1. A pharmaceutical ointment is made in a laboratory scale colloid mill. The formulation consists of mineral oil of viscosity 1 cp dispersed in an aqueous medium of viscosity 10 cp. The viscosity of the continuous phase is provided by the addition of xanthan gum. The emulsion is stabilized by a combination of phospholipids and fatty acid esters. The interfacial tension of the mineral oil in the formulation is 20 mN/m. The average drop size of the ointment is 0.153 micron. Consumer trials indicated that the ointment is too difficult to spread. It was therefore decided to reduce the viscosity of the continuous phase in the second formulation to 1 cp. In addition, the amount of fatty acid esters was also reduced so that the interfacial tension of the second formulation increased to 30 mN/m. The laboratory scale colloid mill is now operated under the same rotational speed.

a. Calculate the average drop size of the second formulation. (d) (c) oil in water

b. What will be the criteria for scaling up the laboratory scale process to industrial scale if you want to maintain the same average drop size for the formulation?

a) Drop size $q_2 = \frac{\mu_d}{\mu_c} = \frac{1cp}{1cp} = 1$ $We_{crit} = 0.7$

$\gamma_2 = 30\times10^{-3} N/m$ $G = \frac{\pi N D}{h}$ $G_1 = G_2$

$r_1 = \frac{2\gamma_1 We_{crit,1}}{G_1 \mu_{c1}} = 0.153\times10^{-6} m$ $q_2 = \frac{1cp}{10cp} = 0.1$

$We_{crit,1} = 0.9$

$G_1 = \frac{2\gamma_1 We_{crit,1}}{r_1 \mu_{c1}} = \frac{2(20\times10^{-3})(0.9)}{(0.153\times10^{-6})10^3} = 235.294 = G_2$ $\gamma_1 = 20\times10^{-3} N/m$

$r_2 = \frac{2\gamma_2 We_{crit,2}}{G_2 \mu_{c,2}} = \frac{2(30\times10^{-3} N/m)}{235.294\ 10^3} = 1.79\times10^{-7} m$

b) $R = \left(\frac{V_2}{V_1}\right)^{1/3} = \frac{DT_2}{DT_1}$ ← you must scaleup all dimensions by the scaleup ratio R. you must scaleup the rpm using $N_2 = N_1\left(\frac{1}{R}\right)^n$ where n=1 for equal liquid motion

The characteristic curve for the metering section of a single screw extruder relating the volumetric flow rate Q in m^3s^{-1} and the pressure difference $(P_2 - P_1)$ in Pa (where P_1 and P_2 are the inlet and outlet pressures for the metering zone respectively) is given by,

$$Q = 1.13\times10^{-5} N - \frac{1.26\times10^{-10}(P_2 - P_1)}{\eta L}$$

where N is the rotational speed of the screw in rps, η is the apparent viscosity of the dough in the metering section and L is the length of the metering section. The pressure P_1 can be taken as 1 atm. The diameter of the screw $D = 6\ cm$, the channel depth $H = 1\ mm$, the screw speed is 100 rpm, the length of the metering section $L = 60\ cm$ and the temperature of the metering section is 70 C. The apparent viscosity of the dough is given by,

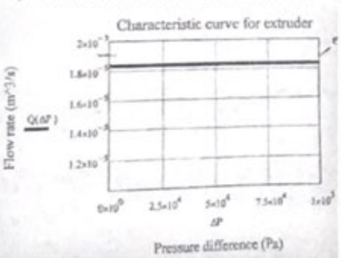
$$\eta = 20(\dot{\gamma})^{-0.4}\exp(2500/T)$$

where $\dot{\gamma}$ is the shear rate and T is the absolute temperature in K. At the end of the metering zone three circular dies are attached. The die diameter is 1 mm, $L_{die}/D_{die} = 8$. The end effects of the die can be accounted for by assuming an equivalent die length of $L_{die}^*/R_{die} = 4$. The characteristic curve for the extruder is shown in the figure below as a plot of Q_{ext}(m^3/s) vs ΔP(Pa).

a) If there are three dies at the end of the metering zone, calculate the volumetric flow rate Q and pressure difference ΔP (operating condition) of the extruder.

b) Calculate the wall shear stress in each die.

