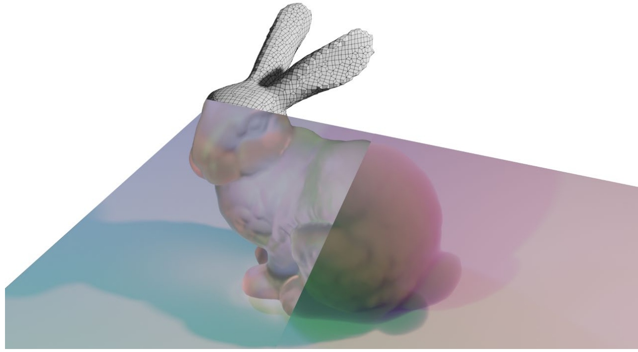


# 1. Radiometry and Light transport

Physically Based Rendering

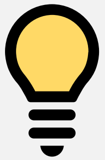
Shinyoung Yi (이신영)

# Rendering

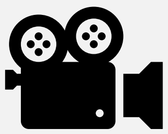
**Scene**

geometry

BRDF



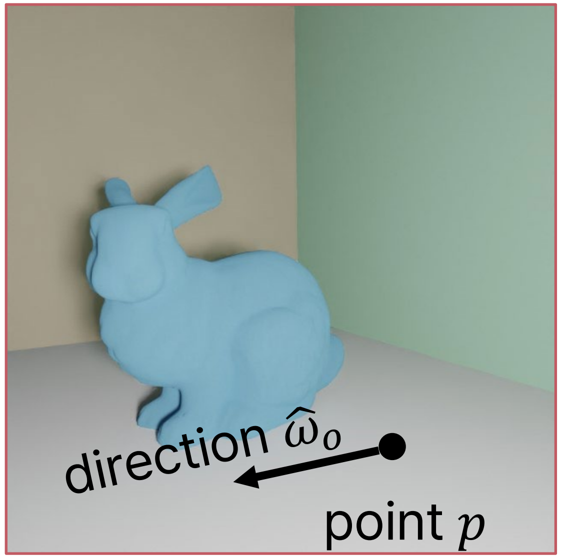
emitter



sensor

**Rendering**

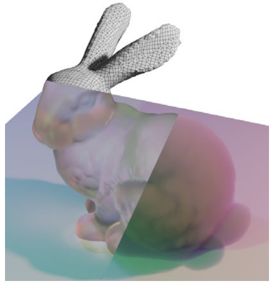
**Image** $\mathbf{I} \in \mathbb{R}^{H \times W}$




**Rendering equation**

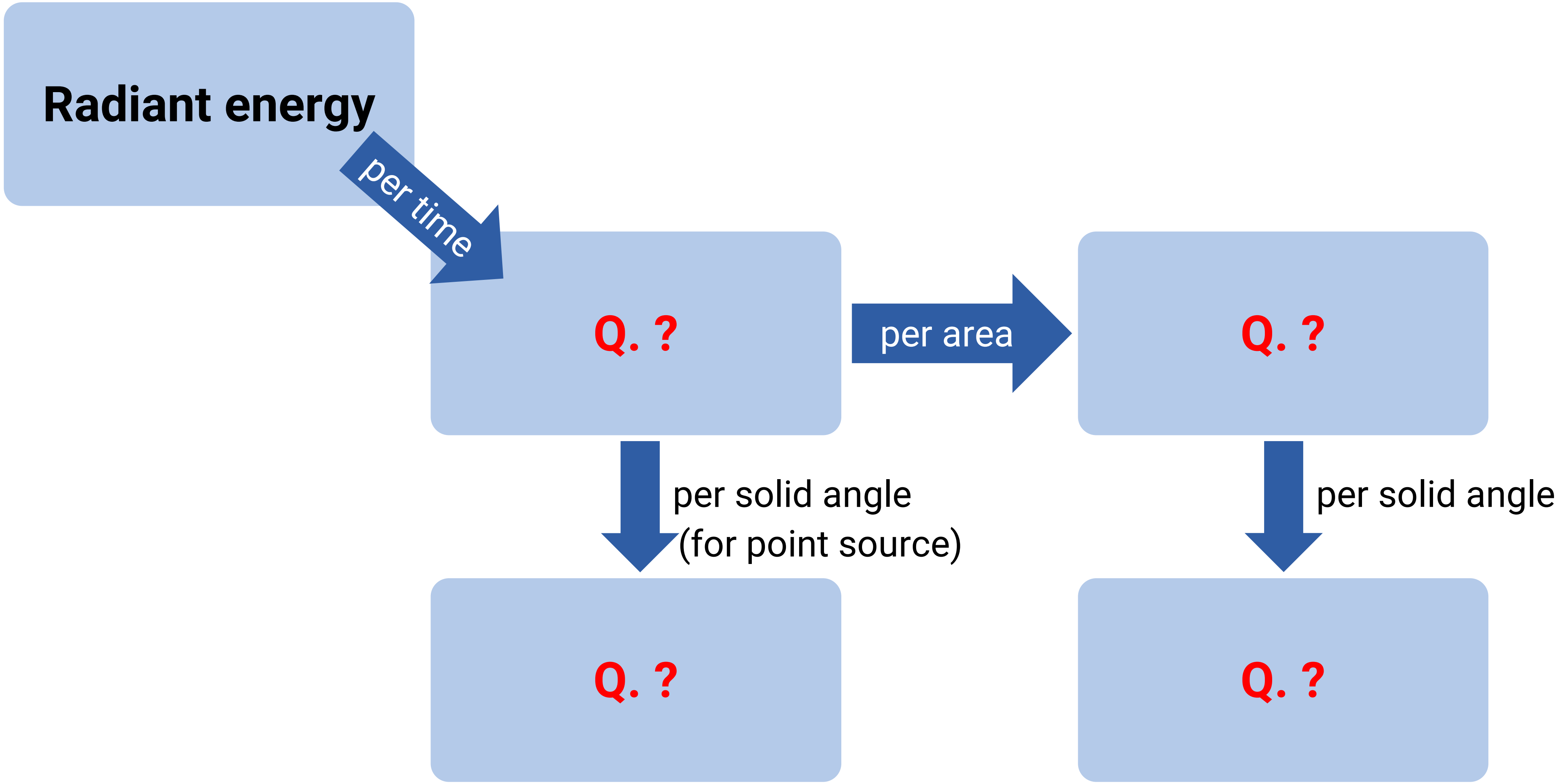
Find scalar $\boxed{\text{radiance } L(p, \hat{\omega})}$ at any point $p$, along any direction $\hat{\omega}$

?

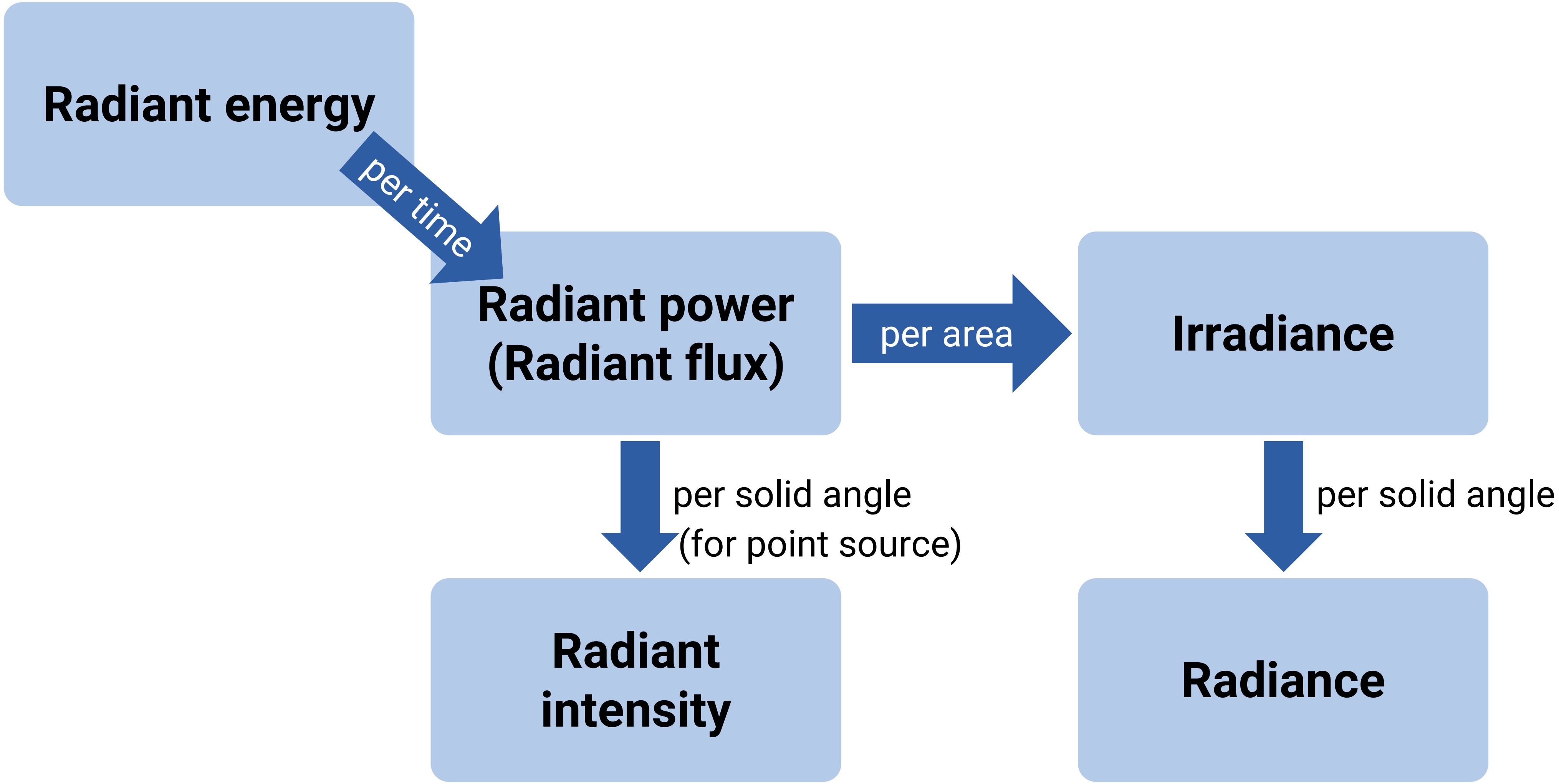


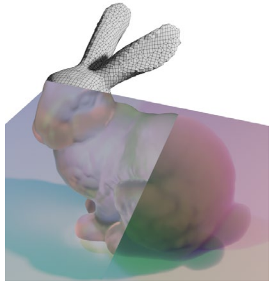
# Quiz & Preview

# Radiometric quantities



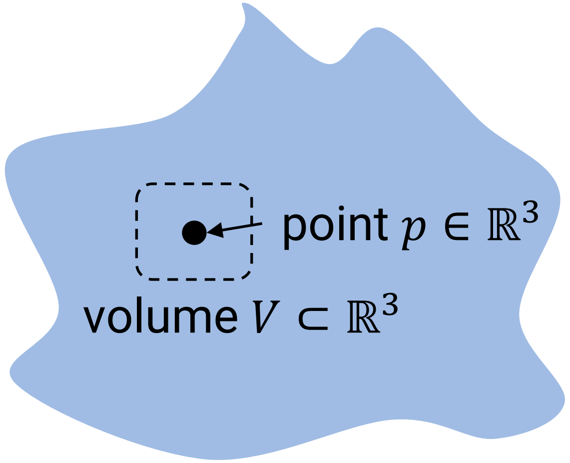
# Radiometric quantities





# Mass vs. Density

# Concepts of mass vs. density

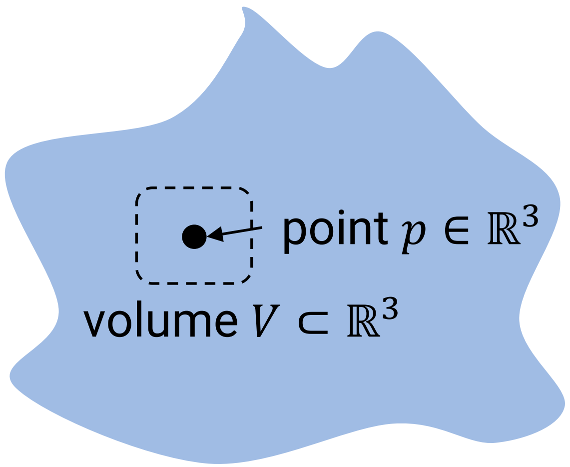



Object $O$

**Mass** → per volume → **Density**

| Mass | Density |
| --- | --- |
| ✓ Mass of the object $O$ | ✗ Density of the object $O$<br>→ illegal or "average density" of the object $O$ |
| ✓ Mass of some region (volume) $V$ | ✗ Density of some region (volume) $V$<br>→ illegal or "average density" of the volume $V$ |
| ✗ Mass at the point $p$<br>→ illegal or meaningless (always zero) | ✓ Density at the point $p$ |

# Concepts of mass vs. density



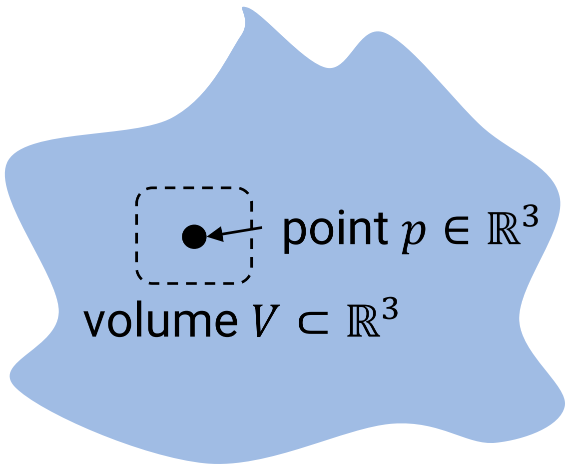

**Mass of** what? $V \subset \mathbb{R}^3$

- “mass of an object $O$”

= “mass of the volume of $O$”

