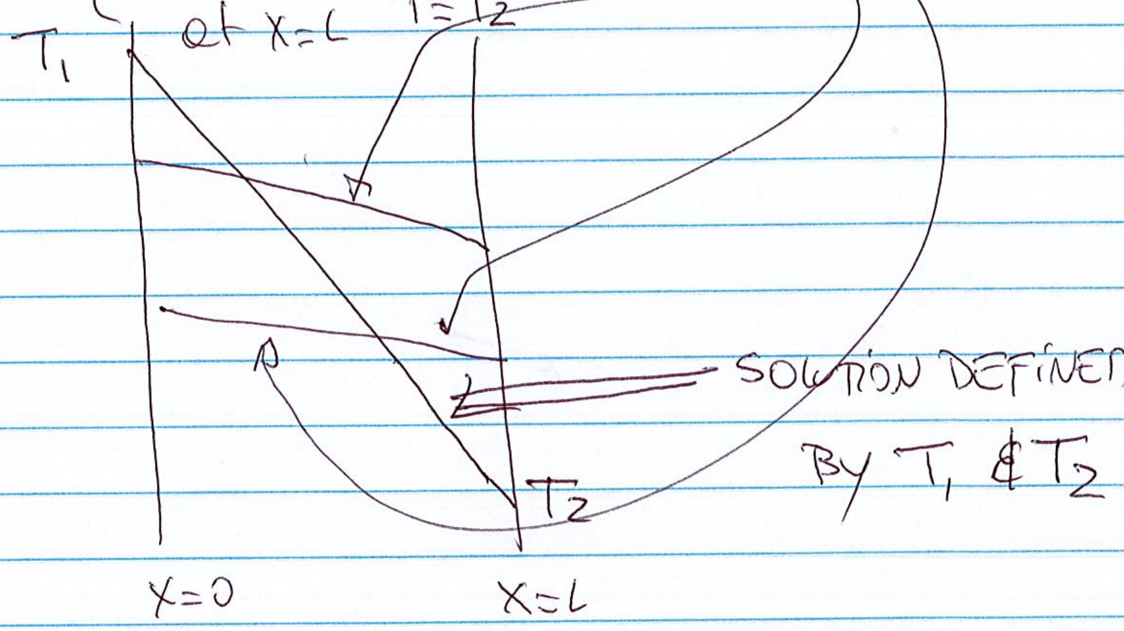
EQUATION FOR STEADY STATE HEAT TRANSFER, 1D, NO HEAT GENERATION

SOLUTION

$$\begin{cases} \dfrac{d^2T}{dx^2} = 0 \\ \text{at } x=0 \quad T=T_1 \\ \text{at } x=L \quad T=T_2 \end{cases} \Longrightarrow \boxed{T(x) = C_1 x + C_2}$$



$$\text{at } x=0 \quad T(x=0) = T_1 = C_1 \times 0 + C_2$$

$$C_2 = T_1$$

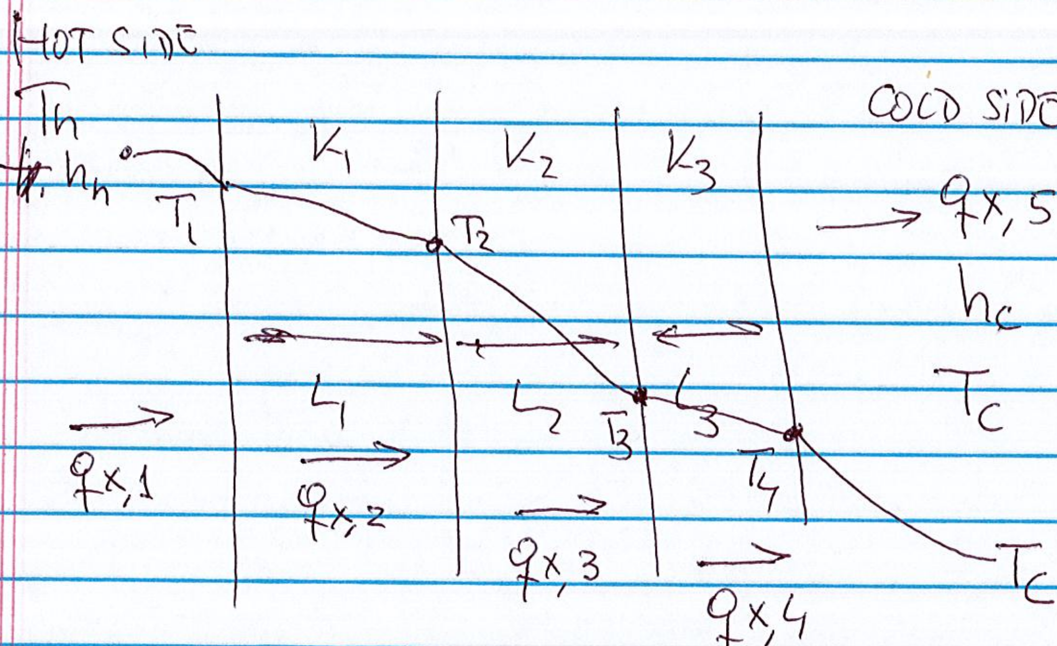
$$\text{at } x=L \quad T(x=L) = T_2 = C_1 \times L + C_2$$

