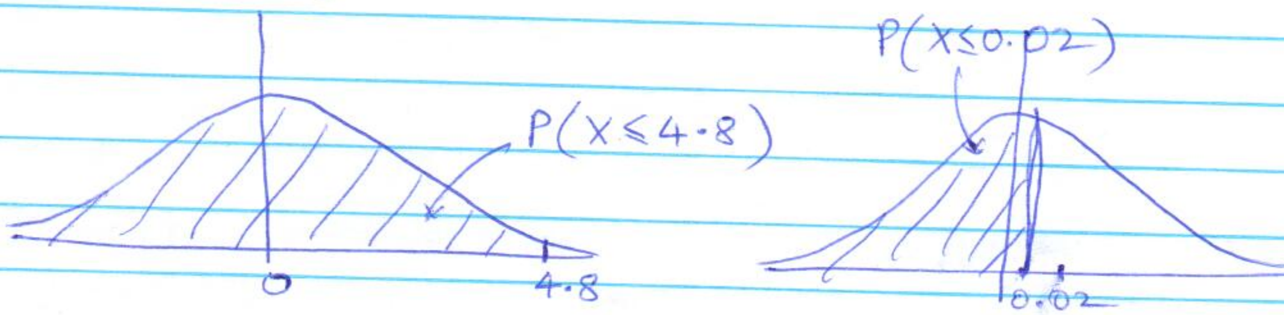
Lab 3.

2. a. $P(0.02 < z < 4.8)$

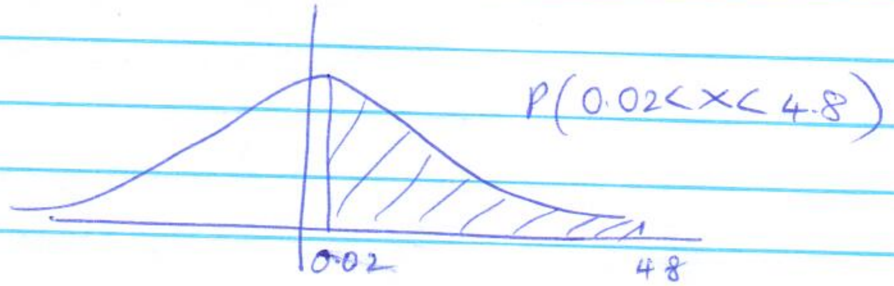
standard normal random variable

$\Rightarrow \sigma = 1 \quad \mu = 0$ (standard normal dist^n)

$= P(X \leq 4.8) - P(X \leq 0.02)$



$\Downarrow$



$= 0.49202$

b. $P(X \leq 13.8) \quad \mu = 12.8\text{ mm} \quad \sigma = 5.2\text{mm}$

$$Z = \frac{13.8 - 12.8}{5.2} = 0.1923$$

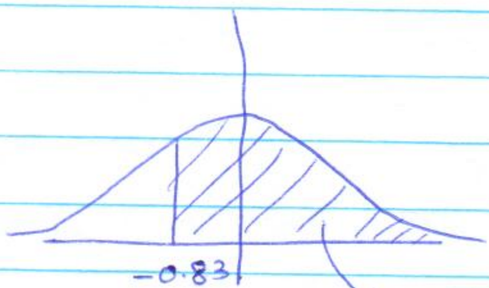
$P(z = 0.19) \Rightarrow$ get from standard normal dist^n table (Appendix A of text)

or =NORM.DIST(13.8,12.8,5.2,TRUE)

2.

c. $P(X > 100\,hr)$ $\quad \mu = 110\,hr \quad \sigma = 12\,hr$.

