Ch. 5 Notes: For 5.20 Assume Ku = 5.0 (It's not given)

Problem 5.8

From setting time equation on p80: $t_s = 4\tau = \frac{4}{\zeta\omega_n}$

For this problem, $t_s = 4\ seconds$

From equations on p97: $\zeta = \cos(\theta) = \frac{|\sigma|}{|\omega_n|}$

Substituting: $t_s = \frac{4}{|\sigma|} = 4\ seconds$

The condition to be met on the s-plane:

- $|\sigma| = 1$
- Since our system does settle out to reach 98% of its final value in 4 seconds, it is stable. Since the system is stable, σ is a negative value $\therefore\ \sigma = -1$

Problem 5.9

From table or equation on p80-81: 5%O.S. → $\xi = 0.7$

From settle time equation on p80: $t_s = 4\sec onds = 4\tau = \dfrac{4}{\xi\omega_n}$

$$\therefore \xi\omega_n = \frac{4}{4} = 1 \Rightarrow \omega_n = \frac{1}{0.7} = 1.43$$

CL TF $$\frac{C(s)}{R(s)} = \frac{\dfrac{K}{s(s+p)}}{1+\dfrac{K}{s(s+p)}} = \frac{K}{s^2+sp+K} = \frac{\omega_n{}^2}{s^2+2s\xi\omega_n+\omega_n{}^2}$$

Set like terms equal

$p = 2\xi\omega_n = 2*0.7*1.43 = 2$

$K = \omega_n{}^2 = 1.43^2 = 2$

Problem 5.13

Plant model transfer function is on p273

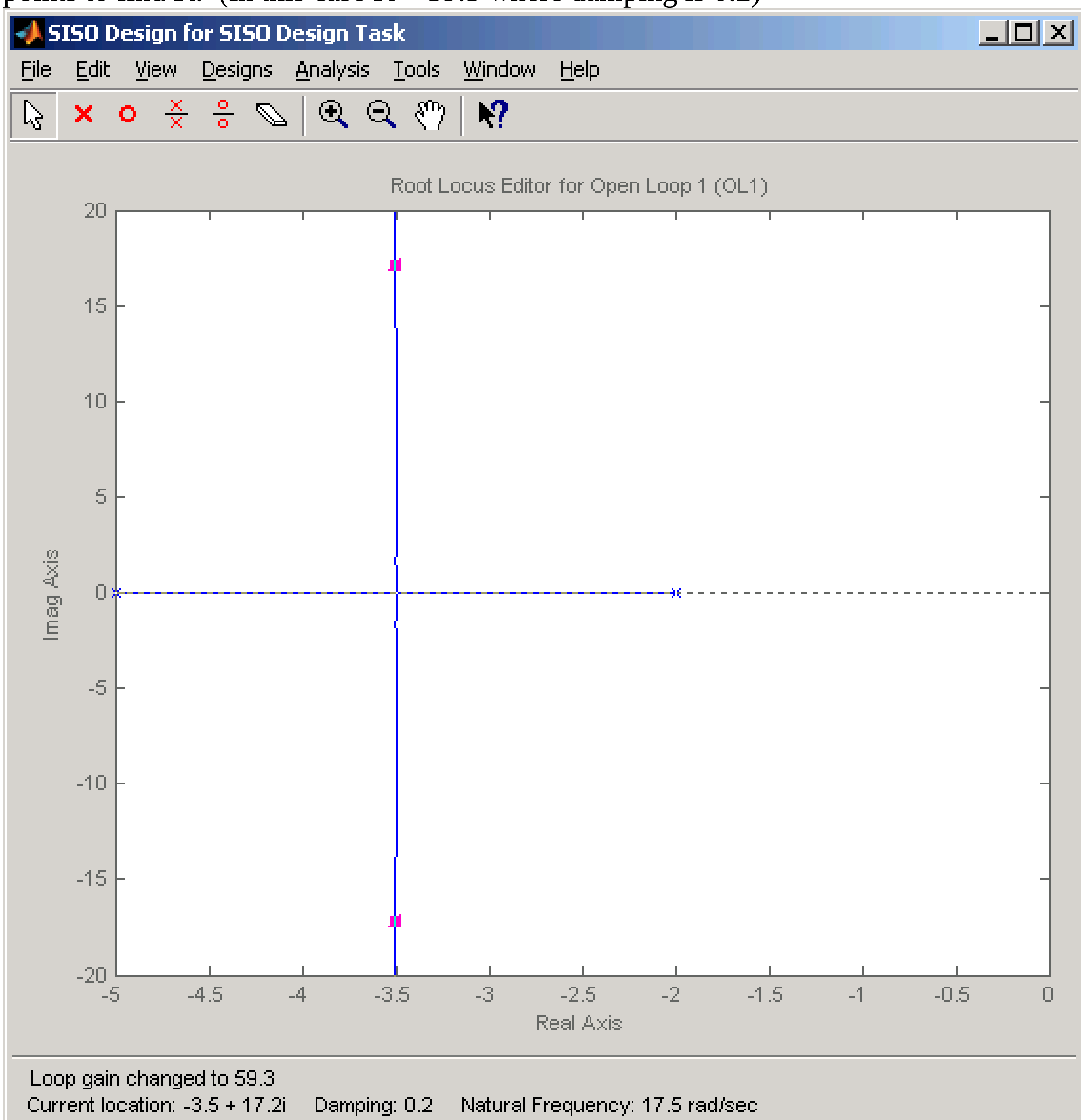
$$G(s) = \frac{5}{s^2 + 7s + 10} = \frac{5}{(s+2)(s+5)}$$

