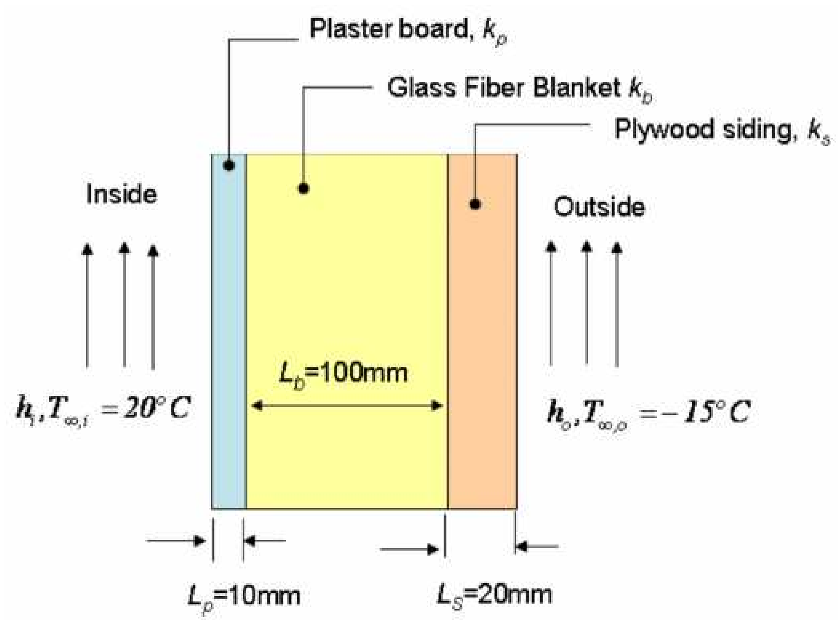
Examples - Steady State heat Transfer Spring 2018

Example 1



Data

$h_i := 20 \cdot \dfrac{W}{m^2 \cdot K}$ $\quad h_o := 150 \cdot \dfrac{W}{m^2 \cdot K}$ $\quad A_{area} := 400 \cdot m^2$ $\quad T_{inf\_o} := -15 \cdot K$

$L_p := 10 \cdot mm$ $\quad L_b := 100 \cdot mm$ $\quad L_s := 20 \cdot mm$ $\quad T_{inf\_i} := 20 \cdot K$

$k_p := 0.1 \cdot \dfrac{W}{m \cdot K}$ $\quad k_b := 0.04 \cdot \dfrac{W}{m \cdot K}$ $\quad k_s := 0.15 \cdot \dfrac{W}{m \cdot K}$

(a)

$$\Sigma Res := \frac{1}{h_i \cdot A_{area}} + \frac{L_p}{k_p \cdot A_{area}} + \frac{L_b}{k_b \cdot A_{area}} + \frac{L_s}{k_s \cdot A_{area}} + \frac{1}{h_o \cdot A_{area}}$$

$$\Sigma Res = 0.007 \cdot \frac{K}{W}$$

$$\boxed{q := \frac{T_{inf\_i} - T_{inf\_o}}{\Sigma Res} = 5017.9\,W}$$

(b) Estimate the temperature at the boundary between the glass fiber blanket and the plywood siding. It will be called T_{bound} and the resistances bewtween the interior of the house will include the convection inside, conduction through the plaster board a the glass fiber blanket

$$\Sigma Res\_new := \frac{1}{h_i \cdot A_{area}} + \frac{L_p}{k_p \cdot A_{area}} + \frac{L_b}{k_b \cdot A_{area}}$$

$$\Sigma Res\_new = 0.007 \cdot \frac{K}{W}$$

The heat flow remains the same but the resistance changes, from the equation considering the driving temperature difference T_{inf_i}-T_{bound} we can estimate T_{bound}

$q = (T_{inf\_i}-T_{bound})/\Sigma Res\_new$ --> $T_{bound} := T_{inf\_i} - q \cdot \Sigma Res\_new$

