ABE 30800 Heat and Mass Transfer - Spring 2018
Homework 3– Due Thursday February 15
Total 200 marks

Question 1

An active MRI (MRI stands for Magnetic Resonance Image) is a small device incorporated into metal surgical implants to make imaging via MRI more effective. However, MRI implants produce heat and it is important to know whether tissue temperature around the implant can rise too high. You are asked to do a simplified analysis to estimate the temperature rise in the tissue due to the MRI implant.

Assume that the MRI implant is spherical with diameter 40mm and is surrounded by the tissue. The implant has a rate of generation of 1Watt that is conducted through the tissue at steady state.

Please note that the main interest is to know how the heat generated in the MRI implant is transferred through the tissue, and from that information to find the highest temperature in the tissue. You can assume that the tissue temperature far away from the implant is maintained at 37^{o}C; and at the boundary where the tissue touches the implant, there is a heat flux from the implant (i.e. 1 W/m^2) and that the generated heat flux is considered equal over the surface of the spherical implant. Assume that the thermal conductivity of the tissue is 0.5 W/m.K.

(a) Draw a schematic of the system

(b) Write the governing equations in the proper coordinate system needed to solve the problem. Do suitable assumptions (and write them) to develop a model that could provide an analytical solution to the problem. In addition, write appropriate boundary conditions that will help you to solve the problem. <u>An important boundary condition</u> is where the tissue touches the implant.

(c) Solve the resulting mathematical model obtained from (b), which is a differential equation + boundary conditions, to obtain the temperature profile in the tissue, i.e. temperature as a function of position (since the geometry is a sphere, position will be indicated by r).

(d) Find the maximum temperature in the tissue (under steady state conditions) due to heat generated in the implant.

(e) How is that maximum temperature associated to the heat generation? For example, answer if the heat generation is doubled, does the maximum temperature rise doubles?

[40 marks]

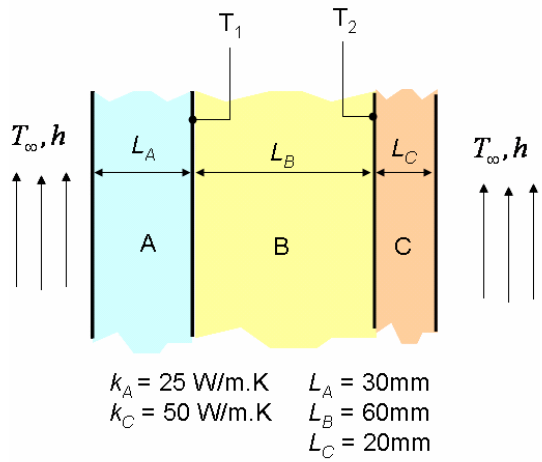
Question 2

Consider one-dimensional heat conduction in a plane composite wall. The outer surfaces are exposed to a fluid at 25^{o}C and a convection heat transfer coefficient of 800 W/m^2.K. The middle wall (B) experiences uniform heat generation Q_B while there is no heat generation in walls A and C. The temperatures of the interfaces are T_1 = 250^{o}C and T_2 = 210^{o}C.

(a) Assuming negligible contact resistances at the interfaces, determine the volumetric heat generation Q_B and the wall conductivity k_B.

(b) Plot the temperature distribution, showing its important features.

(c) Consider conditions corresponding to a loss of coolant at the exposed surface of material A (that is h = 0). Determine the new temperature T_1 and T_2 under this condition and plot the temperature distribution throughout the system. Further data is provided in the figure below.



[40 marks]

Question 3

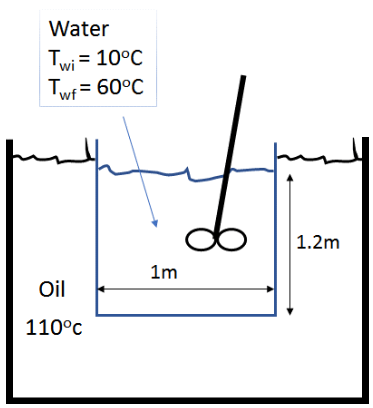
Green coffee beans after harvesting are dried and roasted in a fluidized bed roaster. The roaster works with hot air at 250°C and the air blows through a metal screen above which the beans "float" suspended in the air (that stage is called fluidization). As the beans heat up they get dried and roasted. Assume that the beans are spherical with an average volume of $50mm^3$ and an average surface area of $60mm^3$.

To get an aromatic medium roast flavor the beans need to be heated to 200°C. The beans are initially at 25°C at the time they enter to the roaster and the convective heat transfer coefficient of the heating air is 15 $W/m^2.K$. By considering that the coffee bean thermophysical properties are k = 0.18 W/m.K, c = 2.5kJ/kg.K and $\rho =$ 600 kg/m^3, determine the required roasting time.

[20 marks]

Question 4

A thin-wall cylindrical vessel (1 m in diameter) is filled to a depth of 1.2 m with water at an initial temperature of 15°C. The water is well stirred by a mechanical agitator. Estimate the time required to heat the water to 50°C if the tank is suddenly immersed into oil at 110°C. The overall heat transfer coefficient between the oil and the water is 250 W/m^2 K, and the effective heat transfer surface area is 4.2 m^2.



[20 marks]

Question 5

The depth to which freezing temperature reaches inside the soil is important for plant growth and construction of buildings. Ignore the phase transfer that could happen because of the freezing of the water and focus only in the cooling effect. Assume that at the end of the fall the soil has a uniform temperature of 18^{o}C. In a typical and cold winter day the air temperature drops off to -15^{o}C. The wind over the soil surface produces a convective heat transfer coefficient of 25W/m^2.K. The thermal diffusivity and conductivity of the soil are 6x10^{-7} m^2/s and 0.65 W/m.K, respectively. By writing the proper assumptions that allow to perform relevant calculations for this problem, calculate the following at t = 4hours

(1) The depth to which soil temperature is at -2^{o}C
(2) The surface temperature of the soil
(3) The heat flux from the surface of soil to the cold air

[40 marks]

Question 6

In the inspection of a sample of meat intended for human consumption, it was found that certain undesirable microorganisms were present. To make the meat safe for consumption, it is ordered that the meat be kept at a temperature of at least 121°C for a period of at least 20 min during the preparation. Assume that a 2.5cm thick slab of this meat is originally at a uniform temperature of 25°C, that it is to be heated from both sides in a constant temperature oven; and that the maximum temperature meat can withstand is 155°C. Assume that the surface coefficient of heat transfer remains constant and is 10 W/m^2.K. The following data can be assumed for the sample of meat: c_{meat} = 4184 J/kg K; ρ_{meat} = 1280 kg/m^3; k_{meat} = 0.5 W/m K. Calculate the oven temperature and the minimum total time of heating required to fulfill the safety regulation.
Hint: you may need to use a trial and error approach to solve this problem

[40 marks]