```
% Kathryn Atherton

% ABE 457
% Homework 2

clc;
clear;
```

Problem 1

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fprintf('Problem 1\n');
radius_piston = 2 / 100; % cm -> m
radius_capillary = 2 / 1000; % mm -> m
length_short = 15 / 1000; % mm -> m
length_long = 100 / 1000; % mm -> m
length = length_long - length_short; % m
piston_velocity = [0.5, 2, 5, 20, 50, 100, 200] / (1000 * 60); % mm/
min -> m/s
long_tube_force = [35.9, 54.4, 71.62, 108.56, 142.91, 175.94,
 216.6]; % N
short_tube_force = [7.18, 10.88, 14.33, 21.71, 28.58, 35.19, 43.32]; %
 N

fprintf('Part a)\n');
fprintf('Flow rate = velocity * area\n');
fprintf('Area = pi * (piston radius)^2\n');
flow_rate = piston_velocity * pi * radius_piston ^ 2; % m^3 / s
fprintf('Flow rate = [%f, %f, %f, %f, %f, %f, %f] m^3/s\n',
 flow_rate(1), flow_rate(2), flow_rate(3), flow_rate(4), flow_rate(5),
 flow_rate(6), flow_rate(7));

fprintf('Part b)\n');
fprintf('Effective force = long tube force - short tube force\n');
fprintf('It is necessary to calculate effective force for a capillary
 of effective length\nbecause it corrects for the energy losses at the
 ends of capillaries due to\nexpansion and contraction.\n');
effective_force = long_tube_force - short_tube_force; % N
fprintf('Effective force = [%f, %f, %f, %f, %f, %f, %f] N\n',
 effective_force(1), effective_force(2), effective_force(3),
 effective_force(4), effective_force(5), effective_force(6),
 effective_force(7));

fprintf('Part c)\n');
fprintf('Pressure drop = tube Force / area\n');
area = 2 * pi * radius_capillary * length; % m^2
pressure_drop = effective_force / area; % Pa
fprintf('Pressure drop = [%f, %f, %f, %f, %f, %f, %f] Pa
\n', pressure_drop(1), pressure_drop(2), pressure_drop(3),
 pressure_drop(4), pressure_drop(5), pressure_drop(6),
 pressure_drop(7));

fprintf('Part d)\n');
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fprintf('(Pressure drop * radius) / (2 * length) = shear stress\n');
shear_stress = (pressure_drop * radius_capillary) / (2 * length); % Pa
fprintf('Shear stress = [%f, %f, %f, %f, %f, %f, %f] Pa\n',
 shear_stress(1), shear_stress(2), shear_stress(3), shear_stress(4),
 shear_stress(5), shear_stress(6), shear_stress(7));

fprintf('Part e)\n');
fprintf('(4 * flow rate) / (pi * radius ^ 3) = pseudo-shear rate at
 wall\n');
pseudo_shear_rate = (4 * flow_rate) / (pi * (radius_capillary ^ 3)); %
 s^-1
fprintf('Pseudo-shear rate = [%f, %f, %f, %f, %f, %f, %f] s^-1\n',
 pseudo_shear_rate(1), pseudo_shear_rate(2), pseudo_shear_rate(3),
 pseudo_shear_rate(4), pseudo_shear_rate(5), pseudo_shear_rate(6),
 pseudo_shear_rate(7));

fprintf('Part f)\n');
fprintf('Correction factor = (3n + 1)/4n, where 1/n = slope at each
 flow rate\n')
slope1 = log(pseudo_shear_rate(1)) / log(shear_stress(1));
n1 = 1 / slope1;
cf1 = (3 * n1 + 1) / (4 * n1);
slope2 = log(pseudo_shear_rate(2)) / log(shear_stress(2));
n2 = 1 / slope2;
cf2 = (3 * n2 + 1) / (4 * n2);
slope3 = log(pseudo_shear_rate(3)) / log(shear_stress(3));
n3 = 1 / slope3;
cf3 = (3 * n3 + 1) / (4 * n3);
slope4 = log(pseudo_shear_rate(4)) / log(shear_stress(4));
n4 = 1 / slope4;
cf4 = (3 * n4 + 1) / (4 * n4);
slope5 = log(pseudo_shear_rate(5)) / log(shear_stress(5));
n5 = 1 / slope5;
cf5 = (3 * n5 + 1) / (4 * n5);
slope6 = log(pseudo_shear_rate(6)) / log(shear_stress(6));
n6 = 1 / slope6;
cf6 = (3 * n6 + 1) / (4 * n6);
slope7 = log(pseudo_shear_rate(7)) / log(shear_stress(7));
n7 = 1 / slope7;
cf7 = (3 * n7 + 1) / (4 * n7);
fprintf('Correction factor = [%f, %f, %f, %f, %f, %f, %f]
 (unitless)\n', cf1, cf2, cf3, cf4, cf5, cf6, cf7);

