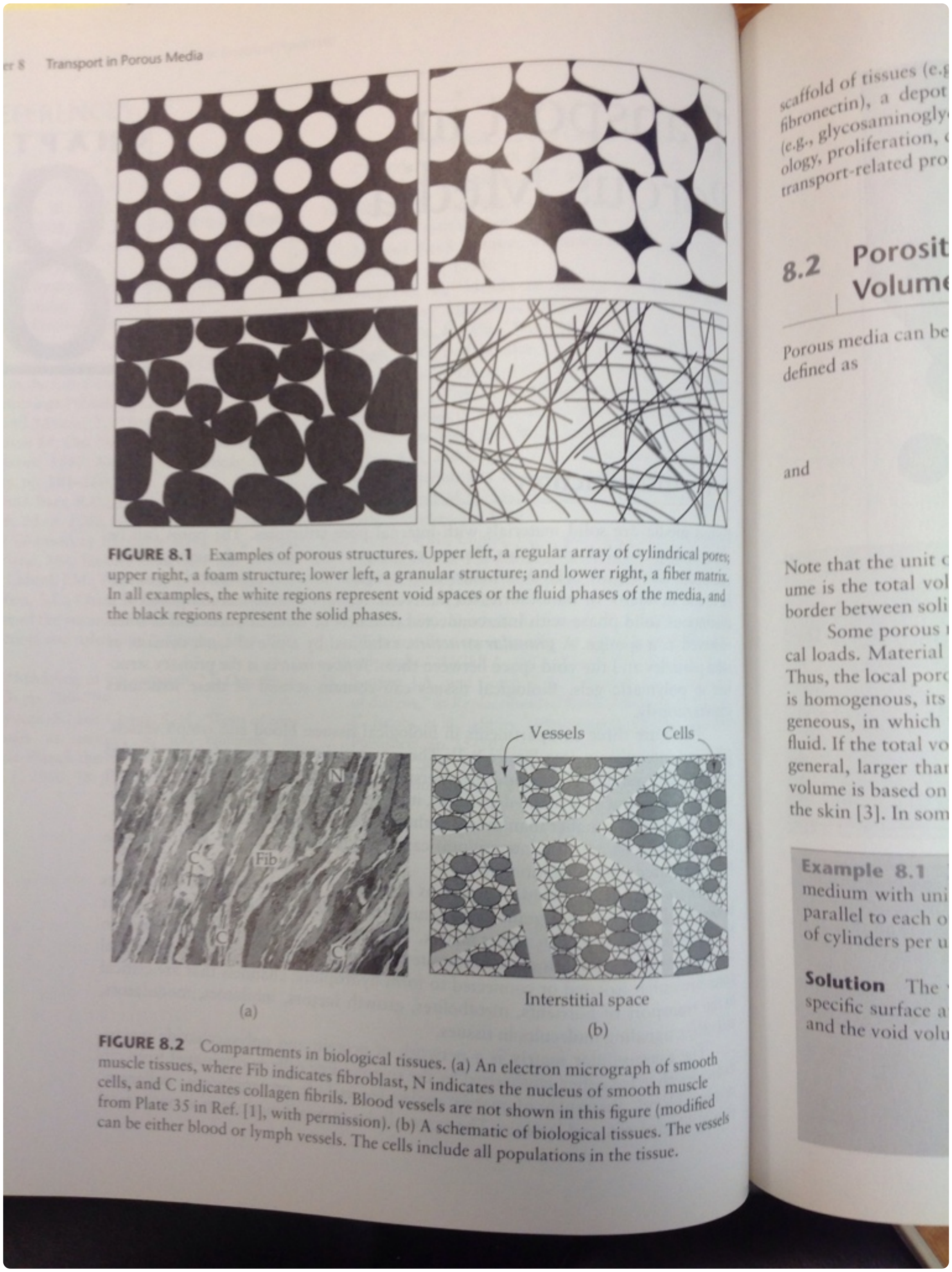
Porous media are solid materials with internal pore structures. Porous structures vary significantly among different media as shown below.



er 8 Transport in Porous Media

FIGURE 8.1 Examples of porous structures. Upper left, a regular array of cylindrical pores; upper right, a foam structure; lower left, a granular structure; and lower right, a fiber matrix. In all examples, the white regions represent void spaces or the fluid phases of the media, and the black regions represent the solid phases.

