NOTES CLASS 2-1-2018 (1)

$$\frac{\partial^{\cancel{2}}\Theta(x,t)}{\partial t} = \alpha \frac{\partial^2\Theta(x,t)}{\partial x^2}$$

$\Theta = 0$ at $x = L$

$\Theta_i = 1$ at $t = 0$

$$\Theta(x,t) = \frac{T(x,t) - T_s}{T_i - T_s}$$

$$\Theta(x,t) = X(x) \cdot T(t)$$

$$\frac{\partial\Theta(x,t)}{\partial t} = X(x)\frac{dT(t)}{dt}$$

$$\frac{\partial^2\Theta(x,t)}{\partial x^2} = \frac{d^2X(x)}{dx^2}T(t)$$

$$X(x)\underbrace{\frac{dT(t)}{dt}}_{T'(t)} = \alpha \underbrace{\frac{d^2X(x)}{dx^2}}_{X''(x)} \cdot T(t)$$

$$\underbrace{\frac{X''(x)}{X(x)}}_{\text{function of } x} = \underbrace{\frac{1}{\alpha}\,\frac{T'(t)}{T(t)}}_{\text{function of time } t} = \text{Constant} = C$$

C can be positive, negative or zero.

$$\underset{\uparrow\ \text{constant}}{C} = \underset{\uparrow\ \text{constant}}{\lambda^2}$$

$$C = \begin{cases} \cancel{0 \quad \text{zero}} \\ +\lambda^2 \quad \text{positive} \\ -\lambda^2 \quad \text{negative} \end{cases}$$

- Zero

$$\frac{X''(x)}{X(x)} = 0 \implies X(x) = 0$$

$$\frac{T'(t)}{T(t)} = 0 \implies T(t) = 0$$

USELESS SOLUTION !!

— $C = +\lambda^2$ [POSITIVE)

$$\frac{T'(t)}{T(t)} = +\alpha\lambda^2$$

$$\frac{1}{T(t)}\frac{dT(t)}{dt} = +\alpha\lambda^2$$

$$\int \frac{dT(t)}{T(t)} = \int \alpha\lambda^2 dt$$

$$\ln T(t) = \alpha\lambda^2 t$$

$$T(t) = c_1 \, l^{\alpha\lambda^2 t}$$

↑ Integration Constant

$t \to \infty \qquad T \to \infty$

positive constant is "<u>unreal</u>"

ONLY CHANCE is C is negative

$$\longrightarrow T(t) = C e^{-\alpha \lambda^2 t}$$

To get $X(x)$

$$\frac{1}{X(x)} \frac{d^2 X(x)}{dx^2} = -\lambda^2$$

$$\frac{d^2 X(x)}{dx^2} + \lambda^2 X(x) = 0$$

↳ Sollution

$$\longrightarrow X(x) = A \cos(\lambda x) + \cancel{B \sin(\lambda x)}$$

This is not a simetric function and the physical problem is symmetric

$$\Theta(x,t) = X(x)\,T(t)$$

$$T(t) \rightarrow C e^{-\alpha\lambda^2 t}$$

$$X(x) = A\cos(\lambda x)$$

λ Constent

at the boundary condition $x = L$

$$T(x=L,t) = T_s$$

$$\Theta(x=L,t) = \frac{T(x,t) - T_s}{T_i - T_s} = 0$$

This $X(x=L) = 0 = A\cos(\lambda L)$

Only if $\cos(\lambda L) = 0 \Rightarrow \lambda L = (2n+1)\frac{\pi}{2}$

there will be many λ that setify the BC

$$\lambda_n = \frac{(2n+1)}{L}\frac{\pi}{2} \quad \text{For all integer}$$

$$X_n(x) = A_n \cos\left[\frac{(2n+1)}{L}\frac{\pi}{2}x\right]$$

$$\Theta_n(x,t) = X_n(x)\,T(t)$$

$$\Theta_n(x,t) = A_n \underbrace{\cos(2n+1)\frac{\pi}{2L}x\,]\cdot C}_{B_n} e^{-\alpha\left((2n+1)\frac{\pi}{2L}\right)^2 t}$$

$$\boxed{\Theta_n(x,t) = B_n \cos\left[(2n+1)\frac{\pi}{2L}x\right] e^{-\left[(2n+1)\frac{\pi}{2L}\right]^2 \alpha t}}$$

solution for one n, so we can take the sum of all n [Integer]

$$\Theta(x,t) = \sum_{n=0}^{\infty} \Theta_n(x,t)$$

B_n is calculate using the concept of the orthogonality of functions.

FOURIER NUMBER

$$F_O = \frac{\alpha t}{L^2}$$

time

DIMENSION OF YOUR SYSTEM

MATERIAL PROPERTY

$$\alpha = \frac{K}{\rho C}$$

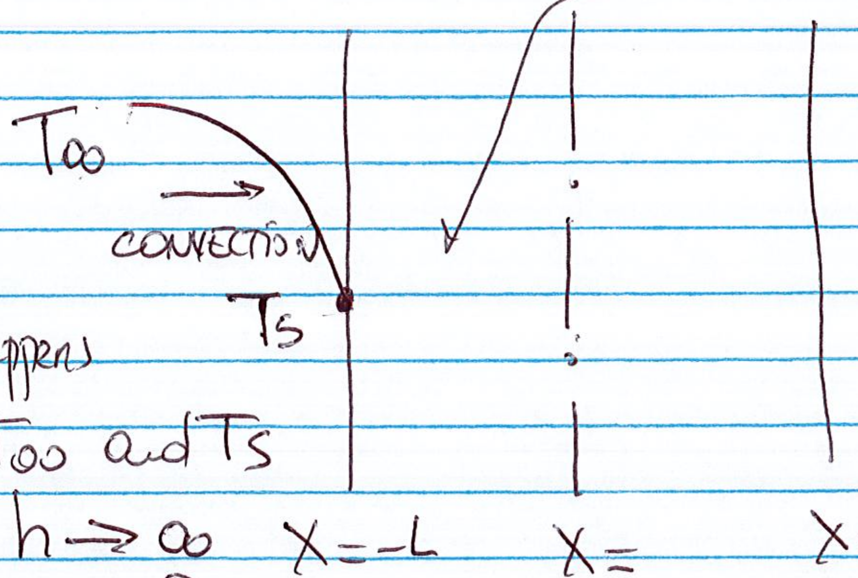
thermal conductivity

density

Heat capacity

SLAB GEOMETRY PLOT

$$\frac{T(x,t) - T_s}{T_i - T_s} \text{ or } \frac{T(x,t) - T_\infty}{T_i - T_\infty}$$



$$m = \frac{K}{hL}$$

When $h \to \infty$ $m \to 0$

What happens with T∞ and Ts when $h \to \infty$ or $m \to 0$?

$T_\infty \to T_s$