Homework 5

Problem 10.1

Given

The air supply to a fermenter was turned off for a short period of time and then restarted.

$C^* = 7.3$ mg/l

Table 1. DO vs. time data for fermenter.

	Time(min)	DO(mg/L)
	-1	3.3
AIR OFF	0	3.3
	1	2.4
	2	1.3
	3	0.3
	4	0.1
	5	0
AIR ON	6	0
	7	0.3
	8	1
	9	1.6
	10	2
	11	2.4
	12	2.7
	13	2.9
	14	3
	15	3.1
	16	3.2
	17	3.2

Find

Estimate the oxygen uptake rate (OUR) and k_La of this system.

Solution

Nomenclature note: DO = C_L

From Schuler chapter 10 on p. 296:

- Plot DO vs. time. The slope of the descending curve (air off) will give the OUR or $-q_{O2}X$.
- A plot of $\left(\frac{dC_L}{dt} + OUR\right) vs. (C^* - C_L)$ results in a line with a slope of k_La

Obtaining OUR:

- For the descending curve, DO was plotted vs. time from time = 0 min to time = 3 min.
- The last couple points at time = 4 min and time = 5 min were excluded to improve the fit of the linear regression.
- From the slope of the line, $\boldsymbol{OUR = 1.01 \frac{mg\ O_2}{L \cdot min}}$

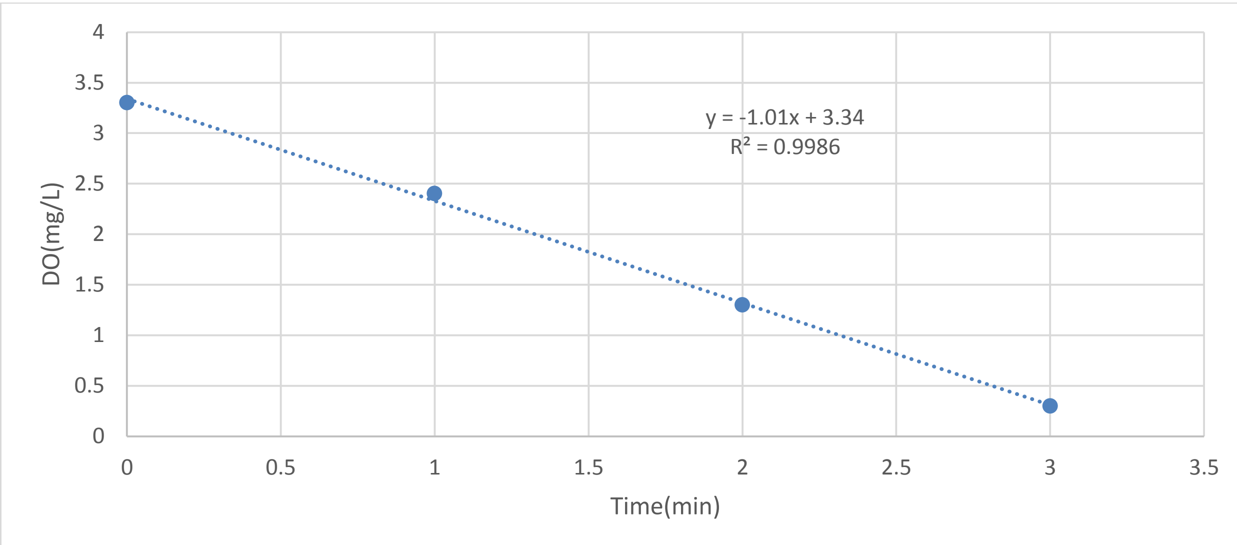

Fig. 1: DO vs. time. Descending curve when air was turned off.

Obtaining k_La:

- As shown in Table 2 and Figure 2 below, Excel was used to calculate and plot $\left(\frac{dC_L}{dt} + OUR\right) vs. (C^* - C_L)$ for the ascending curve (air on).
- The first point was excluded to prevent any effects from start-up conditions immediately after the air was turned on.
- From the slope of the line, $\mathbf{k_La = 0.2337\ min^{-1}}$

Table 2. Ascending curve data (air turned on).

