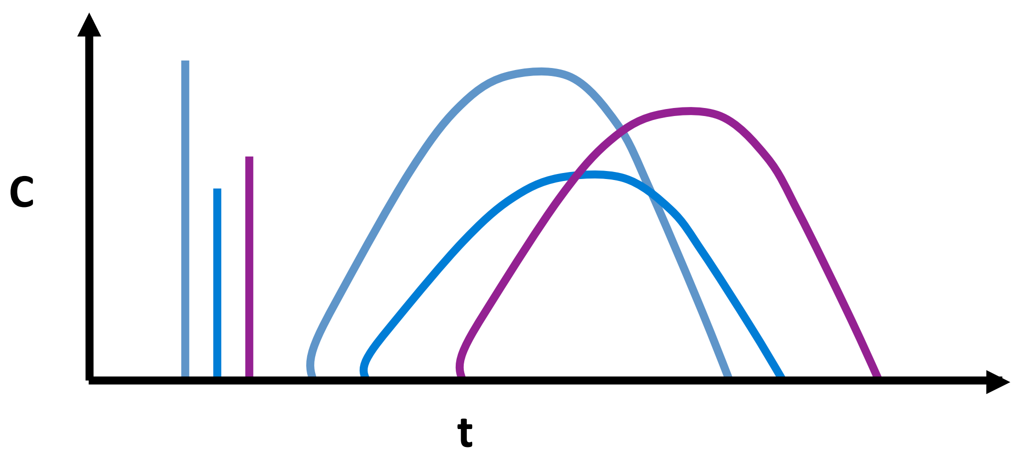
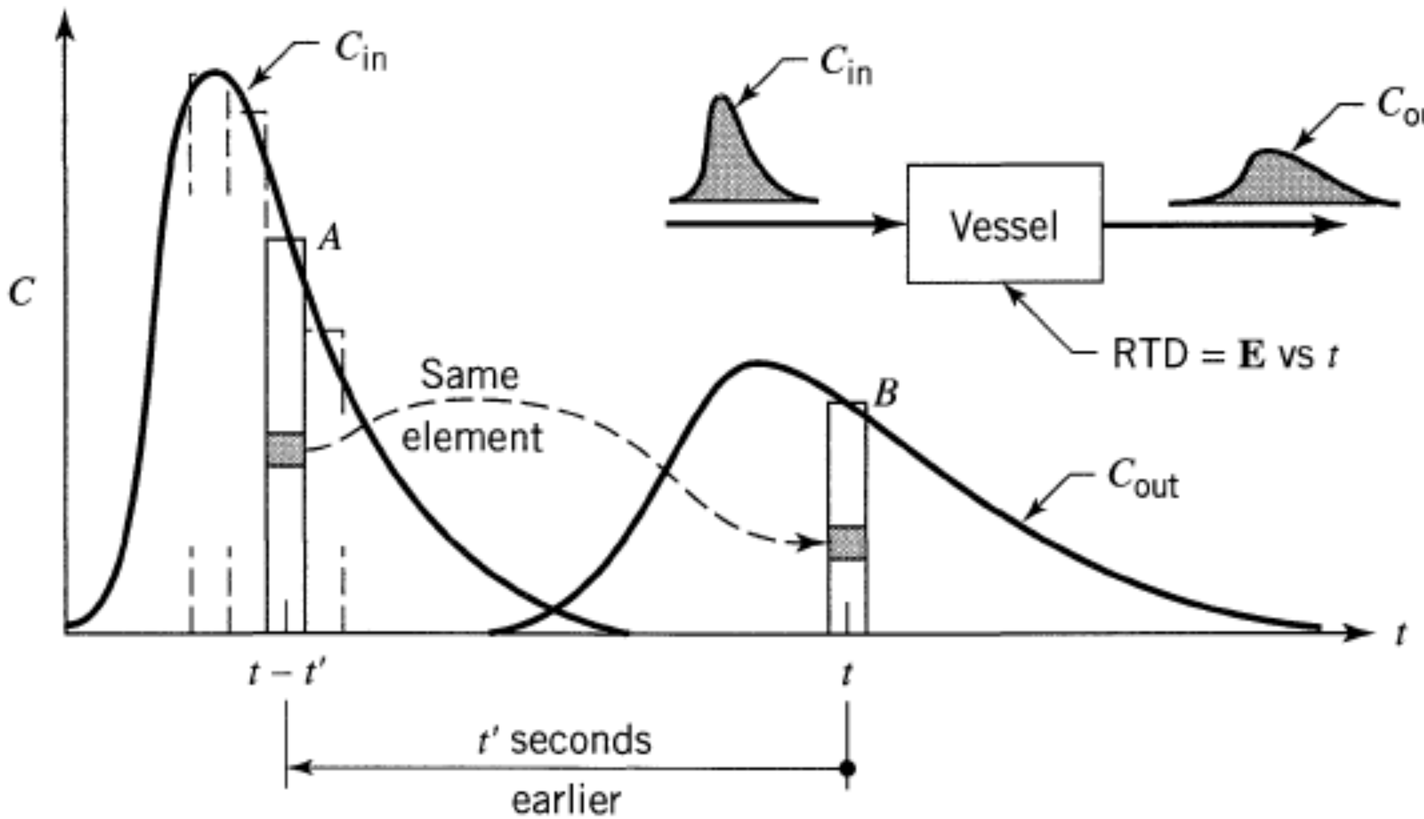
Convolution