1.26×10^{-10}



$P_1 = 1 atm$

$D = 6 = 0.06 m$

$H = 1 = 1\times10^{-3} m$

$N = \frac{100}{60}$

$L = 60cm = 0.6m$

$T = 70°C = 343K$

$D\ d = 1mm = 1\times10^{-3} m$

$\frac{L_d}{D_d} = 8$ $L_d = 0.008m$

$\frac{L_{die}^*}{R_{die}} = 4$ a) $Q_{ex} = 3Q_{die}$ $K = \frac{\pi R^4}{8 L_b}$ $Q_d = \frac{K\Delta P}{\mu_d}$

$\gamma = \frac{3n+1}{4n}\left(\frac{4Q_d}{\pi R^3}\right)$

$\eta = K\gamma^{n-1}$ $n = 0.6$ $\gamma = \frac{\pi N D}{H}$

$\frac{\pi D N}{H} = \frac{3n+1}{4n}\left(\frac{4Q_d}{\pi R^3}\right) \Rightarrow \frac{\pi(0.06)(1.67)}{1\times10^{-3}} = \frac{3(.6)+1\ 4Q_d}{4(.6)\ \pi(0.0005)}$

$Q_d = 1.35\times10^{-5}$ $Q_{ex} = 4.06\times10^{-5} m^3/s$

$4.06\times10^{-5} = 1.13\times10^{-5}\left(\frac{100}{60}\right) - \frac{1.26\times10^{-10}\Delta P}{0.008\eta}$

$\eta = 20\left(\frac{\pi D N}{H}\right)^{-0.4} \exp(2500/T) = 2934.82 Pa.S$

$\Delta P =$

2. concentric cylinder viscometer yield stress - 80Pa

radii inner 1cm outer 1.5cm

length 2cm

inner cyl. rotation @ 0.6 rpm

$T = 1.5\times10^{-3} N.m$

$n = 0.5$

a) wall shear stress @ inner cyl.

$\tau_1 = \frac{T}{2\pi L r^2} = \frac{1.5\times10^{-3}}{2\pi(0.02)(0.01)^2} = 119.36 N/m$

b) calc. shear rate of fluid near inner cylinder

3. viscoelastic material spring constant 2Pa in series w/ dashpot viscosity 5Pa.S. constant strain of 0.1

a) initial stress

$\tau = G\dot{\gamma} \Rightarrow \tau_0 = 2(0.1)$

$\tau_0 = 0.2 Pa$

b) stress after 1 second

$\tau = \tau_0 e^{(-t/\lambda)}$ $\lambda = \frac{\mu}{G}$

$\tau = 0.134 Pa$

c) stress after a long time is 0 (zero)

4. Mechanical analog

G μ_1 $G = 100 Pa$

μ_2 $\mu_1 = 200 Pa.S$ $\mu_2 = 500 Pa.S$

constant stress of 10 Pa

5. data for flow rate in m3/s

Δp (Pa) r=1 mm L=10cm

$Q = 3.662\times10^{-13}\Delta P^{1.429}$

(i) $\tau_0 = 3 Pa$

min ΔP,...

$Q = \frac{\pi n}{(1+3n)}\left(\frac{\Delta P}{2KL}\right)^{1/n} R^{(1/n+3)}$

$\frac{1}{n} = 1.429$ $n = 0.7$

$\Delta P_{min} = \frac{\tau_0 2L}{R} = 600Pa$

$\Delta P = 10^4 Pa$

(ii) shear stress

$\tau_w = \frac{\Delta P}{L}\frac{R}{2} = 50pa$

(iii) pseudo shear rate

$\dot{\gamma}_{Nw} = \frac{4Q}{\pi R^3} = 242.32 1/s$

(iv) correction factor

$C_f = \frac{1}{4}\left(3+\frac{1}{n}\right)$ $C_f = 1.107$

(v) wall shear rate

$\dot{\gamma} = C_f \dot{\gamma}_{Nw} = (1.107)(242.32)$

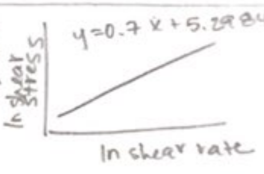
$= 266.759\ 1/s$

$n = 0.7$

$K = e^{5.2984}$

$= 200.0 Pa.S^n$

$y = 0.7x + 5.2984$



A 1.372 m baffled tank with a 0.457 m six-blade open turbine with a liquid depth of 1.372 m. The turbine speed is 75 rpm and the fluid has a viscosity of 3×10^{-3} Pa.s and a density of 1000 kg/m³.

a. Calculate the power input.

b. If the tank is filled with a power law fluid of consistency index of 10^{-2} $Pa\ s^{n}$ and a flow behavior index of 0.5, calculate the power input.

