Medical Application of RTD: Hemodialyzer

- Pictured at right are hollow fiber modules. Each module contains multiple semi-permeable capillaries that have been grouped together.
- These devices are used as hemodialyzers for patients with renal disease.
- RTD is a critical factor that can be used to increase the efficiency of these devices.

Image from: http://members.tripod.com/~anuragv/membrane.html

E(t) Versus F(t) Curves

- F(t) curves can be thought of as the percent of the flow leaving the reactor at a time *t* that is younger than time *t.*
- Mathematically, the E(t) and F(t) curves are related by:

$$\mathbf{F} = \int_0^t \mathbf{E}\, dt \qquad\qquad \frac{d\mathbf{F}}{dt} = \mathbf{E}$$

- Because the derivative of F(t) equals E, a steep slope in an F(t) curve will correspond to a peak in an E(t) curve.

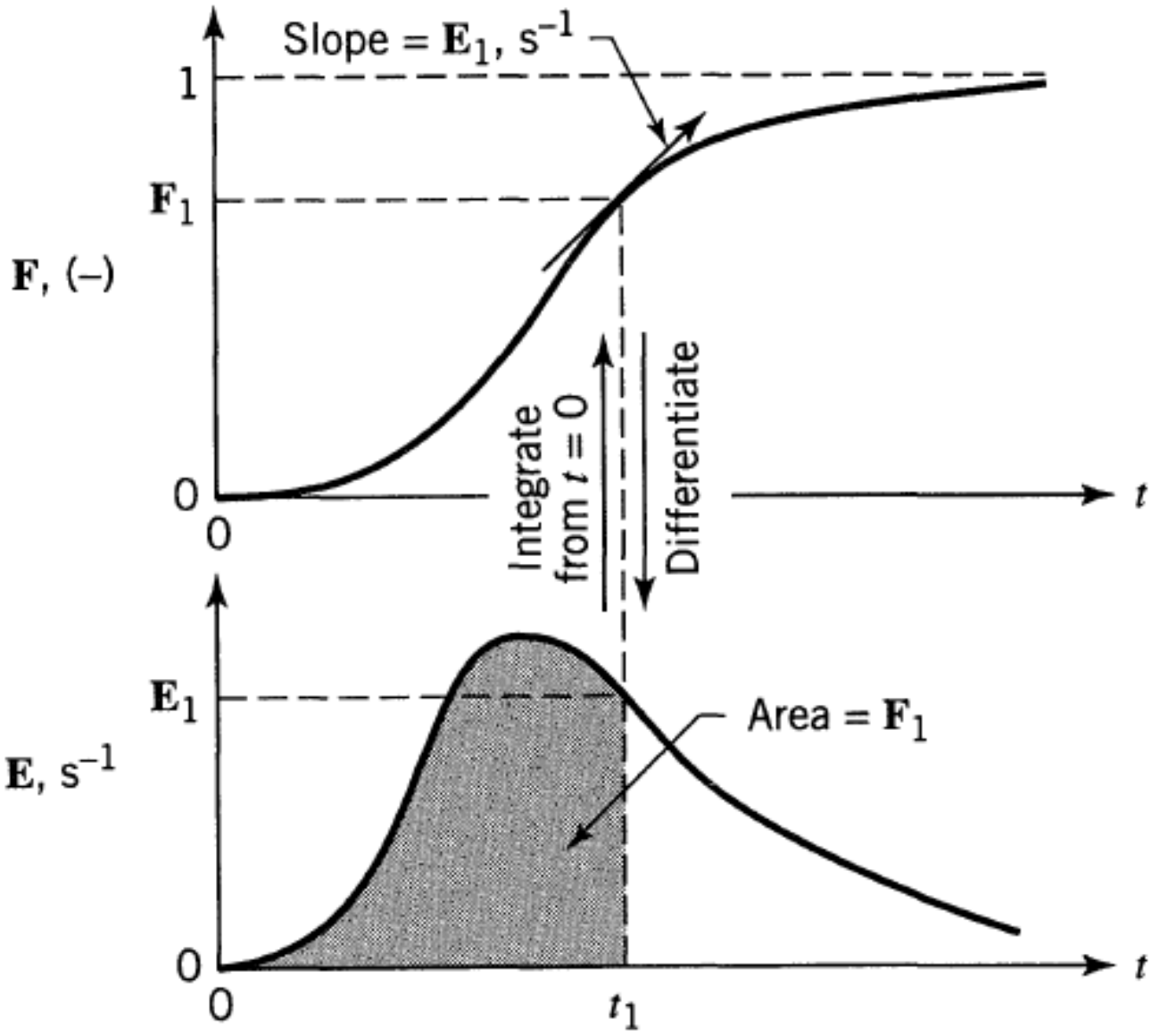
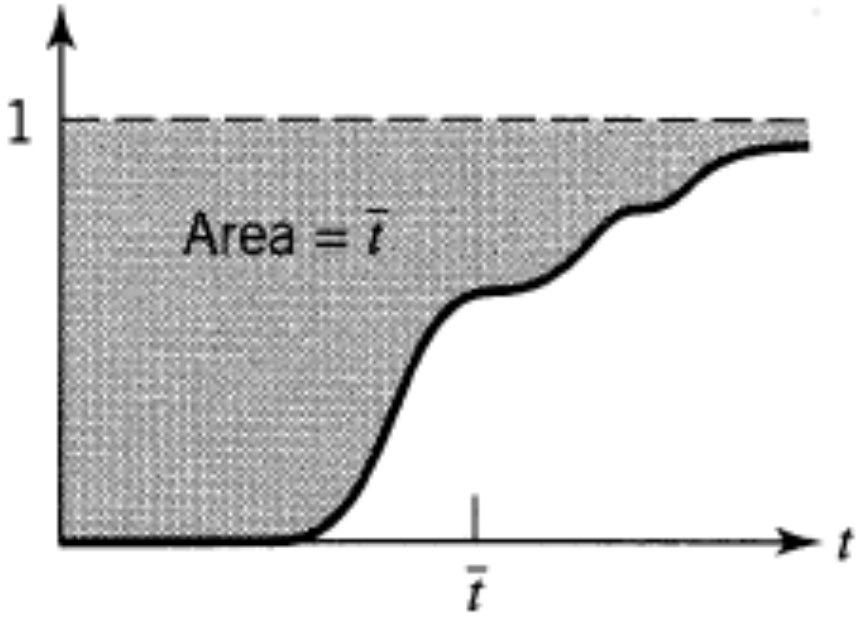


