Given RTD Data (t, C)

1) model using r.p. or c.s.
2) Normalize to get E(t)
3) Zero out terminal ranges => E(t)

Given input data (t, C)

1) model using r.p. or C.S.
2) Zero out terminal ranges => Cin(t)

Integrate convolution of Cin(t) + E(t)

$$C_{out}(t) = \int_0^t C_{in}(t-t')\,E(t')\,dt'$$

$$= \int_0^t C_{in}(t')\,E(t-t')\,dt'$$

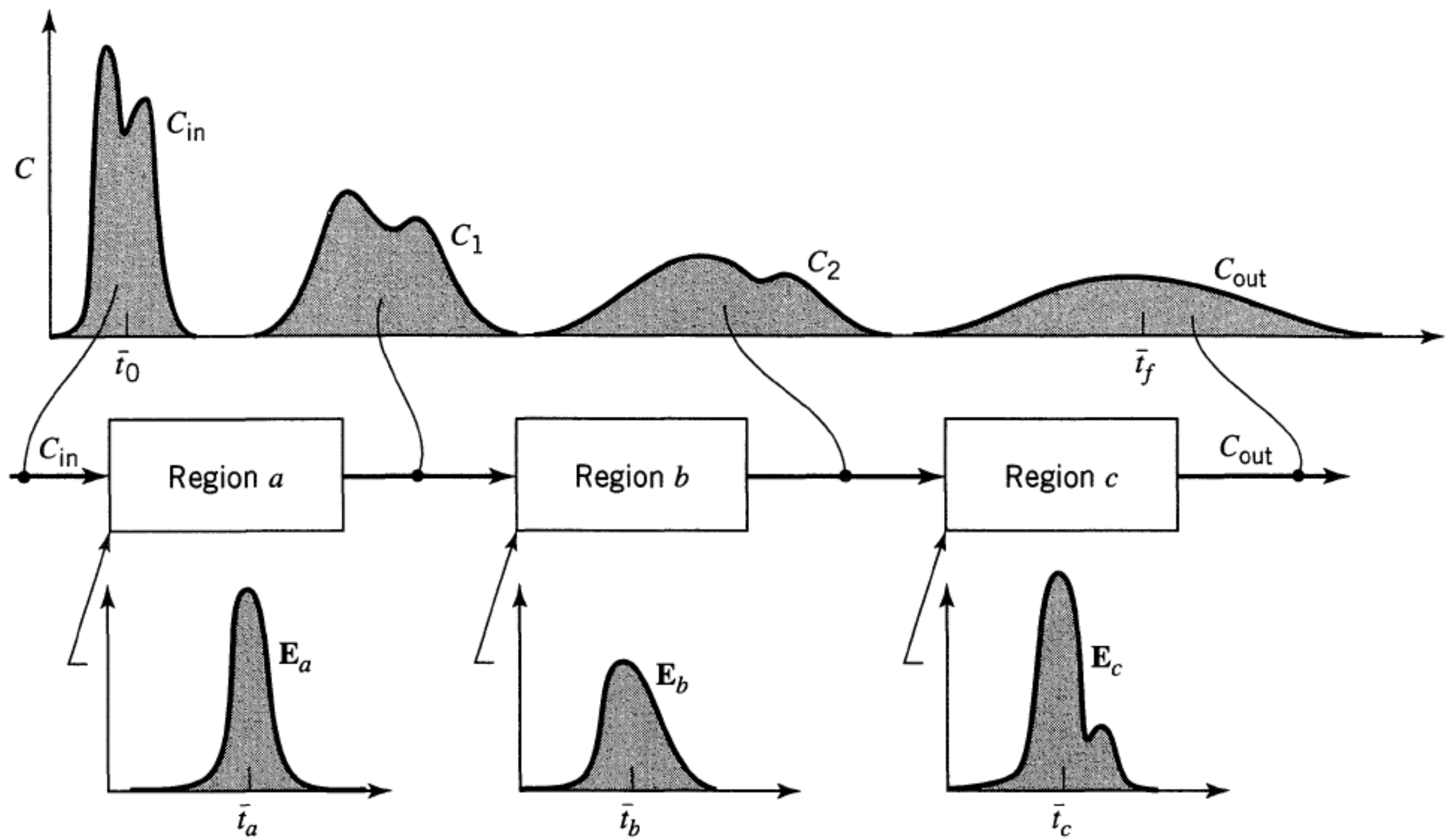


Figure 11.16 Modification of an input tracer signal C_{in} on passing through three successive regions.

