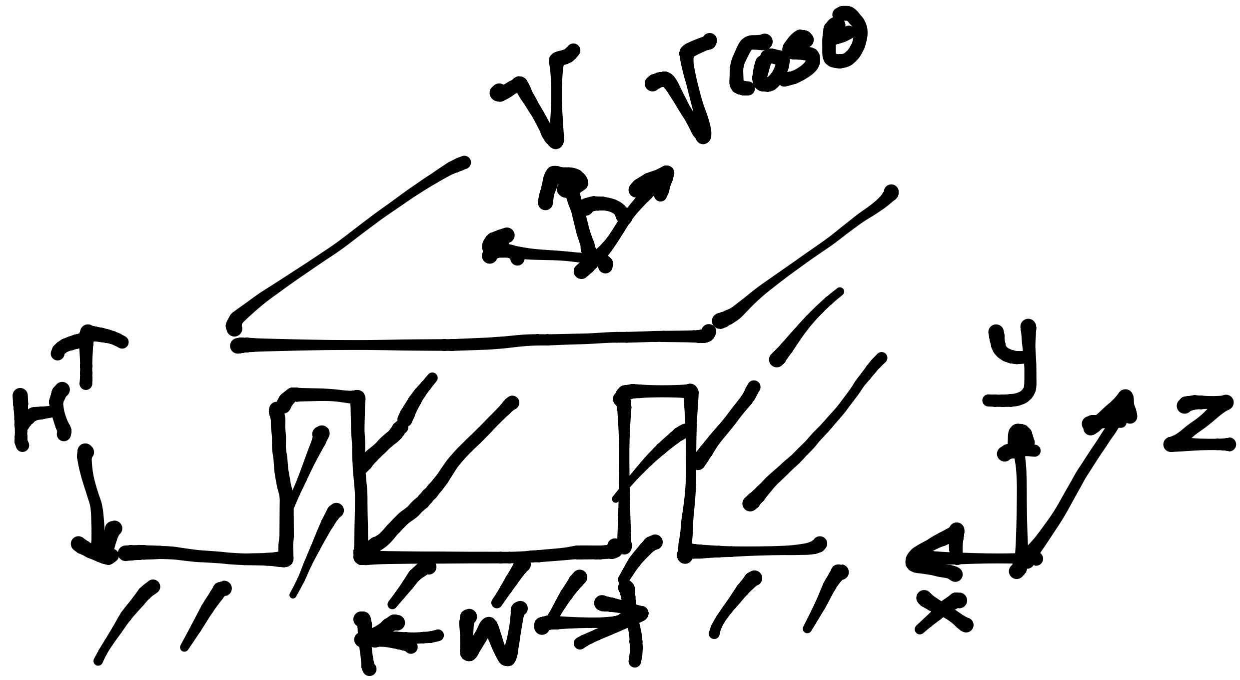


z direction

$$\frac{\partial p}{\partial z} = \mu\left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2}\right)$$

$$H \ll W \Rightarrow \frac{\partial v_z}{\partial y} \gg \frac{\partial v_z}{\partial x}$$

v_z changes from 0 to V

$x \rightarrow W$

$y \rightarrow H$

$$\frac{\partial p}{\partial z} = \frac{(p_L - p_0)}{Z_p} \simeq \mu \frac{d^2 v_z}{dy^2}$$

$$\frac{dv_z}{dy} = \frac{(p_L - p_0)}{\mu Z_p} y + C_1$$

$$v_z = \frac{(p_L - p_0) y^2}{2\mu Z_p} + C_1 y + C_2$$

B.C. $y = 0 \quad v_z = 0$

$y = H \quad v_z = V\cos\theta$

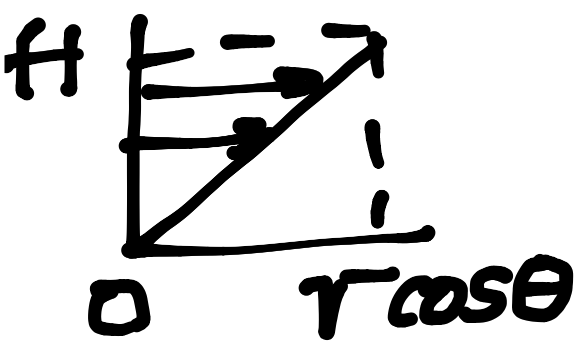
$$0 = C_2$$

$$V\cos\theta = \frac{(p_L - p_0)H^2}{2\mu Z_p} + C_1 H$$

$$C_1 = \frac{V\cos\theta}{H} - \frac{(p_L - p_0)H}{2\mu Z_p}$$

$$v_z = \frac{(p_L - p_0)y^2}{2\mu Z_p} + V\cos\theta\left(\frac{y}{H}\right) - \frac{(p_L - p_0)Hy}{2\mu Z_p}$$

