# Lab Week 2: Analog & Digital Sensors and A/D Conversion

## 0. Objectives:

- Discus analog and digital systems
- Examine the effects of sample time selection in digital controllers.
- Simulate and observe the effects of aliasing as a function of sample time.

## 1. Background

### 1.1 Analog and Digital Signals and Sensors

An analog system is a continuous system that has an infinite set of values within a given range. A digital system is a discrete system that has a finite set of values within a given range. Examples of analog systems include: ambient temperature, the position of a game controller joystick, an analog clock, and garden hose flow rate. Examples of digital systems include: the directional pad of a game controller, a digital thermometer readout, and a light switch. Most controllers today are implemented in the digital domain due to the advent of low cost microprocessors and computers. Because of these common digital controllers, analog systems and signals must be converted to digital signals.

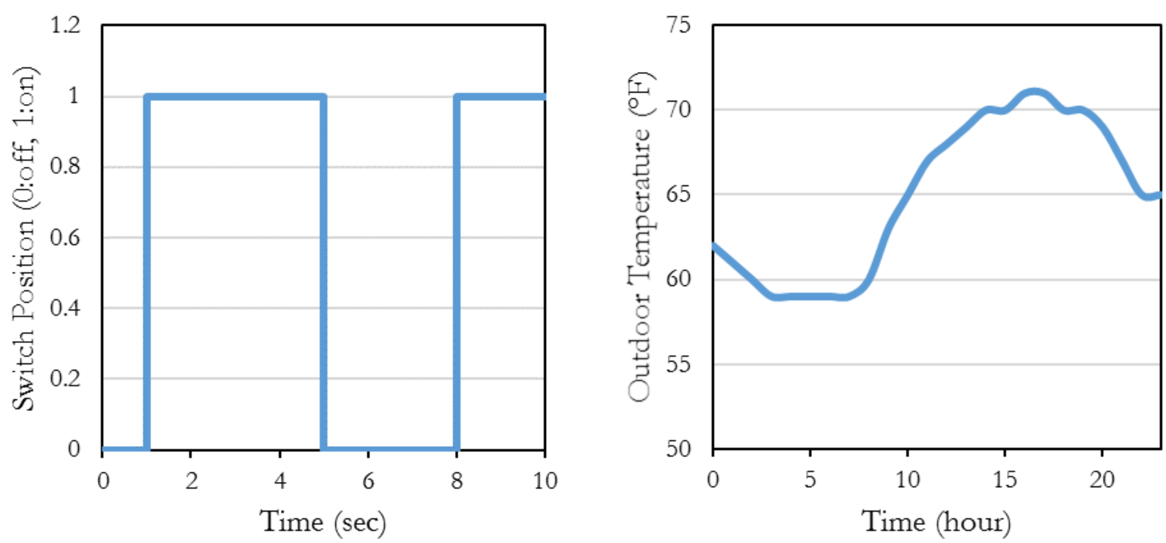


**Figure 1: Examples of digital and analog signals**

## 1.2 Effects of Sample Times, Aliasing, and A/D or D/A Conversion

The measured value of an output signal must be accurate for purposes of dependable feedback control. Parameters that are undergoing change must be measured, or sampled, quickly in order to accurately represent their dynamic properties. If a quickly varying parameter is measured slowly, inaccuracies can result. If the sample rate is too slow, then the signal appears to have a much slower frequency than the actual signal, as in Figure 2(a). If the sample rate is exactly the same as the signal frequency, the sampled signal will appear as a DC signal (zero frequency!), as in Figure 2(b). This apparent change in the frequency due to selection of sampling rate is called **aliasing**.

