Porous media are solid materials with internal pore structures. Porous structures vary significantly among different media as shown below.

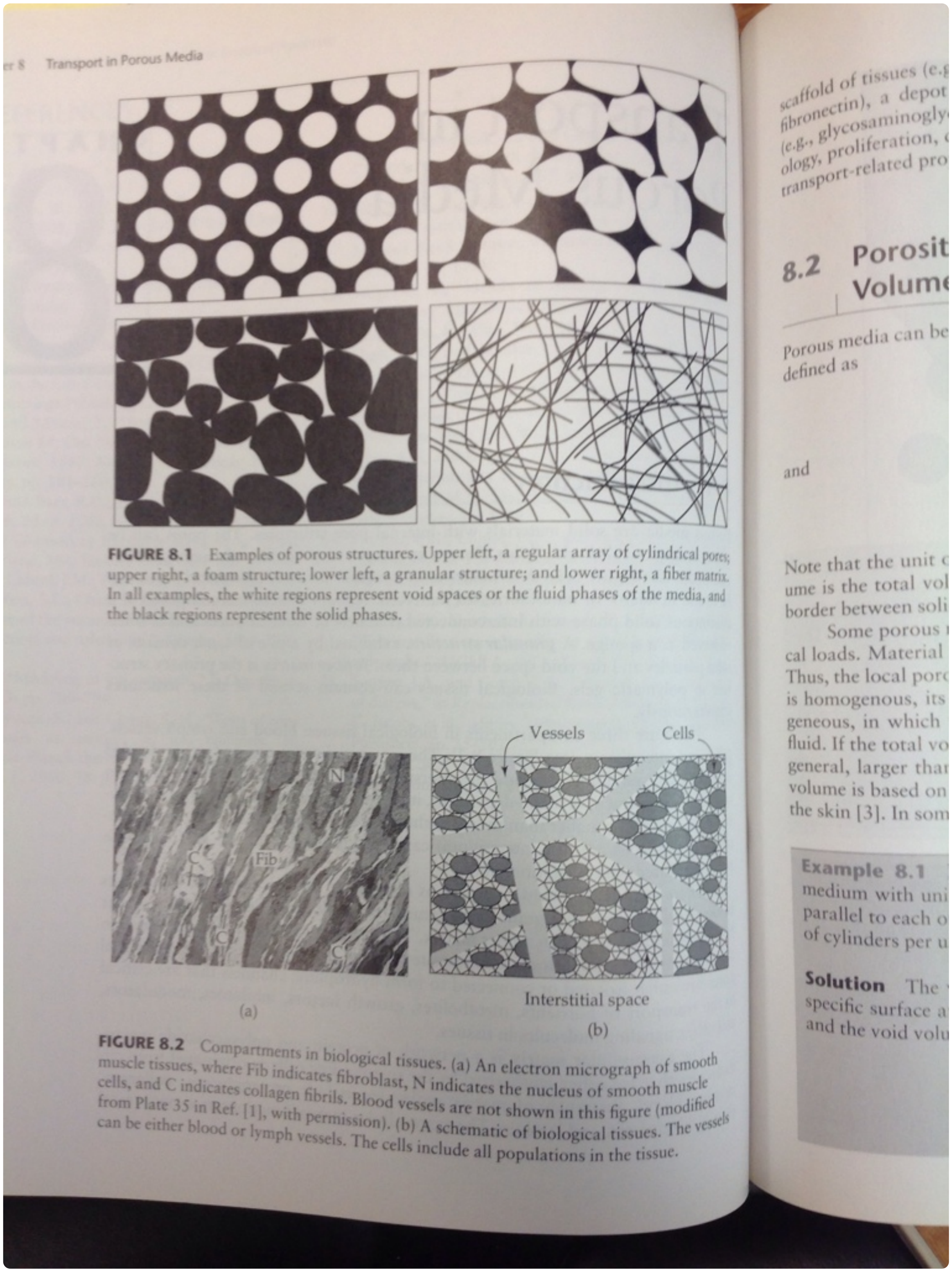


FIGURE 8.1 Examples of porous structures. Upper left, a regular array of cylindrical pores; upper right, a foam structure; lower left, a granular structure; and lower right, a fiber matrix. In all examples, the white regions represent void spaces or the fluid phases of the media, and the black regions represent the solid phases.

