Sterilization Homework

ABE 557

Kathryn Atherton

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Problem 5.2-3: Unsteady-State Heating of a Stirred Tank

A vessel is filled with 0.0283 m^3 of water initially at 288.8 K. The vessel, which is well stirred, is suddenly immersed in a steam bath held at 377.6 K. The overall heat transfer coefficient U between the steam and water is 1136 W/m^2.K and the area is 0.372 m^2. Neglecting the heat capacity of the walls and agitator, calculate the time in hours to heat the water to 338.7 K. [Hint: Since the water is well-stirred, its temperature is uniform. Show that Eq. 5.2-3 holds by starting with Eq. 5.2-1).]

```
volume = 0.0283; % units = m^3
To_water = 288.8; % units = K
T_steam = 377.6; % units = K
U = 1136; % units = W/m^2.K
area = 0.372; % units = m^2
Tf_water = 338.7; % units = K
```

Equation 5.2-1

$$\mathrm{UA}(T_\infty - T)\mathrm{dt} = c_p \rho \mathrm{VdT}$$

$$\frac{\mathrm{dt}}{c_p \rho V} = \frac{\mathrm{dT}}{\mathrm{UA}(T_\infty - T)}$$

$$\frac{t}{c_p \rho V} = -\frac{1}{\mathrm{UA}} \ln\left(\frac{|T_\infty - T_f|}{|T_o - T_\infty|}\right)$$

$$t = -\frac{1}{\mathrm{UA}} \ln\left(\frac{|T_\infty - T_f|}{|T_o - T_\infty|}\right) c_p \rho V$$

```
cp_water = 4.179; % units = J/g.K
rho_water = 997; % units = kg/m^3
rho_water = rho_water * 1000; % units = kg / m^3 * g / kg = g / m^3

t = (-1 / (U * area)) * cp_water * rho_water * volume * ...
    log(abs((T_steam - Tf_water)/(To_water - T_steam))); % units = s
t = t / 3600; % units = s * h / s = h
fprintf('The time required to heat the water from %.1f K to %.1f K is %.2f hours.',...
    To_water,Tf_water,t);
```

```
The time required to heat the water from 288.8 K to 338.7 K is 0.06 hours.
```

Problem 5.4-5: Cooling Beef with Convective Resistance

A large slab of beef is 45.7 mm thick and is at an initial uniform temperature of 37.78 degrees C. It is being chilled at the front surface in a chilled air blast at -1.11 degrees C with a convective heat-transfer coefficient of h = 38.0 W/m^2.K. The rear face of the meat is insulated. The thermal conductivity of the beef is k = 0.498 W/m.K and alpha = 4.464e-4 m^2/h. Using a numerical method with five slices and M = 4.0, calculate the temperature profile after 0.27 h.

```
x = 45.7; % units = mm
x = x / 1000; % units = mm * m / mm = m
Ti_meat = 37.78 + 273.17; % units = K
T_air = -1.11+ 273.15; % units = K
h = 38.0; % W/m^2.K
k = 0.498; % W/m.K
alpha = 4.464e-4; % m^2/h
m = 4.0;
slices = 5;
del_x = x / slices; % units = m
t = 0.27; % units = h
```

Equation 5.4-2: for n = 2, 3, 4

$$_{t+\Delta t}T_n = \frac{1}{M}\left({}_tT_{n+1} + (M-2)\,{}_tT_n + {}_tT_{n-1}\right)$$

Equation 5.4-3

$$M = \frac{\Delta x^2}{\alpha \Delta t}$$

$$\Delta t = \frac{M\alpha}{\Delta x^2}$$

Equation 5.4-7: for n = 1

$$_{t+\Delta t}T_1 = \frac{1}{M}\left(2N\,{}_tT_a + \left(M - (2N+2)\,{}_tT_1 + 2\,{}_tT_2\right)\right)$$

