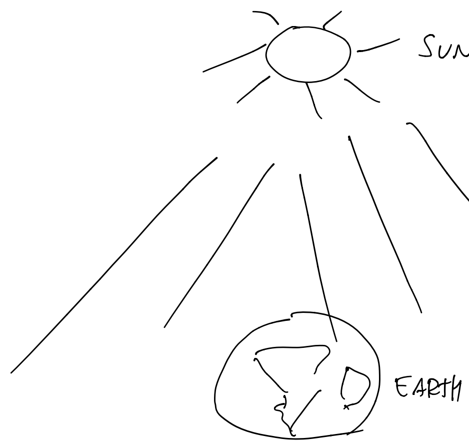
ABE 30800
RADIATION EXAMPLES

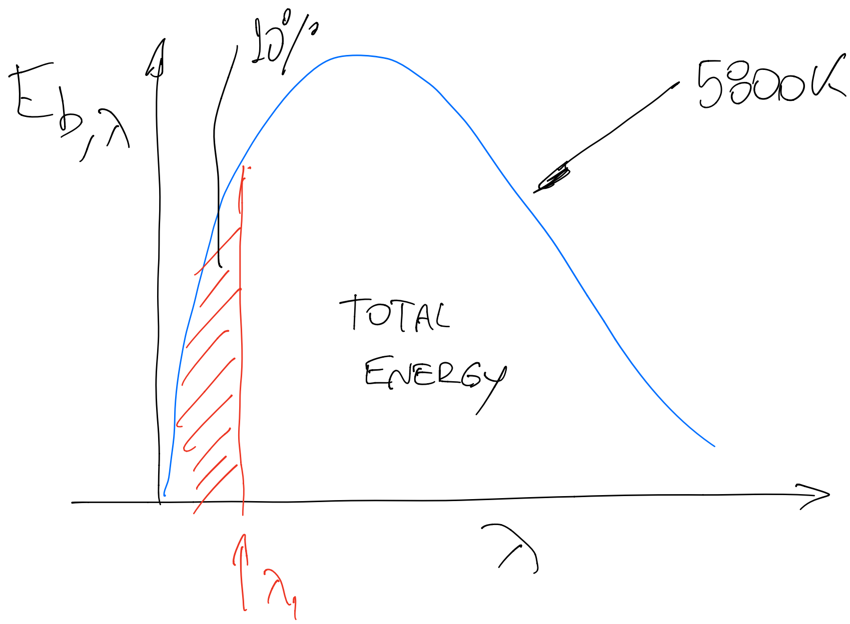
Problem 1



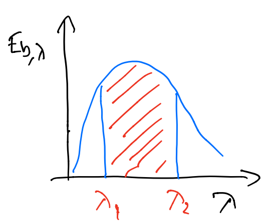
$T_{SUN} = 5800K$

Assumed a Black Body

• Wavelength below which 10% of the solar emission is concentrated?



NOTE THAT TABLES HAVE BEEN CONSTRUCTED TO ESTIMATE THE FRACTION OF ENERGY EMITTED BETWEEN TWO WAVELENGTHS λ_1 AND λ_2 (see below)



FRACTION OF ENERGY BETWEEN λ_1 & λ_2 ⟹ use Tables

λT	$F_{0-\lambda T}$
-	-
-	-
-	-
-	-

So in this case we are looking $F_{0-\lambda_1 T} = 0.10$ (10%) From Tables (2)

Table 8.2: Blackbody radiation functions.

