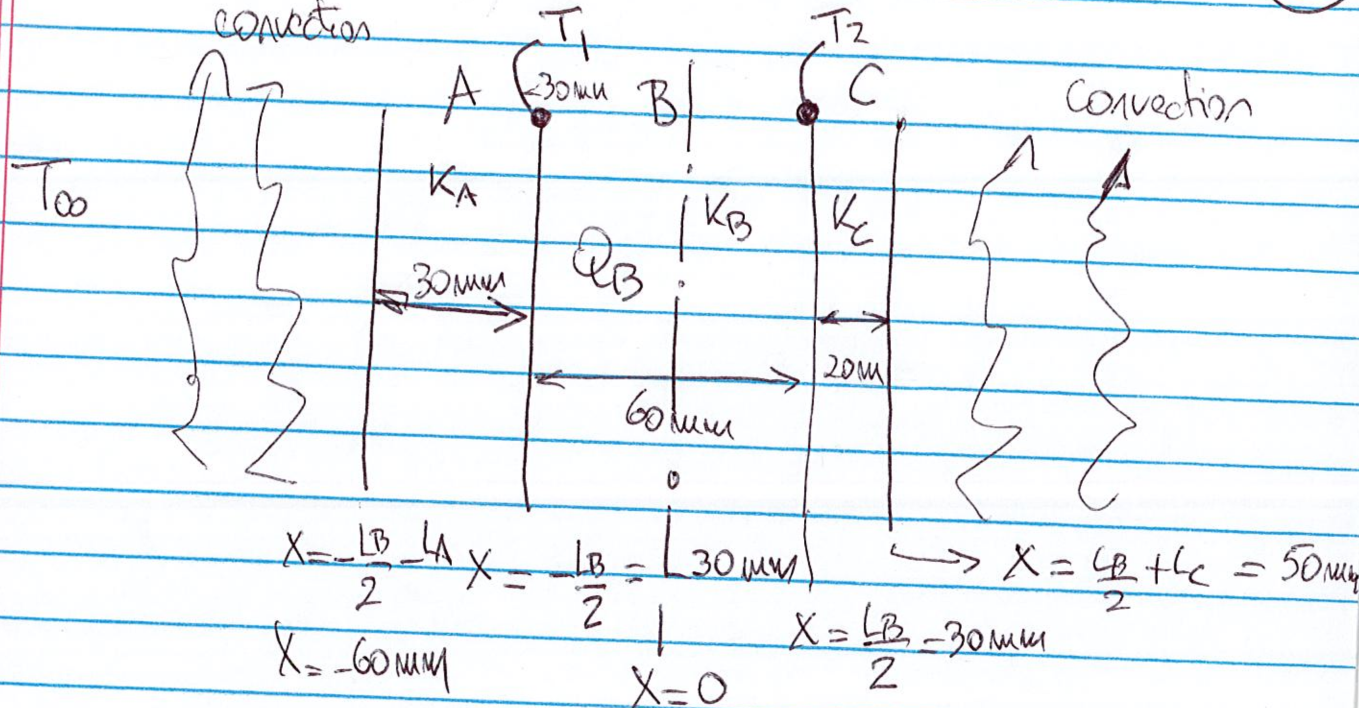
OFFICE HOURS 2-14-18



$T_1 = 250°C$

$T_2 = 210°C$

Assumptions

- 1-D Flow ← X-direction
- steady state
- Q_B = HEAT GENERATED AT B IS UNIFORM $\frac{KW}{m^3}$
- NOT CONVECTION [IN THE DOMAIN, WHICH ARE THE PLATES]

Let's solve the problem in layer B

General Equation [Microscopic Equation]

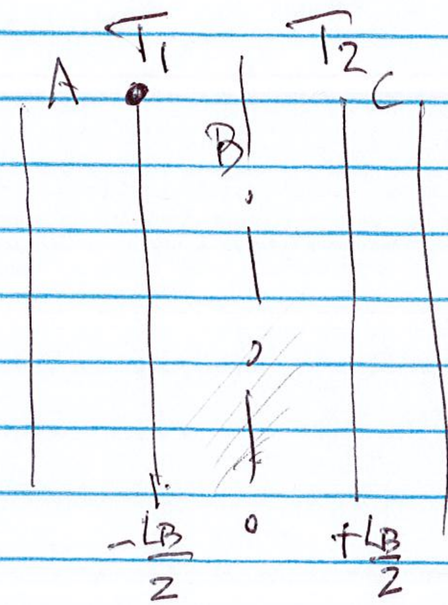
$$\cancel{\frac{\partial T}{\partial t}} + \underbrace{\cancel{u \cdot \nabla T}}_{\text{convection}} = \frac{k}{\rho c}\left[\frac{\partial^2 T}{\partial x^2} + \cancel{\frac{\partial^2 T}{\partial y^2}} + \cancel{\frac{\partial^2 T}{\partial z^2}}\right] + \frac{\dot{q}_B}{\rho c}$$