Equation 5.4-8

$$N = \frac{h\Delta x}{k}$$

Equation 5.4-10: for insulated face, n = 5

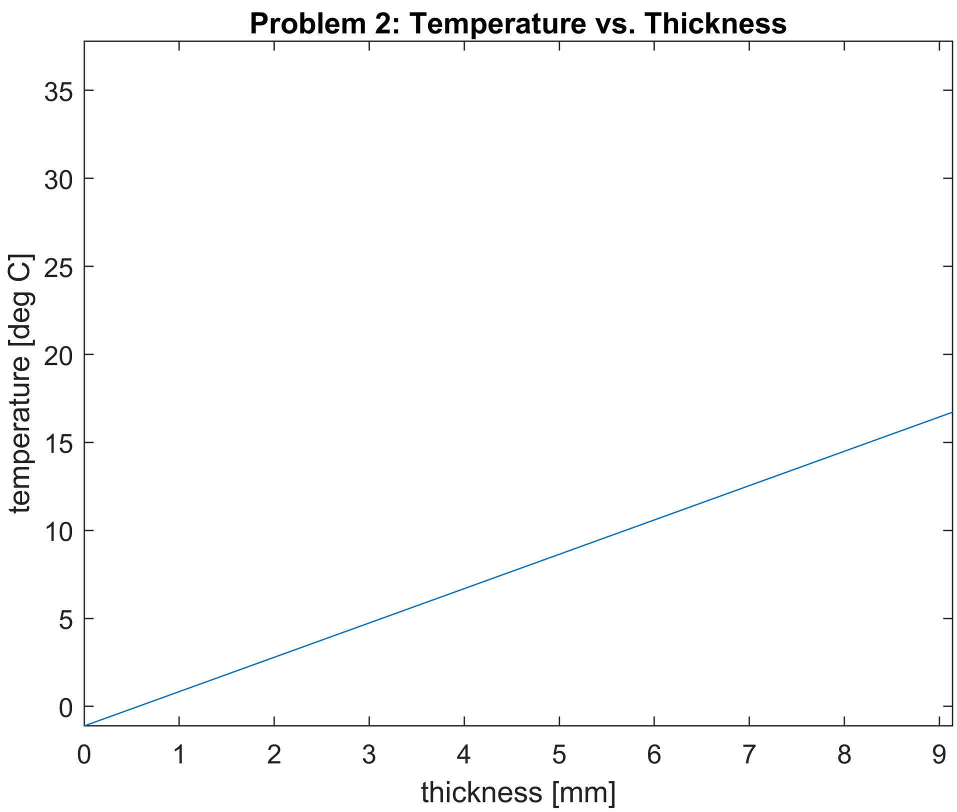
$$_{t+\Delta t}T_f = \frac{1}{M}\left((M-2)\,{}_tT_f + 2\,{}_tT_{f-1}\right)$$

```
t_T1 = Ti_meat;
t_T2 = Ti_meat;
t_T3 = Ti_meat;
t_T4 = Ti_meat;
```

```
t_T5 = Ti_meat;

ti = 0;

del_t = (del_x ^ 2) / (m * alpha); % units = h

N = h * del_x / k;

while ti < t
    ti = ti + del_t;
    tdelt_T1 = (1/m) * (2 * N * T_air + (m - (2 * N + 2)) * t_T1 + 2 * t_T2);
    tdelt_T2 = (1/m) * (t_T3 + (m - 2) * t_T2 + t_T1);
    tdelt_T3 = (1/m) * (t_T4 + (m - 2) * t_T3 + t_T2);
    tdelt_T4 = (1/m) * (t_T5 + (m - 2) * t_T4 + t_T3);
    tdelt_T5 = (1/m) * ((m - 2) * t_T5 + 2 * t_T4);

    t_T1 = tdelt_T1;
    t_T2 = tdelt_T2;
    t_T3 = tdelt_T3;
    t_T4 = tdelt_T4;
    t_T5 = tdelt_T5;
end

x = [0, 1 * del_x * 1000, 2 * del_x * 1000, 3 * del_x * 1000, 4 * del_x * 1000, ...
    5 * del_x * 1000]; % units = mm
T = [T_air - 273.15, t_T1 - 273.15, t_T2 - 273.15, t_T3 - 273.15, t_T4 - 273.15, ...
    t_T5 - 273.15]; % units = deg C
fprintf(['The temperature at node 1 is %.2f, at node 2 is %.2f, at node 3 is %.2f,' ...
    ' at node 4 is %.2f, and \nat node 5 is %.2f.\n'],T(2), T(3), T(4), T(5), T(6));
```

```
The temperature at node 1 is 16.72, at node 2 is 27.59, at node 3 is 34.02, at node 4 is 36.79, and
at node 5 is 37.47.
```

```
plot(x,T);
ylim([T_air - 273.15,Ti_meat - 273.15]);
xlim([0, del_x * 1000]);
xlabel('thickness [mm]')
ylabel('temperature [deg C]')
title('Problem 2: Temperature vs. Thickness')
```



Problem 9.12-5: Process Time and Numerical Integration

The following time-temperature data were obtained for the heating, holding, and cooling of a canned food product in a retort, the temperature being measured in the center of the can. The F0 value used is 2.60 min and z is 10 deg C/18 deg F. Calculate the F0 value for this process and determine if the thermal processing is adequate. Use SI and English units.

```
t = [0, 20, 40, 60, 80, 90, 100]; % units = min
T_c = [43.3, 79.3, 96.1, 108.9, 111.1, 107.2, 71.1]; % units = deg C
T_f = [110, 165, 205, 228, 232, 225, 160]; % units = deg F
F0 = 2.60; % units = min
z_c = 10; % units = deg C
z_f = 18; % units = deg F
```