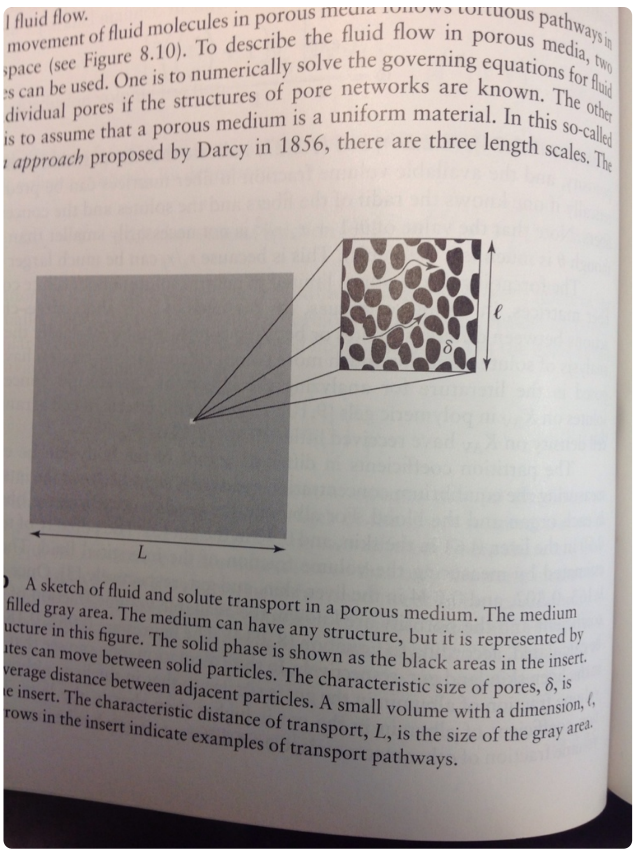
FIGURE 8.2 Compartments in biological tissues. (a) An electron micrograph of smooth muscle tissues, where Fib indicates fibroblast, N indicates the nucleus of smooth muscle cells, and C indicates collagen fibrils. Blood vessels are not shown in this figure (modified from Plate 35 in Ref. [1], with permission). (b) A schematic of biological tissues. The vessels can be either blood or lymph vessels. The cells include all populations in the tissue.

A regular array of cylindrical pores can be found in micro or nanofabricated materials. A foam structure is composed of a continuous solid phase with interconnected channels or isolated pores and is often observed as a sponge. A granular structure exhibited by a pile of sand, consists of solid particles and the void space beteen them. A fiber matrix is the primary structure in polymer gels. Biological tissues can contain seeal of these structures simultaneously.

Definitions:

specific surface s = Total interface area/total volume

porosity ε = void volume/total volume



l fluid flow.
movement of fluid molecules in porous media follows tortuous pathways in
space (see Figure 8.10). To describe the fluid flow in porous media, two
es can be used. One is to numerically solve the governing equations for fluid
dividual pores if the structures of pore networks are known. The other
is to assume that a porous medium is a uniform material. In this so-called
approach proposed by Darcy in 1856, there are three length scales. The