- The convolution integral is a means by which the concentration coming out of a reactor can be determined when the inlet concentration and E curve are know.



Convolution (cont.)

- Although the plots of inlet concentration versus time will not look identical to the plot of outlet concentration versus time it will have the same area.
- This is because the area under the curve is representative of the total concentration which cannot change due to conservation of mass.



C_{in}
C_{in}
C_{out}
Vessel
A
C
RTD = **E** vs t
Same
element
B
C_{out}
t
$t - t'$
t
t' seconds
earlier

$$\begin{pmatrix}\text{tracer leaving}\\ \text{in rectangle } B\end{pmatrix} = \begin{pmatrix}\text{all the tracer entering } t' \text{ seconds earlier than } t,\\ \text{and staying for time } t' \text{ in the vessel}\end{pmatrix}$$

We show the tracer which enters t' seconds earlier than t as the narrow rectangle A. In terms of this rectangle the above equation may be written

$$\begin{pmatrix}\text{tracer leaving}\\ \text{in rectangle } B\end{pmatrix} = \sum_{\substack{\text{all rectangles}\\ A \text{ which enter}\\ \text{earlier then}\\ \text{time } t}} \begin{pmatrix}\text{tracer in}\\ \text{rectangle}\\ A\end{pmatrix}\begin{pmatrix}\text{fraction of tracer in } A\\ \text{which stays for about}\\ t' \text{ seconds in the vessel}\end{pmatrix}$$

In symbols and taking limits (shrinking the rectangles) we obtain the desired relationship, which is called the convolution integral

$$C_{\text{out}}(t) = \int_0^t C_{\text{in}}(t - t')\mathbf{E}(t')dt' \qquad \textbf{(10}\boldsymbol{a}\textbf{)}$$

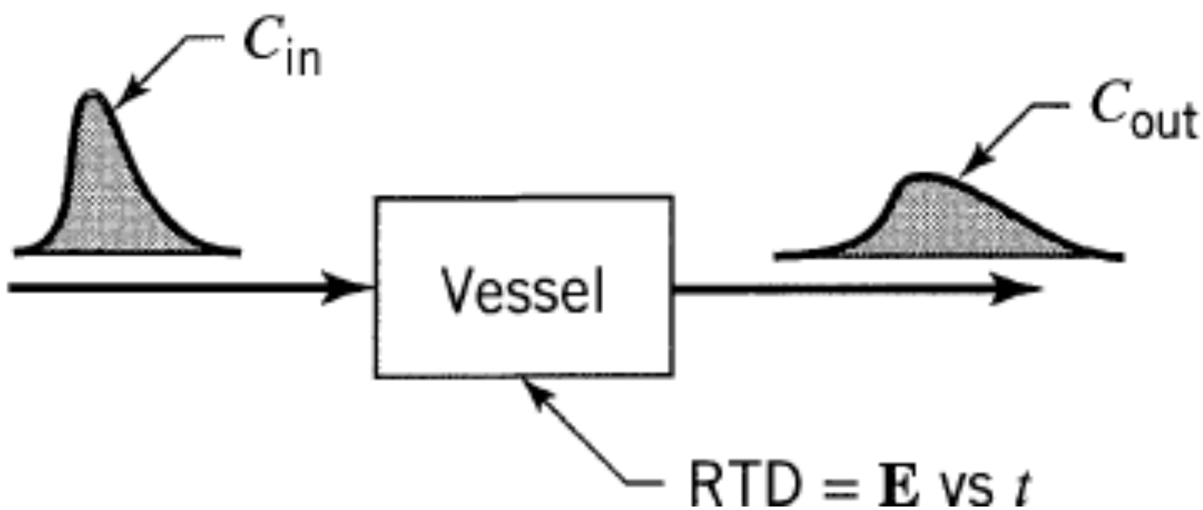
In what can be shown to be equivalent form we also have

