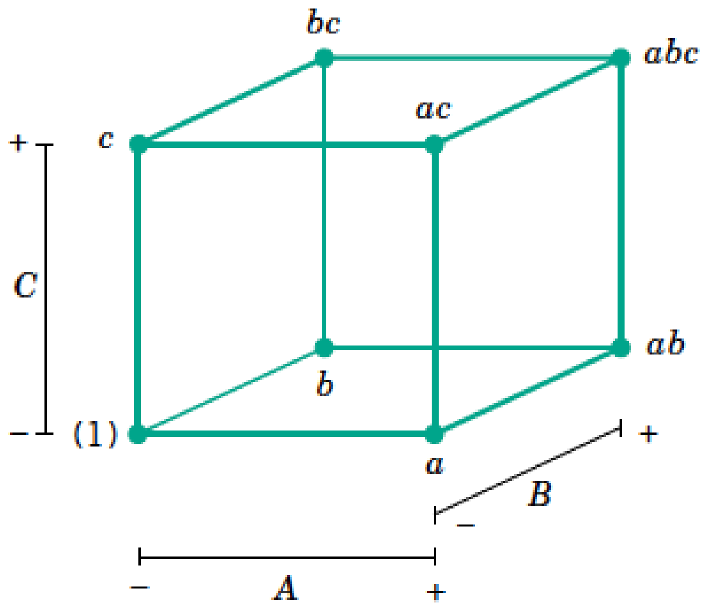
7-3.4 2^k Design for $k \geq 3$ Factors

Consider a 2^3 Design



(a) Geometric view

Run	A	B	C	Label
1	−	−	−	(1)
2	+	−	−	a
3	−	+	−	b
4	+	+	−	ab
5	−	−	+	c
6	+	−	+	ac
7	−	+	+	bc
8	+	+	+	abc

(b) The test matrix

Figure 7-13 The 2^3 design.

