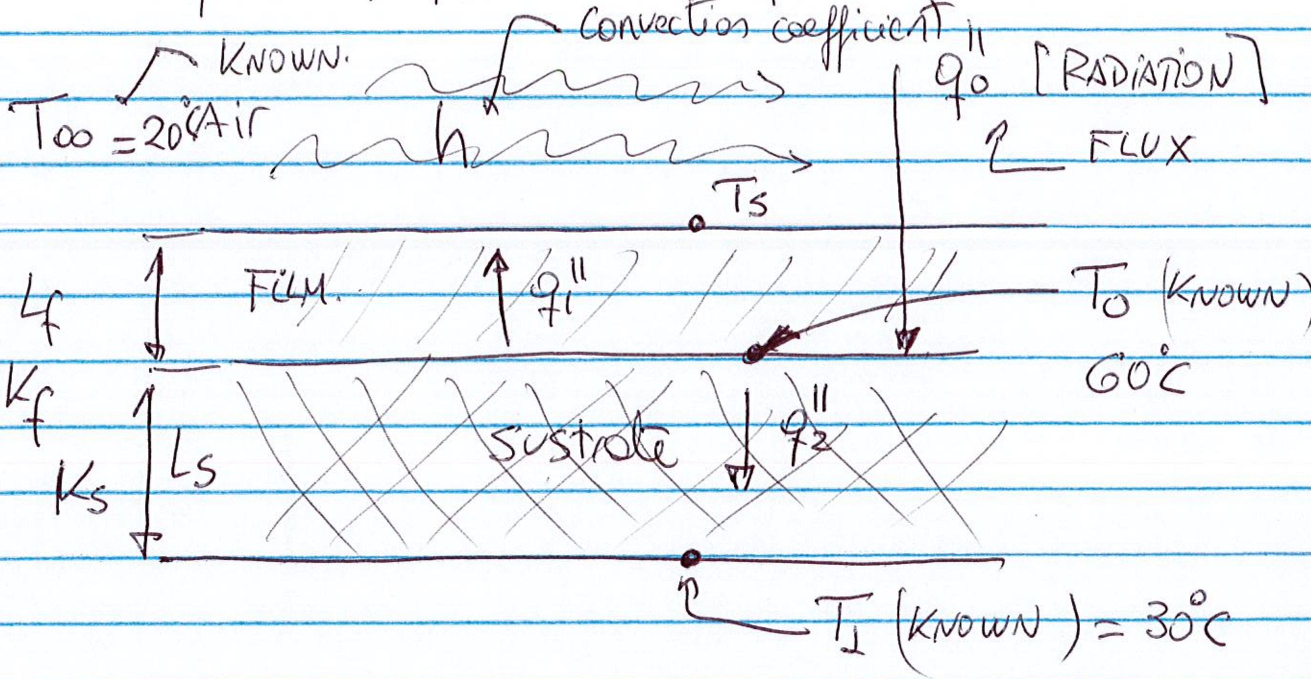
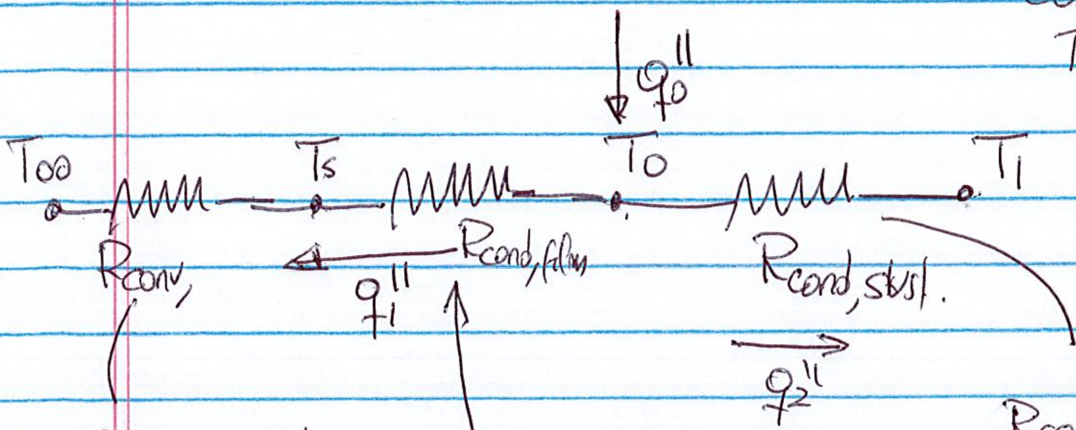
NOTES LECTURE 1-25-2018

STUFF discussed in office hours that could be important, from a concept standpoint.