λT [μmK]	$F_{0-\lambda T}$	λT [μmK]	$F_{0-\lambda T}$	λT [μmK]	$F_{0-\lambda T}$
0	0	10200	0.91823	20400	0.986375
200	7.01016e-27	10200	0.91823	20600	0.986733
400	2.20179e-12	10400	0.92197	20800	0.98707
600	9.96926e-08	10600	0.925491	21000	0.987413
800	1.70667e-05	10800	0.928807	21200	0.987735
1000	0.000328258	11000	0.931932	21400	0.988047
1200	0.00216717	11200	0.9344881	21600	0.988349
1400	0.00787475	11400	0.937665	21800	0.98864
1600	0.0198754	11800	0.942781	22000	0.988922
1800	0.0395777	12000	0.945133	22200	0.989195
2000	0.0670396	12200	0.94736	22400	0.989459
2200	0.10126	12400	0.94947	22600	0.989715
2400	0.140672	12600	0.95147	22800	0.989962
2600	0.183564	12800	0.953368	23000	0.990202
2800	0.228346	13000	0.955169	23200	0.990434
3000	0.273688	13200	0.95688	23400	0.990659
3200	0.318549	13400	0.958506	23600	0.990877
3400	0.362168	13600	0.960052	23800	0.991088
3600	0.404021	13800	0.961523	24000	0.991293
3800	0.443775	14000	0.962924	24200	0.991492
4000	0.481246	14200	0.964258	24400	0.991685
4200	0.51636	14400	0.96553	24600	0.991872
4400	0.549119	14600	0.966743	24800	0.992053
4600	0.579582	14800	0.9679	25000	0.99223
4800	0.607839	15000	0.969004	25200	0.992401
5000	0.634007	15200	0.97006	25400	0.992567
5200	0.658212	15400	0.971068	25600	0.992728
5400	0.680584	15600	0.972032	25800	0.992885
5600	0.701254	15800	0.972954	26000	0.993038
5800	0.720351	16000	0.973836	26200	0.993186
6000	0.737997	16200	0.974681	26400	0.99333
6200	0.754307	16400	0.97549	26600	0.99347
6400	0.769389	16600	0.976265	26800	0.993606
6600	0.783343	16800	0.977008	27000	0.993738
6800	0.796263	17000	0.977721	27200	0.993867
7000	0.808234	17200	0.978405	27400	0.993993
7200	0.819335	17400	0.979061	27600	0.994115
7400	0.829637	17600	0.979691	27800	0.994234
7600	0.839206	17800	0.980297	28000	0.994349
7800	0.848101	18000	0.980879	28200	0.994462
8000	0.856379	18200	0.981438	28400	0.994572
8200	0.864089	18400	0.981976	28600	0.994678
8400	0.871276	18600	0.982494	28800	0.994783
8600	0.877981	18800	0.982992	29000	0.994884
8800	0.884244	19000	0.983471	29200	0.994983
9000	0.890098	19200	0.983933	29400	0.995079
9200	0.895575	19400	0.984378	29600	0.995173
9400	0.900703	19600	0.984807	29800	0.995265
9600	0.90551	19800	0.98522	30000	0.995354
9800	0.910019	20000	0.985619		
10000	0.914252	20200	0.986004		

For $F_{0-\lambda T} \approx 0.10$ (10%)

$\lambda_1 T = 2200\ K.\mu m$

$\lambda_2 T = 9400\ \mu m.K$

From Table 8.2

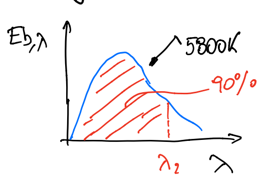
$$\lambda_1 T = 2200\ \mu m.K = \lambda_1 \times 5800\,K$$

$$\lambda_1 = \frac{2200\ \mu m\,K}{5800\,K} \simeq 0.38\ \mu m$$

$$\lambda_2 T = 9400\ \mu m\,K = \lambda_2 \times 5800\,K$$

$$\lambda_2 = \frac{9400\ \mu m\,K}{5800\,K} = 1.62\ \mu m$$

- Wavelength above which 10% of the solar emission is concentrated



From Table 8.2 For $F_{0-\lambda_2 T} = 0.9 \Longrightarrow \lambda_2 T = 9400\ \mu m.K$

$$\lambda_2 = \frac{9400\ \mu m.K}{5800\,K} = 1.62\ \mu m$$

- Determine the maximum spectral emission power of the sun and the wavelength at which this emission occurs

$$\lambda_{max} T = 2897.6\ \mu m.K = \lambda_{max} \cdot 5800\,K$$

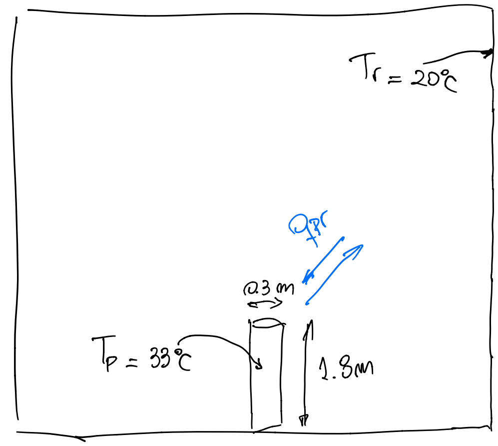
$$\lambda_{max} = \frac{2897.6\ \mu m.\cancel{K}}{5800\cancel{K}} = 0.50\ \mu m$$

$$\boxed{\lambda_{max} = 0.50\ \mu m}$$

Problem 2

$\varepsilon_{skin} = 0.95$

Properties of Air Table C.8



(1) Calculate the net radiative heat transfer from the person and the room

This is a typical case of exchange of radiative energy between a small body (the person) surrounded by an enclosure (the room). For that situation the equation to apply is:

$$q_{pr} = \frac{\sigma (T_p^4 - T_r^4)}{\frac{1-\varepsilon_p}{\varepsilon_p A_p} + \frac{1}{A_p F_{pr}} + \frac{1-\varepsilon_r}{\varepsilon_r A_r}} \quad (1)$$

