600 s} = 0.97h$

and $t = 10{,}434.8 \cancel{s} \times \frac{1h}{3600 \cancel{s}} = 2.9$ hrs

So we need to add 1.9 hours

(3) exposure time to reach same concentration if $\frac{R}{2}$

it has been demonstrated that: $t \propto R^2$

So $t_1 = A R_1^2$ and $t_2 = A R_2^2$

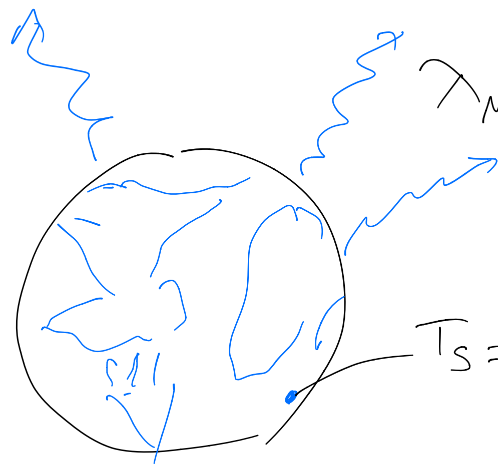
So $\frac{t_1}{t_2} = \left(\frac{R_1}{R_2}\right)^2 \longrightarrow \frac{t_2}{t_1} = \left(\frac{R_2}{R_1}\right)^2 = \left(\frac{R/2}{R}\right)^2 = \left(\frac{1}{2}\right)^2 = 0.25$ (3)

$\boxed{t_2 = 0.25\, t_1}$

$t_2 = 0.25 \times 0.97\,hrs = 0.24\,hrs$

$\boxed{t_2 = 0.24\,hrs}$

Problem 2



λ microwaves $10^4 - 10^5\,\mu m$

$T_s = 14\,°C$

How much the earth in the microwave region $10^4 - 10^5\,\mu m$ the earth emits?

For a black body

$$E_{b,\lambda} = \frac{2\pi c^2 h \lambda^{-5}}{\exp\left(\frac{ch}{K\lambda T} - 1\right)} \qquad (1)$$

$c = 3\times10^8\,m/s$ [Velocity of the light]

$h = 6.625\times10^{-34}\,J.s$

$K = 1.38\times10^{-23}\,J/K$

$T = 14\,°C = 287\,K$

$\lambda = 10^5\,\mu m = 0.1\,m$

substituting values into Eq.(1) For $\lambda = 10^5\,\mu m$

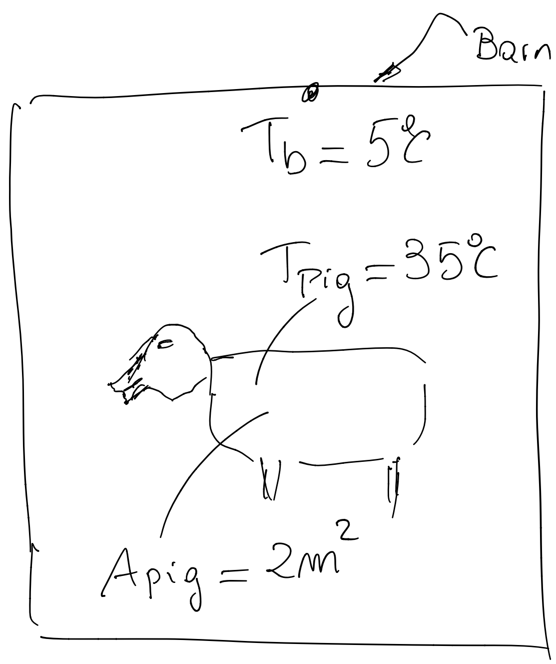
$$E_1 = \frac{2\pi(3\times10^8)^2 \cdot 6.625\times10^{-34}\, 0.1^{-5}}{\exp\left(\frac{3\times10^8 \times 6.625\times10^{-34}}{1.38\times10^{-23}\times 0.1\times 287}\right) - 1} = 7.5\times10^{-14}\,\frac{W}{m^2.\mu m}$$

For $\lambda = 10^4 \mu m$ $\quad E_2 = \dfrac{2\pi(3\times10^8)^2\, 6.625\times10^{-34} \times (10^{-2} m)^{-5}}{\exp\left(\dfrac{3\times10^8 \times 6.625\times10^{-34}}{1.38\times10^{-23} \times 0.01 \times 287}\right) - 1} = 7.5\times10^{-10} \dfrac{W}{m^2 \mu m}$ (4)

$$E_{earth} = \left(\frac{E_1 + E_2}{2}\right)\Delta\lambda = \frac{7.5\times10^{-14} + 7.5\times10^{-10}}{2} \frac{W}{m^2 \cdot \cancel{\mu m}} \cdot [10^5 - 10^4]\, \cancel{\mu m}$$

$$\boxed{E_{earth} = 3.4\times10^{-5} \frac{W}{m^2}}$$

Problem 3



$\varepsilon_{pig} = 0.90$

(1) Calculate the net energy from the pig
We can use the solution of a body inside an enclosure

$$q_{pig-barn} = \frac{\sigma(T_{pig}^4 - T_{barn}^4)}{\dfrac{1-\varepsilon_{pig}}{\varepsilon_{pig} A_{pig}} + \dfrac{1}{A_{pig} F_{pig-barn}} + \dfrac{1-\varepsilon_{barn}}{\varepsilon_{barn} A_{barn}}} \quad (1)$$

$$q_{pig-barn} = \frac{\sigma(T_{pig}^4 - T_{barn}^4)}{\dfrac{1}{\varepsilon_{pig} A_{pig}}\left[1 - \cancel{\varepsilon_{pig}} - \dfrac{\cancel{\varepsilon_{pig} A_{pig}}}{\cancel{A_{pig}} F_{pig-barn}} + \dfrac{(1-\varepsilon_{barn})\,\varepsilon_{pig} A_{pig}}{\varepsilon_{barn} A_{barn}}\right]} \quad (2)$$