	Time (min)	DO (mg/L)	C^*-C_L (mg/L)	dC_L/dt	dC_L/dt + OUR
AIR ON	6	0	7.3	0.3	1.31
	7	0.3	7	0.7	1.71
	8	1	6.3	0.6	1.61
	9	1.6	5.7	0.4	1.41
	10	2	5.3	0.4	1.41
	11	2.4	4.9	0.3	1.31
	12	2.7	4.6	0.2	1.21
	13	2.9	4.4	0.1	1.11
	14	3	4.3	0.1	1.11
	15	3.1	4.2	0.1	1.11
	16	3.2	4.1	0	1.01

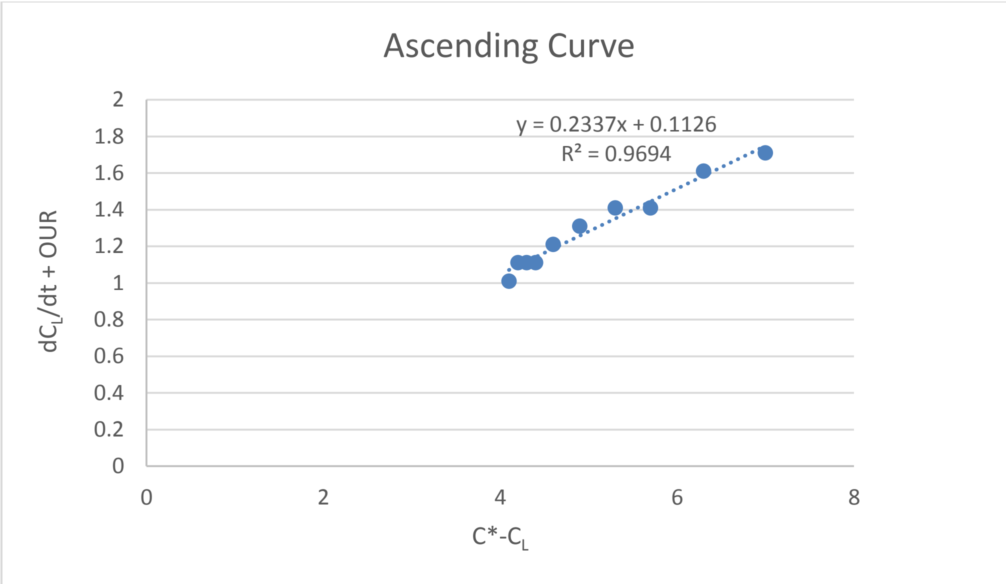

Fig. 2: Plot of $\left(\frac{dC_L}{dt} + OUR\right) vs. (C^* - C_L)$ for the descending curve (air off). From the slope of the line, k_La is equal to 0.2337 min^{-1}.

Problem 10.2

Given

$k_L a = 30\ h^{-1}$

$$Air\ rate = 0.5\ \frac{L\ gas}{L\ reactor\ volume\ \cdot min}$$

$$q_{O2} = 10\ \frac{mmol\ O_2}{(g\ dry\ weight)(h)}$$

Critical DO concentration = C_L = 0.2 mg/L

Oxygen solubility: $C^* = 7.3\frac{mg}{L}$ at 30°C

Find

a) Find the maximum concentration (X) of *E. coli* that can be sustained in this fermenter under aerobic conditions.
b) Find the concentration of *E. coli* that could be maintained if pure oxygen were used to sparge the reactor.

Solution

a)

$OUR = X\, q_{O_2} = k_L a(C^* - C_L)$ (Schuler Equation 10.1 on p. 292)

Solve for concentration, X:

$$X = \frac{k_L a(C^* - C_L)}{q_{O2}} = \frac{(30\ h^{-1})(7.3 - 0.2\ \frac{mg\ O_2}{L})}{\frac{10\ mmol\ O_2}{(g\ dry\ weight)(h)} \cdot \frac{32\ mg\ O_2}{1\ mmol\ O_2}} = \mathbf{0.666\ \frac{g}{L}\ of\ E.coli}$$

b) If pure oxygen were used to sparge the reactor, the partial pressure of oxygen in the air stream would change from P_{p1} = 0.21 atm to P_{p2} = 1 atm.

C* would change according to Henry's Law:

$$P_p = H\, C^*$$

$$H = \frac{P_{p1}}{C_1^*} = \frac{P_{p2}}{C_2^*}$$

$$C_2^* = P_{p2} \cdot \left(\frac{C_1^*}{P_{p1}}\right) = \frac{(1\ atm)\left(7.3\frac{mg}{L}\right)}{0.21\ atm} = 34.762\frac{mg}{L}$$

Solve for new concentration, X_2:

$$X_2 = \frac{k_L a(C_2^* - C_L)}{q_{O2}} = \frac{(30\ h^{-1})(34.762 - 0.2\ \frac{mg\ O_2}{L})}{\frac{10\ mmol\ O_2}{(g\ dry\ weight)(h)} \cdot \frac{32\ mg\ O_2}{1\ mmol\ O_2}} = \mathbf{3.24\ \frac{g}{L}\ of\ E.coli}$$

