# Example

1st order rxn
series mfr

$K = 0.08333/\text{min}$
$q = 25\ L/\text{min}$
$V = 250\ L$
$C_{A0} = 5\ \text{mol}/L$

calculate conversion for:

A. 1 tank

B. 6 tanks in series

A. By calculation $\tau_i = V/q = \dfrac{C_0 - C_1}{KC_1} = \dfrac{1}{K}\left(\dfrac{X}{1-X}\right)$

$K\tau = (0.08333)\left(\frac{250}{25}\right) = (0.8333)$  $X = \dfrac{K\tau_i}{1+K\tau_i} = \boxed{0.4545 \Rightarrow 1\ \text{tank}}$

By chart, $K\tau_i = 0.8333$, $n=1 \Rightarrow nK\tau_i = 0.8333$

Following this at $n=1$ curve, gives

$1-X = 0.55 \Rightarrow \boxed{X = 0.45}$

B. For 6 reactors in series,

By calculation,

$$C_0/C_n = \frac{1}{1-X} = (1+k\tau_i)^6$$

$k\tau_i = 0.8333$

$$= (1+0.8333)^6 = 37.967$$

$$\Rightarrow \boxed{X = 0.9737}$$

By chart, $k\tau_i = 0.8333$

$$nk\tau_i = (6)(0.8333) = 4.9998 = k\tau$$

~~Either~~ Go to $k\tau = 5$ + follow to $N=6$

$$1-X = 0.025 \Rightarrow \boxed{X = 0.975}$$

N = 6
$k_i = 0.08333$
$\tau = 10$

Calc
$k_i\tau = 0.8333$

Calc $Nk_i\tau = 4.9998$

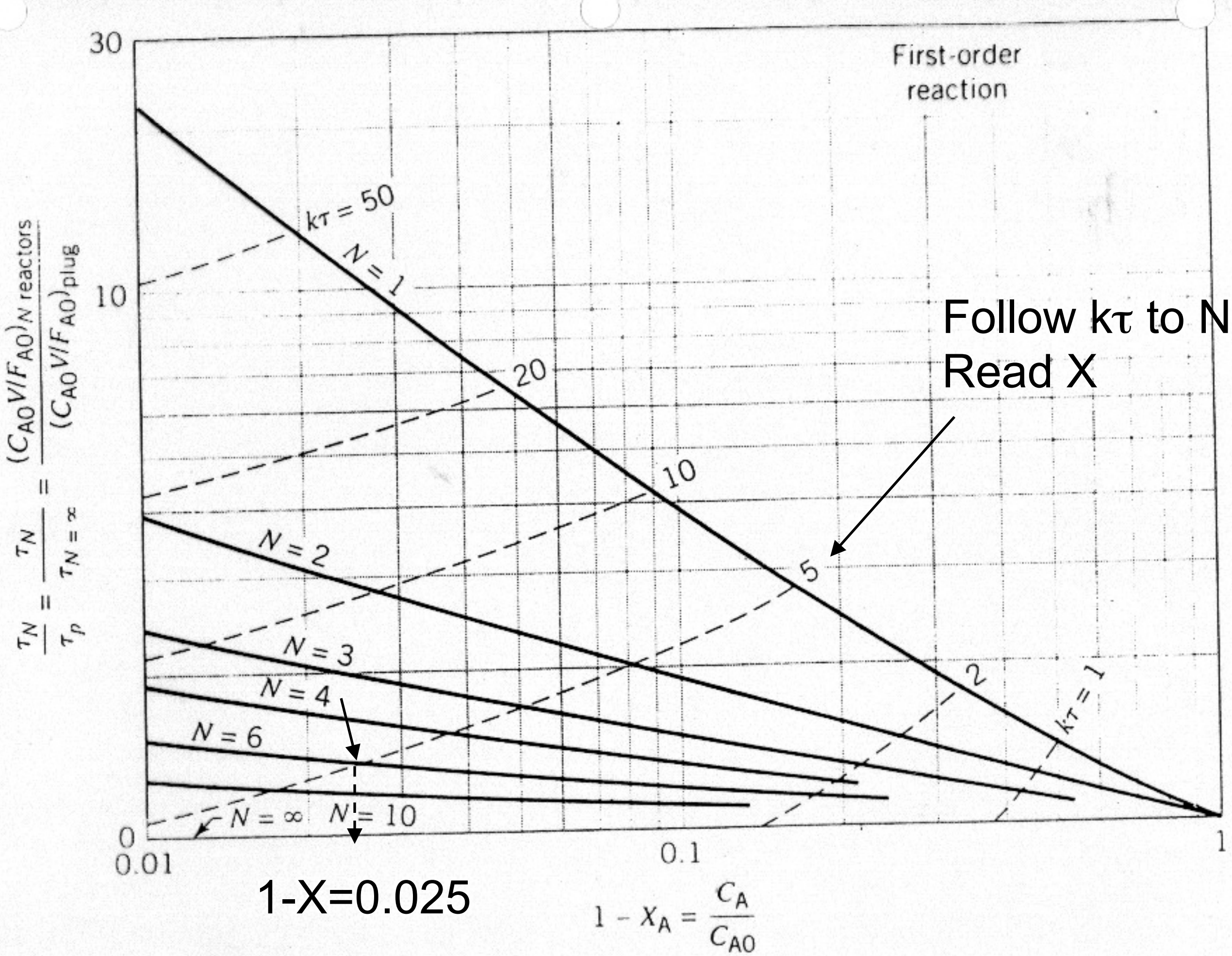



**Figure 6.5** Comparison of performance of a series of $N$ equal-size mixed flow reactors with a plug flow reactor for the first-order reaction