STEADY STATE, T_∞, T_0 and T_1 will not change with time ⟸ PRACTICAL CONSEQUENCE OF THE ASSUMPTION



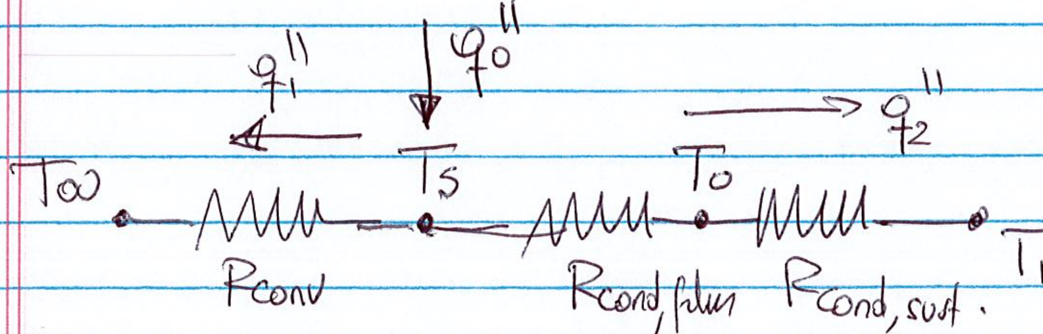
$R_{conv} = \dfrac{1}{h}$ $\qquad R_{cond,film} = \dfrac{L_f}{K_f}$ $\qquad R_{con,sust} = \dfrac{L_s}{K_s}$

$$q''_0 = q''_1 + q''_2 \qquad (2)$$

$$q''_1 = \frac{T_0 - T_\infty}{R_{cond,film} + R_{conv}}$$

$$q''_2 = \frac{T_0 - T_1}{R_{cond,sust}}.$$

(c) what happens if the film is not transparent to radiation?



$$q''_0 = q''_1 + q''_2$$

$$q''_1 \overset{?}{=} \frac{T_s - T_\infty}{R_{conv}} \qquad q''_2 = \frac{T_s - T_1}{R_{cond,film} + R_{cond,sust}}. \quad ?$$

STEADY STATE !!!

$$q''_2 = \frac{T_0 - T_1}{R_{cond,sust}} \; ? \quad \text{YES, BECAUSE} \rightarrow q''_2 \rightarrow T_s$$

$$\underbrace{\frac{d^2T}{dx^2}}_{}\quad \overset{K}{} \quad \underset{m^2}{} = -\frac{\dot{q}}{k} \cdot \frac{\cancel{W}/m^{\cancel{3}\,2}}{\frac{\cancel{W}}{\cancel{m}\cdot K}} = \frac{K}{m^2} \quad (3)$$

