ABE 30800 Heat and Mass Transfer - Spring 2018
Homework 2– Due Thursday February 1
Total 200 marks

Question 1

(a) Compare the heat loss from an insulated and an uninsulated pipe under the following conditions. The pipe has a very high thermal conductivity of k=400 W/m.K, and internal and external diameters of 10cm and 12cm, respectively. Saturated steam flows inside the pipe at 110^{o}C. The pipe is located in an environment at 30^{o}C and the heat transfer coefficient on its outer surface is 15W/m^2.K. The insulation available to reduce heat losses is 5cm thick and it conductivity is 0.20 W/m.K.

(b) Estimate the overall heat transfer coefficients based on the inner area and outer of the pipe U_i and U_o, respectively for both cases, the pipe without and with insulation. Please, write clearly the units of these coefficients

(c) Compare the values of these overall heat transfer coefficient, and briefly explain how they affect the respective heat flows.

[20 marks]

Question 2

It appears to be a logical that by increasing the insulation thickness in a hot cylindrical element decreases its heat losses, however it is practically observed that there is a critical insulation thickness (r_c), below which the heat losses increase rather than decrease when the thickness of insulation thickness increases. To demonstrate that apparently strange phenomenon, consider a circular tube of internal radius r_i maintained at a uniform temperature T_i and covered with a layer of insulation material defined by the radius r_o (see figure below). Heat is lost to an outside cold air that is at a temperature T_∞ ; the heat convection transfer outside is h, the length of the tube is L and the thermal conductivity of the insulation is k.

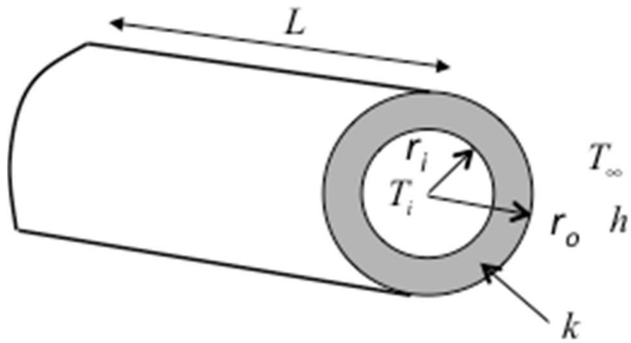
(a) Using a relationship to estimate the heat loss as a function of the tube dimensions (i.e. including the insulation thickness, the insulation thermal conductivity of the material and the convection coefficient show that the insulation critical radius, i.e. the radius beyond which the heat loss to the ambient starts to decrease is equal to $r_c = k/h$

(b) Consider a cylindrical tube with an external diameter of 2 cm that is maintained at a uniform temperature. In order to reduce heat losses, the tube is coated with an insulating coating material having a thermal conductivity k=0.17 W/m.K. Heat is lost by convection with a convection coefficient h=2W/m^2.K. Determine the critical radius and the critical thickness of the insulation. If the temperature at r_i is 100^{o}C and the temperature of the air outside 10^{o}C determine the heat transfer per meter of the cylindrical element

(c) For those temperatures plot the heat loss through the system as a function of radius for radiuses including values lower and higher than the critical radius r_c, briefly explain your results observed in the plot.

(d) Using the same approach than the one used in (a), derive an expression to calculate the critical radius for a spherical geometry. Calculate the critical radius using data given in question (b)

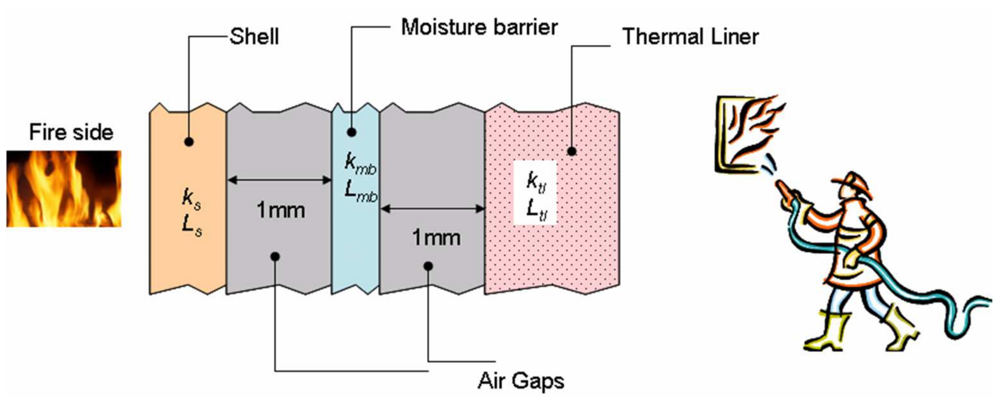
(e) Could you estimate a critical thickness for a slab geometry? Explain your answer



[50 marks]