$\boxed{T_{bound} = -13.2\,K}$ or -13.2C

(c) All the materials thickness will be the same except the fiber glass blanket to reduce the heat flow by 30%

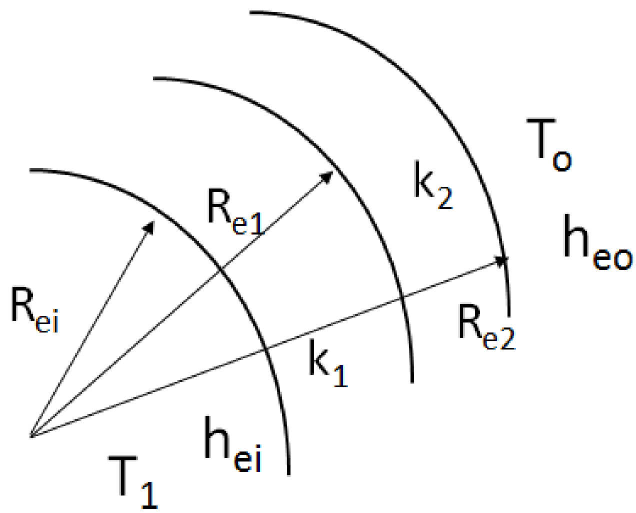
$$q_{new} := 0.30 \cdot q$$

$$R_{ci} := \frac{1}{h_i \cdot A_{area}} \qquad R_{pb} := \frac{L_p}{k_p \cdot A_{area}} \qquad R_{ps} := \frac{L_s}{k_s \cdot A_{area}} \qquad R_{co} := \frac{1}{h_o \cdot A_{area}}$$

$$R_{gb} := \frac{T_{inf\_i} - T_{inf\_o}}{q_{new}} - \left(R_{ci} + R_{pb} + R_{ps} + R_{co}\right) \qquad R_{gb} = 0.023 \cdot \frac{K}{W}$$

$$L_{bnew} := R_{gb} \cdot k_b \cdot A_{area} \qquad \boxed{L_{bnew} = 360.4 \cdot mm}$$

Example 2



$R_{ei} := 10.2mm$ $\quad T_o := 21\cdot K$ $\quad k_1 := 0.35\cdot\frac{W}{m\cdot K}$

$R_{e1} := 12.7\cdot mm$ $\quad T_1 := 37\cdot K$ $\quad k_2 := 0.80\cdot\frac{W}{m\cdot K}$

$R_{e2} := 16.5\cdot mm$ $\quad h_{ei} := 12\cdot\frac{W}{m^2\cdot K}$ $\quad h_{eo} := 6\cdot\frac{W}{m^2\cdot K}$

(a) Heat loss with contact lens (the layer 2 is added)

Assumed is a spherical surface so Resistances will be calculated from the following equations:

$$R_{conv\_i} := \frac{1}{4\cdot\pi\cdot R_{ei}^{2}\cdot h_{ei}} \qquad R_{conv\_i} = 63.7\cdot\frac{K}{W}$$

$$R_{cond\_1} := \frac{R_{e1} - R_{ei}}{4\cdot\pi\cdot k_1\cdot R_{ei}\cdot R_{e1}} \qquad R_{cond\_1} = 4.4\cdot\frac{K}{W}$$

$$R_{cond\_2} := \frac{R_{e2} - R_{e1}}{4\cdot\pi\cdot k_2\cdot R_{e1}\cdot R_{e2}} \qquad R_{cond\_2} = 1.8\cdot\frac{K}{W}$$

$$R_{conv\_o} := \frac{1}{4\cdot\pi\cdot R_{e2}^{2}\cdot h_{eo}} \qquad R_{conv\_o} = 48.7\cdot\frac{K}{W}$$