Problem 10.3

Given

Working volume: V = 80,000 L (in 100,000 L fermenter)

Rate of O_2 consumption = $OUR = \frac{100\ mmol\ O_2}{L \cdot h}$

Desired broth operating temperature = 35°C

Temperature of cooling water = $T_{cool} = 15℃$

Heat capacity: $c_{p,water} = c_{p,broth} = 4.184\ \frac{J}{g\ ℃}$ (from tables)

Minimum allowed ΔT between cooling water and broth = 5°C

- Because the broth temperature remains constant at 35°C, temperature of cooling water can increase to a max of 30°C.
- $\Delta T_{water} = 30℃ - 15℃ = 15℃$

Coil diameter = d = 2.5 cm = 0.025 m

Overall heat transfer coefficient: U =1420 $\frac{J}{s \cdot m^2 \cdot ℃}$

Find

a) Estimate the required cooling-water flow rate through a cooling coil.
b) Estimate the required length of cooling coil.

Solution

a)

$$O_2\ consumption\ rate = \frac{100\ mmol\ O_2}{L \cdot h} \cdot 80{,}000\ L = 8 \times 10^6\ \frac{mmol\ O_2}{h}$$

From Fermentation Design Handout:

$$Q\left[\frac{kcal}{h}\right] = 0.12\ q_{O_2}\ [\frac{mmol\ O_2}{h}]$$

$$Q = 0.12\ \left(8 \times 10^6\ \frac{mmol\ O_2}{h}\right) = 960{,}000\frac{kcal}{h}$$

Plug in known values into the heating equation for cooling water:

$$\Delta H_{total} = Q = mass * c_{p,water} * \Delta T$$

Solve for the mass (i.e. the cooling water flow rate):

$$mass = \frac{\Delta H_{total}}{c_{p,water} * \Delta T} = \frac{960{,}000\frac{kcal}{h} \cdot \frac{4.184\ kJ}{1\ kcal}}{4.184\ {}^{kJ}/_{kg\ °C} \cdot 15°C} = \mathbf{64,000}\ \frac{\boldsymbol{kg\ H_2O}}{\boldsymbol{h}}\ \boldsymbol{required}$$

b) To find the required length of coil, use the following equation:

$$\Delta H_{transfer} = UA(LMTD)$$

where A is the surface area of the coil, U is the overall heat transfer coefficient, and LMTD is the log mean temperature difference of the system.

$$A = 2\pi rL = \pi DL = \pi \cdot 0.025m \cdot L$$

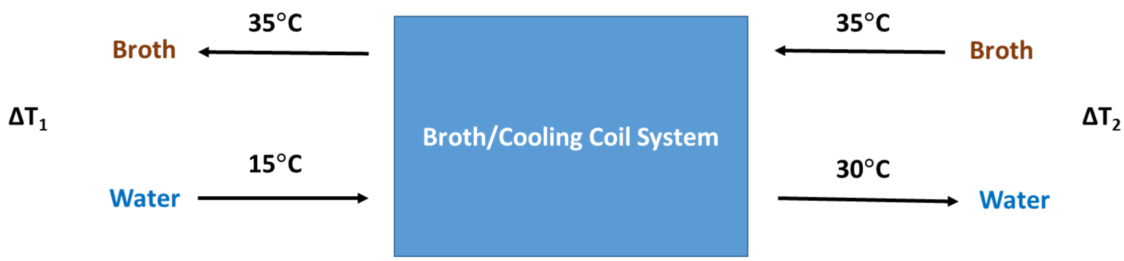

Fig. 3: Temperatures of broth and cooling water when entering or exiting system.

$$LMTD = \frac{\Delta T_1 - \Delta T_2}{\ln({}^{\Delta T_1}/_{\Delta T_2})} = \frac{(35-15)-(35-30)}{\ln({}^{20}/_{5})} = 10.82\text{°C}$$

Plugging in values into the heat transfer equation:

$$\Delta H_{transfer} = (1420\,\frac{J}{s\cdot m^2\cdot\text{°C}})(\pi \cdot 0.025m \cdot L)(10.82\text{°C})$$

Changing units of U:

$$U = 1420\,\frac{J}{s\cdot m^2\cdot\text{°C}} * \frac{1\ kJ}{1000\ J} * \frac{3600\ s}{1\ h} = 5112\,\frac{kJ}{h\cdot m^2\cdot C}$$