$$C_{\text{out}}(t) = \int_0^t C_{\text{in}}(t')\mathbf{E}(t - t')dt' \tag{10b}$$

We say that C_{out} is the *convolution* of $\mathbf{E}$ with C_{in} and we write concisely

$$C_{\text{out}} = \mathbf{E} * C_{\text{in}} \qquad \text{or} \qquad C_{\text{out}} = C_{\text{in}} * \mathbf{E} \tag{10c}$$

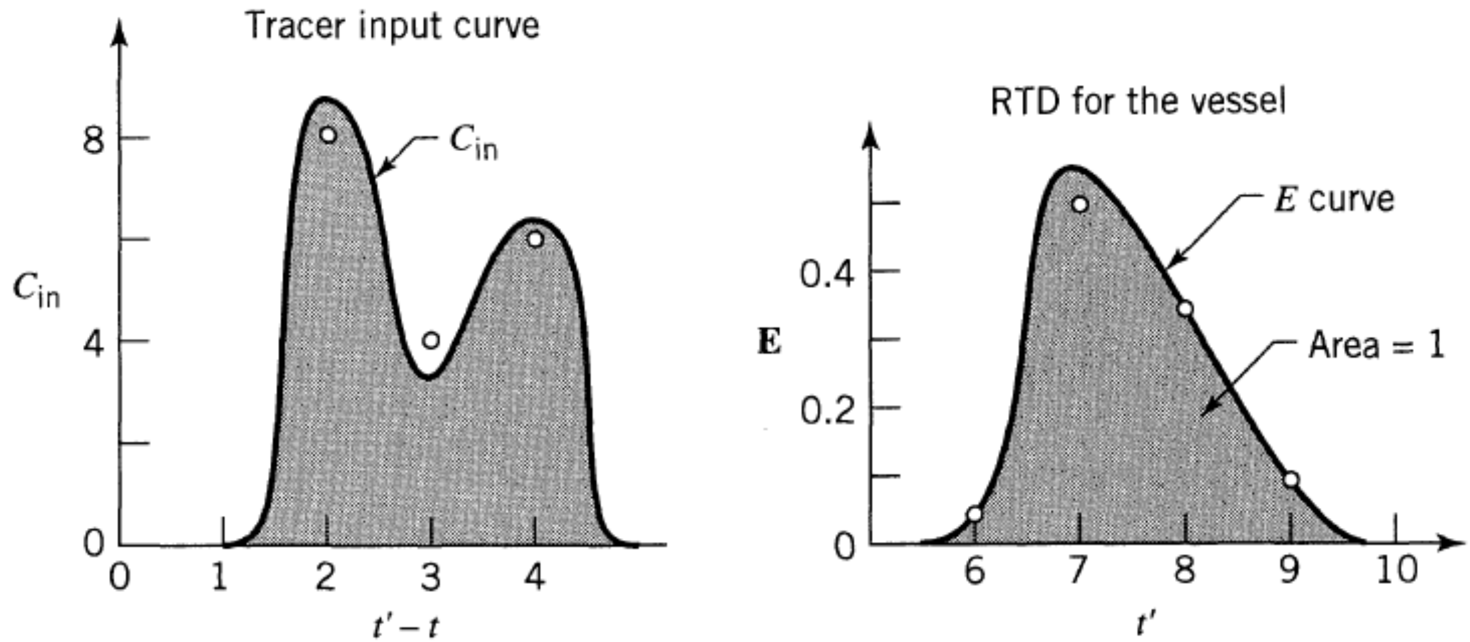
Convolution



How do you hand any input and calculate output curves?

