2.4 RLC Circuit Model

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- Sum the voltage drops:

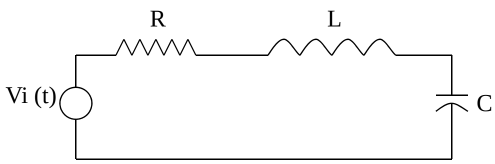
  $V_{in} - V_R - V_L - V_C = 0$

- Use table for relationships:

  $V_{in} - R\,i - L\,di/dt - 1/C \int i\,dt = 0$

- Substitute:

  $i = dq/dt$ q: electric charge [coulomb]



$$\boxed{L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = V_{in}}$$

2.4 RLC, Capacitor voltage as output

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- Using table again:
  - ➢ $V_c = 1/C \int i\,dt = q/C$

- Final differential equation:

$$\boxed{LC\frac{d^2V_C}{dt^2} + RC\frac{dV_C}{dt} + V_C = V_{in}}$$

- Circuit and equation is easily verified in the lab. Remember the basic form.

2.4 Important Modeling Point: Analogies

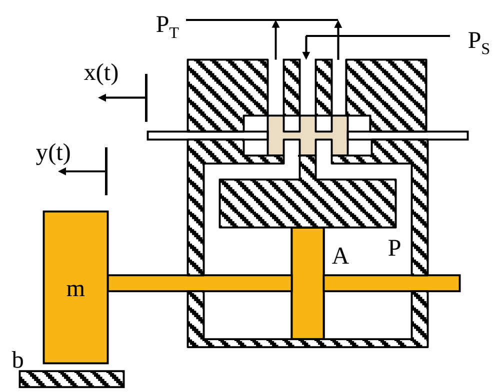
- Notice the RLC and MBK equations:

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{1}{C}q=V_{in} \qquad m\frac{d^2y}{dt^2}+b\frac{dy}{dt}+k\cdot y=F$$

- Same analogies made in table of energy relationships (i.e. mass = inductor)
- Two energy storage devices, both 2nd order systems.
- It is important to feel comfortable with modeling different system types.

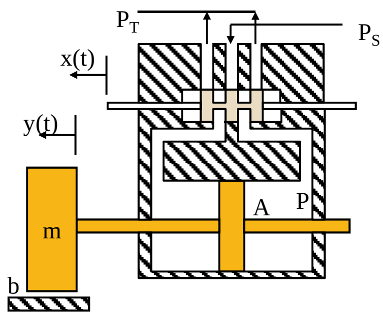
2.4 Hydraulic Positioning Example

- Force Amplifier
  - No feedback for the case shown here.
- Assume basic linear equations.
  Later sections derive them.
- Model includes several domains.



2.4 Hydraulic Servo - Equations

- Valve flow: Q = (dQ/dx) x – (dQ/dP) P = K_x x – K_p P
- Piston flow: Q = A dy/dt
- Force balance: $\sum$ F = m y” = P A – b y’
  - P = load pressure



Solve valve equation for P:

P = -Q/K_P + (K_x/K_P) x

Substitute into force balance:

$\sum$ F = m y” = (-Q/K_P + (K_x/K_P) x) A – b dy/dt

Eliminate Q with piston flow equation:

m y” = (-A y’/K_p + (K_x/K_P) x) A – b y’

Combine inputs and outputs:

$$\boxed{m\frac{d^2y}{dt^2}+\left(\frac{A^2}{K_P}+b\right)\frac{dy}{dt}=\frac{AK_x}{K_P}x}$$

2.4 Hydraulic Servo - Summary

- Equation of motion:
- Type 1 system:
  - y is not present
  - x must be integrated to get y
- Results in motion whenever x $\neq$ 0
- Already has been linearized
  - About what point and how?
  - We will soon see

$$m\frac{d^2y}{dt^2}+\left(\frac{A}{K_P}+b\right)\frac{dy}{dt}=\frac{AK_x}{K_P}x$$

2.4 Hydraulic Servo - Assumptions

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- Linearized about some point.
- Spool dynamics and mass ignored
  - Would require additional force balance
- Mass-less fluid
- Incompressible fluid
- Without these assumptions, result would be a sixth order non-linear differential equation.

