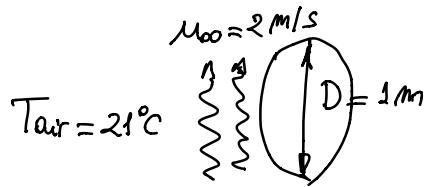


SOLUTION OF CONVECTION PROBLEMS EXTERNAL & INTERNAL CONVECTION

Problem 1

(1)



$$T_{air} = 21^\circ\text{C} = 299\text{ K} \quad A = 0.84\text{ m} \quad D = L = 1\text{ m}$$

We can get the properties of air at 20°C without applying the film temperature
ONLY AN ASSUMPTION IN THIS CASE

$$\rho = 1.22 \frac{\text{kg}}{\text{m}^3} \quad k = 25.5 \times 10^{-3} \frac{\text{W}}{\text{m.K}} \quad \mu = 1.80 \times 10^{-5} \frac{\text{kg}}{\text{m.s}} \quad Pr = 0.71$$

The heat loss through the two ears (heat is transferred from both sides)

$$q = 4A h (T_s - T_{air})$$

The value of h needs to be determined

$$Re = \frac{\rho U_{\infty} D}{\mu} = \frac{1.22 \times 2 \times 1}{1.80 \times 10^{-5}} = 1.35 \times 10^5 \quad \text{Since I use consistent units}$$

the Reynolds number is dimensionless

Flow is laminar so the equation to estimate the Nusselt Number Nu is:

$$Nu = 0.664 Re^{1/2} Pr^{1/3} = 0.664 \times (1.35 \times 10^5)^{1/2} \times 0.71^{1/3} \approx 218$$

$$Nu = 218 = \frac{h D}{k} \Rightarrow h = \frac{218 k}{D} = \frac{218 \times 25.5 \times 10^{-3} \text{ W/m.K}}{1\text{ m}} = 5.6 \frac{\text{W}}{\text{m}^2\text{K}}$$

$$q = 4 \times 0.84 \times 5.6 (T_s - 20) = 18.82 (T_s - T_{air}) \text{ Watts}$$

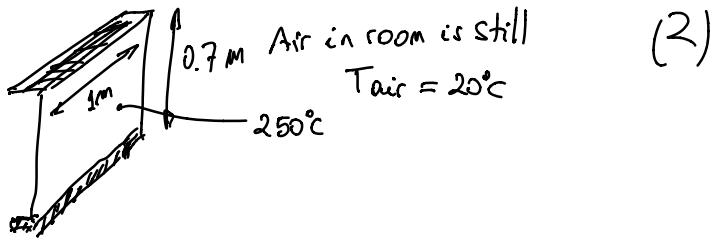
A plot as a function of T_s easily

If the heat production by the elephant is 1650 W and we assume the body temperature 37°C , the heat dissipated by the ears is :

$$\text{percentage} = \frac{18.82(37-20)\text{ W} \times 100}{1650} = 19.4\%$$

About 19.4% of the heat generated is lost through the ears

Problem 2



(2)

(a) Because the air is still the type of convection will be natural convection. Natural convection is produced by the flow created by a difference in densities

(b) Heat transfer to the room

$$q = hA(T_s - T_{air}) \quad \text{so we need to estimate the value of } h \\ \text{For a vertical surface}$$

$$Nu_L = \left(0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + \left(\frac{0.492}{Pr} \right)^{9/16} \right]^{9/27}} \right)^2 \quad (1)$$

This time I remember using the film temperature !!

$$T_f = \frac{250+20}{2} = 135^\circ\text{C} \approx 408\text{K}$$

I will use $T = 400\text{K}$ to get the properties of air so interpolation is avoided

From Table for air at 400K

$$\rho = 0.882 \frac{\text{kg}}{\text{m}^3} \quad \mu = 2.286 \times 10^{-5} \frac{\text{kg}}{\text{m.s}} \quad K = 0.03365 \frac{\text{W}}{\text{m.K}}$$

$$Pr = 0.689 \quad \beta = \frac{1}{T_f} = \frac{1}{408} = 2.451 \times 10^{-3} \frac{1}{\text{K}}$$

$$Gr = \frac{2.451 \times 10^{-3} \times 9.81 \times 0.882^2 \times 0.689^3 \times (250 - 20)}{(2.286 \times 10^{-5})^2} = 2.83 \times 10^9$$