$$C_{out}(t) = \int_0^t C_{in}(t - t')\mathbf{E}(t')dt'$$

Let us illustrate the use of the convolution equation, Eq. 10, with a very simple example in which we want to find C_{out} given C_{in} and the **E** curve for the vessel, as shown in Fig. E11.3*a*.



First of all, take 1 min time slices. The given data are then

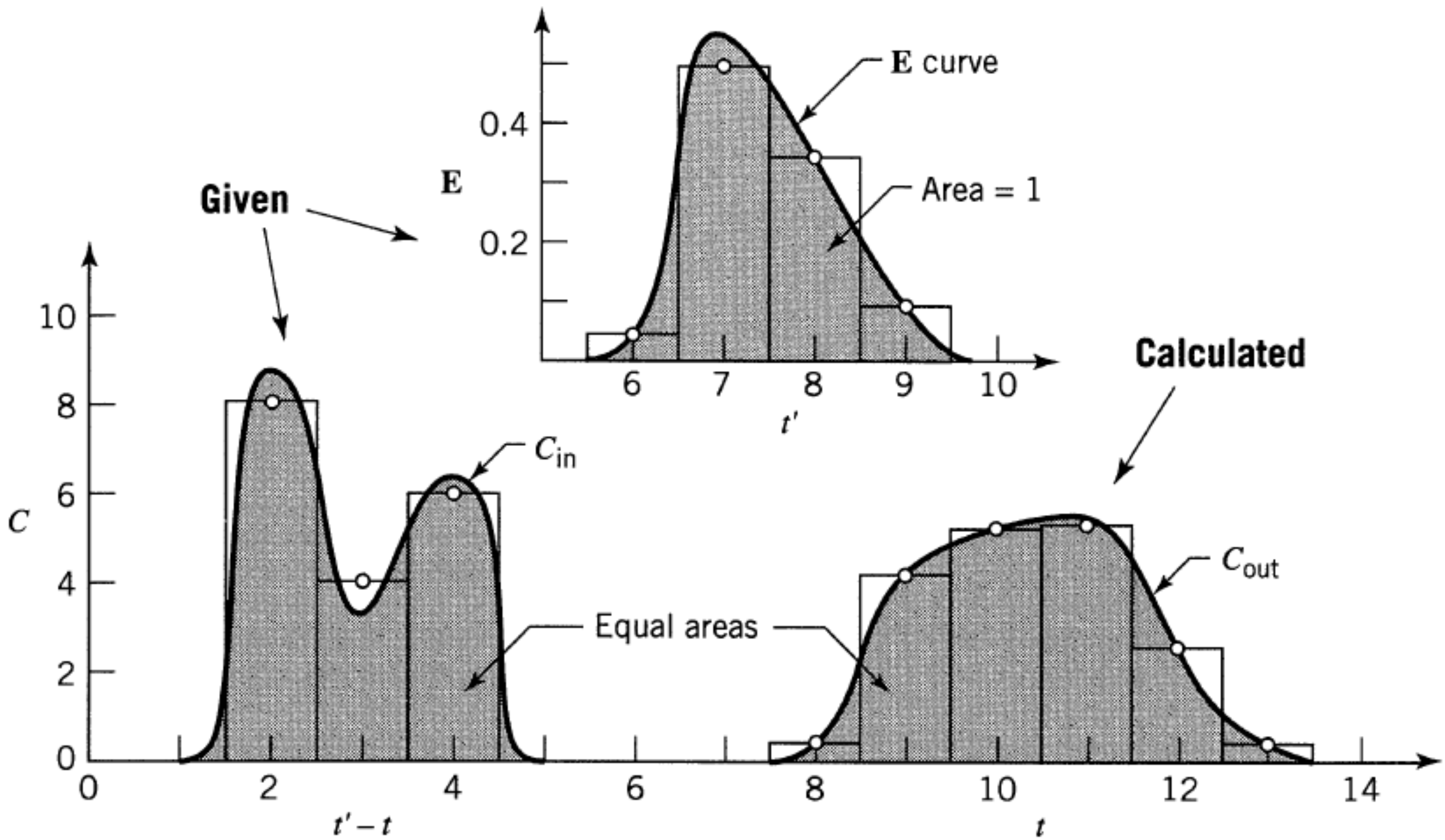
$t' - t$	C_{in}
0	0
1	0
2	8
3	4
4	6
5	0

t'	**E**
5	0
6	0.05
7	0.50
8	0.35
9	0.10
10	0

Note: The area under the **E** curve is unity.

Now the first bit of tracer leaves at 8 min, the last bit at 13 min. Thus, applying the convolution integral, in discrete form, we have