FIGURE 8.2 Compartments in biological tissues. (a) An electron micrograph of smooth muscle tissues, where Fib indicates fibroblast, N indicates the nucleus of smooth muscle cells, and C indicates collagen fibrils. Blood vessels are not shown in this figure (modified from Plate 35 in Ref. [1], with permission). (b) A schematic of biological tissues. The vessels can be either blood or lymph vessels. The cells include all populations in the tissue.

scaffold of tissues (e.g
fibronectin), a depot
(e.g., glycosaminogly
ology, proliferation,
transport-related pro

8.2 Porosit
Volume

Porous media can be
defined as

and

Note that the unit
ume is the total vol
border between soli
Some porous
cal loads. Material
Thus, the local poro
is homogenous, its
geneous, in which
fluid. If the total vo
general, larger than
volume is based on
the skin [3]. In som

Example 8.1
medium with uni
parallel to each o
of cylinders per u

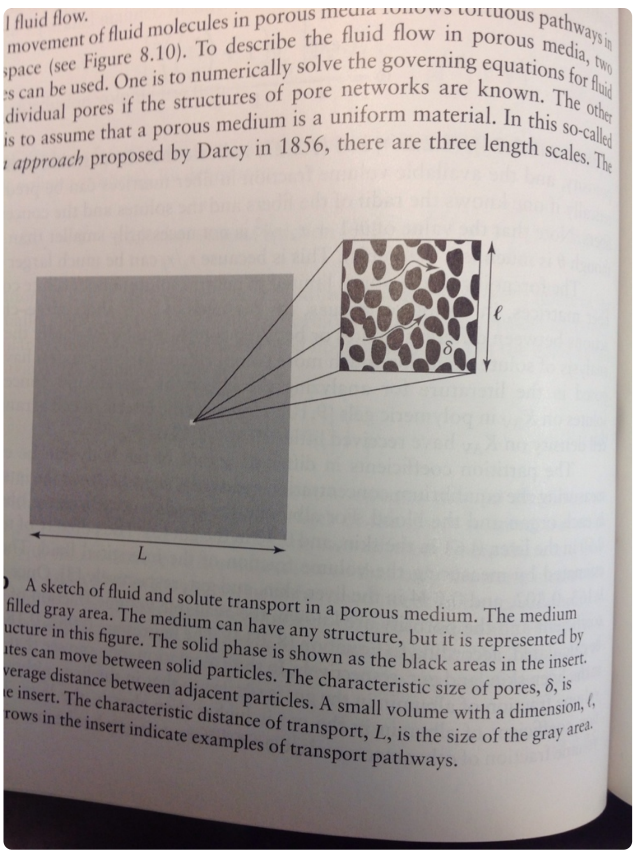
Solution The
specific surface a
and the void volu

A regular array of cylindrical pores can be found in micro or nanofabricated materials. A foam structure is composed of a continuous solid phase with interconnected channels or isolated pores and is often observed as a sponge. A granular structure exhibited by a pile of sand, consists of solid particles and the void space beteen them. A fiber matrix is the primary structure in polymer gels. Biological tissues can contain seeal of these structures simultaneously.

Definitions:

specific surface s = Total interface area/total volume

porosity ε = void volume/total volume



l fluid flow.
movement of fluid molecules in porous media follows tortuous pathways in
space (see Figure 8.10). To describe the fluid flow in porous media, two
es can be used. One is to numerically solve the governing equations for fluid
dividual pores if the structures of pore networks are known. The other
is to assume that a porous medium is a uniform material. In this so-called
approach proposed by Darcy in 1856, there are three length scales. The

A sketch of fluid and solute transport in a porous medium. The medium
filled gray area. The medium can have any structure, but it is represented by
ucture in this figure. The solid phase is shown as the black areas in the insert.
ites can move between solid particles. The characteristic size of pores, δ, is
verage distance between adjacent particles. A small volume with a dimension, ℓ,
e insert. The characteristic distance of transport, L, is the size of the gray area.
rows in the insert indicate examples of transport pathways.

