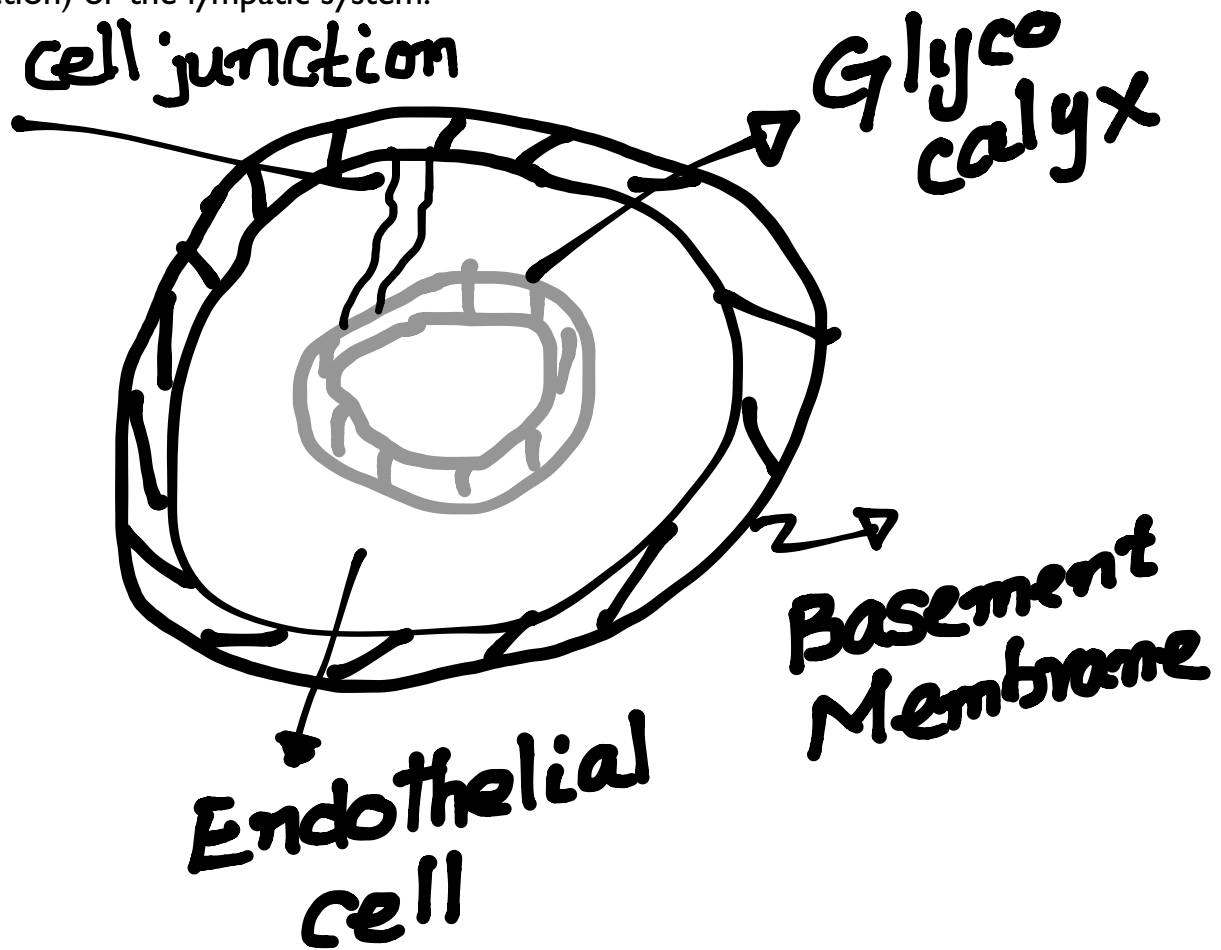


Nutrients and medicines absorbed in the intestine are delivered to individual organs in the body through the systemic blood circulation. In the microcirculation of each organ, the molecules that are involved escape from the lumen of microvessels to enter the interstitial space and then reach individual cells as a result of diffusion and convection. Metabolites and wastes that are released from cells enter the microvessels through either the same pathways (but in the opposite direction) or the lymphatic system.