t	C_{out}	
7	0	$= 0$
8	8×0.05	$= 0.4$
9	$8 \times 0.5 + 4 \times 0.05$	$= 4.2$
10	$8 \times 0.35 + 4 \times 0.5 + 6 \times 0.05$	$= 5.1$
11	$8 \times 0.10 + 4 \times 0.35 + 6 \times 0.5$	$= 5.2$
12	$4 \times 0.10 + 6 \times 0.35$	$= 2.5$
13	6×0.10	$= 0.6$
14		$= 0$



Note that the area under the C_{out} curve equals the area under the C_{in} curve.

Input tracer

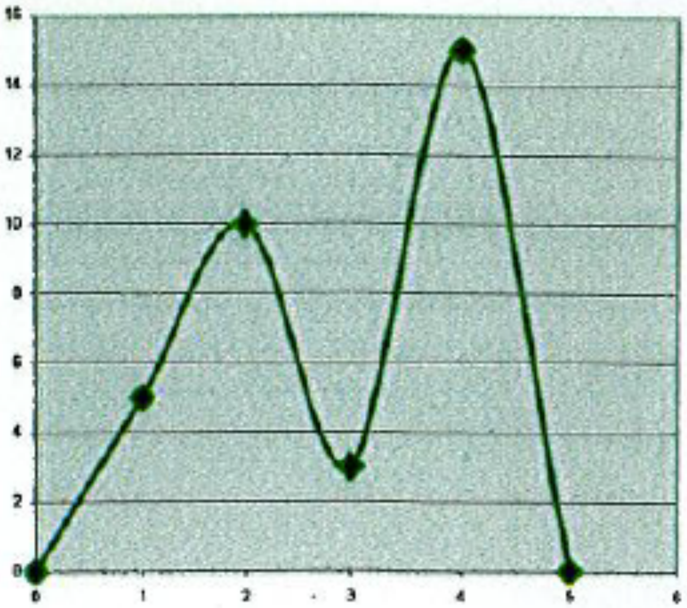
t	Cin(t)	
0	0	
1	5	input1
2	10	input2
3	3	input3
4	15	input4
5	0	

RTD of reactor

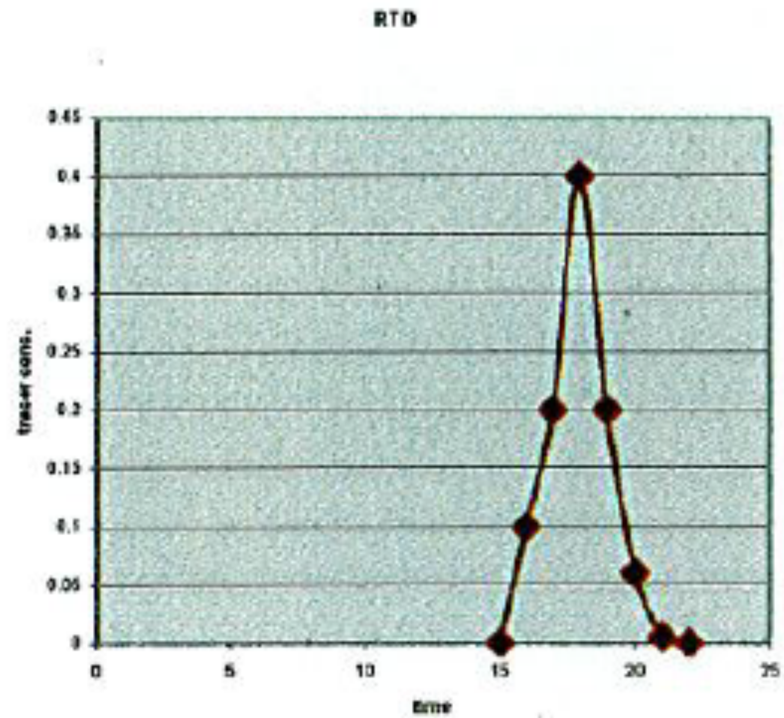
t	E(t)	
15	0	
16	0.1	output1
17	0.2	output2
18	0.4	output3
19	0.2	output4
20	0.06	output5
21	0.005	output6
22	0	

