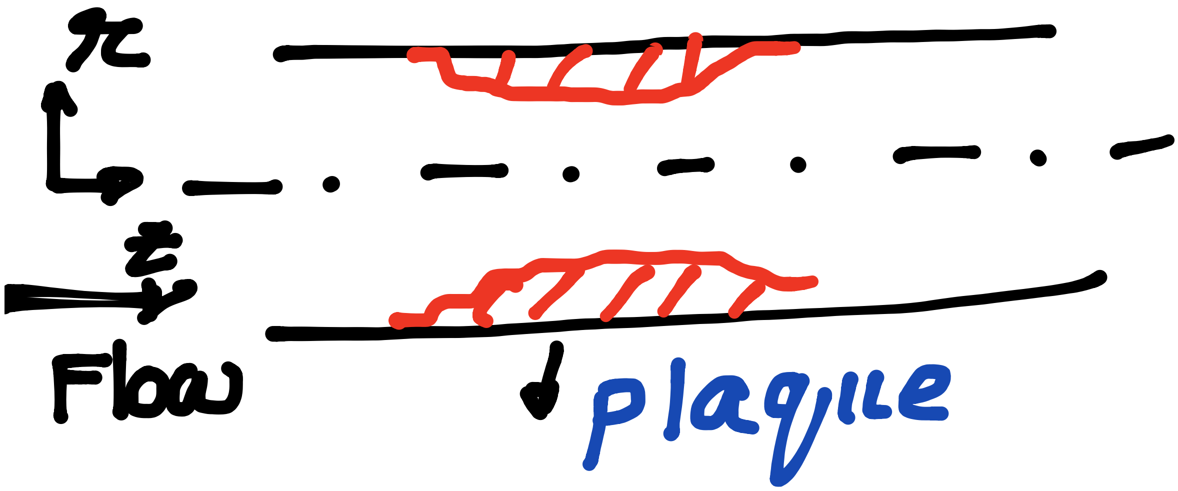
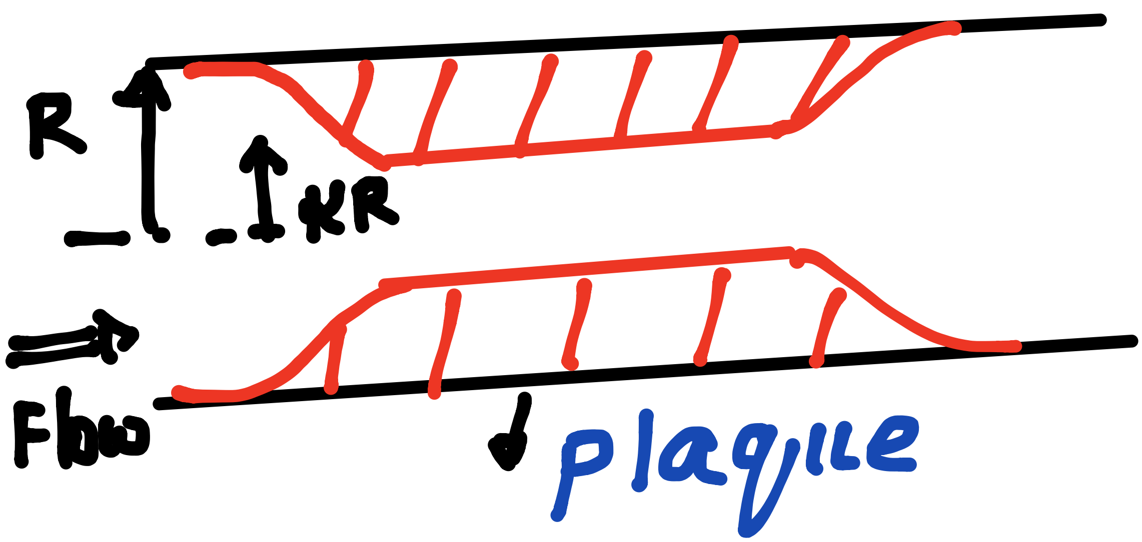
Consider flow of blood through an artery in a region with plaque as shown in the schematic below



As one would expect, because of constriction, the velocity through the plaque region is higher. We would like to explore the consequence. An idealized picture of the plaque region is shown below



One can analyze the flow in the plaque region by making standard assumptions
Steady fully developed flow
Laminar flow
Newtonian fluid

One dimensional flow

$V_z \neq 0 \quad V_r = V_\theta = 0$

Applied pressure gradient is $\frac{\Delta P}{L}$.

