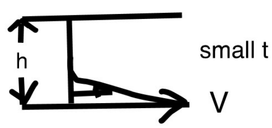
Lecture 1: Viscosity 1/8

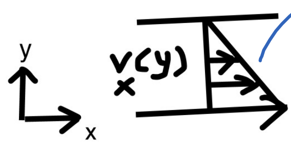
Monday, January 8, 2018 8:49 AM

Viscosity: resistance to flow

Consider a fluid between two parallel plates as shown below. Both plates are stationary. At t=0, the bottom plate is set into motion with a velocity V along x direction. The coordinate perpendicular to the plates is the y coordinate. The fluid next to the bottom plate will also move with the same velocity as that of the plate since there cannot be a relative motion between a solid and fluid adjacent to the solid. This is known as **no slip condition**. The fluid layer in the vicinity of the moving plate has momentum along x direction. It will transfer this momentum to the adjacent layer which in turn will transfer its momentum to its adjacent layer and so on. However, some momentum will be lost in overcoming the resistance to motion by the viscosity of the fluid. Consequently, the velocity of the adjacent layers will progressively decrease thus resulting in a **velocity gradient.** In order to overcome this resistance, one needs to apply a certain force F for the motion of the bottom plate. Also, because of no slip condition, the fluid adjacent to the top stationary plate will be stationary, i.e its velocity will be zero.

Newtonian Fluids Satisfy Newton's Law of Viscosity





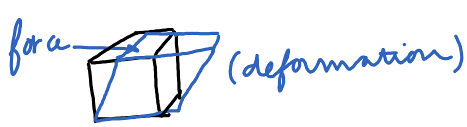
velocity gradient $\left(\frac{dv_x}{dy}\right)$

flow

(force) → rate of deformation – velocity gradient

fluid has same velocity as plate (no slip)

At long time, the system will reach steady state with a linear velocity profile as shown above. The force that needs to be applied to the bottom plate is proportional to the velocity gradient or the rate of deformation of the fluid. It is instructive to compare and contrast the behaviors of fluid and a solid. As you



solid (i.e. rubber)

know, for an elastic solid, the force that is applied is proportional to the deformation of the solid. In other words, at steady state the solid will have a finite deformation. In contrast, the fluid will continue to deform and will reach a steady state at which the rate of deformation will be constant. For a solid, the force is proportional to the deformation, the proportionality constant being the modulus of elasticity. For fluids, however, the force is proportional to the rate of deformation,i.e.

$$\underbrace{\frac{F}{A}}_{\text{shear stress}} = \mu \underbrace{\frac{V}{h}}_{\text{velocity gradient}}$$

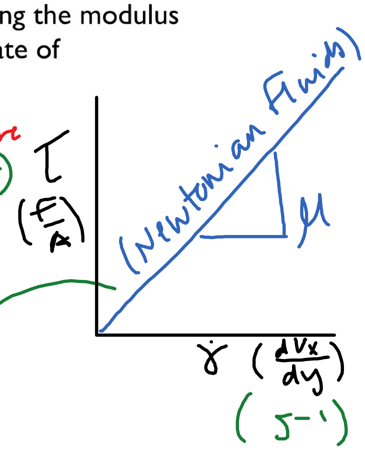
○ = force per unit area (Pa)

○ : viscosity (Pa s) (proportionality constant)

$$\boxed{\tau_{yx} = -\mu \frac{dV_x}{dy}}$$

Newton's Law of Viscosity

follow



The fluids which satisfy the linear relationship between stress and rate of deformation are called **Newtonian fluids**. Many of the commonly encountered fluids are Newtonian. For example, water, air, gases, organic liquids are all Newtonian fluids. In this class, we deal mainly with Newtonian fluids. The shear stress can also be interpreted as the flux of x momentum along y direction "downhill",i.e from region of higher momentum to a region of lower momentum. Units:

$$\tau \sim \frac{N}{m^2} = Pa$$

$$\frac{dV}{dy} = \dot{\gamma} = \frac{m/s}{m} = \frac{1}{s}$$ rate of deformation

$$\mu = \frac{\tau}{\frac{dV}{dy}} = \frac{Pa}{1/s} = Pa \cdot s$$ rate

c.g.s. units

$$\mu \sim \frac{\text{dynes}/\text{cm}^2}{1/\text{s}} = \frac{\text{gm}\cdot\frac{\text{cm}}{\text{s}^2}}{\text{cm}^2}\text{s} = \frac{\text{gm}}{\text{cm}\cdot\text{s}} = \text{Poise}$$

Commonly used c.g.s. unit is centipoise (cp) which is one hundredth of a poise. The viscosity of water at room temperature is 1 cp. Viscosity of gases is much smaller than that for liquids.
Variation of viscosity with temperature: For liquids, viscosity decreases with temperature whereas for gases, viscosity increases with temperature.

solid: deformation

liquid: rate of deformation

shear stress $\propto$ rate of deformation

viscosity $\propto$ shear stress/rate of deformation

most biological fluids do not follow Newton's law of viscosity

→ Non-Newtonian Fluids lecture