7-3.4 2^k Design for $k \geq 3$ Factors

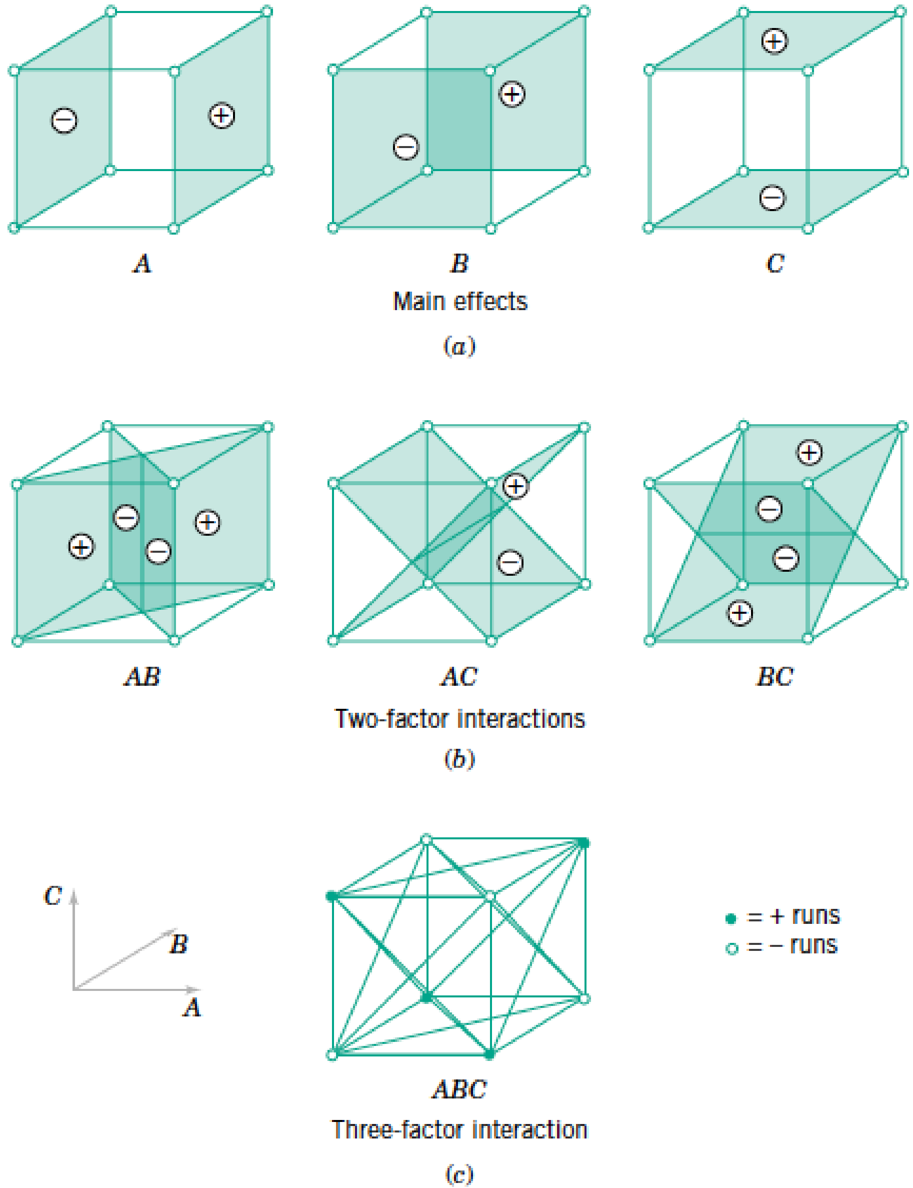


Figure 7-14 Geometric presentation of contrasts corresponding to the main effects and interaction in the 2^3 design. (a) Main effects. (b) Two-factor interactions. (c) Three-factor interaction.

7-3.4 2^k Design for $k \geq 3$ Factors

$$A = \bar{y}_{A+} - \bar{y}_{A-} = \frac{1}{4n}[a + ab + ac + abc - (1) - b - c - bc] \quad (7\text{-}8)$$

$$B = \bar{y}_{B+} - \bar{y}_{B-} = \frac{1}{4n}[b + ab + bc + abc - (1) - a - c - ac] \quad (7\text{-}9)$$

$$C = \bar{y}_{C+} - \bar{y}_{C-} = \frac{1}{4n}[c + ac + bc + abc - (1) - a - b - ab] \quad (7\text{-}10)$$

7-3.4 2^k Design for $k \geq 3$ Factors

AB Interaction – Average of the difference between the average effect of 'A' at the two levels of 'B'

B	Average A Effect
High (+)	$\dfrac{[(abc - bc) + (ab - b)]}{2n}$
Low (−)	$\dfrac{\{(ac - c) + [a - (1)]\}}{2n}$
Difference	$\dfrac{[abc - bc + ab - b - ac + c - a + (1)]}{2n}$

$$AB = \left\{ \frac{[(abc - bc) + (ab - b)]}{2n} - \frac{\{(ac - c) + [a - (1)]\}}{2n} \right\} / 2$$

7-3.4 2^k Design for $k \geq 3$ Factors

$$AB = \frac{1}{4n}[abc - bc + ab - b - ac + c - a + (1)] \quad (7\text{-}11)$$

$$AC = \frac{1}{4n}[(1) - a + b - ab - c + ac - bc + abc] \quad (7\text{-}12)$$

$$BC = \frac{1}{4n}[(1) + a - b - ab - c - ac + bc + abc] \quad (7\text{-}13)$$

$$ABC = \frac{1}{4n}[abc - bc - ac + c - ab + b + a - (1)] \quad (7\text{-}14)$$

Average difference between AB interactions at the two levels of C

Table 7-9 Calculating effects in a 2^3 design

Treatment Combination	Factorial Effect							
	I	A	B	AB	C	AC	BC	ABC
(1)	+	−	−	+	−	+	+	−
a	+	+	−	−	−	−	+	+
b	+	−	+	−	−	+	−	+
ab	+	+	+	+	−	−	−	−
c	+	−	−	+	+	−	−	+
ac	+	+	−	−	+	+	−	−
bc	+	−	+	−	+	−	+	−
abc	+	+	+	+	+	+	+	+

The estimate of any main effect or interaction is obtained by multiplying the first column with the respective sign.