Equation of Motion

$$0 = \frac{\Delta P}{L} + \frac{\mu}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right) \qquad (1)$$

B.C. $\quad r = \kappa R \quad V_z = 0 \quad (2)$

$\quad r = 0 \quad \frac{dv_z}{dr} = 0 \quad (3)$

From (1),

$$\frac{d}{dr}\left(r\frac{dv_z}{dr}\right) = -\frac{\Delta P}{\mu L} r$$

$$r\frac{dv_z}{dr} = -\frac{\Delta P}{2\mu L} r^2 + C_1$$

$$\frac{dv_z}{dr} = -\frac{\Delta P}{2\mu L} r + \cancel{\frac{C_1}{r}} \qquad (4)$$

$$V_z = -\frac{\Delta P}{4\mu L} r^2 + C_2$$

$$0 = -\frac{\Delta P}{4\mu L}\kappa^2 R^2 + C_2$$

$$C_2 = \frac{\Delta P}{4\mu L}\kappa^2 R^2$$

$$\therefore \; V_z = \frac{\Delta P}{4\mu L} \kappa^2 R^2 \left[1 - \left(\frac{r}{\kappa R} \right)^2 \right] \quad (5)$$

$$Q = 2\pi \int_0^{\kappa R} r V_z \, dr$$

$$Q = 2\pi \kappa^2 R^2 \int_0^1 \frac{r}{\kappa R} V_z \, d\left(\frac{r}{\kappa R} \right)$$

$$Q = 2\pi \kappa^2 R^2 \int_0^1 x \frac{\Delta P \kappa^2 R^2}{4\mu L} (1 - x^2) \, dx$$

$$Q = \frac{2\pi \kappa^4 R^4 \Delta P}{4\mu L} \int_0^1 (x - x^3) \, dx$$

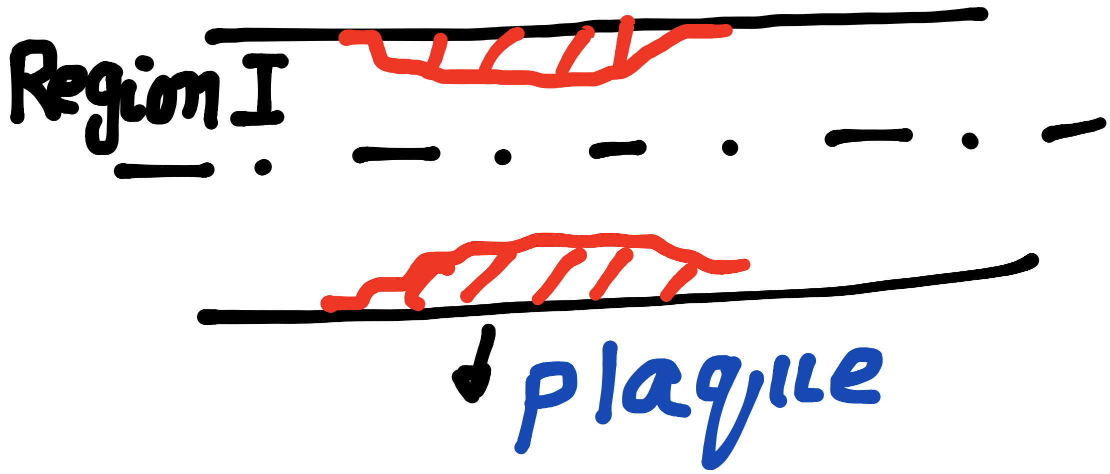
$$Q = \frac{2\pi \kappa^4 R^4 \Delta P}{4\mu L} \left(\frac{x^2}{2} - \frac{x^4}{4} \right) \Big|_0^1$$

