Dialysis:

Kidneys are responsible for the removal of waste products such as urea, for fluid and ion balances and for the regulation of the blood pressure. Filtration of blood plasma and reabsorption of water and ions remove waste products and balance ions and fluids. Kidneys receive blood from the renal arteries, which are branches of the abdominal aorta. Blood from these arteries pass through glomnerular capillaries. The net pressure difference causes fluids and small solutes to flow across the glomerulus, a process known as ultrafiltration.

Between the regions where epithelial cells contact the membranes are narrow gaps through which solute and fluid are transported. The outer surfaces of the epithelial cells are coated with sialic acid, a negatively charged sugar group. These charge groups play a role in the charge selectivity of the membrane. Molecules of radii less than 1.5 nm are litered across the membrane without any restriction. The network structure of these membranes are not uniform. As a result, the membrane allows molecules within a certain size range. Larger molecules are excluded, however. Since the membrane has a sugar coating that is negatively charged, the membrane allows positively charged solutes to pass more easily because of electrostatic attraction. In other words, membrane is charge selective. A variety of diseases of the kidney result in protein in the urine, or proteinurea, a condition caused by a change in filtration due to an alteration in either the charge or the size selectivity of the membrane.

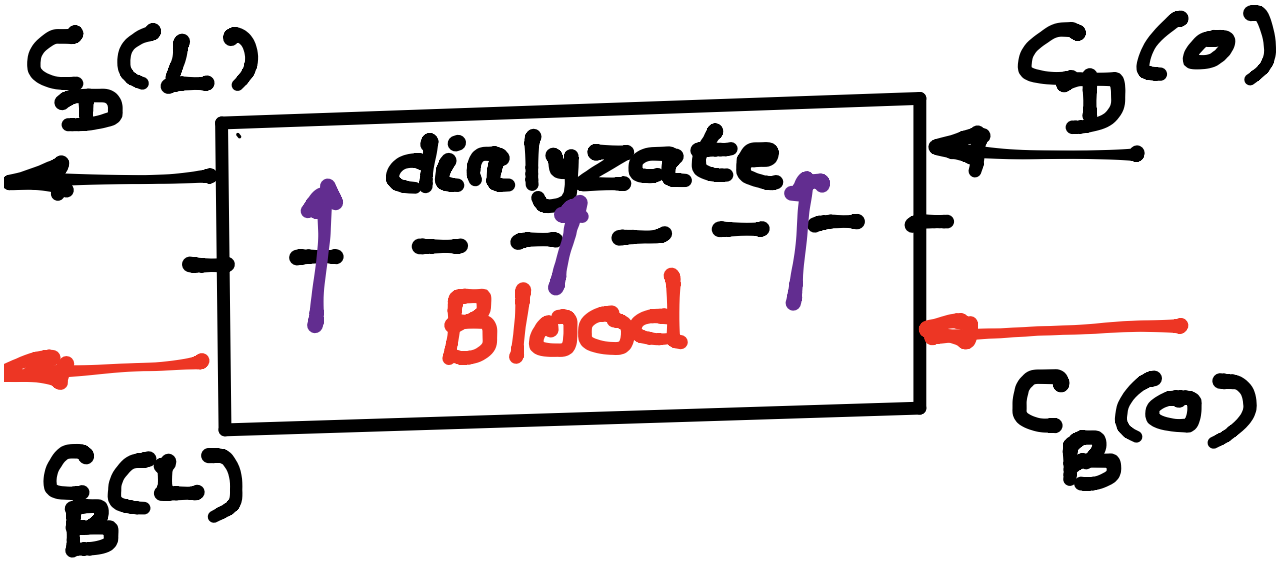
An important application to combat malfunction of kidneys is dialysis which partially recovers kidney function. In dialysis, toxins are removed from blood across a synthetic membrane into a dialysate continuously when blood is pumped from the patient. The purified blood is returned to the blood stream.

