1.) $V_z \neq 0 \quad V_r = V_\theta = 0$

$$\frac{\partial V_z}{\partial z} = 0$$

$$\rho\left(\frac{\partial V_z}{\partial t} + \cancel{V_r}\frac{\partial V_z}{\partial r} + \cancel{\frac{V_\theta}{r}}\frac{\partial V_z}{\partial \theta} + \cancel{V_z}\frac{\partial V_z}{\partial z}\right)$$

$$= -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\left(\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right)\right) + \cancel{\frac{1}{r^2}\frac{\partial^2 V_z}{\partial \theta^2}} + \cancel{\frac{\partial^2 V_z}{\partial z^2}}\right] + \cancel{\rho g_z}$$

$$\rho\frac{\partial V_z}{\partial t} = -\frac{\partial p}{\partial z} + \mu\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{dV_z}{dr}\right)$$

$$-\frac{dp}{dz} = a_0 + \sum_{j=1}^{n} a_j e^{i2\pi jft}$$

$$V_z = V_z^0 + \sum_{j=1}^{n} V_j e^{i(2\pi jft)}$$

$$i\rho(2\pi jf)e^{i(2\pi jft)} = a_0 + a_j e^{i(2\pi jft)} + \mu e^{i(2\pi jft)}\frac{1}{r}\frac{d}{dr}\left(r\frac{dV_j}{dr}\right) + \mu\frac{1}{r}\frac{d}{dr}\left(r\frac{dV_z^0}{dr}\right)$$

$$0 = a_0 + \frac{\mu}{r}\frac{d}{dr}\left(r\frac{dV_z^0}{dr}\right)$$

$$i\rho(2\pi jf) = a_j + \frac{\mu}{r}\frac{d}{dr}\left(r\frac{dV_j}{dr}\right) \qquad j = 1, 2, \ldots n.$$

Boundary conditions

$r = R \quad V_z = 0$

$$V_z^o + \sum_{j=1}^{n} V_j e^{i(2\pi j f t)} = 0 \quad \Rightarrow \quad V_z^o = 0 \quad , \; V_j = 0 \quad j = 1, 2 \dots n$$

$$r = 0 \quad \frac{dV_z}{dr} = 0 \quad \frac{dV_z^o}{dr} + \sum_{j=1}^{n} \frac{dV_j}{dr} e^{i(2\pi j f t)} = 0 \quad \frac{dV_z^o}{dr} = 0 \quad \frac{dV_j}{dr} = 0 \; j = 1, 2 \dots n$$

$$\frac{\mu}{r} \frac{d}{dr}\left(r \frac{dV_z^o}{dr}\right) = -a_0$$

$$\frac{d}{dr}\left(r \frac{dV_z^o}{dr}\right) = \frac{-a_0 r}{\mu}$$

$$r \frac{dV_z^o}{dr} = \frac{-a_0 r^2}{2\mu} + C_1$$

$$C_1 = 0$$

$$\frac{dV_z^o}{dr} = -\frac{a_0 r}{2\mu} \quad \Rightarrow \quad V_z^o = -\frac{a_0 r^2}{4\mu} + C_2.$$

$$0 = \frac{-a_0 R^2}{4\mu} + C_2 \quad \Rightarrow \quad C_2 = \frac{a_0 R^2}{4\mu}$$

$$V_z^o = \frac{a_0}{4\mu}\left(R^2 - r^2\right) = \frac{a_0 R^2}{4\mu}\left(1 - \left(\frac{r}{R}\right)^2\right)$$

$$\frac{d^2 V_j}{dr^2} + \frac{1}{r}\frac{dV_j}{dr} - \frac{i 2\pi f j V_z^o}{\mu/\rho} - \frac{a_j}{\mu} = 0$$

$$V_j' = V_j + \frac{a_j}{2\pi f j \rho i}$$

$$\therefore \quad \frac{d^2 V_j'}{dr^2} + \frac{1}{r}\frac{dV_j'}{dr} + i^3 \frac{2\pi f j}{(\mu/\rho)} V_j = 0$$

$$x = i^{3/2}\sqrt{\frac{2\pi f j}{(\mu/\rho)}}\, r .$$

$$\frac{d^2 V_j'}{dx^2} + \frac{1}{x}\frac{dV_j'}{dx} + V_j' = 0 .$$

$$V_j'(x) = C J_0(x)$$

$$V_j = C J_0\left(i^{3/2}\sqrt{\frac{2\pi f j}{(\mu/\rho)}}\, r\right) + \frac{a_j}{2\pi f j \rho i}$$

