# Lab 5/6: Hypothesis Testing (Chapter 4)

## Objectives

* Identify the differences between populations and samples
* Define the terms related to hypothesis testing
* Learn the algorithm for solving hypothesis testing problems for normally distributed, variance-known data
* Apply more complicated types of hypothesis testing, including tests on mean with variance unknown and tests on variance

## Populations vs. Samples

Our work thus far has been on populations, where we are considering an entire possible range of data. Now, we will work on samples, which are manageable subsets of a population. So before we would be considering population mean and standard deviation (µ and σ), and **now we are considering the sample mean and standard deviation ( and ).**

## Hypothesis Testing

In real life we will mostly be working with samples, but wanting to make inferences about populations. This is where hypothesis testing comes in. **Basically, we will create a hypothesis about parameters of a population (generally means or variances) then use the sample data to see if we can accept or reject the hypothesis.**

The following are a list of terms having to do with hypothesis testing. Some of these terms are not relevant to today’s lab but will be useful when we solve more complicated hypothesis testing problems.

* **Null hypothesis (H0) –** The statement we are testing. Generally, we will be testing whether or not two things are equal to each other: for example, H0: µ = 100.
* **Alternative hypothesis (H1) ­**– An opposite statement to the null. ***If the problem statement suggests we are testing if a mean is exactly equal to a number, we use the alternative hypothesis H1: µ ≠ 100. If the problem suggests we want to know if a mean is less than or greater than a number, we use H1: µ < 100 or H1: µ > 100***. H1 ≠µ is a two-sided hypothesis because are considering if the mean is greater than OR less than a value, or to either side of the mean in the distribution. The other alternative hypotheses are one-sided hypothesis because we are only considering one side of the distribution.
* **Type I error** – This occurs when we reject the null hypothesis, but it is actually true.
* **Significance level (α) –** This is the probability of a type I error occurring, or the probability of rejecting our null hypothesis even though it is true.
* **P-value –** This is the minimum level of significance that would lead to rejecting the null hypothesis. In other words, ***if the P-value is less than the significance level (α), we will reject the null hypothesis.***
* **Two-tailed test –** We perform this test when our alternative hypothesis is two-sided. If it is two-sided, you must consider HALF of the significance level or P-value on either end of the distribution; for example, if your significance level is 0.5, or 5%, you divide that by two to consider 2.5% of the data under the tail on the left and 2.5% of data under the tail on the right.
* **One-tailed test –** We perform this test when our alternative hypothesis is one-sided.
* **Type II error** – This error occurs when we accept the null hypothesis (or fail to reject) but it is actually false.

To solve basic hypothesis testing problems, you essentially:

1. Identify your null and alternative hypothesis and test statistics
2. Normalize your data by converting your test statistic (whatever µ is compared to in your hypotheses) into a Z value (if your variance is known – later we will consider if the variance is unknown).
3. Find the probability corresponding to this Z value using the charts in your book or Excel functions.
4. If the P-value is less than your significance level, you can reject the null hypothesis.

## Lab 5 Exercises

You want to test if the mean of your process data is equal to 100. You know that σ = 3 and n = 9.

1. State your null and alternative hypothesis. Is this a one-sided or two-sided test?
2. What is your P-value if ?
3. If the rejection region is defined as and , what is α?

Now we want to test if the mean of the process data is less than 100 and we still know that σ = 3 and n = 9.

1. State your null and alternative hypothesis. Is this a one-sided or two-sided test?
2. What is your P-value if ?

Now we’ll expand to consider other tests we may do. Essentially, depending on what we are testing and if the variance is known or unknown, we use a different statistic with a different distribution, but the same algorithm that we learned last week to solve problems. For example, for a test on the mean with variance known, we used the Z statistic. The various tests are outlined below. For more information, refer to the section in the book.

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| **Test** | **Book section** | **Statistic** | **Excel functions** |
| Mean, variance known | 4-4 |  | normsdist  normsinv |
| Mean, variance unknown | 4-5 |  | tdist(x,v,tails)  tinv(p,v)   * x = standardized T, using the equation to the left * v = n-1, the degrees of freedom * tails = 1 or 2, depending on the number of tails for the alternative hypothesis * p = probability |
| Variance | 4-6 |  | chidist(x,v)  chiinv(p,v)   * x = standardized , using the equation to the left * v = n-1, the degrees of freedom * p = probability |

## Lab 6 Exercises

For = 61492, s = 3035, and n = 10:

1. State the appropriate hypothesis test to see if the sample mean is greater than 60000.
2. Find the P-value for this test.
3. Find the 90% upper confidence (one sided) interval bound on the mean (μ).

For s = 0.027, and n = 15:

1. State the appropriate hypothesis test to see if the sample standard deviation is greater than 0.02.
2. Find the P-value for this test.
3. Say the 95% confidence interval for the test is . Does the boundary value agree with what you expected from the P-value?