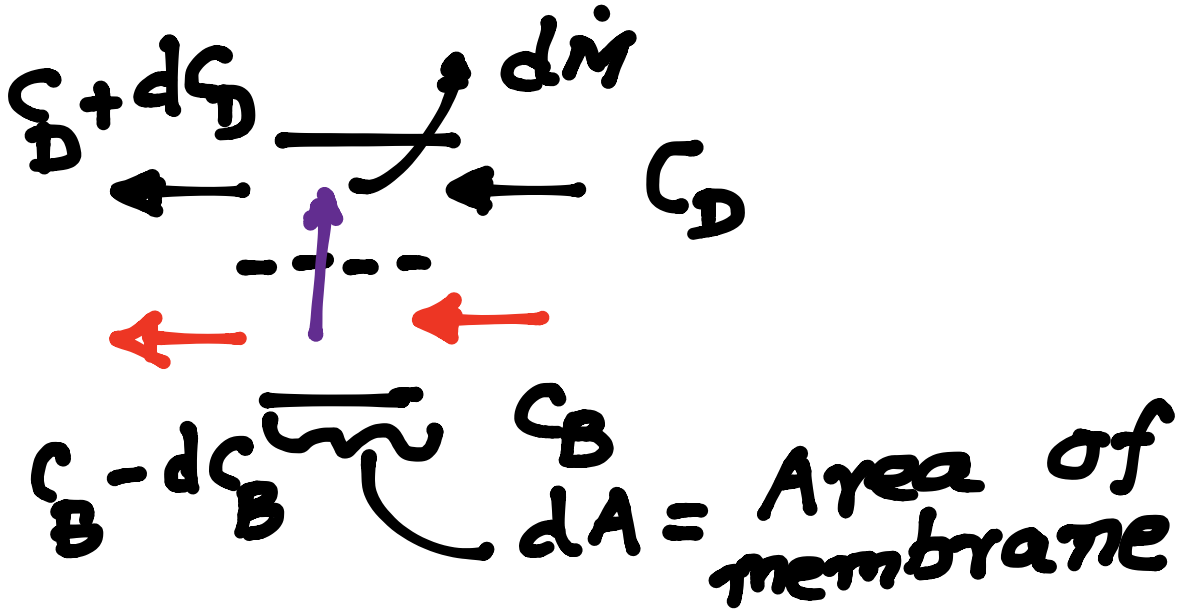
Hemodialysis units are either flat plate or hollow fiber units. The flat-plate design consists of a stach of membrane sheets separated by thin gaps to allow passage of fluids. The blood and dialysate flows in alternate layers. Hollow fiber units contain as many as 11000 fibers with an inner diameter of 200 μm. Blood flows within the hollow fibers and the dialysate flows around the fibers (in the shell). Hollow fiber dialyzers are more compact.

The second major method of removing solutes involves augmenting the concentration difference across membranes with a pressure difference. This procedure, known as hemodiafiltration, produces a convective flow of water from the blood to the dialysate. Because the membrane permits low molecular weight solutes to flow across, they are also transported from blood to dialysate stream. This increases the efficiency of dialysis. The convective flow as a result of applied pressure difference across the membrane can be considered as flow through a porous medium (membrane). Two types of membranes are in widespread use: cellulose and hollow fibers. Cellulose derivatives are used in hemodialyzers. Hollow fiber membranes are asymmetric and have a thin skin (typically 1 μm) on the inner surface and a relatively thick, but porous, foam structure that provides

mechanical support. They are composed of polysulfone or polyamides.

Plate unit:
Plate units can be operated in either co-current or counter current mode. We will consider both of them now.
(i) Co-current flow:





Blood

$$d\dot{M}_B = -Q_B \, dC_B \quad (1)$$

Q_B: Flow Rate of blood

Dialyzate

$$d\dot{M}_D = Q_D \, dC_D \quad (2)$$

Q_D: Flow Rate of dialyzate

$$d\dot{M} = K(C_B - C_D)\, dA \quad (3)$$

$d\dot{M}$: Rate

K: Overall Mass Transfer Coefficient

$$\frac{1}{K} = \frac{1}{R_B} + \frac{1}{P_m} + \frac{1}{R_D} \quad (4)$$

Resistance = Sum of 3 resistances

P_m = Membrane permeability

k_B, k_D = mass transfer coefficients

$$dC_B - dC_D = -d\dot{M}_B\left(\frac{1}{Q_D}+\frac{1}{Q_B}\right) \quad (5)$$

or,

$$dC_B - dC_D = -K\,dA\left(\frac{1}{Q_D}+\frac{1}{Q_B}\right)(C_B - C_D) \quad (6)$$

$$\frac{d(C_B - C_D)}{(C_B - C_D)} = -K\,dA\left(\frac{1}{Q_D}+\frac{1}{Q_B}\right) \quad (7)$$

