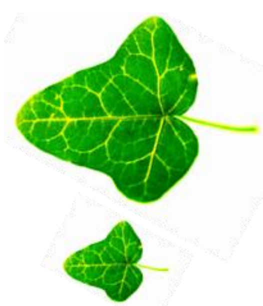
ABE 308 – More Radiation Examples

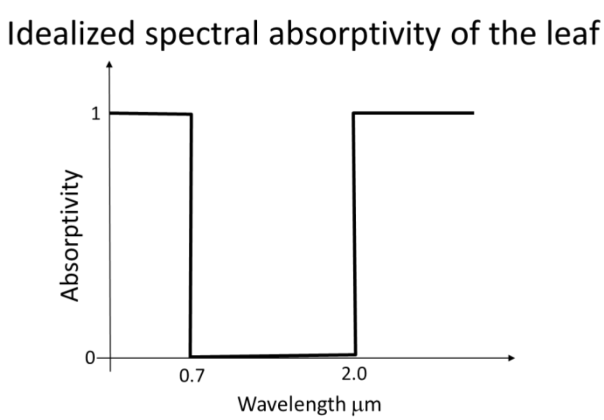
Problem 1

Consider solar radiation incident on a glass plate with air on both sides. The solar radiation can be assumed to be that from a blackbody at 5800 K temperature. The glass absorbs all radiation between 0.3 μm and 3 μm. Outside this range of wavelengths, all radiation is transmitted through the glass without any absorption. The radiation incident to the glass is 700W/m^2 and the ambient air temperature is 20◦C. (1) Calculate the solar energy that is absorbed in the glass, in W/m^2. (2) Write an energy balance for the glass at steady state, from which you can calculate the steady-state temperature. You can consider that there is radiation emitted by the soil that reaches the glass. Assume that all materials are black bodies and that the temperature of the soil is the same than that of the air. (3) For a convective heat transfer coefficient of 50 W/m^2 ·K, what is the steady state temperature? Note: Solution requires use of MathCad or Matlab (root of a non-linear function).

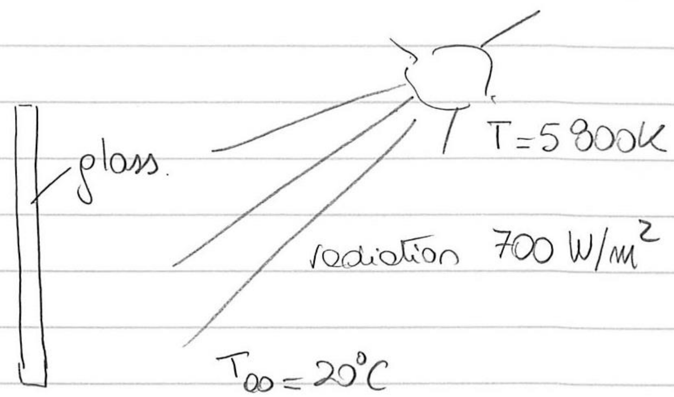
Problem 2

Consider a small leaf completed shaded by a large one as shown in the schematic figure below. The emissivity of the leaf can be idealized as having the values given in the plot below. (1) What fraction of solar radiation (assume sun's surface temperature to be 5800K) reaches the bottom leaf? (2) For photosynthesis all absorption of chlorophyll is below 0.7μm. Comment on how useful the solar radiation reaching the bottom leaf is for the purpose of photosynthesis. (3) If the temperature of the upper leaf is 35ºC, what is the total amount of radiation leaving the top leaf ? Note that is the total energy leaving and not the net amount of energy leaving due to exchange with other bodies. Assume the leaves are gray bodies





Problem 1



Absorption Range $0.3\,\mu m - 3\,\mu m$

(1) Calculate the solar energy that is absorbed in the glass

First we need to know the fraction of the energy absorbed.

