

Homework 1

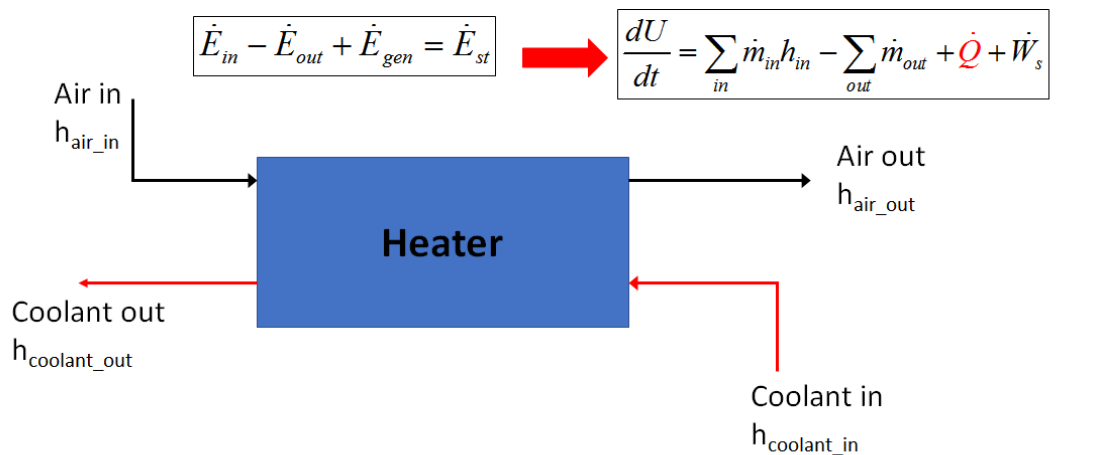
Question 1

Temperatures are all expressed in degree Celcius but to have consistent results in MathCad, they are written in Kelvin, since the calculation of enthalpies are based on temperature differences, results are the same if we express temperatures in either Kelvins or Celsius

$$T_{\text{air_out}} := 30 \cdot \text{K} \quad T_{\text{air_in}} := -10 \cdot \text{K} \quad T_{\text{coolant_in}} := 85 \cdot \text{K} \quad T_{\text{coolant_out}} := 45 \cdot \text{K}$$

$$V_{\text{air}} := 30 \cdot \frac{\text{m}^3}{\text{min}} \quad \text{kJ} := 1000 \cdot \text{J} \quad C_{p_air} := 1 \cdot \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \quad C_{p_coolant} := 3.3 \cdot \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$\rho_{\text{air}} := 1.2 \cdot \frac{\text{kg}}{\text{m}^3}$$



$$\sum_{in} \dot{m}_{in} h_{in} - \sum_{out} \dot{m}_{out} h_{out} \longrightarrow \dot{m}_{air} h_{air_in} + \dot{m}_{coolant} h_{coolant_in} = \dot{m}_{air} h_{air_out} + \dot{m}_{coolant} h_{coolant_out}$$

The equation above is obtained by considering state state $dU/dt = 0$, adiabatic process or not heat loss $Q=0$ (so it is not a energy transport problem) and shaft work $W_s = 0$. Specific enthalpy can be calculated as $h = C_p T$ with units kJ/kg, if T_{ref} is assumed to be 0°C

$$m_{\text{air}} := V_{\text{air}} \cdot \rho_{\text{air}} \quad m_{\text{air}} = 36 \cdot \frac{\text{kg}}{\text{min}}$$

$$m_{\text{coolant}} := \frac{m_{\text{air}} \cdot C_{p_air} \cdot (T_{\text{air_out}} - T_{\text{air_in}})}{C_{p_coolant} \cdot (T_{\text{coolant_in}} - T_{\text{coolant_out}})} \quad m_{\text{coolant}} = 10.9 \cdot \frac{\text{kg}}{\text{min}}$$

Question 2

$$d_{\text{plyw}} := 1 \cdot \text{in}$$

$$T_1 := 100 \cdot \text{F}$$

$$T_2 := 50 \cdot \text{F}$$

$$k_{\text{plyw}} := 0.070 \cdot \frac{\text{BTU}}{\text{hr} \cdot \text{ft} \cdot \text{F}}$$

Assumptions

- 1D heat flow
- Steady State
- No Convection
- No heat generation

Definitely is a energy transfer process so we can use the equations derived in class for the Flux

$$\text{Flux} := k_{\text{plyw}} \cdot \frac{T_1 - T_2}{d_{\text{plyw}}}$$

$$\text{Flux} = 42 \cdot \frac{\text{BTU}}{\text{hr} \cdot \text{ft}^2}$$

Question 3

(a)

- (1) two boundary conditions in x and y and one initial condition
- (2) $T(x,y,t)$
- (3) The equation assumes no convection and no heat generation
- (4) The model is assuming Cartesian coordinates

(b)

- (1) two boundary conditions in r
- (2) $T(r)$
- (3) The equation assumes no convection, no heat generation and steady state
- (4) The model is assuming cylindrical coordinates

(c)

- (1) One boundary condition in x and two boundary conditions in y
- (2) $T(x,y)$
- (3) The equation no heat generation, 1D conduction in direction y and steady state
- (4) The model is assuming Cartesian Coordinates

Question 4

Straight forward proof

Question 5

$$m_p := 85$$

$$h_p := 1.80$$

$$A_p := 2.5$$

$$u_p := 1.1$$

$$h := 25$$

$$T_{\text{air}} := 22 + 273$$

$$\sigma := 5.676 \cdot 10^{-8}$$

Balance of Energy is:

$$\text{Energy In} - \text{Energy Out} + \text{Generation} = \text{Storage}$$

Let's assume steady state so storage = 0

$$\text{Energy_in}=0$$

$$\text{Energy Out} = hA(T_s - T_w) + \sigma A(T_s^4 - T_w^4) + 0.12u_p \times 13.3 \exp(20.4 - 5132/T_s) = 0$$

$$\text{Generation} = 4m_p u_p$$

We can use the balance of Energy and construct an equation to estimate T_s

$$f(T_s) := 4 \cdot m_p \cdot u_p - h \cdot A_p \cdot (T_s - T_{\text{air}}) - \left[\sigma \cdot A_p \cdot (T_s^4 - T_{\text{air}}^4) + 0.12 \cdot u_p^{0.5} \cdot A_p \cdot \left[13.3 \cdot \left(e^{20.4 - \frac{5132}{T_s}} - 0 \right) \right] \right]$$

$$T_s := 300$$

$$T_{\text{Sol}} := \text{root}(f(T_s), T_s) \quad T_{\text{Sol}} = 298.499$$

$$\text{Temp_of_body} := T_{\text{Sol}} - 273$$

$$\text{Temp_of_body} = 25.5$$

In Celsius

(b) We can assume that in order to keep steady state all the heat generated by the person must be lost so:

$$h_{\text{eff}} := \frac{4 \cdot m_p \cdot u_p}{A_p \cdot [26 - (T_{\text{air}} - 273)]}$$

$$h_{\text{eff}} = 37.4$$

Units are W/m²