- If the input signal C_{in} is measured and the exit age distribution functions E_a, E_b, and E_c are known, then C_1 is the convolution of E_a with C_{in} and so on, thus

$$C_1 = C_{\text{in}} * \mathbf{E}_a, \qquad C_2 = C_1 * \mathbf{E}_b, \qquad C_{\text{out}} = C_2 * \mathbf{E}_c$$

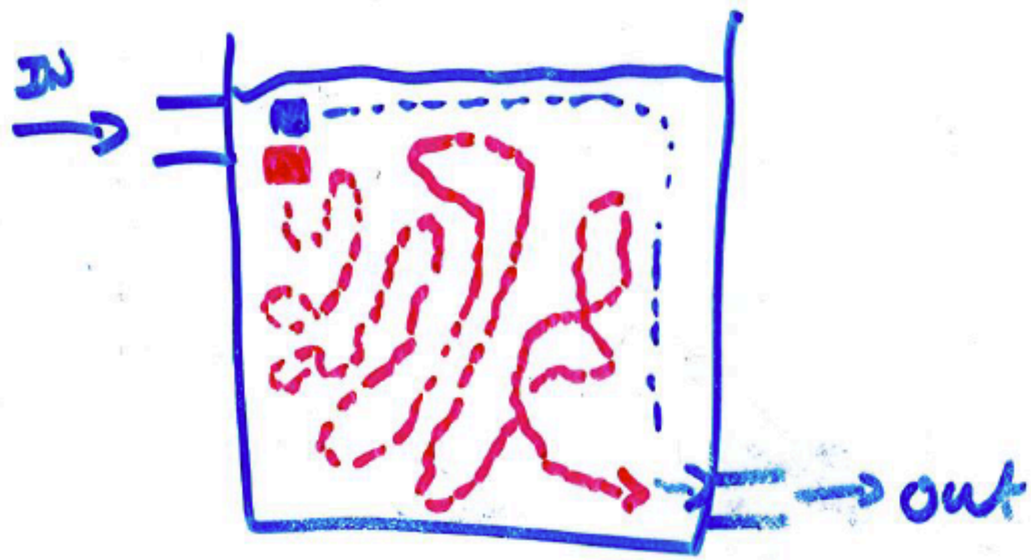
and on combining

$$C_{\text{out}} = C_{\text{in}} * \mathbf{E}_a * \mathbf{E}_b * \mathbf{E}_c \tag{11}$$

Thus we can determine the output from a multiregion flow unit.

RTD

Concept: Elements of fluid entering a reactor follow different paths to exit, ∴ there is a distribution of residence times for outlet flow elements



Non-ideal flow

⇒ How does non-ideal flow affect conversion in reactors?