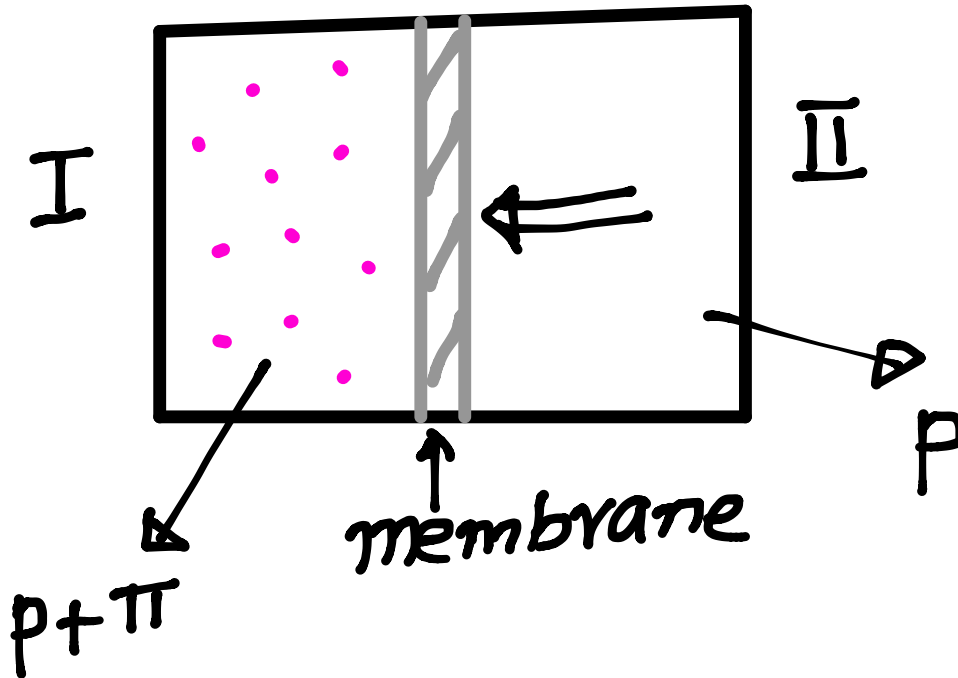
The rate of transport across a microvessel wall are characterized by the hydraulic conductivity and the microvascular permeability coefficient of solutes. The capillary wall consists of three layers - the glycocalyx, endothelium and basement membrane. The glycocalyx acts as a molecular sieve that resists the transport of macromolecules. Endothelial cells are bound together by cell junction that consists of junctional proteins to form a single-cell tube around the glycocalyx. The basement membrane is an electron-dense fiber-matrix layer that contains collagen, proteoglycan, laminin, fibronectin and glycoproteins. Basement membrane allows small molecules to pass through.

To understand transport across microvessel, it is usually modeled as a single porous

membrane of uniform properties even though such an assumption is strictly not valid. Before discussing transport of small molecules across microvessel, we need to understand the concept of osmotic pressure.

Osmotic pressure:

Consider transport of solute molecules across a semi permeable membrane as shown below. At equilibrium, the chemical potential of solvent (water) is equal



in both the chambers. Since the presence of solute molecules decrease the chemical potential of solvent as a result of increase in entropy, initially, the chemical potential of water will be higher in the right chamber. This will result in flow of water from the right to left chamber thereby diluting the solute molecules in the left chamber. In addition, the pressure will increase in the left chamber. The increase in pressure at constant temperature has the effect of increasing the chemical potential of solute. As a result, the system will eventually reach equilibrium. Let p and $p + \pi$ be the pressures in chambers II and I respectively. The chemical potential of solvent in the two chambers are given by,

$$\mu_1^I = \mu_{1,0} + RT \ln x_1^I + \int_p^{p+\pi} \left(\frac{\partial \mu_1}{\partial p} \right)_T dp \quad (1)$$

$$\mu_1^{II} = \mu_{1,0} \quad (2)$$

where $\mu_{1,0}$ is the chemical potential of pure solvent at temperature T. Note that the second term in the right hand side of eq.(1) is the contribution due to the presence of solvent and the third term is contribution due to pressure. Now,

$$\left(\frac{\partial \mu_1}{\partial P}\right)_T = \bar{V}_1 \quad (3)$$

where \bar{V}_1 is the molar volume of the solvent. Since the solvent is incompressible, molar volume is independent of pressure. Also, $x_1 = (1-x_2)$, where x_2 is mole fraction of solute. Therefore,

$$\begin{aligned} \mu_1^I &= \mu_{1,0} + RT \ln(1-x_2) + \bar{V}_1 \pi \\ \mu_1^I &= \mu_{1,0} - RTx_2 + \bar{V}_1 \pi \end{aligned} \quad (4)$$

At equilibrium,

$$\mu_1^I = \mu_1^{II} \quad (5)$$

Therefore,

$$\mu_{1,0} - RTx_2 + \bar{V}_1 \pi = \mu_{1,0}$$

$$\text{or, } \pi = \frac{RTx_2}{\bar{V}_1} \quad (6)$$

The excess pressure in chamber I is known as osmotic pressure. The osmotic pressure increases with solute concentration (mole fraction).