**Density of** what? $p \in \mathbb{R}^3$

# Concepts of mass vs. density




**Mass of** $V \subset \mathbb{R}^3$

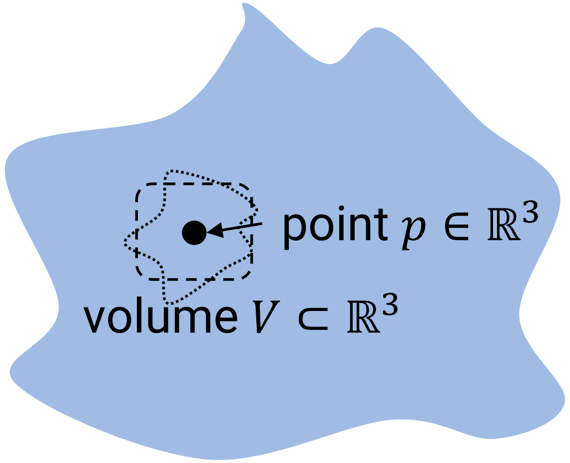
**Density of** $p \in \mathbb{R}^3$

$$\mathrm{mass}(V)$$

per volume

$$\mathrm{density}(p) = \lim_{\substack{\mathrm{vol}(V)\to 0 \\ p \in V}} \frac{\mathrm{mass}(V)}{\mathrm{vol}(V)}$$

# Concepts of mass vs. density




**Mass of** $V \subset \mathbb{R}^3$

**Density of** $p \in \mathbb{R}^3$

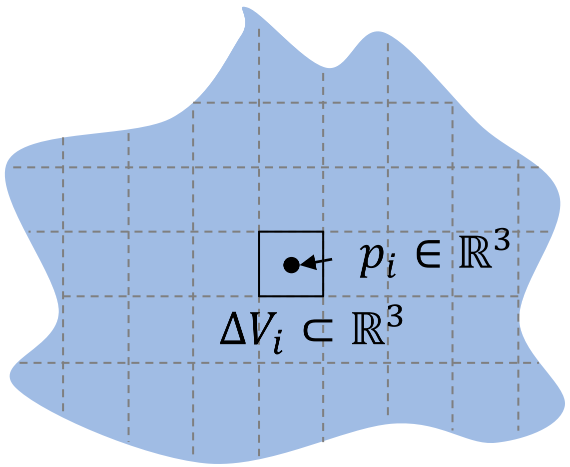
$$\text{mass}(V)$$

per volume →

$$\text{density}(p) = \lim_{\substack{\text{vol}(V)\to 0 \\ p\in V}} \frac{\text{mass}(V)}{\text{vol}(V)} = \lim_{\substack{\text{vol}(V)\to 0 \\ p\in V}} \frac{\text{mass}(V)}{\text{vol}(V)}$$

* The limit converges to the same value whenever $\text{vol}(V) \to 0$ and $p \in V$.

# Concepts of mass vs. density




**Mass of** $V \subset \mathbb{R}^3$

**Density of** $p \in \mathbb{R}^3$

$$\text{mass}(V) = \sum_i \text{mass}(\Delta V_i)$$

$$\approx \sum_i \text{density}(p_i)\,\text{vol}(\Delta V_i)$$

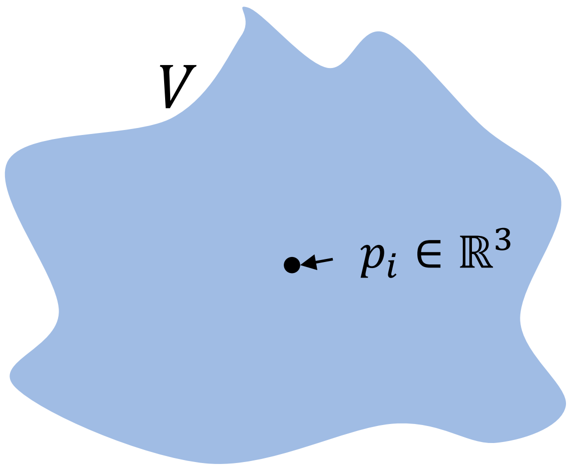
$$\int_V \text{density}(p)\,dp$$

$\text{vol}(\Delta V_i) \to 0$

over volume

$$\boxed{\text{density}(p) = \lim_{\substack{\text{vol}(V)\to 0 \\ p\in V}} \frac{\text{mass}(V)}{\text{vol}(V)}}$$

# Concepts of mass vs. density




**Mass of** $V \subset \mathbb{R}^3$ $[\,\mathrm{kg}\,]$

**Density at** $p \in \mathbb{R}^3$ $[\,\mathrm{kg/m^3}\,]$

**Proposition**

Relations between mass $m$ of a volume $V \subset \mathbb{R}^3$ and density $\rho$ at a point $p \in \mathbb{R}^3$ is that:

$$m(V) = \int_V \rho(p)\mathrm{d}p$$

per volume →

← over volume

$$\rho(p) = \lim_{\substack{|V|\to 0 \\ p\in V}} \frac{m(V)}{|V|}$$

# Concepts of mass vs. density

## Notation comparison

Other text often write $\frac{\mathrm{d}m}{\mathrm{d}V}$ instead of $\lim_{|V|\to 0, p\in V} \frac{m(V)}{|V|}$ but the former notation may give a misunderstanding that $m$ is a function of a real number (volume measure) rather than one of a subset of $\mathbb{R}^3$ (volume region). The formula $\frac{\mathrm{d}m}{\mathrm{d}V}$ can be correctly understood only if it denoted a Radon-Nikodym derivative, which is dealt in *measure theory* (4th grade in Math. major).

We do not assume measure theory as a prerequisite, so we use the latter notation $\lim_{|V|\to 0, p\in V} \frac{m(V)}{|V|}$ for explicitness.

**Mass of** $V \subset \mathbb{R}^3$ $[\mathrm{kg}]$ **Density at** $p \in \mathbb{R}^3$ $[\mathrm{kg}/\mathrm{m}^3]$

**Proposition**

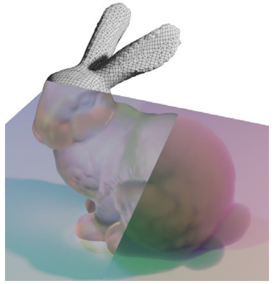
Relations between mass $m$ of a volume $V \subset \mathbb{R}^3$ and density $\rho$ at a point $p \in \mathbb{R}^3$ is that:

$$m(V) = \int_V \rho(p)\mathrm{d}p$$

per volume →

← over volume

$$\rho(p) = \lim_{\substack{|V|\to 0\\ p\in V}} \frac{m(V)}{|V|}$$



# Solid Angles

# Solid angles

We roughly say…

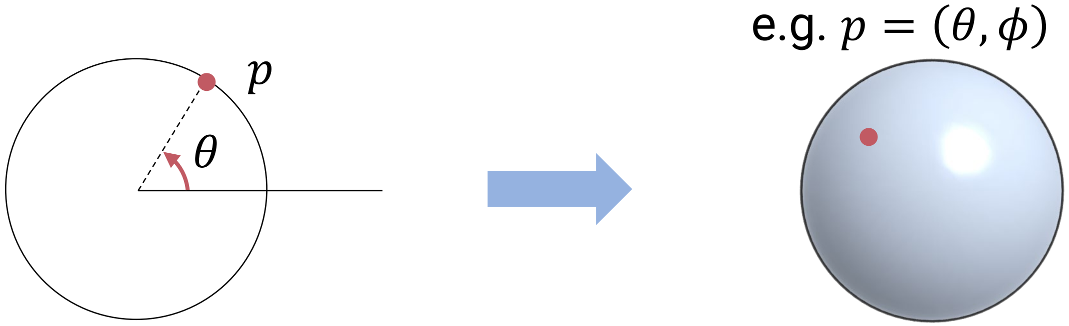
***"Solid angles" are***
***3D versions of "angles"***

How strictly is this sentence correct?

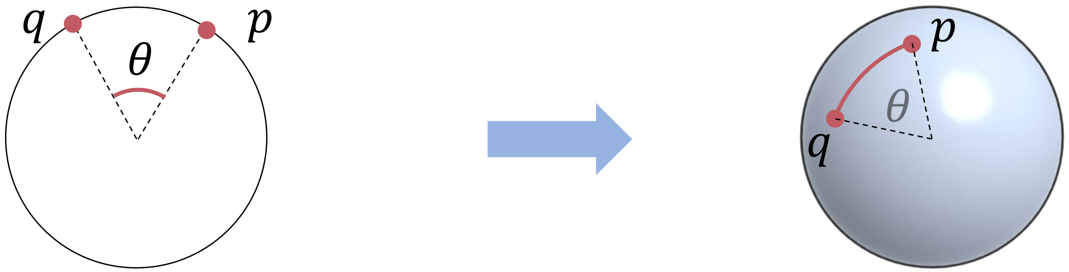
# Solid angles

Can following statements be converted to sphere ($\mathbb{S}^2$) version using “solid angles”?

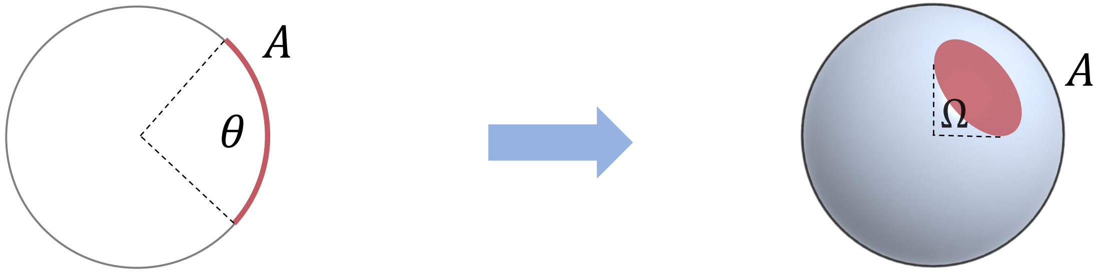
1. The position of a point $p$ in the unit circle ($\mathbb{S}^1$)
   can be represented as the angle $\theta$
   two coordinates
   such as $(\theta, \phi)$




2. How far apart two points $p$ and $q \in \mathbb{S}^1$
   can be represented as the angle $\theta$ between them.
   angle $\theta$




3. The size of a region $A \subset \mathbb{S}^1$
   can be represented as the angle $\theta$
   solid angle $\Omega$



# Solid angles

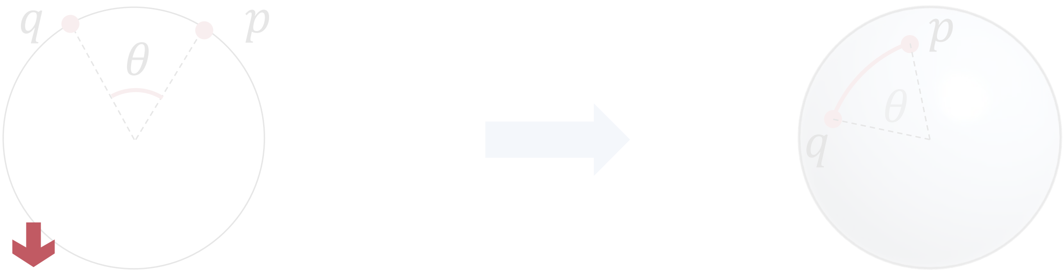
Can following statements be converted to sphere ($\mathbb{S}^2$) version using "solid angles"?

e.g. $p = (\theta, \phi)$

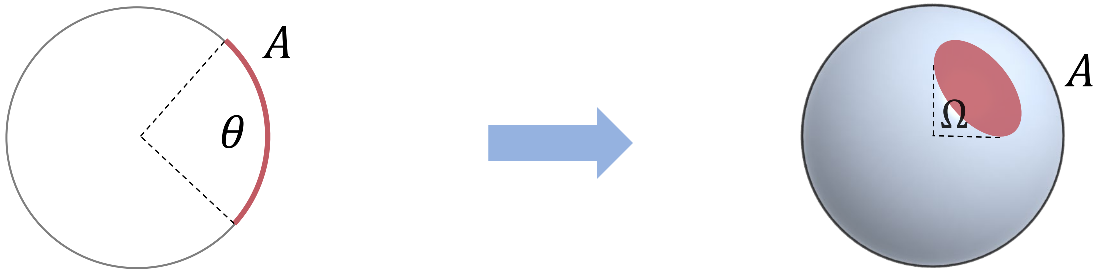
1. The position of a point $p$ in the $\boxed{\text{unit circle } (\mathbb{S}^1)}$ can be represented as the $\boxed{\text{angle } \theta}$
   two coordinates such as $(\theta, \phi)$


**"Solid angles" are only relevant to 3.**

2. How far apart two points $p$ and $q \in \boxed{\mathbb{S}^1}$ can be represented as the $\boxed{\text{angle } \theta}$ between them.
   angle $\theta$




3. The size of a region $A \subset \boxed{\mathbb{S}^1}$ can be represented as the $\boxed{\text{angle } \theta}$
   solid angle $\Omega$



