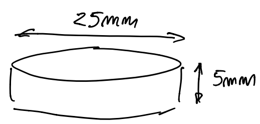
Unsteady State Mass Transfer Examples

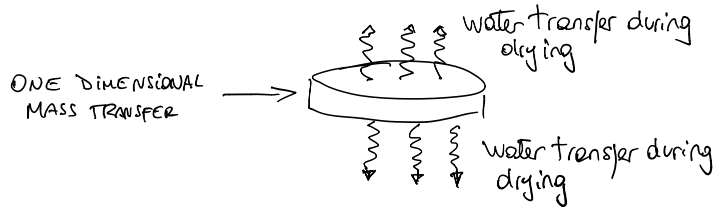
SPRING 2018 (1)

Problem 1




(a) It is clear from the dimensions of the slices that the mass transfer will be done through the two faces of the slice whereas the lateral surface is very small (5mm) so the mass transfer (water) in that area will be negligible.

MAIN ASSUMPTION




(b) $C_i = 3 \frac{\text{Kg water}}{\text{Kg dry matter}}$ $\quad C_f = 0.20 \frac{\text{Kg water}}{\text{Kg dry matter}}$

$C_s = 0.18 \frac{\text{Kg water}}{\text{Kg dry matter}}$

$D_w = 1.2 \times 10^{-10}\ m^2/s$ [Diffusivity of water in the banana slice]

Since there is no location information this final concentration of water (moisture content) is an average value. Please note that in commercial drying that is the more used/practical information

Find out the equation to estimate the desire drying

Since we are using average values we will use the equation for the average concentration. In general the concentration of water is a function of position and time, but since we are using the average concentration of water this value is obtained by integrating the equation given the concentration of water (function of position and time) over the volume of the slice, so the average concentration only varies with time. That average concentration ($C_{avg}$) was obtained and shown in class for a slab and the equation is:

$$\ln \frac{C_{avg}(t) - C_s}{C_i - C_s} = \ln \frac{8}{\pi^2} - D_w \left(\frac{\pi}{2L}\right)^2 t \qquad (1)$$

Eq.(1) shows that $C_{avg}(t)$ is only a function of time, $t$

From Eq.(1) an equation to estimate the time can be obtained as:

$$\ln \frac{C_{avg}(t) - C_s}{C_i - C_s} - \ln\left(\frac{8}{\pi^2}\right) = -D_w\left(\frac{\pi}{2L}\right)^2 t \qquad (2)$$

$$t = -\frac{1}{D_w}\left(\frac{2L}{\pi}\right)^2 \ln\left[\frac{C_{avg}(t) - C_s}{C_i - C_s}\,\frac{\pi^2}{8}\right] \qquad (3)$$

$2L = 5mm$

Substituting values into Eq.(3) we can obtain:

$$t = -\frac{1}{1.2\times10^{-10}\,\frac{m^2}{s}}\left[\frac{5\times10^{-3}}{\pi}\right]^2 \ln\left[\left(\frac{0.2-0.18}{3-0.18}\right)\frac{\pi^2}{8}\right] = 4810.3\,s$$

$$\boxed{t = 4810.3s \cong 1.34\,hr}$$

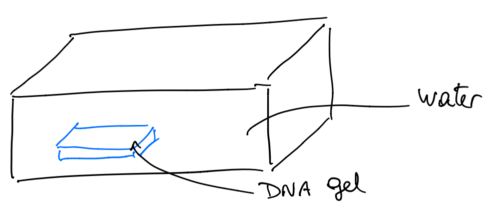
(C) If the thickness is reduced by 60%

$2L_{new} = 5mm \times 0.60 = 3\,mm$

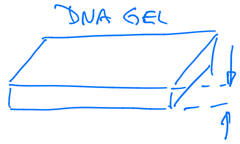
so by using Eq.(3) and $2L_{new} = 3mm$

$$\boxed{t = 1731.7\,s = 0.481\,hr}$$

Problem 2




DNA GEL



thickness = 6mm     $2L = 6mm$     $L = 3mm$

(a) As in the previous problem values given/measured are average values so we can use the equation for the average concentration $C_{avg}$, a little changed

$$\frac{C_{avg}(t) - C_s}{C_i - C_s} = \frac{8}{\pi^2} e^{-D_{Bg}\left(\frac{\pi}{2L}\right)^2 t} \qquad (1)$$