fprintf('Part g)\n');
fprintf('Wall shear rate = pseudo_shear_rate * correction_factor\n');
wall_shear_rate1 = pseudo_shear_rate(1) * cf1;
wall_shear_rate2 = pseudo_shear_rate(2) * cf2;
wall_shear_rate3 = pseudo_shear_rate(3) * cf3;
wall_shear_rate4 = pseudo_shear_rate(4) * cf4;
wall_shear_rate5 = pseudo_shear_rate(5) * cf5;
wall_shear_rate6 = pseudo_shear_rate(6) * cf6;
wall_shear_rate7 = pseudo_shear_rate(7) * cf7;
fprintf('Wall shear rate = [%f, %f, %f, %f, %f,%f, %f]
 s^-1\n', wall_shear_rate1, wall_shear_rate2, wall_shear_rate3,
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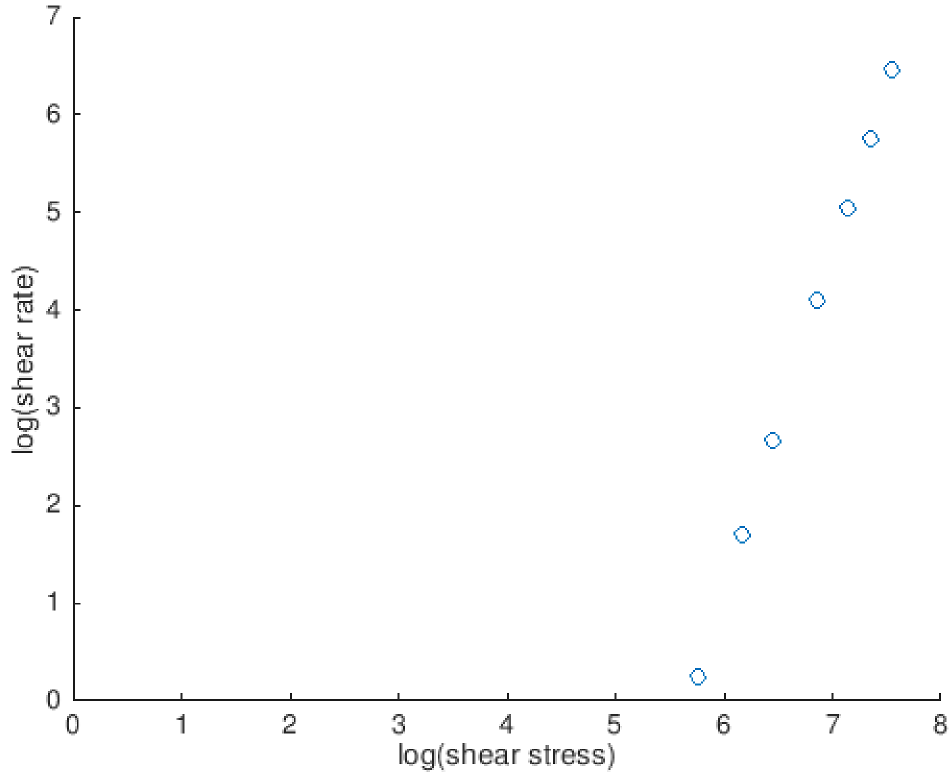
 wall_shear_rate4, wall_shear_rate5, wall_shear_rate6,
 wall_shear_rate7);

fprintf('Part h)\n');
fprintf('Rheological properties:\n');
shear_rate = [wall_shear_rate1, wall_shear_rate2, wall_shear_rate3,
 wall_shear_rate4, wall_shear_rate5, wall_shear_rate6,
 wall_shear_rate7];
shear_rate_log = log(shear_rate);
shear_stress_log = log(shear_stress);
figure
scatter(shear_stress_log, shear_rate_log);
xlim([0,8]);
xlabel('log(shear stress)');
ylabel('log(shear rate)');
p = polyfit(shear_stress_log,shear_rate_log,1);
n = p(1);
k = exp(p(2));
fprintf('n = %f\n', n);
fprintf('k = %f\n\n', k);
```

```
Problem 1
Part a)
Flow rate = velocity * area
Area = pi * (piston radius)^2
Flow rate = [0.000000, 0.000000, 0.000000, 0.000000, 0.000001,
 0.000002, 0.000004] m^3/s
Part b)
Effective force = long tube force - short tube force
It is necessary to calculate effective force for a capillary of
 effective length
because it corrects for the energy losses at the ends of capillaries
 due to
expansion and contraction.
Effective force = [28.720000, 43.520000, 57.290000, 86.850000,
 114.330000, 140.750000, 173.280000] N
Part c)
Pressure drop = tube Force / area
Pressure drop = [26887.823327, 40743.665432, 53635.215822,
 81309.451809, 107036.380257, 131770.930825, 162225.697288] Pa
Part d)
(Pressure drop * radius) / (2 * length) = shear stress
Shear stress = [316.327333, 479.337240, 631.002539, 956.581786,
 1259.251532, 1550.246245, 1908.537615] Pa
Part e)
(4 * flow rate) / (pi * radius ^ 3) = pseudo-shear rate at wall
Pseudo-shear rate = [1.666667, 6.666667, 16.666667, 66.666667,
 166.666667, 333.333333, 666.666667] s^-1
Part f)
Correction factor = (3n + 1)/4n, where 1/n = slope at each flow rate
Correction factor = [0.772184, 0.826839, 0.859092, 0.902975, 0.929175,
 0.947693, 0.965191] (unitless)
Part g)
Wall shear rate = pseudo_shear_rate * correction_factor
```