Given $D_t = 1.372 m$ $D_a = 0.457m$ $N = 75 rpm = 1.25$

$\mu = 3\times10^{-3} Pa.S$ $\rho = 1000 kg/m^3$

$P = N_a N D_a^3 \rho \frac{(\pi N D)^2}{2} = \ldots 1.25(0.457)^3 1000 \cdot \frac{[\pi(1.25)(0.457)]^2}{2}$

$P = 96.1 W$

b. $\mu_a = K(11N)^{n-1} = 10^{-2} Pa.s^{1/2}[11(1.25)]^{0.5-1}$

$= 2.70\times10^{-3} Pa.s$

$N_{Re} = \frac{D_a^2 N \rho}{\mu_a} = \frac{(0.457)^2(1.25)(1000)}{2.7\times10^{-3}} = 9.67\times10^4$

$N_p = 1.3$

$P = N_p N^3 D^5 \rho = (1.3)(1.25)^3(0.457)^5(1000)$

$= 50.612 W$

$R_0 = \left[\frac{\tau_1 r^2}{\tau_0}\right]^{1/2} = \left[\frac{119.36}{80}(0.01)^2\right]^{1/2} = 0.0122m$

$N = \frac{0.6}{60}$ $\Omega = 2\pi N$ $\dot{\gamma} = \frac{2\Omega}{n\left[1-\left(\frac{R_0}{R_1}\right)^{-2/n}\right]} = 0.456 1/s$

1. A flat plate dialyzer is being used to extract a toxin from blood. There are 90 channels for blood and 90 channels for dialysate, and the unit is operated in countercurrent exchange. Each channel has a height $2H$, length L, and width W with $W >> 2H$. The dialysate concentration in the inlet is zero. The toxin level in the blood is 3×10^{-3} mole m^{-3}. The toxin level in the blood stream at the outlet from the dialysis unit is 3×10^{-4} mole m^{-3}. The diffusion coefficient of toxin is 5×10^{-10} m²/s. The overall mass transfer coefficient is 4.3×10^{-6} m/s.

Following are the operating conditions of the dialyzer

$Q_b = 8.33\times10^{-6}$ m³s^{-1}

$Q_D = 1.25\times10^{-5}$ m³s^{-1}

$H = 5\times10^{-4}$ m

$W = 0.1$ m

a. Calculate the toxin concentration in the dialysate stream at the outlet.

b. Calculate the log mean driving force.

c. Calculate the length of the dialyzer.

$K_B = 4.3\times10^{-6} m/s$

$C_0 = 0$

$C_B = 3\times10^{-3} mol/m^3$

$C_{BE} = 3\times10^{-4}$

$\delta = 4.3$

a) $Q_b(C_{bin} - C_{bout}) = Q_d C_{dout}$

$C_{dout} = \frac{Q_b}{Q_d}(C_{bin} - C_{bout}) = 1.8\times10^{-3} mol/m^3$

b) $\Delta C_1 = C_{bin} - C_{dout}$

$\Delta C_2 = C_{bout} - C_{din}$ 0

$LMCD = \frac{\Delta C_1 - \Delta C_2}{\ln\left(\frac{\Delta C_1}{\Delta C_2}\right)} = 6.49\times10^{-4} mol/m^3$

c) $A = \frac{Q_b(C_{bin} - C_{bout})}{K.LMCD} = 8.06 m^2$

$\frac{A}{L} = 2(W + 2H)$

a) strain @ t = 25 $\gamma = \frac{\tau_0}{G}\left(1 - \exp\left[\frac{-Gt}{\mu_1}\right]\right) + \frac{\tau_0 t}{\mu_2}$

$\dot{\gamma} = \dot{\gamma}_s + \dot{\gamma}_s + \dot{\gamma}_d$ $\gamma = 0.103$

b) will the sample reach equilibrium strain - no strain will increase linearly w/ time

2. Vegetable oil is emulsified in water (with emulsifiers and proteins) using a colloid mill at a rotational speed of the rotor of 100,000 rpm. The interfacial tension of vegetable oil in water for the formulation is 15 mN/m. The average drop size was found to be 34 μm. The formulation of proteins and emulsifiers is changed. This new formulation results in interfacial tension of 10 mN/m. Please note that the viscosities of vegetable oil and water phases remain the same as before in this new formulation. It was decided to change the rotational speed of rotor to 80,000 rpm for the second formulation. What is the drop size for the second formulation?