$$A \rightarrow R, \qquad \varepsilon = 0$$

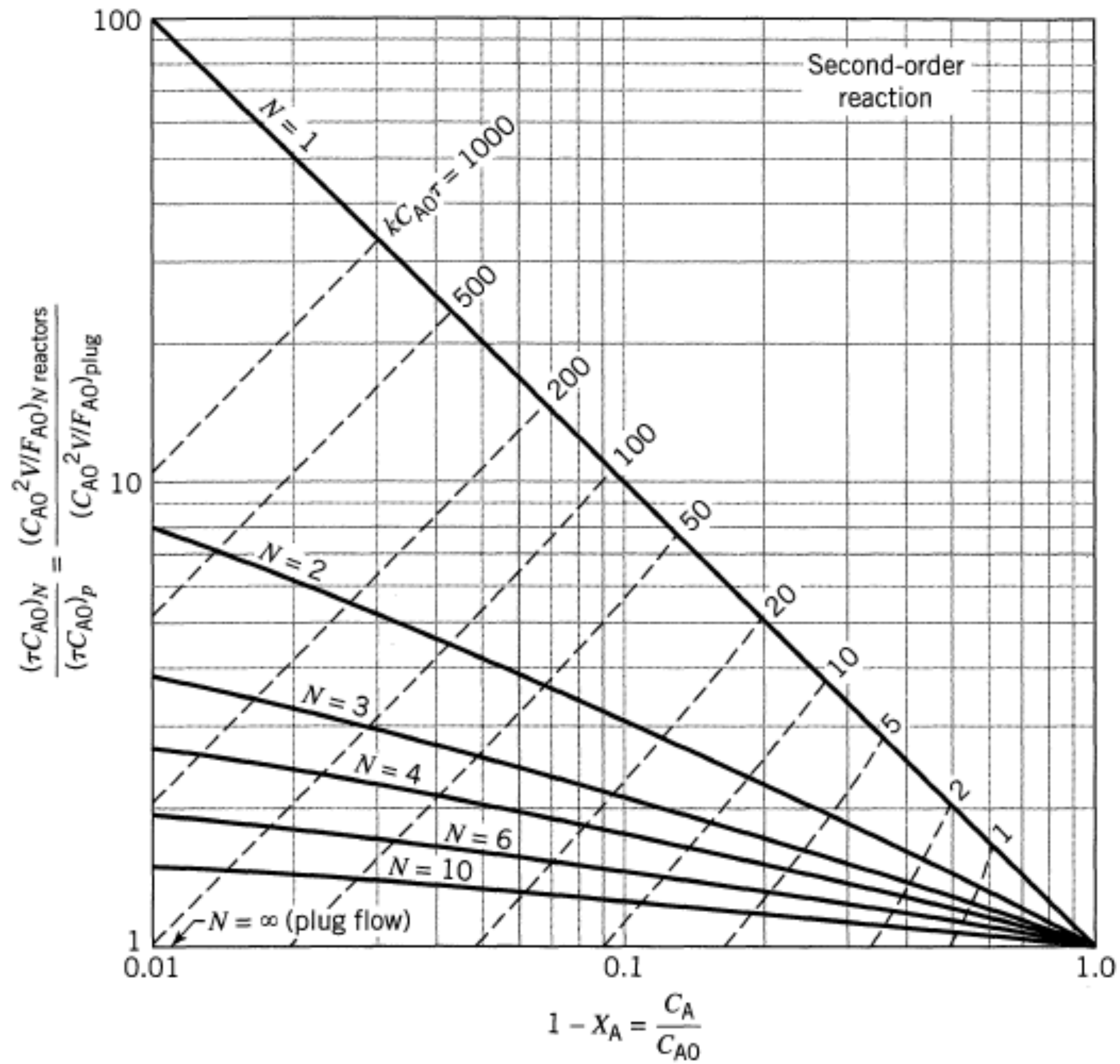



**Figure 6.6** Comparison of performance of a series of $N$ equal-size mixed flow reactors with a plug flow reactor for elementary second-order reactions

$$2A \rightarrow \text{products}$$

$$A + B \rightarrow \text{products}, \qquad C_{A0} = C_{B0}$$

1) Calculate $k\tau_i$ (1st order)

$k\tau_i C_0$ (2nd order)

2) For N reactors in series,

$k\tau = N k\tau_i$ (1st order)

$k\tau C_0 = N k \tau_i C_0$ (2nd order)