(3)

where $D_{Bg}$ is the diffusivity in the gel, which is unknown and in fact is what we want to estimate from the experiment

Since released BSA are given as masses rather than concentrations we get the Masses as: $M_{avg}(t) = C_{avg}(t)V$ [see Hint 1]

According Hint 1 the amount of BSA released from the gels is calculated as:

$$M_r(t) = M_i - M_{avg}(t) \qquad (2)$$

From the experimental data

- at $t_1 = 3$ days $\quad M_{avg_1}(t_1) = C_{avg1}(t_1)V = M_i - M_{r1}(t_1) \qquad (3)$

- at $t_2 = 6$ days $\quad M_{avg2}(t_2) = C_{avg2}(t_2)V = M_i - M_{r2}(t_2) \qquad (4)$

substituting Eqs. (3) and (4) into Eq (1) the following equations at both Times can be obtained:

$$\frac{\frac{M_i - M_{r1}(t_1)}{\cancel{V}}}{\frac{M_i}{\cancel{V}}} = 1 - \frac{M_{r1}(t_1)}{M_i} = \frac{8}{\pi^2} e^{-D_{Bg}\left(\frac{\pi}{2L}\right)^2 t_1} \qquad (5)$$

$$\frac{\frac{M_i - M_{r2}(t_2)}{\cancel{V}}}{\frac{M_i}{\cancel{V}}} = \frac{1 - M_{r2}(t_2)}{M_i} = \frac{8}{\pi^2} e^{-D_{Bg}\left(\frac{\pi}{2L}\right)^2 t_2} \qquad (6)$$

Equations can be rearranged as:

$$\frac{M_{r1}(t_1)}{M_i} = 1 - \frac{8}{\pi^2} e^{-D_{Bg}\left(\frac{\pi}{2L}\right)^2 t_1} \qquad (7)$$

$$\frac{M_{r2}(t_2)}{M_i} = 1 - \frac{8}{\pi^2} e^{-D_{Bg}\left(\frac{\pi}{2L}\right) t_2} \qquad (8)$$

By dividing Eqs. (7) and (8) the following is obtained:

$$\frac{M_{r2}(t_2)}{M_{r1}(t_1)} = \frac{1 - \frac{8}{\pi^2} e^{-D_{Bg}\left(\frac{\pi}{2L}\right)^2 t_2}}{1 - \frac{8}{\pi^2} e^{-D_{Bg}\left(\frac{\pi}{2L}\right)^2 t_1}} \qquad (9)$$

Let's call $A = e^{-D_{Bg}\left(\frac{\pi}{2L}\right)^2 t_1}$ (10) but $t_2 = 3t_1$

So $e^{-D_{Bg}\left(\frac{\pi}{2L}\right)^2 t_2} = e^{-D_{Bg}\left(\frac{\pi}{2L}\right)^2 3t_1} = \left[e^{-D_{Bg}\left(\frac{\pi}{2L}\right)^2 t_1}\right]^3 = A^3$ (11)

Substituting Eqs (10) and (11) into Eq. (9) the following is obtained:

$$\frac{M_{r2}(t_2)}{M_{r1}(t_1)} = \frac{6.25\,\cancel{g}}{4.15\,\cancel{g}} = \frac{1 - \frac{8}{\pi^2} A^3}{1 - \frac{8}{\pi^2} A} \cong 1.5 \qquad (12)$$

$$1.5 - 1.5 \times \frac{8}{\pi^2} \times A = 1 - \frac{8}{\pi^2} A^3 \Longrightarrow \frac{8}{\pi^2} A^3 - \frac{12}{\pi^2} A + 0.5 = 0 \qquad (13)$$

If we obtain the value of A from the cubic equation [Eq.(13)] we can get the diffusivity of BSA in the gel

In MathCad we define $f(A) = \frac{8}{\pi^2} A^3 - \frac{12}{\pi^2} A + 0.5$

and $A_{sol} = \text{root}(f(A), A)$ From MATHCAD $\Longrightarrow A_{sol} = 0.904$

