Practice Problems for Midterm 2

#7, 8 (section 6.2)

PROBLEMS In each of Problems 1 through 10, find the inverse Laplace transform of the given function.

7. $F(s) = \dfrac{2s+1}{s^2-2s+2}$

8. $F(s) = \dfrac{8s^2-4s+12}{s(s^2+4)}$

I will get a table for Laplace transforms. I feel pretty good about these, especially when I have the table. The biggest issue is just recognizing how to manipulate the equations to get the combinations of the Laplace transforms.

#9, 11 (section 6.3)

In each of Problems 7 through 12:

(a) Sketch the graph of the given function.

(b) Express $f(t)$ in terms of the unit step function $u_c(t)$.

9. $f(t) = \begin{cases} 1, & 0 \le t < 2, \\ e^{-(t-2)}, & t \ge 2. \end{cases}$

11. $f(t) = \begin{cases} t, & 0 \le t < 1, \\ t-1, & 1 \le t < 2, \\ t-2, & 2 \le t < 3, \\ 0, & t \ge 3. \end{cases}$

Kind of fuzzy on writing these in terms of the step functions, but with a few practice problems, I shouldn't be too shabby.

#3, 9 (section 6.4)

PROBLEMS

In each of Problems 1 through 13:

(a) Find the solution of the given initial value problem.

(b) Draw the graphs of the solution and of the forcing function; explain how they are related.

3. $y'' + 4y = \sin t - u_{2\pi}(t)\sin(t - 2\pi);\qquad y(0) = 0,\quad y'(0) = 0$

9. $y'' + y = g(t);\qquad y(0) = 0,\quad y'(0) = 1;\qquad g(t) = \begin{cases} t/2, & 0 \le t < 6 \\ 3, & t \ge 6 \end{cases}$

Definitely need work on these, especially drawing graphs.

#3 (section 6.5)

PROBLEMS

In each of Problems 1 through 12:

(a) Find the solution of the given initial value problem.

(b) Draw a graph of the solution.

3. $y'' + 3y' + 2y = \delta(t-5) + u_{10}(t); \qquad y(0) = 0, \quad y'(0) = 1/2$

Again, need work on these, especially drawing the graphs. For the most part, my professor uses a computer to draw the graphs in class and hasn't said if we will need to draw them by hand or not yet.

#4, 10 (section 6.6)

In each of Problems 4 through 7, find the Laplace transform of the given function.

$$4.\ f(t) = \int_0^t (t-\tau)^2 \cos 2\tau \, d\tau$$

In each of Problems 8 through 11, find the inverse Laplace transform of the given function by using the convolution theorem.

$$10.\ F(s) = \frac{1}{(s+1)^2(s^2+4)}$$

The hardest parts of the convolution integral are remembering the formula and doing the actual integrations with rules from all the way back in calc 2.

#3, 11 (section 7.5)

PROBLEMS

In each of Problems 1 through 6:

(a) Find the general solution of the given system of equations and describe the behavior of the solution as $t \to \infty$.

(b) Draw a direction field and plot a few trajectories of the system.

$$3.\ \mathbf{x}' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \mathbf{x}$$

In each of Problems 9 through 14, find the general solution of the given system of equations.

$$11.\ \mathbf{x}' = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix} \mathbf{x}$$

So describing the behaviors and drawing direction fields are where I need the most work. Other than that, I feel fine about finding eigenvalues and eigenvectors.

#3, 7 (section 7.6)

PROBLEMS

In each of Problems 1 through 6:

(a) Express the general solution of the given system of equations in terms of real-valued functions.

(b) Also draw a direction field, sketch a few of the trajectories, and describe the behavior of the solutions as $t \to \infty$.

3. $\mathbf{x}' = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix} \mathbf{x}$

In each of Problems 7 and 8, express the general solution of the given system of equations in terms of real-valued functions.

7. $\mathbf{x}' = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{pmatrix} \mathbf{x}$

So here the same issues as the previous sections, but also I struggle with converting vectors with e^i terms into real-valued terms. I totally get it with regular functions, but for some reason vectors throw me off.

#1, 8, 11 (section 7.8)

PROBLEMS

In each of Problems 1 through 4:

(a) Draw a direction field and sketch a few trajectories.

(b) Describe how the solutions behave as $t \to \infty$.

(c) Find the general solution of the system of equations.

1. $\mathbf{x}' = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} \mathbf{x}$

In each of Problems 7 through 10:

(a) Find the solution of the given initial value problem.

(b) Draw the trajectory of the solution in the x_1x_2-plane, and also draw the graph of x_1 versus t.

10. $\mathbf{x}' = \begin{pmatrix} 3 & 9 \\ -1 & -3 \end{pmatrix} \mathbf{x}, \qquad \mathbf{x}(0) = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

In each of Problems 11 and 12:

(a) Find the solution of the given initial value problem.

(b) Draw the corresponding trajectory in $x_1x_2x_3$-space, and also draw the graph of x_1 versus t.

11. $\mathbf{x}' = \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 3 & 6 & 2 \end{pmatrix} \mathbf{x}, \qquad \mathbf{x}(0) = \begin{pmatrix} -1 \\ 2 \\ -30 \end{pmatrix}$

Finding the nu vector went straight over my head. And again, drawing graphs and describing behaviors.