$$v_z = \underbrace{V\cos\theta\left(\frac{y}{H}\right)}_{\text{DRAG FLOW}} - \underbrace{\frac{(p_L - p_0)H^2}{2\mu Z_p}\left[\left(\frac{y}{H}\right) - \left(\frac{y}{H}\right)^2\right]}_{\text{PRESSURE FLOW}}$$



Flow rate Q is given by

$$Q = W\int_0^H v_z(y)\,dy$$

$$Q = W\int_0^H \left[V\cos\theta\left(\frac{y}{H}\right) - \frac{(p_L - p_0)H^2}{2\mu Z_p}\left[\left(\frac{y}{H}\right) - \left(\frac{y}{H}\right)^2\right]\right] dy$$

$$Q = WH\int_0^1 \left[V\cos\theta\,\xi - \frac{(p_L - p_0)H^2}{2\mu Z_p}\left\{\xi - \xi^2\right\}\right] d\xi$$

$$Q = WH\left[\frac{V\cos\theta}{2} - \frac{(p_L - p_0)H^2}{2\mu Z_p}\left\{\frac{1}{2} - \frac{1}{3}\right\}\right.$$

$$Q = \underbrace{\frac{WHV\cos\theta}{2}}_{\text{DRAG}} - \underbrace{\frac{(p_L - p_0)WH^3}{12\mu Z_p}}_{\text{PRESSURE}} \qquad (\mathrm{I})$$

Use apparent viscosity

shear rate $\frac{\pi DN}{H}$ for extruder

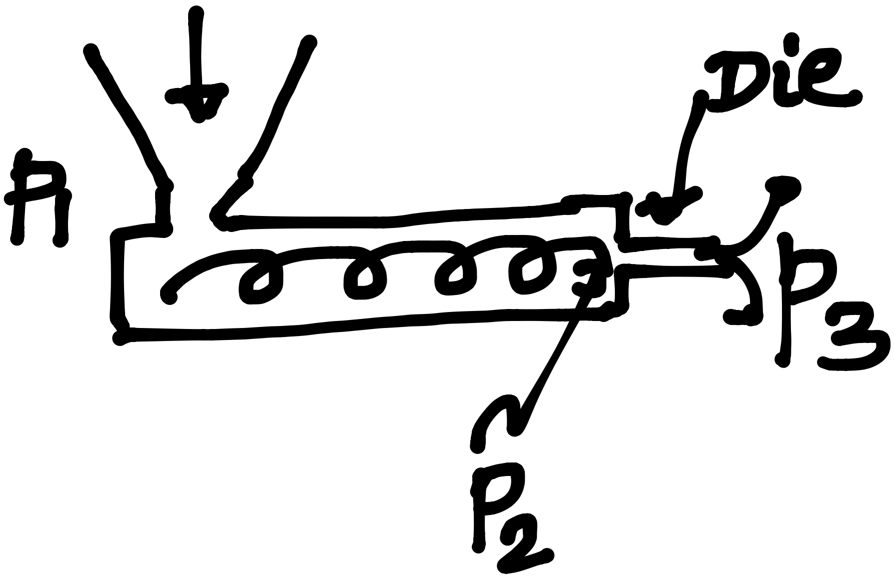
$$\mu = K\left(\frac{\pi DN}{H}\right)^{n-1}$$