$r = f(c)$

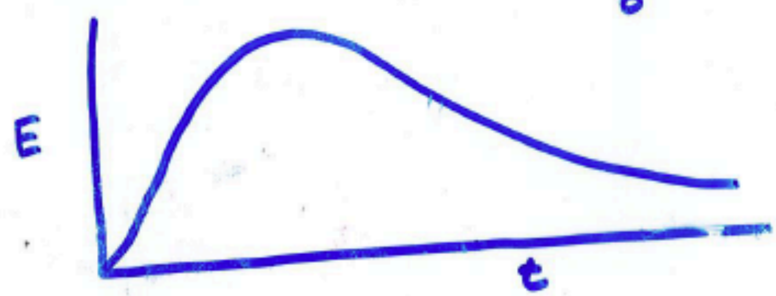
$X = g(c)$

$\tau = h(c)$

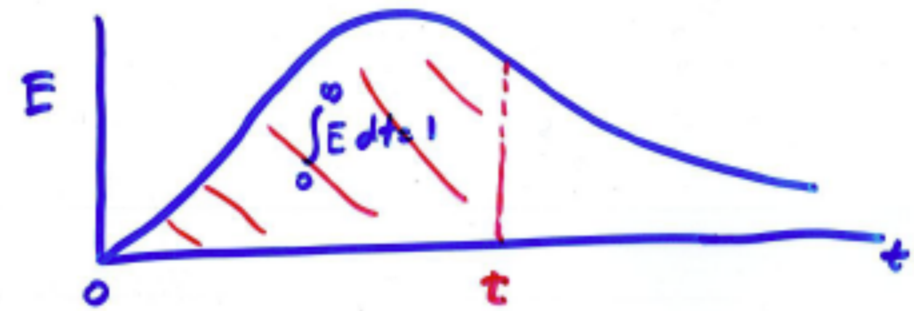
<u>ONLY</u> <u>FLOW</u>

$t < t$

Residence time distribution $\int_0^\infty E\,dt = 1$



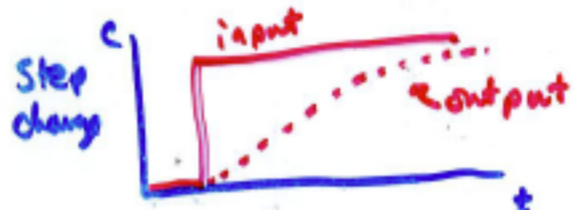
E = exit age distribution

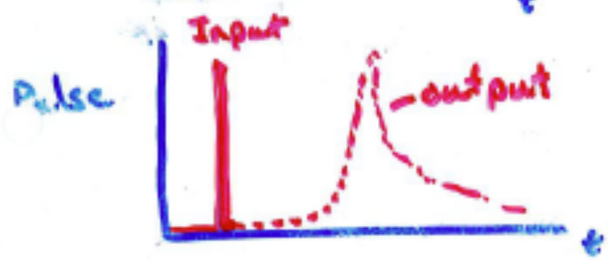


$\int_0^t E\,dt$ = % of exiting fluid that has spent less than t in reactor

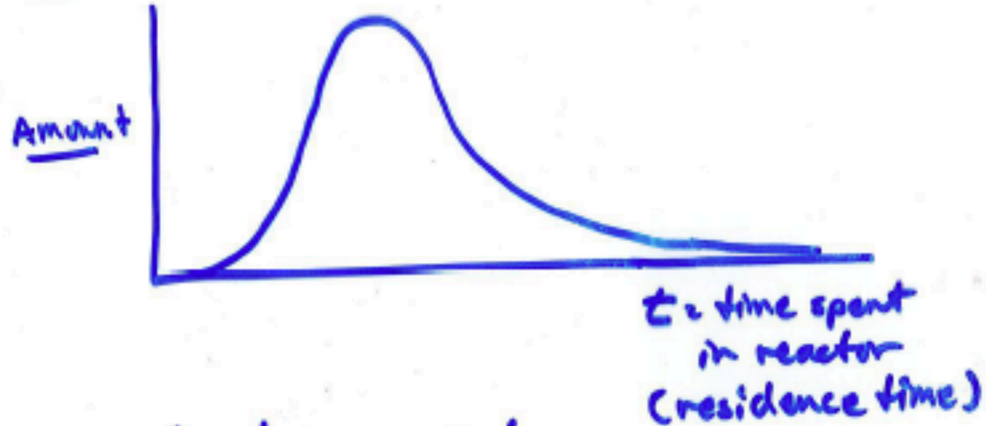
Note: $E [=] \frac{1}{t}$

How to get E(t)





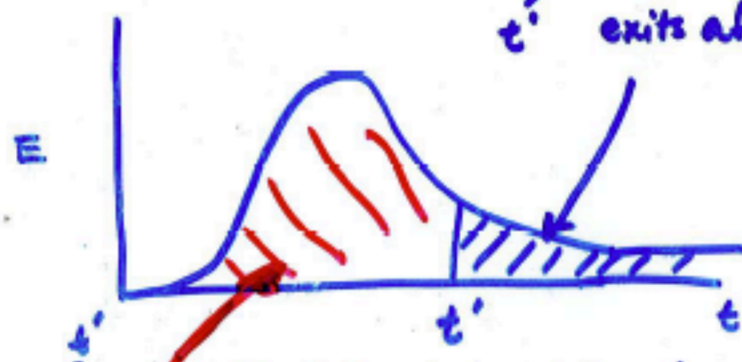
E curve



If normalized so area under curve = 1, this is called the exit age distribution or E curve.

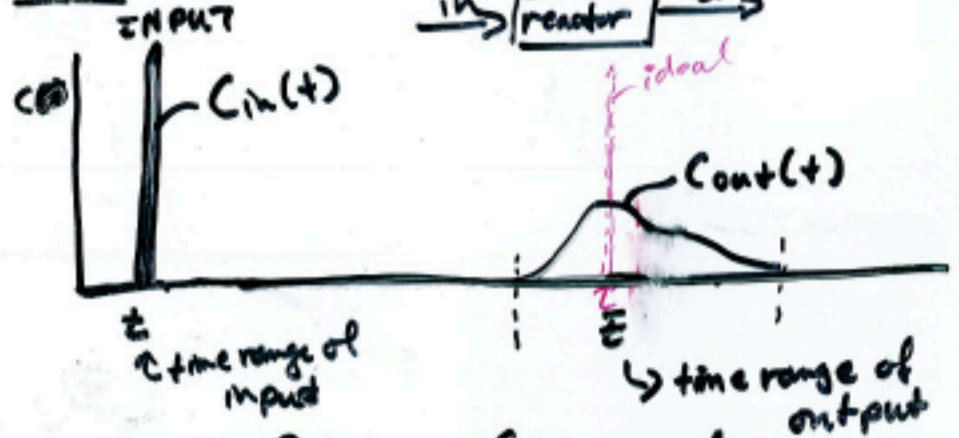
$$\int_0^\infty E(t)\,dt = 1$$

$\int_{t'}^{\infty} E\,dt$ = % fluid that exits after t'



$\int_0^{t'} E\,dt$ = % of fluid that takes less than t' to exit reactor

RTD



mass balance: $\int C_{in}(t)dt = \int C_{out}(t)dt$

$$\bar{t} = \frac{\int t\, C_{out}(t)\,dt}{\int C_{out}(t)\,dt} \quad (= \tau \text{ for ideal reactor})$$

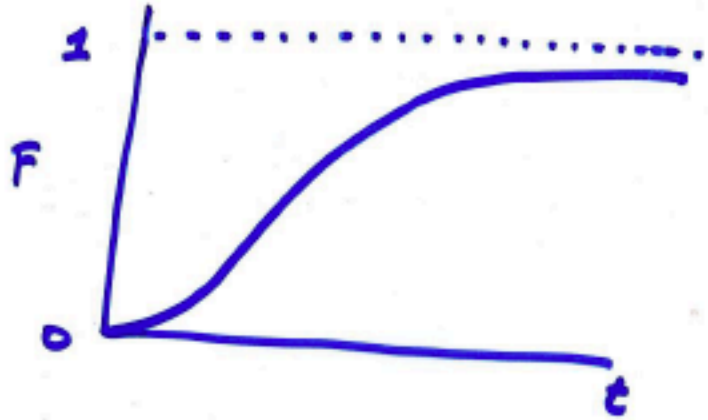
$$E(t) = \frac{C_{out}(t)}{\int C_{out}(t)}$$