$$Z = \frac{100 - 110}{12} = -0.83$$



$\cancel{P(Z = -0.83) = P(Z = 0.83) = 0}$

$$P(Z > -0.83) = 1 - P(Z < -0.83)$$
$$= P(Z < 0.83)$$

$$P(Z < 0.83) = 0.79673$$

d. $P(0.0050 < X < 0.0110)$ $\quad \mu = 0.0099$, $\sigma = 0.0021$

$$Z_- = \frac{0.0050 - 0.0099}{0.0021} = -2.333$$

$$Z_+ = \frac{0.0110 - 0.0099}{0.0021} = 0.523$$

$$\left.\begin{aligned} P(Z_-) &= 0.00982 \\ P(Z_+) &= 0.69979 \end{aligned}\right\} \quad P(0.0050 < X < 0.0110) = 0.6899 \approx 0.69$$

3. $P(z) = 0.95$

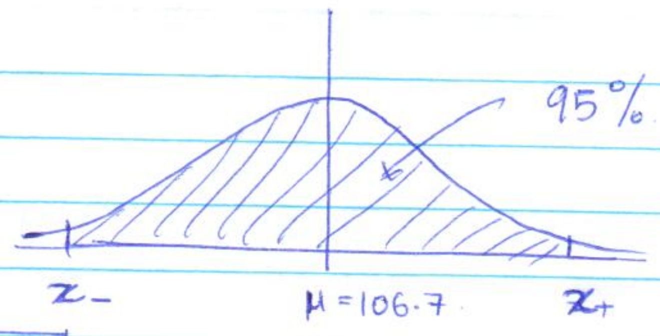
$z = ?$

find nearest value in statistical table (appendix A, table I) pg 486/487 of text.

or $= \text{NORM.S.INV}(0.95) = 1.64485$

Lab 3.

4.



$P(x_- < X < x_+) = 0.95$

$P(X < x_-) = 0.025$ $\qquad$ $P(X > x_+) = 0.025$

∴ ~~NORM.INV~~

$$Z = \frac{X - 106.7}{1.8}$$

$P(z_+) \Rightarrow P(Z < z_+) = \cancel{1.95996}\ 0.975$

$P(z_-) \Rightarrow P(Z < z_-) = 0.025$

$$\left.\begin{aligned} \therefore z_+ &= 1.95996 \\ z_- &= -1.95996 \end{aligned}\right\} \text{observe symmetry}$$

$$\therefore x_- = z_- * \sigma$$
$$x_+ = z_+ * \sigma$$

$$\therefore X = \mu \pm z * \sigma$$
$$\therefore \quad = 106.7 \pm 3.53 \text{ mm}$$

5 log normal distribution

$\ln(x)$ is a normal distribution

$x = 100$.

$$\therefore Z^* = \frac{\ln(100) - \theta}{\omega} = -0.263$$

$$\therefore P(Z < z^*) = 0.3962$$

But distribution greater than 100 required

$\therefore 1 - 0.3962$

$= 0.6038$

6. Binomial distribution

$p = 1:100 = \frac{1}{101}$

a) $n = 100 \quad x = 1$

$P(X=1) = \binom{100}{1}\left(\frac{1}{101}\right)^1\left(1-\frac{1}{101}\right)^{99} = 0.369$

b) $P(X \geq 2) = 1 - P(X \leq 1) = 1 - (P(X=1) + P(X=0))$

$= 1 - \left[\binom{100}{1}\left(\frac{1}{101}\right)^1\left(1-\frac{1}{101}\right)^{99} + \binom{100}{0}\left(\frac{1}{101}\right)^0\left(1-\frac{1}{101}\right)^{100}\right]$

$= 0.261$

7. Poisson distribution

$\lambda = 4.3$ particles / 10 min.

a) $P(X \geq 8) = 1 - P(X \leq 7)$

$= 1 - \sum_{x=0}^{7} \frac{e^{-4.3}(4.3)^x}{x!}$

$= 0.071$

b) $P(X=1) = \frac{e^{-4.3}(4.3)^1}{1!} = 0.058$

$P(X=2) = 0.125$

$P(X=3) = 0.179$

To use Excel to solve binomial distribution problems, use the function `binomdist(x,n,p,true)` where, as in the `normdist` functions, "true" refers to finding the probability (CDF) and "false" refers to finding the f(x) (PDF).