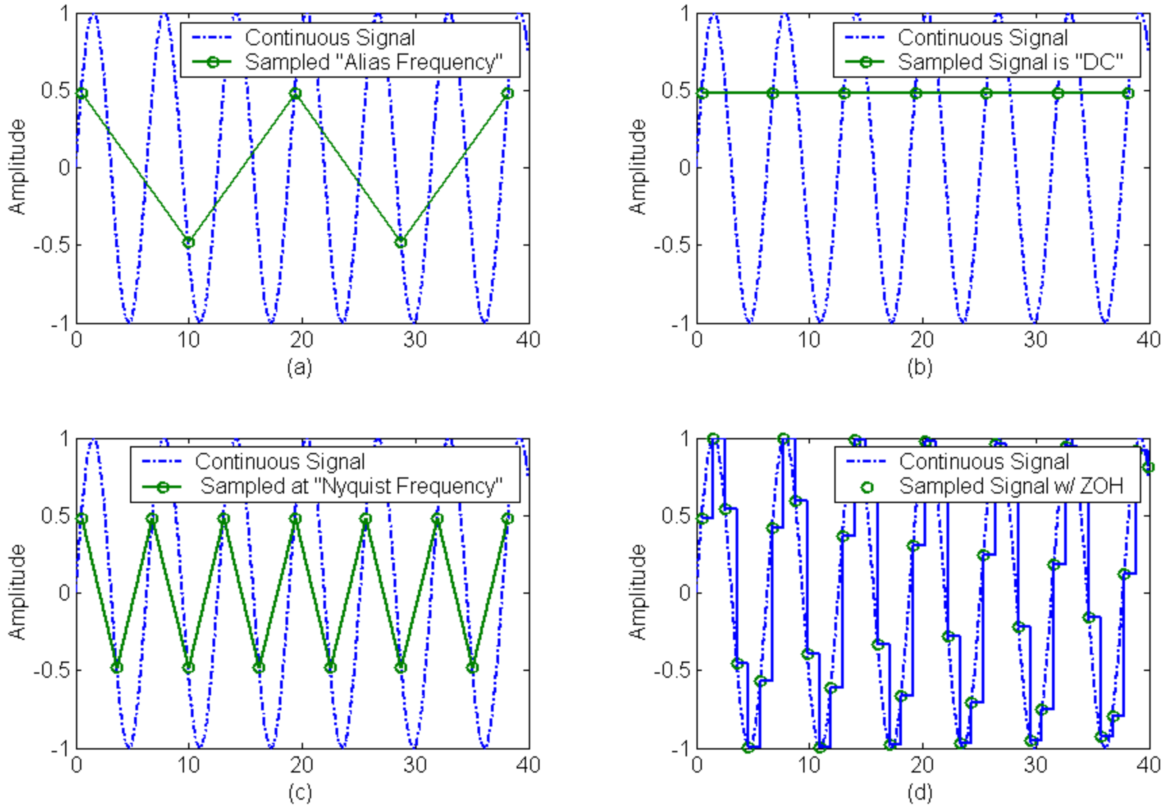


**Figure 2: Sampling effects**

Close examination of Figure 2(c) reveals that there must be at least two samples per cycle in order to detect the true frequency of the signal. In other words, if it is suspected that the signal contains 60 Hz components, or that you need to capture frequencies up to 60Hz, then the sample rate should be at least double that rate, or at least 120 samples per second (i.e. 120 Hz or 1 sample every 0.00833 sec). In order to find the minimum sampling rate that is required to avoid aliasing, the Nyquist Sampling Theorem should be followed. This theorem states:

> *The sampling frequency should be **at least** twice as large as the highest frequency that is contained in the signal that is being measured.*

This **minimum** sample rate is referred to as the "Nyquist Frequency" and is seen in mathematical terms in the equations below:

$$f_{sampling} \geq 2f_{signal,max} \qquad\qquad f_{Nyquist} = 2f_{signal,max}$$

A good example of the application of Nyquist sampling principles is music CDs. The human ear can detect up to approximately 17 or 20 kHz signals. The music industry set the standard sampling rate at 44 kHz in order to digitally reproduce up to 22 kHz signals in the original (analog) recordings. In this example, the highest frequency that is contained in the signal that is being measured is 22 kHz (i.e. $f_{signal,max}$ = 22 kHz). Therefore, the sampling rate (i.e. $f_{sampling}$) must be **at least** twice as large as 22 kHz. This is why a sampling rate of 44 kHz was chosen. This sampling rate of 44 kHz also happens to be the Nyquist Frequency because 44 kHz is twice as large as 22 kHz.

Sampling at the Nyquist Frequency faithfully reproduces <u>frequency</u> content, but it should be clear from Figure 2(a) and 2(b) that <u>amplitude</u> information may be quite inaccurate. **To maintain amplitude reproduction, a much higher sampling rate must be used**. (CDs recorded using DVD-Audio employ sampling rates of 96 kHz, and occasionally higher, to more accurately reproduce high frequency signals.) In Figure 2(d), it can be seen that between samples the signal is assumed held constant. This is known as zero-order-hold (ZOH). It is important to recognize that when it is assumed that the signal is held constant between samples, error is introduced.