$r = R \quad V_j(r) = 0$

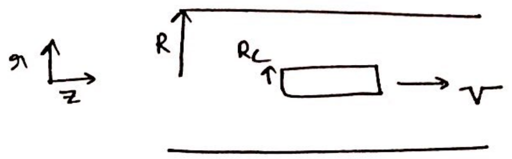
$$\Rightarrow \quad V_j(r) = \frac{a_j}{2\pi f j \rho i}\left(1 - \frac{J_0\left(i^{3/2}\sqrt{\frac{2\pi f j}{(\mu/\rho)}}\, r\right)}{J_0\left(i^{3/2}\sqrt{\frac{2\pi f j}{(\mu/\rho)}}\, R\right)}\right)$$

$$J_0(z) = \frac{1}{\pi}\int_0^{\pi} e^{iz\cos\theta}\, d\theta$$

$$J_0\left(i^{3/2}\sqrt{\frac{2\pi f j}{(\mu/\rho)}}\, r\right) = \frac{1}{\pi}\int_0^{\pi} e^{i^{5/2}\sqrt{\frac{2\pi f j \rho}{\mu}}\, r\cos\theta}\, d\theta .$$

$$V_j(r) = \frac{a_j}{2\pi f j \rho i}\left(1 - \int_0^{\pi} e^{i^{5/2}\sqrt{\frac{2\pi f j \rho}{\mu}}\,(r-R)\cos\theta}\, d\theta\right)$$

2.)



$V_r = V_\theta = 0 \qquad V_z \neq 0$

a.) Equation of continuity.

$$\cancel{\frac{\partial \rho}{\partial t}}^{0} + \frac{1}{r}\frac{\partial}{\partial r}\cancel{(\rho r V_r)}^{0} + \frac{1}{r}\frac{\partial}{\partial \theta}\cancel{(\rho V_\theta)}^{0} + \frac{\partial}{\partial z}(\rho V_z) = 0$$

$$\Rightarrow \frac{\partial V_z}{\partial z} = 0$$

b.) Equation of Motion

$$\rho\left(\cancel{\frac{\partial V_z}{\partial t}}^{0} + V_r \cancel{\frac{\partial V_z}{\partial r}}^{0} + \frac{V_\theta}{r}\cancel{\frac{\partial V_z}{\partial \theta}}^{0} + V_z \cancel{\frac{\partial V_z}{\partial z}}^{0}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) + \frac{1}{r^2}\cancel{\frac{\partial^2 V_z}{\partial \theta^2}}^{0} + \cancel{\frac{\partial^2 V_z}{\partial z^2}}^{0}\right] + \cancel{\rho g_z}^{0}$$

$$0 = -\frac{\partial p}{\partial z} + \mu \frac{1}{r}\frac{d}{dr}\left(r\frac{dV_z}{dr}\right)$$

c.) Boundary Conditions

$r = R \quad V_z = 0$

$r = R_c \quad V_z = V$

d. Velocity profile.

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{dV_z}{dr}\right) = \frac{dp}{dz} = -\frac{\Delta P}{L}$$

$$\frac{d}{dr}\left(r\frac{dV_z}{dr}\right) = \frac{-\Delta P r}{\mu L}$$

$$r\frac{dV_z}{dr} = \frac{-\Delta P r^2}{2\mu L} + C_1$$

$$\frac{dV_z}{dr} = \frac{-\Delta P r}{2\mu L} + \frac{C_1}{r}$$

$$V_z = \frac{-\Delta P r^2}{4\mu L} + C_1 \ln r + C_2$$

B.C

$$0 = \frac{-\Delta P R^2}{4\mu L} + C_1 \ln R + C_2$$

$$V = \frac{-\Delta P R_c^2}{4\mu L} + C_1 \ln R_c + C_2$$

$$-V = \frac{-\Delta P}{4\mu L}(R^2 - R_c^2) + C_1 \ln\left(\frac{R}{R_c}\right)$$

$$C_1 = \left[\frac{\Delta P(R^2 - R_c^2)}{4\mu L} - V\right] \Big/ \ln\left(\frac{R}{R_c}\right)$$

