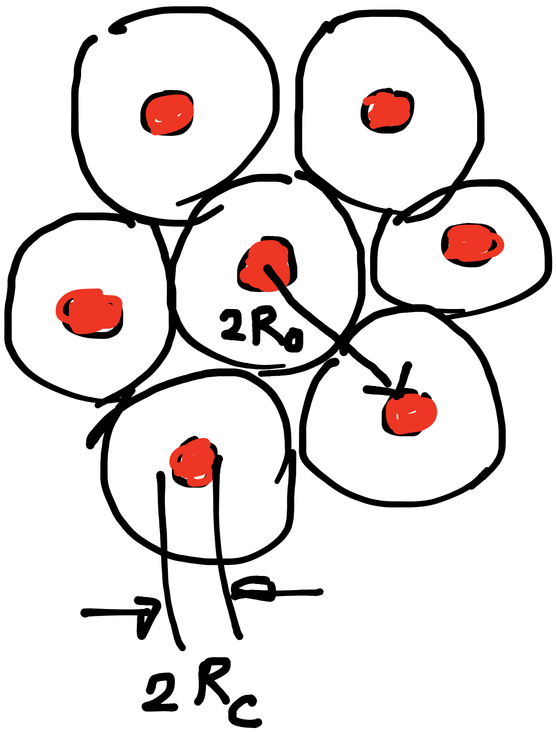
Oxygen delivery to tissues involves the dissociation of oxygen from hemoglobin, the diffusion and convection of oxygen through the plasma, diffusion across endothelium, diffusion through the tissue and cells and finally, the reaction of oxygen in mitochondria as part of aerobic metabolism. Blood vessels in tissues can be modelled as organization of capillaries as shown in schematic below.



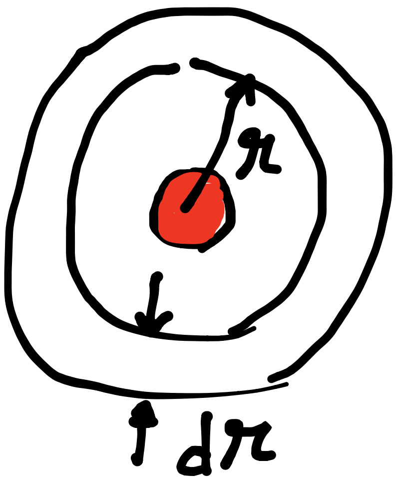
Rc is the radius of the capillary which is surrounded by tissue. The tissue radius R0 represents half the distance between the ceter of two capillaries. Oxygen transport consists of two parts, namely, oxygen transport through blood and oxygen transport through the tissue. We now assume that oxygen concentration is uniform at plasma concentration CRc. Oxygen is transferred radially into the tissue and is consumed by metabolism. The rate of consumption of oxygen RO2 can be written as Micheles-Menton kinetics,

$$R_{O_2} = \frac{R_{max} C_{O_2}}{K_M + C_{O_2}} \quad (1)$$

Km is usually much smaller than CO2. Therefore, the rate of oxygen consumption in tissue is usually assumed to be zero order, i.e.

$$R_{O_2} \simeq R_{max} = \text{Constant} \quad (2)$$

Therefore, radial oxygen transport into tissues is written by the mass balance,



$$-D_{O_2} 2\pi r L \left.\frac{dC_{O_2}}{dr}\right|_{r} + D_{O_2} 2\pi r L \left.\frac{dC_{O_2}}{dr}\right|_{r+dr} - R_{O_2} 2\pi r L \, dr = 0$$

Dividing by $2\pi L \, dr$, we get,

$$D_{O_2}\left[\frac{r\left.\frac{dC_{O_2}}{dr}\right|_{r+dr} - r\left.\frac{dC_{O_2}}{dr}\right|_{r}}{dr}\right] = r R_{O_2}$$

$$\text{or,} \quad \frac{D_{O_2}}{r}\frac{d}{dr}\left[r\frac{dC_{O_2}}{dr}\right] = R_{O_2} \quad (3)$$