It is an average value and does not provide information on how different pores are connected or on how any pores are available for water and solute transport. Pores can further be subdivided into penetrable pores and isolated pores. As the name suggests, isolated pores are not available for solute transport. As shown in the figure above, the path between points A and B in a porous medium is tortuous depending on the nature of the medium. In other words, the length traversed in going from point A to B is greater than the shortest length. One can define tortuosity T as

l^3 .

$$T = (L/L_{short})^2 \quad (1)$$

where L and Lshort refer to the actual path length and shortest pathlength respectively. One can see that tortuosity is always greater than one.
Not all pores are accessible to solutes. Accessibility will depend on molecular properties of the solute. If the solute molecule is larger than the pore size, it will be inaccessible. The partition coefficient of a solute is the ratio of available volume and the total volume, i.e.

$$\phi = \frac{\text{Available Volume}}{\varepsilon\,(\text{total Volume})} \quad (2)$$

For a circular pore and a flexible random coil solute molecule such as protein, the partition coefficient is given by,

$$\phi_{cylinder} = 4\sum_{n=1}^{\infty}\frac{1}{\alpha_n^2}\exp\left[-\alpha_n^2\lambda_c^2\right] \quad (3)$$

$$\lambda_c = \frac{R_g}{R_{pore}} \quad (4)$$

where Rg, the radius of gyration of protein is given by,

$$R_g^2 = \frac{1}{2N^2}\sum_i\sum_j r_{ij}^2 \quad (5)$$

r_{ij} being the distance between residues i and j and N is the total number of residues. In eq. (3), α_n are the zeros of $J_0(\alpha_n)=0$, J_0 being Bessel function of zero order.

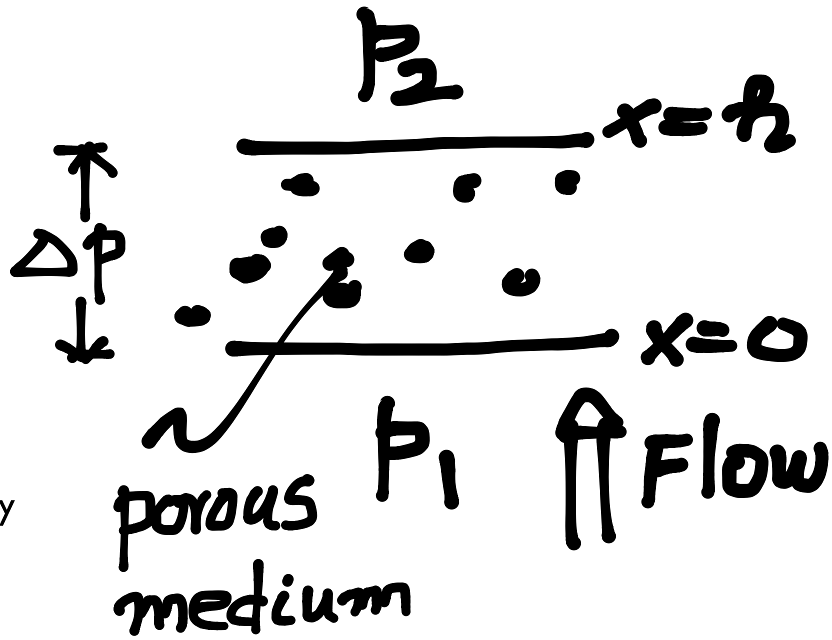
Flow through porous medium:

One can average all the properties over a length scale of l to obtain smooth volume averaged properties. Therefore, one can still consider the porous medium as a continuum over length scale l .

$\underset{\sim}{v} =$ velocity of fluid volume averaged over

$\underset{\sim}{v}_f =$ velocity of fluid volume averaged through pores.

$$\underset{\sim}{v} = \varepsilon \underset{\sim}{v}_f \quad (6)$$



Continuity:

$$\nabla \cdot \underset{\sim}{v} = 0 \quad (7)$$

Darcy's law:

$$\underset{\sim}{v} = -K \nabla p \quad (8)$$

$K =$ hydraulic conductivity

Substituting in equation of continuity, one obtains,

$$\nabla \cdot \{-K \nabla p\} = 0 \quad (9)$$

For constant K (homogeneous medium), one obtains,

$$\nabla \cdot \nabla p = 0$$

$$\boxed{\nabla^2 p = 0} \quad (10) \quad \text{Laplace Equation}$$

One dimensional flow in rectangular coordinates:

