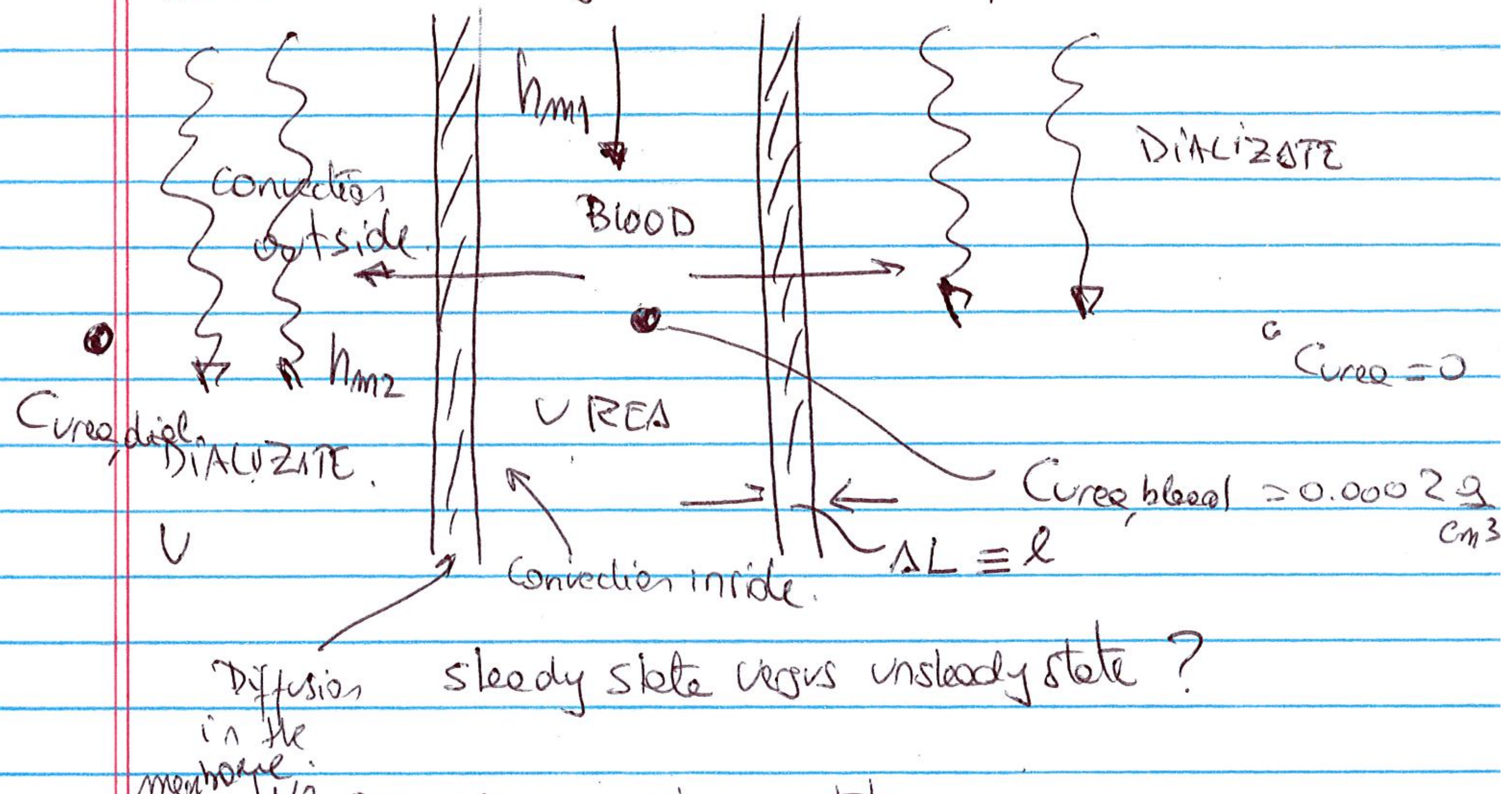
NOTES LECTURE 4_12_18

Problem 1 – Steady state Mass transfer.



steady state versus unsteady state?