1 | 2

$N = 100000 rpm/60$ $N = 80000 rpm/60$

$\gamma = 15 mN/m$ $\gamma = 10 mN/m \rightarrow 10\times10^{-3} N/m$

$r = 34 \mu m$ $r = ?$

$= 3.4\times10^{-6} m$ $r = \frac{2\gamma We_{crit}}{G\mu_c}$ $q = \frac{\mu_d}{\mu_c} = 1$

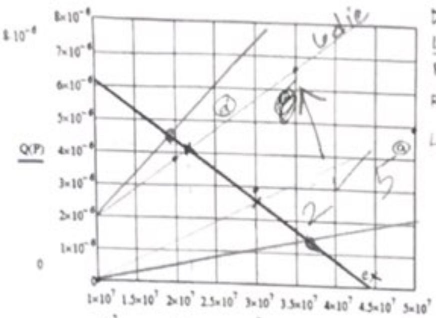
$3.4\times10^{-6} m = \frac{2(15\times10^{-3} N/m)(0.7)}{\left(\frac{\pi N D}{h}\right)\mu_c}$ $We_{crit} = 0.7$

$\Rightarrow \frac{\mu_c}{h}$ plug into

$r = \frac{2(10\times10^{-3} N/m)\ 0.7}{\pi N D\left(\frac{\mu_c}{h}\right)} =$

he characteristic curve for the metering section of a single screw extruder relating the olumetric flow rate Q in $m^3 s^{-1}$ of dough and the pressure difference $P = (P_2 - P_1)$ in Pa where P_1 and P_2 are the inlet and outlet pressures for the metering zone respectively) is lotted below as a plot of volumetric flow rate Q (m^3/s) vs $P = P_2 - P_1$ (Pa). The dough an be considered to be Newtonian with a viscosity of 10^3 Pa.s At the end of the netering zone a circular die is attached. The die diameter is 2 mm, $L_{die}/D_{die} = 5$. The end ffects of the die can be accounted for by assuming an equivalent die length of $L_{die}^*/R_{die} = 4$.

$\mu_d = 10^3$ Pa·s
$D = 2\times10^{-3}$ m
$\frac{L_{die}^*}{R_{die}} = 4$
$R_{die} = 1\times10^{-3}$ m
$L_{die}^* = 4\times10^{-3}$ m



d) $Q_{die} = \left(\dfrac{\frac{\pi(10^{-3})^4}{8(10\times10^{-3})}\Delta P}{3\times10^4\left[\frac{3(0.51)+1}{4(.51)}\left(\frac{4}{\pi(10^{-3})^3}\right)\right]^{.51-1}}\right)^{1/0.51}$ $Q_{die} = (4.44\Delta P)^{1/.51}$

a) If there is one die at the end of the metering zone, calculate the volumetric flow rate Q and pressure difference ΔP (operating condition) of the extruder.
b) Calculate the volumetric flow rate Q and pressure difference ΔP if there are 6 identical dies in the die assembly with the same dimensions as given above.
c) Calculate the wall shear stress experienced by the product for one die and six dies and comment on the effect of number of dies.
d) If the dough can be considered as a power law fluid with the following expression for the apparent viscosity,

$$\eta = 3\times10^4(\dot\gamma)^{-0.49}$$

where $\dot\gamma$ is the shear rate, calculate the volumetric flow rate Q and the pressure difference P for one die at the end of the extruder.

a) $Q_{ex} = G_1 N - \frac{G_2}{\mu}\frac{\Delta P}{L}$ $\boxed{Q_{die} = K_{die}\left(\frac{\Delta P}{\mu_d}\right)}$

$Q_{die} = Q_{ext}$

$K_{die}\left(\frac{\Delta P}{\mu_d}\right) = G_1 N - \frac{G_2}{\mu}\frac{\Delta P}{L}$

$Q_{die} = K_{die}\left(\frac{\Delta P}{\mu_d}\right) = \frac{\pi R^4}{8 L_d}\left(\frac{\Delta P}{10^3}\right) = \frac{\pi(10^{-3})^4}{8(10\times10^{-3})}\left(\frac{\Delta P}{10^3}\right)$