$$\frac{dp}{dx^2} = 0$$

$$\frac{dp}{dx} = c_1 \qquad p = c_1 x + c_2$$

$$x = 0 \quad p = p_1$$

$$x = h \quad p = p_2$$

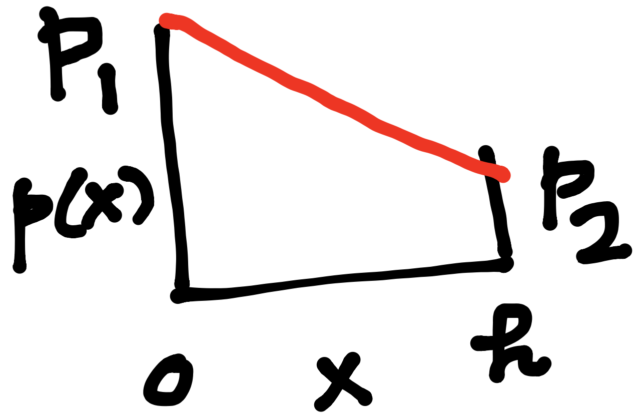
$$\therefore\ p_2 = c_1 h + p_1$$

$$c_1 = (p_2 - p_1)/h$$

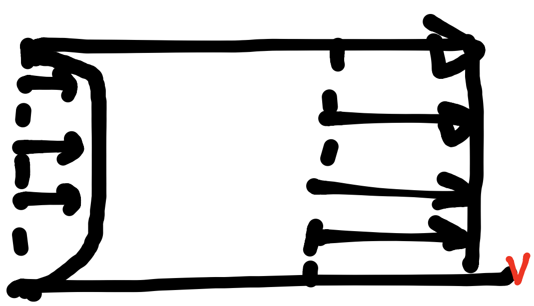
$$\therefore\ p = p_1 - (p_1 - p_2)\left(\frac{x}{h}\right)$$

$$V_x = -K\frac{dp}{dx} = -K c_1$$

$$V_x = K\frac{(p_1 - p_2)}{h} \quad \text{Flux}$$



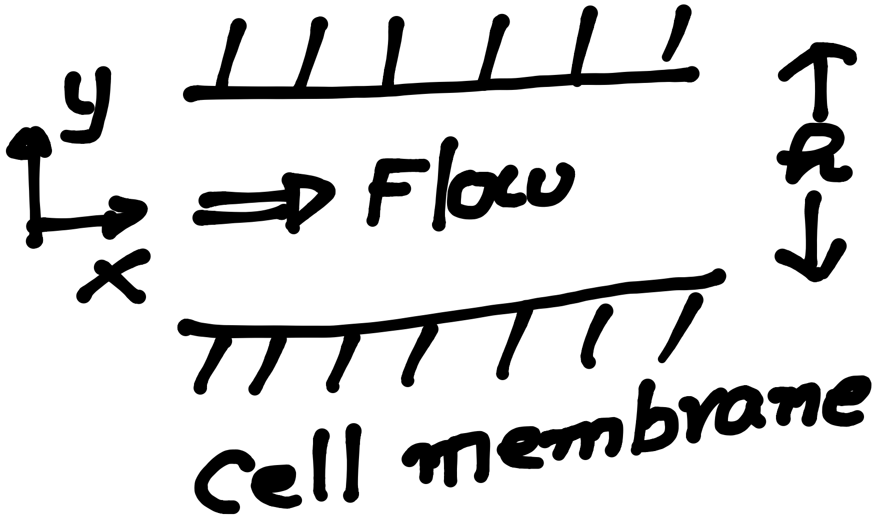
Note that is constant. Therefore, $V_{fx} = \frac{V_x}{\varepsilon}$ is also a constant. In other words, the resistance to flow near the wall is much greater than the viscous resistance. Therefore, if you plot the velocity profile within a pore, you will obtain a flat velocity profile with a steep gradient near the wall. Therefore, Darcy's law does NOT satisfy NO SLIP condition. Darcy's law is valid only for dense porous materials or for low values of hydraulic conductivity K.