Sec 7-3.4 Properties of a 2^k effects table

Table 7-8 has several interesting properties:

1. Except for the identity column I, each column has an equal number of plus and minus signs.
2. The sum of products of signs in any two columns is zero; that is, the columns in the table are **orthogonal.**
3. Multiplying any column by column I leaves the column unchanged; that is, I is an **identity element.**
4. The product of any two columns yields a column in the table, for example $A \times B = AB$, and $AB \times ABC = A^2B^2C = C$, because any column multiplied by itself is the identity column.

Example 7-2 Surface roughness problem

Table 7-10 Surface Roughness Data for Example 7-2

Treatment Combinations	Design Factors A	B	C	Surface Roughness	Total	Average	Variance
(1)	−1	−1	−1	9, 7	16	8	2.0
a	1	−1	−1	10, 12	22	11	2.0
b	−1	1	−1	9, 11	20	10	2.0
ab	1	1	−1	12, 15	27	13.5	4.5
c	−1	−1	1	11, 10	21	10.5	0.5
ac	1	−1	1	10, 13	23	11.5	4.5
bc	−1	1	1	10, 8	18	9	2.0
abc	1	1	1	16, 14	30	15	2.0
Average						11.0625	2.4375

Variance ➔ Eq. 7-6

Standard Error of effect ➔ Eq. 7.5

7-3.4 2^k Design for $k \geq 3$ Factors

Regression Model and Residual Analysis

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \epsilon$$

$$\hat{y} = 11.0625 + \left(\frac{3.375}{2}\right) x_1 + \left(\frac{1.625}{2}\right) x_2 + \left(\frac{1.375}{2}\right) x_1 x_2$$

$$\hat{y} = 11.0625 + \left(\frac{3.375}{2}\right)(-1) + \left(\frac{1.625}{2}\right)(-1) + \left(\frac{1.375}{2}\right)(-1)(-1)$$
$$= 9.25$$