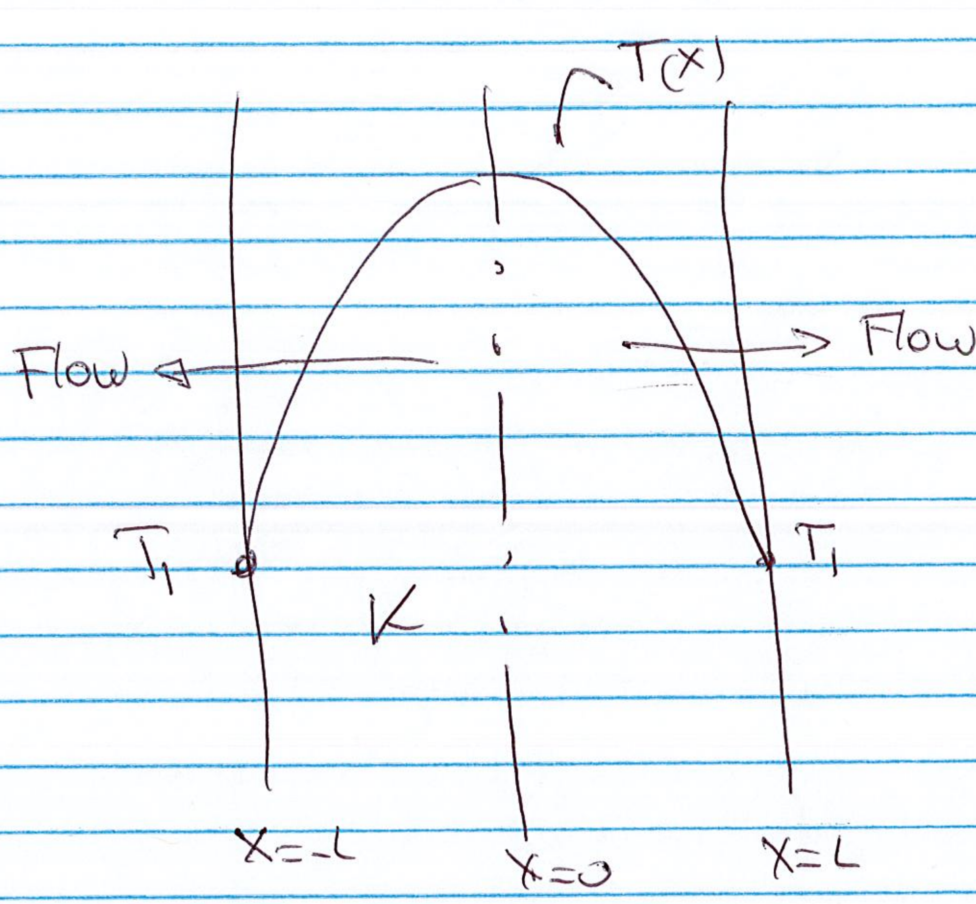
$$\begin{cases} \dfrac{d^2T}{dx^2} = -\dfrac{\dot{q}}{k} \\ x = -L \quad T = T_1 \\ x = +L \quad T = T_1 \end{cases}$$

integrating once $\quad \dfrac{dT}{dx} = -\dfrac{\dot{q}}{k}x + C_1$

integrating once $\quad \boxed{T(x) = -\dfrac{\dot{q}x^2}{2k} + C_1 x + C_2}$

if we use the Boundary conditions we get C_1 & C_2

if we use the BC at $x=0$ immediately we found $C_1 = 0$



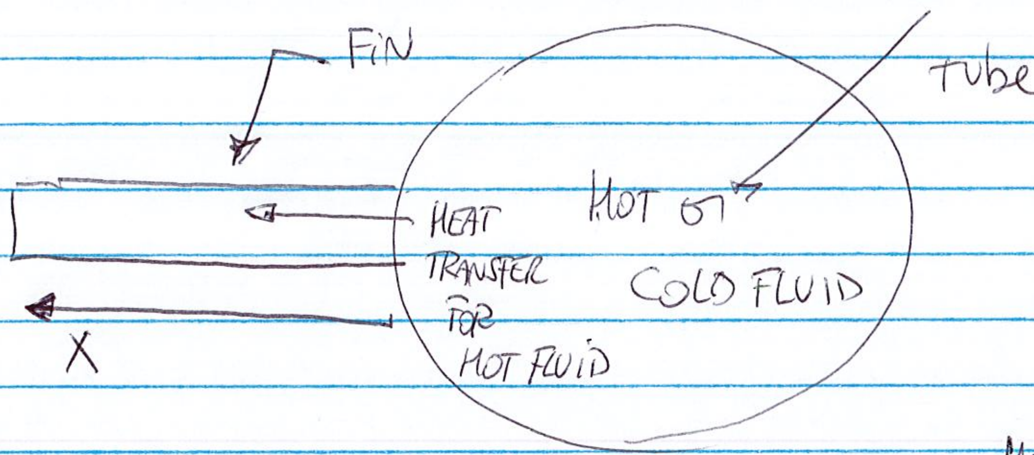
$$q'' = \frac{q}{A} = -K\frac{dT(x)}{dX} = -\cancel{K}\left(-\frac{\dot{q}X}{\cancel{K}}\right) = \dot{q}X$$