$$0 = \frac{k}{\cancel{\rho c}} \frac{d^2T}{dx^2} + \frac{\dot{q}_B}{\cancel{\rho c}} \qquad (2)$$

$$\boxed{\frac{d^2T}{dx^2} = -\frac{\dot{q}_B}{k}}$$

LAYER B $\left[T_B(x) \right]$

$$\begin{cases} \dfrac{d^2T_B}{dx^2} = -\dfrac{\dot{q}_B}{k_B} \\ x = -\dfrac{L_B}{2} \quad T_B = T_1 \\ x = +\dfrac{L_B}{2} \quad T_B = T_2 \end{cases}$$



Integrating once

$$\frac{dT_B}{dx} = -\frac{\dot{q}_B}{k_B} x + C_1$$

Integrating again.

$$T_B(x) = -\frac{\dot{q}_B}{2k_B} x^2 + C_1 x + C_2$$

$x = -\frac{L_B}{2}$ $\qquad T_1 = -\frac{\dot{q}_B}{2k_B} \times \frac{L_B^2}{4} - C_1 \frac{L_B}{2} + C_2 \qquad (1)$

$x = \frac{L_B}{2}$ $\qquad T_2 = -\frac{\dot{q}_B}{2k_B} \frac{L_B^2}{4} + C_1 \frac{L_B}{2} + C_2 \qquad (2)$

Adding Eqs (1) and (2)

$$T_1 + T_2 = -\frac{\dot{q}_B}{2K_B}\underbrace{\left[\frac{L_B^2}{4} + \frac{L_B^2}{4}\right]}_{\frac{L_B^2}{2}} + 2C_2$$

$$\cancel{(T_1 + T_2) + \frac{\dot{q}_B}{2K_B}}$$

$$(T_1 + T_2) + \frac{\dot{q}_B}{2K_B}\frac{L_B^2}{2} = 2C_2$$

$$\boxed{C_2 = \frac{(T_1 + T_2)}{2} + \frac{\dot{q}_B}{4K_B}\frac{L_B^2}{2}}$$

Eq.(1) − Eq. 2

$$T_1 - T_2 = \cancel{-\frac{\dot{q}_B}{2K_B}\frac{L_B^2}{4}} \cancel{+\frac{\dot{q}_B}{2K_B}\frac{L_B^2}{4}} \underbrace{- C_1\frac{L_B}{2} - C_1\frac{L_B}{2}}_{-C_1 L_B}$$

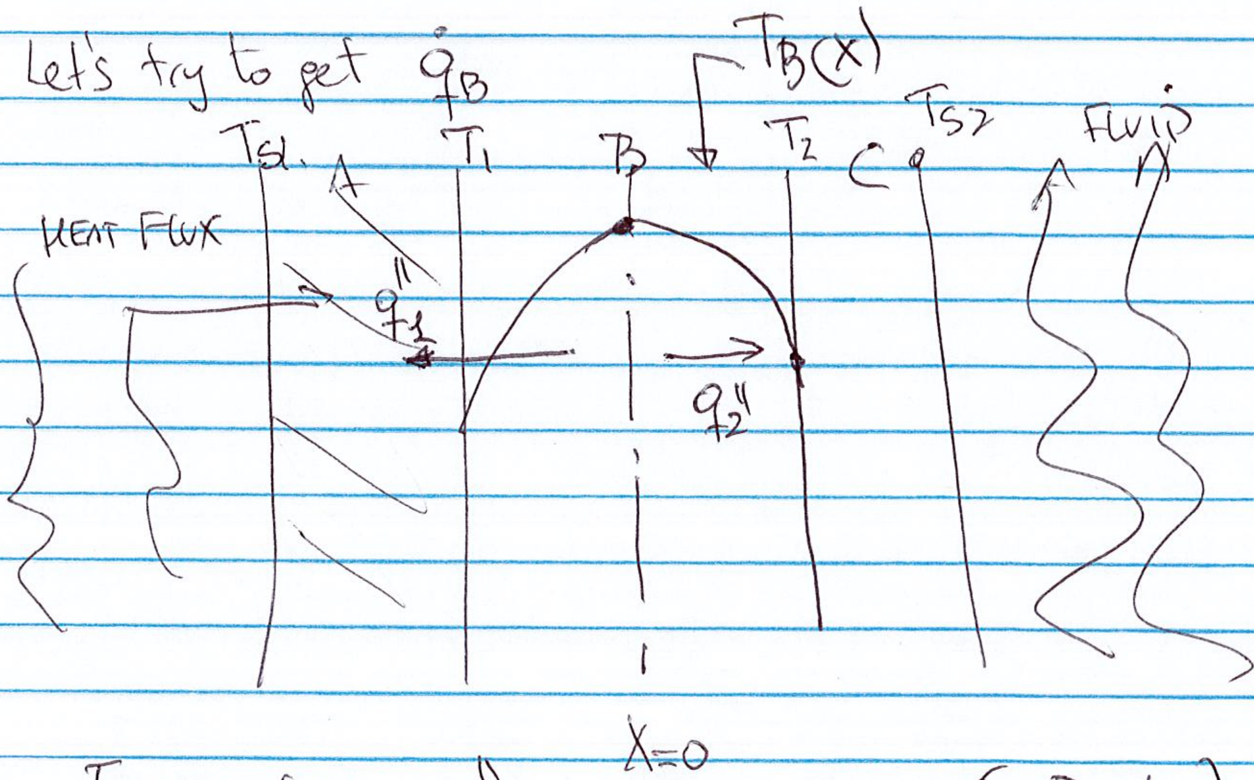
$$\boxed{C_1 = -\frac{T_1 - T_2}{L_B}}$$