Figure 11.13 Relationship between the **E** and **F** curves.

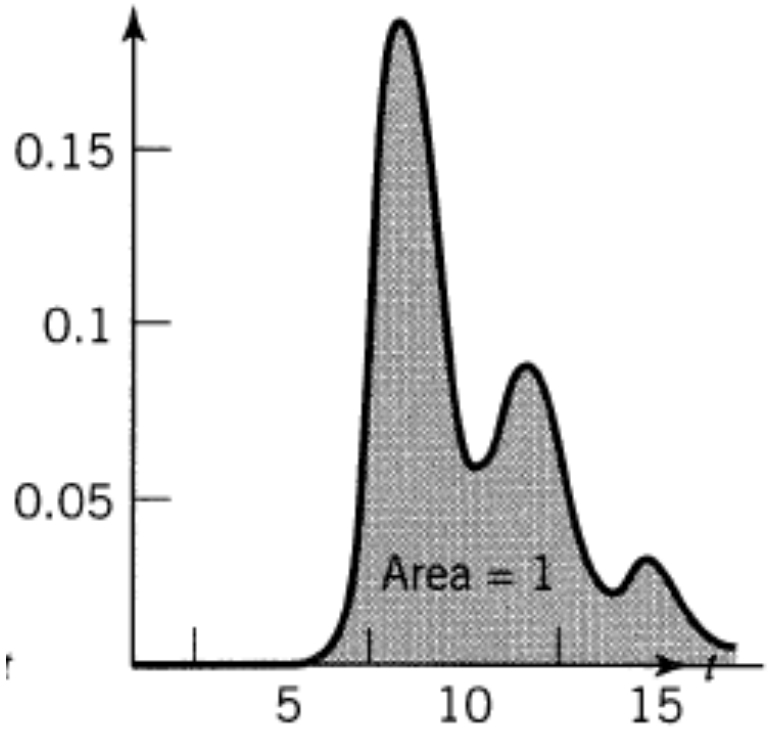
Example 1: E(t) Vs. F(t) Curves

- Problem Statement: Based on the following F(t) curve, predict what the E(t) curve will look like



Example 1: E(t) Vs. F(t) Curves (cont.)

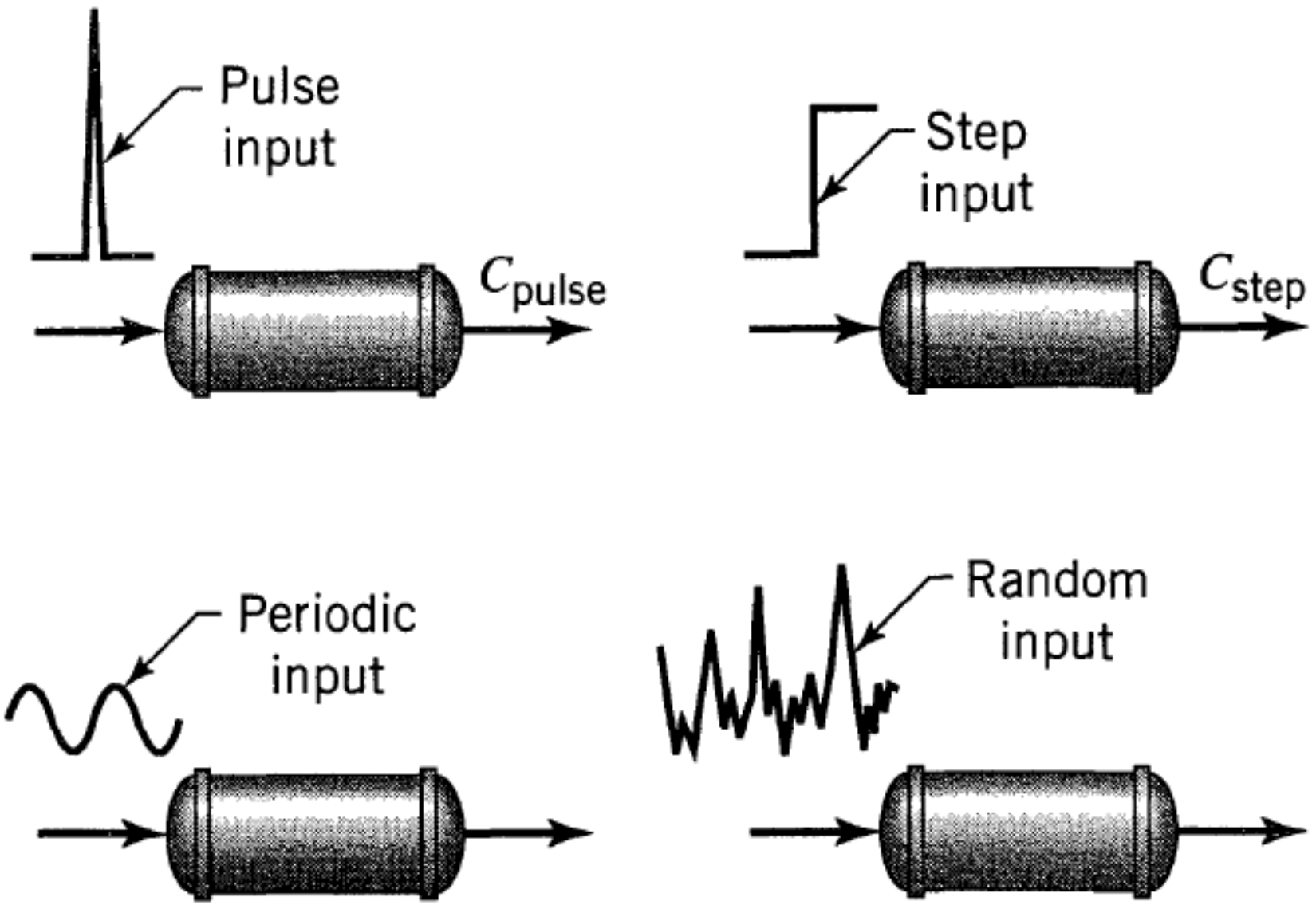
- Solution: Recalling that the a steep slope in an F(t) curve corresponds to a peak in an E(t) the following result is obtained.