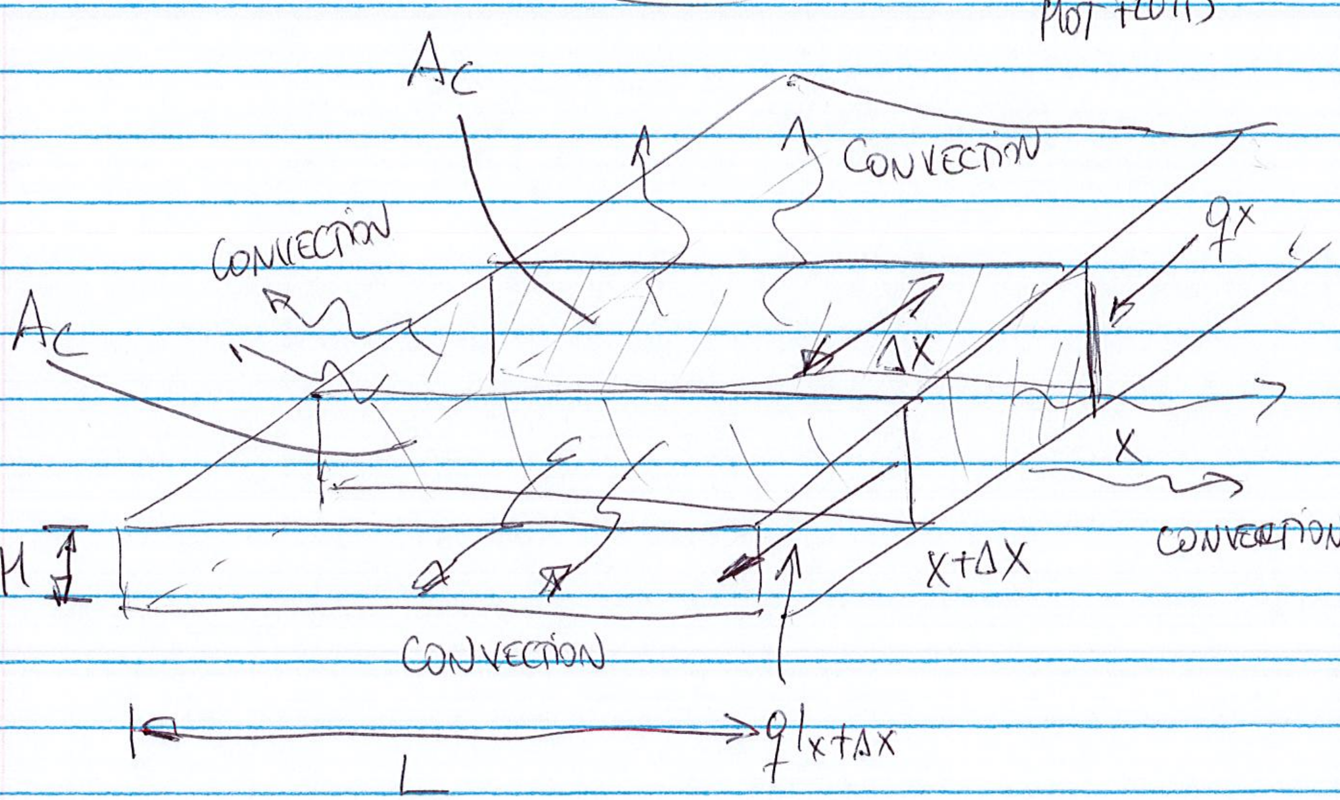
$$T(x) = T_1 + \frac{\dot{q}L^2}{2K}\left(1 - \frac{X^2}{L^2}\right)$$

$$\frac{dT(x)}{dX} = -\frac{\cancel{2}\dot{q}\cancel{L^2}X}{\cancel{2}K\cancel{L^2}}$$

$q''(x) = \dot{q}X$ at $X=L$ $q'' = \dot{q}L$

CONTINUATION OF PAST LECTURE 1/25/2018





[MACROSCOPIC BALANCE] → ENERGY IN − ENERGY OUT = 0 [Steady state]