poles at -2, -5

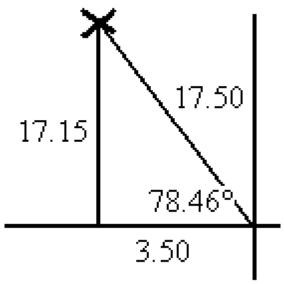
asymptote break away at $\frac{-2-5}{2} = -3.5$

asymptotes at $\pm 180°$

To find K you can use matlab “sys = TF(num,den)” and then “rltool(sys)” and click points to find K. (In this case K = 59.3 where damping is 0.2)



Or you can find K by hand as shown below



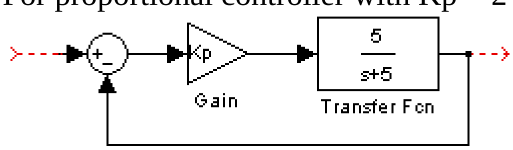
$$\cos(\xi) = \theta$$
$$\cos(0.2) = \theta \Rightarrow \theta = 78.46°$$

$$\text{CL TF} \frac{C(s)}{R(s)} = \frac{\frac{5K}{s^2 + 7s + 10}}{1 + \frac{5K}{s^2 + 7s + 10}} = \frac{5K}{s^2 + 7s + 10 + 5K} = \frac{\omega_n{}^2}{s^2 + 2s\xi\omega_n + \omega_n{}^2}$$

$$2\xi\omega_n = 7$$
$$\omega_n{}^2 = 10 + 5K$$
$$\xi = 0.2$$
$$\omega_n = hypotinuse = 17.50$$
$$or$$
$$\omega_n = \frac{7}{2*0.2} = 17.50$$
$$\therefore 17.50^2 = 10 + 5K \Rightarrow K = 59.25$$

Problem 5.14

For proportional controller with Kp = 2



$$\text{CL TF} \frac{C(s)}{R(s)} = \frac{\frac{5K_p}{s+5}}{1 + \frac{5K_p}{s+5}} = \frac{5K_p}{s + 5 + 5K_p} = \frac{5*2}{s + 5 + 5*2} = \frac{10}{s + 15}$$

Let s → 0

$$C_{SS} = 2 * \frac{10}{15} = \frac{4}{3}$$

$$e_{SS} = R_{SS} - C_{SS} = 2 - \frac{4}{3} = \frac{2}{3}$$

For proportional-integral controller with Kp = 2 and Ti = 1

From p234 $G_C = K_p\left(1+\frac{1}{T_i s}+T_d s\right) = 2+\frac{2}{s}$

CL TF $\frac{C(s)}{R(s)} = \frac{\frac{5\left(2+\frac{2}{s}\right)}{s+5}}{1+\frac{5\left(2+\frac{2}{s}\right)}{s+5}} = \frac{10+\frac{2}{s}}{s+5+10+\frac{2}{s}} = \frac{10s+2}{s^2+15s+2}$