# Solid angles

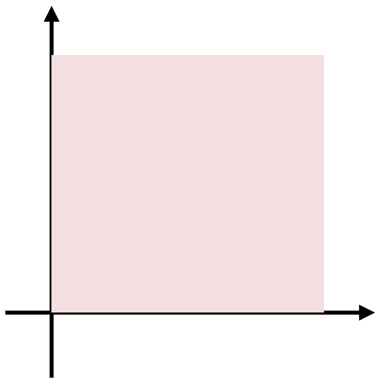
In many times,

several concepts can be treated
as a single concept in lower dimensions,

but they become different in higher dimensions

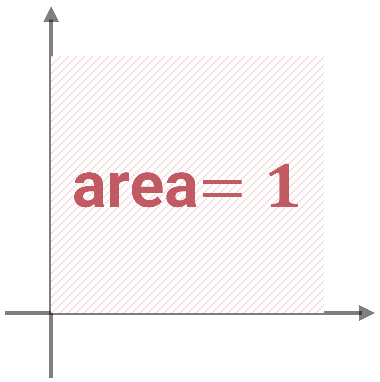
# Solid angles

## We call the both "area". (similarly for "volume")



a 2-dimensional *subset*

$A = \{(x, y) \in \mathbb{R}^2 | 0 \leq x, y \leq 1\}$

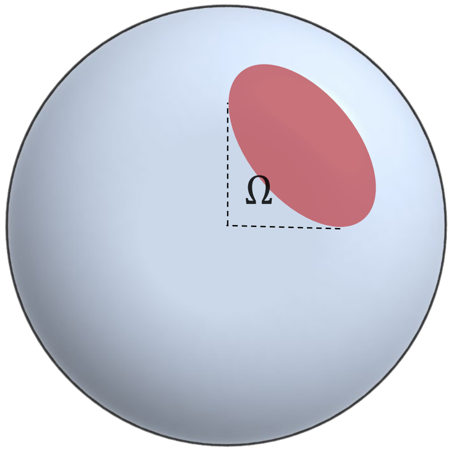



*measure* of a 2-dimensional subset

$|A| = 1$

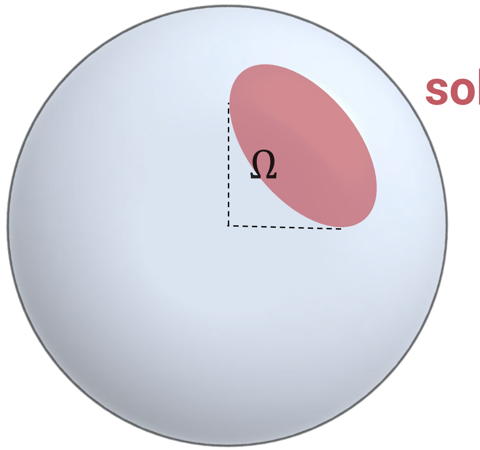
# Solid angles

## Also for “solid angle”




a spherical *subset*

$$\Omega = \{\hat{\omega} \in \mathbb{S}^2 | \hat{u} \cdot \hat{\omega} \geq 0.7\}$$



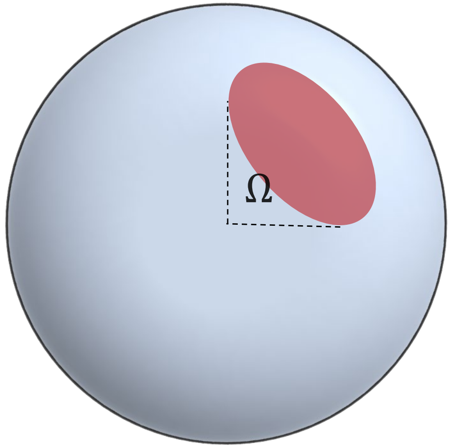

**solid angle**$= 0.2\pi\ sr$

*measure* of a spherical subset

$$|\Omega| = 1$$

# Solid angles

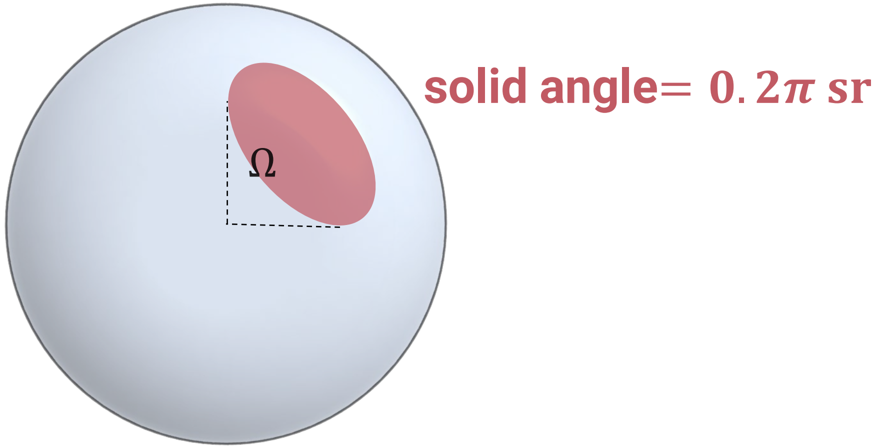
## (not common) terminology in this seminar




a spherical *subset*

$$\Omega = \{\hat{\omega} \in \mathbb{S}^2 | \hat{u} \cdot \hat{\omega} \geq 0.7\}$$

**solid angle *region***



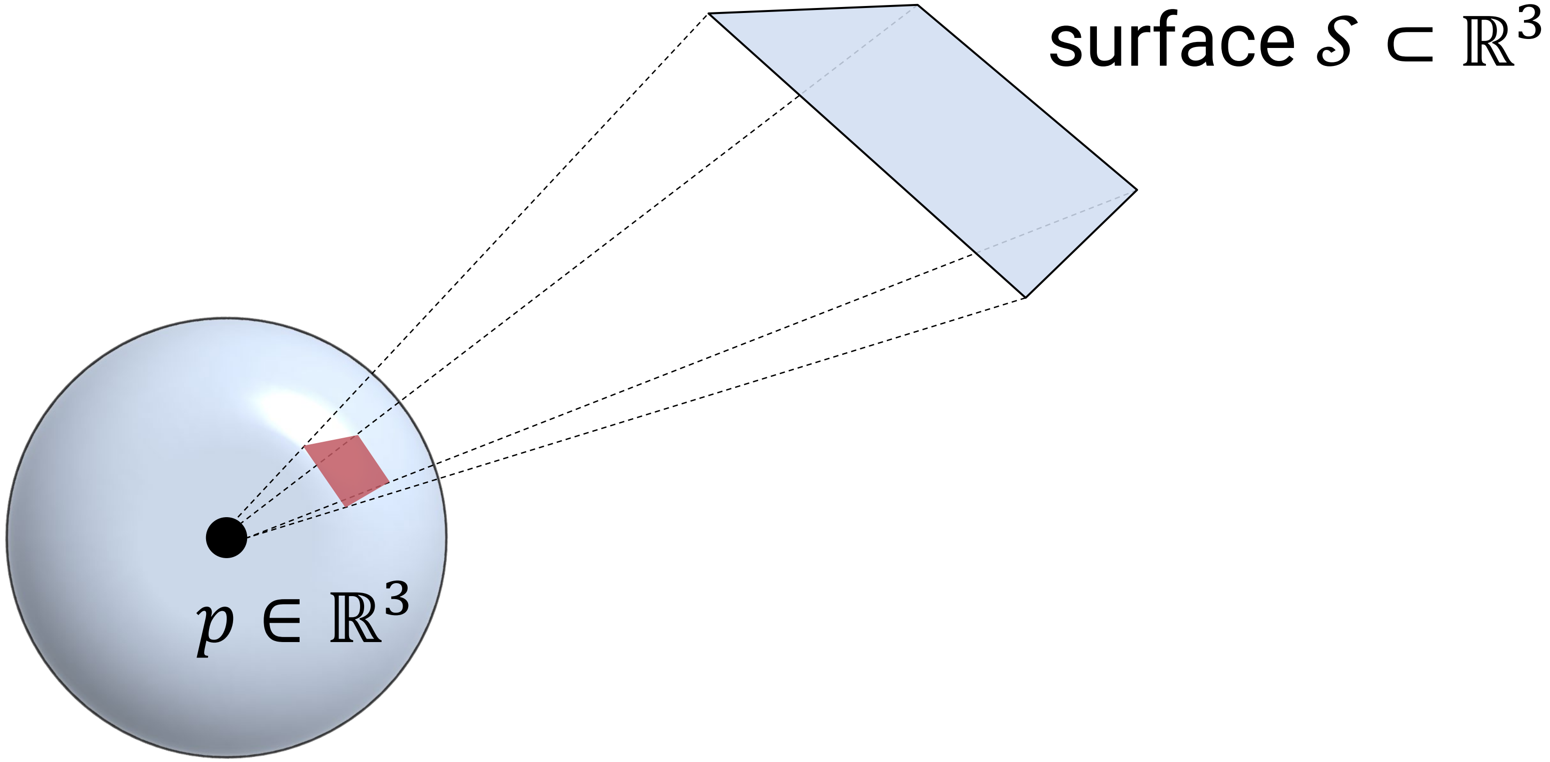

*measure* of a spherical subset

$$|\Omega| = 1$$

**solid angle *measure***

# Surface to solid angle

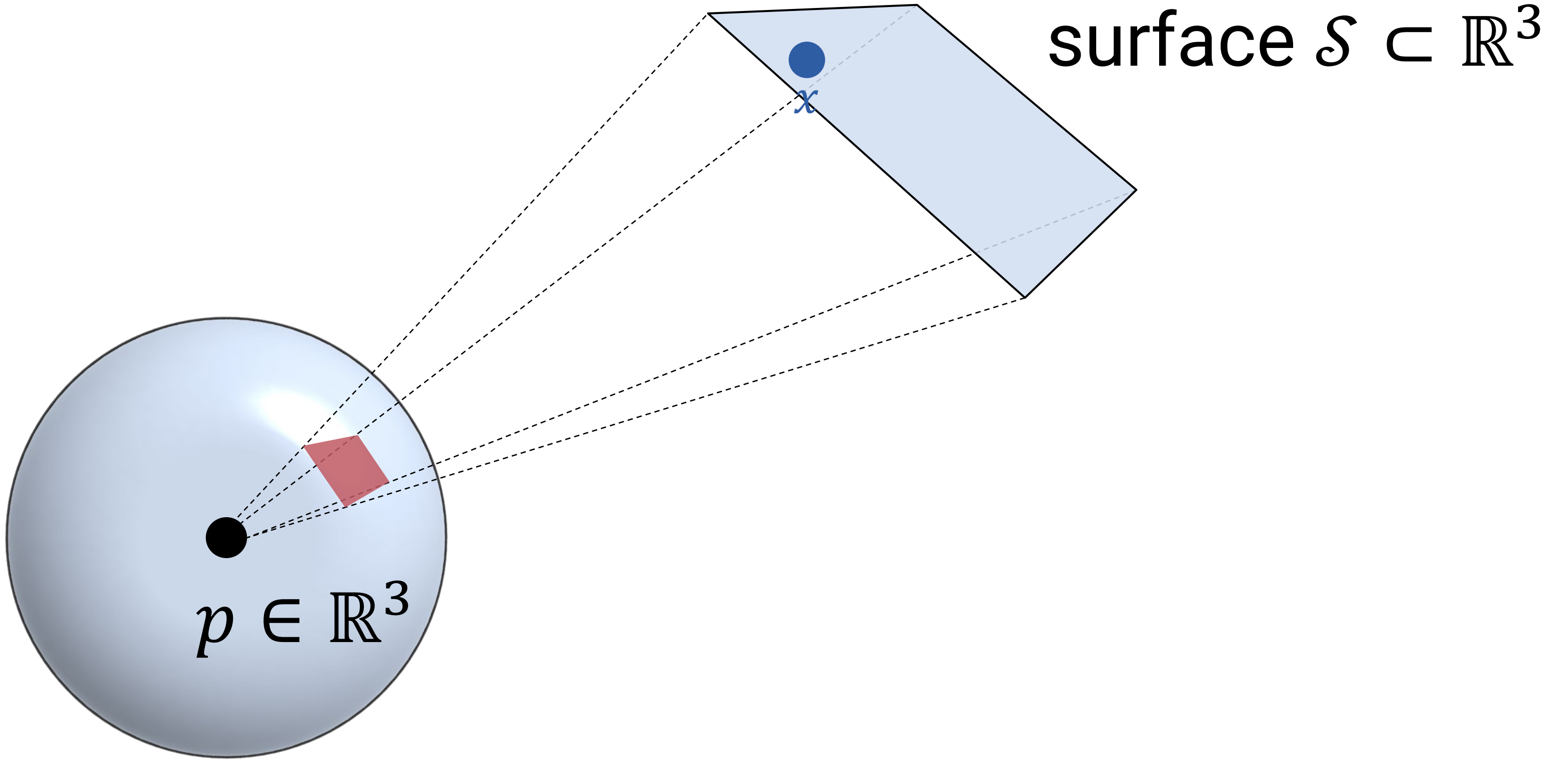
**How large $\mathcal{S}$ appears to and observer at $p$?**




**Solid angle!**

# Surface to solid angle

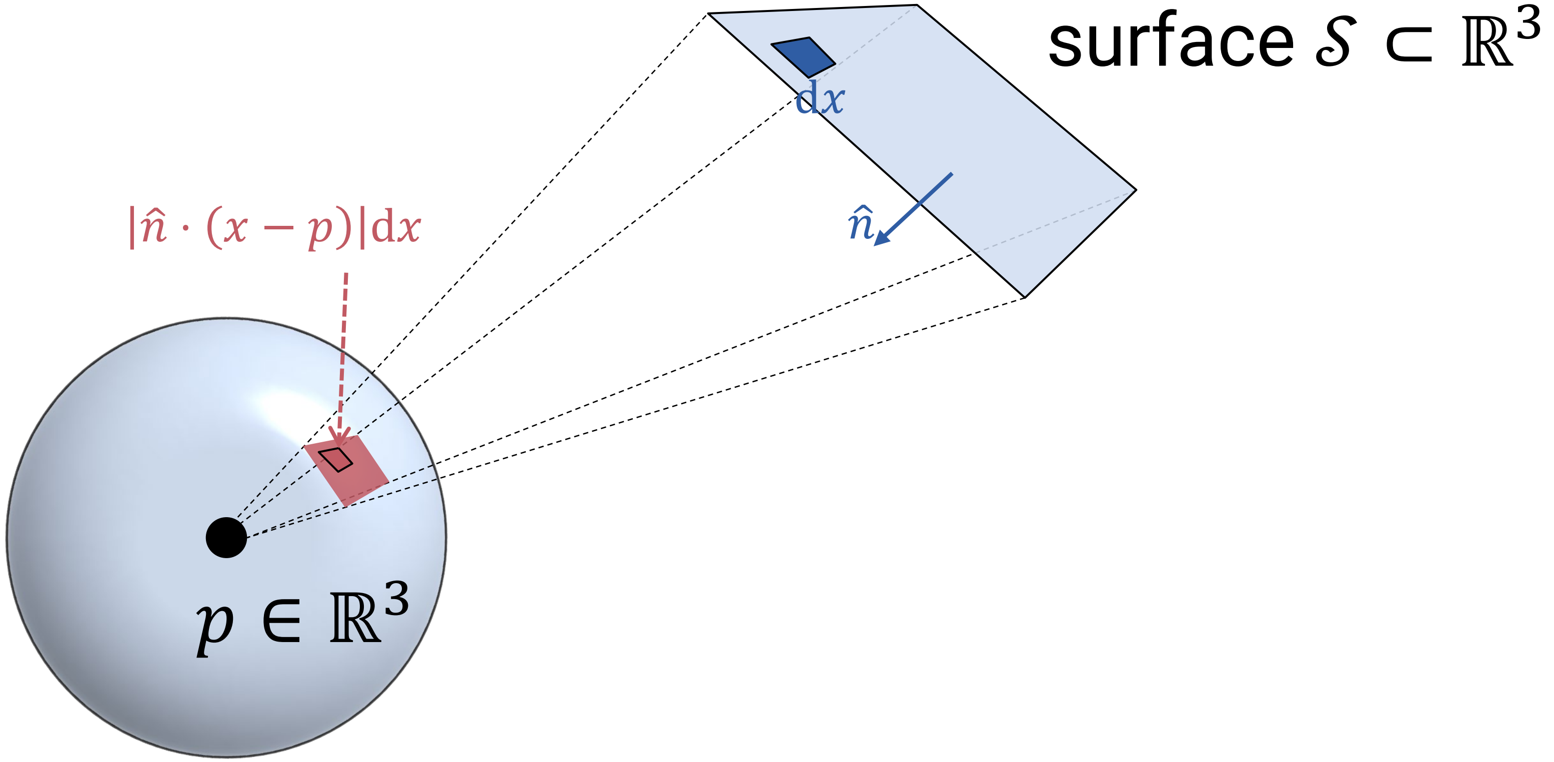
**Try to write in an area integral on $\mathcal{S}$**