We are assuming steady state

$$n_{urea} = \frac{C_{urea,blood} - C_{urea,dial}}{\text{Resist to MT Convection inside} + \text{Resist. to MT diffusion in membrane} + \text{Resist to MT convection outside}}$$

n_{urea}: $\frac{g}{cm^2 \times s}$

the thickness of the membrane $25 \mu m$ is very small compared to the tube diameter (0.5 mm) so we can use rectangular coordinates.

$$n_{vreo} = \frac{0.0002\ g/cm^3 - 0}{\dfrac{1}{h_{m1}} + \dfrac{25\ \cancel{\mu m} \times \frac{1cm}{10^4 \cancel{\mu m}}}{1.5\times10^{-9} \frac{\cancel{m^2}}{s} \times \frac{1cm^2}{10^{-4}\cancel{m^2}} \times 2} + \dfrac{1}{3\times10^{-5} \frac{m}{s} \times \frac{1cm}{10^{-2}m}}}$$

$$\frac{1}{h_{m1}}: \quad 1.5\times10^{-5}\ \cancel{m}/s \times \frac{1cm}{10^{-2}\cancel{m}}\ ?$$

$$K_1^* = 2 = \frac{C_f}{C_s} \quad \text{or} \quad \boxed{K_2^* = \frac{C_s}{C_f}}$$

check when you get data

$$\boxed{n_{vreo} = [\quad]\ \frac{g}{cm^2 s}}$$

We need to calculate h_{m1}, so we will use correlations in slide 12 of lecture 12

so we need to know if the flow is laminar or turbulent.

$$Re = \frac{D \times u \times \rho}{\mu} = \frac{0.5\times10^{-3}\ m \times 2.5\ m/s \times 1020\ kg/m^3}{3\times10^{-3}\ Pa.s}$$

$Re = 425$ LAMINAR.

$$\frac{h_{m1} D}{D_{vreo,m}} = 3.66 \Rightarrow h_{m1} = \frac{3.66 \times 1.5\times10^{-9}\ m^2/s}{0.5\times10^{-3}\ \cancel{m}}$$

$$h_{m1} = 1.1\times10^{-5}\ m/s$$

Calculate the overal mass transfer coefficient (3)

$$n_{urea} = [\quad] \frac{g}{m^2 \cdot s}$$

U_m overall mass transfer coefficient.

$$n_{urea} = \frac{C_{urea,blood} - C_{urea,dial.}}{\underbrace{\frac{1}{h_{m_1}} + \frac{\Delta L}{K^* D_{urea,m}} + \frac{1}{h_{m_2}}}_{1/U_m}}$$

By definition $\longrightarrow$ $$\frac{1}{U_m} = \frac{1}{h_{m_1}} + \frac{\Delta L}{K^* D_{urea,m}} + \frac{1}{h_{m_2}}$$

$$n_{urea} = U_m [C_{urea,blood} - U_{urea,dial}]$$

$$\frac{1}{U_m} = \frac{1}{1{,}5\times10^{-5}\,\frac{m}{s}} + \frac{0{,}25\times10^{-6}\,m}{2\times1{,}5\times10^{-9}\,\frac{m^2}{s}} + \frac{1}{3{,}5\times10^{-5}\,\frac{m}{s}}$$

$$\frac{1}{U_m} = 249242.5\ \frac{s}{m} \longrightarrow U_m = \frac{1}{249242{,}5\,\frac{s}{m}}$$

$$U_m \cong 4\times10^{-6}\ m/s$$

(a) $h_{m1} = 1.5\times10^{-5}\ m/s$

(b) $U_m \cong 4\times10^{-6}\ m/s$

(C) how many tubes we need to remove 5 g/h of urea

FLUX

$$\longrightarrow n_{urea} = U_m\left(C_{urea,blood} - C_{urea,diaz}\right) \quad \frac{g}{m^2 \cdot s}$$

