

Packed  
Bed

Packed beds are important in fixed bed catalytic reactor, adsorption of a solute, absorption, filter bed etc. A packed column can be considered as a bundle of crooked tubes. Since the geometry is irregular, one needs to calculate the equivalent pipe diameter, which is four times the hydraulic radius.

$$R_H = \frac{\text{Cross sectional area}}{\text{Wetted perimeter}} = \frac{\text{Void volume}}{\text{Wetted Surface area}}$$

$\epsilon = \text{Void volume}$

$D_p = \text{Particle dia}$

$$\frac{S_v}{V_p} = \frac{\text{Surface area}}{\text{particle volume}} = \frac{\pi D_p^2}{\frac{\pi D_p^3}{6}} = \frac{6}{D_p}$$

$$\begin{aligned} \circ \circ \frac{\text{Surface area}}{\text{Volume}} &= \frac{\text{Surface area}}{\text{particle vol}} \frac{\text{particle Volume}}{\text{Volume}} \\ &= \frac{6}{D_p} (1-\epsilon) \end{aligned}$$

$$R_H = \frac{\epsilon}{6(1-\epsilon)/D_p} = \frac{\epsilon D_p}{6(1-\epsilon)}$$

$$D_{eq} = 4R_H = \frac{2\epsilon D_p}{3(1-\epsilon)}$$

$v'$  = superficial velocity that is based on empty cross section

$$= \frac{m}{\rho A_{col}} = \frac{G}{\rho}$$

$m$  = Mass flow rate;  $G$  = Mass velocity

$v$  = actual velocity of the liquid through the column

$$= \frac{v'}{\epsilon}$$

$$Re = \frac{D_{eq} v \rho}{\mu} = \frac{2\epsilon D_p}{3(1-\epsilon)} \frac{v'}{\epsilon} \frac{\rho}{\mu}$$

For laminar flow, one can use Hagen Poissuelle equation to calculate the pressure drop

$$\Delta p = \frac{32\mu v L}{D_{eq}^2} = \frac{32\mu (v'/\epsilon) L}{\left\{ \frac{2\epsilon D_p}{3(1-\epsilon)} \right\}^2}$$

$$\Delta p = \frac{72\mu v' L (1-\epsilon)^2}{\epsilon^3 D_p}$$

Blake Kozny equation. Because of tortuous path of the liquid the effective length is greater than  $L$ . The factor 72 is usually replaced by 150, i.e.

$$\Delta p = \frac{150\mu v' L (1-\epsilon)^2}{\epsilon^3 D_p}$$

For turbulent flow,  $Re > 1000$

$$\Delta P = \frac{2fL V^2}{D_{eq}} = \frac{2fL (V'/\epsilon)^2}{\{2\epsilon D_p / 3(1-\epsilon)\}}$$