$$C_2 = \frac{\Delta P R^2}{4\mu L} - \left[\frac{\Delta P(R^2 - R_c^2)}{4\mu L} - V\right]\frac{\ln R}{\ln\left(\frac{R}{R_c}\right)}$$

$$V_z = \frac{-\Delta P r^2}{4\mu L} + c_1 \ln r + c_2 .$$

$$\tau_{rz} = -\mu\left(\frac{dV_z}{dr}\right) = \frac{\Delta P r}{2L} - \frac{\mu c_1}{r} .$$

$$\tau_{rz}\Big|_{r=R} = \frac{\Delta P R}{2L} - \frac{\mu}{R}\frac{\left(\frac{\Delta P (R^2 - R_c^2)}{4\mu L} - V\right)}{\ln\left(\frac{R}{R_c}\right)}$$

$$\tau_w^* = \frac{\tau_w}{\Delta P} = \frac{R}{2L} - \frac{1}{4RL}\frac{(R^2 - R_c^2)}{\ln(R/R_c)} + \frac{\mu V}{(\Delta P R)\ln(R/R_c)}$$

$$\tau_w^* = \frac{1}{2L^*} + \frac{1}{4L^* \ln\left(\frac{R_c}{R}\right)}\left(1 - \left(\frac{R_c}{R}\right)^2\right) + \frac{V^*}{\ln\left(\frac{R_c}{R}\right)}$$

$$\frac{\mu V}{\Delta P R} = V^*$$

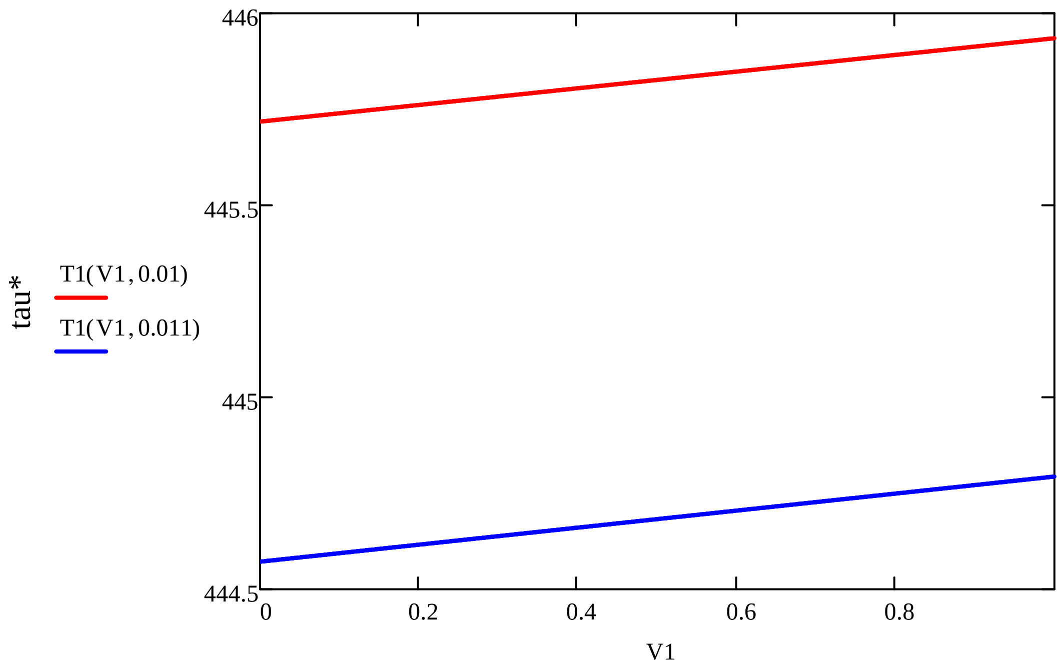
$$\frac{L}{R} = L^* \qquad \frac{R_c}{R} = R_c^*$$

$$\tau_w^* = \frac{1}{2L^*} + \frac{1}{4L^* \ln(R_c^*)}\left(1 - {R_c^*}^2\right) + \frac{V^*}{\ln R_c^*}$$

Scanned by CamScanner

$$T1(V1,R) := \frac{1}{2L} + \frac{1-(R)^2}{4\cdot L\cdot \ln(R)} + \frac{-V1}{\ln(R)}$$

tau* vs V*



V*

Rc*=0.01
Rc*=0.011