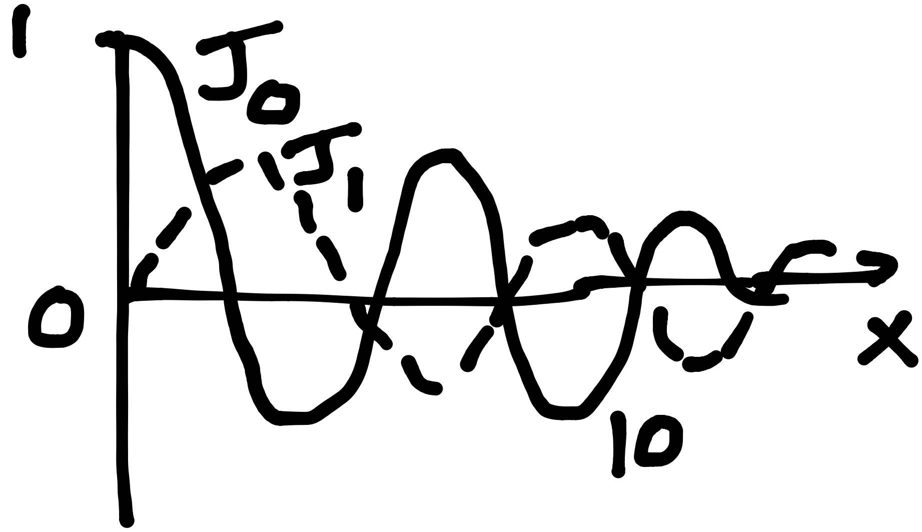
Consider the following differential equation

$$x^2 y'' + x y' + (x^2 - n^2) y = 0 \quad (1)$$

where n is an integer. The general solution of this equation is given by,

$$y(x) = J_n(x) \quad (2)$$

where $J_n(x)$ is Bessel's function of nth order. Typical plots of Bessel's function of zero and first order are shown below.



As can be seen from the above plot, Bessel's function takes up both positive and negative values and exhibits several zeros. This will be useful in solving your problem. You also have to take advantage of the following orthogonality relation in solving for the coefficients.

$$\text{Let } J_n(\lambda_k R) = 0 \quad k = 1, 2, 3, \dots \quad (3)$$

$$\int_0^R x\, J_n(\lambda_k x)\, J_n(\lambda_l x)\, dx = 0 \quad k \neq l \quad (4)$$

$$\int_0^R x\, J_n^2(\lambda_k x)\, dx = \frac{R^2}{2} J_{n+1}^2(\lambda_k R) \quad (5)$$

Properties of Bessel's function:

$$\frac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x) \quad (6)$$

$$J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x) \quad (7)$$

$$J_{n-1}(x) - J_{n+1}(x) = 2 J_n'(x) \quad (8)$$