3) Follow $k\tau$ or $k\tau C_0$ line to

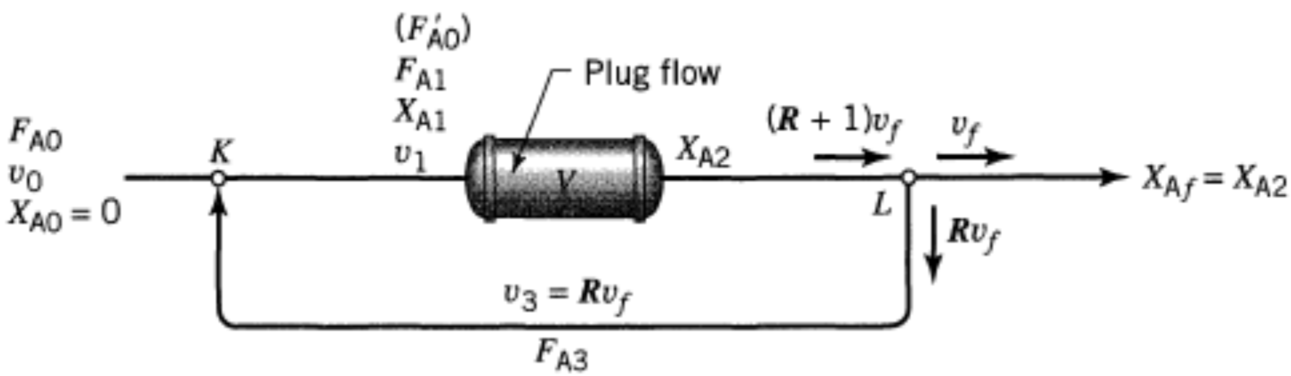
N & get $1-X$ or $\tau_m/\tau_p$

## RECYCLE REACTOR

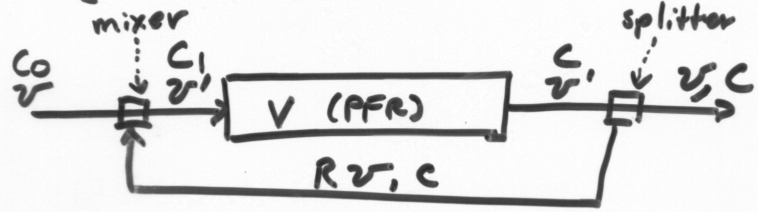
In certain situations it is found to be advantageous to divide the product stream from a plug flow reactor and return a portion of it to the entrance of the reactor. Let the *recycle ratio* $\boldsymbol{R}$ be defined as

$$\boldsymbol{R} = \frac{\text{volume of fluid returned to the reactor entrance}}{\text{volume leaving the system}} \tag{15}$$

This recycle ratio can be made to vary from zero to infinity. Reflection suggests that as the recycle ratio is raised the behavior shifts from plug flow ($\boldsymbol{R} = 0$) to mixed flow ($\boldsymbol{R} = \infty$). Thus, recycling provides a means for obtaining various degrees of backmixing with a plug flow reactor. Let us develop the performance equation for the recycle reactor.



## Recycle reactor (fixed volume)




Splitter mass balance

$$Cv' - Cv - CRv = 0$$

$$v' = v + Rv = (R+1)v$$

mixer m. b.

$$C_0 v + RvC - v'C_1 = 0$$

$$\frac{C_0 v + RvC}{v'} = C_1$$

$$\frac{C_0 v + RvC}{(R+1)v} = C_1 = \frac{C_0 + RC}{R+1}$$

Reactor Perf. EQN