Determining E(t) and F(t) Experimentally

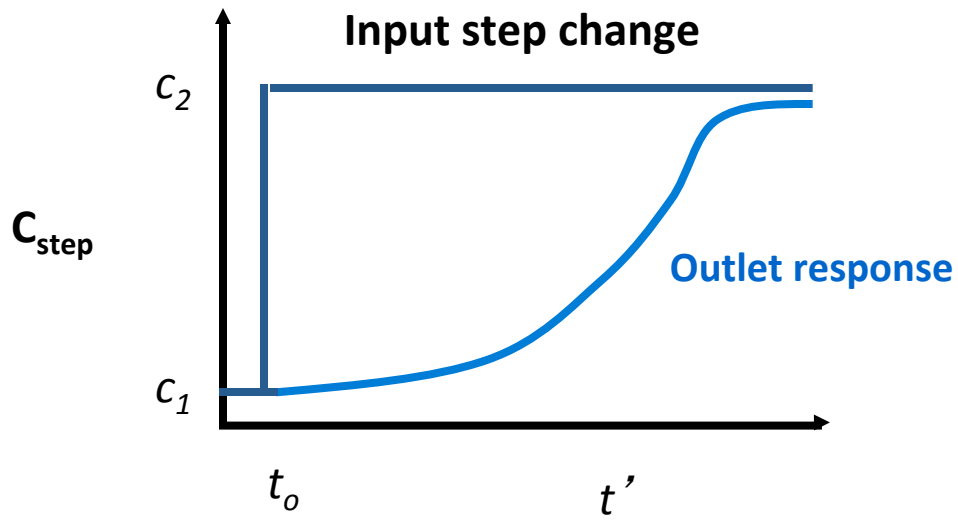
- To analyze the RTD for a particular reactor one must measure the non-ideal flow component, for example by adding a colored dye or radioactive tracer.

- The tracer component is then measured at the outlet of the reactor by monitoring with an appropriate sensor (i.e. colorimeter, electrical conductance, emission of β and γ rays, etc).