Because the eye is approximated as 1/3 of a spherical surface

$$q_{w\_lens} := \frac{1}{3} \cdot \frac{T_1 - T_o}{R_{conv\_i} + R_{cond\_1} + R_{cond\_2} + R_{conv\_o}}$$

$$\boxed{q_{w\_lens} = 45 \cdot mW}$$

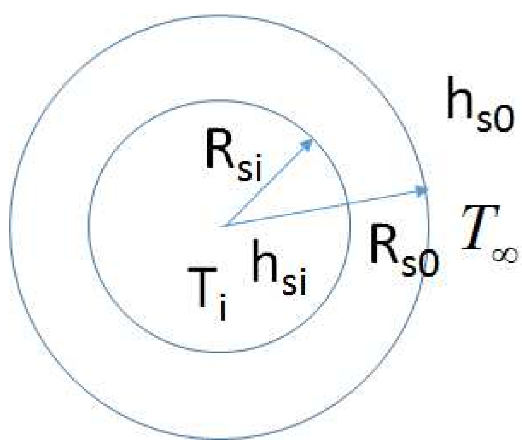
(b) Without contact lens the second conduction layer must be ignored, and a new convection thermal resistance must be recalculated because the external area decreases

$$R_{conv\_o\_new} := \frac{1}{4 \cdot \pi \cdot R_{e1}^{\,2} \cdot h_{eo}}$$

$$q_{wout\_lens} := \frac{1}{3} \cdot \frac{T_1 - T_o}{R_{conv\_i} + R_{cond\_1} + R_{conv\_o\_new}}$$

$$\boxed{q_{wout\_lens} = 35.5 \cdot mW}$$

Example 3



$D_{si} := 36mm$ $\qquad$ $e_s := 2 \cdot mm$ $\qquad$ $k_{ss} := 15 \cdot \frac{W}{m \cdot K}$

$$R_{si} := \frac{D_{si}}{2} \qquad R_{s0} := \frac{D_{si}}{2} + e_s$$

$T_i := 6 \cdot K \qquad T_{inf} := 23 \cdot K \qquad h_{si} := 400 \cdot \frac{W}{m^2 \cdot K} \qquad h_{s0} := 6 \cdot \frac{W}{m^2 \cdot K}$

(a) Heat Gain per unit length

$$R_{conv\_si} := \frac{1}{h_{si} \cdot 2 \cdot \pi \cdot R_{si}} \qquad R_{conv\_si} = 0.022 \cdot \frac{m \cdot K}{W}$$

$$R_{cond\_s} := \frac{\ln\left(\frac{R_{s0}}{R_{si}}\right)}{2\pi \cdot k_{ss}} \qquad R_{cond\_s} = 0.001 \cdot \frac{m \cdot K}{W}$$

$$R_{conv\_s0} := \frac{1}{h_{s0} \cdot 2 \cdot \pi \cdot R_{s0}} \qquad R_{conv\_s0} = 1.326 \cdot \frac{m \cdot K}{W}$$

$$q_p := \frac{T_{inf} - T_i}{R_{conv\_si} + R_{cond\_s} + R_{conv\_s0}} \qquad \boxed{q_p = 12.6 \cdot \frac{W}{m}}$$

(b) Heat gain if a 10mm thick layer of calcium silicate insulation (k_{ins}=0.05 W/m.K) is applied to the tube

New radius can be calculated as: $e_{insul} := 10 \cdot mm \qquad k_{ins} := 0.05 \cdot \frac{W}{m \cdot K}$

$$R_{ext} := R_{s0} + e_{insul}$$

and the new resistance due conduction can be calculated as

$$R_{cond\_insul} := \frac{\ln\left(\frac{R_{ext}}{R_{s0}}\right)}{2 \cdot \pi \cdot k_{ins}} \qquad R_{cond\_insul} = 1.291 \cdot \frac{m \cdot K}{W}$$