$\Rightarrow$ output curve normalized to total area = 1

$$\frac{\int_{t_1}^{t_2} C_{out}(t)}{\int_0^\infty C_{out}(t)} = \% \text{ of material exiting}$$

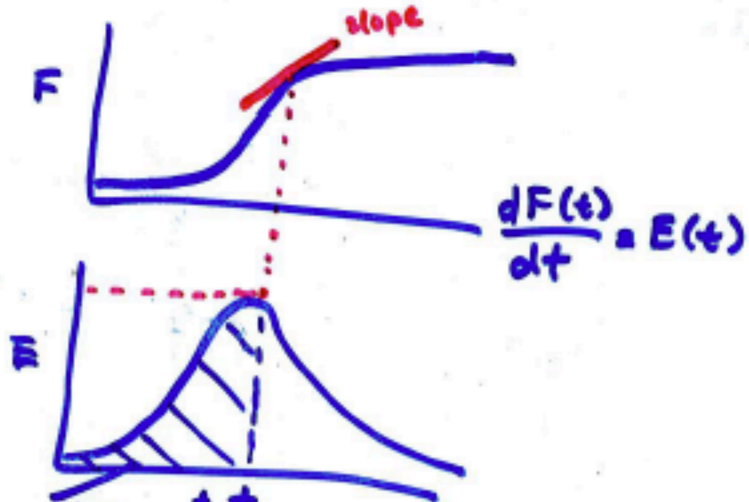
Example 11.1

F curve



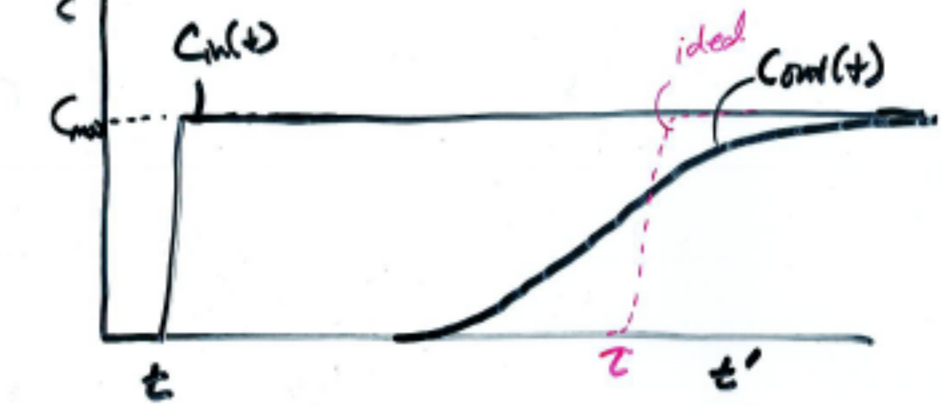
$F(t) = $ % of material entering that has exited by time t

$$F = \int_0^t E \, dt \quad \text{or} \quad \frac{dF}{dt} = E$$



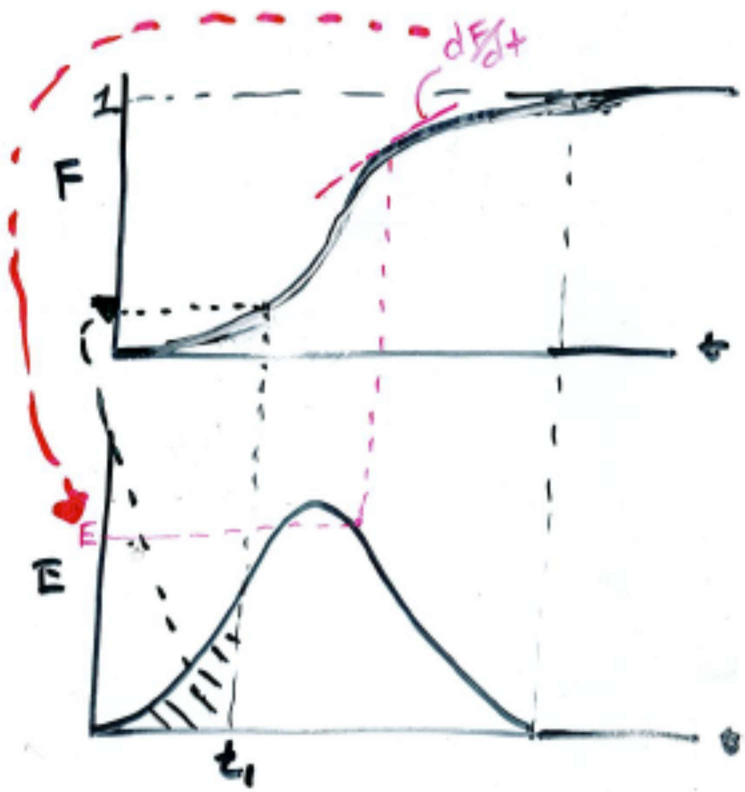
$$\text{Area} = F(t) = \int_0^t E \, dt$$

