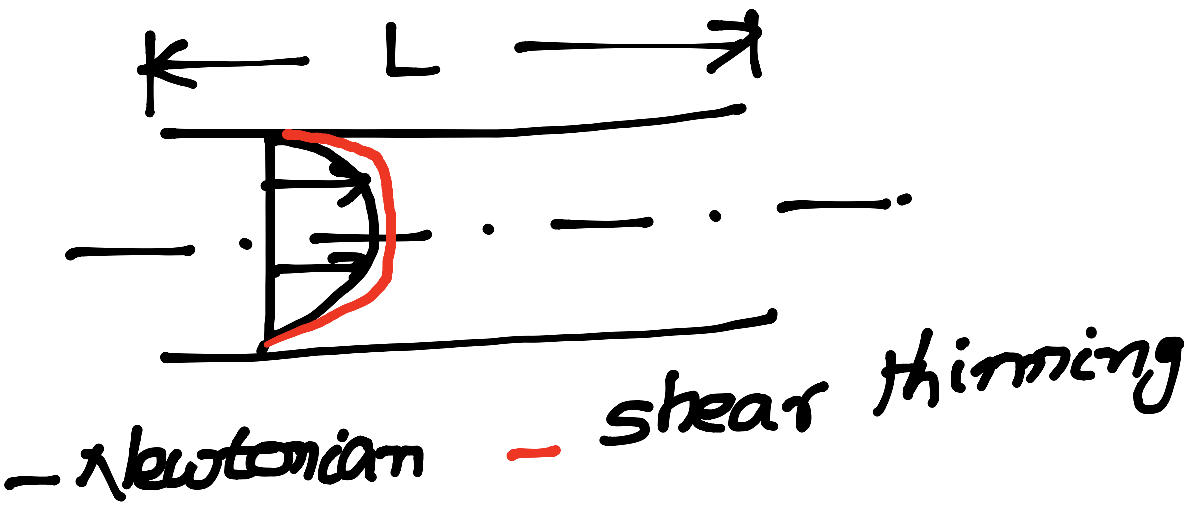
Consider sterilization of liquid flowing continuously through a tube. Let R be the radius of the tube. In general, the fluid can be considered as a power law fluid. The fluid will exhibit a velocity profile when flowing through a tube. For a laminar flow, for a Newtonian fluid, the velocity profile is parabolic. For shear thinning fluid, the profile is flatter than parabolic profile.



For a parabolic velocity profile, vav = vmax/2, where vav and vmax refer to the average and maximum velocities respectively. Because of the velocity profile, different layers of fluid spend different times within the tube (usually referred to as hold tube). An aseptic processing system will consist of a heat exchanger (usually plate or double tube) that heats the fluid from room temperature to the temperature of hold tube. The heated fluid from the heat exchanger will be sent to a hold tube where it is kept at that temperature for a certain residence time. As expected, the residence time will depend on tube size, length and volumetric flow rate. Let T be the temperature of the hold tube. For a fluid spending a residence time θ we can write,

$$\frac{N}{N_0} = 10^{-\theta/D_T} \quad (1)$$

where N and N0 refer to final and initial number of viable microorganism and D_T is the decimal reduction time at T. However, since the residence time is

different for different layers, it tempting to use the average residence time in the above equation to calculate the lethality. It is to be noted that this is incorrect. FDA requires calculation of lethality based on the fastest moving fluid. Since the velocity of the fluid is maximum at the center of the tube, one has to calculate lethality for the fluid at the center of the tube. Therefore, θ has to be evaluated from the equation,

$$\theta = \frac{L}{V_{max}} = \frac{L}{2\bar{V}} \quad (2)$$

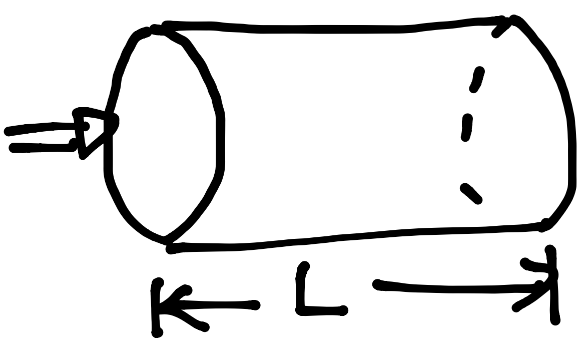
$$\therefore \frac{N_0}{N} = 10^{\frac{L}{2\bar{V} D_T}}$$

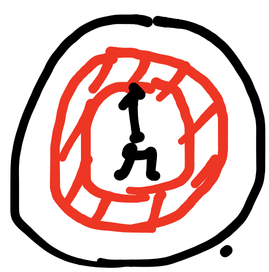
$$\log \frac{N_0}{N} = F = \frac{L}{2\bar{V} D_T} \quad (3)$$

$$D_T = D_{T_0}\, 10^{-\frac{T-T_0}{z}}$$

$$\therefore \log \frac{N_0}{N} = F = \frac{L}{2\bar{V} D_{T_0}}\, 10^{(T-T_0)/z} \quad (4)$$

Extent of overprocessing:
Since most of the fluid spends a residence time that is greater than that required to obtain the lethality, the fluid is overprocessed. The extent of overprocessing can be evaluated as follows.





The velocity profile is given by,

$$V(r) = \bar{V}\left(\frac{3n+1}{n+1}\right)\left[1 - \left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right] \quad (5)$$

$$v(y) = A(1-y^B)\bar{v} \quad (6)$$

$$y = \frac{r}{R} \quad A = \left(\frac{3n+1}{n+1}\right) \quad B = \frac{n+1}{n}$$

Consider the flow of liquid through the shaded cross section between r and r+dr. The residence time of liquid flowing through this cross section is given by

$$\frac{L}{v(r)} = \frac{L}{A(1-y^B)\bar{v}} \quad (7)$$

Let n0 be the number of viable microorganism per unit volume at the inlet. Therefore, the rate N0 at which viable microorganism is entering the hold tube is given by,

$$N_0 = n_0 \int_0^R 2\pi r\, v(r)\, dr$$
$$= n_0 \pi R^2 \bar{v} \quad (8)$$

The number of viable microorganism per unit volume that is leaving the shaded cross sectional area is given by,

$$n_0 V(r)\, 10^{-\left(L/A(1-y^B)\bar{v} D_T\right)}\, 2\pi r\, dr \quad (9)$$

Therefore, the rate N at which viable microorganism is leaving the hold tube is given by

$$N = \int_0^R n_0 V(r)\, 10^{-\left(L/A(1-y^B)\bar{v} D_T\right)}\, 2\pi r\, dr$$
$$N = \int_0^1 n_0 A(1-y^B)\bar{v}\, 10^{-\left(L/A(1-y^B)\bar{v} D_T\right)}\, 2\pi R^2 y\, dy \quad (10)$$

$$\frac{N_0}{N} = \frac{\pi R^2 n_0 \bar{V}}{\int_0^1 n_0 A(1-y^B)\bar{V}\; 10^{-\left(L/A(1-y^B)\bar{v} D_T\right)}\, 2\pi R^2 y\, dy}$$

$$\log \frac{N_0}{N} = -\log\left[\int_0^1 2A(1-y^B)\quad 10^{-\left(L/A(1-y^B)\bar{v} D_T\right)} y\, dy\right] \quad (11)$$

Denote $S_V = \dfrac{L}{\bar{V} D_T}$

$$\log \frac{N_0}{N} = -\log\left[\int_0^1 2A(1-y^B)\quad 10^{-\left(S_V/A(1-y^B)\right)} y\, dy\right] \quad (12)$$