$$\text{FRACTION OF ENERGY ABSORBED} = F_{0-\lambda_2 T} - F_{0-\lambda_1 T}$$

$$\lambda_2 T = 3\,\mu m \times 5800K = 17{,}400\,\mu m.K$$

$$\lambda_1 T = 0.3\,\mu m \times 580K = 1{,}740\,\mu m\,K$$

Using Table 8.2 From Textbook

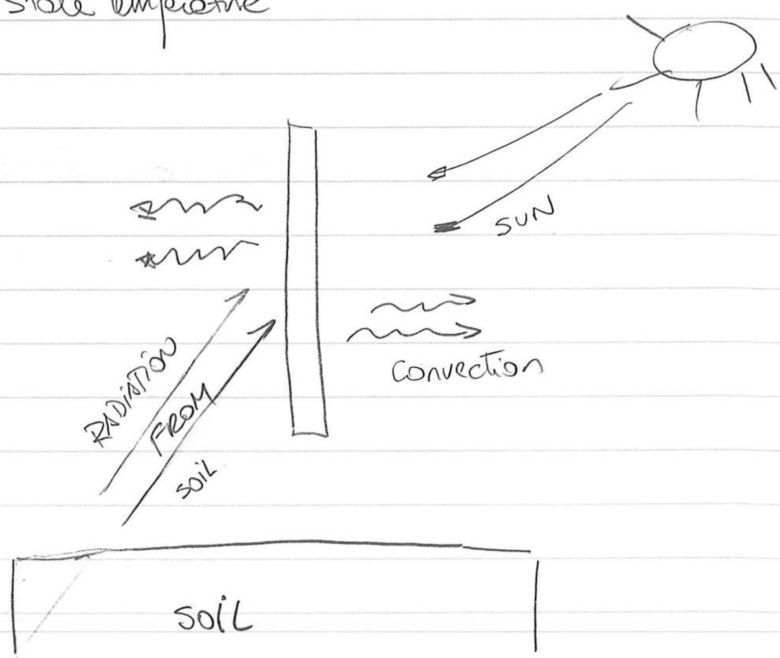
$\lambda_2 T = 17{,}400\,\mu m.K \Rightarrow F_{0-\lambda_2 T} = 0.979061$

$\lambda_1 T = 1{,}740\,\mu m K \Rightarrow F_{0-\lambda_1 T} = 0.03367$ (Interpolation)

Energy Absorbed by the plate $= (0.979061 - 0.03367) \times 700 \frac{W}{m^2}$ (2)

$$\boxed{\text{Energy Absorbed by the plate} = 661.8 \frac{W}{m^2}}$$

(2) Write an energy balance for the plate at steady state from which it is possible to calculate the steady-state temperature



A balance of energy at steady state gives.

Energy In = Energy out. (1)

(3)

Assuming that the sun and the ground are black body

$$\text{Energy In} = \left(661.8 \frac{W}{m^2} + \sigma T_{gr}^4\right) A$$

and Energy Out is

$$\text{Energy Out} = 2A\sigma T^4 + 2Ah\left[T - T_\infty\right]$$

A is the Area of one side of the glass and T is the Temperature of the glass at steady state.
The glass is also assumed a black body with $\varepsilon = 1$

substituting into Eq. (1)

$$\left(661.8 + \sigma T_{gr}^4\right)\cancel{A} = 2\cancel{A}\sigma T^4 + 2\cancel{A}h\left(T - T_\infty\right)$$

$$T^4 + \frac{h}{\sigma}\left(T - T_\infty\right) - \left[\frac{661.8}{2\sigma} + \frac{T_{gr}^4}{2}\right] = 0 \quad (2)$$

substituting the following values into Eq. (2)

$h = 50\ W/m^2 \cdot K$

$\sigma = 5.67 \times 10^{-8}\ \frac{W}{m^2 \cdot K^4}$

$T_\infty = 20^\circ C = 293\ K$

$T_{gr} = 20^\circ C = 293\ K$ Assumed equal than air

$T^4 + 8.82\times10^8 T - 2.58\times10^{11} - (5.84\times10^9 +$ (4)

$+ 3.69\times10^9) = 0$

So the Equation is

$$T^4 + 8.82\times10^8 T - 2.68\times10^{11} = 0$$

Solve it using MathCad

$T = 400$

$f(T) = T^4 + 8.82\times10^8 T - 2.68\times10^{11}$

$T_{sol} = \text{root}(f(T), T)$

$\boxed{T_{sol} = 295.24\ K}$

Due to the high convection coefficient $h = 50\ W/m^2K$

the glass plate is cooling down and reaches a Temperature lower than the air temperature

Problem 2 (5)

(1) What fraction of solar radiation ($T_{sun} = 5800K$) reaches the bottom leaf.

Fraction of solar energy reached by bottom leaf = LOCATED BETWEEN 0.7–2 μm Because is not absorbed by the top leaf.

Fraction reached by bottom leaf $= F_{0-\lambda_2 T} - F_{0-\lambda_1 T}$

Table 8.2

$\lambda_2 T = 2\,\mu m \times 5800K = 11{,}600\,\mu m.K \Rightarrow F = 0.9402$

$\lambda_1 T = 0.7 \times 5800K = 4060\,\mu m.K \Rightarrow F = 0.4918$

Fraction reached by bottom leaf $= 0.9402 - 0.4918 = 0.4484$

(2) From Figure 8.6 Book, all absorption of chlorophil is below 0.7 μm so the small leaf will not absorbe any thing, neither the big one.

(3) The fraction of energy absorbed by the upper leaf is

$1 - 0.4484$

Energy reaching the bottom leaf.

So Fraction of the energy absorbed by the top leaf. (6)

Fraction Absorbed = 1 - 0.4434 = 0.5516

If we assume the leafs are a grey body

Emissillity = $\mathcal{E}$ = absorptivity = α

SO TOTAL RADIATION LEAVING THE LEFT CAN BE CALCULATED AS

TOTAL RADIATION = $\varepsilon \sigma T^4 = 0.5516 \times 5.67 \times 10^{-8} \frac{W}{m^2 K^4} \times 308^4$

$$\boxed{\text{TOTAL RADIATION} = 281.5 \ \frac{W}{m^2}}$$