Integrating,

$$\ln\left\{\frac{C_B(0) - C_D(0)}{C_B(L) - C_D(L)}\right\} = KA\left(\frac{1}{Q_B}+\frac{1}{Q_D}\right) \quad (8)$$

From (1),

$$\dot{M}_B = Q_B\{C_B(0) - C_B(L)\} = -Q_D\{C_D(0) - C_D(L)\} \quad (9)$$

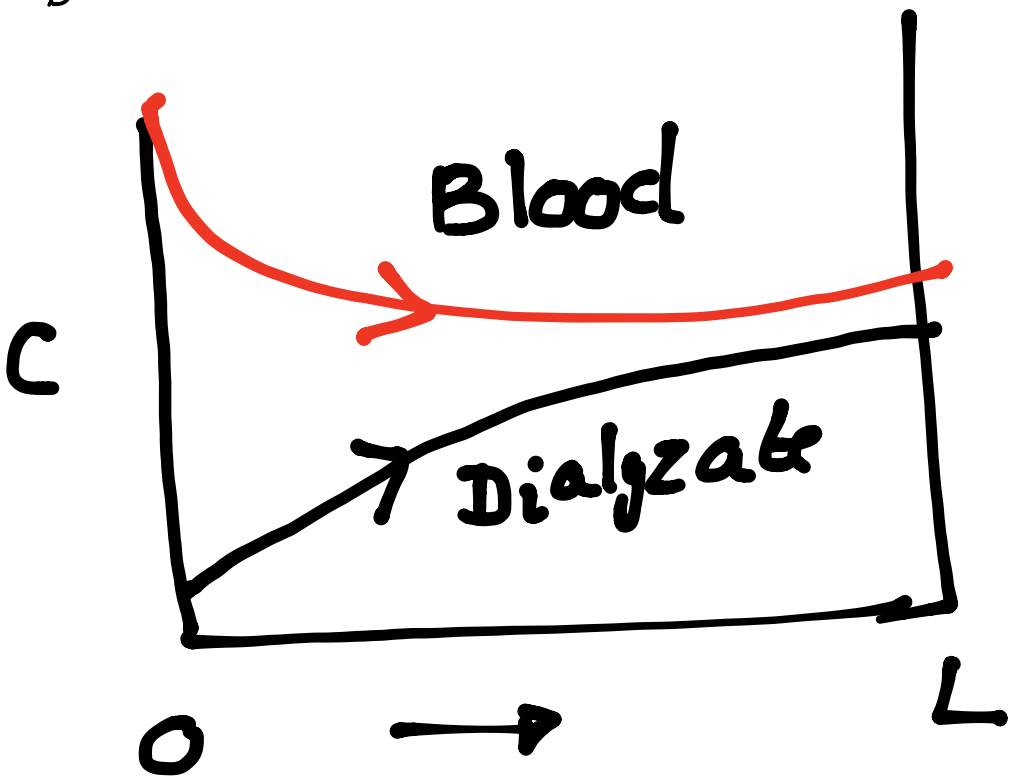
or, $$\frac{1}{Q_B} = \frac{\{C_B(0) - C_B(L)\}}{\dot{M}_B} \quad (10a)$$