$$Q = \frac{\pi \kappa^4 R^4 \Delta P}{8\mu L}$$

Wall shear stress

$$\tau_{rz} \Big|_{r = \kappa R} = -\mu \frac{dV_z}{dr} \Big|_{r = R} = -\mu \left(-\frac{\Delta P}{2\mu L} \kappa R \right) = \frac{\Delta P}{2L} \kappa R$$

$$\tau_w = \frac{\Delta P}{2L} \kappa R = \frac{4\mu Q}{\pi \kappa^3 R^3}$$

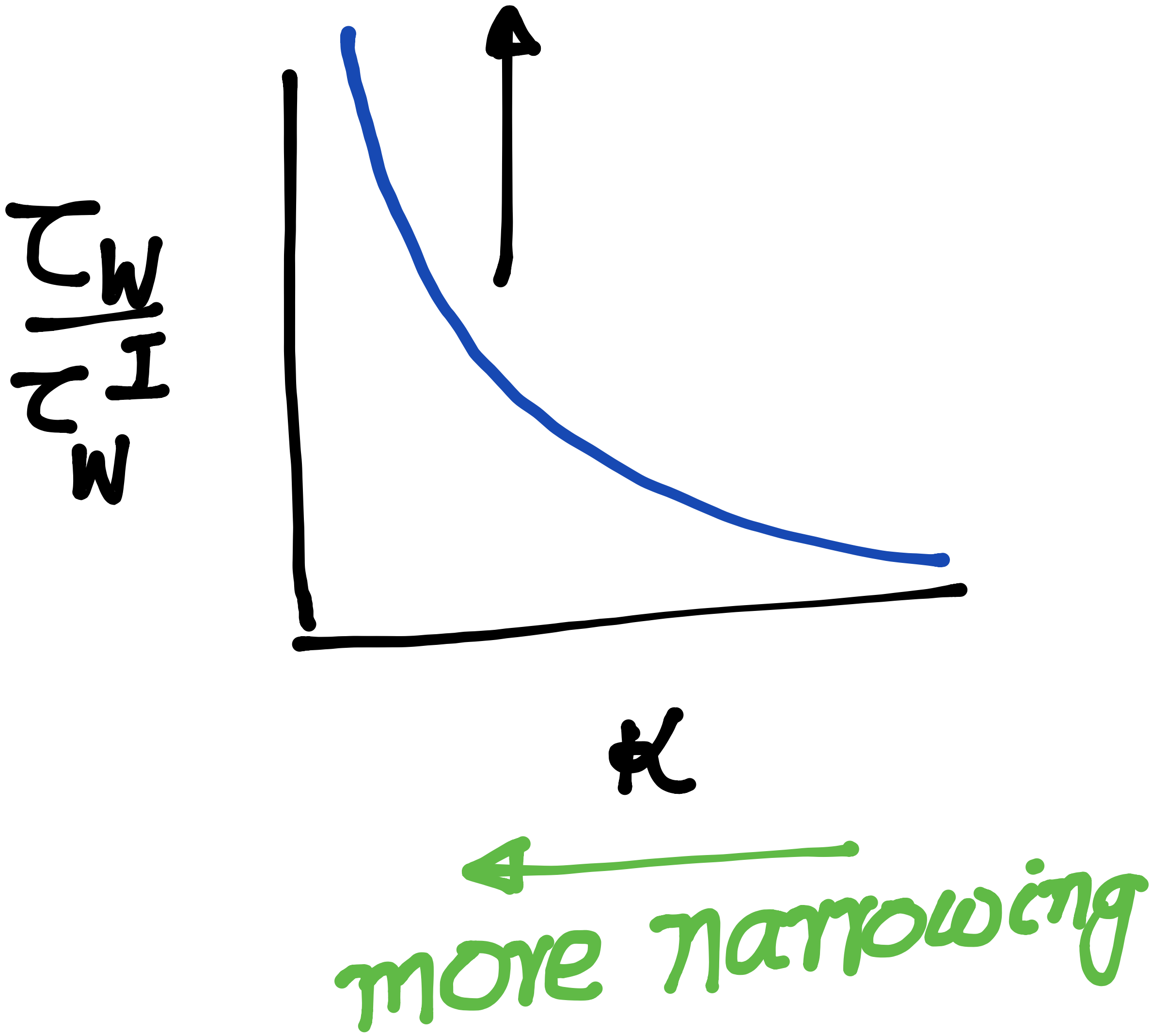


One can now evaluate the wall shear stress in the arterial region without plaque (region I). By mass balance, the flow rate in region I is the same