$$v'C - v'(C + dC) - r\,dV = 0$$

$$v'\,dC = r\,dV$$

$$\int_{C_1}^{C} \frac{dC}{r} = \int \frac{dV}{v'} = \int_0^V \frac{dV}{(R+1)v} = \frac{\tau}{R+1}$$

$$\int_{C_1}^{C} \frac{dC}{r} = \frac{\tau}{R+1}$$

e.g. $r = -kC$

$$\int_{C_1}^{C} \frac{dC}{-kC} = \frac{1}{k} \ln \frac{C_1}{C} = \frac{1}{k} \ln \left\{ \frac{\frac{C_0 + RC}{R+1}}{C} \right\}$$

$$= \frac{1}{k} \ln \left[ \frac{C_0 + RC}{(R+1)C} \right] = \frac{\tau}{R+1}$$

$$\ln \left\{ \frac{C_0 + RC}{(R+1)C} \right\} = \frac{k\tau}{R+1} \equiv \gamma$$

$$\frac{C_0 + RC}{(R+1)C} = e^{\gamma}$$

$$C_0 + RC = (R+1)C e^{\gamma}$$

$$C_0 = (R+1)e^{\gamma}C - RC = \left[(R+1)e^{\gamma} - R\right]C$$

$$C_0 = \left[(R+1)e^{\gamma} - R\right] C$$

$$\frac{C}{C_0} = \frac{1}{(R+1)e^{\gamma} - R} = 1 - X$$

$$X = 1 - \frac{1}{(R+1)e^{\gamma} - R}$$

$$= \frac{(R+1)e^{\gamma} - R - 1}{(R+1)e^{\gamma} - R}$$

$$\boxed{X = \frac{(R+1)e^{\gamma} - (R+1)}{(R+1)e^{\gamma} - R}} \qquad \gamma = \frac{k\tau}{R+1}$$

$R = 0 \quad \therefore$ PFR Performance

$$X = \frac{e^{\gamma} - 1}{e^{\gamma}} = 1 - e^{-\gamma}$$

$$= 1 - e^{-k\tau}$$

$R \to \infty \quad \therefore$ MFR

$$X = \frac{k\tau}{1 + k\tau}$$

Note: $R \Uparrow \; X \Downarrow$