IN THE SHADED VOLUME

$$q''_x|_x A_c - q''_x|_{x+\Delta x} A_c = h\underbrace{[2L\Delta x + 2H\Delta x]}_{A_{convection}}[T(x) - T_\infty]$$

$$\frac{q''_x|_x A_c - q''_x|_{x+\Delta x} A_c}{\Delta x} = h[2L+2H][T(x) - T_\infty]$$

To get a Microscopic balance $\Delta X \to 0$

$$q''_x\big|_x = -K \frac{dT(x)}{dx}\bigg|_x$$

$$q''_x\big|_{x+\Delta x} = -K \frac{dT(x)}{dx}\bigg|_{x+\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{-\left\{K \frac{dT}{dx}\big|_x + K \frac{dT}{dx}\big|_{x+\Delta x}\right\} A_c}{\Delta x} = h \underbrace{[2H+2L]}_{\text{Perimeter of the fin}} (T(x) - T_\infty)$$

$$K A_c \frac{d^2T(x)}{dx^2} = h P [T(x) - T_\infty]$$

$$\begin{cases} \dfrac{d^2T(x)}{dx^2} = \dfrac{hP}{K A_c} [T(x) - T_\infty] \\ x = 0 \quad T(x) = T_b \quad \leftarrow \text{base of the pipe.} \\ x \to \infty \quad T = T_\infty \end{cases}$$

$$\begin{cases} \dfrac{d^2T(x)}{dx^2} - m^2\left(T(x) - T_\infty\right) = 0 \\ T(x) = T_b \text{ at } x = 0 \\ T(x) \to T_\infty \text{ at } x \to \infty \end{cases}$$

assumed Constant

$$\Theta(x) = T(x) - T_\infty$$

$$\frac{d^2\Theta(x)}{dx^2} = \frac{d^2T(x)}{dx^2}$$

$\equiv$ Equivalent

at $x=0$ $\quad T(x) = T_b \quad \Theta = \Theta_b = T_b - T_\infty$

at $x \to \infty$ $\quad T(x) \to T_\infty \quad \Theta \to 0$

$$\begin{cases} \dfrac{d^2\Theta(x)}{dx^2} - m^2\Theta(x) = 0 \\ \Theta = \Theta_b \text{ at } x = 0 \\ \Theta = 0 \text{ at } x \to \infty \end{cases}$$

MA 303 or MA 262 (8)