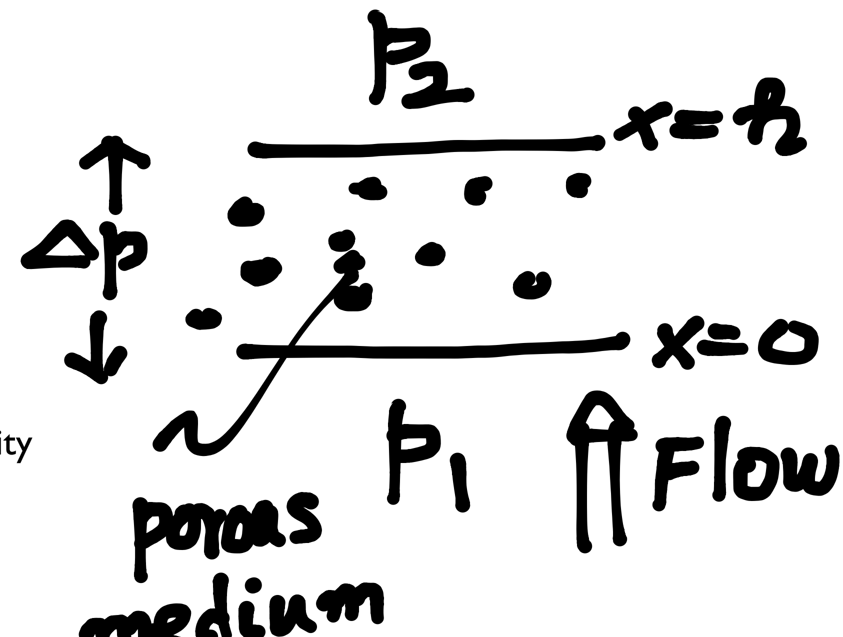
A sketch of fluid and solute transport in a porous medium. The medium
filled gray area. The medium can have any structure, but it is represented by
ucture in this figure. The solid phase is shown as the black areas in the insert.
ites can move between solid particles. The characteristic size of pores, δ, is
verage distance between adjacent particles. A small volume with a dimension, ℓ,
e insert. The characteristic distance of transport, L, is the size of the gray area.
rows in the insert indicate examples of transport pathways.

One can average all the properties over a length scale of l to obtain smooth volume averaged properties. Therefore, one can still consider the porous medium as a continuum over length scale l .

$\underset{\sim}{v}$ = velocity of fluid volume averaged over ℓ^3.

$\underset{\sim}{v}_f$ = velocity of fluid volume averaged through pores.

$\underset{\sim}{v} = \varepsilon \underset{\sim}{v}_f$



Continuity:

$$\nabla \cdot \underset{\sim}{v} = 0$$

Darcy's law:

$$\underset{\sim}{v} = -K \nabla p$$

K = hydraulic conductivity

Substituting in equation of continuity, one obtains,

$$\nabla \cdot \{-K \nabla p\} = 0$$

For constant K (homogeneous medium), one obtains,

$$\nabla \cdot \nabla p = 0$$

$$\boxed{\nabla^2 p = 0}$$ *Laplace Equation*

One dimensional flow in rectangular coordinates:

$$\frac{d^2 p}{dx^2} = 0$$

$$\frac{dp}{dx} = c_1 \qquad p = c_1 x + c_2$$

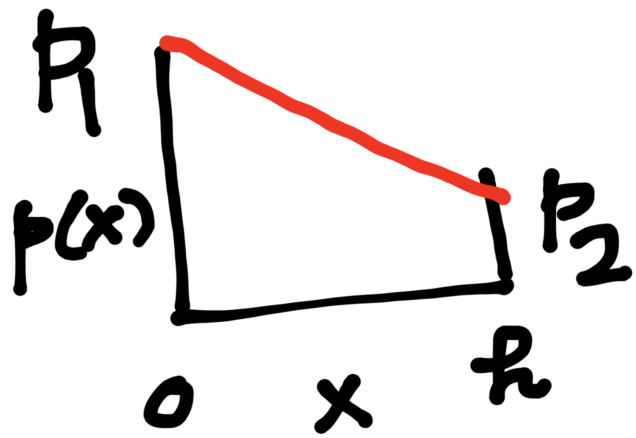
$$x = 0 \quad p = p_1 \Rightarrow c_2 = p_1$$

