

Instructions:

Read the questions carefully and plan accordingly before answering. Use the backside of the pages if extra space is required.

Name: _____

Signature: _____

1. (30 marks) In a study on effect of certain nutrients on the birth weight of guinea pigs record the following data:

Table 1: Birth weight (gms)

421	453	456	495	374
91	111	96	82	102

(Residuals)²

$$\begin{array}{l} 2.34 \times 10^4 \quad 3.42 \times 10^4 \quad 3.53 \times 10^4 \\ 5.14 \times 10^4 \\ 1.12 \times 10^4 \\ 3.14 \times 10^4 \quad 2.47 \times 10^4 \quad 2.96 \times 10^4 \\ 3.46 \times 10^4 \\ 2.76 \times 10^4 \end{array}$$

- (a) Test the hypothesis that mean body weight is 300 grams. Use $\alpha = 0.05$
(b) Why do we say "failed to reject the null hypothesis" instead of "accept the null hypothesis"?
(c) Explain how you could answer the question in part (a) with a two sided confidence interval on mean body weight.

$$\sum (\text{Residual})^2 = 3.03 \times 10^5$$

a) $\bar{x} = 268.1$

$$S = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{3.03 \times 10^5}{9}} = 183.5$$

$$H_0: \mu = 300$$

$$H_1: \mu \neq 300$$

t-statistics to be used

$$t = \frac{268.1 - 300}{183.5 / \sqrt{10}} = -0.55$$

$$D.F = 9$$

α is between 0.40 & 0.25

(t table is symmetric use $t = 0.55$)

Failed to reject H_0 .

(2)

b) See the discussion on page 163 of text.

c) Two sided C.I. on mean with $\alpha = 0.05$

$$\therefore t_{\alpha/2, n} = t_{0.025, 9} = 2.262$$

$$Se(\bar{x}) = \frac{183.5}{\sqrt{10}} = 58.0 \text{ (Std. error)}$$

$$\therefore 268.1 - 2.262 \times 58 \leq \mu \leq 268.1 + 2.262 \times 58$$

$$137 \leq \mu \leq 400$$

Because the range does not include 300, we cannot reject the null hypothesis.

2. (30 marks) The following ANOVA table has been prepared for the experiment where 3 different levels of a factor has been varied. Fill in the blanks and check whether everything given is consistent or not. Some round-off error is expected.

Source	DF	SS	MS	F	P
Factor	—	—	192.3	—	0.005
Error	12	—	22.9		
Total	—	658.9			

of factors = 3

DF factor = 2

Error DF = 12

Total DF = 14

$$SS = MS \times DF$$

$$SS_{\text{Fac}} = 192.3 \times 2 = 384.6$$

$$SS_E = 22.9 \times 12 = 274.8$$

$$\underline{659.4}$$

$$F = 8.4$$

$$F_{0.01, 2, 12} = 6.93$$

Because $F = 8.4$ p value 0.005 is consistent.

3. (40 marks) Use the properties of χ^2 distribution to show that the sample variance is an unbiased estimator of the population variance.

Sample variance is an unbiased estimator if

$$E(S^2) = \sigma^2$$

or
$$E\left(\frac{S^2(n-1)}{\sigma^2}\right) = (n-1)$$

multiply both sides by $\frac{(n-1)}{\sigma^2}$. A constant multiplier can be taken inside the $E()$ according to eqn 3-25

$\frac{S^2(n-1)}{\sigma^2}$ is χ^2 random variable. The expected value of χ^2 random variable is $(n-1)$ (Eqn 4-55) Pg-200.