In Eq.(2) we can assume that $F_{pig-barn} = 1$ [the pig can see the overall barn] (5)

and that $A_{pig}/A_{barn} \approx 0$, so Eq.(2) is simplified to:

$$q_{pig-barn} = \varepsilon_{pig} A_{pig} \sigma \left(T_{pig}^4 - T_{barn}^4\right) \qquad (3)$$

Substituting values into Eq.(3) we can get:

$$q_{pig-barn} = 0.90 \times 2\,\cancel{m^2} \times 5.67 \times 10^{-8} \frac{W}{\cancel{m^2 K^4}} \left[(35+273)^4 - (5+273)^4 \right] \cancel{K^4}$$

$$\boxed{q_{pig-barn} = 264\ W}$$

(2) Calculate the radiation emitted by the pig and explain differences with the one calculated in (1)

$$q_{pig} = \varepsilon_{pig} A_{pig} \sigma T_{pig}^4 = 0.90 \times 2\,\cancel{m^2}\ 5.67 \times 10^{-8} \frac{W}{\cancel{m^2 K^4}} \times (35+273)^4 \cancel{K^4}$$

$$q_{pig} \cong 918\ W$$

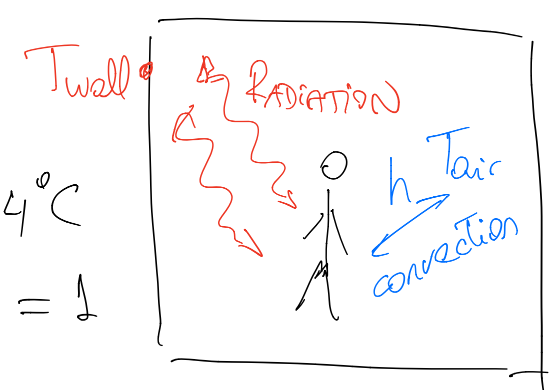
The difference in the net heat calculated in (1) and (2) is that part of the energy returns from the surroundings (the cold barn). This problem shows the importance of thermal radiation coming from another object, and in this case the barn that is cold.

It may explain why we feel cold when we sit near a window in cold weather even if the indoor temperature is adequate

Problem 4

$T_s = 34\,°C$

$F_{p\text{-}wall} = 1$



$q''_{gen} = 90\ \frac{W}{m^2}$

$A = 2\,m^2$

$h = 12\ \frac{W}{m^2 K}$

(1) By ignoring evaporative cooling (sweating) derive an Equation to estimate the air temperature as a function of Twall (6)

Under steady state $q_{generation}$ = heat loss by Convection + net radiation (1)

$$q''_{gen} \cdot A = hA(T_s - T_{air}) + q_{per\text{-}wall} \quad (2)$$

but $q_{per\text{-}wall} = A F_{proom} \sigma [T_s^4 - T_{wall}^4) \quad (3)$

Substituting Eq.(3) into Eq.(2) we can obtain:

$$q''_{gen} \cancel{A} = h\cancel{A}(T_s - T_{air}) + \cancel{A} F_{p\text{-}room} \sigma [T_s^4 - T_{wall}^4]$$

Assuming $F_{p\text{-}room} \cong 1$

$$q''_{gen} = h(T_s - T_{air}) + \sigma(T_s^4 - T_{wall}^4) \quad (4)$$

$$T_s^4 - T_{wall}^4 = (T_s^2 - T_{wall}^2)(T_s^2 + T_{wall}^2) = (T_s - T_{wall})(T_s + T_{wall})(T_s^2 + T_{wall}^2) \quad (5)$$

but since $T_s \approx T_{wall}$ $\quad T_s + T_{wall} \approx 2T_s \quad (6)$

and $\quad T_s^2 + T_{wall}^2 \approx 2T_s^2 \quad (7)$

substituting Eqs (5), (6) and (7) into Eq.(4)

$$q''_{gen} = h(T_s - T_{air}) + 4\sigma T_s^3 (T_s - T_{wall}) \quad (8)$$

$$h(T_{air} - T_s) = 4\sigma T_s^3 - q''_{gen} - 4\sigma T_s^3 T_{wall}$$

$$T_{air} = \frac{1}{h}[4\sigma T_s^3 + hT_s - q''_{gen}] - \frac{4\sigma T_s^3 T_{wall}}{h} \quad (9)$$

substituting values into Eq.(9) we can obtain the following equation

$$T_{air} = \frac{1}{12}[4 \times 5.67 \times 10^{-8} \times 307^3 + 12 \times 307 - 90] - \frac{4 \times 5.67 \times 10^{-8} \times 307^3 \, T_{wall}}{12}$$

$$T_{air} = 468.4K - 0.55\, T_{wall} \quad (10)$$

(2) The slope of the line in the plot is -0.57, so the model given by Eq. 10 appears to be accurate. The dashed line has a similar slope but a lower intercept. The intercept of the line is controlled by the heat generation, which in this case is $90 \frac{W}{m^2}$. A large generation of heat, e.g. a person running, may require a lower wall temperature to be in the comfort zone (7)