(4)

$$T_B(x) = -\frac{\dot{q}_B}{2K_B} x^2 + C_1 x + C_2$$

$$\boxed{T_B(x) = -\frac{\dot{q}_B}{2K_B} x^2 - \left(\frac{T_1 - T_2}{L_B}\right) x + \frac{T_1 + T_2}{2} + \frac{\dot{q}_B L_B^2}{8K_B}}$$

Let's try to get $\dot{q}_B$

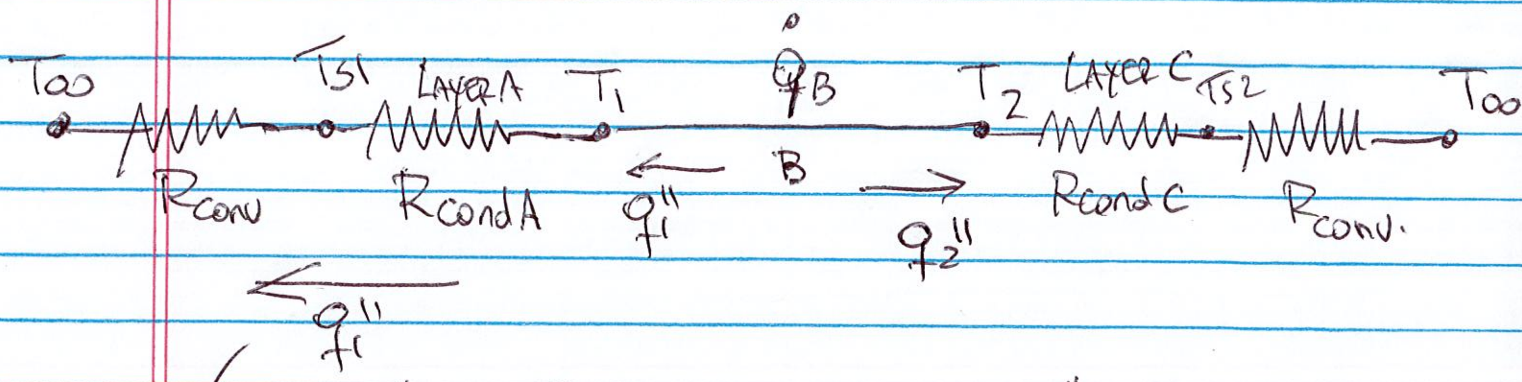


FOURIER LAW $\quad q'' = -K\frac{dT}{dx} \quad \frac{W}{m\,\cancel{K}} \cdot \frac{\cancel{K}}{m} \left[\frac{W}{m^2}\right]$

$L_B \dot{q}_B = \underbrace{q_1'' + q_2''}_{W/m^2}$ ✓ ?? Yes if we have steady state!! Which was assumed

$\dot{q}_B = Q_B = \frac{W}{m^3}$

Electrical diagram



$$q_1'' = \frac{T_1 - T_\infty}{R_{cond\,A} + R_{conv}} \qquad q_2'' = \frac{T_2 - T_\infty}{R_{cond\,C} + R_{conv.}}$$

for example $\quad q_1'' = \dfrac{T_{S1} - T_\infty}{R_{conv.}} \qquad q_2'' = \dfrac{T_{S2} - T_\infty}{R_{conv.}}$

NOT CONVENIENT BECAUSE WE DO NOT KNOW T_{S1} & T_{S2}

$$R_{cond\,A} = \frac{L_A}{K_A}$$ ← we don't use A [Area] because we are calculating the flux

