Problem 3.2

Estimate from graph:

- Percent Overshoot: 56%
- Settling Time:
  - Textbook does not specify whether it is 2% or 5% settling time
  - 2%: 37 sec
  - 5%: 46 sec
- Rise Time: 4 sec

Problem 3.3

$$TF = \frac{Y(s)}{G(s)} = \frac{10}{s^3+3s^2+4s+9}$$

Problem 3.8

$$U(s) = \frac{2}{s}$$

$$Y(s) = \frac{2}{s^2+3s+2} \cdot \frac{2}{s} = \frac{2}{s} + \frac{2}{s+2} - \frac{4}{s+1}$$

$$y(t) = 2 + 2e^{-2t} - 4e^{-t}$$

Problem 3.12

Steady state value = 2

$\tau = 5$

$$G(s) = \frac{2}{5s+1}$$

Problem 3.13

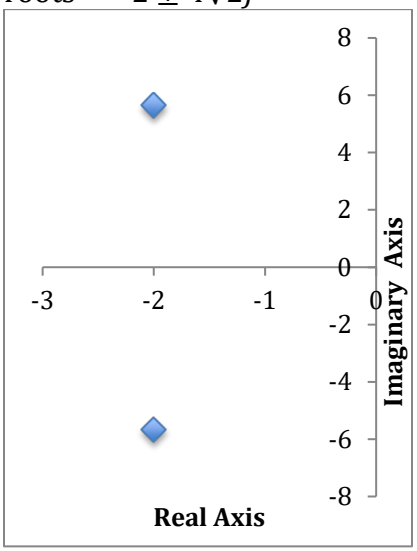
$\omega_n \approx 6$

$\xi \approx 0.333$

$$G(s) = \frac{36}{s^2+4s+36}$$

Problem 3.14

roots = $-2 \pm 4\sqrt{2}j$



zeta = 0.3333
natural frequency = 6 rad/s

Problem 3.18