7-3.4 2^k Design for $k \geq 3$ Factors

Projection of a 2^k Design

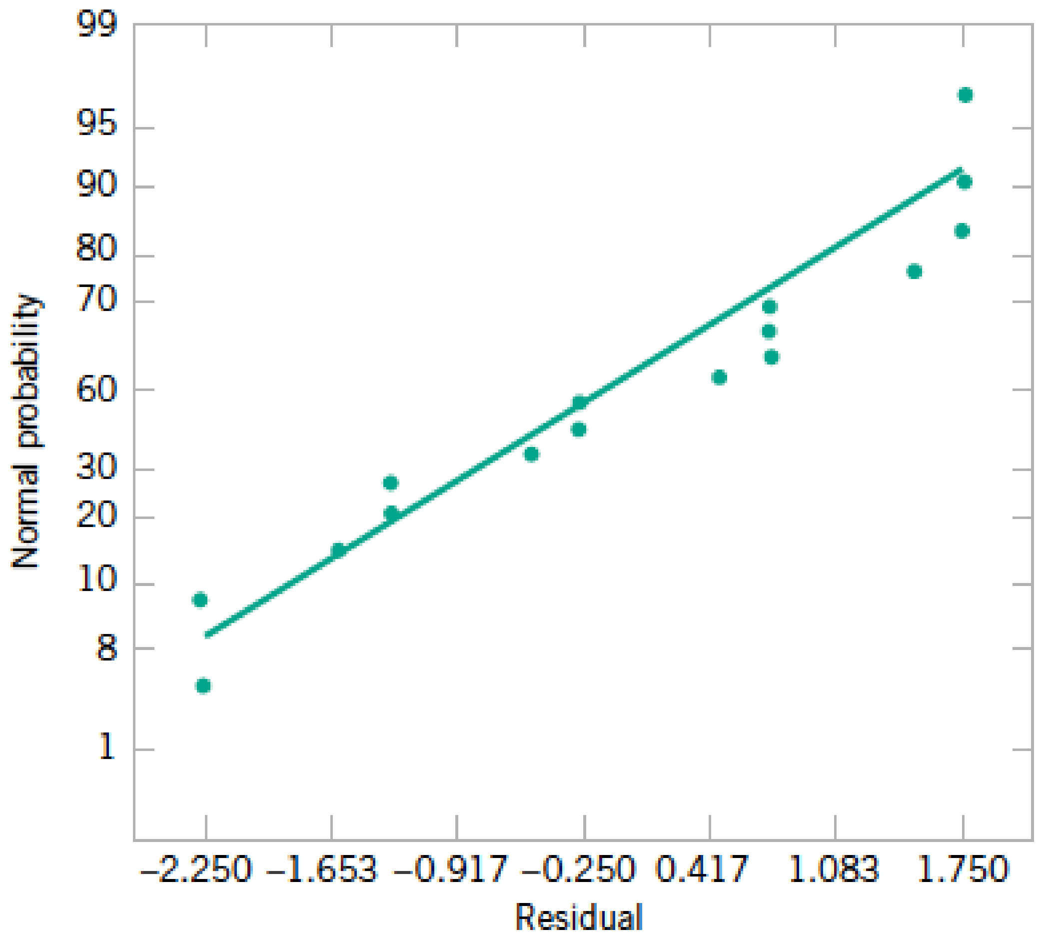


Figure 7-15 Normal probability plot of residuals from the surface roughness experiment.

7-3.5 Single Replicate of a 2^k Design

- As the number of factors (variables eg. pressure, temperature, reaction time, flow rate, etc) in a factorial experiment grows, the number of effects that can be estimated also grows.

- In most situations the **sparsity of effects principle** applies; ie., the main effects and low-order interactions are dominant. The 3-factor and higher-order interactions are usually negligible.

- A simple method of analysis called a **normal probability plot of effects** can be used – Negligible effects will **fall in** the straight line. Significant effects will **not fall** along the straight line.

7-3.5 Single Replicate of a 2^k Design

Table 7-11 The 2^4 Design for the Plasma Etch Experiment

A (gap)	B (pressure)	C (C_2F_6 flow)	D (power)	Etch Rate (Å/min)
−1	−1	−1	−1	550
1	−1	−1	−1	669
−1	1	−1	−1	604
1	1	−1	−1	650
−1	−1	1	−1	633
1	−1	1	−1	642
−1	1	1	−1	601
1	1	1	−1	635
−1	−1	−1	1	1037
1	−1	−1	1	749
−1	1	−1	1	1052
1	1	−1	1	868
−1	−1	1	1	1075
1	−1	1	1	860
−1	1	1	1	1063
1	1	1	1	729

7-3.5 Single Replicate of a 2^k Design

Table 7-12 Contrast Constants for the 2^4 Design

	Factorial Effect														
	A	*B*	*AB*	*C*	*AC*	*BC*	*ABC*	*D*	*AD*	*BD*	*ABD*	*CD*	*ACD*	*BCD*	*ABCD*
(1)	−	−	+	−	+	+	−	−	+	+	−	+	−	−	+
a	+	−	−	−	−	+	+	−	−	+	+	+	+	−	−
b	−	+	−	−	+	−	+	−	+	−	+	+	−	+	−
ab	+	+	+	−	−	−	−	−	−	−	−	+	+	+	+
c	−	−	+	+	−	−	+	−	+	+	−	−	+	+	−
ac	+	−	−	+	+	−	−	−	−	+	+	−	−	+	+
bc	−	+	−	+	−	+	−	−	+	−	+	−	+	−	+
abc	+	+	+	+	+	+	+	−	−	−	−	−	−	−	−
d	−	−	+	−	+	+	−	+	−	−	+	−	+	+	−
ad	+	−	−	−	−	+	+	+	+	−	−	−	−	+	+
bd	−	+	−	−	+	−	+	+	−	+	−	−	+	−	+
abd	+	+	+	−	−	−	−	+	+	+	+	−	−	−	−
cd	−	−	+	+	−	−	+	+	−	−	+	+	−	−	+
acd	+	−	−	+	+	−	−	+	+	−	−	+	+	−	−
bcd	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−
abcd	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

