

Problem 1.

You are developing a process to extract exciting new flavors from MCO's novel cola nuts for a new soft drink.

The nuts are basically spherical, with a radius of 2 cm. The density of flavoring agents in the nuts is 0.02 gm/cm^3 . Using pure water as the extraction solvent, the effective diffusivity through nut is $D_{\text{eff}} = 8.7 \times 10^{-3} \text{ cm}^2/\text{min}$. The aqueous solubility of the flavoring agents is 50 gm/L of water. Assume that the extraction is controlled by diffusion of the flavoring agents through the nut matrix and that the flavor solution leaving the nut is saturated.

- A. Calculate the time (min) needed to extract 100% of the flavoring agents.
- B. Calculate the time (min) needed to extract 90% of the flavoring agents.

Part A

$$\tau = \frac{\rho R^2}{\text{solubility} \cdot 6 D \cdot C_w}$$

$$\rho = 0.02 \text{ g/cm}^3 \quad R = 2 \text{ cm} \quad D = 8.7 \times 10^{-3} \text{ cm}^2/\text{min}$$

$$\text{solubility} = 50 \text{ g/L} \quad C_w = 1 \text{ g/cm}^3$$

$$\tau = \frac{0.02 \text{ g/cm}^3 \cdot 2^2 \text{ cm}^2}{\text{cm}^3} \cdot \frac{\text{L}}{50 \text{ g}} \cdot \frac{\text{min}}{6 \cdot 8.7 \times 10^{-3} \text{ cm}^2} \cdot \frac{\text{cm}^3}{1 \text{ g}}$$

$$= 0.03065 \frac{\text{L} \cdot \text{min}}{\text{g}}$$

Must convert solubility to g/g not L/g

Assume sol'n has density of water (1 g/cm³)

$$\therefore 50 \text{ g flav.} / 1000 \text{ g sol'n} = 0.05 \frac{\text{g flav.}}{\text{g sol'n}}$$

$$\tau = 30.65 \text{ min}$$

Part B

$$t = \tau \left[1 - 3(1-x)^{2/3} + 2(1-x) \right]$$

$$x = 0.9$$

$$t = 17 \text{ min}$$

Problem 2.

For a 1st order reaction in a catalyst particle, calculate the effectiveness factors for a flat plate (thickness 2 cm), a cylinder (R=2 cm) and a sphere (R=2 cm). For the cylinder, use the chart. For the flat plate and the sphere, use both the analytical equation and the chart.

$$De = 5.31 \times 10^{-3} \text{ cm}^2/\text{s}$$

$$k_e = 0.18 \text{ s}^{-1}$$

1st order rxn, calc eff factor for flat plate, cylinder, and sphere. For cylinder, use the chart. For flat plate and sphere, use both analytical equation and chart.

$$\begin{aligned} De &:= 5.31 \cdot 10^{-3} \frac{\text{cm}^2}{\text{s}} & t_{\text{plate}} &:= 2 \text{ cm} & L_{\text{plate}} &:= \frac{t_{\text{plate}}}{2} & L_{\text{plate}} &= 1 \text{ cm} \\ k_e &:= 0.18 \text{ s}^{-1} & R_{\text{cyl}} &:= 2 \text{ cm} & L_{\text{cyl}} &:= \frac{R_{\text{cyl}}}{2} & L_{\text{cyl}} &= 1 \text{ cm} \\ & & R_{\text{sph}} &:= 2 \text{ cm} & L_{\text{sph}} &:= \frac{R_{\text{sph}}}{3} & L_{\text{sph}} &= 0.667 \text{ cm} \end{aligned}$$

CYLINDER

volume to surface area ratio for a cylinder = $r/2$

$$\phi_{\text{cyl}} := \frac{R_{\text{cyl}}}{2} \cdot \sqrt{\frac{k_e}{De}}$$

$$\phi_{\text{cyl}} = 5.822$$

$$\eta_{\text{cyl}} := 0.18 \quad \text{This value was obtained from Fig 18.6 of the text.}$$

FLAT PLATE

$$mL_{\text{plate}} := L_{\text{plate}} \cdot \sqrt{\frac{k_e}{De}}$$

$$mL_{\text{plate}} = 5.822$$

$$\eta_{\text{platechart1}} := 0.18$$

$$\eta_{\text{plate1eqn}} := \frac{\tanh(mL_{\text{plate}})}{mL_{\text{plate}}}$$

$$\eta_{\text{plate1eqn}} = 0.172$$

These two values are reasonably close to one another.

SPHERE

$$\phi_{\text{sph}} := \frac{R_{\text{sph}}}{3} \cdot \sqrt{\frac{k_e}{De}}$$

$$\phi_{\text{sph}} = 3.881$$

$$\eta_{\text{sphchart}} := 0.236$$

$$\eta_{\text{sph1eqn}} := \frac{1}{3 \cdot \phi_{\text{sph}}^2} \cdot (3 \cdot \phi_{\text{sph}} \cdot \coth(3 \cdot \phi_{\text{sph}}) - 1)$$

$$\eta_{\text{sph1eqn}} = 0.236$$

Problem 3.

A catalyzed chemical reaction (A->B) has an effective 1st order reaction constant $k_e = 0.2/\text{sec}$. The effective diffusivity of A in the catalyst particle is $7.2 \times 10^{-4} \text{ cm}^2/\text{sec}$. It has been proposed to use either spherical or cylindrical catalyst particles with a radius of 0.3 cm. Calculate the difference in effectiveness between these alternative catalyst particle shapes.

1st order rxn. Calculate the difference in effectiveness between spherical and cylindrical catalysts.

$$k_e := 0.2 \quad \text{s}^{-1}$$

$$D_e := 7.2 \cdot 10^{-4} \quad \frac{\text{cm}^2}{\text{s}}$$

$$R := 0.3 \quad \text{cm}$$

SPHERE

$$\phi_{\text{sph}2} := \frac{R}{3} \cdot \sqrt{\frac{k_e}{D_e}} = 1.66$$

$$\eta_{\text{sph}2} := \frac{1}{3 \cdot \phi_{\text{sph}2}^2} \cdot (3 \cdot \phi_{\text{sph}2} \cdot \coth(3 \cdot \phi_{\text{sph}2}) - 1) = 0.45$$

CYLINDER

$$\phi_{\text{cyl}2} := \frac{R}{2} \cdot \sqrt{\frac{k_e}{D_e}} = 2.5$$

$$\eta_{\text{cyl}2\text{chart}} = 0.32$$

$$\text{difference} := \eta_{\text{sph}2} - \eta_{\text{cyl}2\text{chart}} = 0.45 - 0.32 = 0.13$$