and the new resistance to the convection with he new radius is

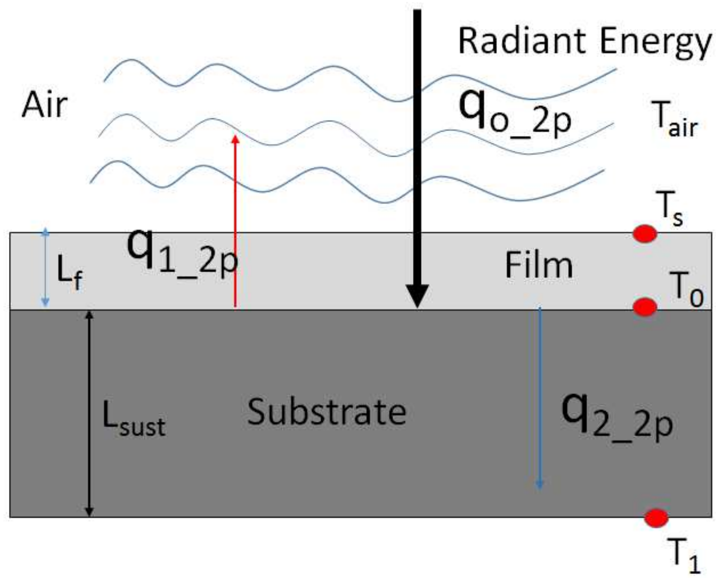
$$R_{conv\_new} := \frac{1}{h_{s0} \cdot 2\pi \cdot R_{ext}} \qquad R_{conv\_new} = 0.884 \cdot \frac{m \cdot K}{W}$$

And the new gain per unit length is

$$q_{p\_new} := \frac{T_{inf} - T_i}{R_{conv\_si} + R_{cond\_s} + R_{cond\_insul} + R_{conv\_new}}$$

$$\boxed{q_{p\_new} = 7.73 \cdot \frac{W}{m}}$$

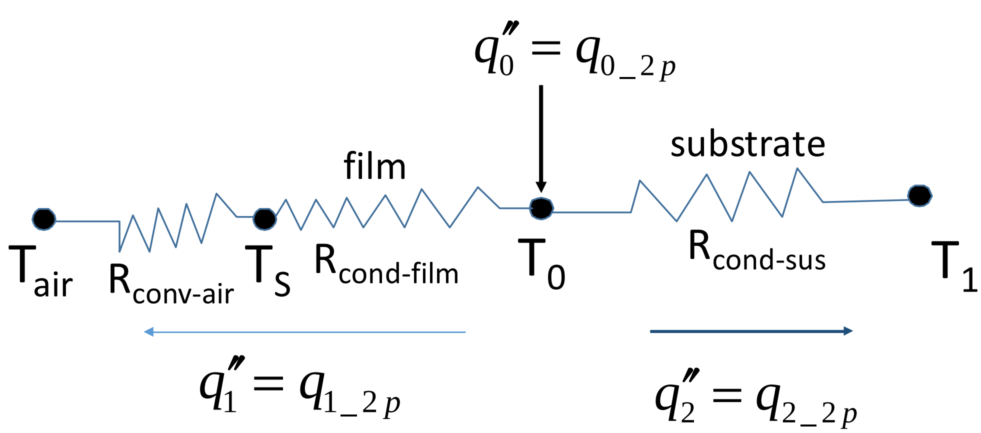
Example 4



$L_f := 0.25 \cdot mm$ $\quad L_{sus} := 1 \cdot mm$ $\quad k_f := 0.025 \cdot \frac{W}{m \cdot K}$ $\quad k_{sus} := 0.05 \cdot \frac{W}{m \cdot K}$

$T_{air} := 20 \cdot K$ $\quad T_0 := 60 \cdot K$ $\quad T_{1s} := 30 \cdot K$ $\quad h_{air} := 50 \cdot \frac{W}{m^2 \cdot K}$