### 1.3 Resolution

Another source of error is quantization error (or 'dynamic range' in audio terms). Analog to digital converters, or discrete devices, provide discrete output levels based on the number of bits used to report the value. There are two basic operations for analog to digital conversion:

1. Quantization: map the analog signal into discrete ranges
2. Coding: assign binary code to the quantized signal.

For example, a 3-bit analog to digital converter (ADC) that has eight discrete levels ($2^3$ discrete levels) will break up the analog signal into eight discrete ranges and assign binary code to the quantized signal following standard binary counting:

**Table 1: Counting in binary**

| Number | Binary Code |
|---|---|
| 0 | 000 |
| 1 | 001 |
| 2 | 010 |
| 3 | 011 |
| 4 | 100 |
| 5 | 101 |
| 6 | 110 |
| 7 | 111 |

The number of discrete levels is dependent on the bit number of the ADC and is given by the equations below where *n* is the bit number of the ADC. Therefore, resolution of the ADC **in terms of bits** shows the number of discrete levels available. Since the first level is zero, the number of intervals will be one less than the total number of discrete levels. This will become clearer on the following page.

$$Number\ of\ discrete\ levels = 2^n$$
$$Number\ of\ intervals = 2^n - 1$$

Figure 3 below shows how the analog signal is mapped to a discrete/binary value using an ideal ADC. The x-axis represents the analog value. This starts at the lowest analog value (0 FS) and ends at the full-scale analog value (FS). For example, if an analog signal varies from 0 to 8 V, the lowest analog value is 0 V, the full-scale analog value (FS) is 8 V, and the full-scale measurement range is 8 V. Similarly, if the analog signal varies from -10 to +10 V, the x-axis will vary from -10 V (i.e. 0 FS), +10 V (i.e. FS), and the full-scale measurement <u>range</u> is 20 V. The y-axis represents the binary code that is given to each analog value. The number of codes/levels is dependent on the bit number of the ADC, as seen in the equations above. Figure 3 shows a 3-bit ADC which results in eight discrete levels (number of levels equals $2^3$ for a 3-bit ADC); however, there are only seven vertical intervals ($2^3$ - 1).

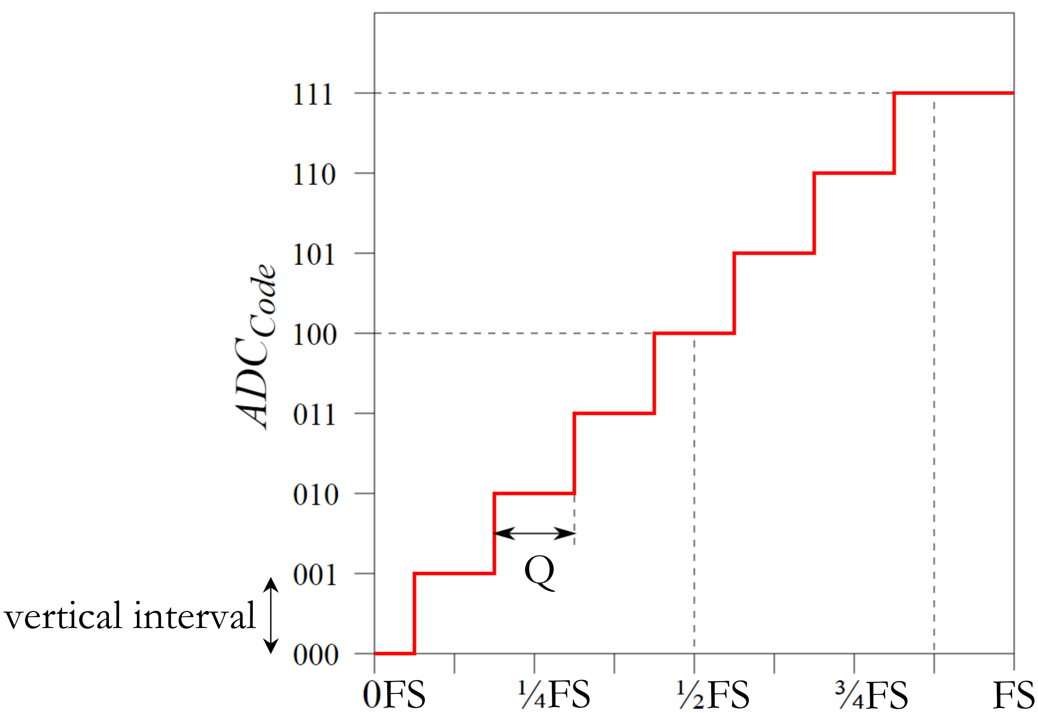


**Figure 3: Ideal 3-bit ADC**

Another way to express the resolution of the ADC is by the analog signal that is being measured. This is called the quantization interval (Q) and is equal to the analog full-scale measurement range divided by the number of vertical intervals.