Darcy's law is applicable only when the porous medium is dense, I.e. The permeability is low. When the resistance of fluid flow is not large, the approximation of no slip at the wall breaks down. Consequently, one cannot use Darcy's law. The equation of motion has to be modified as

$$\underbrace{\mu \nabla^2 \underset{\sim}{V}}_{\text{viscous resistance}} - \frac{1}{K}\underset{\sim}{V} - \nabla p = 0 \qquad (11)$$

This is known as Brinkman's equation.
Let us examine interstitial flow through cells. The two cell membranes are separated by h (see schematic below).



$$\frac{\partial v_x}{\partial x} = 0 \quad (11)$$

$$\frac{\partial p}{\partial y} = 0 \quad (12)$$

Lubrication Approximation

$$\mu \frac{\partial^2 v_x}{\partial y^2} - \frac{\mu}{k} v_x - \frac{\partial p}{\partial x} = 0 \quad (13)$$

or,

$$\underbrace{\mu \frac{\partial^2 v_x}{\partial y^2} - \frac{\mu}{k} v_x}_{\text{function of } y} = \underbrace{\frac{\partial p}{\partial x}}_{\text{function of } x} = \underset{B}{\text{Constant}}$$

$$\therefore \quad \frac{d^2 V_x}{dy^2} - \frac{1}{k} V_x = \frac{B}{\mu} \quad (4)$$

The solution to the above equation is

$$V_x = C_1 \sinh\left(\frac{y}{\sqrt{k}}\right) + C_2 \cosh\left(\frac{y}{\sqrt{k}}\right) - \frac{k}{\mu} B \quad (5)$$

B.C.:- $y = 0 \qquad \frac{dV_x}{dy} = 0 \quad (6)$

dy

$$y = h \qquad V_x = 0 \qquad (7)$$

$$V_x = -\frac{k}{\mu} B \left[1 - \frac{\cosh\left(y/\sqrt{k}\right)}{\cosh\left(h/2\sqrt{k}\right)} \right] \qquad (8)$$

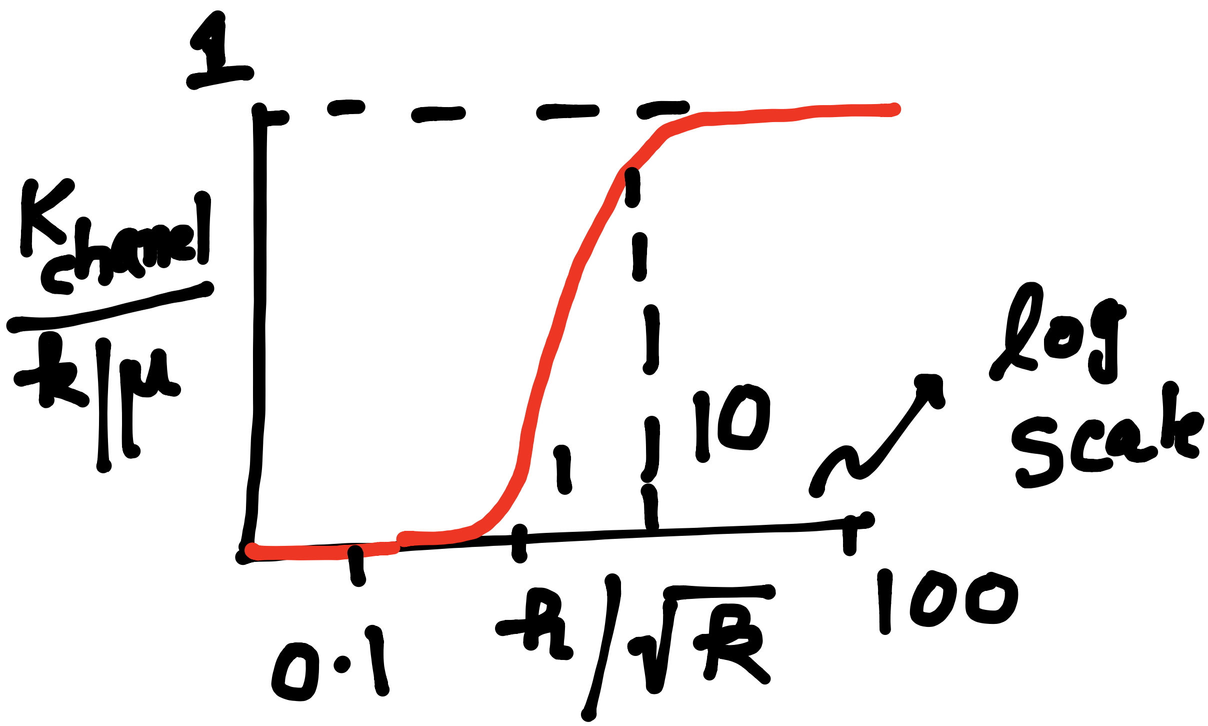
One can evaluate the volumetric flow rate per unit cross sectional area from

$$q = \frac{1}{h} \int_{-h/2}^{h/2} V_x \, dy$$

$$= -\frac{k}{\mu} B \left[1 - \frac{2\sqrt{k}}{h} \tanh\left(\frac{h}{2\sqrt{k}} \right) \right]$$