(a) Thermal circuit when the film is transparent



$$R_{conv\_air} := \frac{1}{h_{air}} \qquad R_{conv\_air} = 0.02 \cdot \frac{K \cdot m^2}{W}$$

$$R_{cond\_film} := \frac{L_f}{k_f} \qquad R_{cond\_film} = 0.01 \cdot \frac{K \cdot m^2}{W}$$

$$R_{cond\_sus} := \frac{L_{sus}}{k_{sus}} \qquad R_{cond\_sus} = 0.02 \cdot \frac{K \cdot m^2}{W}$$

$$q_{1\_2p} := \frac{T_0 - T_{air}}{R_{cond\_film} + R_{conv\_air}} \qquad q_{1\_2p} = 1.333 \cdot \frac{kW}{m^2}$$

$$q_{2\_2p} := \frac{T_0 - T_{1s}}{R_{cond\_sus}} \qquad q_{2\_2p} = 1.5 \cdot \frac{kW}{m^2}$$

$$q_{o\_2p} := q_{1\_2p} + q_{2\_2p} \qquad \boxed{q_{o\_2p} = 2.83 \cdot \frac{kW}{m^2}}$$

(b) the film is not transparent and the radiant heat s absorbed at the interface film/air

Since we do not know Ts, we can calculate the heat flux between To and T1 that for steady state is

$$q_{2\_2p\_new} := \frac{T_0 - T_{1s}}{R_{cond\_sus}} \qquad q_{2\_2p\_new} = 1.5 \cdot \frac{kW}{m^2}$$

and for steady state the temperature Ts can be calculated as;

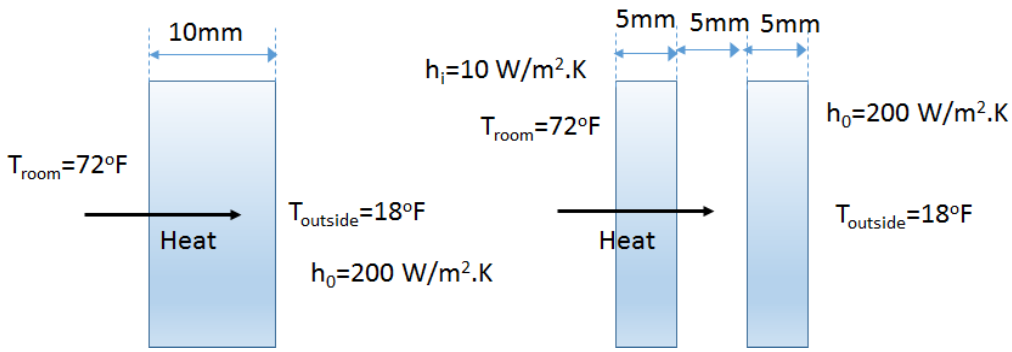
$$T_S := T_{1s} + q_{2\_2p\_new} \cdot \left(R_{cond\_film} + R_{cond\_sus}\right)$$

$T_S = 75\,K$ or T_S=75C

$$q_{1\_2p\_new} := \frac{T_S - T_{air}}{R_{conv\_air}} \qquad q_{1\_2p\_new} = 2.75 \cdot \frac{kW}{m^2}$$

$$q_{0\_2p\_new} := q_{1\_2p\_new} + q_{2\_2p\_new} \qquad \boxed{q_{0\_2p\_new} = 4.25 \cdot \frac{kW}{m^2}}$$

Example 5



Data

- Window 1

$$k_{glass} := 0.9 \cdot \frac{W}{m \cdot K} \qquad x_1 := 10 \cdot mm \qquad h_{in} := 10 \cdot \frac{W}{m^2 \cdot K} \qquad h_{out} := 200 \cdot \frac{W}{m^2 \cdot K}$$

(a) **Units for the data temperatures are in Farenheit**

$$T_{room} := 72 \qquad T_{out} := 18$$

Conversion of Temperatures from Farenheit to Kelvins

$$T_{room\_K} := \frac{T_{room} - 32}{1.8} \cdot K \qquad T_{out\_K} := \frac{T_{out} - 32}{18} \cdot K$$

