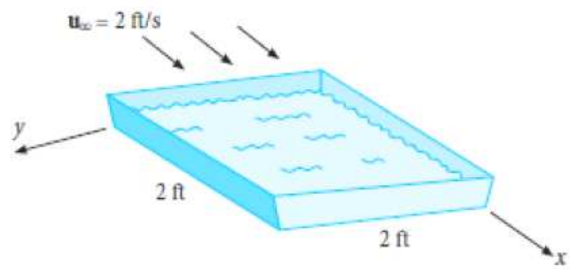
Homework 4 - Solutions
ABE 30800 - Spring 2018

Question 1



There will be some heat transfer between the pan and the water, that energy is used to vaporize part of the water in the water pan. This problem is similar than the example problem given in class, but in this case we only need to estimate the heat transfer coefficient. As we know the coefficient is a function of the Reynolds number

Data

Temperatures are expressed in degree Farehnheit and transformed in degree Celsius. Absolute temperature in units of Kelvins (K)

$T_{w1} := 95 \qquad T_{surr1} := 80 \qquad kJ := 1000 \cdot J$

$T_{w1C} := \dfrac{T_{w1} - 32}{1.8} \qquad T_{surr1C} := \dfrac{T_{surr1} - 32}{1.8}$

$T_{w1C} = 35 \qquad T_{surr1C} = 26.7$

$T_{film1} := \dfrac{T_{w1C} + T_{surr1C}}{2} \qquad T_{film1} = 30.8$

$T_{film1K} := (T_{film1} + 273) \cdot K \qquad T_{film1K} = 303.8 \cdot K$

Values of the air properties are taken at 300K (it will not change much respect to 303.8K)

$\rho_1 := 1.17 \cdot \dfrac{kg}{m^3} \qquad \mu_1 := 1.85 \cdot 10^{-5} \cdot \dfrac{kg}{m \cdot s} \qquad u_{inf1} := 2 \cdot \dfrac{ft}{s}$

$k_1 := 0.026 \cdot \dfrac{W}{m \cdot K} \qquad c_1 := 1 \cdot \dfrac{kJ}{kg \cdot K} \qquad Pr_1 := 0.706$

$L_1 := 2 \cdot ft \qquad Re_{L1} := \dfrac{L_1 \cdot \rho_1 \cdot u_{inf1}}{\mu_1} \qquad Re_{L1} = 2.4 \times 10^4$ Flow is laminar

$Nu_{L1} := 0.664 \cdot Re_{L1}^{\frac{1}{2}} \cdot Pr_1^{\frac{1}{3}} \qquad Nu_{L1} = 90.6$

$h_{L1} := \dfrac{k_1 \cdot Nu_{L1}}{L_1} \qquad \boxed{h_{L1} = 3.9 \cdot \dfrac{W}{m^2 \cdot K}}$

Question 2

The building is square with sides of 20m. Assume that the air surrounds the building so the heat transfer is through 4 sides. Let's calculate first the heat transfer coefficient h_{L2}

$u_2 := 30\frac{km}{hr}$ $\quad L_2 := 20\cdot m$ $\quad T_{air2} := -10$ $\quad T_{w2} := 10$

$$T_{film2} := \frac{(T_{air2}+273)+(T_{w2}+273)}{2}\cdot K \qquad T_{film2} = 273K$$

The properties of the air at 273k (let's use 270K) in the air table

$\rho_2 := 1.30\cdot\frac{kg}{3}$ $\quad \mu_2 := 1.70\cdot 10^{-5}\cdot\frac{kg}{m\cdot s}$ $\quad k_2 := 0.0238\frac{W}{m\cdot K}$

$c_2 := 1\cdot\frac{kJ}{kg\cdot K}$ $\quad Pr_2 := 0.716$

$$Re_{L2} := \frac{u_2\cdot\rho_2\cdot L_2}{\mu_2} \qquad Re_{L2} = 1.3\times 10^7$$ Turbulent flow

$$Nu_{L2} := 0.036\cdot Re_{L2}^{\frac{4}{5}}\cdot Pr_2^{\frac{1}{3}} \qquad Nu_{L2} = 1.6\times 10^4$$

$$h_{L2} := \frac{k_2\cdot Nu_{L2}}{L_2} \qquad \boxed{h_{L2} = 18.5\cdot\frac{W}{m^2\cdot K}}$$

$$Q_{2\_per\_length} := h_{L2}\cdot(4\cdot L_2)\cdot(T_{w2}-T_{air2})\cdot K$$

$$\boxed{Q_{2\_per\_length} = 3\times 10^4\cdot\frac{W}{m}}$$

Question 3

$m3\_dot := 200\cdot\frac{gm}{s}$ $\quad D_3 := 2\cdot cm$ $\quad T_{w3} := 80$ $\quad T_{w3K} := (T_{w3}+273)\cdot K$