**Solid angle=** $\int_{\mathcal{S}} \; ? \;\; \mathrm{d}x$

# Surface to solid angle

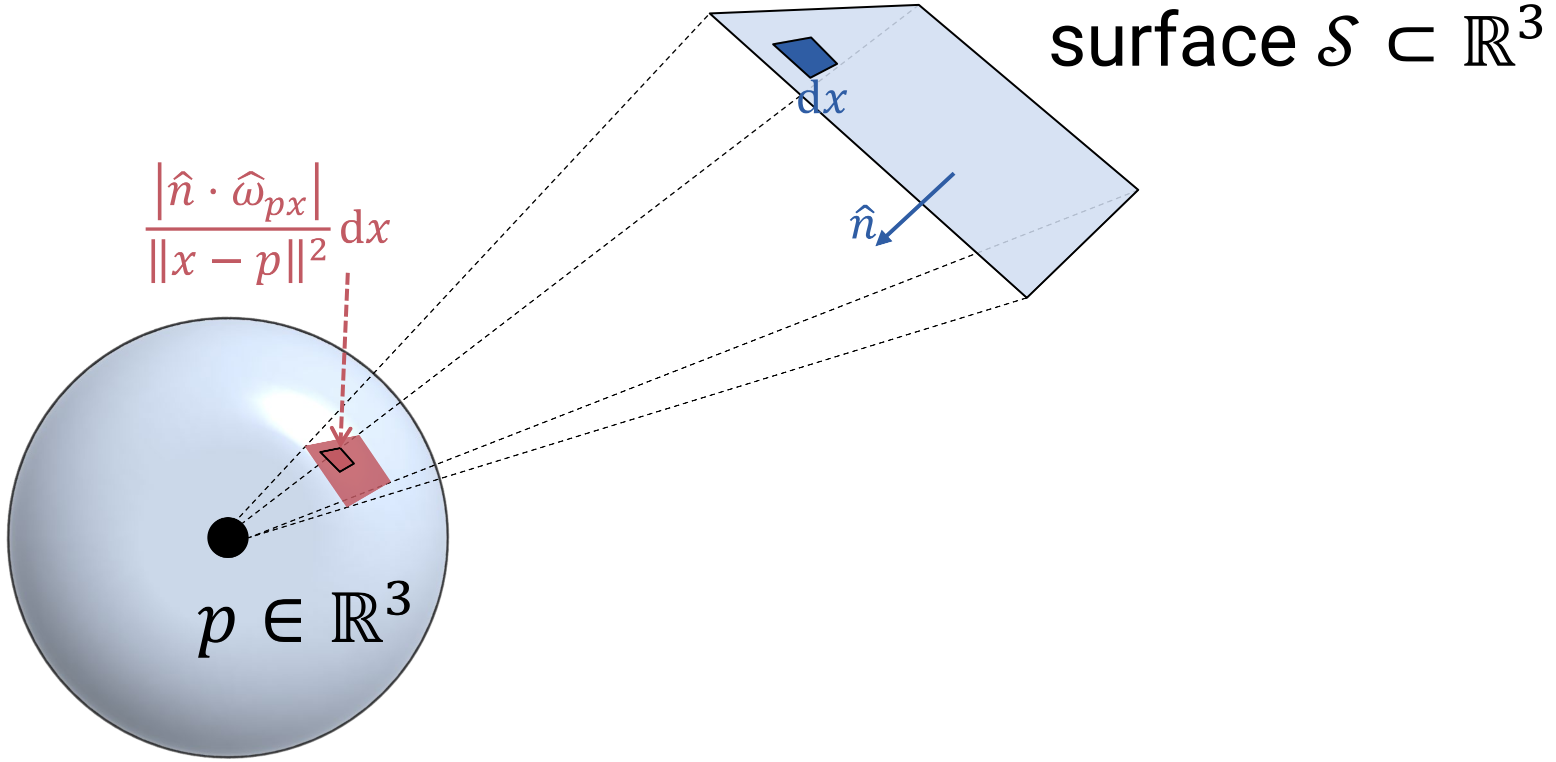
**Try to write in an area integral on $\mathcal{S}$**