$\frac{\pi R^4}{8 L_d}$ $Q_{die} \approx 3.927\times10^{-14}\Delta P$

$Q_{die} \approx 1.5\times10^{-6}\ m^3/s$ $\Delta P = 3.7\times10^7$ Pa

b) $Q_{ex} = N Q_{die}$ $Q_{ex} = 6 Q_{die}$ $= 6(3.927\times10^{-14})\Delta P$

$Q_{ex} = 6 Q_{die}$

New operat cond. $Q \approx 4.5\times10^{-6}\ m^3/s$
$\Delta P \approx 1.9\times10^7$ Pa

c) $\tau_W = \frac{R\Delta P}{2L_d}$ $\tau_{W,1} = \frac{(1\times10^{-3})(3.7\times10^7)}{2(4\times10^{-3})}$ N/m^2

$\tau_W = \frac{\Delta P}{2\left(\frac{L}{R}+\frac{L^*}{R}\right)}$ $\tau_{W,6} = \frac{(1\times10^{-3})(1.9\times10^7)}{2(4\times10^{-3})}$

more dies
lower τ_W

d) $\eta = K\dot\gamma^{n-1}$ $n = 0.51$ $Q_{ex} = G_1 N - \frac{G_2 \Delta P}{\mu L}$

$Q_{die} = \frac{K_{die}\Delta P}{K\left[\left(\frac{3n+1}{4n}\right)\frac{4Q_{die}}{\pi R_{die}^3}\right]^{n-1}}$ $Q_{die} = \left(\frac{\frac{\pi R^4}{8L_e}\Delta P}{K\left(\frac{3n+1}{4n}\left[\frac{4}{\pi R^3_{die}}\right]\right)^{n-1}}\right)^{1/n}$

A high pressure homogenizer is employed to homogenize milk. The homogenizer pressure is 10^7 Pa. The viscosity of the continuous phase is 10^{-3} Pa.s. The density of the continuous phase is 1000 $kg\,m^{-3}$. The interfacial tension of fat globules in aqueous phase is 15 mN/m. The path length of the homogenizer valve is 1 cm. The size of fat globules in the homogenized milk is found to be 1.85 microns. What will be the fat globule size if the homogenizer pressure is decreased to 2×10^6 Pa?

$P_h = 10^7$ Pa $P_{h_2} = 2\times10^6$ Pa
$\mu_c = 10^{-3}$ Pa.s
$\rho = 1000\ kg/m^3$
$\gamma = 15\times10^{-3}$ N/m
$z = 0.01$ m
$r_1 = 1.85\times10^{-6}$ m

$\bar u_1 = \left(\frac{P_h}{\rho}\right)^{1/2} = \left(\frac{10^7}{1000}\right)^{1/2}$ → 100 m/s

$\theta_1 = \frac{z}{\bar u} = \frac{0.01\,m}{100\,m/s} = 1\times10^{-4}$ s

$\varepsilon_1 = \frac{P_h}{\theta} = \frac{10^7\,Pa}{10^{-4}\,s} = 1\times10^{11}$ Pa.s

decrease P_h

$\bar u_2 = \left(\frac{2\times10^6}{1000}\right)^{1/2} = 44.721$ m/s $\theta_2 = \frac{z}{\bar u} = 2.24\times10^{-4}$ s

$\varepsilon_2 = \frac{P_h}{\theta} = \frac{2\times10^6\,Pa}{2.24\times10^{-4}\,s} = 8.94\times10^9\ Pa\,s^{-1}$

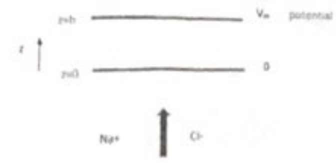
$\lambda_1 = \mu^{3/4}\rho^{-1/2}\varepsilon_1^{-1/4} = 3.16\times10^{-7}$

$r_1 > \lambda_1$ $d_{max} = 4^{3/5}\varepsilon^{-2/5}\gamma^{3/5}\rho^{-1/5}$

$r_2 = 4^{3/5}(8.94\times10^9)^{-2/5}(15\times10^{-3})^{3/5}(10^3)^{-1/5}$

$r_2 = 4.86\times10^{-6}$ m

Consider transport of Na+ and Cl- ions across a membrane as shown schematically below.