The two boundary conditions are

$r = R_c , \quad C_{O_2} = C_{R_c} \quad (4)$ Plasma Concentration

$r = R_o \quad -D\frac{dC_{O_2}}{dr} = 0 \quad (5)$ No flux

$$\frac{d}{dr}\left(r\frac{dC_{O_2}}{dr}\right) = \frac{R_{O_2}}{D_{O_2}} r$$

$$r\frac{dC_{O_2}}{dr} = \frac{R_{O_2}}{2D_{O_2}} r^2 + C_1$$

$$\frac{dC_{O_2}}{dr} = \frac{R_{O_2}}{2D_{O_2}} r + \frac{C_1}{r} \quad (6)$$

$$C_{O_2} = \frac{R_{O_2}}{4D_{O_2}} r^2 + C_1 \ln r + C_2 \quad (7)$$

From (5) and (6),

$$0 = \frac{R_{O_2}}{2D_{O_2}} R_o + \frac{C_1}{R_o}$$

$$\text{or,} \quad C_1 = -\frac{R_{O_2}}{2D_{O_2}} R_o^2 \quad (8)$$

$$\therefore C_{O_2} = \frac{R_{O_2}}{4D_{O_2}} r^2 - \frac{R_{O_2}}{2D_{O_2}} R_o^2 \ln r + C_2 \quad (9)$$

From (4) and (9),

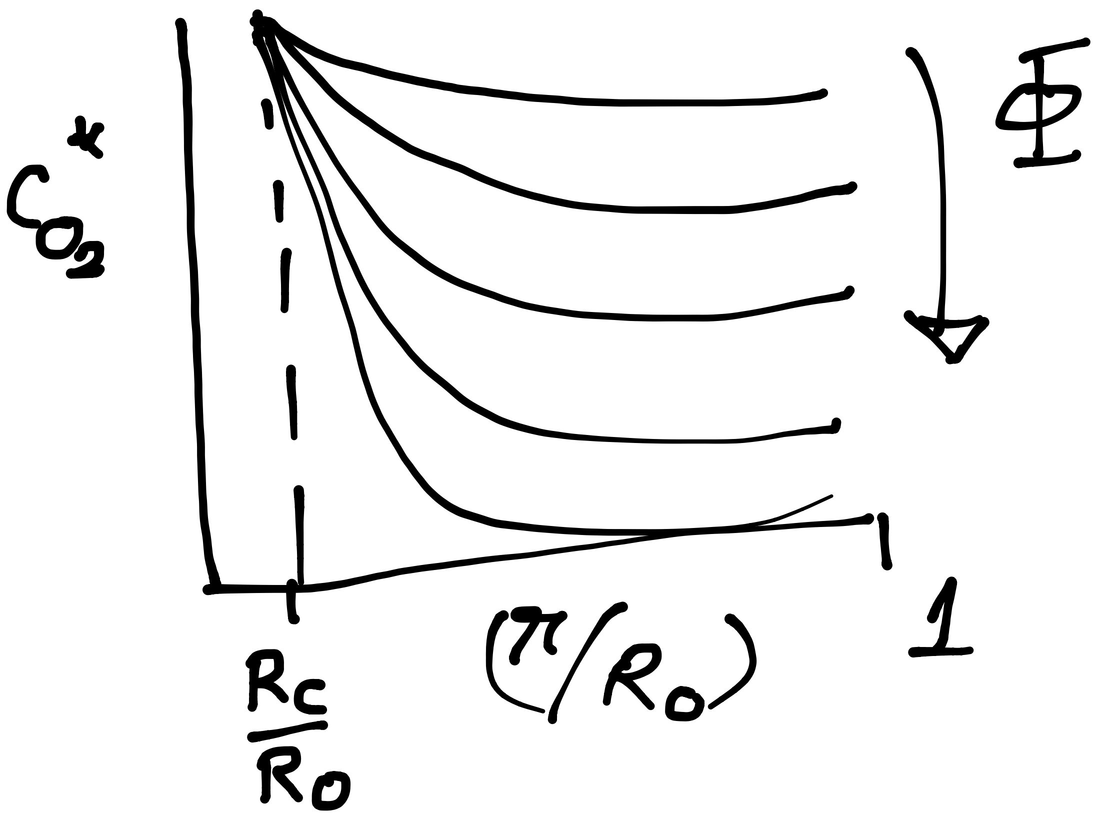
$$C_{R_c} = \frac{R_{O_2}}{4D_{O_2}} R_c^2 - \frac{R_{O_2}}{2D_{O_2}} R_o^2 \ln R_c + C_2$$