$$r^2 - m^2 = 0 \Longrightarrow r_{1,2} = \pm m = \pm\sqrt{m^2}$$

Solution

$$\theta(x) = A_1 e^{r_1 x} + B e^{r_2 x} = \cancel{A_1 e^{mx}} + B e^{-mx}$$

For $x \to \infty$ $\theta(x \to \infty) \longrightarrow \infty$

NOT PHYSICALLY POSSIBLE

$$\theta(x) = B e^{-mx}$$

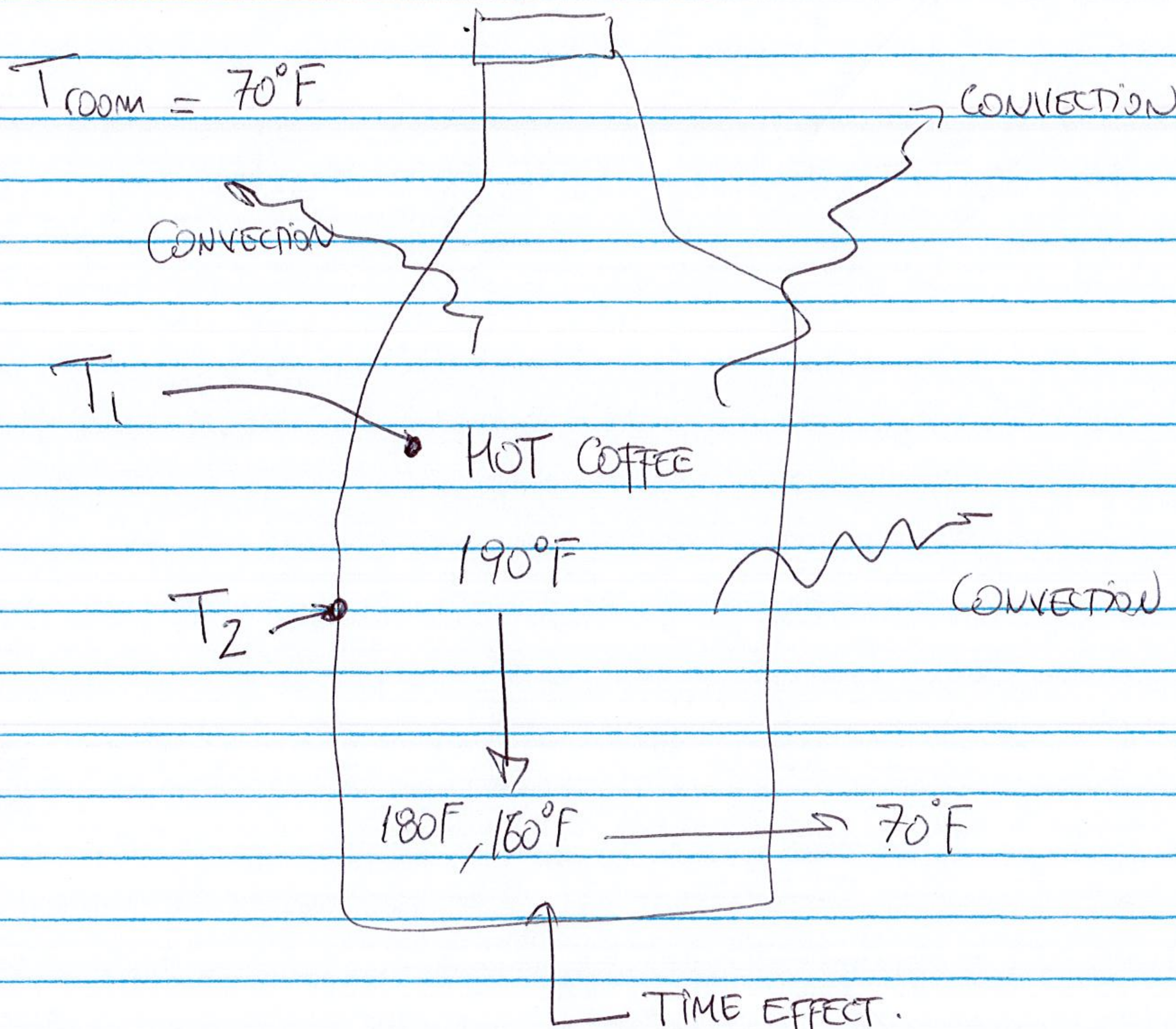
at $x = 0$ $\theta_b = B$

$$\theta(x) = \theta_b e^{-mx}$$

For $x \to \infty$ $\theta(x) = 0 \Longrightarrow T(x \to \infty) = T_\infty$

$$\boxed{T(x) - T_\infty = (T_b - T_\infty) e^{-\frac{hP}{KA_c} x}}$$

UNSTEADY STATE HEAT TRANSFER



$$T_1 = f[\text{Time}, \text{position}]$$

$$T_2 = f[\text{Time}, \text{position}]$$

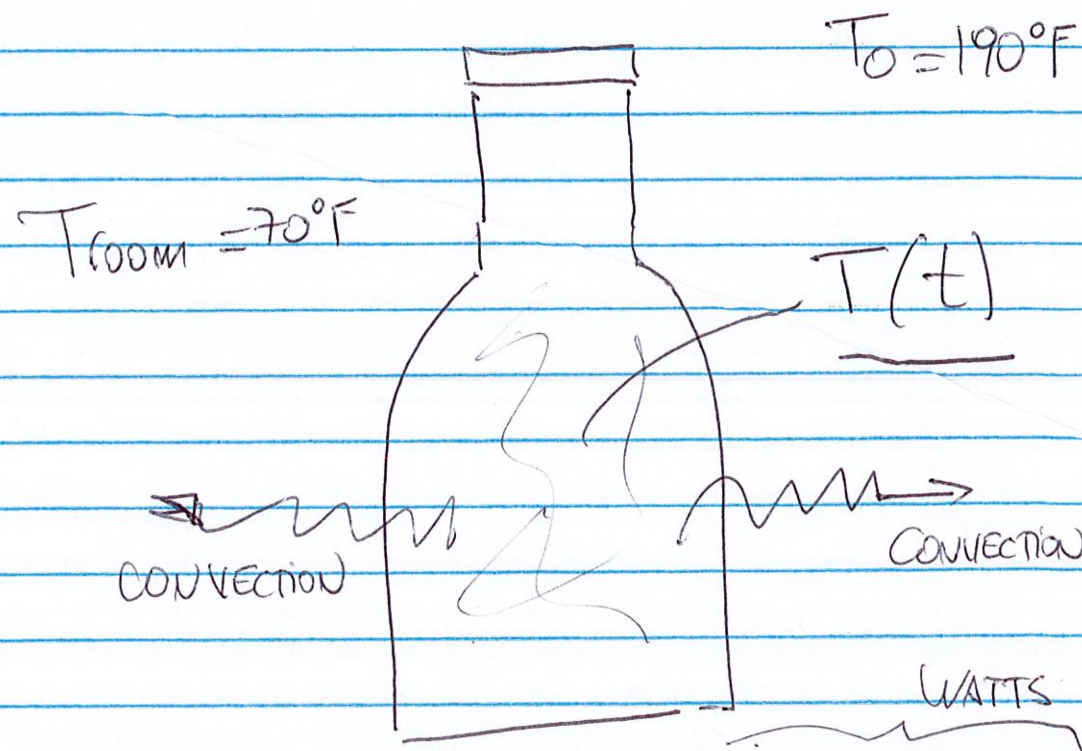
If the bottle ~~is~~ is made of metal

LUMPED PARAMETER MODEL $T_1 \approx T_2$ [POSITION IS NOT IMPORTANT]

if the bottle is made of plastic

UNSTEADY STATE WITH TIME & POSITION TO BE CONSIDERED. Probabley $T_1 \neq T_2$ [POSITION IS IMPORTANT]

FOR BOTH TIME IS IMPORTANT



$$-\underset{kg \,\cdot\, \frac{J}{\cancel{kg\,K}} \times \cancel{K}}{mc\Delta T} = \underbrace{\overbrace{hA[T(t) - T_{room}]}^{\text{WATTS}}}_{\text{CONVECTION}} \times \overset{s}{\Delta t}$$