Solving for L:

$$L = \frac{960{,}000\,\frac{kcal}{h}\cdot\frac{4.184\ kJ}{1\ kcal}}{\left(5112\,\frac{kJ}{h\cdot m^2\cdot C}\right)(\pi \cdot 0.025m)(10.82\text{°}C)}$$

$$\boldsymbol{L = 924.6\ m\ required\ for\ cooling\ coil}$$

Problem 10.10

Given

E. coli maximum respiration rate: q_{O2max} = 240 mg O_2/(g-dry-wt-h)

Desired cell mass: X = 20 g dry wt/l

$k_La = 120\ h^{-1}$

Working volume: V = 800 L (in a 1000 L reactor)

Gas stream has 80% O_2

Oxygen solubility: C* value of 28 mg/L is used.

For an oxygen limited situation:

$$q_{O2} = \frac{q_{O2max} C_L}{0.2\frac{mg}{l} + C_L}$$

Find

Find C_L (dissolved oxygen concentration in the fermenter) when the cell mass is X = 20 g/l?

Solution

The oxygen uptake rate is:

$$OUR = X\, q_{O_2} = k_L a(C^* - C_L) \qquad \text{(Schuler Equation 10.1 on p. 292)}$$

Plugging in the given values:

$$(20\frac{g}{l})\left(\frac{240\ {}^{mg}/_{gh}\, C_L}{0.2\, {}^{mg}/_{l} + C_L}\right) = (120\ h^{-1})\left(28\, {}^{mg}/_{l} - C_L\right)$$

Rearranging:

$$120C_L^2 + 1464C_L - 672 = 0$$

Finding the positive root using the Mathcad root() function:

$X := 20$ $k := 120$ $qmax := 240$ $Cstar := 28$

$$f(Cl) := k \cdot Cl^2 + (X \cdot qmax - k \cdot Cstar + 0.2 \cdot k) \cdot Cl - (0.2 \cdot k \cdot Cstar)$$

$Cl := 28$

$Cl\_value := root(f(Cl), Cl) = 0.443$

$$\boldsymbol{C_L = 0.443 \frac{mg}{L}}$$

Problem 10.14

Given

A stirred-tank reactor is to be scaled down from $V_1 = 10\ m^3$ to $V_2 = 0.1\ m^3$.

Dimensions of the large tank:

$D_t = 2$ m

$D_i = 0.5$ m

N = 100 rpm

Find

a) Determine the dimensions of the small tank (Dt, Di, H) by using geometric similarity.
b) Determine the required rotational speed of the impeller (N) in the small tank for the following criteria:
    1. Constant tip speed
    2. Constant impeller Re number

Solution

a) Assume cylindrical geometry.

From Schuler p. 304

Scale-up factor = cube root ratio of tank volumes

$$= \left(\frac{10}{0.1}\right)^{\frac{1}{3}} = 100^{\frac{1}{3}} = 4.6416$$

Small tank dimensions:

$$\boldsymbol{D_t = \frac{2\ m}{4.6416} = 0.431\ m}$$

$$\boldsymbol{D_i = \frac{0.5\ m}{4.6416} = 0.108\ m}$$

Volume: $V = \pi r^2 H$

Large height, H_1:

$$H_1 = \frac{V_1}{\pi r^2} = \frac{V_1}{\pi(\frac{D_{t,1}}{2})^2} = \frac{10\ m^3}{\pi(1\ m)^2} = 3.183\ m$$

Small height, H_2:

$$\boldsymbol{H_2} = \frac{V_2}{\pi r^2} = \frac{V_2}{\pi(\frac{D_{t,2}}{2})^2} = \frac{0.1\ m^3}{\pi(\frac{0.431\ m}{2})^2} = \boldsymbol{0.685\ m}$$

b) N_2 of small tank?

1. Constant tip speed

   According to Schuler p. 304, ND_t must be the same in both vessels.

   Solving for N_2:

   $$N_2 = N_1\left(\frac{D_{t,1}}{D_{t,2}}\right) = (100\ rpm)\left(\frac{2\ m}{0.431\ m}\right) = \boldsymbol{464.16\ rpm}$$

2. Constant impeller Re number

   According to Schuler p. 304, ND_t^2 must be the same in both vessels.

   $$N_2 = N_1\left(\frac{D_{t,1}}{D_{t,2}}\right)^2 = (100\ rpm)\left(\frac{2\ m}{0.431\ m}\right)^2 = \boldsymbol{2,154.4\ rpm}$$