The effective hydraulic conductivity is defined as the ratio of fluid flux to pressure gradient and is given by

$$K_{channel} = \frac{k}{\mu}\left[1 - \frac{2\sqrt{k}}{h}\tanh\left(\frac{h}{2\sqrt{k}}\right)\right] \tag{9}$$



If $\frac{R}{\sqrt{k}} > 10$, $K_{channel} \simeq \frac{k}{\mu}$

Darcy's Law Valid

Solute transport through porous media:

For neutral solutes, diffusion is governed by Fick's law, i.e diffusive flux is proportional to concentration gradient of solute.

1. Within the pore, the diffusion of solute is retarded because of interaction with the pore walls. One has to use an effective diffusion coefficient.
2. The convective velocity of solute is less than that of solvent.
3. In the continuum approach, the concentration of solute at the boundary between solution and porous medium is discontinuous. The boundary conditions are:

Continuity of flux at the interface

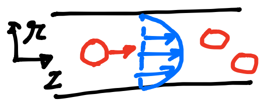
$$N_1 = N_2$$

Concentrations at the interface are related by partition coefficient,

$$\frac{C_1}{K_1} = \frac{C_2}{K_2}$$

where K1 and K2 are area fractions available at the interface.

Diffusion through liquid filled pore:



General mass balance for solute is given by,

$$\frac{\partial c}{\partial t} + \nabla \cdot (\underset{\sim}{v} c) = D_{eff} \nabla^2 c$$

For a cylindrical pore, at steady state, we have,

$$N_z = -D_{eff} \frac{\partial c(r,z)}{\partial z} + c(r,z)\, v_z(r)$$

$$N_z = -D_{eff} \frac{\partial c(r,z)}{\partial z} + c(r,z)\, \overset{\text{parabolic}}{v_{max} \left\{1 - \left(\frac{r}{R}\right)^2\right\}}$$

For low hydraulic conductivity, i.e. dense porous medium, viscous resistance to fluid flow is much smaller than the resistance due to porous medium, i.e the velocity profile can be considered to be flat. In this case,

$$N_z = -D_{eff} \frac{dc(z)}{dz} + c(z)\, v_{max} = \text{Constant}$$

$$\text{or,} \quad \frac{dc}{dz} - \frac{v_{max}}{D_{eff}} c = -\frac{N_z}{D_{eff}}$$

$$\text{Integrating factor} \quad e^{-\frac{v_{max} z}{D_{eff}}}$$

$$e^{-\frac{v_{max} z}{D_{eff}}} \frac{dc}{dz} - \frac{v_{max}}{D_{eff}} c\, e^{-\frac{v_{max} z}{D_{eff}}} = -\frac{N_z}{D_{eff}} e^{-\frac{v_{max} z}{D_{eff}}}$$

or,

$$\frac{d}{dz}\left[c e^{-\frac{V_{max}}{D_{eff}}z} \right] = -\frac{N_z}{D_{eff}} e^{-\frac{V_{max}z}{D_{eff}}}$$

Integrating,

$$c(z)\exp\left\{-\frac{V_{max}}{D_{eff}}z\right\} - c_0 = -\frac{N_z}{D_{eff}}\int_0^z \exp\left\{-\frac{V_{max}}{D_{eff}}z'\right\}dz'$$

$$= \frac{N_z}{D_{eff}}\frac{D_{eff}}{V_{max}}\left[\exp\left\{-\frac{V_{max}}{D_{eff}}z\right\} - 1\right]$$

$$c(z) = c_0\exp\left\{\frac{V_{max}}{D_{eff}}z\right\} + \frac{N_z}{V_{max}}\left[1 - \exp\left\{\frac{V_{max}}{D_{eff}}z\right\}\right]$$