$$Ra_L = Gr \times Pr = 2.83 \times 10^9 \times 0.689 = 1.95 \times 10^9$$

Substituting values into Eq.(1)

$$Nu_L = \left(0.825 + \frac{0.387 \times (1.95 \times 10^9)^{1/6}}{\left(1 + \left(\frac{0.492}{0.689} \right)^{9/16} \right)^{9/27}} \right)^2$$

$$N_{UL} = 150.5 \Rightarrow h = \frac{KN_{UL}}{L} = \frac{0.03365 \times 150.5}{0.7} \frac{W}{m^2 K} = 7.24 \frac{W}{m^2 K} \quad (3)$$

$$h = 7.24 \frac{W}{m^2 K}$$

$$Q = 7.24 \frac{W}{m^2 K} \times 1 \times 0.7 \text{ m}^2 [250 - 20] K = 1,165 \text{ W}$$

$$\boxed{Q = 1,165 \text{ W}}$$

Problem 3

$$q_{w,av} = 0.001 \text{ kg/s}$$

$$q_{w,av} = h_m A (P_{w,sat} - P_{w,air}) \quad (1)$$

$$\frac{\bar{h}}{h_m} = \rho C \left(\frac{\alpha}{D_{w,air}} \right)^{2/3} \quad D_{w,air} = 26 \times 10^{-6} \frac{m^2}{s}$$

$$T_{air} = 290K \quad T_{pool} = 310K \quad T_{film} = \frac{290 + 310}{2} = 300 \\ RH = 30\%$$

Let's assume that the water vapor behaves as an ideal gas, thus

$$PV = nRT = \frac{m}{M_w} RT \Rightarrow \frac{P_{H_2O}}{RT} = \frac{m}{V} = \rho_w$$

Thus

$$P_{w,air} = \frac{P_w M_w}{RT} \quad \rho_{w,sat} = \frac{P_{sat} M_w}{RT} \quad \text{but} \quad RH = \frac{P_w}{P_{sat}} \quad (2)$$

Substituting Eq.(2) into Eq.(1) we get :

$$q_{w,av} = h_m A \left[\frac{P_{sat} M_w}{RT} - \frac{P_w M_w}{RT} \right] = h_m A \left[\frac{P_{sat} M_w - P_w M_w}{RT} \right]$$

$$q_{w,av} = h_m A \frac{P_{sat} M_w}{RT} [1 - RH] \quad (3)$$

at 310K for water

$$P_{sat} = 0.0622 \text{ bar} \times 101.3 \frac{\text{kPa}}{1 \text{ bar}} = 6.3 \text{ kPa}$$

Substituting values into Eq.(3)

$$h_m A = \frac{0.001 \text{ kg/s}}{\frac{P_{sat} M_w}{RT} [1 - RH]}$$

(4)

$$h_{mA} = \frac{0.001 \frac{kg}{s}}{\frac{6.3 \text{ kPa} \times 18 \frac{kg}{kg}}{8.31 \frac{KJ}{kg}} \times 310K} = \frac{0.001 \frac{kg}{s}}{0.044 \frac{\text{kPa}}{KJ}} = 0.023 \frac{KJ}{\text{kPa.s}}$$

$$\frac{KJ}{\text{kPa}} = m^3 \quad \text{so} \quad h_{mA} = 0.023 \frac{m^3}{s} \quad \text{and} \quad A\bar{h} = h_{mA} \times 8c \left(\frac{\alpha}{D_{w,air}} \right)^{2/3} (4)$$

$$\begin{aligned} \text{at } 300K & \quad c = 1.0063 \frac{KJ}{kg.K} \quad K = 0.0262 \frac{W}{m.K} \quad \rho = 1.177 \frac{kg}{m^3} \quad \alpha = \frac{0.0262}{1 \times 10^3 \times 1.177} \\ \text{for air} & \end{aligned}$$

$$\alpha = 2.23 \times 10^{-5} \frac{m^2}{s}$$

Substituting into Eq.(4)

$$A\bar{h} = 0.023 \frac{m^3}{s} \times 1.177 \times 1 \times 10^3 \left(\frac{2.23 \times 10^{-5}}{26 \times 10^{-6}} \right)^{2/3} \approx 24.4 \frac{W}{W}$$