For leakage
shear rate $\frac{\pi DN}{\delta}$

$$\mu_\delta = K\left(\frac{\pi DN}{\delta}\right)^{n-1}$$

<u>Die</u>

$$Q = K_{die}\frac{\Delta P}{\mu_d} \qquad (\mathrm{II})$$



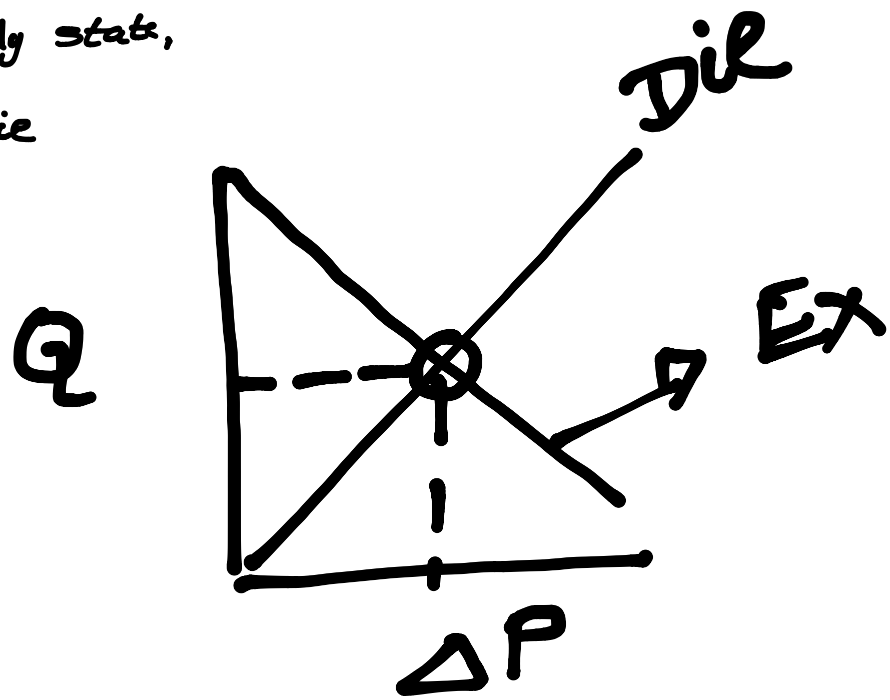
$\Delta p_{die} = p_2 - p_3$

$p_3 = p_{atm} \quad p_1 = p_{atm}$

$\therefore \Delta p_{die} = p_2 - p_{atm} = p_2 - p_1$

At steady state,

$Q_{ex} = Q_{die}$



Continuity:-

$$\cancel{\frac{\partial V_x}{\partial x}} + \frac{\partial V_y}{\partial y} = 0$$

Motion:-

y:

$$\frac{\partial p}{\partial y} = \mu\left(\cancel{\frac{\partial^2 V_y}{\partial x^2}} + \frac{\partial^2 V_y}{\partial y^2}\right)$$

$$\frac{\partial p}{\partial y} = \mu \cancel{\frac{\partial^2 V_y}{\partial y^2}} \Rightarrow \frac{\partial p}{\partial y} = 0$$ Lubrication Approximation

x:

$$\frac{\partial p}{\partial x} = \mu\left(\cancel{\frac{\partial^2 V_x}{\partial x^2}} + \frac{\partial^2 V_x}{\partial y^2}\right)$$

$$\frac{\Delta P_x}{W} = \mu \frac{d^2 V_x}{dy^2}$$

Integrate,

$$\frac{dV_x}{dy} = \frac{\Delta P_x}{\mu W} y + C_1$$

$$V_x = \frac{\Delta P_x y^2}{2\mu W} + C_1 y + C_2$$

B.C. $y=0 \quad V_x = 0$

$y = H \quad V_x = V\sin\theta$

$$C_2 = 0 \qquad C_1 = \frac{V\sin\theta}{H} - \frac{\Delta P_x H}{2\mu W}$$

$$V_x = V\sin\theta \frac{y}{H} - \frac{\Delta P_x H y}{2\mu W} + \frac{\Delta P_x y^2}{2\mu W}$$

$$Q_x = \int_0^H V_x \, dy \quad \text{per unit length}$$

$Q_x = 0 \Rightarrow$ Solve for ΔP_x

$$Q_x = \frac{V\sin\theta\, H}{2} - \frac{\Delta P_x H^3}{2\mu W}\frac{1}{2} + \frac{\Delta P_x H^3}{2\mu W}\frac{1}{3}$$