$$Q = \frac{full\ scale\ measurement\ range}{2^n - 1} = \frac{High\ analog\ value - Low\ analog\ value}{2^n - 1}$$

Because the ADC cannot perfectly match the analog signal, this gives rise to quantization error. For an ideal ADC, the quantization error is as follows:

$$Quantization\ error = e_{quant} = \frac{Q}{2}$$

For example, a temperature sensor has a measurement range of 0 to 100 °C and outputs an analog voltage signal with a range of -5 to +5 volts. This means that if the sensor measures 0 °C, it will output -5 V, and if the sensor measures 100 °C, it will output +5 V. Let the temperature sensor be sampled using an 8-bit ADC. Using the equations given above, the output from the ADC has a voltage resolution of 0.0392 V and a quantization error of 0.0196 V.

$$Q = \frac{5V - (-5V)}{2^8 - 1} = \frac{10V}{255} = 0.0392\ V$$

$$e_{quant} = \frac{Q}{2} = \frac{0.0392\ V}{2} = 0.0196\ V$$

Since the temperature sensor has a measurement range of 0 to 100°C and an output voltage signal range of -5 to +5 V, its output can be expressed as being 1 V for every 10°C. Since the ADC has a voltage resolution of 0.0392 V, and 1 V is 10°C, the ADC has a temperature resolution of 0.392 °C and a quantization error of 0.196 °C. This means that the digital/discrete/binary output from the ADC has an uncertainty of ±0.196 °C.

The higher the bit resolution of an ADC, the higher its dynamic range. A 4-bit ADC has $2^4 = 16$ discrete output levels and is said to have higher dynamic range than a 3-bit ADC. Typical electronic components employ 8-bit or 16-bit (e.g. CD-players) devices. Advances in computer technologies continue to increase the available dynamic range.

## 2. Procedure

1. Build the Simulink diagram shown in Figure 4. Your initial scope should look like Figure 5.

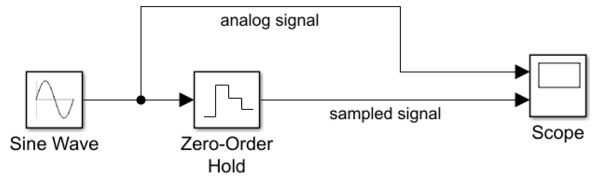


**Figure 4: Simulink Diagram**

Sine Wave: Frequency = 1 Hz = 2π rad/sec and Amplitude = 1
Zero-Order Hold: Sample time = 0.01 sec (which is 100 Hz)
Simulation Parameters: Simulation time = 3 sec