Pulse
input
C_{pulse}
Step
input
C_{step}
Periodic
input
Random
input

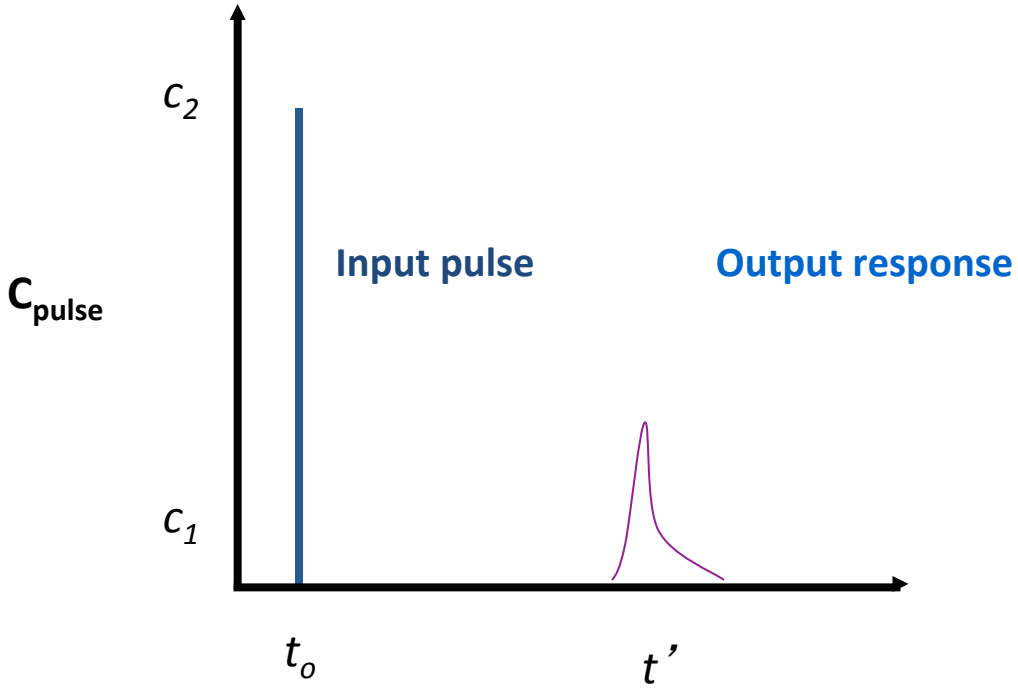
Determining E(t) and F(t) Experimentally: Step Experiment, PFR

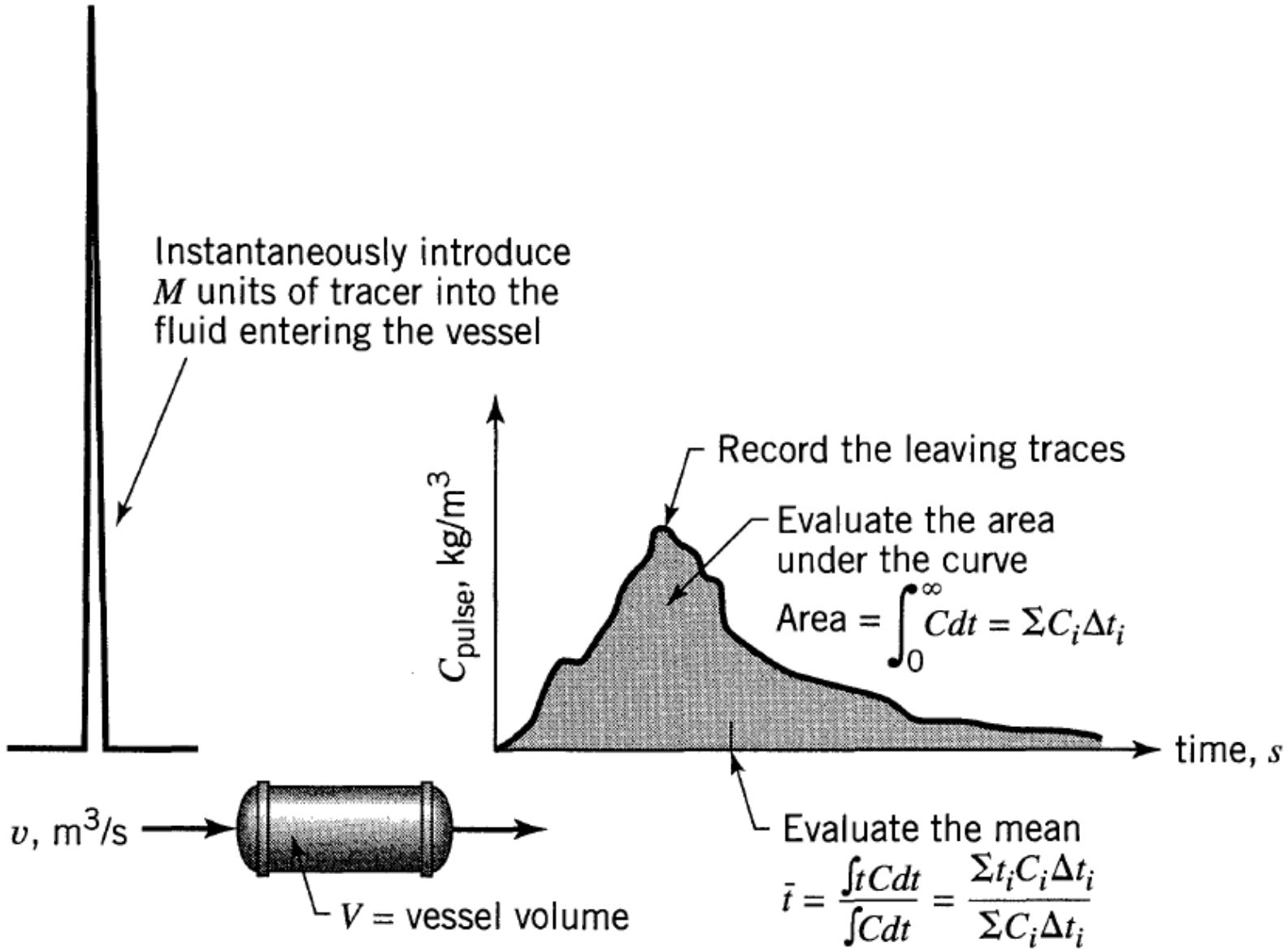
- For a step function test, the concentration of a component in the feed stream is instantly changed from a concentration c_1 to a concentration of c_2, where it is held constant. The outlet concentration is then measured (see figure below)