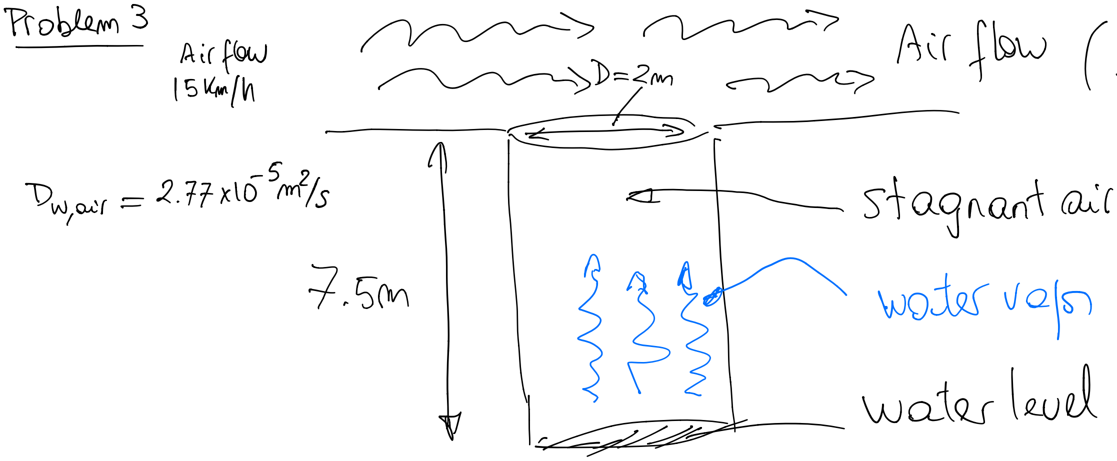
So $e^{-D_{Bg}\left(\frac{\pi}{2L}\right)^2 t_1} = 0.904 \Longrightarrow D_{Bg} = -\frac{1}{\left(\frac{\pi}{2L}\right)^2 t_1} \ln(0.904)$ (14)

Substituting values into Eq. (14):

$$D_{Bg} = -\frac{\ln(0.904) \times 4 \times (3\times10^{-3})^2\, m^2}{\pi^2 \times 1\,\cancel{day} \times \frac{24\,\cancel{h}}{\cancel{day}} \times \frac{3600\,s}{1\,\cancel{h}}} = 4.22\times10^{-12}\ \frac{m^2}{s}$$

$$\boxed{D_{Bg} = 4.22\times10^{-2}\ m^2/s}$$

Problem 3




(5)

- Determine the rate of water loss in $g/m^2 \cdot s$

Since there is an air flow on the top of the tank we can use the following equation:

$$N_w = h_m A (C_{w,sat} - C_{w,air}) \Rightarrow \frac{N_w}{A} = n_w = h_m (C_{w,sat} - C_{w,air}) \qquad (1)$$

$C_{w,sat}$ is the concentration of water at the water surface, which is assumed at saturation conditions, whereas $C_{w,air}$ is the concentration of water vapor in the air which is related to the humidity of air. To determine the flux by Eq.(1) we have to estimate $h_m$. For mass convection the value of $h_m$ can be determined from a correlation $Sh = \frac{h_m L}{D_{w,air}} = f(Re, Sc)$

$$Re = \frac{u_{flow} D \rho_{air}}{\mu_{air}} \qquad Sc = \frac{\mu_{air}}{\rho_{air} D_{w,air}}$$

- Properties of air at 18°C

$\mu_{air} \cong 1.8 \times 10^{-5}\, Pa.s \qquad \rho_{air} = 1.22\, kg/m^3$

$$Sc = \frac{\mu_{air}}{\rho_{air} D_{w,air}} = \frac{1.8 \times 10^{-5}\, Pa.s}{1.22 \frac{kg}{m^3} \times 2.77 \times 10^{-5} \frac{m^2}{s}} = 0.53$$

$$Re = \frac{\frac{15000\,m}{3600\,s} \times 2m \times 1.22\, kg/m^3}{1.8 \times 10^{-5}\, Pa.s} \cong 5.7 \times 10^4 \qquad \text{LAMINAR FLOW}$$

$$Sh = \frac{h_m D}{D_{w,air}} = 0.664\, Re^{1/2}\, Sc^{1/3} \qquad (6)$$

$$\frac{h_m D}{D_{w,air}} = 0.664 \times (5.7\times10^4)^{1/2} \times 0.53^{1/3} = 128.3$$

$$h_m = \frac{D_{w,air}}{D} \times 128.3 = \frac{2.77\times10^{-5}\, m^2/s \times 128.3}{2\, m} \cong 1.78\times10^{-3}\, m/s$$