$$T_2 = C_1 L + T_1 \Longrightarrow C_1 = \frac{T_2 - T_1}{L}$$

$$T(x) = \frac{T_2 - T_1}{L}\, x + T_1 \qquad (2)$$

$$\boxed{T(x) = T_1 - \left(\frac{T_1 - T_2}{L}\right) x}$$

DERIVATION OF EQUATION IN SLIDE 7



HEAT BY CONVECTION IN ~~THE~~ THE HOT FLUID

$$q_{x,1} = A h_h [T_h - T_1] = \underbrace{\frac{T_h - T_1}{\frac{1}{A h_h}}}_{\text{Res conv hot side}}$$

CONDUCTION IN LAYER 1

$$q_{x,2} = \frac{K_1 A}{L_1}(T_1 - T_2) = \underbrace{\frac{T_1 - T_2}{\frac{L_1}{K_1 A}}}_{\text{Resist conduction layer 1}}$$

CONDUCTION IN LAYER 2

$$q_{x,3} = \frac{K_2 A}{L_2}[T_2 - T_3] = \frac{T_2 - T_3}{\frac{L_2}{K_2 A}} \qquad (3)$$

CONDUCTION IN LAYER 3

$$q_{x,4} = \frac{K_3 A}{L_3}(T_3 - T_4) = \frac{T_3 - T_4}{\frac{L_3}{K_3 A}}$$

CONVECTION IN COLD SIDE

$$q_{x,5} = h_c A[T_4 - T_c] = \frac{T_4 - T_c}{\frac{1}{h_c A}}$$

$$T_h - \cancel{T_1} = q_{x,1}\left[\frac{1}{A h_h}\right]$$

$$+$$

$$\cancel{T_1} - \cancel{T_2} = q_{x,2}\left[\frac{L_1}{K_1 A}\right]$$

$$+$$

$$\cancel{T_2} - \cancel{T_3} = q_{x,3}\left[\frac{L_2}{K_2 A}\right]$$

$$+$$

$$\cancel{T_3} - \cancel{T_4} = q_{x,4}\left[\frac{L_3}{K_3 A}\right)$$

$$+$$

$$\cancel{T_4} - T_c = q_{x,5}\left[\frac{1}{A h_c}\right)$$

$T_h - T_c$

how are

$q_{x,1}$, $q_{x,2}$, $q_{x,3}$

$q_{x,4}$ and $q_{x,5}$

Under our assumptions

Assuming steady state

Assuming steady state

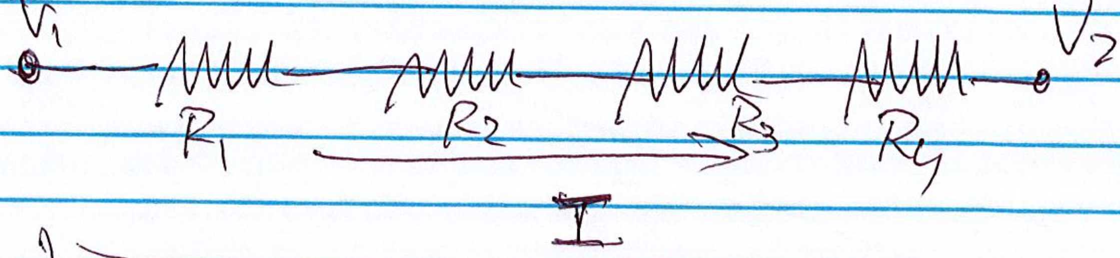
$$q_{x,1} = q_{x,2} = q_{x,3} = q_{x,4} = q_{x,5} = q_x$$

$$T_h - T_c = q_x \left[\frac{1}{Ah_h} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{1}{Ah_c} \right]$$

$$q_x = \frac{T_h - T_c}{\frac{1}{h_h A} + \frac{K_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{1}{Ah_c}}$$

$$q_x = \frac{T_h - T_c}{R_{conv,h} + R_{C,1} + R_{C,2} + R_{C,3} + R_{Conv,C}}$$