Determining E(t) and F(t) Experimentally: Pulse Experiment, PFR

- For a pulse input experiment, a single injection of tracer component is added to the feed stream. Ideally, this occurs instantaneously.
- The figure below shows the input and output.




Instantaneously introduce
M units of tracer into the
fluid entering the vessel
C_{pulse}, kg/m^3
Record the leaving traces
Evaluate the area
under the curve
Area = $\int_0^\infty Cdt = \Sigma C_i \Delta t_i$
time, s
v, m^3/s
V = vessel volume
Evaluate the mean
$\bar{t} = \frac{\int tCdt}{\int Cdt} = \frac{\Sigma t_i C_i \Delta t_i}{\Sigma C_i \Delta t_i}$

Example 1: Modeling E(t) Curve

- Given concentration readings for a continuous response to a pulse input into a closed vessel which is to be used as a chemical reactor

Time t, min	Tracer Output Concentration, C_{pulse} gm/liter fluid
0	0
5	3
10	5
15	5
20	4
25	2
30	1
35	0

- Solution: Create a model (E(t) curve) for the input data

The mean residence time, from Eq. 4, is

$$\bar{t} = \frac{\sum t_i C_i \Delta t_i}{\sum C_i \Delta t_i} \overset{\Delta t = \text{constant}}{=\!=} \frac{\sum t_i C_i}{\sum C_i}$$

$$= \frac{5 \times 3 + 10 \times 5 + 15 \times 5 + 20 \times 4 + 25 \times 2 + 30 \times 1}{3 + 5 + 5 + 4 + 2 + 1} = 15 \text{ min}$$