2. A Poisson distribution can be used to calculate the number of events that occur randomly over an interval. A Poisson distribution is represented by:

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

   Where λ is the mean and variance of X. To use Excel for Poisson distribution problems, use the function `poisson(x,λ,true)` where, as in the `normdist` functions, "true" refers to finding the probability (CDF) and "false" refers to finding the f(x) (PDF).

Lab 3 Exercises

You use JMP or Excel as needed for today's problems, or if you prefer, you may do them by hand using the charts in your textbook.

Continuous Distributions

1. Using the data for Exercise 3-87 (from the textbook site), from your choices of normal, lognormal, or Weibull, which distribution fits the data the best and why?
2. Find the following probabilities:
   a. P(0.02 < Z < 4.8)
      Normdist(4.8,0,1,true)-normdist(0.02,0,1,true)
      We are specifying "true" since we are finding the cumulative probability here.
   b. P(X $\leq$ 13.8 mm) where μ = 12.8 mm and σ = 5.2 mm
      Normdist(13.8,12.8,5.2,true)
      Specifying "true" since we are finding the cumulative probability. Also, mean and variance is specified.
   c. P(X > 100 hr) where μ = 110 hr and σ = 12 hr
      1-normdist(100,110,12) since normdist finds the cumulative probability of P(X<100 hr)
   d. P(0.0050 mg/mL < X < 0.0110 mg/mL) where μ = 0.0099 mg/mL and σ = 0.0021 mg/mL
      Normdist(0.011,0.0099,0.0021,true)-normdist(0.005,0.0099,0.0021)
3. What is the z value that corresponds to a probability of 95%?
   P(x<z)=0.95
   Use the excel function normsinv to find the z value. Normsinv(0.95) = 1.65.

4. If a process creates a part with a diameter that follows a normal distribution with a mean value of 106.7 mm and a standard deviation of 1.8 mm, in what range of diameters will 95% of the parts produced occur? Give your answer in the form of 106.7 ± *x* mm.

   P(106.7-x < X < 106.7+x) = 0.95

Subtracting 106.7, we get,

$P(-x < X-106.7 < x) = 0.95$

Dividing by the standard deviation, we get,

$P(-x/1.8 < (X-106.7)/1.8 < x/1.8) = 0.95$

$(X-106.7)/1.8$ will follow a normal distribution with mean 0 and variance 1.

$P(-x/1.8 < z < x/1.8) = 0.95$

$\Phi(x/1.8) - \Phi(-x/1.8) = 0.95$

$\Phi(x/1.8) - (1- \Phi(x/1.8)) = 0.95$

$2\ \Phi(x/1.8) = 1.95$

$\Phi(x/1.8) = 0.975$

x/1.8 = normsinv(0.975) = 1.95

x = 1.8*normsinv(0.975) = 3.52

5. For a lognormal distribution with $\theta = 5$ and $\omega = 1.5$, what proportion of the distribution is greater than 100?

   1-lognormdist(100,5,1.5)

Discrete Distributions

6. A game sells tokens for $1 a pack. Each pack has a 1:100 chance of having a special token included.
   a. What are the odds of getting a single special token when purchasing 100 packs?

      P = 1/101 to be used in the binomial distribution

      Ans: binomdist(1,100,1/101,false)

      Here, we are not finding the cumulative probability, hence, the input will be "false".

   b. What are the odds of getting two or more special tokens when purchasing 100 packs?

1-binomdist(1,100,1/101,true)

7. You are monitoring the output of a reactor for catalyst particles escaping by counting the particles over a period of 10 minutes. Historically, the average number of particles per 10 minutes is 4.3 particles.
    a. What is the probability of finding 8 or more particles escaping the reactor in 10 minutes?

       1-poisson(7,4.3,true)

       Here, we subtract the probability that we will get 7 or less particles, from 1.

    b. What is the probability of finding 1, 2, or 3 particles over a 10 minute span?

       Poisson(1,4.3,false) + poisson(2,4.3,false) + poisson(3,4.3,false)

       OR

       Poisson(3,4.3,true)-poisson(0,4.3,false)