Let s → 0

$C_{SS} = 2*\frac{2}{2} = 2$

$e_{SS} = R_{SS} - C_{SS} = 2-2 = 0$

Problem 5.19

From p235 Figure 46

$D \approx 0.125$

$T \approx 0.75 - 0.125 = 0.625$

$K_p = 1.2\frac{T}{D} = 1.2\frac{0.625}{0.125} = 6$

$T_i = 2D = 2*0.125 = 0.25$

$T_d = 0.5D = 0.5*0.125 = 0.0625$

From p234 $G_C = K_p\left(1+\frac{1}{T_i s}+T_d s\right) = 6\left(1+\frac{1}{0.25s}+0.0625s\right) = K_p + \frac{K_i}{s} + K_d s$

$K_p = 6$

$K_i = 6/0.25 = 24$

$K_d = 6*0.0625 = 0.375$

Problem 5.20

P236 fig 47 and table 2

T_u= 0.42 (estimate)

K_u=20

K_P=0.6K_u = (0.6*20) = 12

T_i= 0.5T_u = 0.21

T_d = 0.125T_u = 0.0525

K_i = K_p/T_i = 57.1

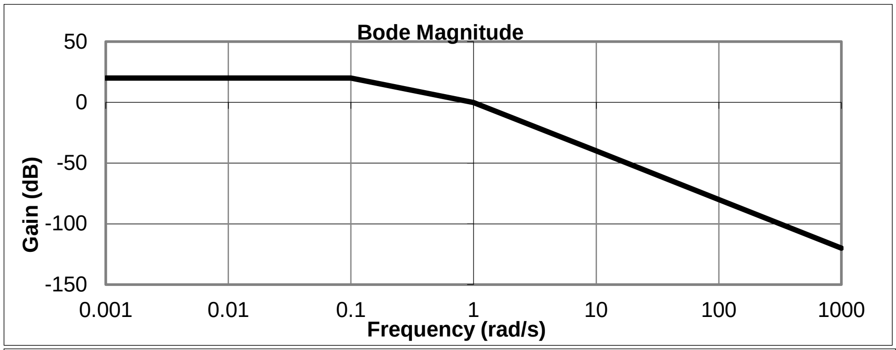
K_d= K_p*T_d = 0.63

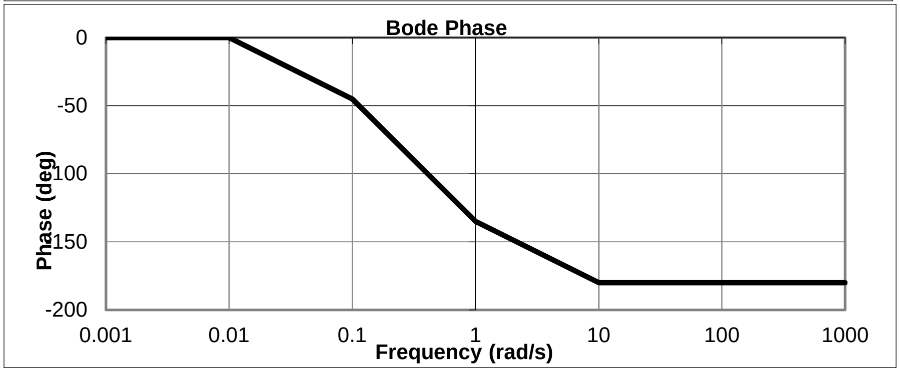
Problem 5.21

Gain margin of 45° means that when the magnitude plot crosses 0dB the phase must be -135° (-180°+45° = -135°). A phase angle of -135° occurs at 1rad/s. With a gain of 1 the magnitude at 1rad/s is -20dB therefore the gain K must be 20dB to create a gain of 0dB at 1rad/s. A gain of 20dB means K = 10. The book answer does not use straight line approximations and is therefore different. Straight line approximations are appropriate for these solutions.