C_{out}?

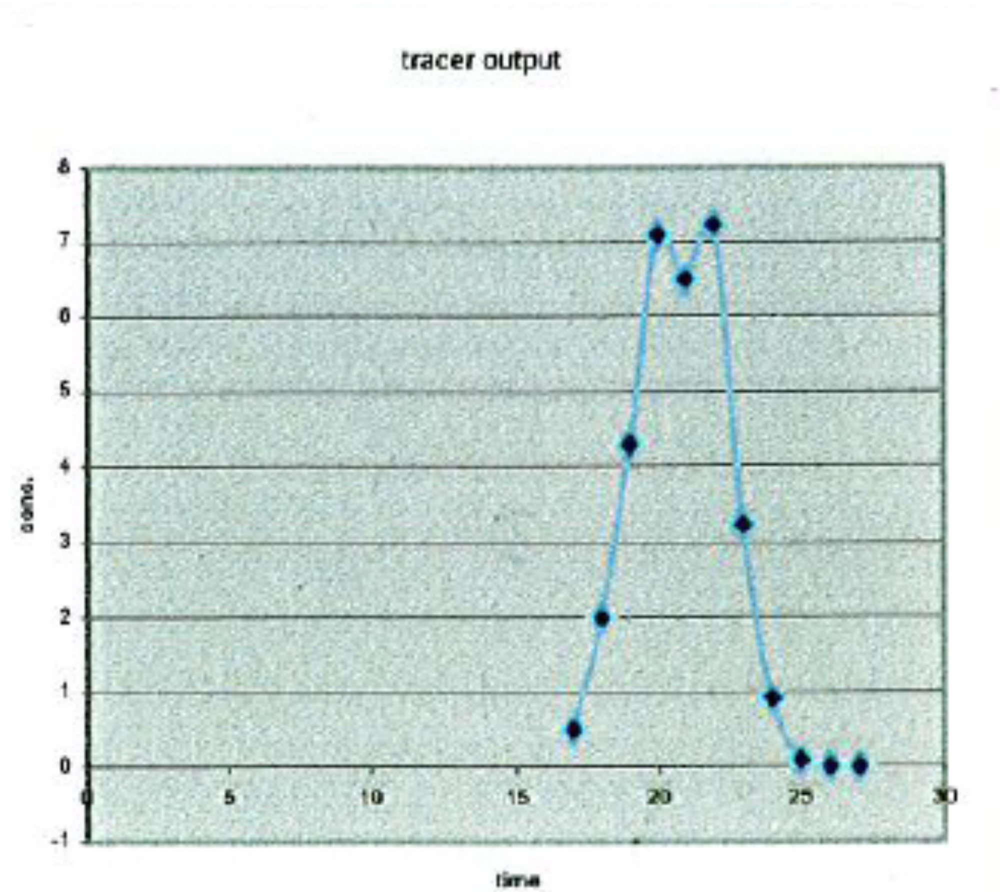
Input



16
14
12
10
8
6
4
2
0
0
1
2
3
4
5
6



RTD
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
tracer conc.
0
5
10
15
20
25
time


tracer output
conc.
time
8
7
6
5
4
3
2
1
0
-1
0
5
10
15
20
25
30