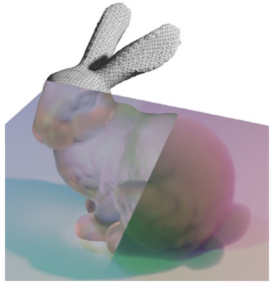

**Solid angle=** $\int_{\mathcal{S}} \ ? \ \ \mathrm{d}x$

# Surface to solid angle

**Try to write in an area integral on $\mathcal{S}$**

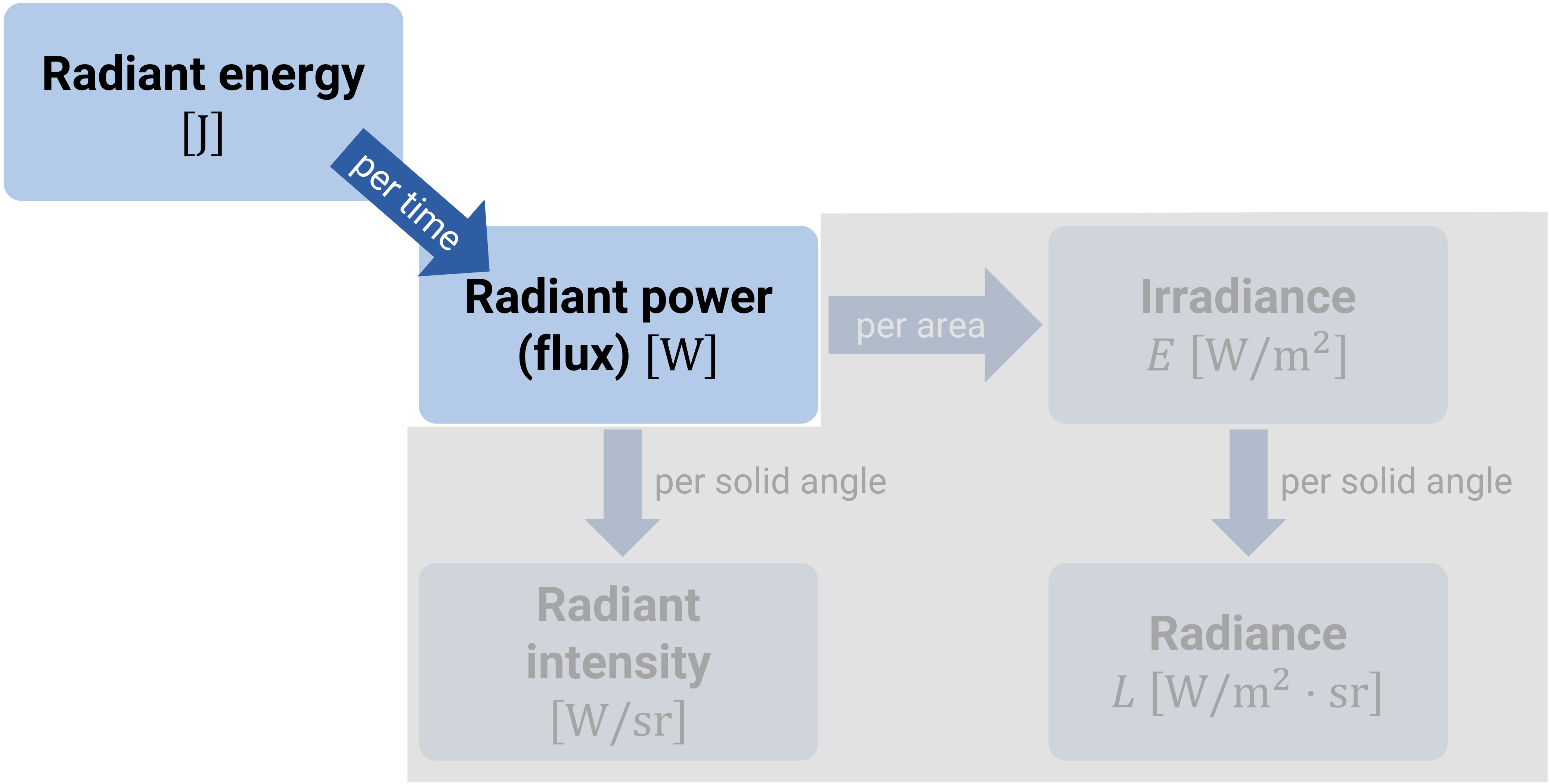



**Solid angle=** $\int_{\mathcal{S}} \frac{|\hat{n} \cdot \widehat{\omega}_{px}|}{\|x-p\|^2} \mathrm{d}x$



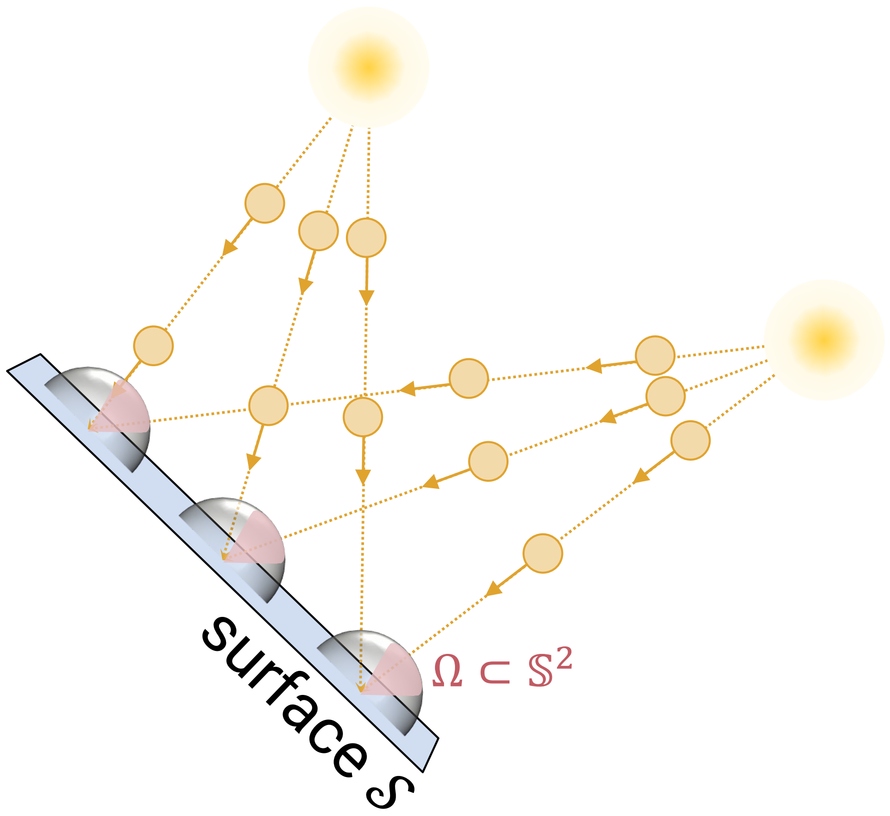
# Radiometric Quantities

# Radiometric quantities



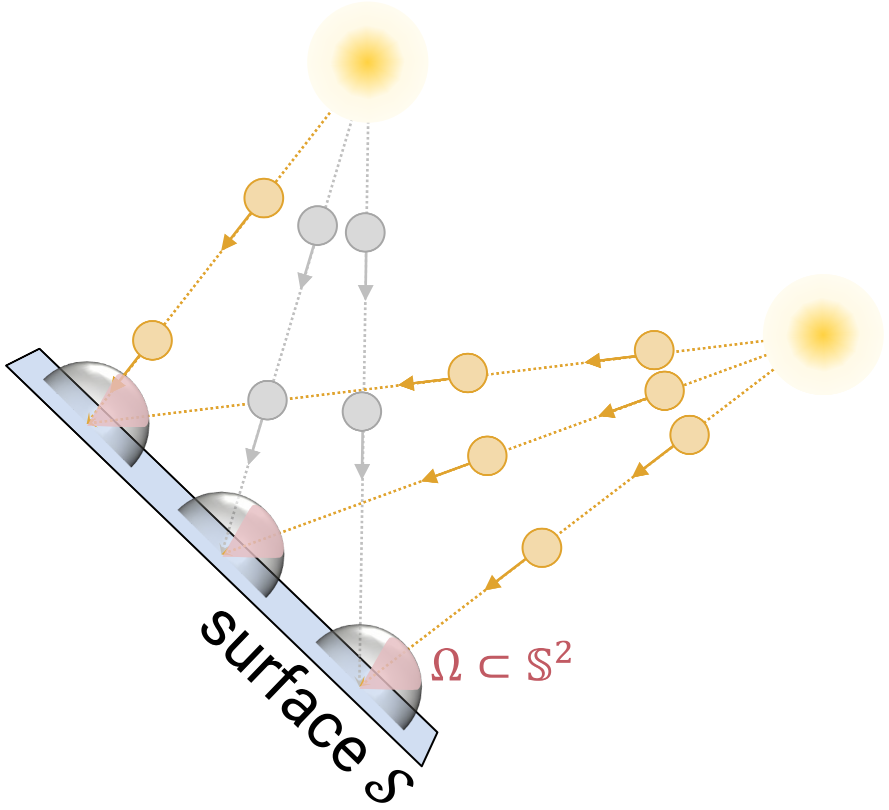
# Radiant energy

**Radiant energy of what?** surface $S \subset \mathbb{R}^3$,
solid angle $\Omega \subset \mathbb{S}^2$,




- "Energy" is "energy"!
- *Number of "hits" of photons on the surface*

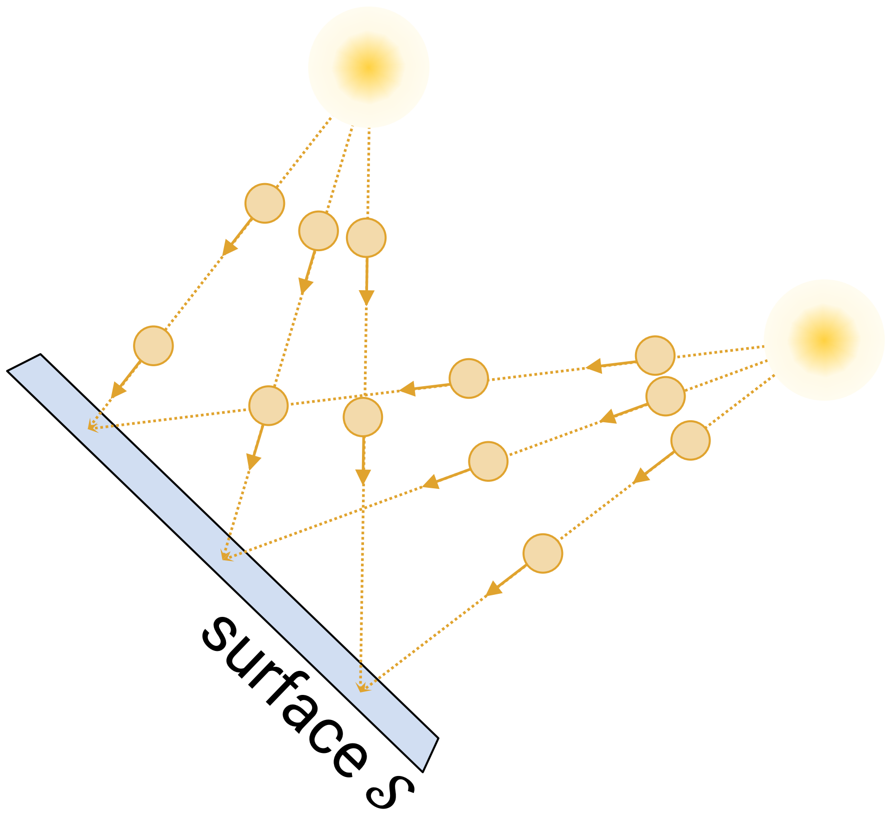
# Radiant energy




**Radiant energy of** surface $\mathcal{S} \subset \mathbb{R}^3$,
solid angle $\Omega \subset \mathbb{S}^2$,
time interval $[t_1, t_2] \subset \mathbb{R}$

- "Energy" is "energy"!
- *Number of "hits" of photons on the surface*

# Radiant energy

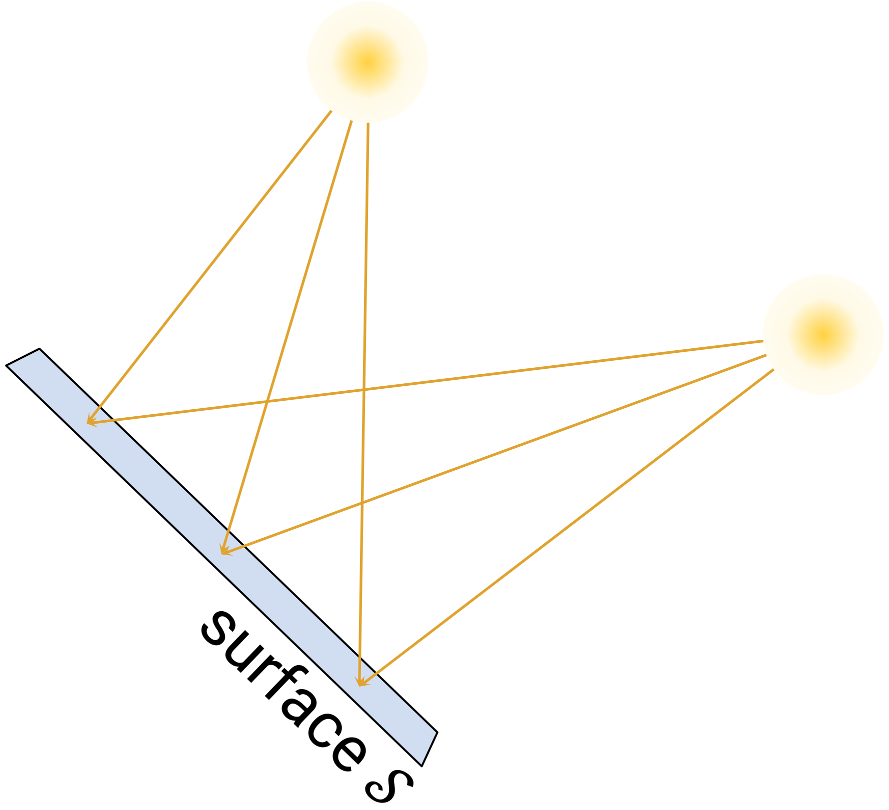



**Radiant energy of** surface $\mathcal{S} \subset \mathbb{R}^3$,
solid angle $\Omega \overset{\text{default}}{\in} \mathbb{S}^2$,
time interval $[t_1, t_2] \subset \mathbb{R}$

$$Q(\mathcal{S}, \Omega, [t_1, t_2]) \quad [\mathrm{J}]$$

- "Energy" is "energy"!
- *Number of "hits" of photons on the surface*

# Radiant flux (radiant power)




**Radiant flux (Radiant power)** **of what?**

surface $\mathcal{S} \subset \mathbb{R}^3$,

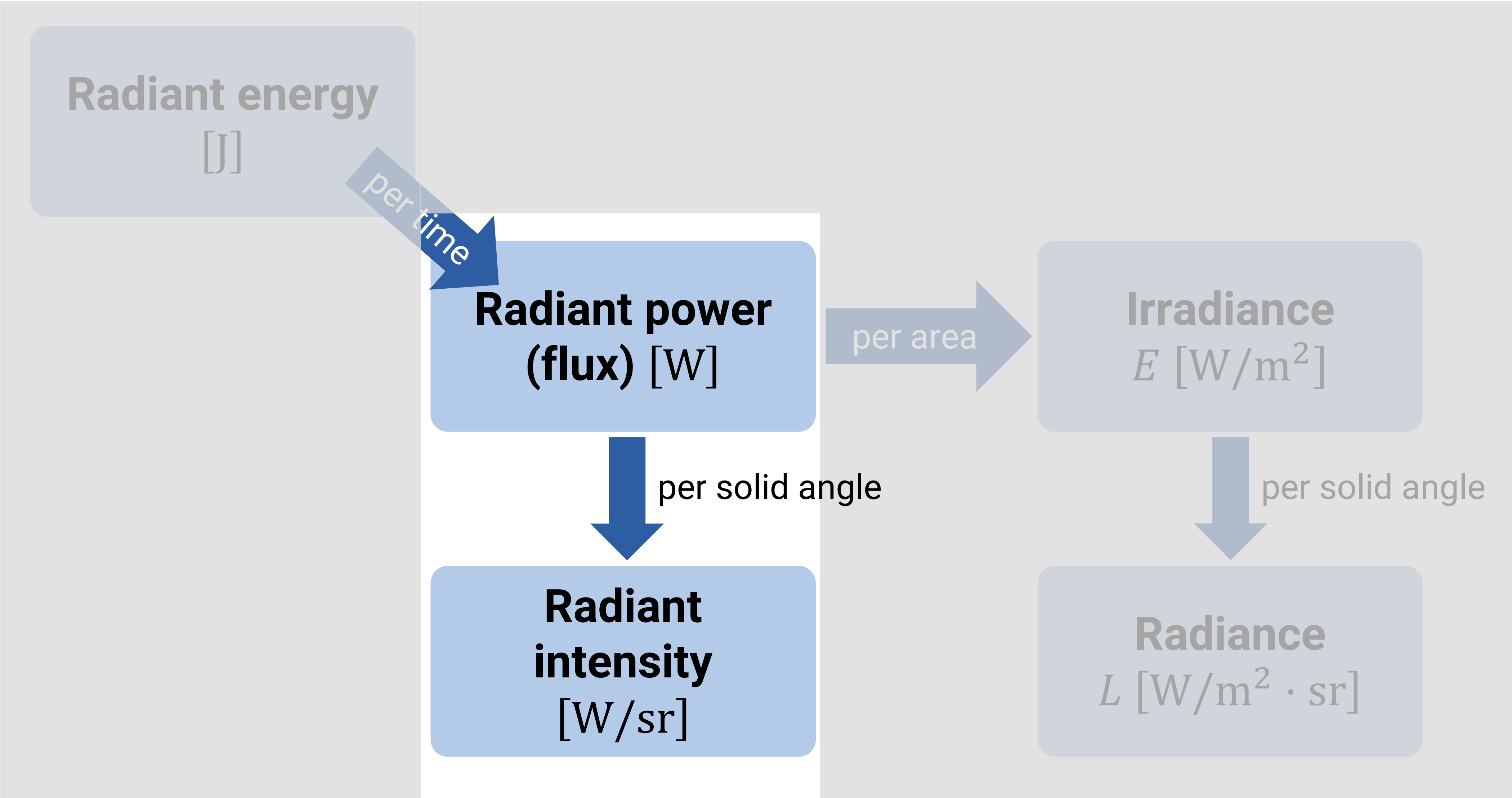
solid angle $\Omega \subset \mathbb{S}^2$

time $t \in \mathbb{R}$ (steady state)

$$\Phi(\mathcal{S}, \Omega, t) \quad [\mathrm{J/s} = \mathrm{W}]$$

- “Power” is “energy per time”!
- *Number of “intersecting rays” on the surface*

# Radiant intensity

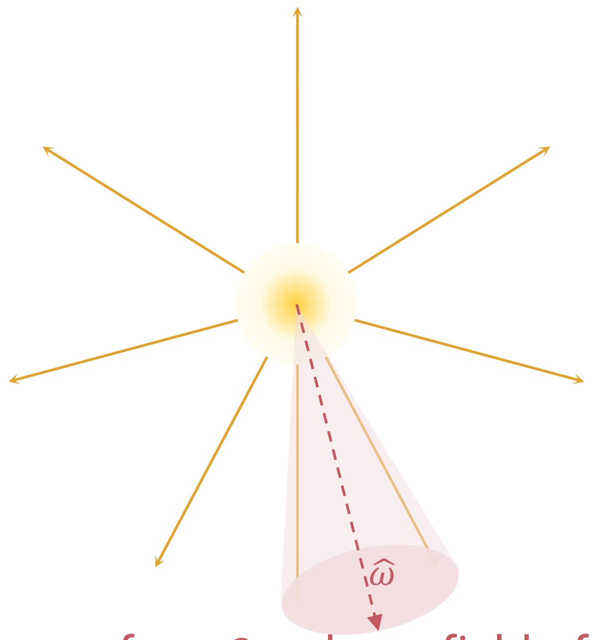


# Radiant intensity

**Radiant flux** **of** surface $\mathcal{S} \subset \mathbb{R}^3$,
solid angle $\Omega \subset \mathbb{S}^2$

↓ per solid angle

**Radiant intensity** **of** point source, ?
direction $\hat{\omega}$




surface $\mathcal{S}_\Omega$ whose field of view from the point source is $\Omega \subset \mathbb{S}^2$

**Proposition**

Relations between radiant flux $\Phi$ of a surface $\mathcal{S} \subset \mathbb{R}^3$ and radiant intensity of a point source at a position $p \in \mathbb{R}^3$ is that:

$$\Phi(\mathcal{S}_\Omega) = \int_\Omega I(\hat{\omega})\mathrm{d}\hat{\omega}$$

per solid angle →

← over solid angle

$$I(\hat{\omega}) = \lim_{\substack{|\Omega|\to 0 \\ \hat{\omega}\in\Omega}} \frac{\Phi(\mathcal{S}_\Omega)}{\boxed{|\Omega|}},$$

solid angle measure

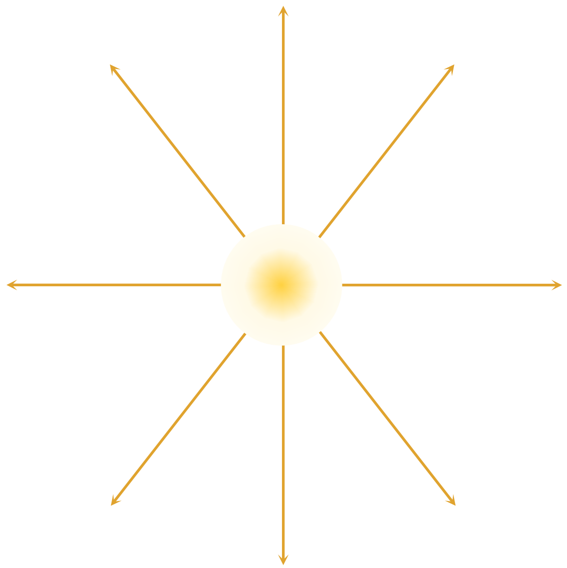
where $\Omega$ is the solid angle region