$T_{tw3} := 200$ $\quad T_{tw3K} := (T_{tw3}+273)\cdot K$

$$T_{film3K} := \frac{T_{w3K}+T_{tw3K}}{2} \qquad T_{film3K} = 413K$$

$$T_{film3} := \frac{T_{film3K}}{1\cdot K} - 273 \qquad T_{film3} = 140$$ degree Celsius

Properties of Saturated liquid water at 140ºC are:

$\rho_{w3} := 921.7\cdot\frac{kg}{m^3}$ $\quad c_3 := 4.38\frac{kJ}{kg\cdot K}$ $\quad k_{w3} := 0.683\cdot\frac{W}{m\cdot K}$ $\quad Pr_3 := 1.24$

$$\mu_3 := 0.197 \cdot 10^{-3} \cdot \frac{kg}{m \cdot s} \qquad u_3 := \frac{4 \cdot m3\_dot}{\rho_{w3} \cdot \pi \cdot D_3^{\,2}} \qquad u_3 = 0.7 \frac{m}{s}$$

$$Re_{D3} := \frac{u_3 \cdot \rho_{w3} \cdot D_3}{\mu_3} \qquad Re_{D3} = 6.5 \times 10^4$$

Turbulent Flow

$$Nu_{D3} := 0.023 \cdot Re_{D3}^{\frac{4}{5}} \cdot Pr_3^{\,0.4} \qquad Nu_{D3} = 176.8$$

$$h_{D3} := \frac{k_{w3} \cdot Nu_{D3}}{D_3} \qquad \boxed{h_{D3} = 6 \times 10^3 \cdot \frac{W}{m^2 \cdot K}}$$

$$Q_3 := h_{D3} \cdot \pi \cdot D_3 \cdot \left(T_{tw3K} - T_{w3K}\right) \qquad \boxed{Q_3 = 45.5 \cdot \frac{kW}{m}}$$

(b)

$$D_{new} := 1.9 \cdot cm \qquad u_{new} := \frac{4 \cdot m3\_dot}{\rho_{w3} \cdot \pi \cdot D_{new}^{\,2}}$$

$$Re_{D\_new} := \frac{u_{new} \cdot \rho_{w3} \cdot D_{new}}{\mu_3} \qquad Re_{D\_new} = 6.8 \times 10^4$$

Turbulent Flow

$$Nu_{D\_new} := 0.023 \cdot Re_{D\_new}^{\frac{4}{5}} \cdot Pr_3^{\,0.4} \qquad Nu_{D\_new} = 184.2$$

$$h_{D\_new} := \frac{k_{w3} \cdot Nu_{D\_new}}{D_{new}} \qquad \boxed{h_{D\_new} = 6.6 \times 10^3 \cdot \frac{W}{m^2 \cdot K}}$$

$$Q_{3\_new} := h_{D\_new} \cdot \pi \cdot D_{new} \cdot \left(T_{tw3K} - T_{w3K}\right) \qquad \boxed{Q_{3\_new} = 47.4 \frac{1}{m} \cdot kW}$$

The heat gain is increased because the velocity of the fluid is increased when the diameter decreases. However, the result woud be different if the scale formed has a low thermal conductivity but no information is giving so we cannot evaluate this. The increase of roughness may promote turbulence and increase the heat transfer coefficient.