$$\frac{1}{Q_D} = -\frac{\{C_D(0) - C_D(L)\}}{\dot{M}_B} \quad (10b)$$

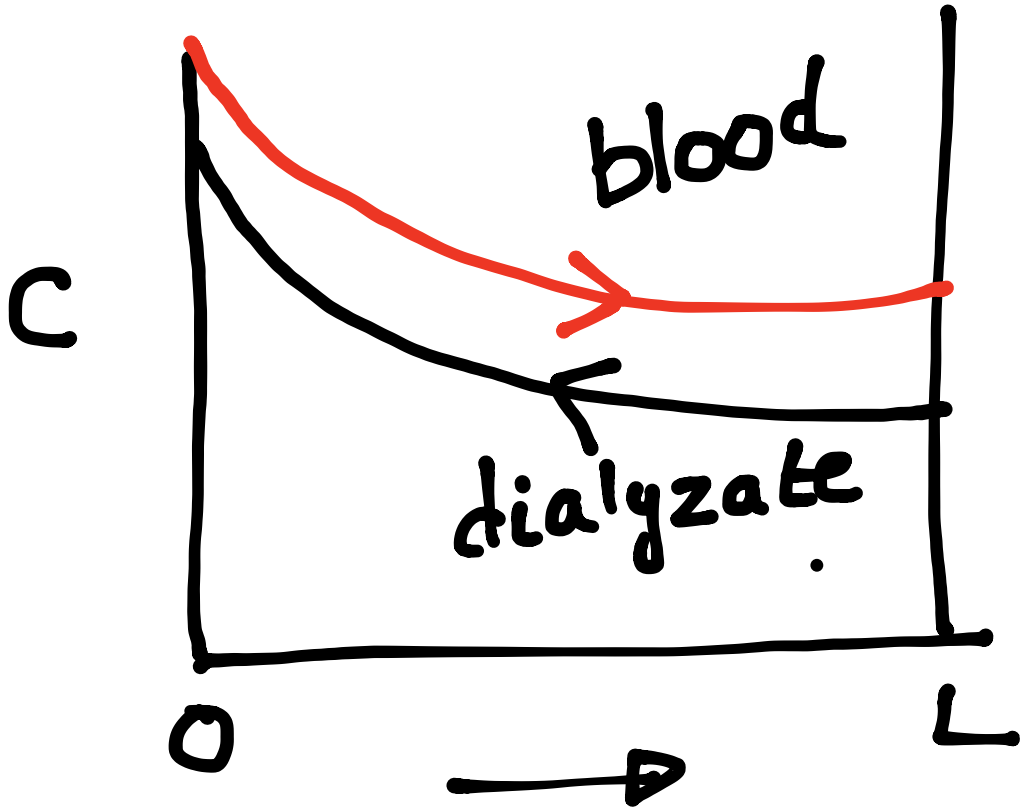
From (8) and (10), we get,

$$\dot{M}_B = KA \frac{\{C_B(0) - C_D(0)\} - \{C_B(L) - C_D(L)\}}{\ln\left\{\frac{C_B(0) - C_D(0)}{C_B(L) - C_D(L)}\right\}}$$

$$\dot{M}_B = KA \, \Delta C_{LM}$$

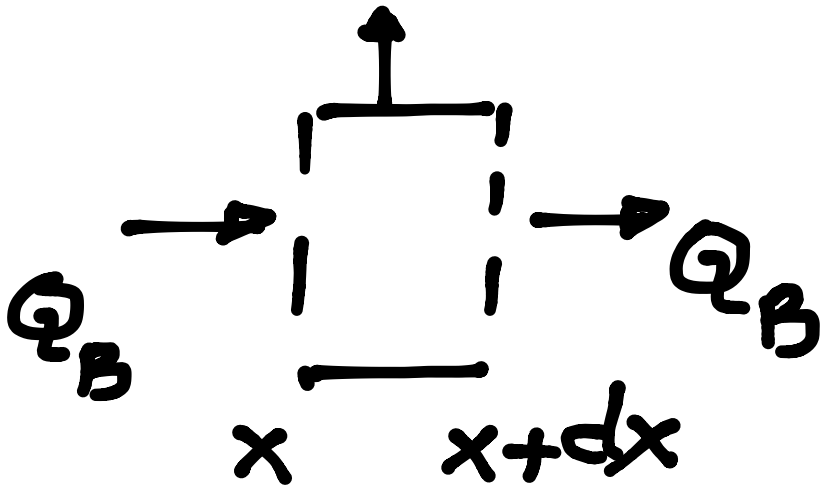


Countercurrent flow:
The final expression for the rate of dialysis is the same as before. However, the concentration profiles within the dialyzer will be different as shown below



Note that the concentration difference (driving force) is more or less constant throught the dialyzer. As a result, the log mean driving force is much higher in countercurrent compared to cocurrent flow. Therefore, countercurrent configuration is preferred to cocurrent. Also, as shown above, the exit concentration of toxins in dialyzate can be greater than the exit concentration in blood. This can never be the case for cocurrent configuration.