RTD



Normalize: $F = \frac{c}{c_{max}} \Rightarrow 0 \leq F \leq 1$

F(t) = % of new fluid



$F = \int_0^{t_1} E\,dt$

or

$\frac{dF}{dt} = E$

F = % of material exiting

$= \int E\,dt$

Step input

$$\bar{t} = \frac{\int_0^{C_{max}} t\,dC}{\int_0^{C_{max}} dC} = \frac{\int_0^1 t\,dF}{\int_0^1 dF} = \int_0^1 t\,dF$$

CAN model $E(t)$ & $F(t)$ from

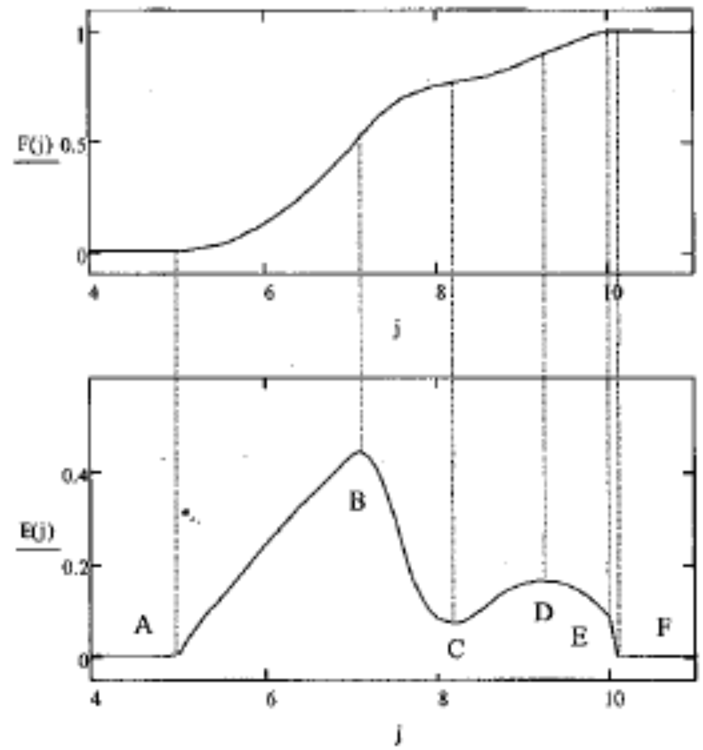
$C(t)$ data.

$\Rightarrow$ Regression polynomials $\Rightarrow R(t)$

$\Rightarrow$ Cubic splines $\Rightarrow S_i(t)$

Can differentiate & integrate

both $R(t)$ & $S_i(t)$ models.



A – start of output response, note relative shapes of E and F curves with time

B – maximum point of E ($dE/dt = 0$), inflection point of F ($dF/dt = dF^2/dt^2=0$)

C – minimum point of E ($dE/dt = 0$), inflection point of F ($dF/dt = dF^2/dt^2=0$)

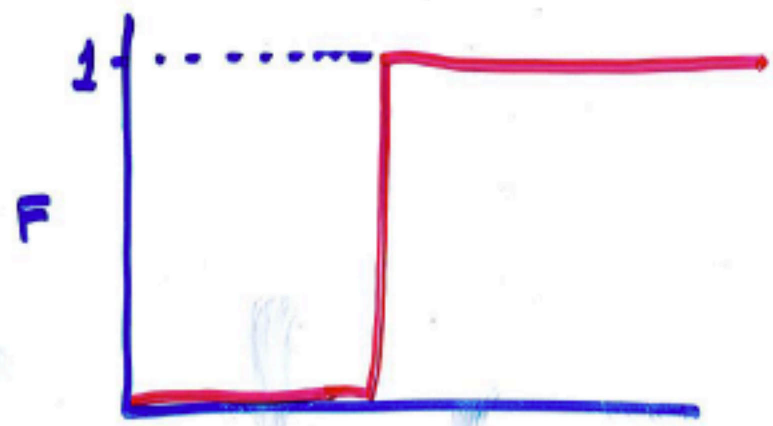
D - maximum point of E ($dE/dt = 0$), inflection point of F ($dF/dt = dF^2/dt^2=0$)

E – sharp drop in E curve, flattens out F curve

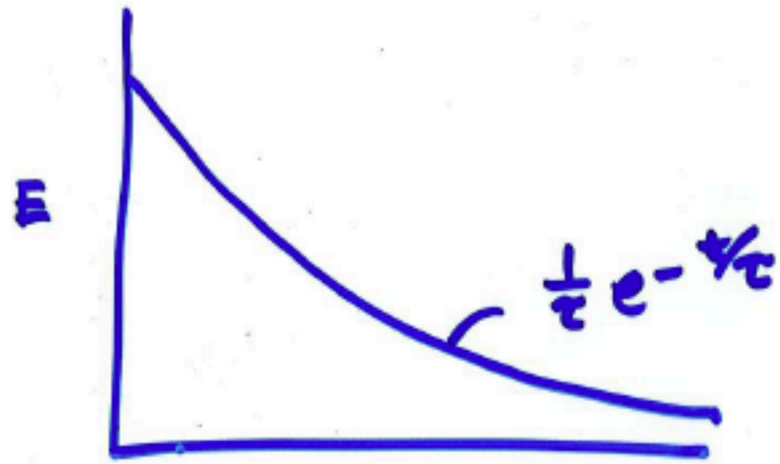
F – output response stops, E goes to zero, F goes to 1 (remains there for increasing t)