$$x = h \quad p = p_2$$

$$\therefore \quad p_2 = c_1 h + p_1$$

$$c_1 = (p_2 - p_1)/h$$

$$\therefore \quad p = p_1 - (p_1 - p_2)\left(\frac{x}{h}\right)$$

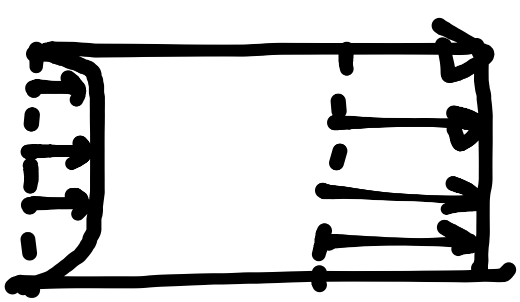


$$V_x = -K\frac{dp}{dx} = -Kc_1$$

$$V_x = K\frac{(p_1 - p_2)}{h} \quad \text{Flux}$$

Note that V_x is constant. Therefore, $V_{fx} = \frac{V_x}{\varepsilon}$ is also a constant. In other

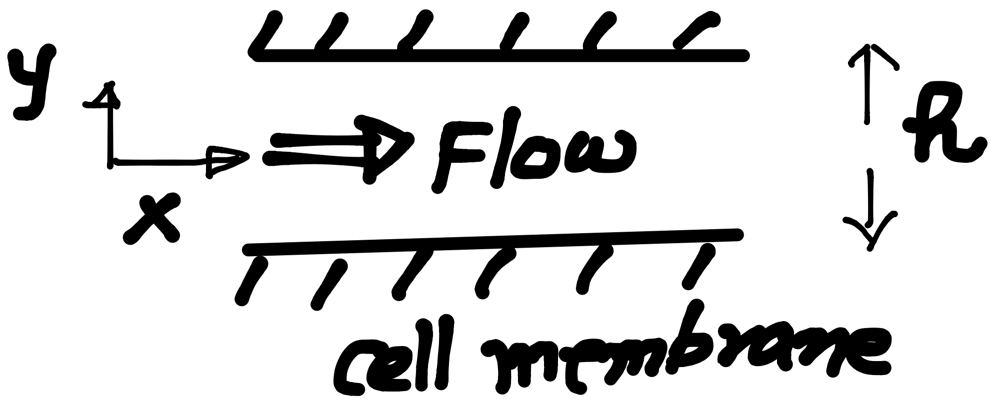
the resistance to flow near the wall is much greater than the viscous resistance. Therefore, if you plot the velocity profile within a pore, you will obtain a flat velocity profile with a steep gradient near the wall. Therefore, Darcy's law does NOT satisfy NO SLIP condition. Darcy's law is valid only for dense porous materials or for low values of hydraulic conductivity K.



Darcy's law is applicable only when the porous medium is dense, I.e. The permeability is low. When the resistance of fluid flow is not large, the approximation of no slip at the wall breaks down. Consequently, one cannot use Darcy's law. The equation of motion has to be modified as

$$\underbrace{\mu \nabla^2 \underset{\sim}{v}}_{\text{Viscous resistance}} - \frac{1}{K}\underset{\sim}{v} - \nabla p = 0$$

Let us examine interstitial flow through cells. The two cell membranes are separated by h (see schematic below).



$$\frac{\partial v_x}{\partial x} = 0 \qquad (1)$$

$$\frac{\partial p}{\partial y} = 0 \qquad (2)$$

Lubrication approximation

$$\mu \frac{\partial^2 v_x}{\partial y^2} - \frac{\mu}{k} v_x - \frac{\partial p}{\partial x} = 0 \qquad (3)$$

or,

$$\underbrace{\mu \frac{\partial^2 v_x}{\partial y^2} - \frac{\mu}{k} v_x}_{\text{function of } y} \qquad (3a)$$

$$= \underbrace{\frac{\partial p}{\partial x}}_{\text{function a } x}$$

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$$\therefore \mu \frac{\partial v_x}{\partial y^2} - \frac{\mu}{k} v_x$$

$$= \frac{\partial p}{\partial x} = B$$ constant

$$\therefore \quad \frac{d^2 V_x}{dy^2} - \frac{1}{R} V_x = \frac{B}{\mu} \qquad (4)$$

The solution to the above equation is

$$V_x = C_1 \sinh\left(\frac{y}{\sqrt{R}}\right) + C_2 \cosh\left(\frac{y}{\sqrt{R}}\right) - \frac{R}{\mu} B \qquad (5)$$

