ABE 457
Homework 5

Due: 2/21/18

1. A realistic physiological pressure pulse can be represented as

$$-\frac{dp}{dx} = a_0 + \sum_{j=1}^{n} a_j \cos\left(2\pi jft\right) \quad (1)$$

Where j is the harmonic and f is the frequency in Hz (cycles per second. The solution procedure that is developed for a complex pulse $\exp\left(i\omega t\right) = \cos\left(\omega t\right) + i\sin\left(\omega t\right)$ can be applied to an arbitrary Fourier series by adapting the solution for the different terms in the series and the change in Wormersley parameter as

$$\alpha_j = R\left[\frac{j2\pi f\rho}{\mu}\right]^{1/2} \quad (2)$$

(a) Determine the velocity profile for a pressure gradient represented by the equation (1) with the Wormsley number for the harmonics represented by eq. (2). You need to take the real part of the solution for periodic pressure gradient

$$a_0 + \sum_{j=1}^{n} a_j \exp(i2\pi jft)$$

(b) Calculate the pressure and velocity profile if the pressure gradient is represented by only first three harmonics with $f = 72$ cycles/min, $a_0 = 10^{12}$ $a_1 = 10^{11}$ $a_2 = 10^{11}$, $a_3 = 10^{11}$. Compare this velocity profile with that when you include only the first harmonic (leading term in the summation).

2. Consider a catheter of radius R_c placed in a small artery of radius R as shown in the figure below. The catheter moves at a constant speed V. In addition, blood flows through the annular region between R_c and R under a pressure gradient $\Delta p/L$. The capillary can be assumed to be horizontal. We want to determine the effect of the catheter upon the shear stress at $r = R$. Assume steady, fully developed flow of a Newtonian fluid.
    a. Write the equation of continuity.
    b. Write the equation of motion along the axial direction.
    c. Identify the boundary conditions
    d. Solve for the velocity profile.
    e. Calculate the wall shear stress. Discuss the effect of catheter velocity on wall shear stress.