Solution to Homework 7
ABE 457
Spring 2017

Problem 1 : Plot the characteristic curves of the metering zone and the die vs P

The dimensions of the metering section of the single screw extruder

$D := 5\cdot 10^{-2}$ m $\qquad e := 4.52\cdot 10^{-3}$ m $\qquad N := \frac{100}{60}$ $\qquad L := 0.2($ m

$H := 5\cdot 10^{-3}$ m $\qquad \theta := 18\cdot\frac{\pi}{180}$ $\qquad \delta := 3\cdot 10^{-4}$ m $\qquad \pi := 3.1$

Cooked corn dough: $\qquad M := 2$ % $\qquad t := 4($ C

$n := 0.$

$\sin(\theta) = 0.309 \qquad T := 273 + t$

$\cos(\theta) = 0.951 \qquad W := \pi\cdot D\cdot\sin(\theta) - e$

$$\mu := 60.0\cdot\left(\frac{\pi\cdot D\cdot N}{H}\right)^{n-1}\cdot\exp\left(\frac{2500}{T}\right)\cdot\exp(-0.06M)$$

$$\mu\delta := 60.0\cdot\left(\frac{\pi\cdot D\cdot N}{\delta}\right)^{n-1}\cdot\exp\left(\frac{2500}{T}\right)\cdot\exp(-0.06M)$$

$\mu = 1.355\times 10^{4}$ Pa.s

$\mu\delta = 5.827\times 10^{3}$ Pa.s

$$G1 := \frac{\pi}{2}\cdot D^2\cdot H\cdot\left(1 - \frac{e}{\pi\cdot D\cdot\sin(\theta)}\right)\cdot\sin(\theta)\cdot\cos(\theta)$$

$$G2 := \frac{\pi}{12}\cdot D\cdot H^3\cdot\left(1 - \frac{e}{\pi\cdot D\cdot\sin(\theta)}\right)\cdot\sin(\theta)^2$$

$$G3 := \left(\frac{\delta}{H}\right)^3\cdot\frac{e}{W} \qquad G4 := 1 + \frac{e}{W}$$

$$G5 := \frac{-6\cdot\pi\cdot L\cdot D\cdot(H-\delta)}{H^3\cdot\tan(\theta)}$$

$$G6 := \frac{1+\frac{e}{W}}{\tan(\theta)^2}$$

$$G7 := \left(\frac{H}{\delta}\right)^3\cdot\frac{e}{W}$$

$$Q(P) := G1\cdot N\cdot\left(1-\frac{\delta}{H}\right)-\frac{G2}{\mu}\left(\frac{P}{L}\right)\cdot\left[1+G3\cdot\frac{\mu}{\mu\delta}+G4\cdot\frac{\left(\frac{-G5\cdot\mu\cdot N}{P}\right)+G6}{1+G7\cdot\frac{\mu\delta}{\mu}}\right]$$

$Ld := 1.0\cdot10^{-2}$ m *Length of die*

$Dd := 2.0\cdot10^{-3}$ m *Diameter of cylindrical die*

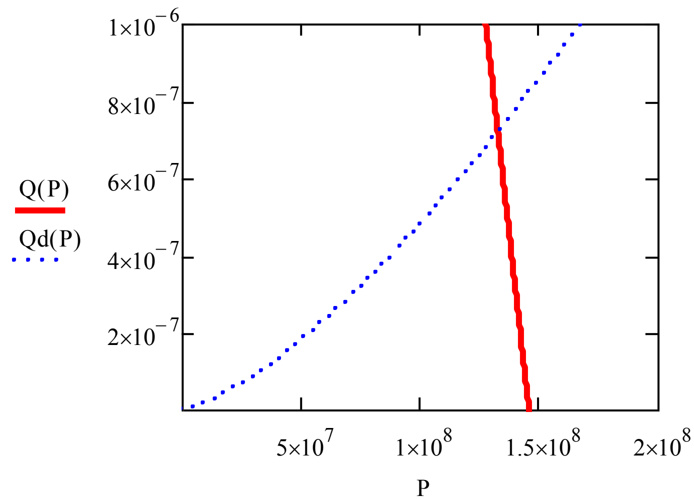
$$Rd := \frac{Dd}{2}$$

$$Qd(P) := \frac{\pi^{\frac{1}{n}}\cdot Rd^{\frac{4}{n}}}{8^{\frac{1}{n}}\cdot Ld^{\frac{1}{n}}}\cdot\frac{P^{\frac{1}{n}}}{78.5^{\frac{1}{n}}\cdot\left(\frac{3\cdot n+1}{\pi\cdot n\cdot Rd^3}\right)^{\frac{n-1}{n}}\cdot\exp\left(\frac{2500}{n\cdot T}\right)\cdot\exp\left(\frac{-0.06\cdot M}{n}\right)}$$