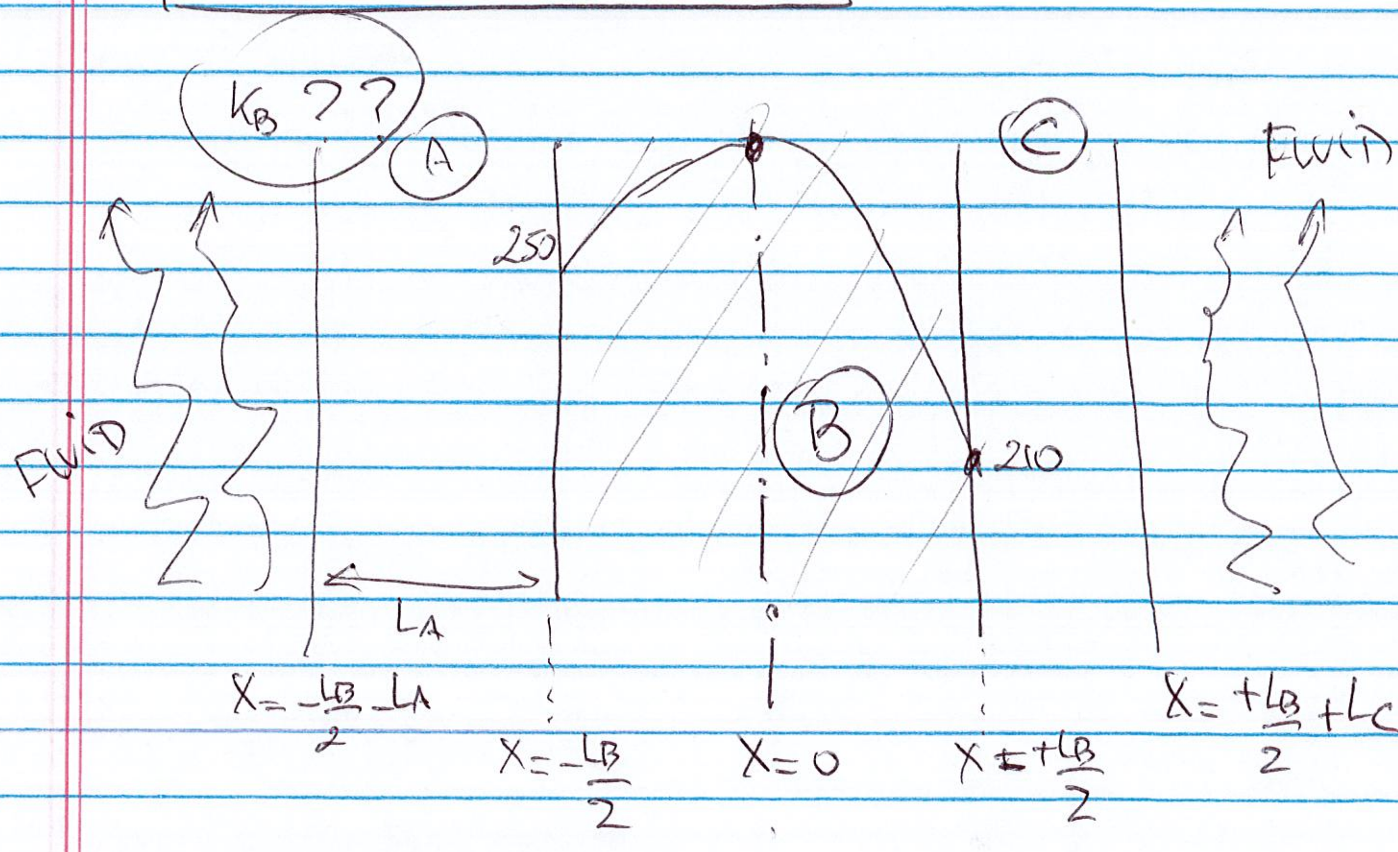
$$R_{conv} = \frac{1}{h} \qquad \frac{q_1}{A} = q_1''$$

$$q_1'' = \frac{[250 - 25]\cancel{K}}{\dfrac{30\times10^{-3}\,m}{25\,W/m\cancel{K}} + \dfrac{1}{800\,W/m^2\cancel{K}}} = (\quad)\frac{W}{m^2}$$