Question 4

(a)

$$D_{per} := 30 \cdot cm \qquad L_{per} := 1.8 \cdot m \qquad T_{body} := 37 \cdot K$$

$$u_{wind} := 5 \cdot \frac{m}{s} \qquad T_{air4} := 35 \cdot K$$

$$T_{film4} := \frac{\left(T_{body} + 273 \cdot K\right) + \left(T_{air4} + 273 \cdot K\right)}{2} \qquad T_{film4} = 309\,K$$

Let's use the properties of air at 310K

$\rho_4 := 1.138 \cdot \frac{kg}{m^3}$ $c_4 := 1.007 \cdot \frac{kJ}{kg \cdot K}$ $\mu_4 := 1.893 \cdot 10^{-5} \cdot \frac{kg}{m \cdot s}$

$k_4 := 0.027 \cdot \frac{W}{m \cdot K}$ $Pr_4 := 0.705$

$Re_{D4} := \frac{u_{wind} \cdot \rho_4 \cdot D_{per}}{\mu_4}$ $Re_{D4} = 9 \times 10^4$

From Table $B := 0.027$ $n := 0.805$

$Nu_{D4} := B \cdot Re_{D4}{}^{n} \cdot Pr_4{}^{\frac{1}{3}}$ $\boxed{Nu_{D4} = 234.2}$ $h_4 := \frac{k_4 \cdot Nu_{D4}}{D_{per}}$ $h_4 = 21.1 \cdot \frac{W}{m^2 \cdot K}$

$q_4 := h_4 \cdot \pi \cdot D_{per} \cdot L_{per} \cdot \left(T_{body} - T_{air4}\right)$ $\boxed{q_4 = 71.5\,W}$

for calm day $h_{calm} := 3.6 \cdot \frac{W}{m^2 \cdot K}$ $q_{4calm} := h_{calm} \left[\pi \cdot D_{per} \cdot L_{per} \cdot \left(T_{body} - T_{air4}\right)\right]$

$\boxed{q_{4calm} = 12.2\,W}$ much lower value of heat loss

If the velocity of the air increases to 10 m/s and the temperatures decreases to 10ºC

$u_{wind\_new} := 10 \cdot \frac{m}{s}$ $T_{air\_new} := 10 \cdot K$

$T_{film\_new} := \frac{\left(T_{air\_new} + 273 \cdot K\right) + \left(T_{body} + 273 \cdot K\right)}{2}$ $T_{film\_new} = 296.5\,K$

Let's use values of tables at 295K

$\rho_{new} := 1.19 \cdot \frac{kg}{m^3}$ $\mu_{new} := 1.82 \cdot 10^{-5} \cdot \frac{kg}{m \cdot s}$ $k_{new} := 0.026 \cdot \frac{W}{m \cdot K}$ $Pr_{new} := 0.706$

$Re_{new} := \frac{u_{wind\_new} \cdot \rho_{new} \cdot D_{per}}{\mu_{new}}$ $Re_{new} = 2 \times 10^5$

From the Table B =0.027 and n = 0.805

$Nu_{D\_4new} := B \cdot Re_{new}{}^{n} \cdot Pr_{new}{}^{\frac{1}{3}}$ $Nu_{D\_4new} = 438$

$h_{new} := \frac{k_{new} \cdot Nu_{D\_4new}}{D_{per}}$ $\boxed{h_{new} = 38 \cdot \frac{W}{m^2 \cdot K}}$

$q_{new} := h_{new} \cdot \pi \cdot D_{per} \cdot L_{per} \cdot \left(T_{body} - T_{air\_new}\right)$ $\boxed{q_{new} = 1.7 \cdot kW}$ Significantly larger loss in a windy day

Question 5

$T_{air5} := 15 \cdot K$ $\quad T_m := -0.5 \cdot K$ $\quad k_{unf} := 0.55 \cdot \frac{W}{m \cdot K}$

$\rho_{unf} := 950 \cdot \frac{kg}{m^3}$ $\quad \Delta H_f := 335 \cdot \frac{kJ}{kg}$ $\quad h := 20 \cdot \frac{W}{m^2 \cdot K}$

Scenary 1 - there is convection from one side $\quad x_{1\_side} := 2.5 \cdot cm$

$$t_{F1} := \frac{\Delta H_f \cdot \rho_{unf}}{T_{air5} - T_m} \cdot \left(\frac{x_{1\_side}^2}{2 \cdot k_{unf}} + \frac{x_{1\_side}}{h} \right) \qquad \boxed{t_{F1} = 10.4 \cdot hr}$$

Scenary 2 - there is convection from one side $\quad x_{2\_side} := 1.25 \cdot cm$

$$t_{F2} := \frac{\Delta H_f \cdot \rho_{unf}}{T_{air5} - T_m} \cdot \left(\frac{x_{2\_side}^2}{2 \cdot k_{unf}} + \frac{x_{2\_side}}{h} \right) \qquad \boxed{t_{F2} = 4.4 \cdot hr}$$