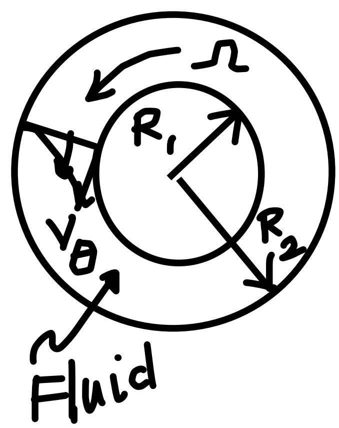
Sample placed between two concentric cylinders. One cylinder is rotated while the other is stationary. The torque required to rotate the cylinder is measured at different rpms. Torque is converted to wall shear stress and rpm is converted to wall shear rate.




Assumptions:

1. steady flow.
2. one dimensional flow $V_\theta \neq 0,\ V_r = V_z = 0$
3. Power law fluid.

Constitutive equation

$$\tau_{r\theta} = -\eta\left[r\frac{\partial}{\partial r}\left(\frac{V_\theta}{r}\right)\right]$$

$\eta$ is apparent viscosity

$$\eta = K\left|r\frac{\partial}{\partial r}\left(\frac{V_\theta}{r}\right)\right|^{n-1}$$

Therefore,

$$\tau_{r\theta} = -K\left|r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right|^{n-1}\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right]$$

$$\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right] < 0$$

$$\therefore \left|r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right| = -r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)$$

$$\therefore \tau_{r\theta} = K\left[-r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right]^{n}$$

Equation of motion:

$\theta$ component

$$\rho\left(\cancel{\frac{\partial v_\theta}{\partial t}} + \cancel{v_r}\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\cancel{\frac{\partial v_\theta}{\partial \theta}} + \frac{\cancel{v_r} v_\theta}{r} + \cancel{v_z}\frac{\partial v_\theta}{\partial z}\right)$$

$$= -\frac{1}{r}\cancel{\frac{\partial p}{\partial \theta}} - \left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\tau_{r\theta}\right) + \frac{1}{r}\cancel{\frac{\partial \tau_{\theta\theta}}{\partial \theta}} + \cancel{\frac{\partial \tau_{\theta z}}{\partial z}}\right) + \rho \cancel{g_\theta}$$

$$\frac{d}{dr}\left(r^2\tau_{r\theta}\right) = 0$$

$$\tau_{r\theta} = \frac{C_1}{r^2}$$

$$K\left[-r\frac{\partial}{\partial r}\left(\frac{V_\theta}{r}\right)\right]^n = \frac{c_1}{r^2}$$

$$\frac{d}{dr}\left(\frac{V_\theta}{r}\right) = -\frac{c_1^{1/n}}{K^{1/n} r^{2/n}}\frac{1}{r} = -\frac{c_1^{1/n}}{K^{1/n} r^{1+2/n}}$$

Integrating, we get,

$$\frac{V_\theta}{r} = \frac{c_1^{1/n}}{K^{1/n}}\frac{n}{2}\frac{1}{r^{2/n}} + c_2$$

$$V_\theta = \frac{c_1^{1/n}}{K^{1/n}}\frac{n}{2} r^{1-\frac{2}{n}} + c_2 r$$

B.C.

$r = R_2 \quad V_\theta = 0$

$r = R_1 \quad V_\theta = \Omega R_1$

substitute and solve for $c_1$ and $c_2$

$$c_1 = K\left[\frac{2\Omega}{n\{R_1^{-2/n} - R_2^{-2/n}\}}\right]^n$$

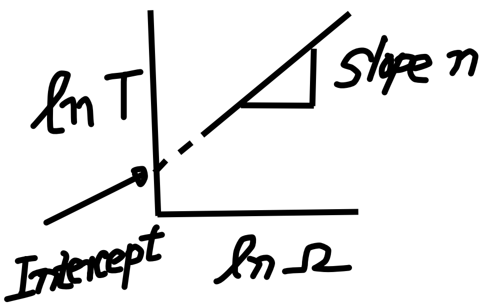
$$c_2 = -\frac{\Omega R_2^{-2/n}}{R_1^{-2/n} - R_2^{-2/n}}$$

$$V_\theta = \frac{\Omega r (r/R_2)^{-2/n}}{\left(\frac{R_1}{R_2}\right)^{-2/n} - 1} - \frac{\Omega r}{\left(\frac{R_1}{R_2}\right)^{-2/n} - 1}$$

Torque $T = 2\pi R_2 L \, \tau_{r\theta}\Big|_{r=R_2} R_2$

$$T = 2\pi R_2 L \frac{C_1}{R_2^2} R_2 = 2\pi L C_1$$

$$T = 2\pi L K \left[\frac{2\Omega}{n\{R_1^{-2/n} - R_2^{-2/n}\}}\right]^n$$




$$\tau_{r\theta} = \frac{C_1}{r^2}$$

$$\tau_{r\theta}\Big|_{r=R_2} = \frac{C_1}{R_2^2} = \frac{K}{R_2^2}\left[\frac{2\Omega}{n\{R_1^{-2/n} - R_2^{-2/n}\}}\right]^n$$

$$= K\left[\frac{2\Omega}{n\left\{\left(\frac{R_1}{R_2}\right)^{-2/n} - 1\right\}}\right]^n$$

$$\tau_{r\theta}\Big|_{r=R_2} = \frac{T}{2\pi R_2^2 L}$$

$$\dot{\gamma} = -r\frac{\partial}{\partial r}\frac{V_\theta}{r} = \left(\frac{1}{K}\frac{C_1}{r^2}\right)^{1/n}$$

$$\dot{\gamma} = \frac{1}{r^{2/n}}\frac{2\Omega}{n\left\{R_1^{-2/n} - R_2^{-2/n}\right\}}$$

$$\dot{\gamma}\Big|_{r=R_2} = \frac{2\Omega}{n\left\{\left(\frac{R_1}{R_2}\right)^{-2/n} - 1\right\}}$$