ELECTRICAL PROBLEM



$$\boxed{I = \frac{V_1 - V_2}{R_1 + R_2 + R_3 + R_4}}$$

Since we are assuming that the geometry is rectangular A is the same

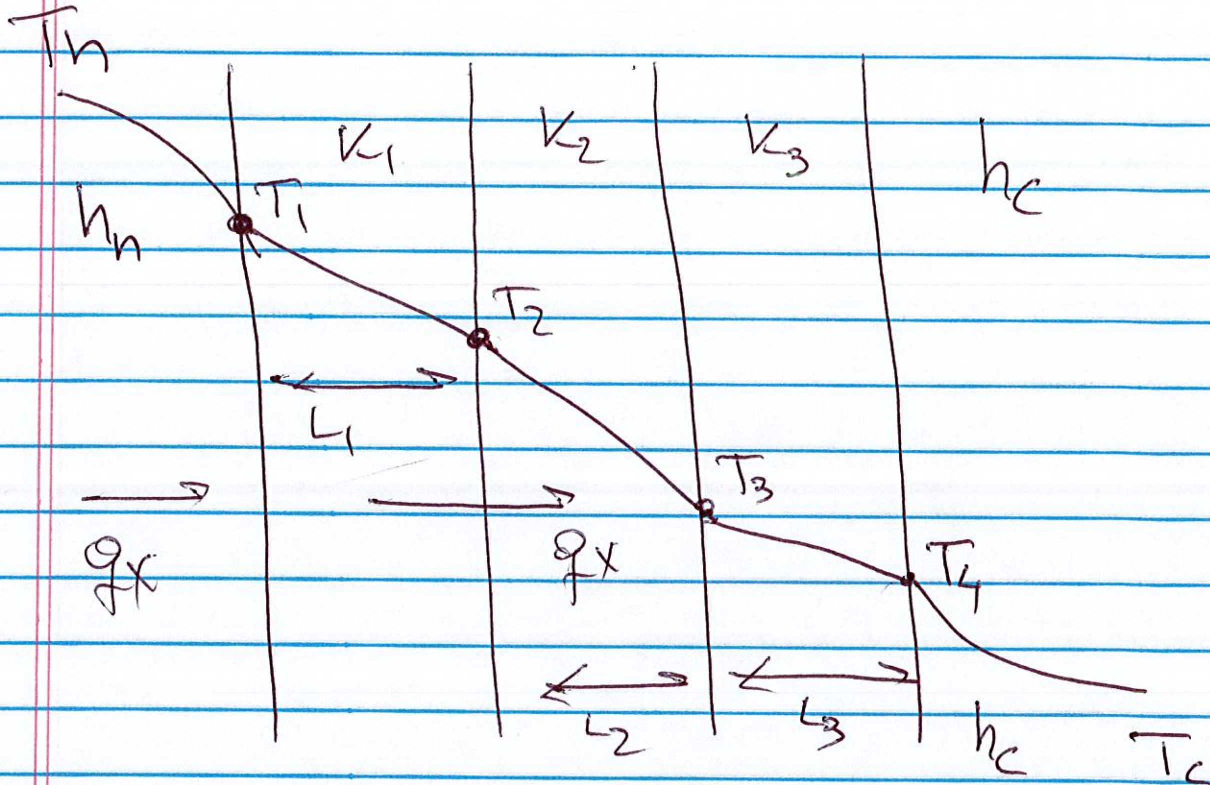
$$q_x = \frac{T_h - T_c}{\frac{1}{h_h A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{1}{h_c A}}$$

HEAT FLOW

$$q_x = \frac{A\,[T_h - T_c]}{\left[\frac{1}{h_h} + \frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3} + \frac{1}{h_c}\right]}$$

HEAT FLUX

$$q_x'' = \frac{q_x}{A} = \frac{T_h - T_c}{\left(\frac{1}{h_h} + \frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3} + \frac{1}{h_c}\right)}$$



$$\frac{q_x}{A} = \frac{T_h - T_c}{\frac{1}{h_h} + \frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3} + \frac{1}{h_c}}$$

I want to calculate T_2, could we do?

Let's consider the driving force $T_h - T_2$

$$\frac{q_x}{A} = \frac{T_h - T_2}{\frac{1}{h_h} + \frac{L_1}{K_1}} \Rightarrow T_2$$

Driving force is $T_2 - T_c$

$$\frac{q_x}{A} = \frac{T_2 - T_c}{L_2/K_2 + L_3/K_3 + 1/h_c} \longrightarrow T_2$$

Assuming radial heat flow [1D], (7)
Steady state and no heat generation

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

integrating once $\quad r\frac{dT}{dr} = C_1$

$$\frac{dT}{dr} = \frac{C_1}{r}$$

integrating again

$$T(r) = \int dT(r) = \int \frac{C_1}{r}\,dr = C_1 \ln r + C_2$$

$$\boxed{T(r) = C_1 \ln r + C_2}$$

at $r = r_i \quad T = T_i$

$$T(r = r_i) = C_1 \ln r_i + C_2 = T_i$$

at $r = r_o \quad T = T_o$

$$T(r = r_o) = C_1 \ln r_o + C_2 = T_o$$

$$T_i = C_1 \ln r_i + C_2$$

$$T_o = C_1 \ln r_o + C_2$$

$$T_i - T_o = C_1 \ln \frac{r_i}{r_o}$$

$$C_1 = \frac{T_i - T_o}{\ln r_i/r_o}$$

$$C_2 = T_i - \frac{T_i - T_o}{\ln r_i/r_o}$$

substitute C_1 & C_2 in

$$T(r) = C_1 \ln r + C_2$$