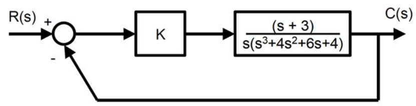
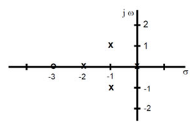
Example 4-9 (textbook pg 177)



Step 1: From OLTF (i.e. G(s)H(s)) find poles and zeros

- Poles: 0, -2, -1±1j ∴ n = 4
- Zeros: -3 ∴ m = 1

Step 2: Draw the poles and zeros on the s-plane. **X** for poles, **O** for zeros

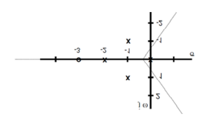


Step 3: The number of asymptotes equals n – m

- We have 4 – 1 = 3 asymptotes

Step 4: For 3 asymptotes, the angles are ±60° and 180° from the pos. real axis.

Step 5: Asymptotes intersect real axis at σ



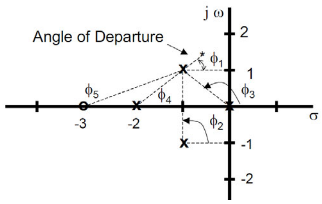
- $\sigma = \frac{sum\ of\ poles - sum\ of\ zeros}{number\ of\ asymptotes} = \frac{(0-2-1-1)-(3)}{3} = -\frac{1}{3}$
- Only real portion is added

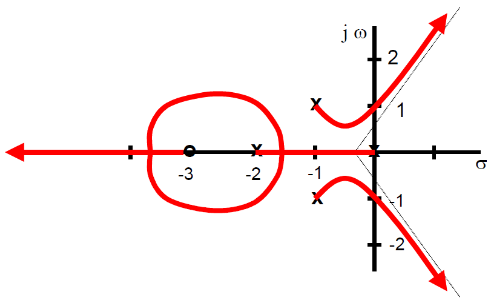
Step 6: The loci paths include all portions of the real axis that are to the left of an odd number of poles and/or zeros that are on the real axis. Between 0 and -2 and to the left of -3

Step 7: There is one break-away point since the root locus paths begin on the real axis between 0 and -2 and end along the asymptotes. For this example, there is also a break-in point since the zero lies on real axis and must have one path approach it as K goes to infinity. To the left of the zero is also part of the root locus plot (an asymptote) and two paths must come together at this break-in point.

- Set CE equal to zero: $s^4 + 4s^3 + 6s^2 + 4s + Ks + 3K = 0$
- Solve for K: $K = \frac{-s^4 - 4s^3 - 6s^2 - 4s}{s+3}$
- $\frac{dK}{ds} = \frac{-3s^4 - 20s^3 - 42s^2 - 36s - 12}{(s+3)^2} = 0$
- $s = -1.54, -3.65, -0.74 \pm 0.41j$. Two of the four roots are valid and which coincide with the expected locations along the real axis. The breakaway point will be -1.54 and the break in point will be -3.65

Step 8: Departure angles are clear except for the complex conjugate pair at -1±1j

- 

- Angles must add up to an odd multiple of ±180º
- $-\phi_1 - \phi_2 - \phi_3 - \phi_4 + \phi_5 = \pm 180°$
- $-\phi_1 - 90° - 135° - 45° + \tan^{-1}(0.5) = \pm 180°$
- $\phi_1 = -63.4°$
- 

Step 9: The asymptotes cross the Imaginary axis and become stable for a particular value of K

- K can be founding using Routh-Hurwitz stability criterion (Example 4-5); but not covered in this course. Can also be found with Magnitude Condition or using CE to find K with a given desired pole location. Or using Bode Plot

Step 10: Find value of K that will place our poles at a desired location.
Step 11: Can confirm with Matlab that Gain = 2.33 produces marginal stability