# Radiant intensity

**Radiant flux of** surface $\mathcal{S} \subset \mathbb{R}^3$,
solid angle $\Omega \subset \mathbb{S}^2$

per solid angle

**Radiant intensity of** point source,
direction $\hat{\omega}$

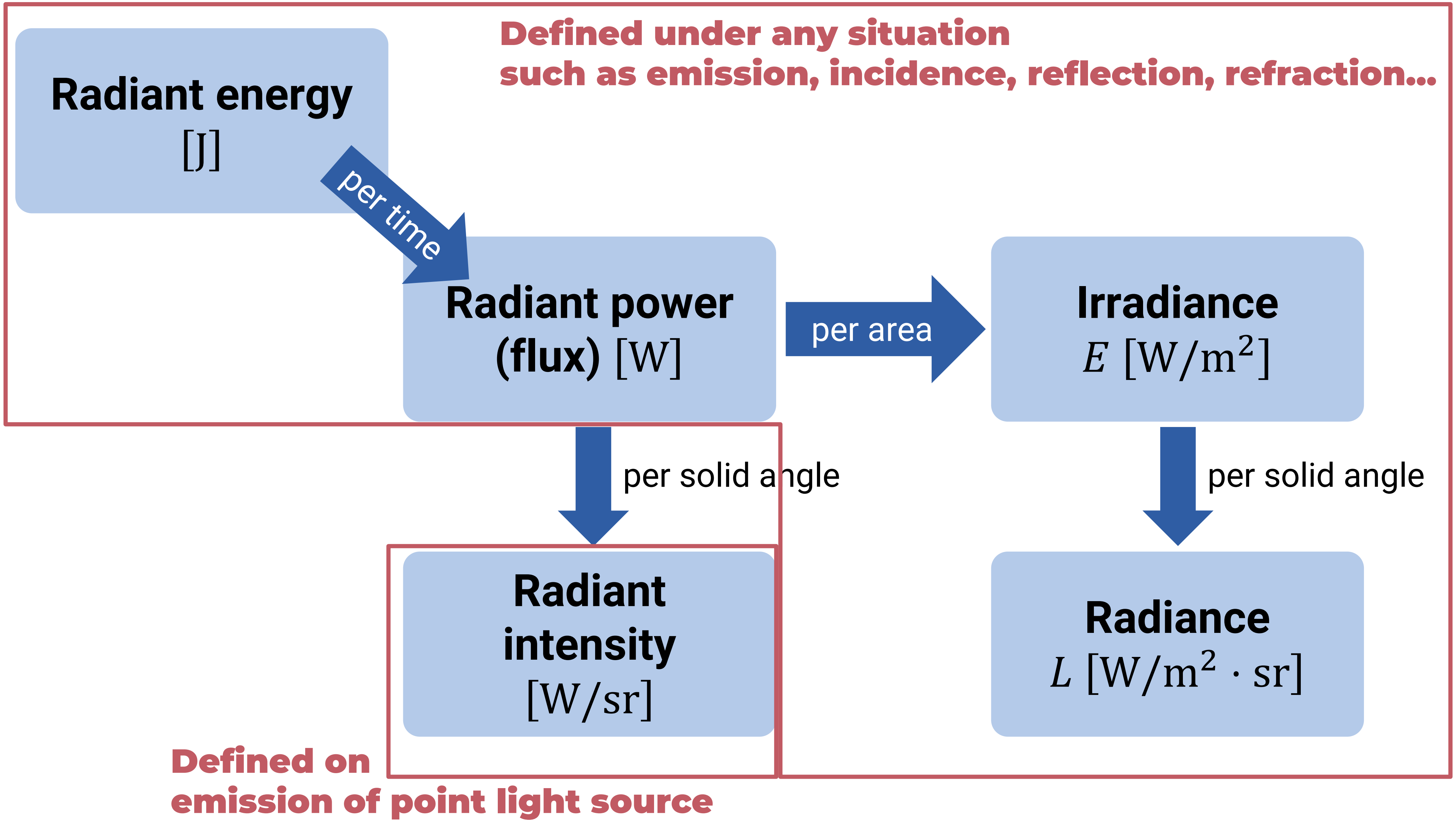


Practice

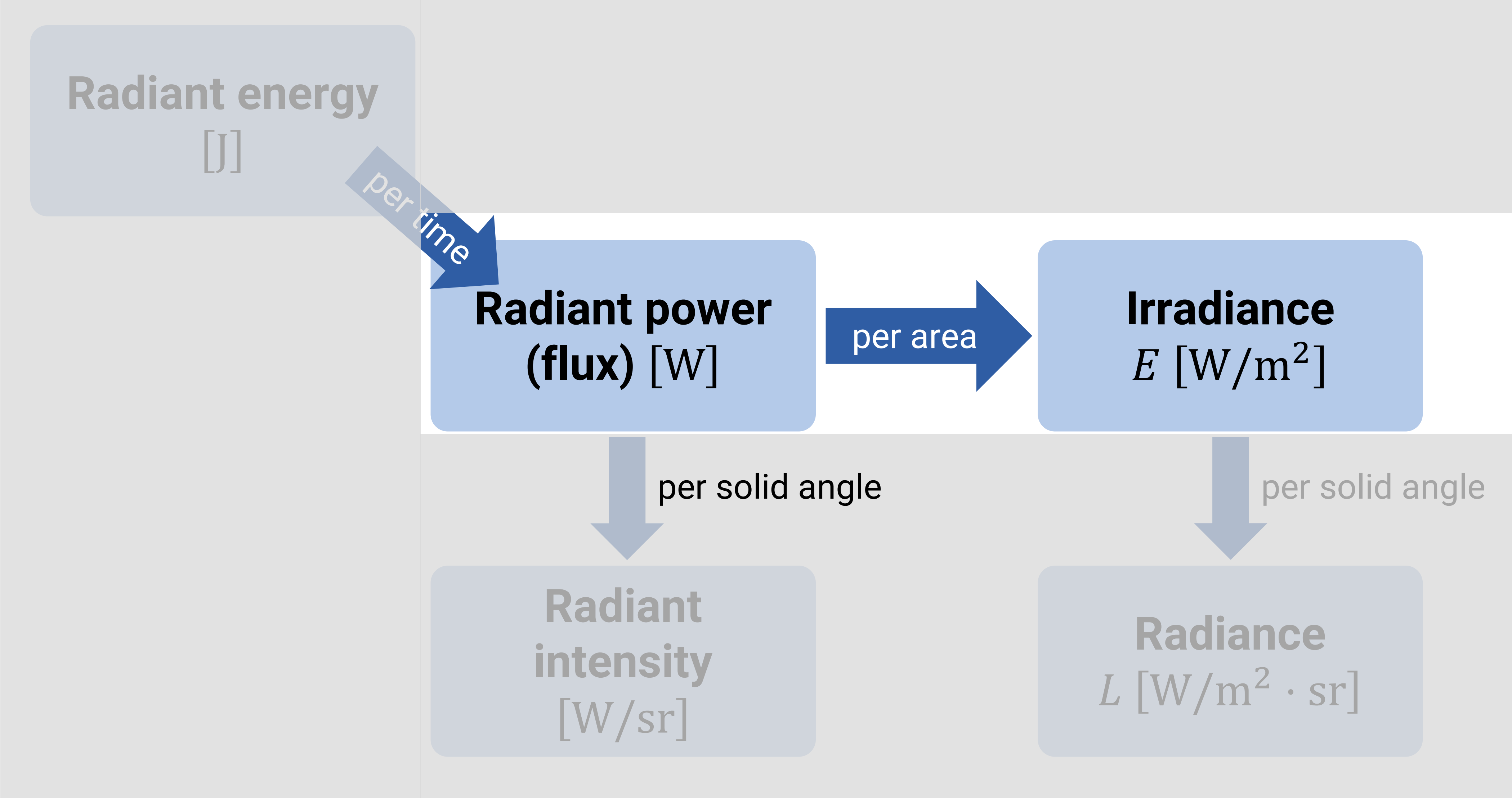
There is an isotropic point light source with radiant flux $\Phi$.
The radiant intensity of the source is?

$$I(\hat{\omega}) = \frac{\Phi}{4\pi}$$

# Radiant intensity



# Irradiance



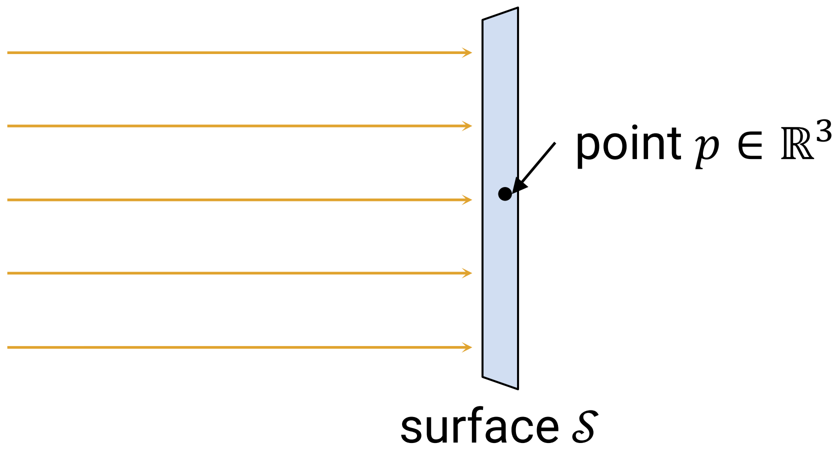
Radiant energy
[J]
per time
Radiant power
(flux) [W]
per area
Irradiance
$E$ [W/m$^2$]
per solid angle
per solid angle
Radiant
intensity
[W/sr]
Radiance
$L$ [W/m$^2 \cdot$ sr]

# Irradiance

**Radiant flux** **of** surface $\mathcal{S} \subset \mathbb{R}^3$,
solid angle $\Omega \subset \mathbb{S}^2$

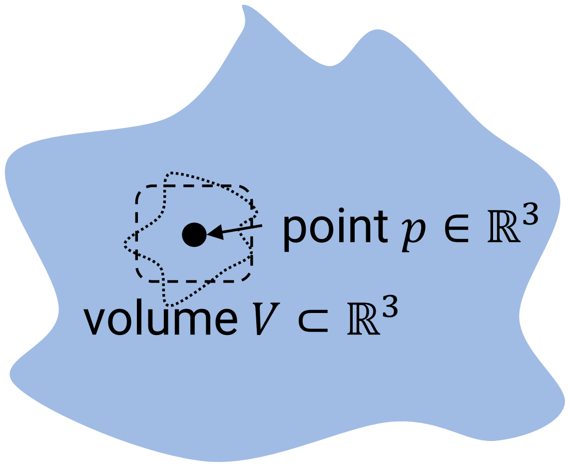
per area

**irradiance** **of** point $p \in \mathbb{R}^3$, **?**
solid angle $\Omega \subset \mathbb{S}^2$




**?** $$E(p, \Omega) = \lim_{\substack{\text{area}(\mathcal{S}) \to 0 \\ p \in \mathcal{S}}} \frac{\Phi(\mathcal{S}, \Omega)}{\text{area}(\mathcal{S})}$$ **is it enough?**

# Review: Concepts of mass vs. density




**Mass of** $V \subset \mathbb{R}^3$

**Density of** $p \in \mathbb{R}^3$

$$\text{mass}(V)$$

per volume

$$\text{density}(p) = \lim_{\substack{\text{vol}(V)\to 0 \\ p\in V}} \frac{\text{mass}(V)}{\text{vol}(V)}$$

$$= \lim_{\substack{\text{vol}(V)\to 0 \\ p\in V}} \frac{\text{mass}(V)}{\text{vol}(V)}$$

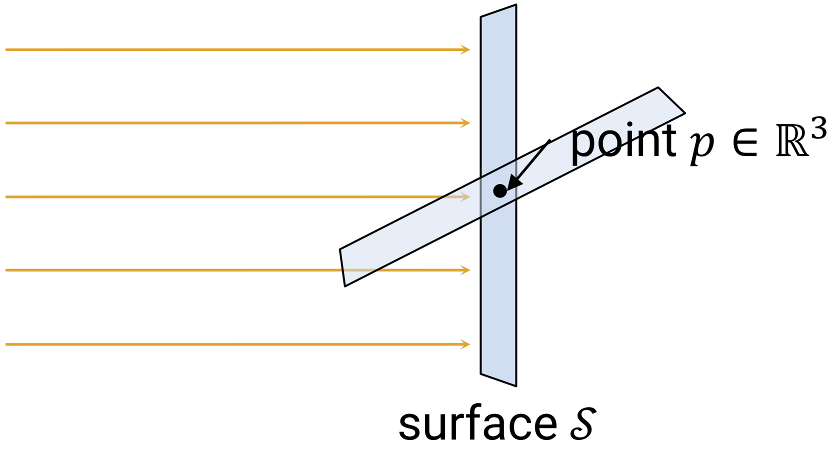
*** The limit converges to the same value whenever** $\text{vol}(V) \to 0$ **and** $p \in V$.