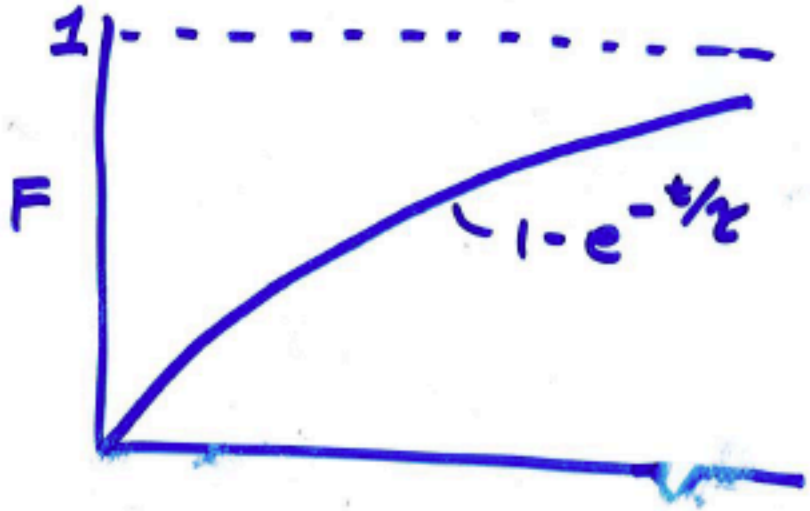
Ideal flow systems

PFR



MFR



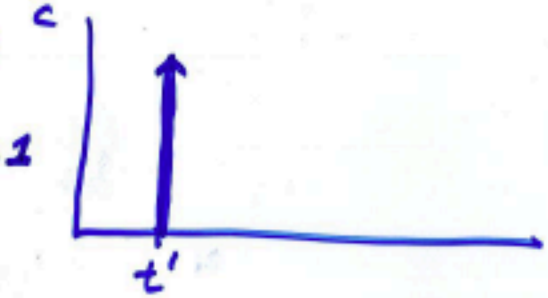


Ideal PFR

Pulse input

$t \neq t'$ $C=0$

$t=t'$ pulse $\int C\,dt = 1$

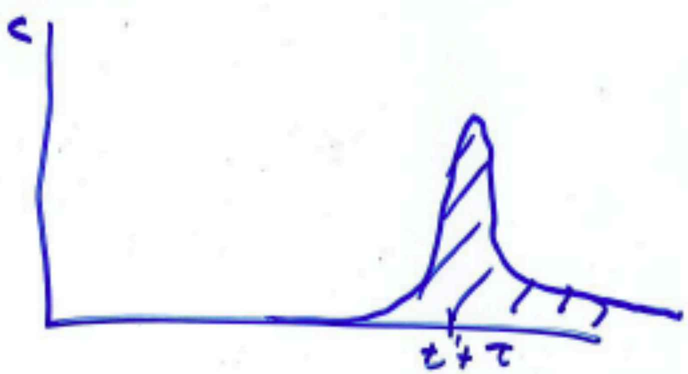


$t \neq t'+\tau$
$C=0$

$t = t'+\tau$
$\int C\,dt = 1$

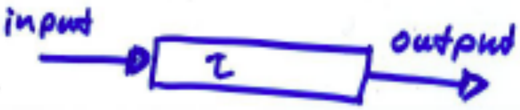
What would non-ideal flow look like?

$\int C\,dt = ?$



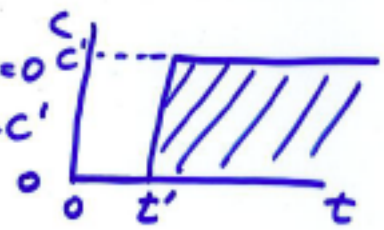
Methods to test/visualize nonideality (mixing) only