$$C_2 = C_{R_c} - \frac{R_{O_2}}{4D_{O_2}} R_c^2 + \frac{R_{O_2}}{2D_{O_2}} R_o^2 \ln R_c \quad (10)$$

$$C_{O_2} = \frac{R_{O_2}}{4D_{O_2}} r^2 - \frac{R_{O_2}}{2D_{O_2}} R_o^2 \ln r + C_{R_c} - \frac{R_{O_2}}{4D_{O_2}} R_c^2 + \frac{R_{O_2}}{2D_{O_2}} R_o^2 \ln R_c$$

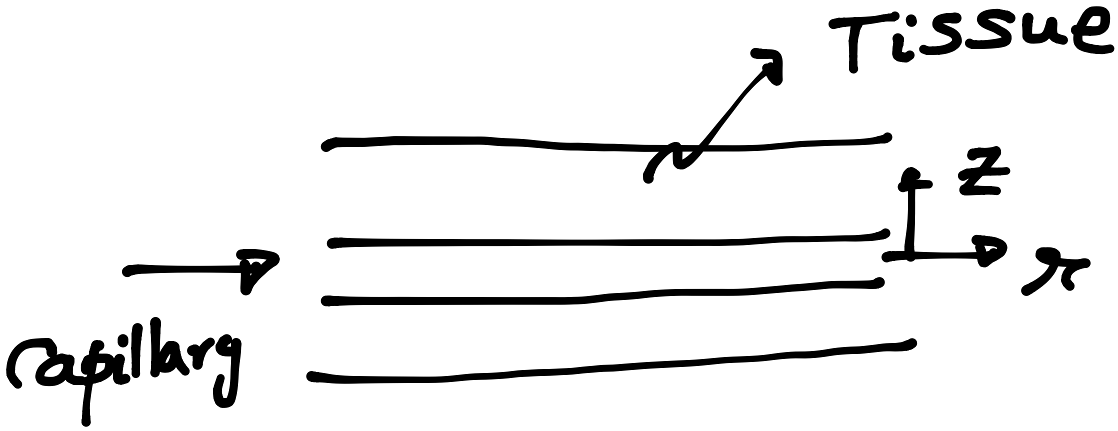
$$C_{O_2} = C_{R_c} + \frac{R_{O_2} R_o^2}{4 D_{O_2}} \left\{ \left(\frac{r}{R_o}\right)^2 - \left(\frac{R_c}{R_o}\right)^2 \right\} - \frac{R_{O_2}}{2 D_{O_2}} R_o^2 \ln\left(\frac{r}{R_c}\right)$$

$$C_{O_2}^{*} = \frac{C_{O_2}}{C_{R_c}} = 1 + \frac{R_{O_2} R_o^2}{4 D_{O_2} C_{R_c}} \left[\left\{ \left(\frac{r}{R_o}\right)^2 - \left(\frac{R_c}{R_o}\right)^2 \right\} - 2 \ln\left(\frac{r}{R_c}\right) \right]$$



$$\phi = \frac{R_{O_2} R_o^2}{4 D_{O_2} C_{R_c}}$$

One can see that the oxygen concentration in tissue decreases faster as dimensionless group Φ increases. Therefore, as expected, an increase in metabolic rate leads to faster oxygen depletion. At very high metabolic rates, the concentration can become zero at large radial distance. In other words, outer layers of tissue can be starved of oxygen. This is known as anoxic region. So far, we have neglected axial variation of oxygen within the capillary. We will consider that in the following.



$$\pi R_c^2 V_z \frac{dC_{R_c}}{dt} = -\pi (R_o^2 - R_c^2) R_{O_2}$$

$$\frac{dC_{R_c}}{dz} = -\left[\left(\frac{R_o}{R_c}\right)^2 - 1\right] \frac{R_{O_2}}{V_z}$$

$$C_{R_c} = C_{R_c,o} - \left[\left(\frac{R_o}{R_c}\right)^2 - 1\right] \frac{R_{O_2}}{V_z} z$$