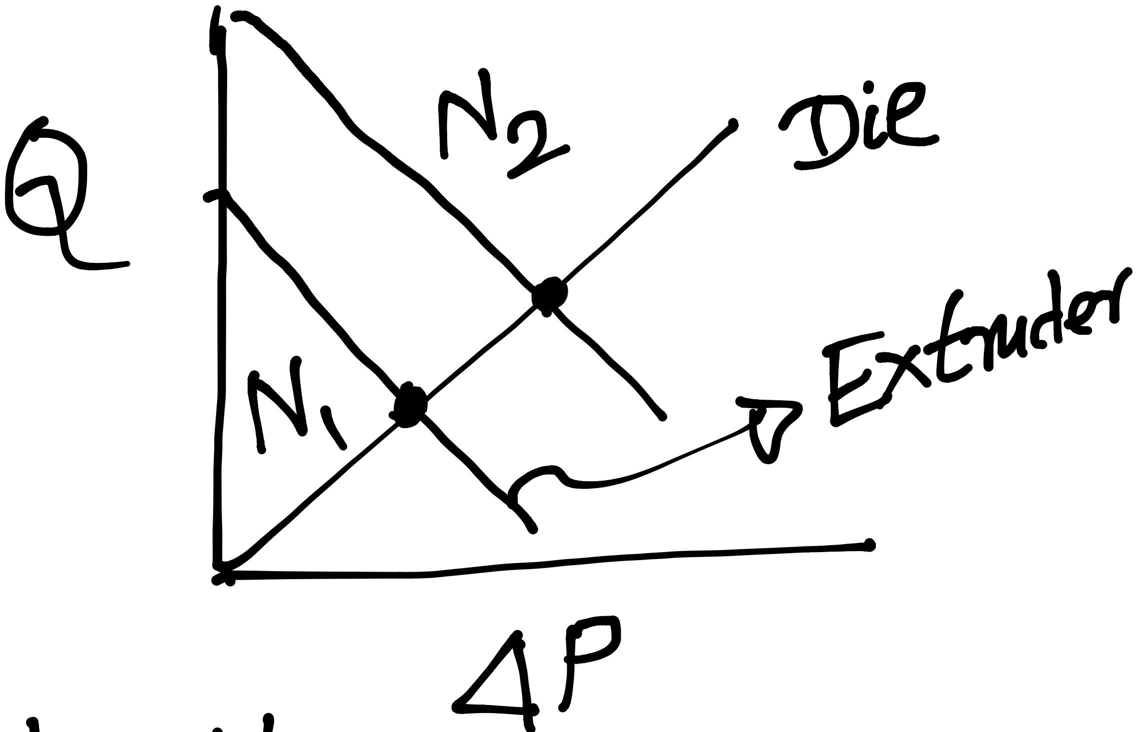
$$Q_x = \frac{V\sin\theta\, H}{2} - \frac{\Delta P_x H^3}{12\mu W} = 0$$

$$\Rightarrow \Delta P_x = \frac{6V\sin\theta\,\mu W}{H^2}$$

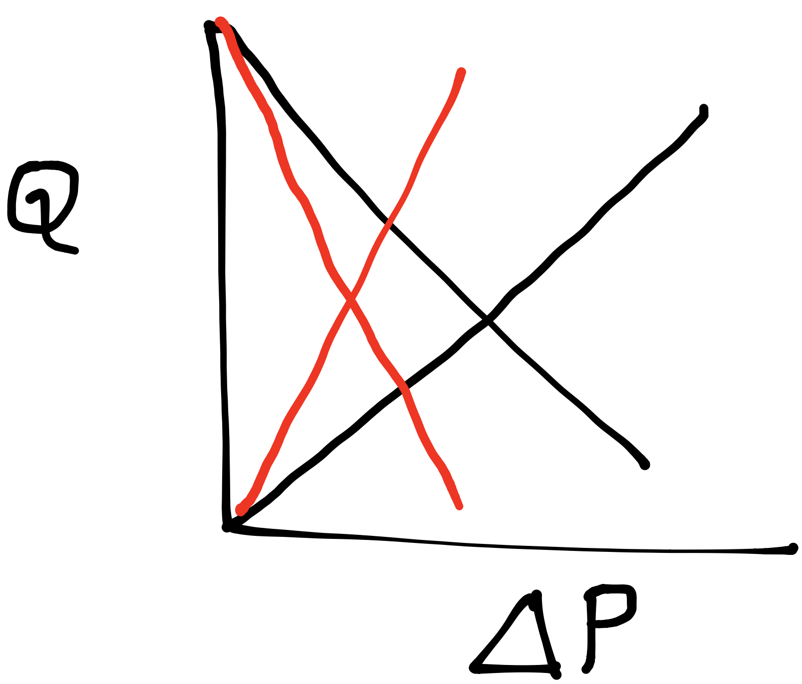
$$V_x = V\sin\theta\frac{y}{H} + \frac{H^2}{2\mu W}\left[\left(\frac{y}{H}\right)^2 - \left(\frac{y}{H}\right)\right]\frac{6V\sin\theta\,\mu W}{H^2}$$

