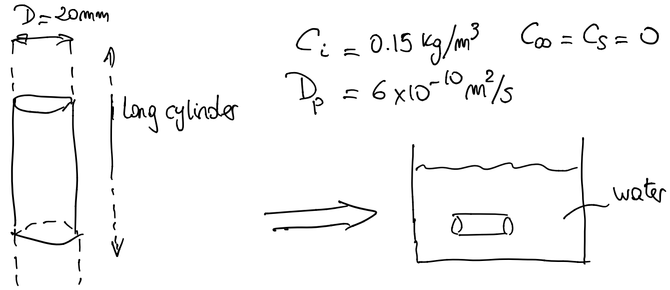
Homework 6 Solutions

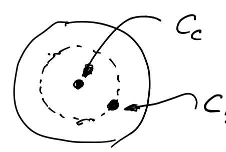
Problem 1



$C_i = 0.15\ kg/m^3 \quad C_\infty = C_s = 0$

$D_p = 6\times10^{-10}\ m^2/s$

(a) Concentration of the polymer at the center and at 5mm from the center after 20 hours of immersion



We will assume that the length of the cylinder is much larger than the diameter, so we will be use solutions for an infinite cylinder. We can use the Heisler charts or tables. The later only suitable for Fourier Number $F_o > 0.2$

$$F_o = \frac{D_p t}{R^2} = \frac{6\times10^{-10}\ m^2/s \times 20h \times 3600 s/h}{(15\times10^{-3} m)^2\ m^2} = 0.19$$

It is very close to 0.2 so let's use the table

The table is a function of the Biot number which in this case can be considered ∞ because the external mass transfer coefficient is very large

So $Bi_m \to \infty$ and $m = \frac{1}{Bi_m} \Rightarrow 0$

So for an infinite cylinder $\Longrightarrow$ $\gamma_1(rad) = 2.405$ and $C_1 = 1.6018$

The solution for a cylinder is $\frac{C(r,t) - C_\infty}{C_i - C_\infty} = C_1 e^{-\gamma_1^2 F_o} J_0\left(\gamma_1 \frac{r}{R}\right)$ (1)

At the center of the cylinder $r = 0$ and

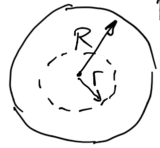
$$\frac{C_c - C_\infty}{C_i - C_\infty} = C_1 e^{-\gamma_1^2 F_o} J_0(0) = C_1 e^{-\gamma_1^2 F_o}$$ because $J_0(0) = 1$ [see tables]

$$\frac{C_c - 0}{0.15 - 0} = 1.6018 \exp(-2.405^2 \times 0.19) = 0.53$$

$$C_c = 0.53 \times 0.15 \frac{kg}{m^3} = 0.08\ kg/m^3$$

$$\boxed{C_c = 0.08\ kg/m^3}$$

– at 5mm from the center



$R = 15\,mm$
$r = 5\,mm$

Values of C_1 and γ_1 remain the same but $n = \frac{r}{R} = \frac{5}{15} = 0.33$, so:

$$\frac{C_1 - 0}{0.15 - 0} = C_1 e^{-\gamma_1^2 F_o} J_o\left(\gamma_1 \frac{r}{R}\right)$$

$$\frac{C_1}{0.15} = 1.6018\, e^{-2.405^2 \times 0.19} \times J_o(2.405 \times 0.33) = 0.53\, J_o(0.79) = 0.53 \times 0.81$$

$J_o(0.79) \approx 0.81$ [From Bessel Tables]

$C_1 = 0.15 \times 0.43\ Kg/m^3$

$$\boxed{C_1 = 0.06\ Kg/m^3}$$

(b) If the diameter is doubled what is the concentration of the center of the cylinder at 20h

The new Fourier Number F_o will be

$$F_o = \frac{6 \times 10^{-10}\, m^2/s \times 20h \times 3600\, s/h}{(30 \times 10^{-3})^2} = 0.05$$

F_o is very small and $F_o < 0.2$ so we cannot use the approximated solution and we need to use the charts

From the Heisler charts for $F_o = 0.05$, $n = \frac{r}{R} = 0$ and $m = 0$

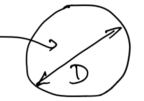
$$Y = \frac{C_c - C_s}{C_i - C_s} \cong 0.92 \text{ [hard to see in the chart]}$$

$$C_c = 0.92 \times 0.15\ Kg/m^3 = 0.14\ Kg/m^3$$

$$\boxed{C_c \cong 0.14\ Kg/m^3}$$

Problem 2

(3)

alveolus 

$D = 0.1\,mm$

at $t=0$ $C=C_i$

at >0 $C = C(R,t) = C_\infty$

$D_o = 2.4\times10^{-9}\,m^2/s$

How long does it take for the concentration change $(C_c - C_i)$ at the center to be 80% of the final concentration change?

