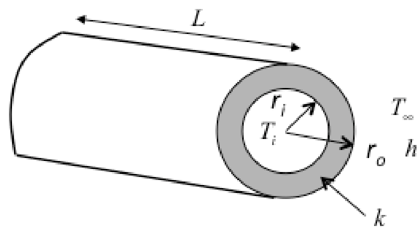
(a) Using a relationship to estimate the heat loss as a function of the tube dimensions (i.e. including the insulation thickness, the insulation thermal conductivity and the convection coefficient show that the insulation critical radius, i.e. the radius that minimizes the heat loss to the ambient is equal to $r_c = \frac{k}{h}$



The heat flow through the radial geometry at a radius *r* can be estimated by the following equation:

$$\frac{q}{L} = q' = \frac{T_i - T_\infty}{R_{cond,wall} + R_{conv}} \tag{1}$$

$$q' = \frac{T_i - T_\infty}{\frac{\ln r/r_i}{2\pi k} + \frac{1}{2\pi hr}} \tag{2}$$

where *r* is the external radius of the cylinder once insulation is added to reduce the heat flow. However, when an insulation layer (r-r_i) is added, although the heat flow may be reduced because the thermal resistance increases, the external area of the cylinder is also increased, and the heat flow may increase. Thus, there are two opposite effects competing. For a small radius, an increase of the insulation thickness has more influence on the increase of area than on the resistance to heat transfer so the heat flow per unit length *L* increases. But when the insulation layer increases beyond a critical radius (r_c) the resistance to heat transfer starts to increase and the heat flow decreases. If we plot *q/L* versus the external radius *r*, we would find a maximum value, which can be obtained finding the maximum of the expression given by Eq. (2):

$$\frac{dq'}{dr} = \frac{0 - \left(\frac{1}{2\pi kr} - \frac{1}{2\pi hr^2}\right)}{\left(\frac{\ln r/r_i}{2\pi k} + \frac{1}{2hr}\right)^2} = 0 \tag{3}$$

From where:

$$\left(\frac{1}{2\pi kr} - \frac{1}{2\pi hr^2}\right) = 0 \quad \rightarrow \quad r_c = \frac{k}{h} \tag{4}$$

(b) Now, you will need to prove that the belief is not right. For that consider a cylindrical tube with an external diameter of 2.0 cm that is maintained at a uniform temperature. In order to reduce heat losses, the tube is coated with an insulating coating material having a thermal conductivity k=0.17 W/m.K. Heat is lost by convection with a convection coefficient h=2W/m^2.K. Determine the critical radius and the critical thickness of the insulation.

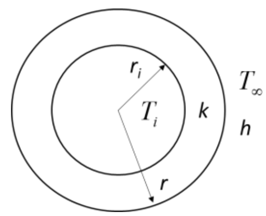
The critical radius is now calculated as:

$$r_c = \frac{k}{h} = \frac{0.17 W/m \cdot K}{2 W/m^2 \cdot K} = 0.085 m = 8.5 cm$$

The critical thickness of the insulation is 8.5cm – 1cm = 7.5cm

(d) Using the same approach than the one used in (a) estimate the critical radius for a spherical geometry.

Check the spherical surface illustrated below:



$$q_r = \frac{T_i - T_\infty}{R_{cond} + R_{conv}} = \frac{T_i - T_\infty}{\frac{r - r_i}{4\pi k r_1 r_i} + \frac{1}{4\pi r^2 h}} \tag{11}$$

As before, we need to find the value of r_c (critical radius) where q_r is maximum, as before in this case we can get rid of the factor 4π.

$$\frac{dq_r}{dr} = 4\pi \frac{\frac{1}{k r^2} - \frac{2}{r^3 h}}{\left(\frac{r - r_i}{k r r_i} + \frac{1}{r^2 h}\right)^2} = 0 \quad \rightarrow \frac{1}{k r^2} - \frac{2}{r^3 h} = 0 \rightarrow r_{critical} = \frac{2k}{h}$$

(e) Could you estimate a critical thickness for a slab geometry? Explain your answer

There is no point to define a critical thickness for a slab/rectangular geometry because it does not exist since the area of heat transfer does not change when more insulation thickness is added so the resistance to heat transfer always increase whereas the heat exchange does not change.