If the person is completely enclosed (by the room) $F_{pr} \to 1$. Also $A_r >> A_p$

By rearranging Eq.(1)

$$q_{pr} = \frac{\sigma (T_p^4 - T_r^4)}{\frac{1}{\varepsilon_p A_p} \left[1 - \varepsilon_p + \varepsilon_p + (1-\varepsilon_r) \underbrace{\frac{\varepsilon_p \cancel{A_p}}{\cancel{\varepsilon_r A_r}}}_{\approx 0 \text{ because } \frac{A_p}{A_r} \to 0} \right]} \quad (2)$$

$$q_{pr} = A_p \varepsilon_p \sigma (T_p^4 - T_r^4) \quad (3)$$

$A_p = \pi D L$ [We are assuming no radiation from head and feets

$\varepsilon_p = \varepsilon_k = 0.95$

Substituting values into Eq.(3)

$$q_{pr} = \pi \times 0.3 \times 1.8\, m^2 \times 0.95 \times 5.67 \times 10^{-8} \frac{W}{m^2 K^4} [306^4 - 293^4] K^4$$

$$\boxed{q_{pr} = 128\, W}$$

(2) what is the wavelength λ_{max} where the maximum amount of energy will be radiated? (4)

Since we are considering radiated energy from the person the temperature is

$$T_p = 33°C + 273 = 306K$$

$$\lambda_{max} T = 2897.6\ \mu m.K \Rightarrow \lambda_{max} = \frac{2897.6\ \mu m.K}{306K} \cong 9.5\mu m$$

$$\boxed{\lambda_{max} = 9.5\mu m}$$

(3) Region of spectrum where the radiation calculated in (2) is:

The region is mostly in the IR region [i.e Heat]

(4) A detector is available to detect in the range $\lambda_{max} \pm 5\mu m$, what fraction of the total energy from humans will this detector will be sensitive to?

$$F_{\lambda_1 - \lambda_2} = F_{0-\lambda_1 T} - F_{0-\lambda_2 T} \qquad (1)$$

$$\lambda_1 T = (9.5+5)\mu m \times 306K = 4428\mu m.K \Longrightarrow F_{0-\lambda_1 T} \cong 0.55$$

$$\lambda_2 T\ (9.5-5)\mu m \times 306K = 1638\mu mK \Longrightarrow F_{0-\lambda_2 T} \cong 0.007$$

From Eq.(1) FRACTION DETECTED BETWEEN λ_1 & λ_2 $= F_{0-\lambda_1 T} - F_{0-\lambda_2 T} = 0.53 - 0.007 \approx 0.543\ (54.3\%)$

(5) Calculate the natural convection to the air

We can use the Following correlation:

$$Nu_L = \left(0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{9/16}\right]^{8/27}}\right)^2 \qquad (2)$$

Film temperature $T_f = \frac{20+33}{2}°C \approx 26.5°C \cong 300K$

From Air Tables at 300K $\quad Pr = 0.71 \quad \mu_{air} = 1.85\times10^{-5}\ Pa.s \quad \rho_{air} = 1.18\ kg/m^3$

$K_{air} = 0.026\ \frac{W}{m.K}$

For $T_f = 300K$

$$Gr = \frac{\beta g L^3 \Delta T}{\left(\frac{\mu_{air}}{\rho_{air}}\right)^2} = \frac{\frac{1}{300K} \times 9.81 \, m/s^2 \times 1.8^3 m^3 \times (33-20) K}{\frac{1.85 \times 10^{-5} \, Pa.s}{1.18 \, kg/m^3}} \cong 1 \times 10^{10}$$

$$Gr = 1 \times 10^{10}$$

$$Ra = Gr \times Pr = 1 \times 10^{10} \times 0.708 \cong 7.1 \times 10^{9}$$

Substituting values into Eq.(2)

$$Nu_L = \left[0.825 + \frac{0.387 \times (7.1 \times 10^9)^{1/6}}{\left(1 + \left(\frac{0.492}{0.708}\right)^{9/16}\right)^{8/27}}\right]^2 \cong 226$$

$$Nu_L = \frac{hL}{k_{air}} \cong 226 \Rightarrow h = \frac{226 \, k_{air}}{L} = \frac{226 \times 0.026 \, W/m.K}{1.8 \, m} = 3.3 \, \frac{W}{m^2 K}$$

