ABE 457
Homework 7
Spring 2018

Due: 4/2/2018

1. Consider a capillary of radius R_c and length L carrying blood at a flow rate of Q. The surrounding tissue can be modelled as a concentric cylinder of radius R_t surrounding the capillary. The rate of consumption of oxygen R_{O_2} in the tissue is given by the following Micheles-Menton equation

$$R_{O_2} = \frac{R_{\max} c_{O_2}}{K_M + c_{O_2}}$$

As a result of oxygen transfer from the blood to the surrounding tissue, oxygen concentration in plasma is decreasing along the length of the capillary. You can assume that oxygen concentration in blood at any fixed axial distance z does not vary with radial distance r within the capillary. However, please note that oxygen concentration in the tissue varies both with r and z. This variation is a combined effects of consumption and diffusion of oxygen in tissues.

(a) Write a mass balance for oxygen for a volume element between radii $r, r+dr$ and $z, z+dz$. Take the limit as $dr \to 0$ and $dz \to 0$. Convert the mass balance to partial differential equation for c_{O_2} in terms of r and z.

(b) Identify the boundary conditions for the partial differential equation you derived in part (a).

(c) Simplify the partial differential equation in part (b) by neglecting axial diffusion. Solve for the radial oxygen profile, i.e. $c_{O_2}(r, L_1)$ at a fixed axial distance L_1 for two simplifying cases, namely, (i) case 1: In Micheles-Menton equation, $K_M << c_{O_2}$; case (ii) In Micheles-Menton equation, $K_M >> c_{O_2}$.

(d) Obtain the rate of consumption of oxygen at axial distance L_1 in terms of (yet unknown) oxygen concentration in plasma at L_1.

(e) Write a mass balance for oxygen concentration in blood for a volume element between $z, z+dz$ recognizing that it varies only with axial distance z. Take the limit as $dz \to 0$ and convert the mass balance into an ordinary differential equation in oxygen concentration, Identify the boundary condition . Combine this with the result in part (d) and solve for oxygen profile in the blood.

2. A flat plate dialyzer is being used to extract a toxin from blood. To enhance flow, the unit is connected to the femoral artery in the leg and a new high-permeability membrane is being used. There are 90 channels for blood and 90 channels for dialysate, and the unit is operated in countercurrent exchange. Each channel has a height $2H$, length L, and width W with $W >> 2H$. The dialysate concentration in the inlet is zero. The toxin level in the blood is c_0. The toxic concentration at the dialysate outlet is $0.769c_0$. Following are the operating conditions of the dialyzer

$D = 5 \text{ x } 10^{-6} \text{ cm}^2/\text{s}$
$\text{v}_{blood} = 0.03 \text{ cm/s}$
$\text{v}_{dialysate} = 0.009 \text{ cm/s}$
$c_0 = 3 \text{ x } 10^{-9} \text{ mol/cm}^3$
$H = 0.005 \text{ cm}$
$W = 10 \text{ cm}$
$P_m = 0.005 \text{ cm/s}$

The local mass transfer coefficient k_{local} for flow through the rectangular channel is given by

$$\frac{k_{loc} 4H}{D} = 3.770$$

Where D is the diffusion coefficient of toxin in plasma, v is the average velocity through the channel and L is the length of the channel. You can take the diffusion coefficient to be the same in dialysate stream.

a. Calculate the exit concentrations of toxin in blood for a single pass of both streams.
b. Calculate the overall mass transfer coefficient K .
c. For countercurrent operation, calculate the log mean concentration difference and the length of the dialyzer.
d. Repeat part (c) for concurrent operation. Compare this result with that in part (c) and comment on the difference.
e. Calculate the flow rate of blood and residence time of blood for one pass.
f. Assuming a semi batch operation in which all the blood is collected after it passes through the dialyzer once (single pass) and then recycled the second time (second pass) and so on. What will be the toxin concentration in the blood after 5 passes if the total volume of blood that is dialyzed is 1 liter. Calculate the total time for this operation.

3. Consider the steady state one dimensional transport of a binary electrolyte across an uncharged membrane of thickness L . There is no applied potential across the membrane and transport is assumed to occur only due to diffusion, i.e. convection is unimportant. However, a potential occurs across the membrane due to differences in diffusion coefficient of ions. Let D_+ and D_- be the diffusion coefficients of cation and anion respectively. Let N_+ and N_- be the flux of cation and anion respectively.

a. Write a mass balance for cation and anion across the membrane.
b. Write the expression for N_+ and N_- in terms of concentration gradient and potential gradient across the membrane.
c. Write the equation for electrical neutrality.
d. Obtain the expression for the potential difference across the membrane.
e. For the electrolyte NaCl, $D_{Na+} = 1.334 \text{ x } 10^{-5} m^2/s$, $D_{Cl-} = 2.032 \text{ x } 10^{-5} m^2/s$. Calculate the potential difference at 298 K if $c_L/c_0 = 0.6$ where c_0 and c_L are the electrolyte concentrations at two sides of the membrane. Repeat the calculation for KCl noting that $D_{K+} = 1.957 \text{ x } 10^{-5} m^2/s$.