$$\frac{Y(s)}{R(s)} = \frac{k1 + bs}{ms^2 + bs + (k1 + k2)}$$

Problem 3.22

From 2.29

$$\frac{dh_1}{dt} = \frac{Q_i - Q_b}{C_1}$$

$$\frac{dh_2}{dt} = \frac{Q_b - Q_o}{C_2}$$

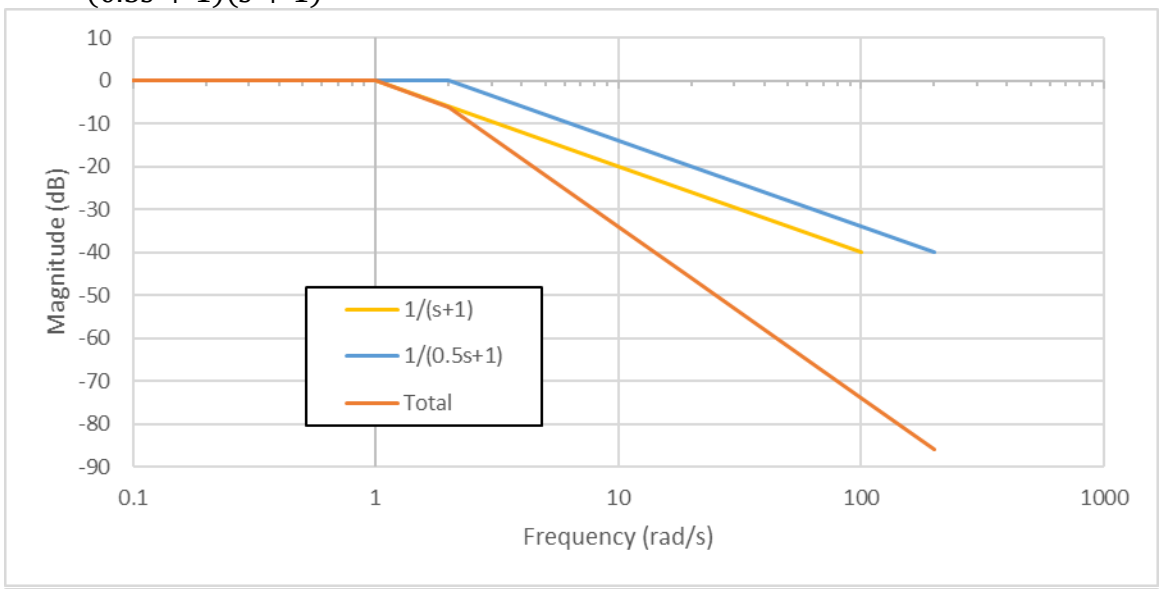
$$Q_b = \frac{h_1 - h_2}{R_1}$$

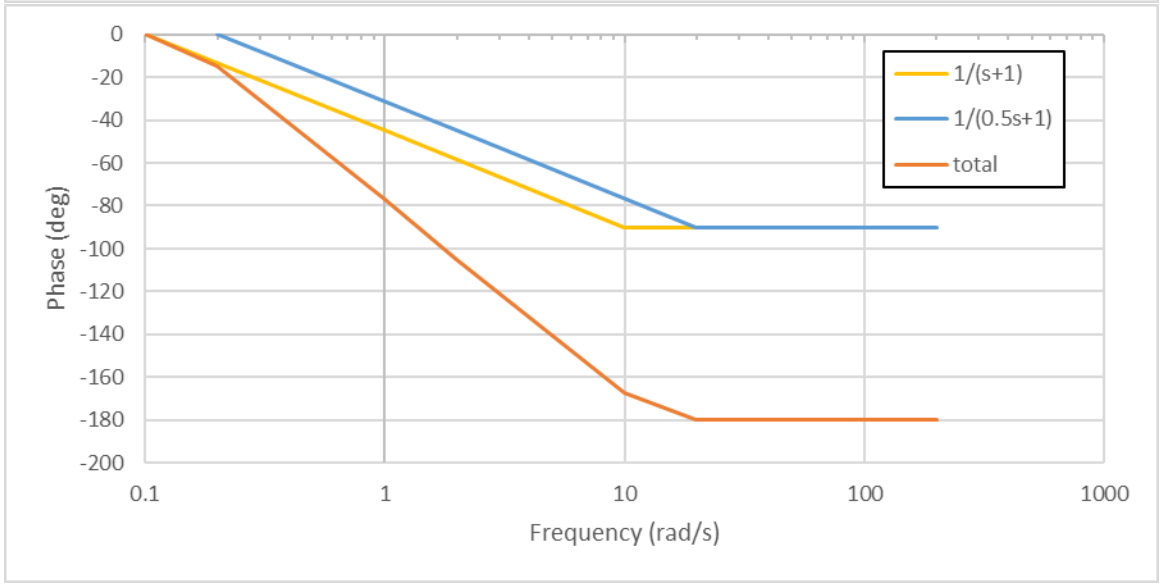
$$Q_o = \frac{h_2}{R_2}$$

$$\frac{H_2}{Q_i} = \frac{R_1}{\left[\frac{(R_1R_2C_2s + R_2 + R_1)(R_1C_1s + 1)}{R_2}\right] - 1} \rightarrow \frac{R_2}{s^2(R_1R_2C_1C_2) + s(R_2C_2 + R_1C_1 + R_2C_1) + 1}$$

Problem 3.25

$$TF = \frac{1}{(0.5s+1)(s+1)}$$





Problem 3.27

The plot is easiest using Excel, but it can be done by hand by creating a table.

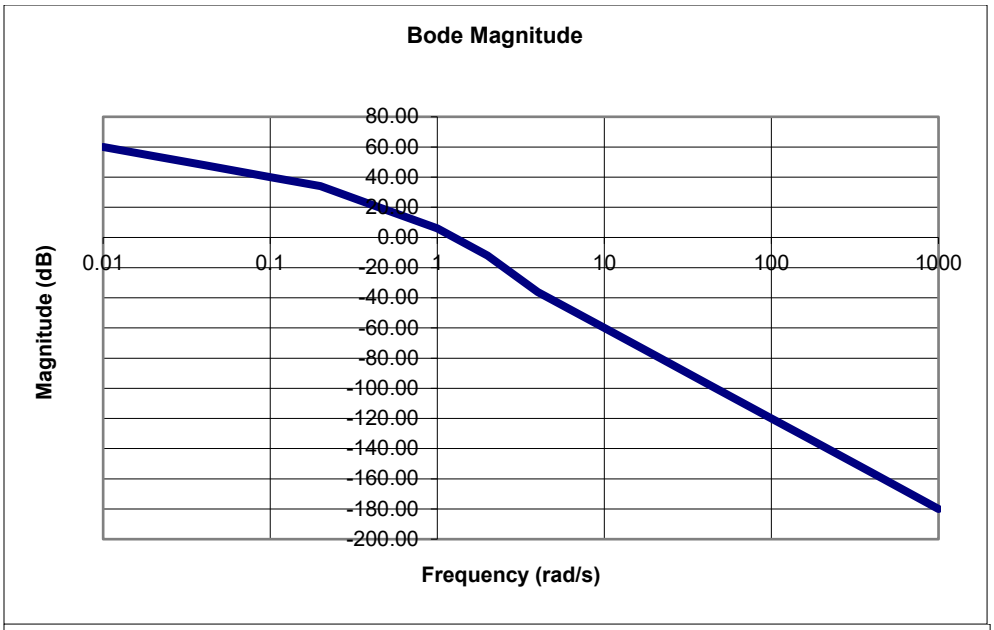
$$G(s)=\frac{5(s+4)}{s(s+1)(s+2)(5s+1)}=\frac{10(0.25s+1)}{s(s+1)(0.5s+1)(5s+1)}$$