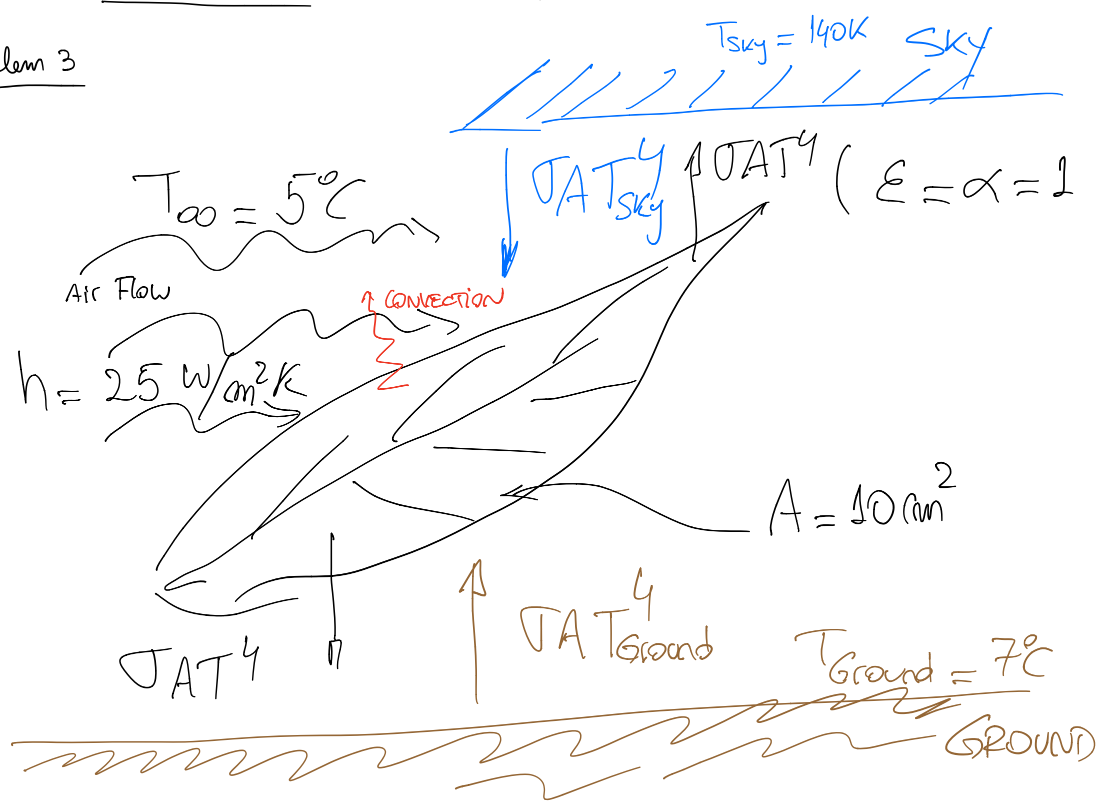
$$\boxed{h \cong 3.3 \, \frac{W}{m^2 K}}$$

So Heat loss by Convection [Natural because there is no a forced air flow]

$$Q = hA[T_p - T_r] = 3.3 \, \frac{W}{m^2 K} \times \pi \times 0.3 \times 1.8 \, m^2 (33-20) K = 72.8 \, W$$

$\underline{Q = 72.8 \, W}$ [Almost half of the radiative heat loss]

Problem 3



Balance of Energy [Let's Assume All black bodies) on the leaf

Energy From Sky + Energy From Ground = Energy out to Sky + Energy out to Ground + Convection both sides of leaf

$$\sigma \cancel{A} T_{sky}^4 + \sigma \cancel{A} T_{Ground}^4 = \sigma \cancel{A} T^4 + \sigma \cancel{A} T^4 + 2h\cancel{A}[T - T_{air}] \quad (1)$$

$$\sigma(T_{sky}^4 + T_{Ground}^4 - 2T^4] = 2h(T - 278)$$

$$5.67 \times 10^{-8} \frac{W}{m^2 \cancel{K^4}} [110^4 + 280^4 - 2T^4] \cancel{K^4} = 50(T - 278)$$