~~250+273~~ ~~25+273~~

$$\dot{q}_2'' = \frac{(210 - 25)\,\cancel{K}}{\dfrac{20\times10^{-3}\,m}{50\,\frac{W}{m\cdot \cancel{K}}} + \dfrac{1}{800\,\frac{W}{m^2 \cancel{K}}}} = (\quad)\ \frac{W}{m^2} \quad (6)$$

$$\boxed{\dot{q}_B = \frac{q_1'' + q_2''}{L_B}} \quad \frac{W}{m^3}$$



Let's see what layer A

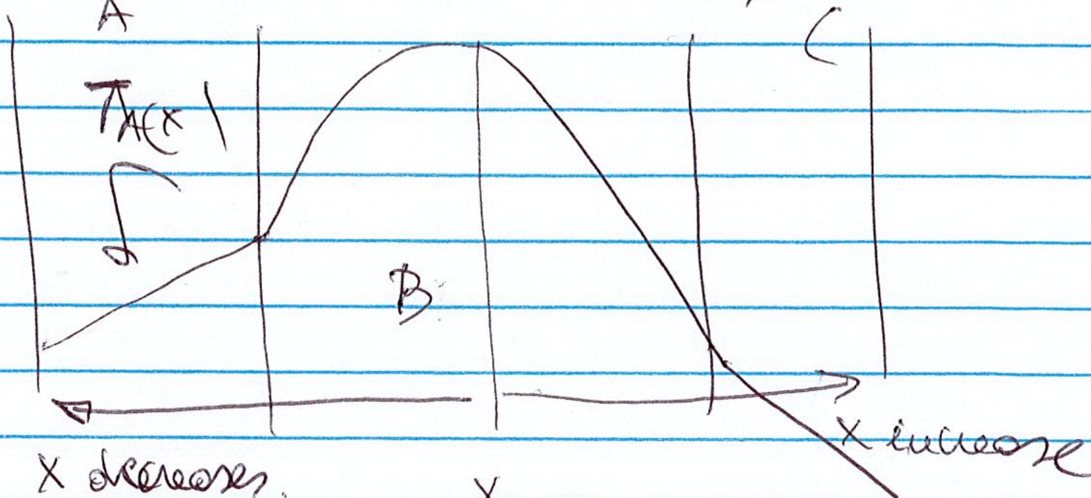
$$-K_A \frac{d^2 T_A(x)}{dx^2} = 0$$

Because there is no heat generation in A

(7)

$$\begin{cases} \dfrac{d^2 T_A(x)}{dx^2} = 0 \quad (3) \\ \text{at } x = -\dfrac{L_B}{2} \quad T_A(x) = T_1 \\ \text{at } x = -\dfrac{L_B}{2} - L_A \quad \underbrace{+K_A \dfrac{dT_A}{dx}}_{\text{Conductive Flux}} = h\left[T_A\big|_{-\frac{L_B}{2}-L_A} - T_\infty\right] \end{cases}$$

LAYER A



$\dfrac{dT_A}{dx}$ (+)

integrating Eq. (3) once. $\dfrac{dT_A(x)}{dx} = B_1$

Integrating above Equation again $T_A(x) = B_1 x + B_2$

Let's use Boundary conditions.

(8)