ideal PFR



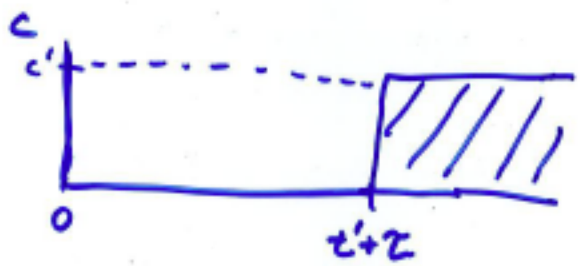
Ideal

Step input
at $t < t'$ $C = 0$
$t \geq t$ $C = C'$

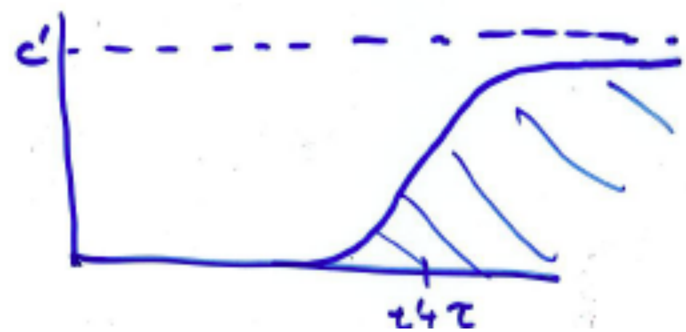


output
$t < t' + \tau$
$C = 0$

$t \geq t' + \tau$
$C = C'$



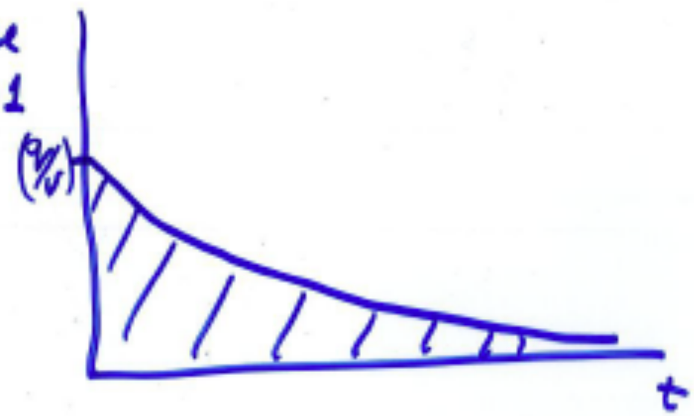
What would Non-ideal flow look like?



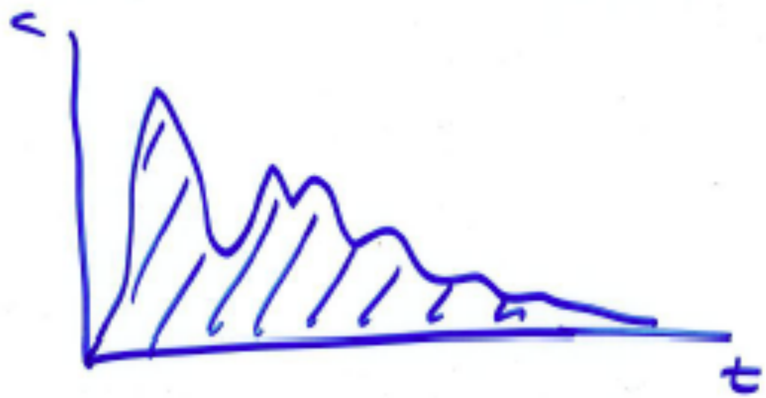
Pulse input, MFR

$t=0$ pulse
$\int C\,dt = 1$

$t<$ $C=0$



what would non-ideal look like?



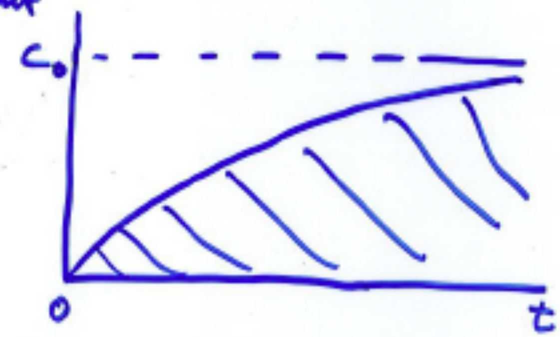
MFR

step input

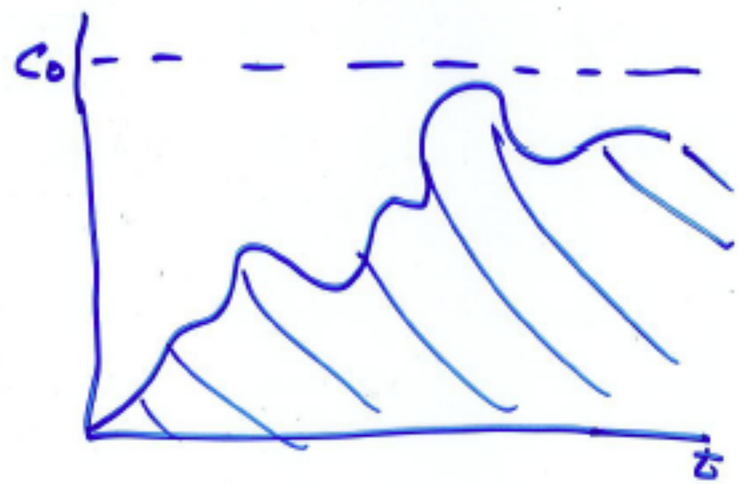
$t = 0 \quad C = 0$

$\qquad\quad C_{in} = C_0$

$t > 0 \quad C = C = C_{out}$



What would non-ideal mixing look like?