A potential gradient is applied across the membrane so that the potential difference across the membrane of thickness h is V_m as shown in the schematic. The diffusion coefficient of cation and anion are D+ and D- respectively. The valence number of cation and anion are z+ and z- respectively. Assume the mass transfer to be one dimensional along coordinate.

a. Write the expression for the flux of Na+ ion across the membrane.
b. Write the electrical neutrality condition.
c. Write mass balance equations for Na+ and Cl- ions.

a) $N_i = -D_i\left(\frac{\partial C_i}{\partial z} - \frac{z_i F C_i}{RT}\frac{\partial \psi}{\partial z}\right)$

$N_{a+} = -D_+\left[\frac{\partial C_{Na^+}}{\partial z} - \frac{z_+ C_+ F}{RT}\frac{\partial\psi}{\partial z}\right]$

b) $z_+ N_+ = z_- N_-$

$z_+ D_+\left(\frac{dC_+}{dz} + \frac{z_+ C_+ F}{RT}\frac{d\psi}{dz}\right) = -z_- D_-\left(\frac{dC_-}{dz} + \frac{z_- C_-}{RT}\frac{d\psi}{dz}\right)$

c) $\frac{dN_{a^+}}{dz} = 0$ $\frac{dN_{Cl^-}}{dz} = 0$

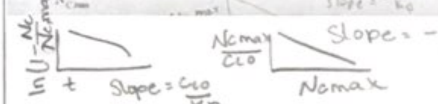
d) $\frac{d\psi}{dz} = -\left(\frac{D_+ - D_-}{z_+D_+ - z_-D_-}\right)\frac{RT}{F}\frac{1}{C_+}\frac{dC_+}{dz}$

$\psi(L) - \psi(0) = \left(\frac{D_+ - D_-}{D_+ + D_-}\right)\frac{RT}{F}\ln\left(\frac{C_0}{C_L}\right)$

Ligand binding to receptors is described by the following

$$L + R \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} C$$

Where L is the ligand, R is the receptor and C is the complex. k_1 and k_{-1} are the forward and backward reaction rate constants. The equilibrium constant $K_D = k_{-1}/k_1$. $k_{-1} = 0.005\ min^{-1}$, $K_D = 1.6\times10^{-10}$ M. The ligand concentration $c_{L0} = 10^{-10}$ M. $N_c(t)$ is the concentration of the complex at time t. $N_{C\,max}$ is the maximum concentration of the complex. Calculate the time it takes for the concentration of the complex to reach a value at which $\frac{N_c(t)}{N_{C\,max}} = 0.9$.



$\ln\left(1 - \frac{N_c}{N_{cmax}}\right) = -k_{-1}\left(1 + \frac{C_{L0}}{K_D}\right)t$

$\ln(1 - 0.9) = -0.005\left(1 + \frac{10^{-10}}{1.6\times10^{-10}}\right)t$

$t \sim 283$ min

A tower having a diameter of 0.1524 m is being fluidized with water at room temperature. The density and viscosity of water are 1000 kg/m^3 and 10^{-3} Pa.s respectively. The beads in the tower bed have a diameter of 4.42 mm and a density of 1603 kg/m^3. The uniform spherical fraction at minimum fluidization is 0.43.

a. Estimate the minimum fluidization superficial velocity using the simplified equation.
b. Set up the equation for the solution of the porosity of the bed at 4 times the minimum fluidization velocity? You do not have to solve the equation.

a) $N_{Re,mf} = \left[(33.7)^2 + 0.0408\,\frac{D_p^3\,\rho(\rho_p-\rho)g}{\mu^2}\right]^{1/2} - 33.7$

particles: $D_p = 4.42\times10^{-3}$ m, $\rho_p = 1603\ kg/m^3$, $\varepsilon_{mf} = 0.43$
water: $\rho = 1000\ kg/m^3$, $\mu = 10^{-3}$ Pa.s

$N_{Re,mf} = \left(33.7^2 + 0.0408\,\frac{(4.42\times10^{-3})^3(1000)(1603-1000)(9.8)}{(10^{-3})^2}\right)^{1/2} - 33.7$

$N_{Re,mf} = 114.545 = \frac{D_p V'_{mf}\rho}{\mu}$

$V'_{mf} = \frac{N_{Re,mf}\,\mu}{D_p\rho} = \frac{114.545\,(10^{-3})}{4.42\times10^{-3}\,m\ 1000\ kg/m^3} = 0.025\ m/s$

b) $V' = K_1\frac{\varepsilon^3}{1-\varepsilon}$ $V' = 4V'_{mf}$

$V'_{mf} = K_1\frac{(\varepsilon_{mf})^3}{1-\varepsilon_{mf}}$ $K_1 = \frac{(V'_{mf})(1-\varepsilon_{mf})}{\varepsilon_{mf}^3} =$

Eqn $4(V'_{mf}) = \left[\frac{V'_{mf}(1-\varepsilon_{mf})}{(\varepsilon_{mf})^3}\right]\frac{\varepsilon^3}{1-\varepsilon}$