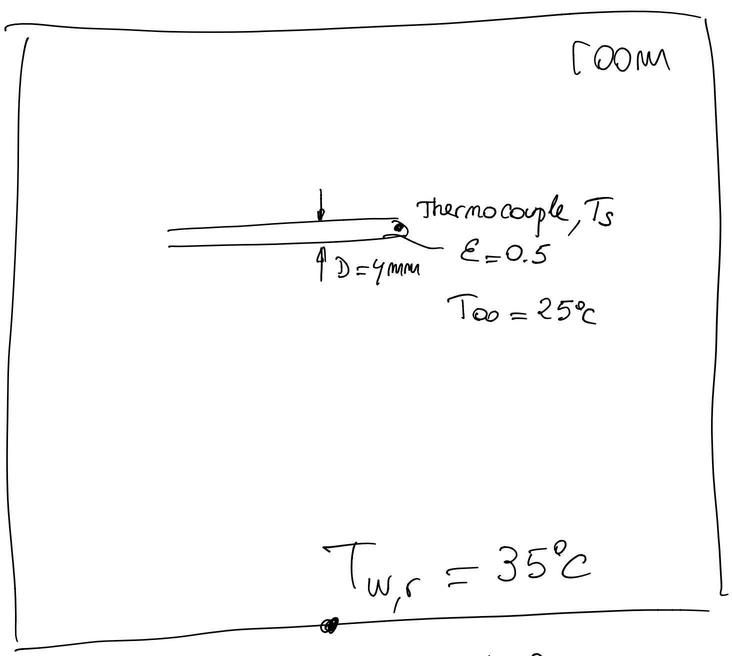
The equation can be solved numerically, for example, using MathCad (root)

Solution $T = 272.8K \Rightarrow \underline{T = -0.2°C}$

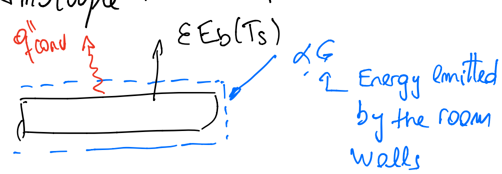
If h is increased $T \to T \to T_\infty$ so leaves will not freeze.

In Florida that is achieved by using fans to increase air velocity and thus the value of h

Problem 4



(1) what temperature would the thermocouple indicate?



The balance of energy is

$$q''_{conv} + \varepsilon E_b(T_s) = \alpha G \qquad \left[\text{Assume } \varepsilon_{room} \approx 1\right] \qquad (7)$$

$h(T_s - T_\infty)$ $\qquad$ σT_s^4 ← Thermocouple temperature $\qquad$ $\sigma T_{w,r}^4$

$$h(T_s - T_\infty) + \varepsilon \sigma T_s^4 = \alpha \sigma T_{w,r}^4 \qquad (1)$$

If we assume that the thermocouple is a gray body $\varepsilon = \alpha$

$$\varepsilon \sigma (T_{w,r}^4 - T_s^4) = h(T_s - T_\infty) \qquad (2)$$

T_s is the temperature measured by the thermocouple, it can be calculated from Eq.(2) provided the value of h is known

It is assumed natural convection so:

$$Nu_D = \frac{hD}{K} = \left[0.60 + \frac{0.387\, Ra_D^{1/6}}{\left(1 + \left(\frac{0.492}{Pr}\right)^{9/16}\right)^{8/27}}\right]^2 \qquad (3)$$

$$Ra_D = \frac{\beta g \Delta T D^3}{\nu_{air}\, \alpha_{air}}$$

The problem is that we do not know T_s (it is what to be evaluated. let's assume for example a value; for instance $T_s = 25°C$

$$T_f = \frac{25 + 20}{2} \approx 23°C = 296\,K$$

$$Ra_D = \frac{\left(\frac{1}{296}\right) \times 9.81 \times (25 - 20)(4 \times 10^{-3})^3}{15.5 \times 10^{-6} \times 22 \times 10^{-6}} \cong 31$$

at $23°C$ $\quad Pr = 0.708$

Substituting values into Eq(3)

$$Nu_D = \left[0.60 + \frac{0.387 \times 31^{1/6}}{\left(1 + \left(\frac{0.492}{0.708}\right)^{9/16}\right)^{8/27}}\right]^2 \cong 1.4$$

(8)

$$Nu_D = \frac{hD}{K_{air}} = 1.4 \Rightarrow h = \frac{1.4 \times K_{air}}{D} = \frac{1.4 \times 0.023}{4 \times 10^{-3}} = 8.1 \ \frac{W}{m^2K}$$

by substituting into Eq.(2)

$$0.5 \times 5.67 \times 10^{-8} \frac{W}{m^2 \cancel{K^4}} [303^4 - T_s^4] \cancel{K^4} = 8.1 [T_s - 293]$$

By using MathCad $T_s = 295.7 \Rightarrow T_s = 22.7°C$ [Error ~ 2.3°C]