$$\Delta P = \frac{3fL V'^2 (1-\epsilon)}{\epsilon^3 D_p}$$

$$3f \approx 1.75$$

$$\Delta P = \frac{1.75 L V'^2 (1-\epsilon)}{\epsilon^3 D_p}$$

Burke-Plummer equation

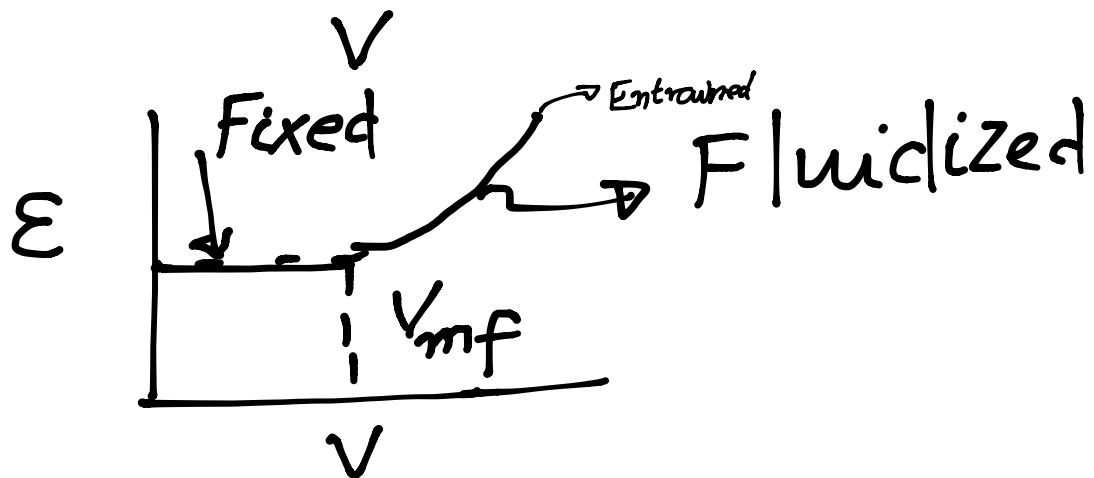
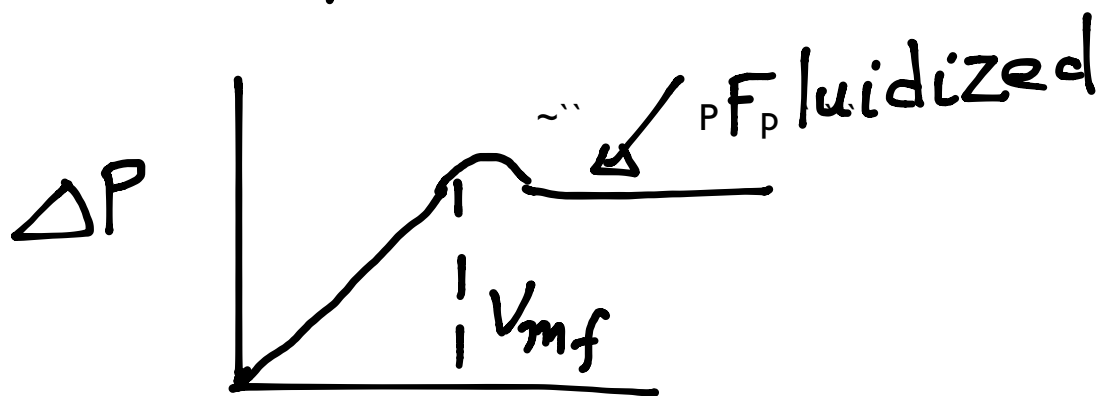
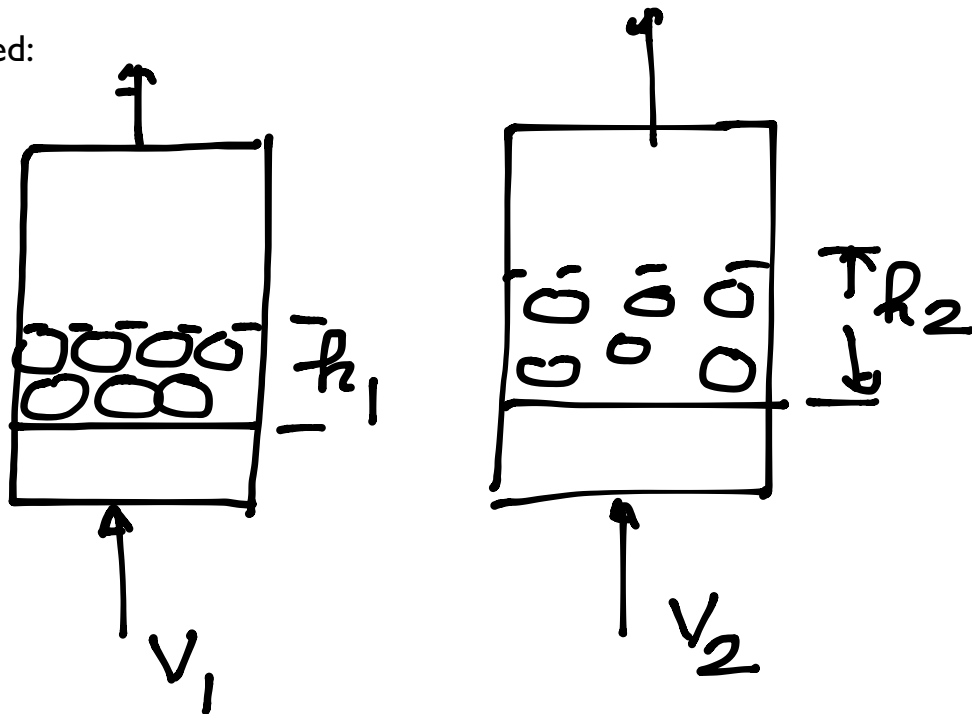
For intermediate Reynolds number, we have,

$$\Delta P = \frac{150 \mu V' L (1-\epsilon)^2}{\epsilon^3 D_p} + \frac{1.75 L V'^2 (1-\epsilon)}{\epsilon^3 D_p}$$

$$\frac{\Delta P}{(\frac{V'}{L})^2} \frac{D_p}{\epsilon^3} = \frac{150}{Re} + 1.75$$