setting up equations 30 points

$$Qd\left(10^7\right) = 1.774\times10^{-8}$$

$$Q\left(10^8\right) = 2.538\times10^{-6}$$

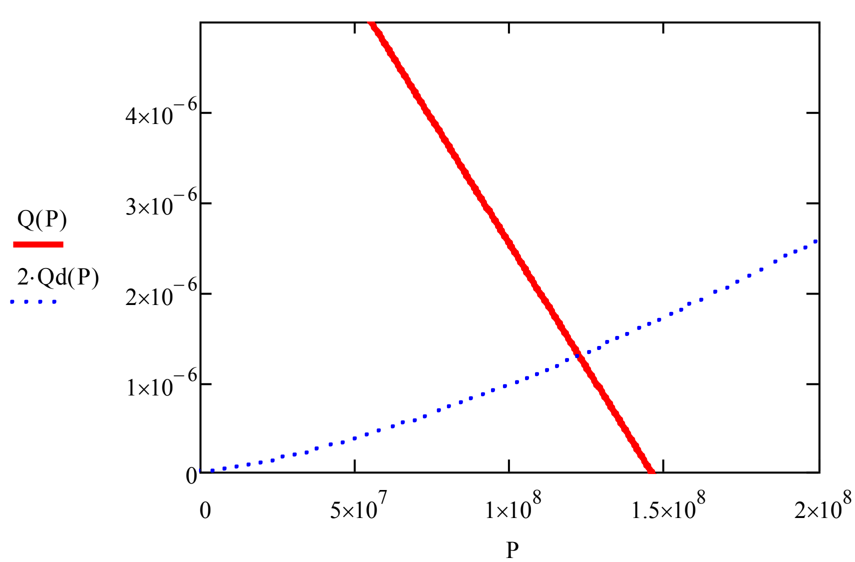


10 points for plot
5 point for discussion

<u>Problem 2: Different numbers of die</u>

5. If there are two dies at the end of the metering zone, the flow rate through a die is one half the flow rate through the metering zone. Alternatively, the flow rate through the metering zone is twice the flow rate through the die. In order to find out the operating condition of the extruder, we need to plot Q vs deltaP and 2Qdie versus deltaP and determine the point of intersection. A sample calculation for a die of 2 mm dia is shown below. Similar plots can be made for the other two die diameters.

10 points for plot
5 point for discussion



Comparing this figure with the earlier figure, one can see that the flow rate through the extruder has increased to 10^-6 m^3/s with a corresponding decrease in pressure drop compared to the case of a single die. For a maximum shear stress of 5 x 10^5 Pa one can calculate the corresponding maximum shear rate and flow rate.

$\tau max := 5 \cdot 10^{5}$ Pa $M = 23$ %

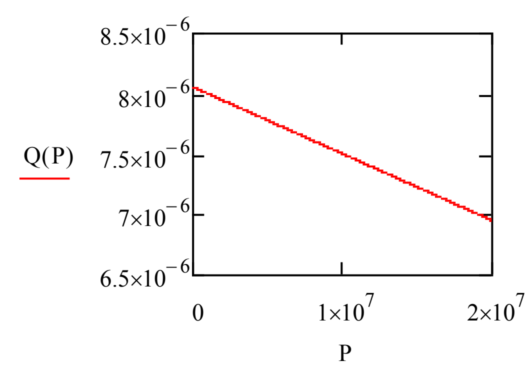
$T = 313$ K

$K := 60.0 \cdot \exp\left(\frac{2500}{T}\right) \cdot \exp(-0.06 \cdot M)$

$Rd = 1 \times 10^{-3}$

$K = 4.443 \times 10^{4}$ Pa.s^n

$\gamma max := \left(\frac{\tau max}{K}\right)^{\frac{1}{n}}$



$\gamma max = 31.762$ s^-1

*gamma = (4Q/pi*R**3)*(3n+1)/4n. Therefore,*

$Qmax := \gamma max \cdot \left(\frac{4 \cdot n}{3 \cdot n + 1}\right) \cdot \left(\frac{\pi \cdot Rd^{3}}{4}\right)$