Please Note that the D was used in the Reynolds and the Sherwood numbers, which is an approach commonly used and does not change much the result. Also convection is originated by a flow Parallel to the diameter so D is the characteristic length

Let's assume that $C_{w,air} \cong 0$. $C_{w,s}$ can be determined from saturation conditions assuming that the water vapor behaves as an ideal gas:

$$P_{w,s} = \frac{n_{ws}}{V} RT = C_{ws} RT \qquad (2)$$

at 18°C $P_{ws} = 2.085\times10^3\, Pa$ From Tables and $R = 8.31\, \frac{J}{mol\, K}$

Substituting in Eq.(2)

$$C_{ws} = \frac{P_{ws}}{RT} = \frac{2.085\times10^3\, Pa}{8.31\, \frac{J}{mol\, K} \times 291 K} = 0.861\, \frac{mol}{m^3}$$

Substituting values into Eq.(1)

$$n_w = 1.78\times10^{-3}\, \frac{m}{s} \times 0.861\, \frac{mol}{m^3} = 1.53\times10^{-3}\, \frac{mol}{m^2 s} = 1.53\times10^{-3}\, \frac{mol}{m^2 s} * \frac{18 g}{1 mol} = 2.75\times10^{-2}\, \frac{g}{m^2 s}$$

$$\boxed{n_w = 2.75\times10^{-2}\, \frac{g}{m^2 s}}$$

TOTAL FLOW OF WATER CAN BE CALCULATED AS:

$$N_w = n_w A = 2.75\times10^{-2}\, \frac{g}{\cancel{m^2} s} \times \frac{\pi \times 2^2}{4}\, \cancel{m^2} = 8.64\times10^{-2}\, g/s$$

$$\boxed{N_w = 8.64\times10^{-2}\, \frac{g}{s}}$$

Problem 4 (7)

(a) The lizard eye can be regarded as circular plate, which for laminar flow, the external mass transfer can be determined by the following equation:

$$\frac{h_m D}{D_w} = 0.664\, Re^{1/2}\, Sc^{1/3} \qquad (1)$$

(b) For a wind velocity giving a heat transfer coefficient of 11 $W/m^2 K$ calculate the average mass transfer coefficient for the air flow over the eye

For heat transfer under a laminar flow:

$$\frac{hD}{K} = 0.664\, Re^{1/2}\, Pr^{1/3} \qquad (2)$$

By dividing Eqs (1) and (2) we obtain:

$$\frac{h_m}{h}\frac{K}{D_w} = \left(\frac{Sc}{Pr}\right)^{1/3} \qquad (3)$$

$$h_m = \frac{D_w}{K} \times h \left( \frac{\frac{\cancel{\nu_{air}}}{D_w}}{\frac{\cancel{\nu_{air}}\, \rho_{air}\, c_{air}}{K}} \right)^{1/3} = \frac{D_w}{K}\, h \left(\frac{K}{\rho_{air}\, c_{air}\, D_w}\right)^{1/3} \qquad (4)$$

Substituting values into Eq.(4)

$$h_m = \frac{2.6\times10^{-5} \times 11}{0.027} \times \left(\frac{0.027}{1.14 \times 1000 \times 2.6\times10^{-5}}\right)^{1/3} = 0.01\ \frac{m}{s}$$

$$h_m = 0.01\ m/s$$

(c) Find the rate of evaporative water loss (EWL)

$$n_w = h_m\,(C_{w,sat} - C_{w,air}) \qquad (5)$$

At 37°C $P_{ws} = 5.9\times10^{3}\,Pa$

by assuming ideal gas behavior for the water vapor:

$$C_{w,sat} = \frac{P_{ws}}{RT} = \frac{5.9\times10^{3}\,Pa}{8.31\,\frac{J}{mol\,\cancel{K}} \times 310\,\cancel{K}} = 2.29\,\frac{mol}{m^3}$$

Substituting into Eq.(5):

$$n_w = 0.01\,\frac{m}{s}\left(2.29\,\frac{mol}{m^3} - 0\right) = 0.0229\,\frac{mol}{m^2 s}$$

[Assume habitat very dry so $C_{w,air} = 0$]