7-3.5 Single Replicate of a 2^k Design

Projection of a 2^k Design

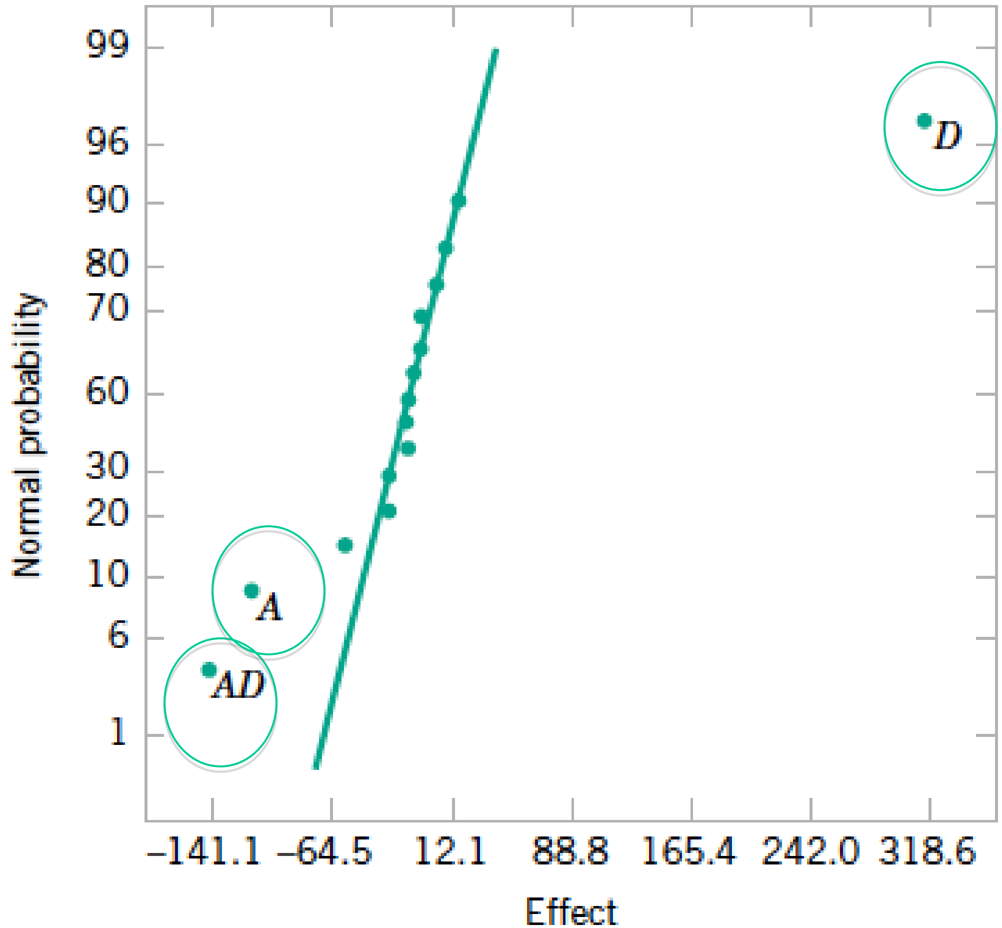


Figure 7-16 Normal probability plot of effects from the plasma etch experiment.