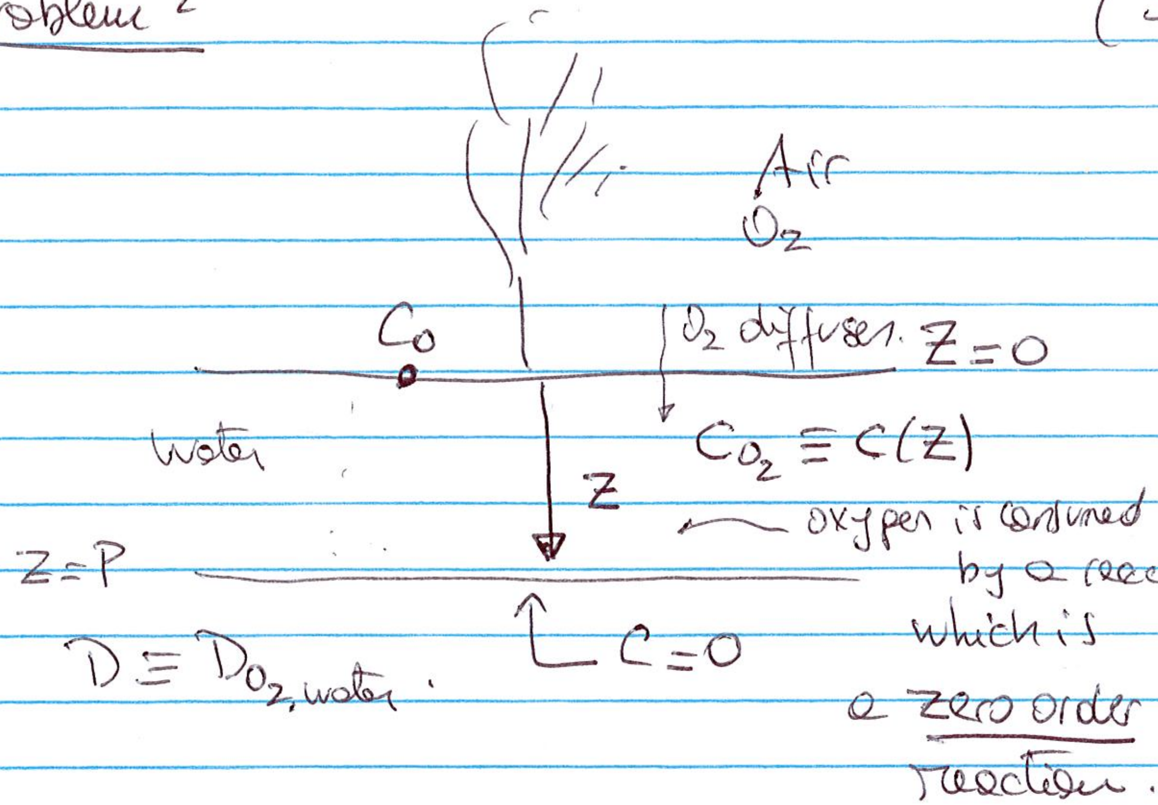
$$N_{urea} = \frac{5g}{h} = \underbrace{4\times10^{-6}\ m/s\left[0.0002\ \frac{g}{cm^3}\times\frac{10^6\ cm^3}{1\ m^3} - 0\right]}_{Flow/m^2} \times$$

$$\times\ \pi \overset{0.5\times10^{-3}m}{D} \underset{0.24\ m}{L} \times N_T\ \frac{g}{s} \times \frac{3600\ s}{1\ h}$$

FLOW

$$\boxed{N_T = 4588\ \text{tubes}}$$

Problem 2



We will assume steady state

$$0 = D\frac{d^2C(Z)}{dZ^2} - r_0$$

$$r_0 = +K$$

$$\begin{cases} \dfrac{d^2C(Z)}{dZ^2} = \dfrac{K}{D} & (1) \\ C = C_0 \text{ at } Z=0 & (1a) \\ C=0 \text{ at } Z=P \text{ [We don't know P]} & (1b) \\ \dfrac{dC}{dZ} = 0 \text{ at } Z=P \text{ [Added one]} & (1c) \end{cases}$$

$$\frac{dC(z)}{dz} = \frac{K}{D} z + A \quad (2)$$

$$C(z) = \frac{K}{2D} z^2 + Az + B \quad (3)$$

we use first boundary condition $B = C_0$

$$C(z) = C_0 + \frac{K}{2D} z^2 + A z \quad (4)$$

$$\cancel{\text{at } z = P \quad 0 = C_0 + \frac{K}{2D} P^2 + AP}$$

$$\cancel{Az = -C_0 \quad A = \frac{-C_0}{[P + \frac{K}{2D} P^2]} = \frac{-C_0}{P[1 + \frac{K}{2D} P]}}$$

$$\cancel{C(z) = C_0 - \frac{C_0}{P[1 + \frac{K}{2D} P]}}$$

Let's use Eq. (1c) to get A from Eq (2)

$$0 = \frac{K}{D} P + A \Longrightarrow A = -\frac{K}{D} P$$

$$\boxed{C(z) = C_0 + \frac{K}{2D} z^2 - \frac{K}{D} P z}$$

to calculate P we can use the boundary condition (1b)

$$C(Z=P)=0=C_0+\frac{K}{2D}P^2-\frac{K}{\cancel{2}D}P^2$$

$$\frac{KP^2}{D}-\frac{KP^2}{2D}=C_0$$

$$\frac{K}{2D}P^2=C_0 \Longrightarrow P=\sqrt{\frac{2DC_0}{K}}$$

$$\boxed{P=\sqrt{\frac{2DC_0}{K}}}$$