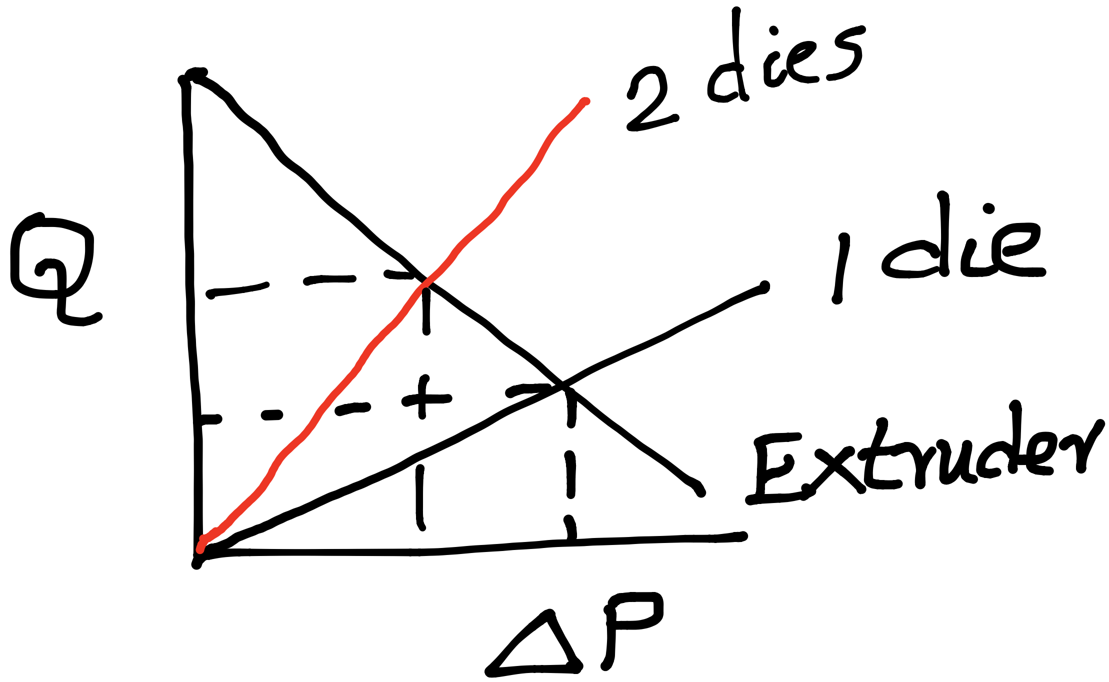
$$V_x = V\sin\theta\frac{y}{H} + 3V\sin\theta\left(\frac{y}{H}\right)^2 - 3V\sin\theta\left(\frac{y}{H}\right)$$

$$V_x = 3V\sin\theta\left(\frac{y}{H}\right)^2 - 2V\sin\theta\left(\frac{y}{H}\right)$$



$N_2 > N_1$





What happens for 2 dies ?

Wall shear stress

$$\tau_w = \frac{R_d \Delta P}{2 L_d}$$

τ_{max} = Maximum wall shear stress

If $\tau_w > \tau_{max}$, cracks appear – Loss of quality

$$\Delta P_{max} = \frac{2 \tau_{max} L_d}{R_d}$$

$$Q_{max} = \frac{K}{\mu_d} \frac{2 \tau_{max} L_d}{R_d}$$

This limits min # dies.