The final concentration change at the center will hapen when $C_c = C_\infty$

So written in terms of concentration changes the final change of concentration in the center of the alveolus will be $C_\infty - C_i$. Thus we need to find the time for $\frac{C_c - C_\infty}{C_\infty - C_i} = 0.80$. But we can see that the solution of the partial differential equation is written using a different concentration ratio, i.e. $\frac{C_c - C_\infty}{C_i - C_\infty}$. However if we use the following relationship

$$\underbrace{\frac{C_c - C_i}{C_\infty - C_i}}_{0.8} = 1 - \underbrace{\frac{C_c - C_\infty}{C_i - C_\infty}}_{\text{what we get from either tables or Heisler charts}} = \frac{C_i - \cancel{C_\infty} - C_c + \cancel{C_\infty}}{C_i - C_\infty} = \frac{C_i - C_c}{C_i - C_\infty} = \frac{C_c - C_i}{C_\infty - C_i}$$

So $\frac{C_c - C_\infty}{C_i - C_\infty} = 1 - 0.8 = 0.2$

We don't know the Fo number but let's assume a priori that will be larger than 0.2, so let's use the tables. Again since the surface of the alveolus has a fixed concentration, $Bi_m \to \infty$, so from tables for a sphere

$C_1 = 2.0$ $\quad \delta_1 = 3.1415$

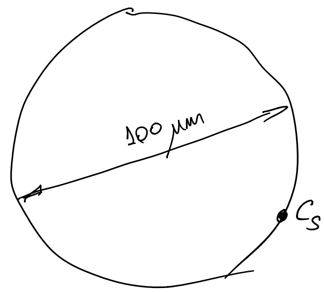
Using Equation $\frac{C_c - C_\infty}{C_i - C_\infty} = 0.2 = 2 \cdot \exp(-3.1415^2\, Fo)\, \frac{1}{\cancel{\delta_1}} \cancel{\delta_1}$

So $Fo = -\frac{1}{3.1415^2} \ln \frac{0.2}{2} = 0.23$ [So the approximation was right]

$$Fo = \frac{D_o t}{R^2} \Rightarrow t = \frac{Fo R^2}{D_o} = \frac{0.23 \times (0.05\times10^{-3})^2\, \cancel{m^2}}{2.4\times10^{-9}\, \cancel{m^2}/s} = 0.24\,\text{Seconds}$$

$$\boxed{t = 0.24\,s}$$

Problem 3 (4)



$C_i = 65 mg/100 mg$ resin

$D_d = 3.4 \times 10^{-9} m^2/s$

$C_s = 0$

90% of the average drug concentration is released in 3 hours

and $D = \frac{100}{\sqrt{2}} = 70.71 \mu m$

(a) For long times [Normal in controlled release] we can estimate the average concentration as:

$$\ln \frac{C_{avg} - C_s}{C_i - C_s} = \ln \frac{8}{\pi^2} - D_d \left(\frac{\pi}{2R}\right)^2 t \quad (1)$$

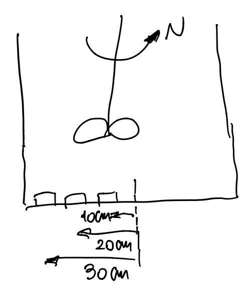
Note that Eq.(1) was derived for a slab but the average concentration for a Sphere has to be obtained using a complex integration.

If we inspect Eq.(1) we can see that the C_{avg} depends on D_d, R and time. Provided the time t is the same the value of C_{avg} will depend on D_d and R therefore the group A multiplying the time t in Eq.(1) is:

$A = D_d \left(\frac{\pi}{2R}\right)^2$ with the new values of D_d and R $A = \frac{D_d}{2}\left(\frac{\pi}{2\frac{R}{\sqrt{2}}}\right)^2 = D_d \left(\frac{\pi}{2R}\right)^2$

So the value of the C_{avg} will be the same for these new values of D_d and R

Problem 4



$N = 50 rpm$

$u = 2\pi N R$

R = radial location of the tablet

$\mu = 0.8 cP = 0.8 \times 10^{-3} Pa.s$

$\rho = 1000 kg/m^3$

$D_d = 1.5 \times 10^{-9} m^2/s$

(a) Mass transfer coefficient in position 1

$$u_1 = 50 \frac{rev}{min} \times \frac{1 min}{60 s} \times \frac{2\pi rad}{1 rev} \times 0.10 m = 0.52 \frac{m}{s}$$

$$Re_1 = \frac{u_1 L \rho}{\mu} = \frac{0.52\,m/s \times 0.01m \times 1000\,kg/m^3}{0.8\times10^{-3}\,Pa.s} = 6{,}500$$