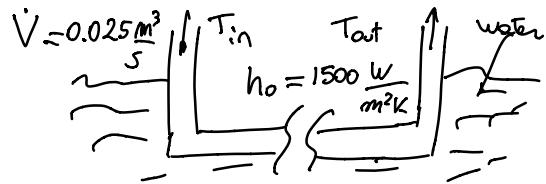
$$Q = A\bar{h} (310 - 290) = 24.4 \frac{W}{W} \times 20K = 488W$$

the energy lost due to evaporation is

$$Q_{w,ev} \times \Delta H_{vap} = 0.001 \frac{kg}{s} (2568.14 - 155) \frac{KJ}{kg} = 2.41 \text{ kWatts}$$

Problem 4

$$\text{Pipe} \quad \begin{cases} D_i = 0.15m \\ D_o = 0.17m \\ K = 0.15 \frac{W}{m.K} \end{cases}$$



Since the temperature of the water is constant we can use the equation for a surface temperature constant. We know h_o but we need to estimate the heat transfer coefficient within the tube, h_i :

$$Re = \frac{D_i \rho v P}{\mu} \quad \mu_{ao} = \frac{V}{\pi D_i^2} = \frac{0.025 \frac{m^3}{s}}{\pi \times 0.15^2 \frac{m^2}{s}} = 1.415 \frac{m}{s}$$

The properties of the air, let's assume

$$\text{at } T_{avg} \approx 300K, \text{ are: } \rho = 1.18 \frac{kg}{m^3} \quad \mu = 1.85 \times 10^{-5} \frac{kg}{m.s} \quad Pr = 0.708 \frac{mK}{W}$$

$$Re = \frac{0.15 \times 1.415 \times 1.18}{1.85 \times 10^{-5}} = 13540 \quad \text{Turbulent flow}$$

For Cooling $n \approx 0.3$ (5)

$$Nu = 0.023 \times 13540^{4/5} \times 0.708^0.3 = 41.9$$

$$Nu = 41.9 \Rightarrow h_i = \frac{K \alpha \nu Nu}{D} = \frac{0.02624 \times 41.9}{0.15} = 7.3 \frac{W}{m^2 K}$$

Let's consider all resistances in the system

$$\text{Resistance inside} = \frac{1}{2\pi L \times \frac{0.15}{2} \times 7.3} = \frac{0.29}{L} \frac{K}{W} m$$

$$\text{Resistance outside} = \frac{1}{2\pi L \times \frac{0.17}{2} \times 1500} = \frac{1.25 \times 10^{-3}}{L} \frac{K}{W} m$$

$$\text{Resistance well} = \frac{\ln 0.17/0.15}{2\pi L \times 0.15 \frac{W}{m \cdot K}} = \frac{0.132}{L} \frac{K}{W} m$$

Obviously the resistance outside can be neglected

$$\frac{1}{U} = \text{Resistance inside} + \text{Resistance well} = \frac{0.29 + 0.132}{L} = \frac{0.422}{L} \frac{K}{W} m$$

$$\boxed{U = 2.4 L \frac{W}{m \cdot K}}$$

$$\frac{T_w - T_{out}}{T_w - T_{in}} = \exp\left(-\frac{P \cdot L \cdot U}{m \cdot c}\right)$$

Taking logarithm in both sides of the above equation and by rearranging :

$$\frac{P \cdot 2.4 L^2}{m \cdot c} = - \ln \frac{T_w - T_{in}}{T_w - T_{out}}$$

$$L = \sqrt{-\frac{m \cdot c}{2.4 P} \ln \left(\frac{T_w - T_{in}}{T_w - T_{out}} \right)}$$

$$\dot{m} = \dot{V} \times \rho = 0.025 \frac{m^3}{s} \times 1.18 \frac{kg}{m^3} \approx 0.03 \frac{kg}{s}$$

$$c \approx 1 \frac{KJ}{kg} \quad P = \pi D = \pi \times 0.15 m = 0.47 m$$

Substituting above :

$$L = \sqrt{-\frac{0.03 \times 1000}{0.47 \times 2.4} \ln \left(\frac{17 - 21}{17 - 30} \right)} = 5.6 m \quad L = 5.6 m$$

(b) Substituting in the resistances the value of L , we have: (6)

$$\text{Resistance inside} = \frac{0.29}{5.6} = 5.2 \times 10^{-2} \frac{\text{K}}{\text{W}}$$

$$\text{Resistance outside} = \frac{1.25 \times 10^3}{5.6} = 2.23 \times 10^{-4} \frac{\text{K}}{\text{W}}$$

$$\text{Resistance wall} = \frac{0.132}{5.6} = 2.4 \times 10^{-2} \frac{\text{K}}{\text{W}}$$