# Irradiance

**Radiant flux** **of** surface $\mathcal{S} \subset \mathbb{R}^3$,
solid angle $\Omega \subset \mathbb{S}^2$

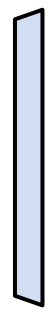
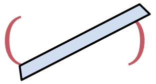
per area

**irradiance** **of** point $p \in \mathbb{R}^3$, **?**
solid angle $\Omega \subset \mathbb{S}^2$




$$? \quad E(p,\Omega) = \lim_{\substack{\text{area}(\mathcal{S})\to 0 \\ p\in\mathcal{S}}} \frac{\Phi(\mathcal{S},\Omega)}{\text{area}(\mathcal{S})}$$

**is it enough?**

$\mathcal{S} = ($  $)$ and $\mathcal{S} = ($  $)$ yield different limits

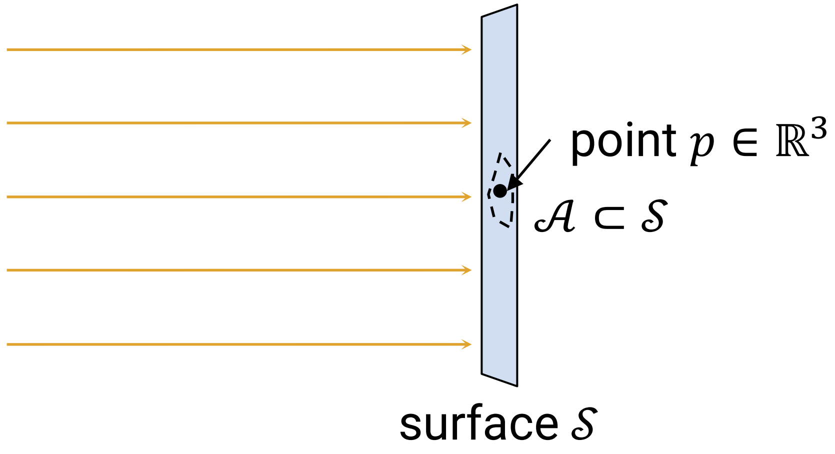
# Irradiance

**Radiant flux** **of** surface $\mathcal{S} \subset \mathbb{R}^3$,
solid angle $\Omega \subset \mathbb{S}^2$

per area

**irradiance** **of** point $p \in \mathbb{R}^3$, **?**
solid angle $\Omega \subset \mathbb{S}^2$

$$E(p, \Omega) = \lim_{\substack{\text{area}(\mathcal{S}) \to 0 \\ p \in \mathcal{S}}} \frac{\Phi(\mathcal{S}, \Omega)}{\text{area}(\mathcal{S})}$$

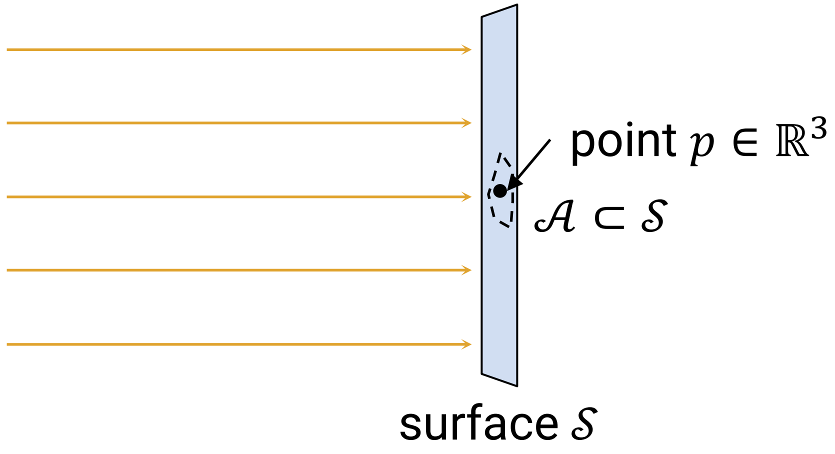


# Irradiance

**Radiant flux** of surface $\mathcal{S} \subset \mathbb{R}^3$,
solid angle $\Omega \subset \mathbb{S}^2$

per area

**irradiance** of point $p \in \mathbb{R}^3$, ?
solid angle $\Omega \subset \mathbb{S}^2$




$$E_{\mathcal{S}}(p, \Omega) = \lim_{\substack{\text{area}(\mathcal{A}) \to 0 \\ p \in \mathcal{A} \subset \mathcal{S}}} \frac{\Phi(\mathcal{A}, \Omega)}{\text{area}(\mathcal{A})}$$

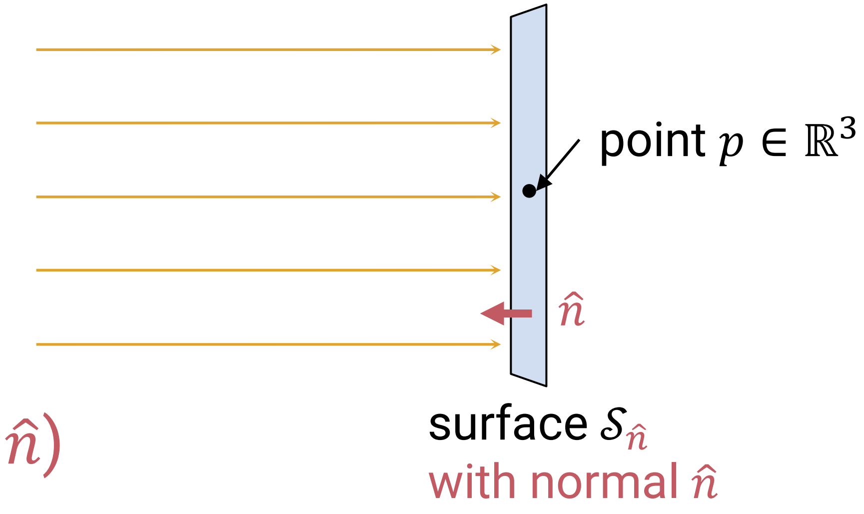
Irradiance defined as the limit about a subset of given fixed surface

# Irradiance

**Radiant flux** of surface $\mathcal{S} \subset \mathbb{R}^3$,
solid angle $\Omega \subset \mathbb{S}^2$

per area

**irradiance** of point $p \in \mathcal{S}$ (or $p \in \mathbb{R}^3$ and $\hat{n}$)
solid angle $\Omega \subset \mathbb{S}^2$




$$E_{\hat{n}}(p, \Omega) = \lim_{\substack{\text{area}(\mathcal{S}_{\hat{n}}) \to 0 \\ p \in \mathcal{S}_{\hat{n}}}} \frac{\Phi(\mathcal{S}_{\hat{n}}, \Omega)}{\text{area}(\mathcal{S}_{\hat{n}})}$$

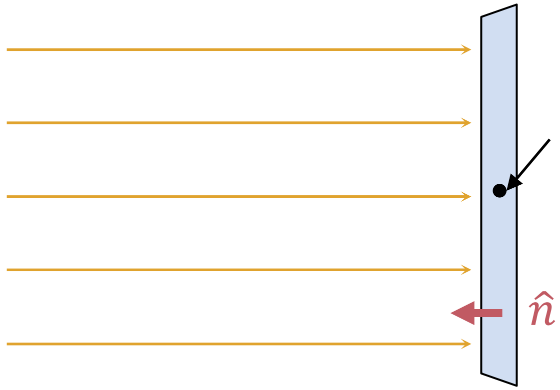
Irradiance defined as the limit about a subset of given fixed surface,
or a surface with given fixed normal

# Irradiance

**Radiant flux** of surface $\mathcal{S} \subset \mathbb{R}^3$,
solid angle $\Omega \subset \mathbb{S}^2$

per area

**irradiance** of point $p \in \mathcal{S}$ (or $p \in \mathbb{R}^3$ and $\hat{n}$)
solid angle $\Omega \subset \mathbb{S}^2$




$$E_{\hat{n}}(p, \Omega) = \lim_{\substack{\mathrm{area}(\mathcal{S}_{\hat{n}}) \to 0 \\ p \in \mathcal{S}_{\hat{n}}}} \frac{\Phi(\mathcal{S}_{\hat{n}}, \Omega)}{\mathrm{area}(\mathcal{S}_{\hat{n}})}$$

Irradiance defined as the limit about a subset of given fixed surface,
or a surface with given fixed normal

We don't say just "irradiance at $p$" when $p$ is not on any surface

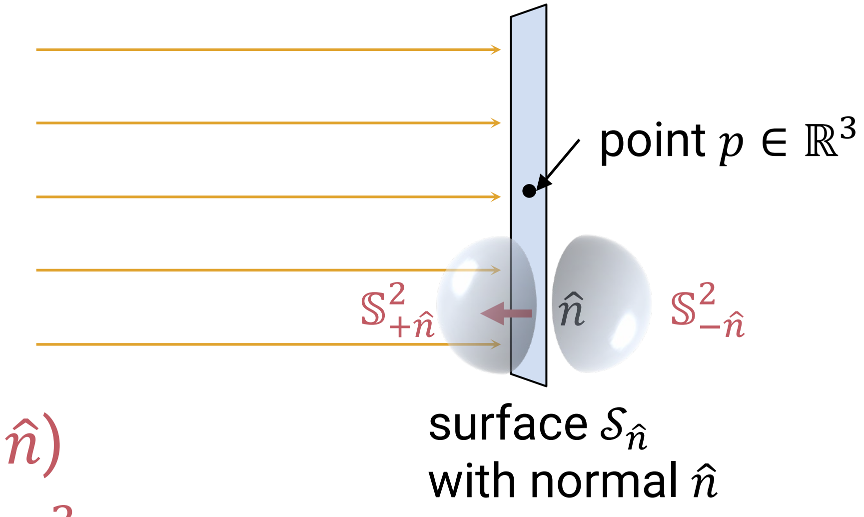
# Irradiance

**Radiant flux** of surface $\mathcal{S} \subset \mathbb{R}^3$,
solid angle $\Omega \subset \mathbb{S}^2$

per area

**irradiance** of point $p \in \mathcal{S}$ (or $p \in \mathbb{R}^3$ and $\hat{n}$)

default: solid angle $\Omega = \mathbb{S}^2, \mathbb{S}^2_{+\hat{n}}$, or $\mathbb{S}^2_{-\hat{n}}$




$$E_{\hat{n}}(p, \Omega) = \lim_{\substack{\text{area}(\mathcal{S}_{\hat{n}}) \to 0 \\ p \in \mathcal{S}_{\hat{n}}}} \frac{\Phi(\mathcal{S}_{\hat{n}}, \Omega)}{\text{area}(\mathcal{S}_{\hat{n}})}$$

The default solid angle changes depending on the context

***How?***

# Irradiance

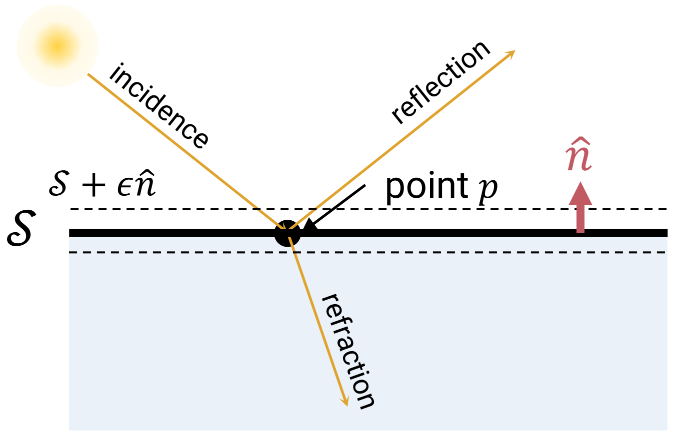
**Radiant flux** **of** surface $\mathcal{S} \subset \mathbb{R}^3$,
solid angle $\Omega \subset \mathbb{S}^2$

per area

**irradiance** **of** point $p \in \mathcal{S}$ (or $p \in \mathbb{R}^3$ and $\hat{n}$)

default: solid angle $\Omega = \mathbb{S}^2, \mathbb{S}^2_{+\hat{n}},$ or $\mathbb{S}^2_{-\hat{n}}$

Some details depending on context…




- "incoming" irradiance: $E_{\mathcal{S}}^{(\text{in})}(p) = E_{\mathcal{S}+\epsilon\hat{n}}\left(p, \mathbb{S}^2_{-\hat{n}}\right)$
- "reflected" irradiance: $E_{\mathcal{S}}^{(\text{refl})}(p) = E_{\mathcal{S}+\epsilon\hat{n}}\left(p, \mathbb{S}^2_{+\hat{n}}\right)$
- "refracted" irradiance: $E_{\mathcal{S}}^{(\text{refr})}(p) = E_{\mathcal{S}-\epsilon\hat{n}}\left(p, \mathbb{S}^2_{-\hat{n}}\right)$
- "outgoing" irradiance: $E_{\mathcal{S}}^{(\text{out})}(p) = E_{\mathcal{S}}^{(\text{refl})}(p) + E_{\mathcal{S}}^{(\text{refr})}(p)$

**Our intuition easily can do this!**

# Irradiance

**Radiant flux** **of** surface $\mathcal{S} \subset \mathbb{R}^3$,
solid angle $\Omega \subset \mathbb{S}^2$

per area

**irradiance** **of** point $p \in \mathcal{S}$ (or $p \in \mathbb{R}^3$ and $\hat{n}$)
solid angle $\Omega \subset \mathbb{S}^2$

$$\Phi(\mathcal{S}, \Omega) = \int_{\mathcal{S}} E_{\mathcal{S}}(p, \Omega) \mathrm{d}p$$

per area

over area

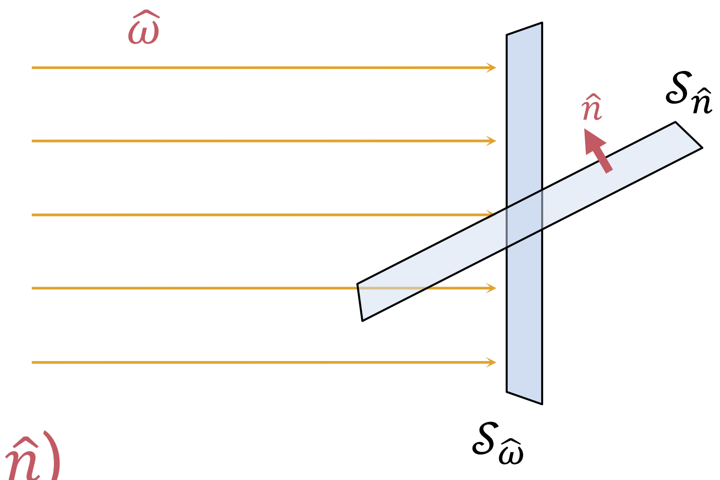
$$E_{\mathcal{S}}(p, \Omega) = \lim_{\substack{\mathrm{area}(\mathcal{A}) \to 0 \\ p \in \mathcal{A} \subset \mathcal{S}}} \frac{\Phi(\mathcal{A}, \Omega)}{\mathrm{area}(\mathcal{A})}$$

# Irradiance

**Radiant flux of** surface $\mathcal{S} \subset \mathbb{R}^3$,
solid angle $\Omega \subset \mathbb{S}^2$

per area

**irradiance of** point $p \in \mathcal{S}$ (or $p \in \mathbb{R}^3$ and $\hat{n}$)
solid angle $\Omega \subset \mathbb{S}^2$




Practice

There is directional light with $\hat{\omega}$.
What is the relationship between $E_{\hat{\omega}}(p)$ and $E_{\hat{n}}(p)$?