Total water loss through the lizard eyes assuming 60% open

$$N_w = n_w \times A \times 0.60 = 0.0229\,\frac{\cancel{mol}}{\cancel{m^2} s} \times \frac{18g}{\cancel{mol}} \times 8.5\times10^{-7}\,\cancel{m^2} \times 0.60 = 2.10\times10^{-7}\,\frac{g}{s}$$

$$N_w = 2.10\times10^{-7}\,g/s = 2.10\times10^{-7}\,\frac{g}{\cancel{s}} \times \frac{1000\,mg}{1g} \times \frac{3600\,\cancel{s}}{hr} = 0.76\,\frac{mg}{h}$$

(d) Fraction of water loss from lizard eyes:

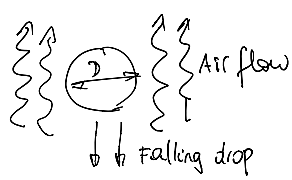
$$\%\,\text{Fraction} = \frac{0.76\,mg/hr}{1.77\,mg/hr} \times 100 \cong 43\%$$

## Problem 5

$D_{w,air} = 0.273\times10^{-4}\,\frac{m^2}{s}$ at 35°C

$P_{ws} = 5.62\times10^{3}\,Pa$ at 35°C

$D = 1\,mm = 1\times10^{-3}\,m$

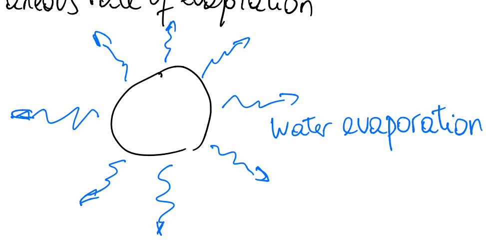



$u = 3\,\frac{m}{s}$

Properties of air at 35°C

$\mu_{air} = 1.9\times10^{5}\,Pa.s$ $\rho_{air} = 1.14\,\frac{kg}{m^3}$

Calculate the instantaneous rate of evaporation




$$N_w = h_m\left(C_{w,sat} - C_{w,air}\right) \qquad (1)$$

We have to estimate $h_m$. There are many correlations to be used but in chapter 14 of the textbook we can find one which is suitable for a sphere in a moving fluid For laminar flow

$$Sh_D = 2 + \left(0.4\, Re_D^{1/2} + 0.06\, Re_D^{2/3}\right) Sc^{0.4} \qquad (2)$$ (9)

The subscript "D" in the sherwood and Reynolds numbers are indicating that the charasteristic length of the dropplet is its diameter D

$$Re_D = \frac{D\, u\, \rho_{air}}{\mu_{air}} = \frac{1\times10^{-3} \times 3 \times 1.14}{1.9\times10^{-5}} = 180 \quad \text{LAMINAR FLOW}$$

$$Sc = \frac{\mu_{air}}{\rho_{air}\, D_{w,air}} = \frac{1.9\times10^{-5}}{1.14 \times 0.273\times10^{-4}} = 0.611$$

Substituting values into Eq.(2)

$$Sh_D = \frac{h_m D}{D_{w,air}} = 2 + \left(0.4\times180^{1/2} + 0.06\times180^{2/3}\right) 0.611^{0.4} \cong 8$$

$$h_m = \frac{Sh_D\, D_{w,air}}{D} = \frac{8 \times 0.273\times10^{-4}\, m^2/s}{0.001\, m} = 0.218\, \frac{m}{s}$$

$$h_m = 0.218\, \frac{m}{s}$$

Since there is no further information let's assume $C_{w,air} = 0$

and $C_{w,sat} = \frac{P_{ws}}{RT} = \frac{5.62\times10^{3}\, Pa}{8.31 \frac{J}{mol\, K} \times 308\, K} \cong 2.2\, \frac{mol}{m^3}$

Substituting into Eq.(1)

$$n_w = 0.218\, \frac{m}{s} \left(2.2\, \frac{mol}{m^3} - 0\right) = 0.48\, \frac{mol}{m^2 s}$$

At the instant that the droplet has a diameter of 1mm the water vapor flux is 0.48 $\frac{mol}{m^2 s}$. Please note that the diameter of the droplet will be decreasing due to the water evaporation until the droplet dissapears.

The total Flow of water evaporation in g/s at that instant is:

$$N_w = n_w \times A = 0.48\, \frac{\cancel{mol}}{m^2 s} \times \pi \times (1\times10^{-3})^2\, \cancel{m^2} \times 18\, \frac{g}{\cancel{mol}} = 2.71\times10^{-5}\, g/s$$

$$\boxed{N_w = 2.71\times10^{-5}\, g/s}$$