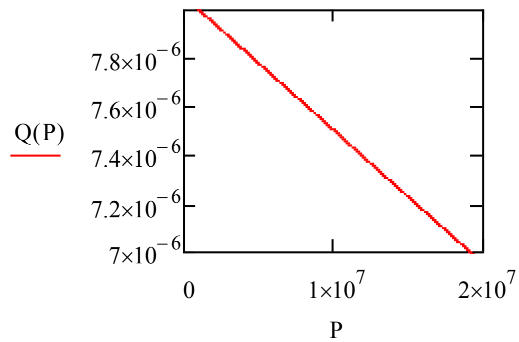
5 points

$Qmax = 2.252 \times 10^{-8}$ m^3/s

Corresponding to maximum wall shear stress, one can calculate the maximum pressure drop through the die as,

$$\Delta Pmax := \frac{\tau max \cdot 2 \cdot Ld}{Rd}$$

$$\Delta Pmax = 1 \times 10^{7} \quad \text{Pa}$$

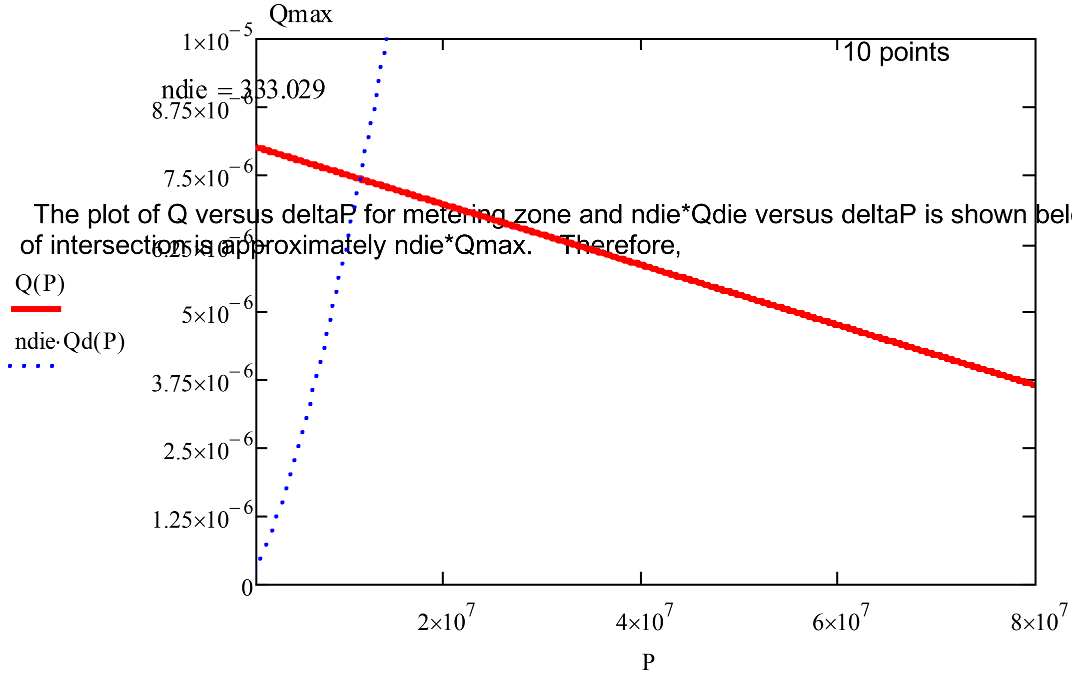


From the characteristic curve of extruder, as shown above, the corresponding flow rate through the extruder is 7.5*10^-6 m^3/s. Therefore, the minimum number of dies ndies can be calculated as,

$$ndie := \frac{7.5 \cdot 10^{-6}}{Qmax}$$

$$ndie = 333.029$$

The plot of Q versus deltaP for metering zone and ndie*Qdie versus deltaP is shown below. The point of intersection is approximately ndie*Qmax. Therefore,



$$Qdie := \frac{3.6 \cdot 10^{-6}}{ndie}$$

5 points

$$Qdie = 1.081 \times 10^{-8}$$

7. derivation - 20
Plot for extruder-10
Plot for die-5