Equation 9.12-16

$F_0 = \int_{t=0}^{t=t} 10^{\frac{T-250}{z}} \mathrm{dt}$ (English Units)

$F_0 = \int_{t=0}^{t=t} 10^{\frac{T-121.1}{z}} \mathrm{dt}$ (SI Units)

```
F0_f = ((10^((T_f(1) - 250)/z_f) + 10^((T_f(2) - 250)/z_f)/2) * (t(2)-t(1))) + ...
    ((10^((T_f(2) - 250)/z_f) + 10^((T_f(3) - 250)/z_f)/2) * (t(3)-t(2))) + ...
    ((10^((T_f(3) - 250)/z_f) + 10^((T_f(4) - 250)/z_f)/2) * (t(4)-t(3))) + ...
    ((10^((T_f(4) - 250)/z_f) + 10^((T_f(5) - 250)/z_f)/2) * (t(5)-t(4))) + ...
```

```
    ((10^((T_f(5) - 250)/z_f) + 10^((T_f(6) - 250)/z_f)/2) * (t(6)-t(5))) + ...
    ((10^((T_f(6) - 250)/z_f) + 10^((T_f(7) - 250)/z_f)/2) * (t(7)-t(6))); % Trapezoidal Metho
F0_c = ((10^((T_c(1) - 121.1)/z_c) + 10^((T_c(2) - 121.1)/z_c)/2) * (t(2)-t(1))) + ...
    ((10^((T_c(2) - 121.1)/z_c) + 10^((T_c(3) - 121.1)/z_c)/2) * (t(3)-t(2))) + ...
    ((10^((T_c(3) - 121.1)/z_c) + 10^((T_c(4) - 121.1)/z_c)/2) * (t(4)-t(3))) + ...
    ((10^((T_c(4) - 121.1)/z_c) + 10^((T_c(5) - 121.1)/z_c)/2) * (t(5)-t(4))) + ...
    ((10^((T_c(5) - 121.1)/z_c) + 10^((T_c(6) - 121.1)/z_c)/2) * (t(6)-t(5))) + ...
    ((10^((T_c(6) - 121.1)/z_c) + 10^((T_c(7) - 121.1)/z_c)/2) * (t(7)-t(6))); % Trapezoidal Me
fprintf('The F0 value in English units is %.2f mins and in SI units is %.2f mins.', ...
    F0_f, F0_c)
```

```
The F0 value in English units is 4.51 mins and in SI units is 4.52 mins.
```

```
if ((F0_f + F0_c)/2) >= F0
    fprintf('Therefore, the thermal process is adequate.')
else
    fprintf('Therefore the thermal process is inadequate.')
end
```

```
Therefore, the thermal process is adequate.
```

Problem 9.12-6: Sterility Level of Fermentation Medium

The aqueous medium in a fermentor is being sterilized and the time-temperature data obtained are as follows. The reaction velocity constant k in min^-1 for ___ can be represented as (see below) where T = K. The contamination level N0 = 1e12 spores. Calculate the sterility level N at the end and V.

```
t = [0, 10, 20, 25, 30, 35]; % units = min
T_c = [100, 110, 120, 120, 110, 100]; % units = deg C
T_k = T_c + 273.15; % units = K
k = 7.94e38 * exp((-68.7e3) ./ (1.987 .* T_k)); % units = min^-1
N0 = 1e12; % units = spores
```

Equation 9.12-1

$$\frac{\mathrm{dN}}{\mathrm{dt}} = -\mathrm{kN}$$

Equation 9.12-3

$$\ln \frac{N_0}{N} = \mathrm{kt}$$

Equation 9.12-4

$$N = N_0 e^{-\mathrm{kt}}$$

```
N_final = N0 * exp(-k(6) * t(6)); % units = spores
```

Equation 9.12-17

$$\nabla = \ln \frac{N_0}{N}$$

```
V = log(N0/N_final);
fprintf("The final sterility level is %.2d and the V is %.2f.",N_final,V)
```

```
The final sterility level is 2.02e+11 and the V is 1.60.
```

Problem 9.12-7: Time for Pasteurization of Milk

Calculate the time in min at 62.8 degrees C for pasteurization of milk. The F0 value to be used at 65.6 degrees C is 9.0 min. The z value is 5 degrees C.

```
T = 62.8; % units = deg C
F0 = 9.0; % units = minutes
T1 = 65.6; % units = deg C
z = 5; % units = deg C
```

Equation 9.12-19

$$F_0 = t10^{\frac{T-T_1}{z}}$$

$$t = \frac{F_0}{10^{\frac{T-T_1}{z}}}$$

```
t = F0 / (10 ^ ((T-T1)/z)); % units = minutes
fprintf('The time to pasteurize is %.2f min.',t)
```

```
The time to pasteurize is 32.68 min.
```

Rederive Equation 5.4-20 for a cylinder with convection not ignoring cp.

