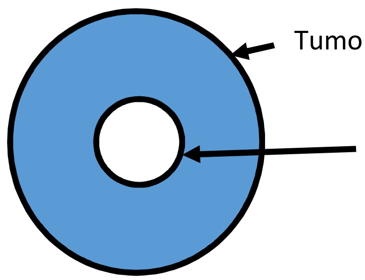
ABE 457
Homework 6
Spring 2018

Due: 3/21/2018

1. Therapeutic agents can be infused directly into solid tumors. Intratumoral infusion enhances the convective transport of drugs, which is critical to the delivery of macromolecules and nanoparticles. To simplify the analysis of convective transport in a tumor, we consider only the fluid flow and assume that the tumor is removed from the body. Thus, the tumor has no blood and lymph circulation, and the pressure at its surface is equal to zero. The needle tip is inserted into the center of the tumor after which a small fluid cavity forms around the tip. The radius of the cavity is δ, which can be taken to the radius of the needle. We assume that solid tumors are spherical, that the radius of the tumor is a , and that the hydraulic conductivity is K . If the infusion rate is Q and is held constant during injection, determine the pressure profile in the tumor at steady state.



2. A Newtonian fluid flows through a cylindrical pipe filled with a porous medium. The length of the pipe is L . The flow is one dimensional and at steady state. The pressures at two ends of the pipe are p_1 and p_2 . The specific hydraulic permeability of the porous edium is k and the viscosity of the fluid is μ .

   (i) Determine the velocity profile.

   (ii) Plot the nondimensionalized velocity $\dfrac{\mathrm{v}(r)}{(kB/\mu)}$ versus $\dfrac{r}{R}$ where $B=\dfrac{(p_1-p_2)}{L}$. Assume that $k=400\ \mathrm{nm}^2$. Plot the velocity profiles for values of $R=20,60,100\ \text{and}\ 400\ \mathrm{nm}$.

   (iii) Under what conditionscan viscous effects be neglected? That is, under what conditions is the error in the flow rate that is predicted by Darcy's law less than 10% of the flow rate that is determined by Brinkman's equation?

3. We are interested in evaluating the normal stress that is developed as a result of growth of clump of tumor cells that are separated by plasma. One can model the region between two neighboring tumor cells as liquid between two circular disks. The initial gap between the two cells (disks) is 1 mm. The two

tumor cells are growing at the rate of $0.1\ \mathrm{nm/s}$. The viscosity of plasma between the two growing tumor cells is $10^{-3}\ \mathrm{Pa.s}$.

(i) To model the effect of tumor cell size on normal stress, calculate the force $F(t)$ on the tumor cells for the different radii of the disk, 1 mm, 2 mm, 5 mm, 1 cm as a function of time. Also plot the normal stress $N(t)$ on the tumor cells as a function of time for different radii. Discuss the effect of radius and time on normal stress.

(ii) Repeat the calculations as described in (i) for a fixed radius of 1 mm but for different growth rate (velocity) of the tumor cells, 0.1 nm/s, 0.5 nm/s, 1 nm/s, 2 nm/s, 5 nm/s and 10 nm/s. Plot the normal stress $N(t)$ for these velocities. Discuss the effect of time and growth rate of tumor cells on $N(t)$