```
Wall shear rate = [1.286973, 5.512259, 14.318207, 60.198361,
 154.862475,315.897646, 643.460650] s^-1
Part h)
Rheological properties:
n = 3.455698
k = 0.000000
```



Problem 2

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fprintf('Problem 2\n');
radius = 2 / 1000; % mm -> m
length = 50 / 1000; % mm -> m

flow_rate = [0.000001, 0.000005, 0.00001, 0.00005, 0.0001, 0.0005]; %
 m^3/s
pressure_drop = [285000, 589000, 804000, 1660000, 2270000, 4680000]; %
 Pa
fprintf('apparent viscosity = consistency index * shear rate ^ (flow
 index - 1)\n');

apparent_viscosity = (pressure_drop .* pi .* radius ^ 4) ./ (8 .*
 flow_rate .* length);
shear_stress = (pressure_drop * radius) / (2 * length);
pseudo_shear_rate = (4 * flow_rate) / ( pi * radius ^ 3);
```

```
log_shear_stress =
 [log(shear_stress(1)),log(shear_stress(2)),log(shear_stress(3)),log(shear_stress(
log_pseudo =
 [log(pseudo_shear_rate(1)),log(pseudo_shear_rate(2)),log(pseudo_shear_rate(3)),lo
slope = log_pseudo ./ log_shear_stress;
n = 1 ./ slope;
cf = (3 * n + 1) / (4 * n);
shear_rate = pseudo_shear_rate .* cf;
log_rate =
 [log(shear_rate(1)),log(shear_rate(2)),log(shear_rate(3)),log(shear_rate(4)),log(
log_viscosity =
 [log(apparent_viscosity(1)),log(apparent_viscosity(2)),log(apparent_viscosity(3))
figure
scatter(log_rate, log_viscosity);
xlabel('ln(shear rate)');
ylabel('ln(apparent viscosity)');
xlim([0,12]);
p = polyfit(log_rate,log_viscosity,1);
n = p(1);
k = exp(p(2));
fprintf('flow index = %f\n', n);
fprintf('consistency index = %f\n\n', k);
```

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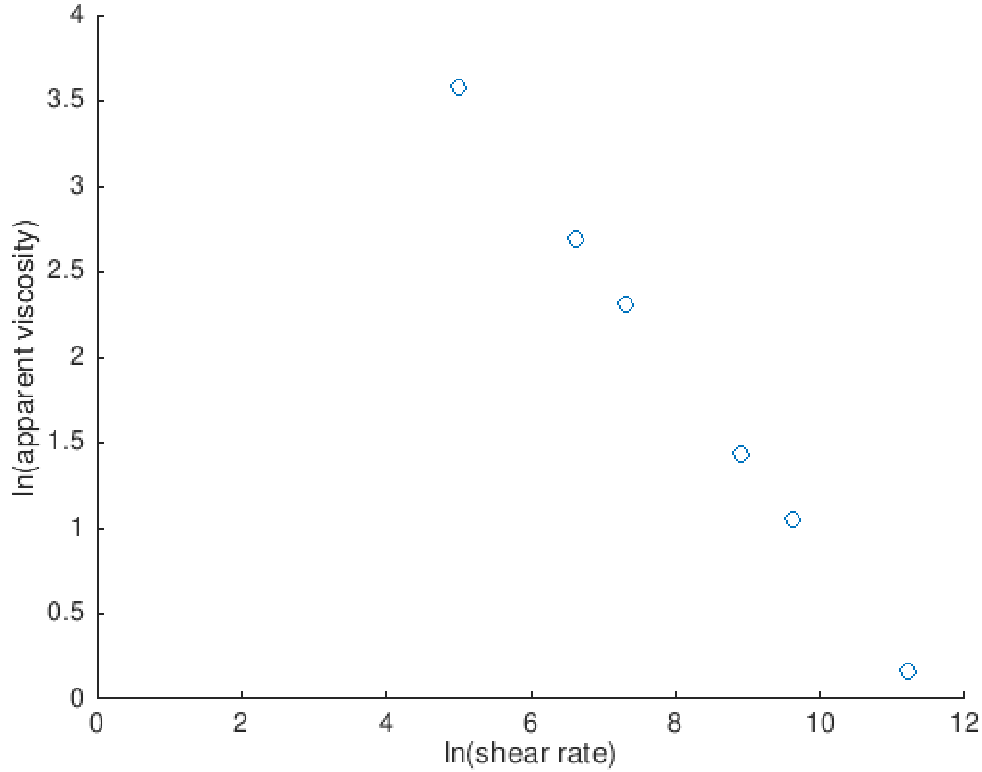
Problem 2
apparent viscosity = consistency index * shear rate ^ (flow index - 1)
flow index = -0.549669
consistency index = 561.614922
```



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