The area under the concentration-time curve,

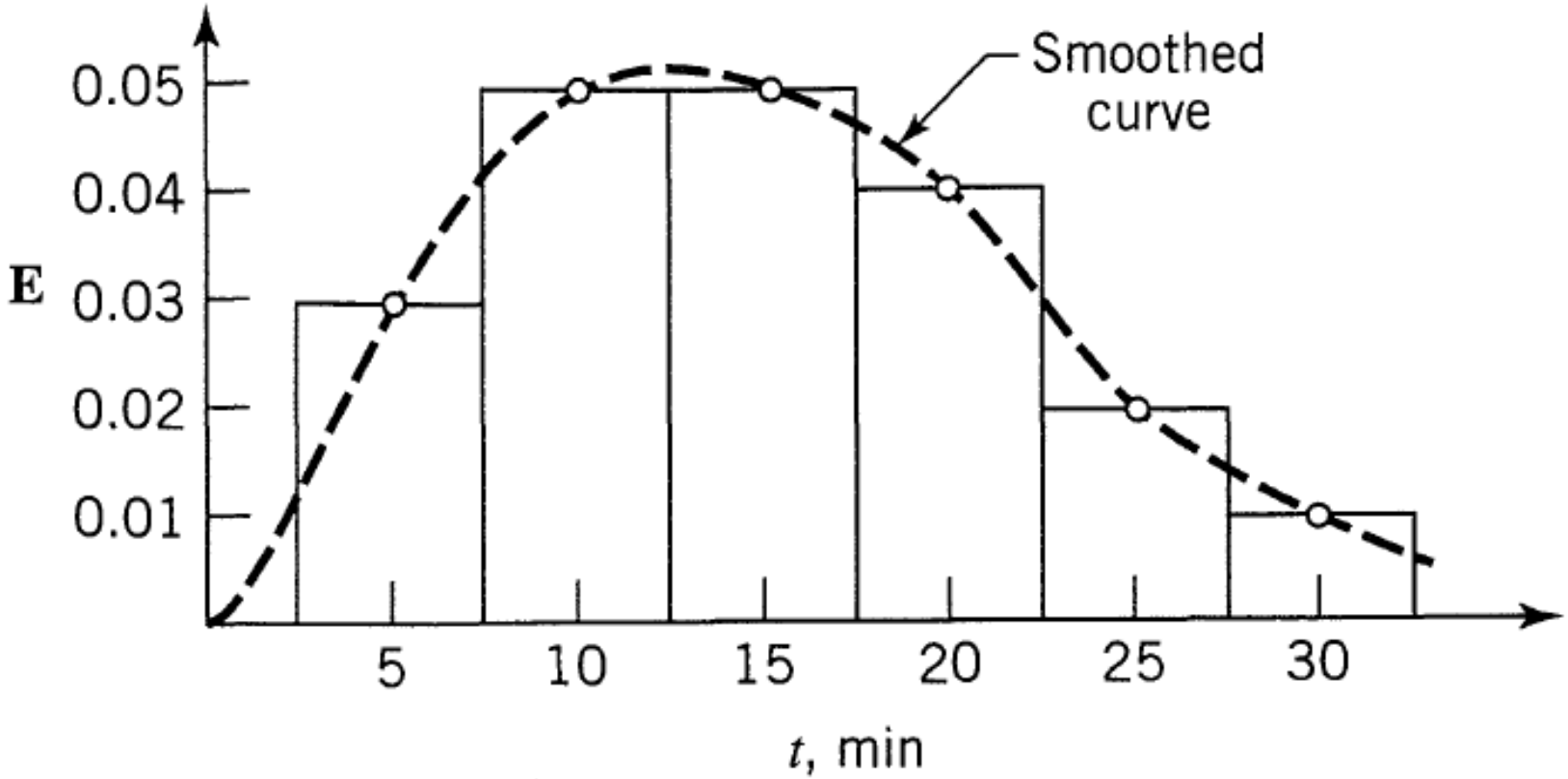
$$\text{Area} = \sum C\,\Delta t = (3 + 5 + 5 + 4 + 2 + 1)5 = 100\ \text{gm} \cdot \text{min/liter}$$

gives the total amount of tracer introduced. To find **E**, the area under this curve must be unity; hence, the concentration readings must each be divided by the total area, giving

$$\mathbf{E} = \frac{C}{\text{area}}$$

Thus we have

t, min	0	5	10	15	20	25	30
$\mathbf{E} = \frac{C}{\text{area}}$, min^{-1}	0	0.03	0.05	0.05	0.04	0.02	0.01


Smoothed
curve
E
0.05
0.04
0.03
0.02
0.01
5
10
15
20
25
30
t, min