$$X = -\frac{L_B}{2} \qquad T_1 = -B_1 \frac{L_B}{2} + B_2$$

at $X = -\frac{L_B}{2} - L_A$ $\quad K_A B_1 = h\left[T_{S1} - T_\infty\right]$

$$T_A\left(X = -\frac{L_B}{2} - L_A\right)$$

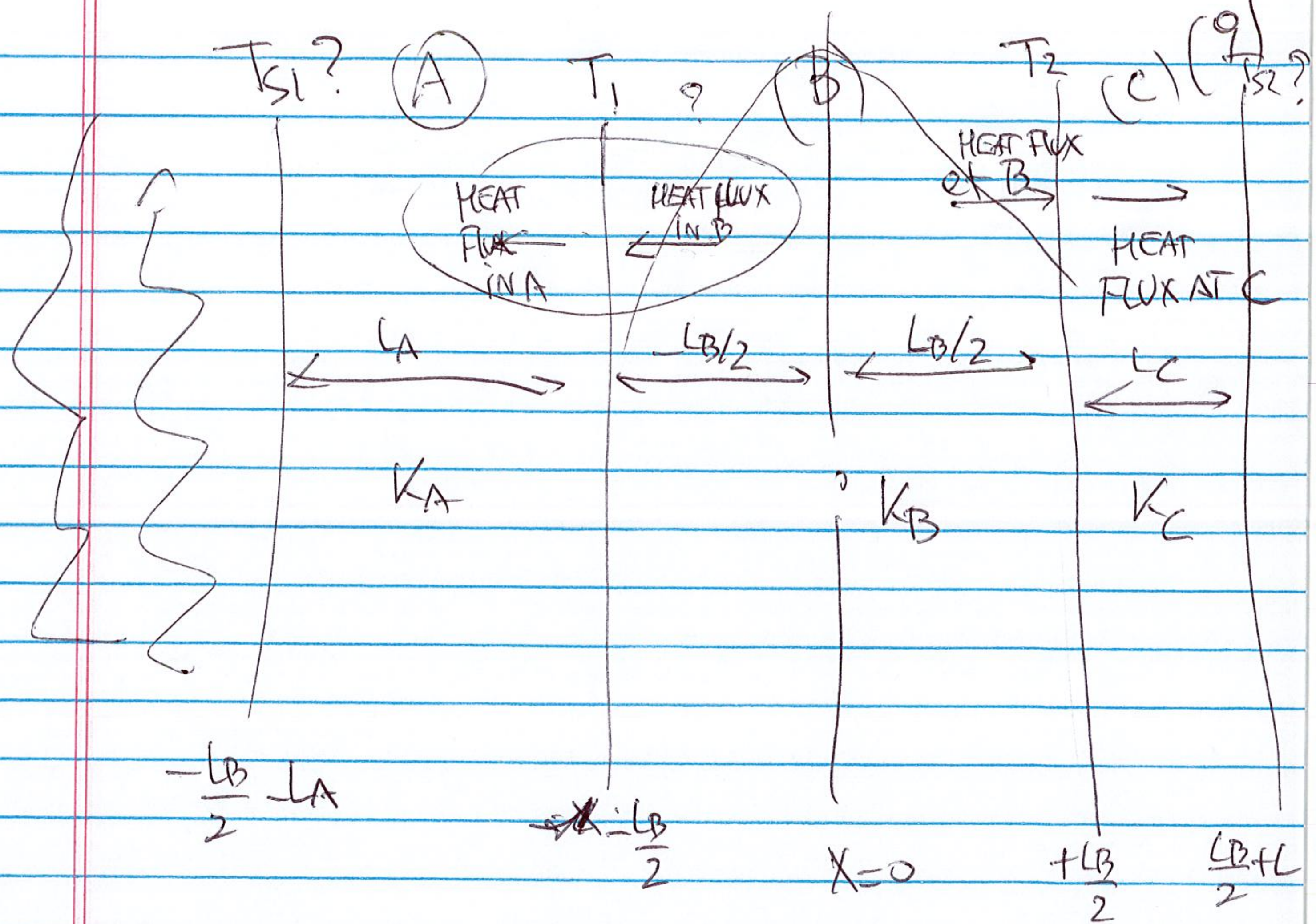
$$B_1 = \frac{h}{K_A}\left[T_{S1} - T_\infty\right]$$

$$\cancel{T_1 = -B_1 \frac{L_B}{2} +}$$

$$B_2 = T_1 + B_1 \frac{L_B}{2}$$

$$T_A(x) = \frac{h}{K_A}\left[T_{S1} - T_\infty\right] X + T_1 + \frac{h}{K_A}\left(T_S - T_\infty\right) X$$

$$\boxed{T_A(X) = \frac{h}{K_A}\left(T_{S1} - T_\infty\right)\left(1 + X\right) + T_1}$$