Equation 5.4-20

$${}_{t+\Delta t}T_n = \frac{1}{M}\left(\frac{2n+1}{2n}{}_tT_{n+1} + (M-2)\,{}_tT_n + \frac{2n-1}{2n}{}_tT_{n-1}\right)$$

Equation 5.4-19

$$\frac{k\left[2\pi\left(n+\frac{1}{2}\right)\Delta x\right]}{\Delta x}\left({}_tT_{n+1} - {}_tT_n\right) - \frac{k\left[2\pi\left(n-\frac{1}{2}\right)\Delta x\right]}{\Delta x}\left({}_tT_n - {}_tT_{n-1}\right) = \frac{2\pi\,\Delta x^2 \rho c_p}{\Delta t}\left({}_{t+\Delta t}T_n - {}_tT_n\right)$$

Adding Convection assuming only radial motion

$$\frac{(k+h\Delta x)\left[2\pi\left(n+\frac{1}{2}\right)\Delta x\right]}{\Delta x}\left({}_tT_{n+1}-{}_tT_n\right)-\frac{(k+h\Delta x)\left[2\pi\left(n-\frac{1}{2}\right)\Delta x\right]}{\Delta x}\left({}_tT_n-{}_tT_{n-1}\right)=\frac{2\pi\Delta x^2\rho c_p}{\Delta t}\left({}_{t+\Delta t}T_n-{}_tT_n\right)$$

$$(k+h\Delta x)\left[\left(n+\frac{1}{2}\right)\right]\left({}_tT_{n+1}-{}_tT_n\right)-(k+h)\left[\left(n-\frac{1}{2}\right)\right]\left({}_tT_n-{}_tT_{n-1}\right)=\frac{\Delta x^2\rho c_p}{\Delta t}\left({}_{t+\Delta t}T_n-{}_tT_n\right)$$

$$\frac{(k+h\Delta x)\Delta t}{\Delta x^2\rho c_p}\left(\left[\left(n+\frac{1}{2}\right)\right]\left({}_tT_{n+1}-{}_tT_n\right)-\left[\left(n-\frac{1}{2}\right)\right]\left({}_tT_n-{}_tT_{n-1}\right)\right)=\left({}_{t+\Delta t}T_n-{}_tT_n\right)$$

$${}_{t+\Delta t}T_n=\frac{(k+h\Delta x)\Delta t}{\Delta x^2\rho c_p}\left(\left(\frac{2n+1}{2n}\right){}_tT_{n+1}-\left(\frac{4n^2-1}{4n^2}\right){}_tT_n+\left(\frac{2n-1}{2n}\right){}_tT_{n-1}\right)+{}_tT_n$$

$${}_{t+\Delta t}T_n=\frac{(k+h\Delta x)\Delta t}{\Delta x^2\rho c_p}\left(\left(\frac{2n+1}{2n}\right){}_tT_{n+1}+\left(\frac{\Delta x^2\rho c_p}{(k+h\Delta x)\Delta t}-\frac{4n^2-1}{4n^2}\right){}_tT_n+\left(\frac{2n-1}{2n}\right){}_tT_{n-1}\right)$$

$${}_{t+\Delta t}T_n=\left(\frac{1}{M}+\frac{h\Delta t}{\Delta x\rho c_p}\right)\left(\left(\frac{2n+1}{2n}\right){}_tT_{n+1}+\left(M+\frac{\Delta x\rho c_p}{h\Delta t}-\frac{4n^2-1}{4n^2}\right){}_tT_n+\left(\frac{2n-1}{2n}\right){}_tT_{n-1}\right)$$

$${}_{t+\Delta t}T_n=\left(\frac{1}{M}+\frac{h\Delta t}{\Delta x\rho c_p}\right)\left(\left(\frac{2n+1}{2n}\right){}_tT_{n+1}+\left(M+\frac{\Delta x\rho c_p}{h\Delta t}-2\right){}_tT_n+\left(\frac{2n-1}{2n}\right){}_tT_{n-1}\right)$$