The transport of solute across microvessel due to pressure difference is usually described by the following equation

$$j_v = L_p S \Delta P \quad (7)$$

where j_v is the flux across the membrane, L_p is the hydraulic conductivity, S is the surface area and ΔP is the applied pressure difference across the membrane. This equation is valid only for very low concentration of the solute which does not pass

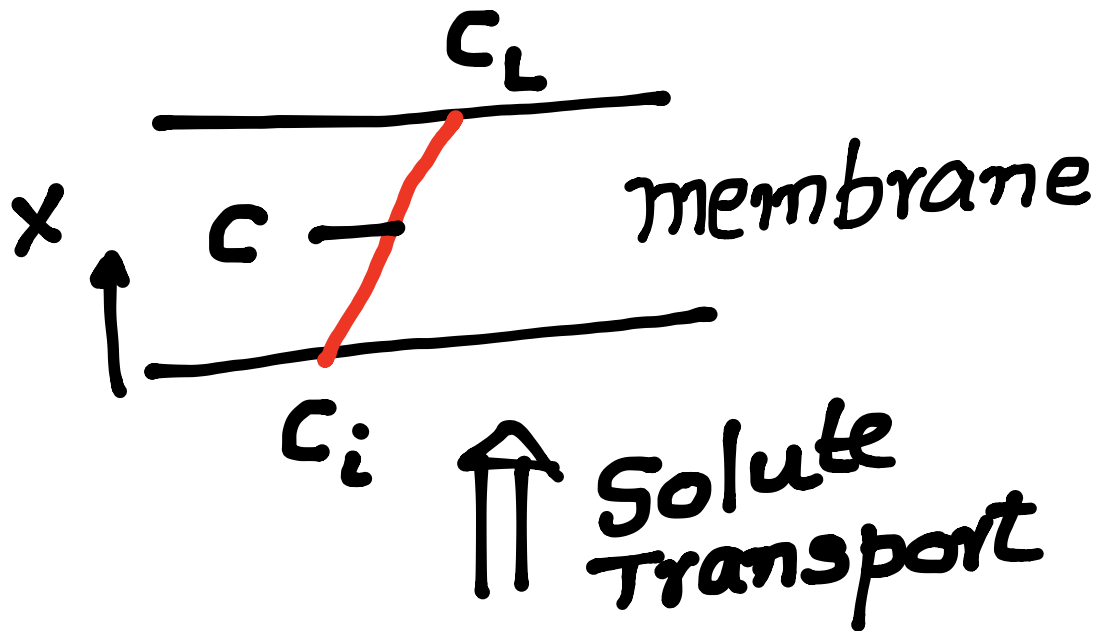
through the membrane. At sufficiently high concentration of solute, the above equation has to be corrected to account for osmotic pressure to give,

$$j_v = L_p S (\Delta p - \pi) \quad (8)$$

Because of distribution of pore sizes in the membrane, all the solute molecules are not excluded by the membrane. To correct for that, we introduce an osmotic reflection coefficient σ_f in the above equation to give,

$$j_v = L_p S (\Delta p - \sigma_f \pi) \quad (9)$$

The value of the reflection coefficient varies between 0 and 1.
Rate of solute transport:



The rate of solute transport is both due to transport due to pressure difference as well as due to diffusion. This can be written as

$$j_s = j_v c - D_{eff} S \frac{dc}{dx} = a_1 \quad (10)$$

where D_{eff} is the effective diffusion coefficient and a_1 is the steady state flux.

$$\circ \circ \frac{dc}{dx} - \frac{j_v}{D_{eff} S} c = -\frac{a_1}{D_{eff} S}$$

$$\frac{d}{dx} \left[c \exp \left\{ -\frac{j_v}{D_{eff} S} x \right\} \right] = \frac{-a_1}{D_{eff} S} \exp \left\{ -\frac{j_v}{D_{eff} S} x \right\}$$

$$x=0 \quad C=C_i$$

$$x=L \quad C=C_L$$

Integrating,

$$\int_0^L \frac{d}{dx} \left[c \exp \left\{ -\frac{j_v}{D_{eff} S} x \right\} \right] dx = \frac{-a_1}{D_{eff} S} \int_0^L \exp \left\{ -\frac{j_v}{D_{eff} S} x \right\} dx$$

Define Peclet # $Pe = \frac{j_v L}{D_{eff} S}$

$$\circ \circ C_L \exp \{-Pe\} - C_i = \frac{-a_1}{j_v} \{1 - \exp(-Pe)\}$$

$$-a_1 = j_v \left[\frac{C_L e^{-Pe} - C_i}{(1 - e^{-Pe})} \right]$$

$$a_1 = j_v \left[\frac{C_i - C_L e^{-Pe}}{(1 - e^{-Pe})} \right] \quad (11)$$