- Windows 2

$$k_{air} := 0.03 \cdot \frac{W}{m \cdot K} \qquad x_2 := 5 \cdot mm$$

- Heat Transfer for Window 1 - Calculated as Flux, that is heat flow per unit area

$$R_{cv\_room} := \frac{1}{h_{in}} \qquad R_{cd\_glass} := \frac{x_1}{k_{glass}} \qquad R_{cv\_out} := \frac{1}{h_{out}}$$

$$q_1 := \frac{T_{room\_K} - T_{out\_K}}{R_{cv\_room} + R_{cd\_glass} + R_{cv\_out}} \qquad \boxed{q_1 = 198.09 \cdot \frac{W}{m^2}}$$

- Heat Transfer for Window 2 - Calculated as Flux, that is heat flow per unit area without considering radiation and only conduction through the air gap

$$R_{cd\_glass1} := \frac{x_1}{k_{glass}} \qquad R_{cd\_air\_gap} := \frac{x_1}{k_{air}} \qquad R_{cd\_glass2} := \frac{x_1}{k_{glass}}$$

$$q_2 := \frac{T_{room\_K} - T_{out\_K}}{R_{cv\_room} + R_{cd\_glass1} + R_{cd\_air\_gap} + R_{cd\_glass2} + R_{cv\_out}}$$

$$\boxed{q_2 = 49.94 \cdot \frac{W}{m^2}}$$

(b)

The equation for radiation is:

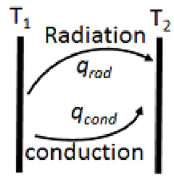
$$q = \sigma A\left(T_1^4 - T_2^4\right)$$

where σ is the stefan-Boltzman constant and A is the area of heat exchange. T_1 and T_2 are absolute temperatures (in degree Kelvin or Rankine). By using algebra the equation can be modified as follows:

$$\frac{q}{A} = q'' = \sigma\left(T_1^4 - T_2^4\right) = \sigma\left(T_1^2 - T_2^2\right)\left(T_1^2 + T_2^2\right) = \sigma\left(T_1 + T_2\right)\left(T_1^2 + T_2^2\right)\left(T_1 - T_2\right)$$

if we define $h_r = \sigma\left(T_1 + T_2\right)\left(T_1^2 + T_2^2\right)$ the radiation equation is $q = h_r\left(T_1 - T_2\right)$

When we consider conduction and radiation through the air gap we have the following:



If we use the concept of resistance

$$\frac{q_{rad}}{A} = q''_{rad} = \frac{T_1 - T_2}{R_{rad}} = \frac{T_1 - T_2}{\frac{1}{h_r}}$$

And

$$\frac{q_{cond}}{A} = q''_{rad} = \frac{T_1 - T_2}{R_{cond}} = \frac{T_1 - T_2}{\frac{x_2}{k_{air}}}$$

$$q'' = \dot{q}_{rad} + \dot{q}_{cond} = (T_1 - T_2)\left(\frac{1}{\frac{1}{h_r}} + \frac{1}{\frac{x_2}{k_{air}}}\right) = (T_1 - T_2)\left(h_r + \frac{k_{air}}{x_2}\right) = \frac{(T_1 - T_2)}{\frac{1}{\left(h_r + \frac{kair}{x_2}\right)}} = \frac{(T_1 - T_2)}{R_{eq}}$$

$$R_{eq} = \frac{1}{h_r + \frac{k_{air}}{x_2}}$$

(c) So now rather that having a resistance due to conduction in the air gap will be conduction plus radiation

First let's calculate h_r $\qquad \sigma := 5.678 \cdot 10^{-8} \cdot \frac{W}{m^2 \cdot K^4}$

$$T_{1\_K} := T_{room\_K} + 273 \cdot K \qquad T_{2\_K} := T_{out\_K} + 273 \cdot K$$

$$T_{avg} := \frac{T_{1\_K} + T_{2\_K}}{2} \qquad h_r := 4\sigma \cdot T_{avg}{}^3 \qquad h_r = 5.187 \frac{1}{K} \cdot \frac{W}{m^2}$$