B.C.:- $y = 0 \qquad \frac{dV_x}{dy} = 0 \quad (6)$

$$y = h \qquad V_x = 0 \qquad (7)$$

$$V_x = -\frac{k}{\mu} B \left[1 - \frac{\cosh\left(y/\sqrt{k}\right)}{\cosh\left(h/2\sqrt{k}\right)} \right] \qquad (8)$$

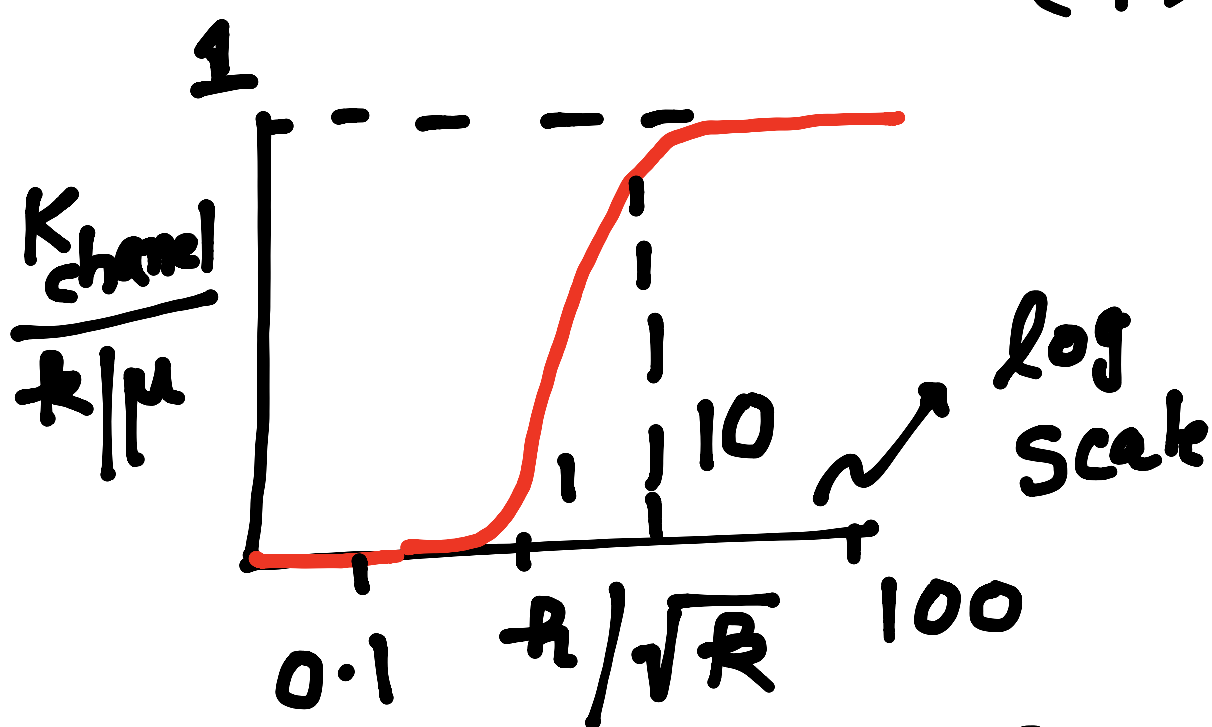
One can evaluate the volumetric flow rate per unit cross sectional area from

$$q = \frac{1}{h} \int_{-h/2}^{h/2} V_x \, dy$$

$$= -\frac{k}{\mu} B \left[1 - \frac{2\sqrt{k}}{h} \tanh\left(\frac{h}{2\sqrt{k}}\right) \right]$$

The effective hydraulic conductivity is defined as the ratio of fluid flux to pressure gradient and is given by

$$K_{channel} = \frac{k}{\mu}\left[1 - \frac{2\sqrt{k}}{h}\tanh\left(\frac{h}{2\sqrt{k}}\right)\right] \quad (9)$$



If $\frac{h}{\sqrt{k}} > 10$, $K_{channel} \simeq \frac{k}{\mu}$

Darcy's Law Valid