2.4 Thermal System Example

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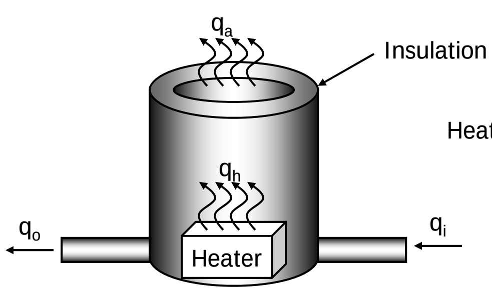
- Common home water heater model
  - Assume uniform water temperature inside the heater.
  - See textbook pg 55



Heat added – removed = Heat stored

$$heat\ flow = \frac{\Delta Temperature}{R}$$

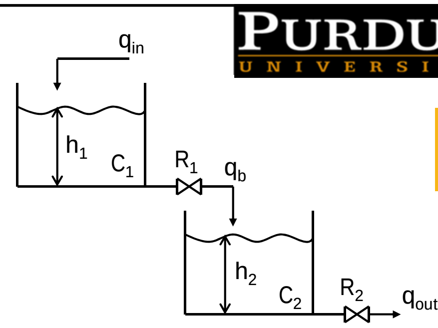
$$heat\ stored = C \cdot \Delta Temperature$$

2.4 Liquid Level System Example

- Common system in many industries
  - Simplified case of hydraulic systems
    - Flow in – flow out = Rate of change in stored volume

$$q_{in} - q_{out} = C\frac{dh}{dt}$$

    - Governing equation for tank 1: $q_{in} - q_b = C_1\frac{dh_1}{dt}$
    - Governing equation for tank 2: $q_b - q_{out} = C_2\frac{dh_2}{dt}$

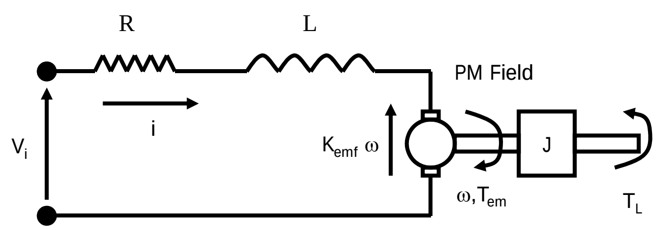


$$q_b = \frac{h_1}{R_1} \qquad q_{out} = \frac{h_2}{R_2}$$

$$R_1C_1\frac{dh_1}{dt} + h_1 = R_1 q_i \qquad R_2C_2\frac{dh_2}{dt} + h_2 = \frac{R_2}{R_1}h_1$$

2.4 DC Permanent Magnet Motor Model

- Common DC motor with inertial load
  - R = armature resistance
  - L = armature inductance
  - K_{emf} = back emf constant
  - K_T = torque constant
  - V_i = applied voltage
  - i = armature current
  - T_{em} = electromagnetic torque
  - J = motor and load inertia
  - ω = rotor rotational velocity



2.4 DC Permanent Magnet Motor Model

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- Voltage Drops
  - Around loop = 0
- Newton' Law
  - Summation of Torque

$$L\frac{di}{dt} = V_i - R\cdot i - K_{emf}\cdot\omega$$

$$J\frac{d\omega}{dt} = T_{em} - T_L = K_T\cdot i - T_L$$

Variables are coupled via back emf, State Space is the easiest:

$$\begin{bmatrix} \dot{i} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{K_{emf}}{L} \\ +\frac{K_T}{J} & 0 \end{bmatrix} \begin{bmatrix} i \\ \omega \end{bmatrix} + \begin{bmatrix} +\frac{1}{L} & 0 \\ 0 & +\frac{1}{J} \end{bmatrix} \begin{bmatrix} V_{in} \\ T_L \end{bmatrix}$$

2.4 DC Motor Summary

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- Same basics applied as before
- Two domains represented
  - Electrical and Mechanical
  - Both equations formed by $\sum$efforts = 0
- Larger systems can usually be broken down into the basic physical relationships illustrated in the table.