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Pre-Quiz 1: Membrane Separation

A membrane system is being designed to concentrate a 10 kg glucose/m^3 solution at 25°C (Mw glucose = 180.16 g/mol). The membrane pure water permeability constant Aw = 4.1e-4 kg water/s.m^2.atm and the glucose permeability constant As = 1.01e-7 m/s. Since the mass transfer constant kc = 1e-7 m/s the osmotic pressure will be affected by concentration polarization. Assume the transmembrane pressure is maintained at 50 atm. R = 82.075e-3 m^3.atm/kg mol.K.

Show how you would calculate the initial permeate flux, solute flux, and concentration of glucose on both sides of the membrane.

Given Equations:

1. $N_w = k_c \rho \ln\left(\frac{c_3 - c_2}{c_1 - c_2}\right)$
2. $N_w = A_w(\Delta P - \Delta \Pi)$
   a. $\Delta P = P_1 - P_2$
3. $N_w = \frac{N_s c_{w2}}{c_2}$
4. $N_s = A_s(c_3 - c_2)$
5. $\Delta \Pi = \Pi_{c3} - \Pi_{c2}$
   a. $\Pi = \frac{nRT}{V}$
   b. $\Delta \Pi = \frac{c_3 RT}{m} - \frac{c_2 RT}{m}$

Solution:

- From Equation 5b: $\Delta \Pi = (c_3 - c_2)\frac{RT}{m}$
- Substitute into Equation 2: $N_w = A_w(\Delta P - (c_3 - c_2)\frac{RT}{m})$
- From Equation 4: $c_2 = c_3 - \frac{N_s}{A_s}$
- From Equation 3: $\frac{N_s}{N_w} = \frac{c_2}{c_{w2}}$
- Solve for Ns: $N_s = \frac{c_2 N_w}{c_{w2}}$
- Rearrange result from Equation 4: $c_2 = \frac{c_3}{1 + \frac{N_w}{c_2 A_s}}$
- From Equation 1: $(\frac{c_3 - c_2}{c_1 - c_2}) = e^{\frac{N_w}{k_c \rho}}$
- Rearrange: $c_3 - c_2\left(1 - e^{\frac{N_w}{k_c \rho}}\right) = c_1 e^{\frac{N_w}{k_c \rho}}$
- From Okos: Check everything from here on for accuracy!!!
- Solve for c3: $c_3 = \frac{c_1 e^{\frac{N_w}{k_c \rho}}}{1 - (1 - \frac{e^{\frac{N_w}{k_c \rho}}}{1 + \frac{N_w A_s}{c_{w2}}})}$

- Solve for c2: $c_2 = \frac{c_1 e^{\frac{N_w}{k_c \rho}}}{1 - \frac{(1 - \frac{e^{\frac{N_w}{k_c \rho}}}{1 + \frac{N_w A_s}{c_{w2}}})}{(1 + \frac{N_w}{c_2 A_s})}}$
- Substitute into result of Equation 5b: $\Delta\Pi = \left(\frac{c_1 e^{\frac{N_w}{k_c \rho}}}{1 - \left(1 - \frac{e^{\frac{N_w}{k_c \rho}}}{1 + \frac{N_w A_s}{c_{w2}}}\right)} - \frac{c_1 e^{\frac{N_w}{k_c \rho}}}{1 - \frac{\left(1 - \frac{e^{\frac{N_w}{k_c \rho}}}{1 + \frac{N_w A_s}{c_{w2}}}\right)}{\left(1 + \frac{N_w}{c_2 A_s}\right)}} \right) \frac{RT}{m}$
- Substitute into Equation 2: $N_w = A_w \left(\Delta P - \left(\frac{c_1 e^{\frac{N_w}{k_c \rho}}}{1 - \left(1 - \frac{e^{\frac{N_w}{k_c \rho}}}{1 + \frac{N_w A_s}{c_{w2}}}\right)} - \frac{c_1 e^{\frac{N_w}{k_c \rho}}}{1 - \frac{\left(1 - \frac{e^{\frac{N_w}{k_c \rho}}}{1 + \frac{N_w A_s}{c_{w2}}}\right)}{\left(1 + \frac{N_w}{c_2 A_s}\right)}} \right) \frac{RT}{m} \right)$