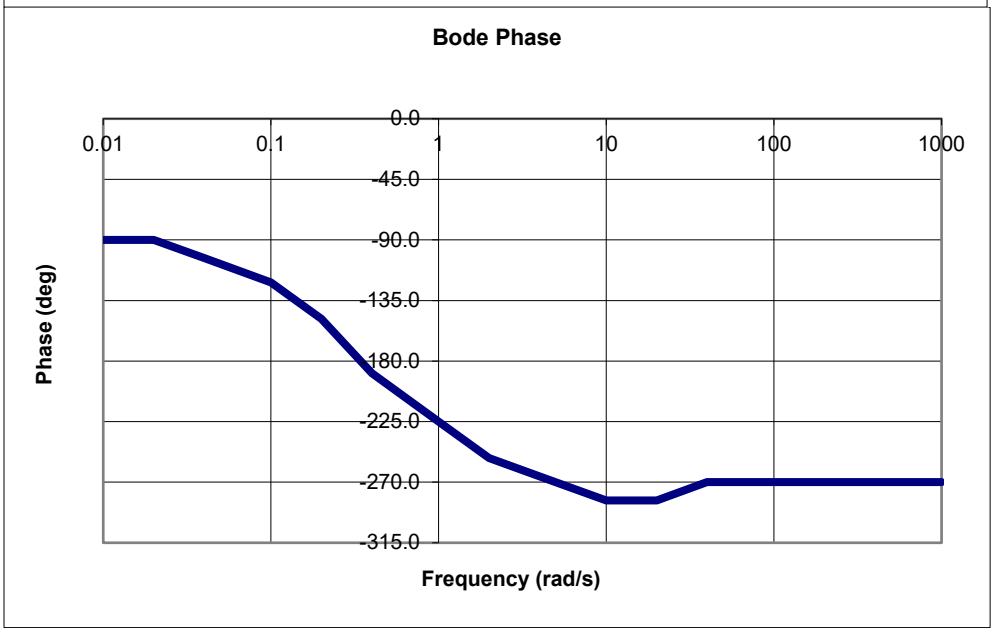
Magnitude (dB)

		tau = 4	tau = 1	tau = 1	tau = 2	tau = 0.2	
w (rad/s)	10	0.25s+1	1/s	1/(s+1)	1/(0.5s+1)	1/(5s+1)	total
0.01	20.00	0.00	40.00	0.00	0.00	0.00	60.00
0.02	20.00	0.00	33.98	0.00	0.00	0.00	53.98
0.04	20.00	0.00	27.96	0.00	0.00	0.00	47.96
0.1	20.00	0.00	20.00	0.00	0.00	0.00	40.00
0.2	20.00	0.00	13.98	0.00	0.00	0.00	33.98
0.4	20.00	0.00	7.96	0.00	0.00	-6.02	21.94
1	20.00	0.00	0.00	0.00	0.00	-13.98	6.02
2	20.00	0.00	-6.02	-6.02	0.00	-20.00	-12.04
4	20.00	0.00	-12.04	-12.04	-6.02	-26.02	-36.12
10	20.00	7.96	-20.00	-20.00	-13.98	-33.98	-60.00
20	20.00	13.98	-26.02	-26.02	-20.00	-40.00	-78.06
40	20.00	20.00	-32.04	-32.04	-26.02	-46.02	-96.12
100	20.00	27.96	-40.00	-40.00	-33.98	-53.98	-120.00
200	20.00	33.98	-46.02	-46.02	-40.00	-60.00	-138.06
400	20.00	40.00	-52.04	-52.04	-46.02	-66.02	-156.12
1000	20.00	47.96	-60.00	-60.00	-53.98	-73.98	-180.00

Phase (deg)

		tau = 4	tau = 1	tau = 1	tau = 2	tau = 0.2	
w (rad/s)	10	0.25s+1	1/s	1/(s+1)	1/(0.5s+1)	1/(5s+1)	total
0.01	0	0.00	-90.00	0.00	0.00	0.00	-90.0
0.02	0	0.00	-90.00	0.00	0.00	0.00	-90.0
0.04	0	0.00	-90.00	0.00	0.00	-13.55	-103.5
0.1	0	0.00	-90.00	0.00	0.00	-31.45	-121.5
0.2	0	0.00	-90.00	-13.55	0.00	-45.00	-148.5
0.4	0	0.00	-90.00	-27.09	-13.55	-58.55	-189.2
1	0	17.91	-90.00	-45.00	-31.45	-76.45	-225.0
2	0	31.45	-90.00	-58.55	-45.00	-90.00	-252.1
4	0	45.00	-90.00	-72.09	-58.55	-90.00	-265.6
10	0	62.91	-90.00	-90.00	-76.45	-90.00	-283.5
20	0	76.45	-90.00	-90.00	-90.00	-90.00	-283.5
40	0	90.00	-90.00	-90.00	-90.00	-90.00	-270.0
100	0	90.00	-90.00	-90.00	-90.00	-90.00	-270.0
200	0	90.00	-90.00	-90.00	-90.00	-90.00	-270.0
400	0	90.00	-90.00	-90.00	-90.00	-90.00	-270.0
1000	0	90.00	-90.00	-90.00	-90.00	-90.00	-270.0





A: Gain margin = -20 dB
B: Phase margin = -55°
C: Bandwidth = 1.3 rad/s
D: Steady state gain = infinite (because there is an integrator and the loop isn't closed)