	k (dB)	1/(10s+1)	1/(s+1)	total
w				
0.001	20	0.0	0	20
0.01	20	0.0	0	20
0.1	20	0.0	0	20
1	20	-20.0	0	0
10	20	-40.0	-20	-40
100	20	-60.0	-40	-80
1000	20	-80.0	-60	-120

phase				
0.001	0	0.0	0	0
0.01	0	0.0	0	0
0.1	0	-45.0	0.0	-45
1	0	-90.0	-45.0	-135
10	0	-90.0	-90.0	-180
100	0	-90.0	-90.0	-180
1000	0	-90.0	-90.0	-180

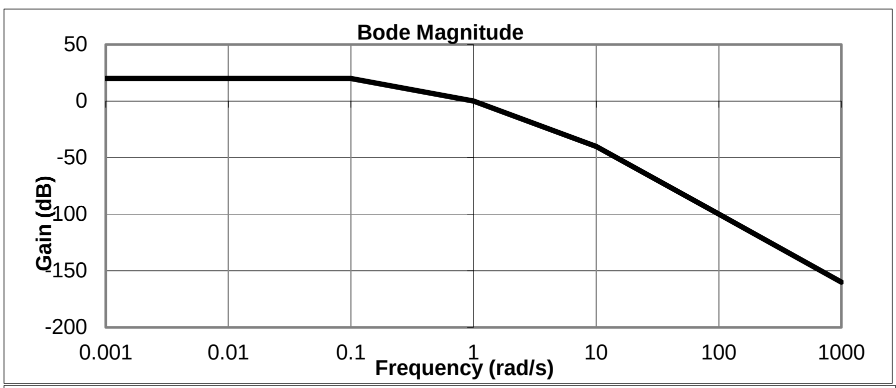


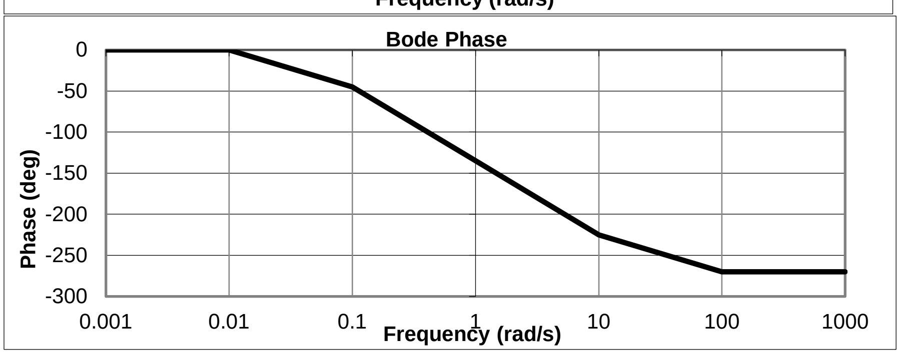


Problem 5.22

mag (dB)	K = 1	10	1/(10s+1)	1/(s+1)	1/(0.1s+1)	total
w						
0.001	0	20	0.0	0	0	20
0.01	0	20	0.0	0	0	20
0.1	0	20	0.0	0	0	20
1	0	20	-20.0	0	0	0
10	0	20	-40.0	-20	0	-40
100	0	20	-60.0	-40	-20	-100
1000	0	20	-80.0	-60	-40	-160

phase (deg)	K = 1	10	1/(10s+1)	1/(s+1)	1/(0.1s+1)	total
w						
0.001	0	0	0.0	0	0	0
0.01	0	0	0.0	0	0	0
0.1	0	0	-45.0	0.0	0	-45
1	0	0	-90.0	-45.0	0.0	-135
10	0	0	-90.0	-90.0	-45.0	-225
100	0	0	-90.0	-90.0	-90.0	-270
1000	0	0	-90.0	-90.0	-90.0	-270





A: At 0dB the phase angle is -135° therefore phase margin = 180°-135° = 45°

- If performed in MATLAB or other program, phase margin is 55.

B: At -180° the gain is -20dB therefore the gain margin is +20dB

C: Instability occurs when gain or phase margin = 0 or are negative. If the gain margin is 20dB then a 20dB gain will yield a gain margin of 0 and cause instability.

Therefore K = 20dB = 10