For problems 2.4 and 2.6, there are two ways to solve the problems.

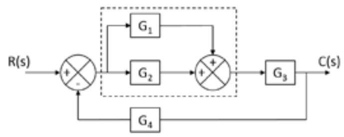
- You can find the equivalent transfer function by combining transfer functions step by step. Use the rule for a closed loop:

$$\frac{Output}{Input} = \frac{FowardPathGain}{1+TotalLoopGain}$$

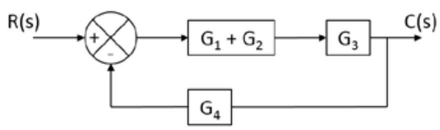
- You can create many intermediate variables, set up a system of equations, and then solve them for C/R in terms of the constants (eliminating all of the intermediate variables you created).

<u>Problem 2.4</u>

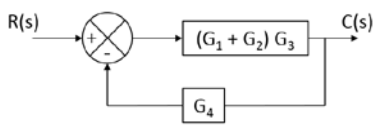
- Begin by simplifying one section of the block diagram.
  - Draw a dotted box around G_1, G_2, and the ++ summing junction.



  - Label the input to the dotted box as X and the output as Y. Now use equations to simplify.
    - $Y = G_1X + G_2X \rightarrow Y = (G_1 + G_2)X$
- Redraw the block diagram.



- Simplify by combining $G_1 + G_2$ and G_3.

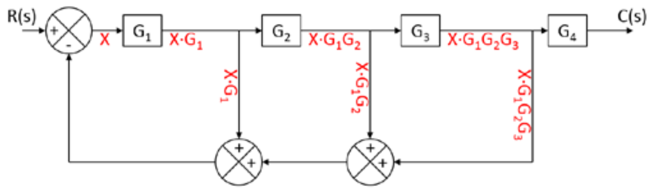


- Now you have a simple negative feedback loop and can use the equations:
  - $\frac{Output}{Input} = \frac{Forward\ Path\ Gain}{1+Total\ Loop\ Gain}$
  - $Forward\ Path\ Gain = (G_1 + G_2)G_3$
  - $Total\ Loop\ Gain = (G_1 + G_2)G_3G_4$
- The final answer is:

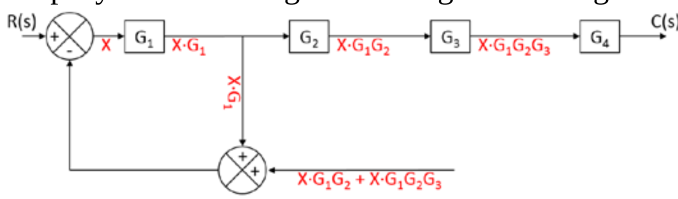
$$TF = \frac{C(s)}{R(s)} = \frac{(G_1 + G_2)G_3}{1 + (G_1 + G_2)G_3G_4}$$

Problem 2.6

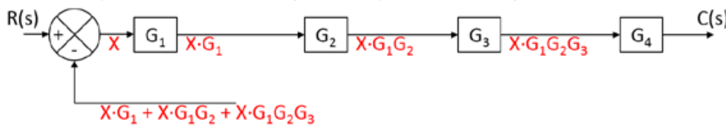
- Begin by labeling the signal flows. Assume that the arrow leaving the $+\,-$ summing junction is X



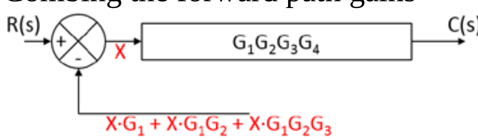
- Simplify the block diagram starting with the right $+\,+$ summing junction



- Simplify the block diagram by removing the last $+\,+$ summing junction



- Combing the forward path gains



- Using equations, solve for the transfer function

$$X = R - [XG_1 + XG_1G_2 + XG_1G_2G_3]$$
$$R = X + XG_1 + XG_1G_2 + XG_1G_2G_3$$
$$R = X(1 + G_1 + G_1G_2 + G_1G_2G_3)$$
$$X = R\frac{1}{1 + G_1 + G_1G_2 + G_1G_2G_3}$$

$$C = X(G_1G_2G_3G_4)$$

$$C = R\frac{1}{1 + G_1 + G_1G_2 + G_1G_2G_3} \cdot (G_1G_2G_3G_4)$$

- The final answer is:

$$TF = \frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1 + G_1 + G_1G_2 + G_1G_2G_3}$$

Problems 2.15, 2.16, and 2.17 are all solved in the same way by using the equation shown in section 2.3.4.

Problem 2.15

$Y = 10.181 + 7.923(X - 2)$

Linear range is subjective, but about from 1 to 3.

COMMON MISTAKE: Unless specified usually all angles, sine, and cosine are in radians not degrees.

Problem 2.16

$Z = 27 + 21(X - 2) + 11(Y - 3)$

Problem 2.17

$Y = -10X_1 + 9X_2 + 5$

Problems 2.22, 2.25, and 2.29 are all modeling problems. Solve them by writing the differential equations for each mass, tank, or circuit loop in terms of the inputs and outputs. You always need to be very careful with the signs of your terms.