INTERFACE BETWEEN A & B → at $X = -\frac{L_B}{2}$ HEAT FLUX IN LAYER B = HEAT FLUX IN LAYER A

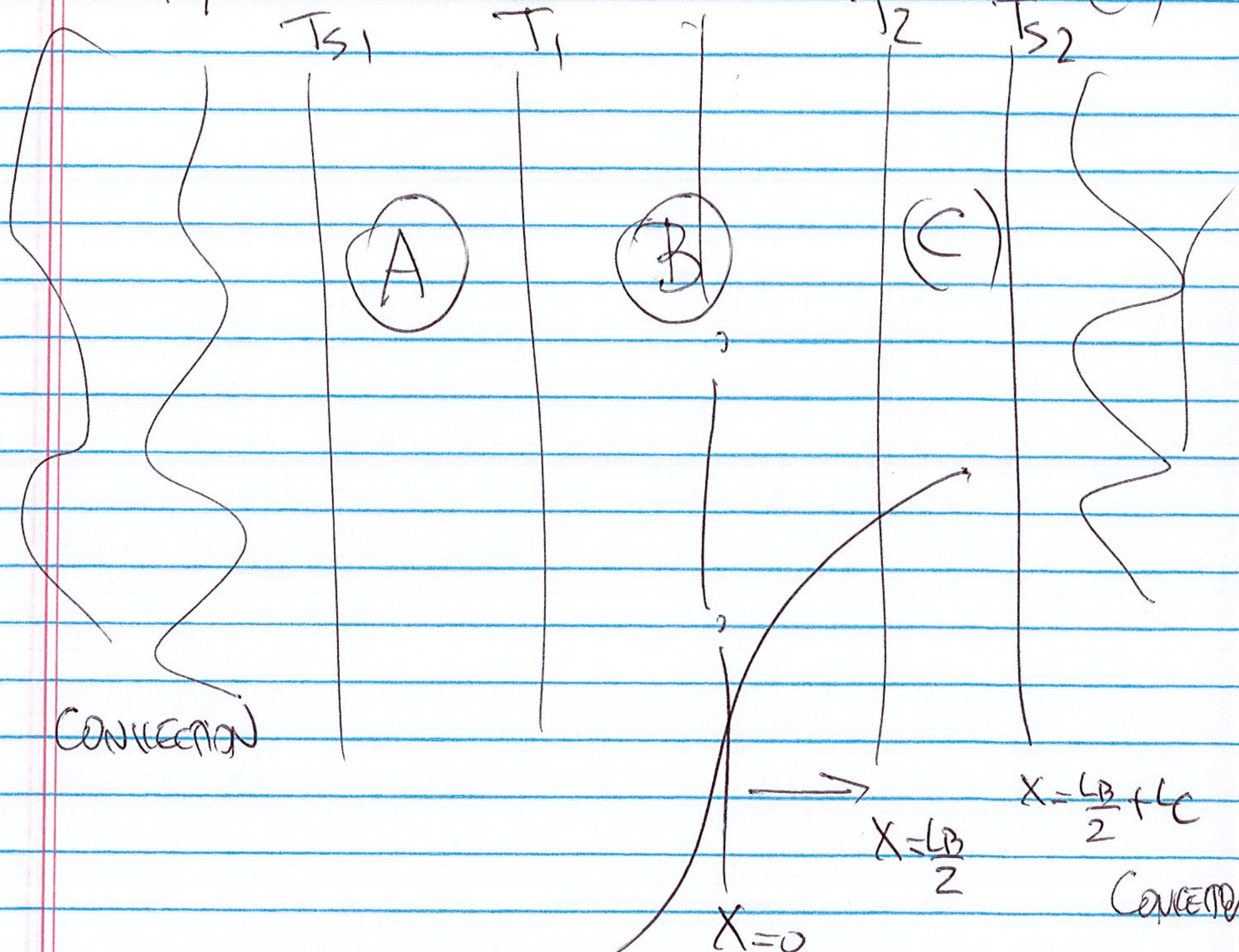
[Because Steady state]

$$K_B \frac{dT_B}{dX} = K_A \frac{dT_A}{dX}$$

INTERFACE BETWEEN B & C at $X = +\frac{L_B}{2}$

$$K_B \frac{dT_B}{dX} = K_C \frac{dT_C}{dX}$$

LAYER C



$$\frac{d^2 T_C(x)}{dx^2} = 0$$

$X = +\frac{L_B}{2}$ $\quad T = T_2$

$X = +\frac{L_B}{2} + L_C$ $\quad T = T_{S2}$

or $-K_C \frac{dT_C}{dX} = h\left[T_{S2} - T_{\infty}\right]$

$$T_C(x) = D_1 x + D_2$$ (11)

at $x = \frac{L_B}{2}$ $\quad T_2 = D_1 \frac{L_B}{2} + D_2$

at $x = \frac{L_B}{2}$ $\quad \underbrace{K_B \frac{dT_B}{dx}} = K_C \underbrace{\frac{dT_C(x)}{dx}}_{D_1}$

$$T_B(x) = -\frac{\dot{q}_B x^2}{2K_B} - \left(\frac{T_1 - T_2}{L_B}\right) x + \left(\frac{T_1 + T_2}{2}\right) + \frac{\dot{q}_B L_B^2}{8K_B}$$

$$\left.\frac{dT_B(x)}{dx}\right|_{x=\frac{L_B}{2}} = -\frac{\dot{q}_B}{\cancel{2}K_B} \cancel{2}x - \left(\frac{T_1 - T_2}{L_B}\right)$$

$$\left.\frac{dT_B(x)}{dx}\right|_{x=\frac{L_B}{2}} = -\frac{\dot{q}_B}{K_B} \frac{L_B}{2} - \left(\frac{T_1 - T_2}{L_B}\right)$$

$$-K_B \left[+\frac{\dot{q}_B}{} \frac{L_B}{2} + \frac{T_1 - T_2}{L_B} \right] = K_C D_1$$

(12)

$$\begin{cases} \dfrac{d^2T_c}{dx^2} = 0 \\ x = \dfrac{L_B}{2} \quad T = T_2 \\ x = \dfrac{L_B}{2} + L_c \quad -K_c \dfrac{dT_c}{dx} = h\left[T_{s2} - T_\infty\right] \\ x = \dfrac{L_B}{2} \quad K_B \dfrac{dT_B}{dx} = K_c \dfrac{dT_c}{dx} \end{cases}$$