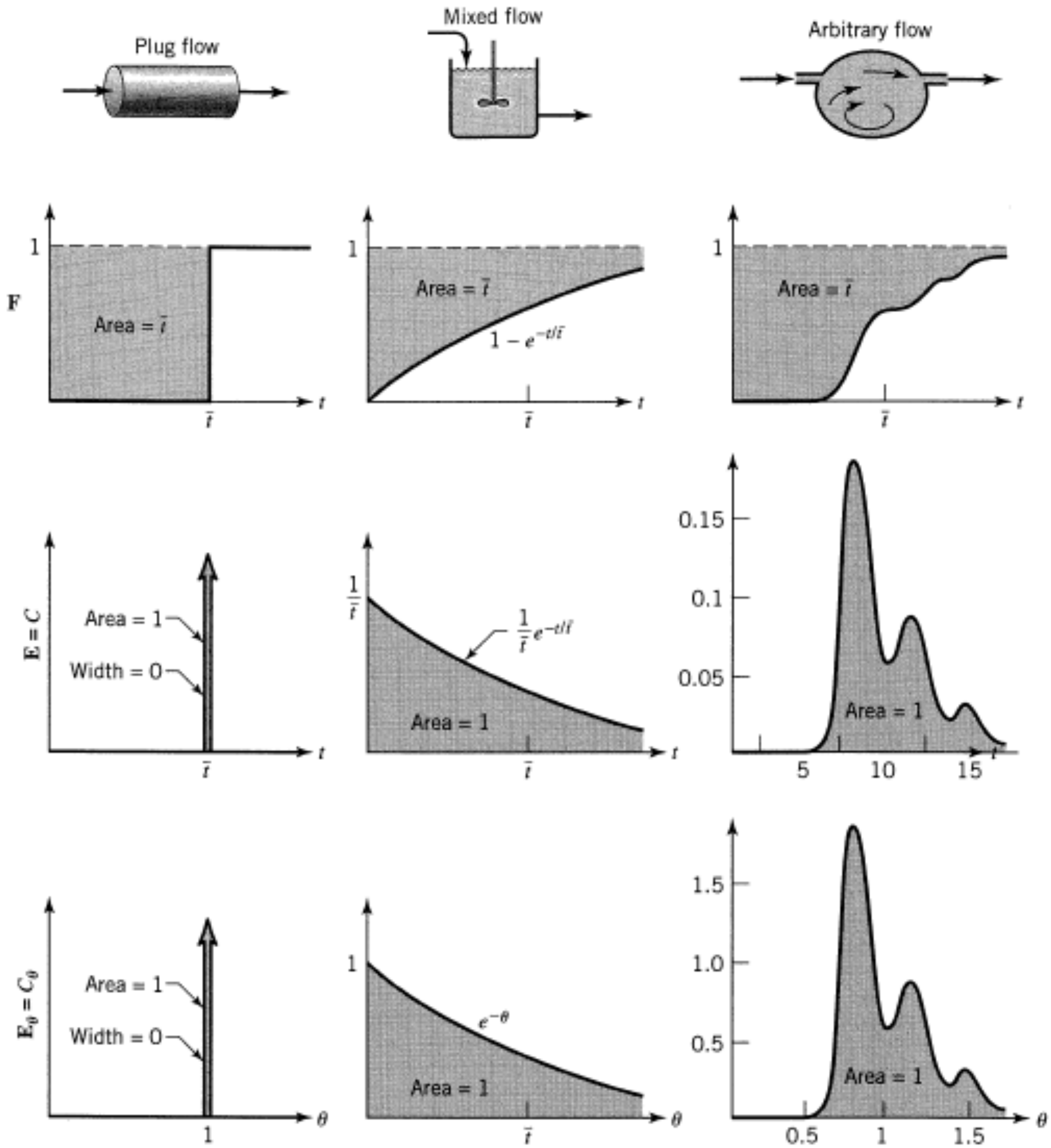


Figure 11.14 Properties of the **E** and **F** curves for various flows. Curves are drawn in terms of ordinary and dimensionless time units. Relationship between curves is given by Eqs. 7 and 8.

Developing models for ideal reactor mixing

PFR → trivial, input signal only time displaced

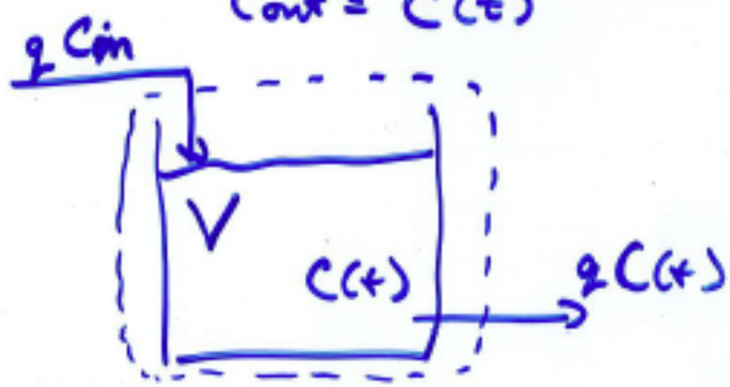
MFR

step input $t=0$ $C=0$ $C_{in}=C_0$

Ideal MFR

Reactor contents homogenous, perfectly mixed

$C_{out} = C(t)$



Derive $C(t) = f(t, \tau)$

t

step function $\quad t=0 \quad C=0$ (start C_0)

$\qquad\qquad t=t \quad C=C$

$$q\,C_0 - q\,C = V\frac{dC}{dt}$$

(No rxn! just mixing)

$$\int_0^t dt = \int_0^C \frac{V}{q}\frac{dC}{(C_0-C)}$$

$$t = -\tau \ln(C_0-C)\Big|_0^C$$

$$= -\tau \ln\left(\frac{C_0-C}{C_0}\right)$$

$$e^{-t/\tau} = \frac{C_0-C}{C_0} = 1 - C/C_0$$

$$C/C_0 = 1 - e^{-t/\tau}$$