Question 3

A firefighter's protective clothing, known as a turnout coat, is constructed as an ensemble of three layers separated by air gaps as shown schematically in the figure below:



The air gaps between the layers are 1mm thick, and heat is transferred by conduction and radiation exchange through the stagnant air. The radiation coefficient can be approximated to $h_{rad} \approx 4\sigma T_{avg}^3$, where T_{avg} is the average temperature of the surfaces at each side of the gap. The radiation flux can be then calculated as $q''_{rad} = h_{rad}(T_1 - T_2)$.
Data for this problem are given in the table below:

Layer	Thickness (mm)	k (W/m.K)
Shell (s)	1	0.06
Moisture barrier (mb)	0.7	0.015
Thermal liner (tl)	4	0.045

(a) Represent the coat by a thermal circuit indicating all resistances, temperatures and hear fluxes.

(b) Calculate the thermal resistances per unit area (m^2.K/W) for each of the layers, as well as for the radiation and conduction processes in the gaps. Conduction and radiation in the air gap can be calculated considering those heat flows in the air gap are transferred in parallel. Assume that T_{avg}=470°C is in both gaps. Comment on the relative magnitude of the resistances.

(c) For a fire environment in which firefighters often work, the typical radiant heat flux on the fire side of the turncoat is 2,500 W/m^2. What would be the outer surface temperature of the turncoat if the inner surface temperature of the coat, i.e. the one in contact with the firefighter skin is 50^{o}C, a maximum temperature after which it would result in body injury?

[50 marks]

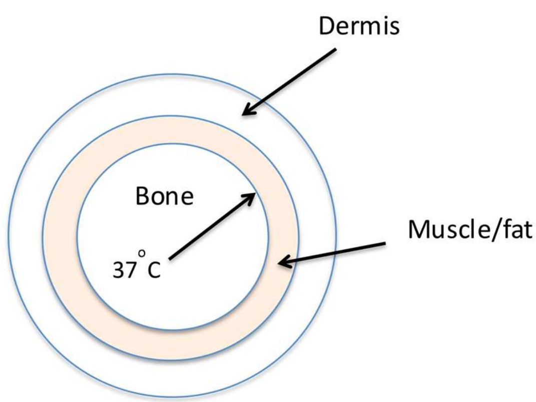
<u>Question 4</u>

Frostbite is damage to the skin from freezing due to prolonged exposure to cold temperatures. Assume that the finger <u>can be modeled</u> as a <u>long solid cylinder</u> (only radial flow exists) where the inner boundary of the muscle/fat layer remains at 37^{o}C and the heat loss to the cold air reaches a steady state. Assume frostbite occurs when any part of the tissue in the finger drops below -1^{o}C, that the outside temperature is -25^{o}C and the heat transfer coefficient in a windy winter day is h=100 W/m^2.K.

(a) How serious a frostbite is usually determined from the depth in the tissue at which the temperature reaches -1^{o}C. Under those climate conditions calculate the depth in the tissue where the temperature is -1^{o}C to estimate how serious the frostbite will be when you are not using gloves

(b) Now, determine how serious the frostbite will be when you are wearing any gloves

(c) Which of the three layers (see table and figure below) has the lowest thermal resistance. Assume that the bone diameter is 25mm; other dimensions and properties are given in the table and figure below?



	Thickness (mm)	*k* (W/m.K)
Muscle/fat	2.1	0.3
Dermis	0.4	0.1
Glove	1.4	0.02

[40 marks]

<u>Question 5</u>

In Lecture 3 a model was used to describe the heat transfer in an ***infinite fin***, and the ratio of heat conducted through the fin attached to the circular pipe and the heat conducted by an area of the circular pipe equal to the cross-section of the rectangular fin can be calculated as:

$$\frac{q_{fin}}{q_{non-fin}} = \sqrt{\frac{k}{h} \cdot \frac{P}{A_c}}$$

An electronic device that produces excess heat needs to be cooled and circular pins are added to improve the cooling. Consider a copper cylindrical pin fin 2.5mm in diameter, that protrudes from a casing wall at 95^{o}C into ambient air at 25^{o}C. There is heat transfer from the fin to the air by natural convection with a heat convection coefficient of 10 W/m^2.K and the thermal conductivity of the material is 400 W/m.K. Calculate the heat loss assuming:

(a) An infinite fin

(b) A fin of length 25mm. Note that for this case the ratio of the heat loss with and without a fin can be calculated as:

$$\frac{q_{fin}}{q_{non-fin}} = \sqrt{\frac{k}{h} \cdot \frac{P}{A_c}} \cdot \frac{\sinh(mL) + \left(\frac{h}{mk}\right) \cdot \cosh(mL)}{\cosh(mL) + \left(\frac{h}{mk}\right) \cdot \sinh mL}$$

where $m^2 = \frac{hP}{kA_c}$

(c) How long would the fin have to be for the infinite long pin solution be correct within 5%?

[40 marks]