ABE 30800 Heat and Mass Transfer - Spring 2018
Homework 1 – Due Thursday January 18
Total 100 marks

Question 1

You must solve the following problem: An automobile heater is required to provide 30m^3/min air at 30^{o}C to the passengers when the outside temperature is -10^{o}C. If a coolant is available at 85^{o}C, determine the mass flow of coolant needed to heat the air if the coolant cannot leave the heater at a temperature lower than 45^{o}C.

(a) Would you need transport equations or equilibrium thermodynamics assumptions to solve this problem? Briefly give your reasons that justify your election. Also state useful assumptions that could help you to solve the problem.

(b) Solve the problem

[20 marks]

Note: Assume that the heat capacity of the coolant and the air are 3.3 kJ/kg.K and 1 kJ/kg.K, respectively and the density of air 1.2 kg/m^3

Question 2

You must determine the heat transfer per unit through a sheet of 1 inch of thick plywood if one slide of the plywood has a surface temperature of 100^{o}F and the other side is 50^{o}F. The thermal conductivity of plywood is 0.070 BTU/hr.ft.F. Briefly discuss if you are using an equilibrium thermodynamics or energy transfer approach to solve this problem. Also state some of the assumptions you are using to solve this problem. Express your result in units of BTU/hr.ft^2

[20 marks]

Question 3

The microscopic energy balance equation derived in Lecture 1 is a model that involves all input and output of energy in a point of the domain under study. Mathematically the model is represented by a Partial Differential Equation (PDE) whose solution would provide the value of the temperature in a location of the domain under study and at a given time. The solution of the equation will depend on the geometry used (described in Lecture 1), and some assumptions you could make to get a closed/analytical solution of the problem. Regardless, the solution of the equation will require boundary conditions and/or an initial condition that will depend on the problem/model being solved. In the following set of equations determine:

(1) the number of boundary conditions and/or initial conditions to obtain a solution to the following differential equations
(2) the variables of which the resulting temperature will be a function
(3) terms and physical meaning of terms that were dropped in the general equation
(4) the coordinate system used to solve the problem.

(a)
$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\dot{q}}{\rho c}$$

(b)
$$\frac{d^2 T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0$$

(c) $$\rho c\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}$$

[20 marks]

Note: The velocity of a fluid is a vector that in general has three components. For instance, in a 3D Cartesian system the fluid velocity components in the directions *x*, *y* and *z* are denoted by u_x, u_y and u_z, respectively. In many textbooks, and to reduce the number of letters used, these three velocity components in directions *x*, *y* and *z* are denoted as *u*, *v* and *w*, respectively. That notation in Equation (c).

Question 4

The differential equation $\frac{d^2T(x)}{dx^2}=-m^2\,T(x)$ is a differential equation whose solution describes the temperature *T* at different locations *x*. The equation is derived from the general equation describing a microscopic balance of energy after many assumptions and 1D heat transfer (x-direction) are assumed. *m* is a constant/parameter, whose definition you will understand later. The fact that the constant/parameter is squared is to assure that the value will be positive (you will find out later why you need that requirement). Try to find out if the potential solutions *T(x) = sin (mx) +cos (mx)* and *T(x) = sin(mx) – cos(mx)* will satisfy the equation.

[20 marks]

Question 5

Improving sport performance requires an understanding of the person heat generation and heat loss. A racquetball player is playing, and it would be necessary to perform a simple heat balance considering the player as the system and counting for heat generation, convection, radiation and heat lost by evaporation of the sweat. The player is 1.80m tall and his weight is 85kg with an average surface of 2.5m^2. Average speed of running during the game is 1.1 m/s, which is the velocity of air relative to his body (no air movement in the court). Air temperature T_∞ and wall temperature T_w are both 22^{o}C. The convective heat transfer coefficient is 25 W/m^2. K. There is also radiative heat transfer given by the equation $\sigma A\left(T_S^4-T_w^4\right)$ where T_S is the body temperature of the player; both temperatures are in Kelvins. The evaporative heat loss is given in units of *Watts* and can be calculated by the following equation: $0.12v^{0.5}A(p_s\text{-}p_\infty)$ where v is the air velocity in m/s, p_s is the surface vapor pressure in Pa, and can be calculated as $p_s=13.3e^{20.4-5132/T_S}$, p_∞ is the pressure in the court away from the player and can be considered as atmospheric pressure so if gauge pressure is used then $p_\infty=0$. The rate of heat generation can be calculated as $4mv$ where *m* is the mass of the player in kg.

1. Calculate the temperature of the player body T_S
2. By assuming that the total heat loss can be written in terms of an effective heat transfer coefficient that includes all modes of heat transfer, i.e. $q_{total}=h_{eff}A\left(T_S-T_\infty\right)$, what is the value of h_{eff} for the steady state situation assuming that T_s is 26^{o}C.

[20 marks]