$Re_1 < 2\times10^5$ so

$$Sh_1 = 0.664 \times Re_1^{1/2} Sc^{1/3}$$

$$Sc = \frac{\mu}{\rho D_d} = \frac{0.8\times10^{-3}\,Pa.s}{1000\,\frac{kg}{m^3} \times 1.5\times10^{-9}\,\frac{m^2}{s}} = 533.3$$

$$Sh_1 = 0.664 \times 6500^{1/2} \times 533.3^{1/3} = 434.13$$

$$Sh_1 = \frac{h_{m1} L}{D_d} \Longrightarrow h_{m1} = \frac{Sh_1 D_d}{L} = \frac{434.13 \times 1.5\times10^{-9}\,m^2/s}{0.01\,m} = 6.5\times10^{-5}\,\frac{m}{s}$$

$$\boxed{h_{m1} = 6.5\times10^{-5}\,\frac{m}{s}}$$

(b) location 2

$$u_2 = \frac{50\,rev}{min} \times \frac{1\,min}{60\,s} \times \frac{2\pi\,rad}{1\,rev} \times 0.20 = 1.05\,m/s$$

$$Re_2 = \frac{1.05\times0.01\times1000}{0.8\times10^{-3}} = 13{,}125 < 2\times10^5$$

$$Sh_2 = 0.664 \times 13{,}125^{1/2} \times 533.3^{1/3} = 616.9$$

$$h_{m2} = \frac{Sh_2 D_d}{L} = \frac{616.9 \times 1.5\times10^{-9}}{0.01} = 9.25\times10^{-5}\,\frac{m}{s}$$

$$\boxed{h_{m2} = 9.25\times10^{-5}\,m/s}$$

- Location 3:

$$u_3 = 50\,\frac{\cancel{rev}}{\cancel{min}} \times \frac{1\,\cancel{min}}{60} \times \frac{2\pi\,rad}{1\,\cancel{rev}} \times 0.30m = 1.57\,m/s$$

$$Re_3 = \frac{1.57\times0.01\times1000}{0.8\times10^{-3}} = 19{,}625 < 2\times10^5$$

$$Sh_3 = 0.664 \times 19{,}625^{1/2} \times 533.3^{1/3} = 754.3$$

$$h_{m3} = \frac{Sh_3 D_d}{L} = \frac{754.3\times1.5\times10^{-9}}{0.01} = 1.13\times10^{-4}\,\frac{m}{s}$$

$$\boxed{h_{m3} = 1.13\times10^{-4}\,m/s}$$

(C) Since For all the tablets the flow is laminar we can get the following relationships, for example for tablets 1 and 2 (6)

$$\frac{h_{m1}}{h_{m2}} = \frac{\cancel{0.664} \times \left(\frac{u_1 \times \cancel{\rho} \times \cancel{L}}{\cancel{\mu}}\right)^{1/2} \times \cancel{Sc}^{1/3}}{\cancel{0.664} \times \left(\frac{u_2 \times \cancel{\rho} \times \cancel{L}}{\cancel{\mu}}\right)^{1/2} \cancel{Sc}^{1/3}} = \left(\frac{u_1}{u_2}\right)^{1/2} \quad (1)$$

So h_{m2} and h_{m3} calculated in (b) could be calculated by Eq.(1) using corresponding velocities

Problem 5

(a) $q = hA\left[T_{air} - T_{sh}\right]$ (1) UNITS $q = \frac{W}{\cancel{m^2}\cancel{K}} \times \cancel{m^2}\cancel{K} = $ Watts

(b) $q_m = h_m A\left[C_{ws} - C_{air}\right] \Delta H_{vap}$ (2) UNITS $q_m = \frac{\cancel{m}}{s}\cancel{m^2}\frac{\cancel{kg}}{\cancel{m^3}} \times \frac{J}{\cancel{kg}} = \frac{J}{s} = $ Watts

(c) At steady state $q = q_m$

$$h\left[T_{air} - T_{sh}\right] = h_m\left[C_{ws} - C_{air}\right]$$

$$T_{sh} = T_{air} - \frac{h_m}{h}\left(C_{ws} - C_{air}\right) \quad (3)$$

The concentration of water in the air can be expressed as $C = \frac{\text{kg water in air}}{\text{m}^3 \text{ of dry air}}$ and

the humidity $\bar{H}$ as $\frac{\text{kg water in air}}{\text{kg dry air}}$. If ρ_o is the density of dry air, in $\frac{\text{kg of dry air}}{\text{m}^3 \text{ dry air}}$

$C = \bar{H}\rho_o$ (4)

So substituting Eq.(4) into Eq(3) we obtain:

$$T_{SH} = T_{air} - \frac{h_m}{h}\rho_o\left[\bar{H}_s - \bar{H}_{air}\right]$$

Clearly when $\bar{H}_{air}$ increases T_{SH} will increase