as that in the plaque region. Therefore, the wall shear stress in region I is given by

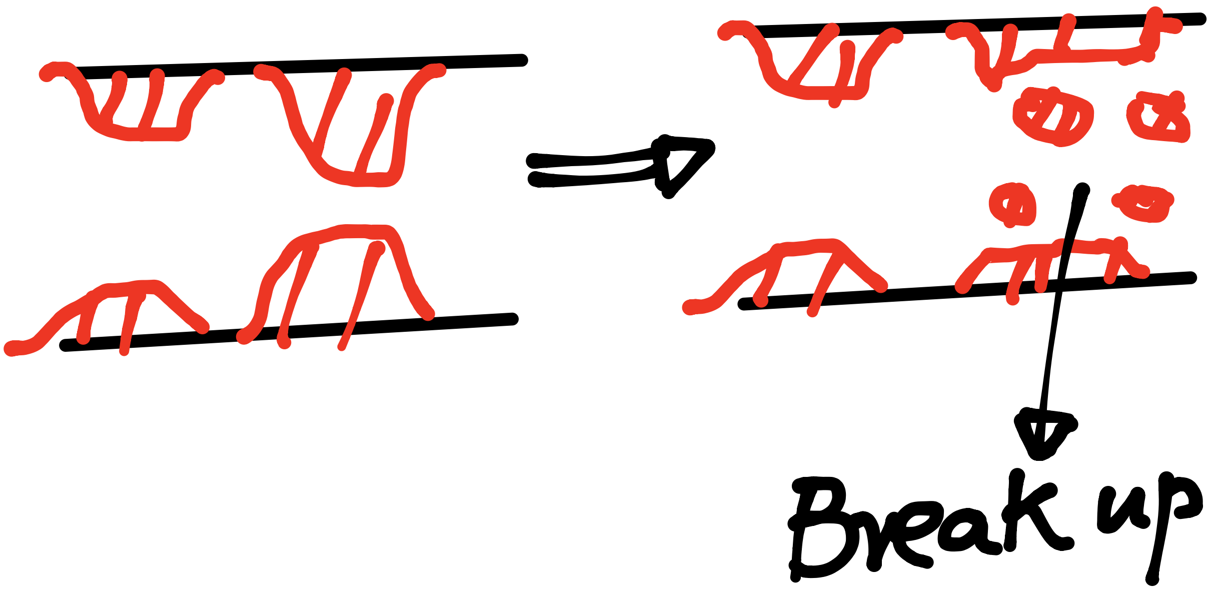
$$\tau_w^I = \frac{4\mu Q}{\pi R^3}$$

The ratio of wall shear stress in two regions is given by

$$\therefore \frac{\tau_w}{\tau_w^I} = \frac{1}{\kappa^3} > 1 \quad \text{since } \kappa < 1$$



$\frac{\tau_W^I}{\tau_W}$
κ
more narrowing



Because of very high wall shear stress, the narrow plaque breaks up and release it into bloodstream. This can result in stroke.