$$T_{avg} = 283.722\,K$$

$$R_{eq} := \frac{1}{h_r + \frac{k_{air}}{x_2}}$$

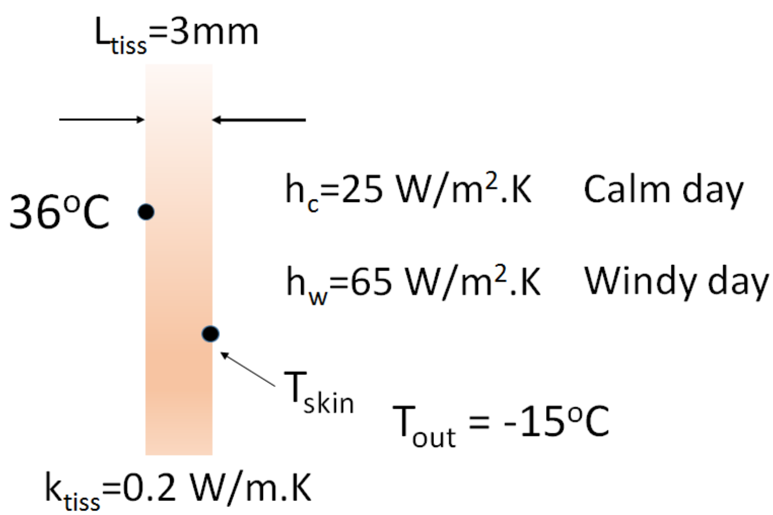
So now the new heat flux including radition is:

$$q_3 := \frac{T_{room\_K} - T_{out\_K}}{R_{cv\_room} + R_{cd\_glass1} + R_{eq} + R_{cd\_glass2} + R_{cv\_out}}$$

$$\boxed{q_3 = 106.182 \cdot \frac{W}{m^2}}$$

The heat flux is larger than the once calculated considering conduction alone

Example 6



$L_{tiss} := 3 \cdot mm$ $\quad k_{tiss} := 0.2 \cdot \frac{W}{m \cdot K}$ $\quad h_c := 25 \cdot \frac{W}{m^2 \cdot K}$ $\quad h_w := 65 \cdot \frac{W}{m^2 \cdot K}$

$T_{tiss} := 36 \cdot K$ $\quad T_{air\_out} := -15 \cdot K$

(a)

$$R_{calm} := \frac{L_{tiss}}{k_{tiss}} + \frac{1}{h_c} \qquad R_{calm} = 0.055 \cdot \frac{m^2 \cdot K}{W}$$

$$R_{windy} := \frac{L_{tiss}}{k_{tiss}} + \frac{1}{h_w} \qquad R_{windy} = 0.03 \cdot \frac{m^2 \cdot K}{W}$$

$$Ratio_{calm\_over\_windy} := \frac{R_{windy}}{R_{calm}}$$ $$\boxed{Ratio_{calm\_over\_windy} = 0.552}$$

(b) Skin Temperatures T_{skin} in calm and windy day

calm day

$$q_{c\_2p} := \frac{T_{tiss} - T_{air\_out}}{R_{calm}}$$ $$q_{c\_2p} = 927.273 \cdot \frac{W}{m^2}$$

$$R_{conv\_calm} := \frac{1}{h_c}$$

and the temperature can be calculated as:

$$T_{skin\_calm} := T_{air\_out} + q_{c\_2p} \cdot R_{conv\_calm}$$

$$\boxed{T_{skin\_calm} = 22.1\,K}$$

In fact is in Celcius

Windy day

$$q_{w\_2p} := \frac{T_{tiss} - T_{air\_out}}{R_{windy}}$$ $$q_{w\_2p} = 1678.481 \cdot \frac{W}{m^2}$$

and the temperature can be calculated as:

$$R_{conv\_windy} := \frac{1}{h_w}$$ $$T_{skin\_windy} := T_{air\_out} + q_{w\_2p} \cdot R_{conv\_windy}$$

$$\boxed{T_{skin\_windy} = 10.8\,K}$$

In fact is in Celcius

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