$$T_c(x) = D_1 x + D_2$$

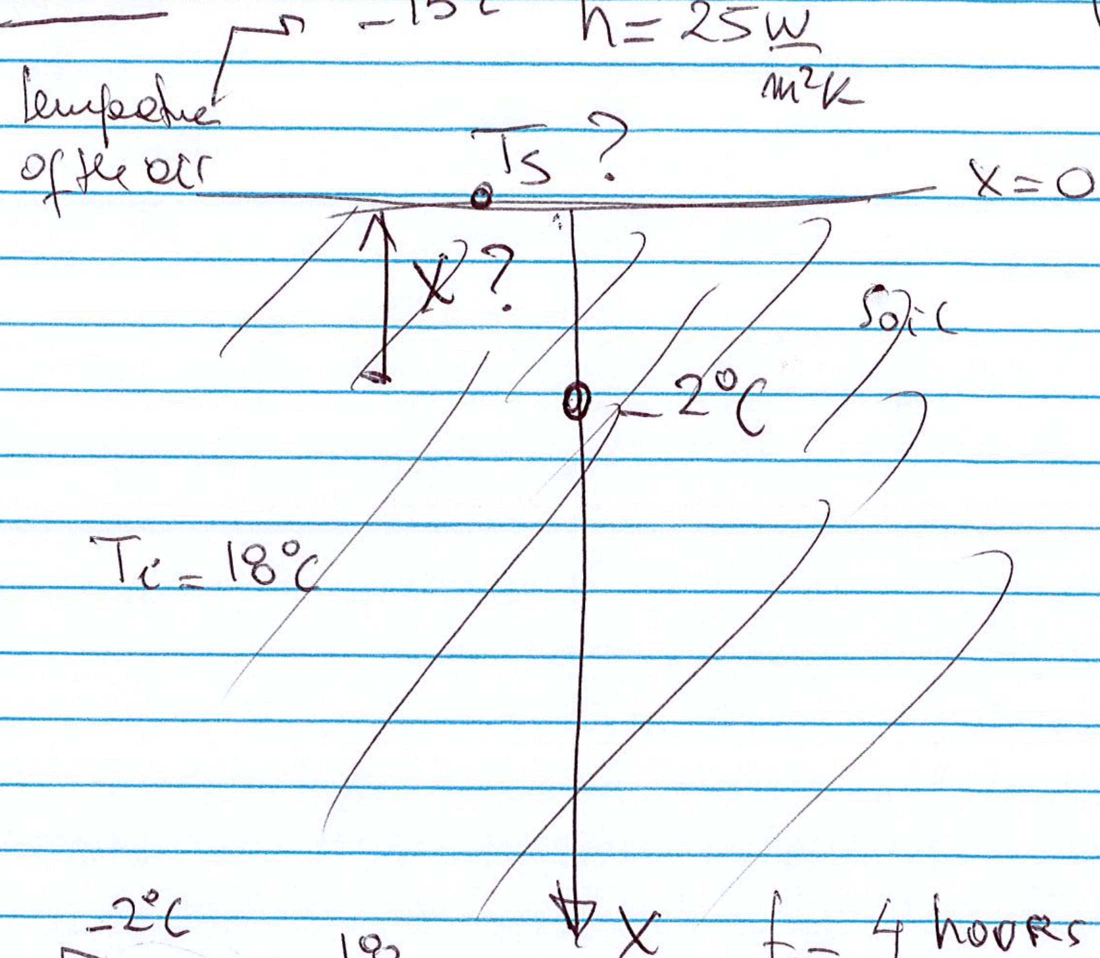
$$T_2 = D_1 \frac{L_B}{2} + D_2$$

$$\frac{dT_c(x)}{dx} = D_1$$

$$K_c D_1 = -K_B \left[\dot{q}_B \frac{L_B}{2} + \frac{T_1 - T_2}{L_B} \right]$$

$$\boxed{D_1 = -\frac{K_B}{K_c}\left(\dot{q}_B \frac{L_B}{2} + \frac{T_1 - T_2}{L_B} \right)}$$

Problem 5



$t = 4$ hours

$$\frac{\overset{-2°C}{T(x,t)} - \overset{18}{T_i}}{\underset{-15°}{T_\infty} - T_i} = 1 - \operatorname{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right) \cdot \left(1 - \operatorname{erf}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)\right)$$

(14)

~~Eq~~ $f(x) = \frac{T(x,t) - T_i}{T_\infty - T_i} - 1 + \mathrm{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)$

$+ \exp\left(\quad \right)\left(1 - \mathrm{erf}\left(x \ldots\right)\right)$

$= 0$

if you use Mathcad. use Root

~~set~~ $x_{guess} = 0.1$

$x = \mathrm{root}(f(x), x)$

if you use ~~Ex~~ Excel use goal seek

IN QUESTION 2 WHEN THERE IS NO MORE COOLING [COOLANT IS OFF]

SO $q_1'' = 0$

~~AB~~ AND $\dot{q}_B L_B = q_2''$

The Temperature Profile will be