Problem 2.22

$$\sum F_{m_1} = m_1\ddot{y}_1 = -F_{k_1} - F_{k_2} - F_{b_2} - F_{k_3} - F_{b_3}$$
$$m_1\ddot{y}_1 = -k_1y_1 - k_2(y_1 - y_2) - b_2(\dot{y}_1 - \dot{y}_2) - k_3(y_1 - y_3) - b_3(\dot{y}_1 - \dot{y}_3)$$
$$m_1\ddot{y}_1 + (b_2 + b_3)\dot{y}_1 + (k_1 + k_2 + k_3)y_1 = k_2y_2 + b_2\dot{y}_2 + k_3y_3 + b_3\dot{y}_3$$

$$\sum F_{m_2} = m_2\ddot{y}_2 = -F_{k_2} - F_{b_2} - F_{k_4} + F_{u_2}$$
$$m_2\ddot{y}_2 = -k_2(y_2 - y_1) - b_2(\dot{y}_2 - \dot{y}_1) - k_4(y_2 - y_3) + u_2$$
$$m_2\ddot{y}_2 + b_2\dot{y}_2 + (k_1 + k_4)y_2 = k_2y_1 + b_2\dot{y}_1 + k_4y_3 + u_2$$

$$\sum F_{m_3} = m_3\ddot{y}_3 = -F_{k_3} - F_{b_3} - F_{k_4} + F_{u_3}$$
$$m_3\ddot{y}_3 = -k_3(y_3 - y_1) - b_3(\dot{y}_3 - \dot{y}_1) - k_4(y_3 - y_2) + u_3$$
$$m_3\ddot{y}_3 + b_3\dot{y}_3 + (k_3 + k_4)y_3 = k_3y_1 + b_3\dot{y}_1 + k_4y_2 + u_2$$

Final answer:

$$m_1\ddot{y}_1 + (b_2 + b_3)\dot{y}_1 + (k_1 + k_2 + k_3)y_1 = k_2y_2 + b_2\dot{y}_2 + k_3y_3 + b_3\dot{y}_3$$
$$m_2\ddot{y}_2 + b_2\dot{y}_2 + (k_1 + k_4)y_2 = k_2y_1 + b_2\dot{y}_1 + k_4y_3 + u_2$$
$$m_3\ddot{y}_3 + b_3\dot{y}_3 + (k_3 + k_4)y_3 = k_3y_1 + b_3\dot{y}_1 + k_4y_2 + u_2$$

Problem 2.25

$$M\ddot{Y} = K_1(-Y + R\theta) + K_2(-Y) + B(-\dot{Y})$$
$$J\ddot{\theta} = T + K_1R(Y - R\theta)$$
$$X_1 = Y, X_2 = \dot{Y}, X_3 = \theta, X_4 = \dot{\theta}$$
$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\dfrac{K_1 + K_2}{M} & -\dfrac{B}{M} & \dfrac{K_1R}{M} & 0 \\ 0 & 0 & 0 & 1 \\ \dfrac{K_1R}{J} & 0 & -\dfrac{K_1R^2}{J} & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{1}{J} \end{bmatrix} T$$

Problem 2.29

Assuming positive flow, Q, is to the right.

$$q_i - q_b = C_1 \frac{dh_1}{dt}$$
$$q_b - q_o = C_2 \frac{dh_2}{dt}$$

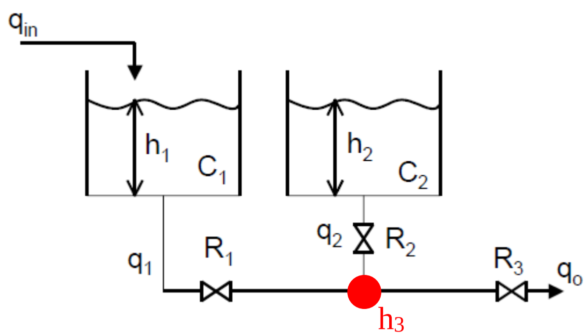
$$q_b = \frac{h_1 - h_2}{R_1}$$
$$q_o = \frac{h_2}{R_2}$$

$$q_i - \frac{h_1 - h_2}{R_1} = C_1 \frac{dh_1}{dt}$$
$$\frac{h_1 - h_2}{R_1} - \frac{h_2}{R_2} = C_2 \frac{dh_2}{dt}$$

$$R_1 C_1 \frac{dh_1}{dt} + h_1 = h_2 + R_1 q_i$$
$$R_1 R_2 C_2 \frac{dh_2}{dt} + (R_1 + R_2) h_2 = R_2 h_1$$

Problem 2.30

2.30 Determine the equations describing the system given in Figure 2.41. Formulate as time derivatives of h_1 and h_2.



Hint: create a new virtual pressure head (h_3) to help with creating the equations

$$q_{in} - q_1 = C_1 \frac{dh_1}{dt}$$
$$-q_2 = C_2 \frac{dh_2}{dt}$$
$$q_o = q_1 + q_2$$

$$q_1 = \frac{h_1 - h_3}{R_1}$$
$$q_2 = \frac{h_2 - h_3}{R_2}$$
$$q_o = \frac{h_3}{R_3} \rightarrow h_3 = q_o R_3$$

$$q_{in} - \frac{h_1 - q_o R_3}{R_1} = C_1 \frac{dh_1}{dt}$$
$$-\frac{h_2 - q_o R_3}{R_2} = C_2 \frac{dh_2}{dt}$$

$$R_1 C_1 \frac{dh_1}{dt} + h_1 = R_1 q_{in} - q_o R_3$$
$$R_2 C_2 \frac{dh_2}{dt} + h_2 = -q_o R_3$$