Dialysis with ultrafiltration: Consider dialysis in a plate as described above. However, in this case, we also apply external pressure to the blood stream so as to force the blood serum through the membrane. Therefore, the dialysis unit also performs as an ultrafiltration unit. The net transfer of toxin in this case is the sum of diffusion and ultrafiltration. Mass balance over a volume element as before yields,



$$Q_B C_B\Big|_x - Q_B C_B\Big|_{x+dx} - K(C_B - C_D)\, a\, dx - \frac{k_p \Delta P}{\mu \Delta l} C_B = 0$$

$$\frac{Q_B}{a}\left[\frac{C_B|_{x+dx} - C_B|_x}{dx}\right] = -K(C_B - C_D)\, a\, dx - \frac{k_p \Delta P}{\mu \Delta l} C_B$$

$$-\frac{Q_B}{a}\frac{dC_B}{dx} = K(C_B - C_D)\, a\, dx + \frac{k_p \Delta P}{\mu \Delta l} C_B$$

similarly, for dialyzate,

$$\frac{Q_D}{a}\frac{dC_D}{dx} = K(C_B - C_D)\,a\,dx + \frac{k_p \Delta P}{\mu \Delta l} C_B$$

$$\therefore \frac{1}{a}\left[Q_B \frac{dC_B}{dx} + Q_D \frac{dC_D}{dx}\right] = 0$$

$$\frac{dC_D}{dx} = -\frac{Q_B}{Q_D}\frac{dC_B}{dx}$$

Integrating and noting that $C_{D,0} = 0$,

$$C_D(x) = -\frac{Q_B}{Q_D}\left(C_B(x) - C_{B,0}\right)$$

Substituting, we get,

$$K\left[C_B + \frac{Q_B}{Q_D}\left(C_B - C_{B,0}\right)\right] + \frac{k_p \Delta P}{\mu \Delta l} C_B = -\frac{Q_B}{a}\frac{dC_B}{dx}$$

or,

$$\frac{dC_B}{dx} + C_B\left[aK\left(\frac{1}{Q_B} + \frac{1}{Q_D}\right) + \frac{k_p \Delta p}{\mu \Delta l}\frac{a}{Q_B}\right] = \frac{aK}{Q_D} C_{B,0}$$

Define $K_a^* = \frac{Ka}{Q_B}$; $Q_d^* = \frac{Q_d}{Q_B}$

$$P^* = \frac{K_p \Delta P}{\mu \Delta \ell K}\left(1 + \frac{1}{Q_d^*}\right)$$

$$B^* = K_a^*\left(1 + \frac{1}{Q_d^*}\right)(1 + P^*)$$

Therefore,

$$\frac{dC_B}{dx} + B^* C_B = \frac{aK}{Q_D} C_{B,0}$$

Solution is

$$C_B(x) = C_{B,0} e^{-B^* x} + \frac{aK}{Q_D}\frac{C_{B,0}}{B^*}\left[1 - e^{-B^* x}\right]$$

The total rate of toxin removed in the dialyzer is given by

$$\dot{M} = Q_B (C_{B,0} - C_{B,L})$$

$$\dot{M} = Q_B C_{B,0}\left[1 - e^{-B^* L} - \frac{Ka^*}{Q_d^* B^*}(1 - e^{-B^* L})\right]$$

$$\dot{M}^* = \frac{\dot{M}}{Q_B C_{B,0}} = \left[1 - e^{-B^* L} - \frac{Ka^*}{Q_d^* B^*}(1 - e^{-B^* L})\right]$$