Ergun equation

Fluidized bed:



$$L_1 A (1 - \epsilon_1) = L_2 A (1 - \epsilon_2)$$

$$\frac{L_1}{L_2} = \frac{(1 - \epsilon_2)}{(1 - \epsilon_1)} \quad (1)$$

As you increase the gas flow rate, the void fraction remains constant until the force due to pressure drop is equal to buoyancy of the particles. Beyond this point, further increase in gas flow rate will result in fluidization of particles and the bed will expand.

$$\Delta P A = L_{mf} A (1 - \epsilon_{mf}) (\rho_p - \rho) g \quad (2)$$

From Ergun's equation, we have,

$$\frac{\Delta P}{L} = \frac{150 \mu v' (1 - \epsilon)^2}{\phi_s^2 d_p^2 \epsilon^3} + \frac{1.75 \rho v'^2 (1 - \epsilon)}{\phi_s d_p \epsilon^3} \quad (3)$$

From (2) and (3), we have,

$$\frac{150 \mu v' (1 - \epsilon)^2}{\phi_s^2 d_p^2 \epsilon^3} + \frac{1.75 \rho v'^2 (1 - \epsilon)}{\phi_s d_p \epsilon^3} = (1 - \epsilon_{mf}) (\rho_p - \rho) g$$

which can be rewritten as,

$$\begin{aligned} \frac{1.75 d_p^2 v_{mf}'^2 \rho}{\phi_s \epsilon_{mf}^3 \mu^2} + \frac{150 (1 - \epsilon_{mf}) d_p v_{mf}' \rho}{\phi_s^2 \epsilon_{mf}^3 \mu} - \frac{d_p^3 \rho (\rho_p - \rho) g}{\mu^2} &= 0 \\ N_{Re, mf} &= \frac{d_p v_{mf}' \rho}{\mu} \\ \frac{1.75 (N_{Re, mf})^2}{\phi_s \epsilon_{mf}^3} + \frac{150 (1 - \epsilon_{mf}) N_{Re, mf}}{\phi_s^2 \epsilon_{mf}^3} - \frac{d_p^3 \rho (\rho_p - \rho) g}{\mu^2} &= 0 \quad (4) \end{aligned}$$

$$\phi_s \varepsilon_{mf}^3 \approx \frac{1}{14} \quad \frac{1 - \varepsilon_{mf}}{\phi_s^2 \varepsilon_{mf}^3} \approx 11 \quad (5)$$

$$N_{Re, mf} = \left[ (33.7)^2 + 0.0408 \frac{D_p^3 \rho (\rho_p - \rho) g}{\mu^2} \right]^{1/2} - 33.7 \quad (6)$$

Either eq (4) or simplified eq (6) can be solved for minimum fluidization velocity.

Eq.(6) is valid when  $N_{Re, mf} < 20$

Expansion of fluidized bed:

For  $\frac{D_p v' \rho}{\mu} < 20$ , we can estimate the variation of porosity with bed height  $L$  as follows. Neglecting the first term in eq. (4), we get,

$$N_{Re} = \frac{D_p v' \rho}{\mu} = \frac{D_p^3 \rho (\rho_p - \rho) g}{\mu^2} \frac{\phi_s^2 \varepsilon^3}{150(1 - \varepsilon)}$$

$$\text{or, } v' = \frac{D_p^2 (\rho_p - \rho) g}{150 \mu} \frac{\varepsilon^3}{(1 - \varepsilon)} = K_1 \frac{\varepsilon^3}{(1 - \varepsilon)} \quad (7)$$

$$\frac{L}{L_0} = \frac{(1 - \varepsilon_0)}{(1 - \varepsilon)} \quad (8)$$

Solve eq (7) for  $\varepsilon$  For a given  $v'$ . Solve eq (8) for bed height.  $L$ .