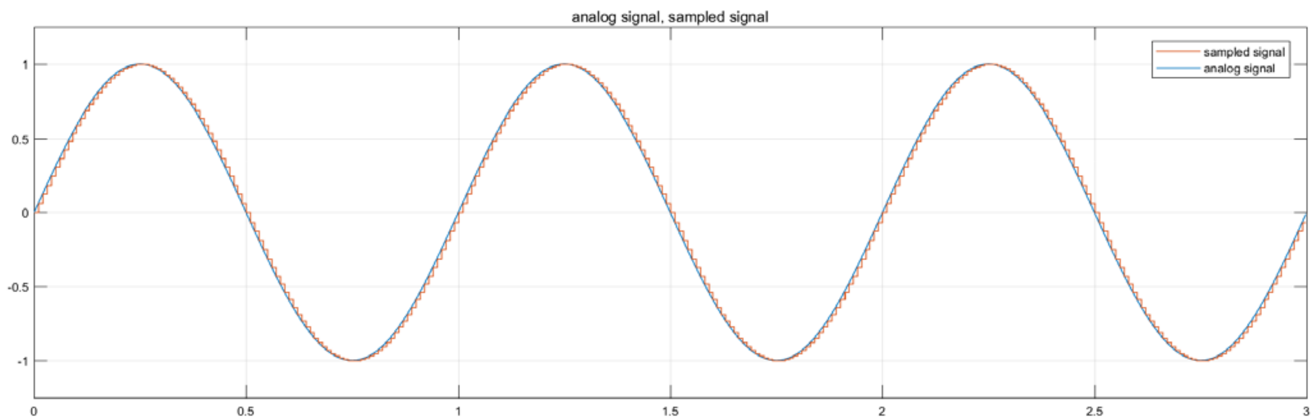


**Figure 5: Initial Scope Output**

2. Leave the input signal (sine wave) at 1 Hz (2π rad/sec). Begin to increase the sample time of the zero-order hold until the effects of aliasing are very evident (try sampling time of 0.02, 0.05, 0.1, etc.). Every iteration is not required in the report, but include enough plots to show what happens as the sample time is increased.
3. With the sine wave at 1 Hz, and the sample time of the zero-order hold at 1 second, vary the sine wave phase and observe what happens (Phase is in units of radians, so try values of π/4, π/2, π, etc.). Describe what you observe and why it is happening.
4. With the sine wave at 1 Hz and the sample time of the zero-order hold at 0.5, vary the sine wave phase and observe what happens. (Phase is in units of radians, so try values of π/4, π/2, π, etc.). Describe what you observe and why it is happening.

## 3. Assignment

Write a one-page maximum (not counting figures) report describing sample rates and the aliasing portion of this laboratory exercise in general, and answers to the specific questions below.

a) Discuss the effects of the speed of a physical system versus the need to sample that system at fast rates. What effects do these phenomena have on controlling physical systems?

b) Determine the Nyquist Frequency for the input signal in the aliasing experiment (i.e., Section 2.1, 2.2, 2.3, and 2.4). Is it helpful to add some phase to the input signal of $\pi/4$ or $\pi/2$ in order to better visualize the frequency of the output signal? What occurred when the sampling frequency (i.e., the sampling time of the zero order hold) was slower than the Nyquist frequency? Determine a 'good' sampling frequency in terms of Nyquist, i.e., the minimum sampling frequency should be 'X' times the Nyquist rate; what is a good 'X'? (Note: for this part, also change the phase value in the Sine wave block; observe and comment on the effects).

c) Considering Figure 6, indicate at each signal location (designated by a letter) whether it is analog or digitally sampled. Re-draw Figure 6 for you report and add in A/D or D/A blocks where necessary.

d) Calculate the quantization error (i.e., the ± uncertainty) in inches that is introduced by the digitizing of a position sensor signal on a cylinder. The ADC is 8-bit and the position sensor has an output of -10 to +10 volts. Assume the cylinder's travel is 0 to 20 inches.

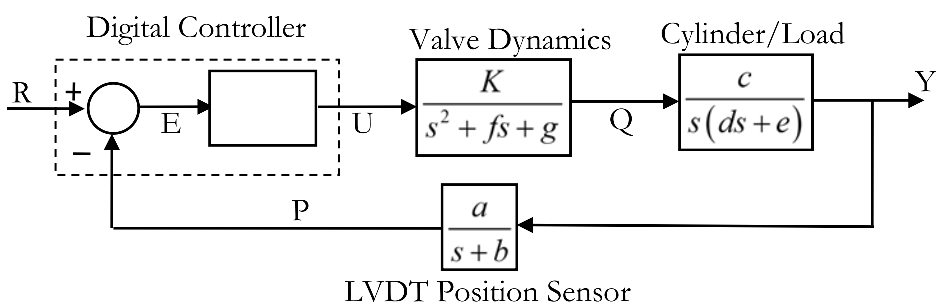


**Figure 6: Typical Cylinder Position Control System**

## 4. Deliverables

Write a one-page maximum (not counting figures) report describing:

- A general discussion of sampling rates, aliasing, and signal conversion
- Answers to the specific questions from Section 3

Any results/figures from the Simulink Scope block that are used in your report **must** have a white background (e.g., Figure 5 above). Do **not** use a screenshot/image of the Scope output that has a black background. To copy the Scope output results to your lab report, go to the 'File' menu in the Scope block and select 'Copy to Clipboard'.

For this lab only, the copied Scope graphs do not need a chart title or labels on the x- or y-axis. Just be sure to include a descriptive figure caption for each plot (frequency, phase shift, sample time, etc.).

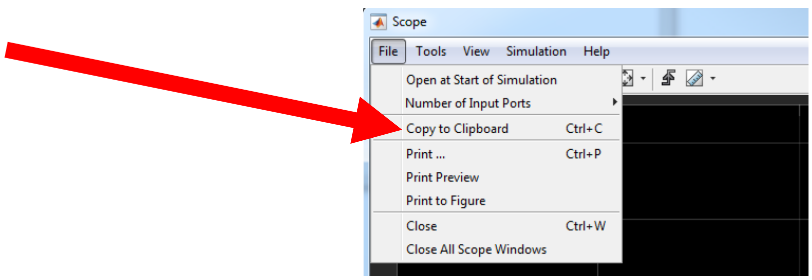


**Figure 7: Copying Scope results for use in lab report**