7-3.5 Single Replicate of a 2^k Design

Projection of a 2^k Design

Table 7-13 Analysis for Example 7-3 Plasma Etch Experiment

Analysis of Variance					
Source	Sum of Squares	Degrees of Freedom	Mean Square	f_0	*P*-Value
Model	521234	10	52123.40	25.58	0.000
Error	10187	5	2037.40		
Total	531421	15			

Independent Variable	Effect Estimate	Coefficient Estimate	Standard Error of Coefficient	t for H_0 Coefficient = 0	*P*-Value
Intercept		776.06	11.28	68.77	0.000
A	⇒ −101.63	−50.81	11.28	−4.50	⇒ 0.006
B	−1.63	−0.81	11.28	−0.07	0.945
C	7.38	3.69	11.28	0.33	0.757
D	⇒ 306.12	153.06	11.28	13.56	⇒ 0.000
AB	−7.88	−3.94	11.28	−0.35	0.741
AC	−24.87	−12.44	11.28	−1.10	0.321
AD	⇒ −153.62	−76.81	11.28	−6.81	⇒ 0.001
BC	−43.87	−21.94	11.28	−1.94	0.109
BD	−0.62	−0.31	11.28	−0.03	0.979
CD	−2.12	−1.06	11.28	−0.09	0.929

7-3.5 Single Replicate of a 2^k Design

Projection of a 2^k Design

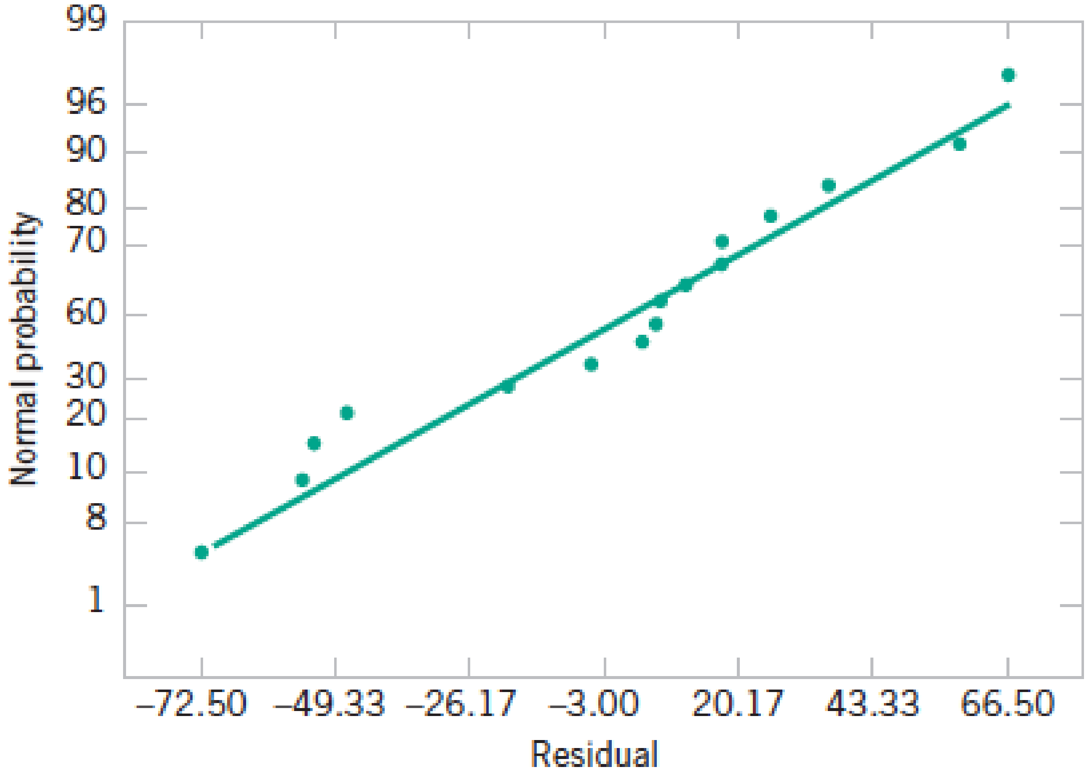


Figure 7-18 Normal probability plot of residuals from the plasma etch experiment.