$$\frac{W}{\cancel{m^2}\cancel{K}} \cdot \cancel{m^2} \cdot \cancel{K}$$

MACROSCOPIC BALANCE $-mc\frac{\Delta T}{\Delta t} = hA\left[T(t) - T_{room}\right]$ (11)

MICROSCOPIC BALANCE WHEN $\Delta t \to 0$

$$-mc\frac{dT(t)}{dt} = hA\left[T(t) - \underbrace{T_{room}}_{T_\infty}\right]$$

$$\begin{cases} \dfrac{dT(t)}{dt} = -\dfrac{hA}{mc}\left[T(t) - T_\infty\right] \\ \text{at } t=0 \quad T = T_0 \end{cases}$$

$$dT(t) = d\left[T(t) - T_\infty\right]$$

$$\int_{T_0}^{T(t)} \frac{d\left[T(t) - T_\infty\right]}{T(t) - T_\infty} = -\int_0^t \frac{hA}{mc}\,dt$$

$$\ln\frac{T(t) - T_\infty}{T_0 - T_\infty} = -\frac{hA}{mc}t$$

$$\boxed{\frac{T(t) - T_\infty}{T_0 - T_\infty} = e^{-\frac{hA}{mc}t}}$$ (12)

Initial temperature

room temperature

LUMPED PARAMETER MODEL

UNSTEADY STATE HEAT FLOW BY CONDUCTION

- NO HEAT GENERATION
- NO ~~&~~ CONVECTION
- 1D FLOW [X-DIRECTION]

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

BC [We need three BC

$$\alpha = \frac{K}{\rho c}$$

$$
\begin{cases}
\dfrac{\partial T}{\partial t} = \alpha \dfrac{\partial^2 T}{\partial x^2} \\
\text{at } t=0 \quad T=0 \\
\text{at } x=\pm L \quad T=T_1 \quad t\geq 0
\end{cases}
$$