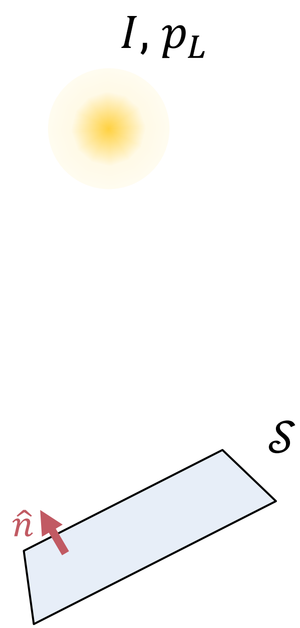
$$E_{\hat{n}}(p) = E_{\hat{\omega}}(p)|\hat{n} \cdot \hat{\omega}|$$

# Irradiance

**Radiant flux** **of** surface $\mathcal{S} \subset \mathbb{R}^3$,
solid angle $\Omega \subset \mathbb{S}^2$

per area

**irradiance** **of** point $p \in \mathcal{S}$ (or $p \in \mathbb{R}^3$ and $\hat{n}$)
solid angle $\Omega \subset \mathbb{S}^2$




Practice

There is a point light source at $p_L$ with the radiant intensity $I(\widehat{\omega})$.
What is the incident irradiance at $p$ on a surface $\mathcal{S}$, $E_{\mathcal{S}}(p) = ?$

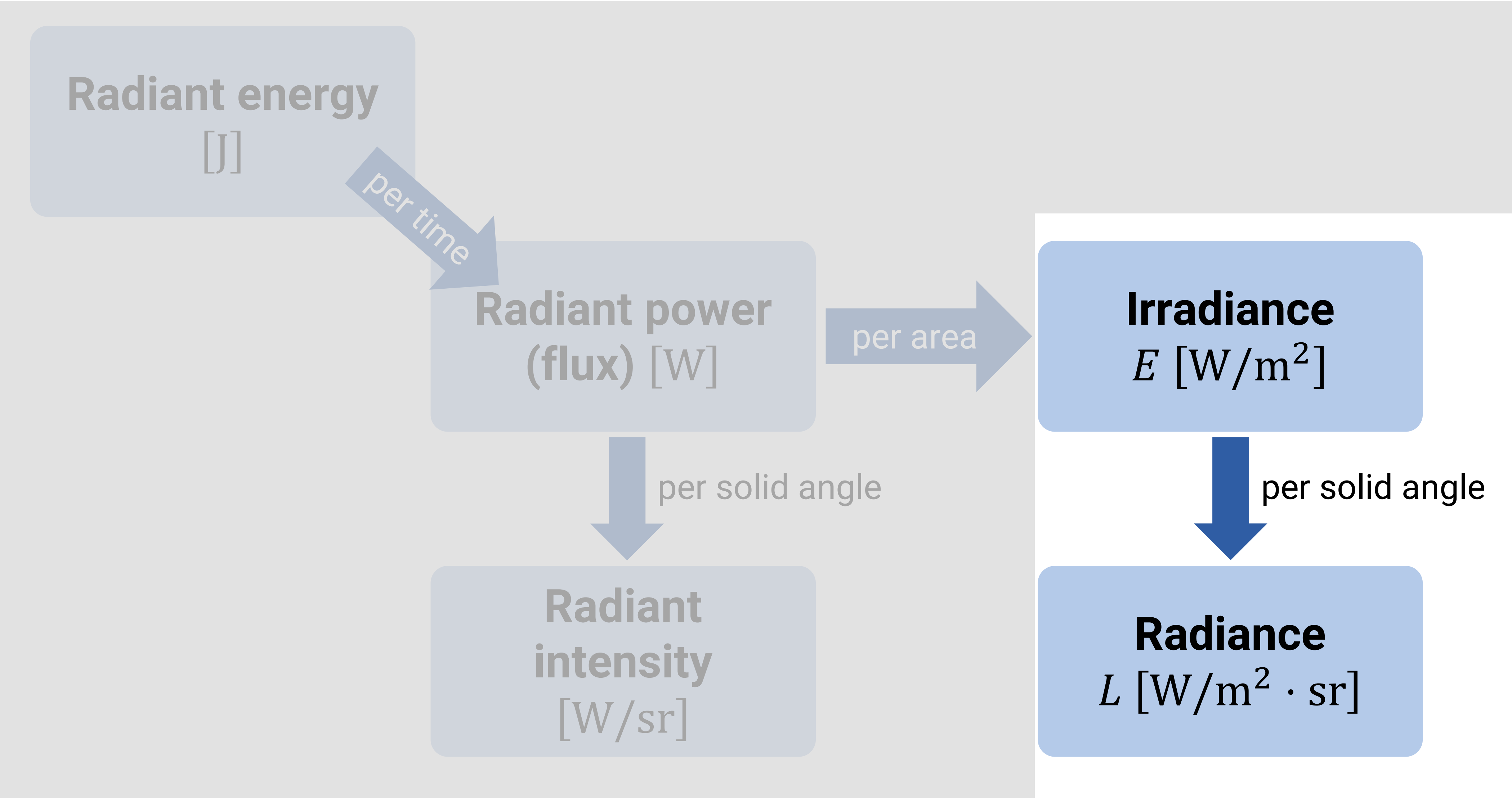
$$E_{\mathcal{S}}(p) = \lim_{\substack{\mathrm{area}(\mathcal{A}) \to 0 \\ p \in \mathcal{A} \subset \mathcal{S}}} \frac{\Phi(\mathcal{A})}{\mathrm{area}(\mathcal{A})} = \lim_{\substack{\mathrm{area}(\mathcal{A}) \to 0 \\ p \in \mathcal{A} \subset \mathcal{S}}} \frac{I(\widehat{\omega}_{p_L p})\,\mathrm{sol.ang.}(\mathcal{A}, p_L)}{\mathrm{area}(\mathcal{A})} = \frac{I(\widehat{\omega}_{p_L p}) |\hat{n} \cdot \widehat{\omega}_{p p_L}|}{\|p - p_L\|^2}$$

Definition of irradiance

Definition of radiant intensity, small areal $\mathcal{A}$

Relation between an area and a solid angle, small areal $\mathcal{A}$

# Radiance

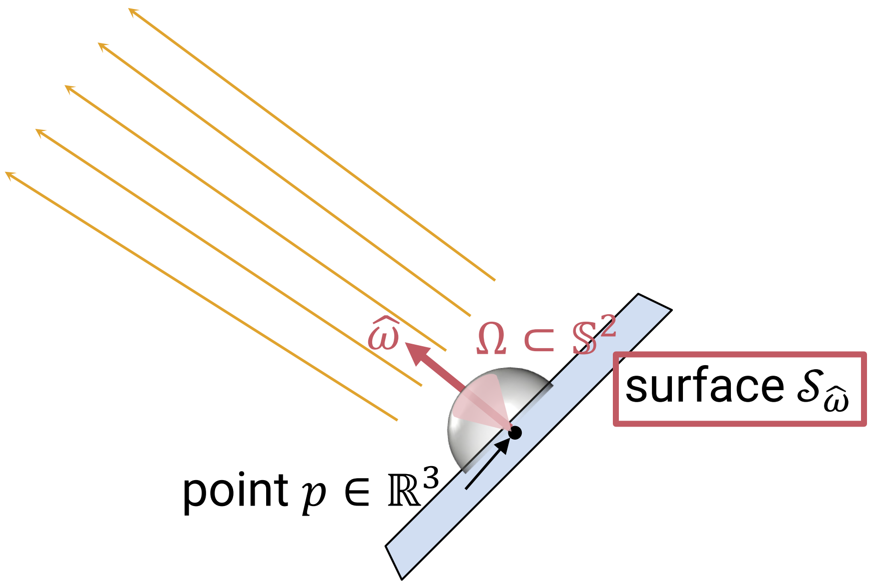

Radiant energy
[J]
per time
Radiant power
(flux) [W]
per area
Irradiance
$E$ [W/m$^2$]
per solid angle
Radiant
intensity
[W/sr]
per solid angle
Radiance
$L$ [W/m$^2 \cdot$ sr]

# Radiance

**Irradiance of** point $p \in \mathcal{S}$,
solid angle $\Omega \subset \mathbb{S}^2$

per solid angle

**Radiance of** point $p \in \mathbb{R}^3$,
direction $\hat{\omega} \in \mathbb{S}^2$




$$L(p, \hat{\omega}) = \lim_{\substack{\text{sol.ang.}(\Omega) \to 0 \\ \hat{\omega} \in \Omega}} \frac{E_{\boxed{\mathcal{S}_{\hat{\omega}}}}(p, \Omega)}{\text{sol. ang.}(\Omega)}$$

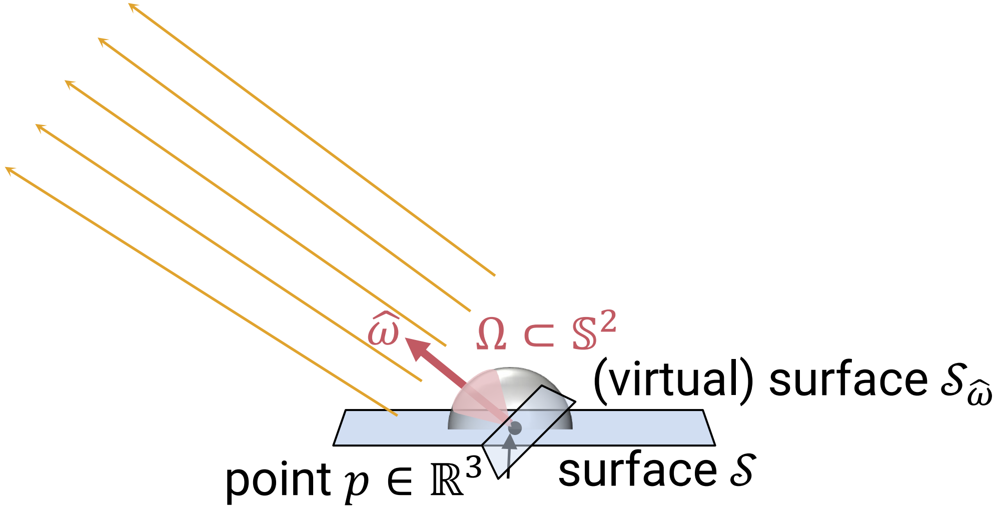
For a point $p \in \mathbb{R}^3$ (on or not on a surface),
the radiance $L(p, \hat{\omega})$ is defined as the limit about a virtual surface facing $\hat{\omega}$

# Radiance

**Irradiance** **of** point $p \in \mathcal{S}$,
solid angle $\Omega \subset \mathbb{S}^2$

per solid angle

**Radiance** **of** point $p \in \mathbb{R}^3$,
direction $\hat{\omega} \in \mathbb{S}^2$




*small solid angle around $\hat{\omega}$!*

$$L(p, \hat{\omega}) = \lim_{\substack{\text{sol.ang.}(\Omega) \to 0 \\ \hat{\omega} \in \Omega}} \frac{\boxed{E_{\mathcal{S}_{\hat{\omega}}}(p, \Omega)}}{\text{sol. ang. }(\Omega)} = \lim_{\substack{\text{sol.ang.}(\Omega) \to 0 \\ \hat{\omega} \in \Omega}} \frac{1}{|\hat{n} \cdot \hat{\omega}|} \frac{E_{\mathcal{S}}(p, \Omega)}{\text{sol. ang. }(\Omega)}$$

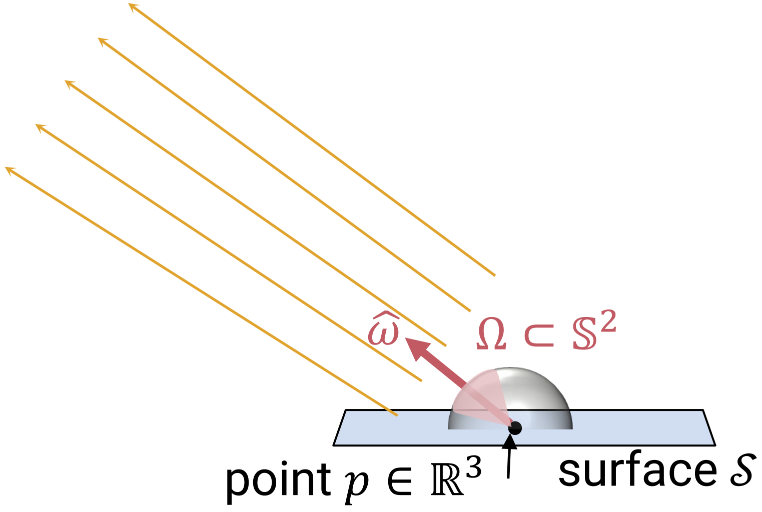
For a point $p$ on given surface $\mathcal{S}$,
the radiance $L(p, \hat{\omega})$ can also be written as the limit about the irradiance on $\mathcal{S}$

# Radiance

**Irradiance** **of** point $p \in \mathcal{S}$,
solid angle $\Omega \subset \mathbb{S}^2$

per solid angle

**Radiance** **of** point $p \in \mathbb{R}^3$,
direction $\hat{\omega} \in \mathbb{S}^2$




$$E_{\mathcal{S}}(p,\Omega) = \int_{\Omega} L(p,\hat{\omega})|\hat{n}\cdot\hat{\omega}|\mathrm{d}\hat{\omega}$$

per solid angle

over solid angle

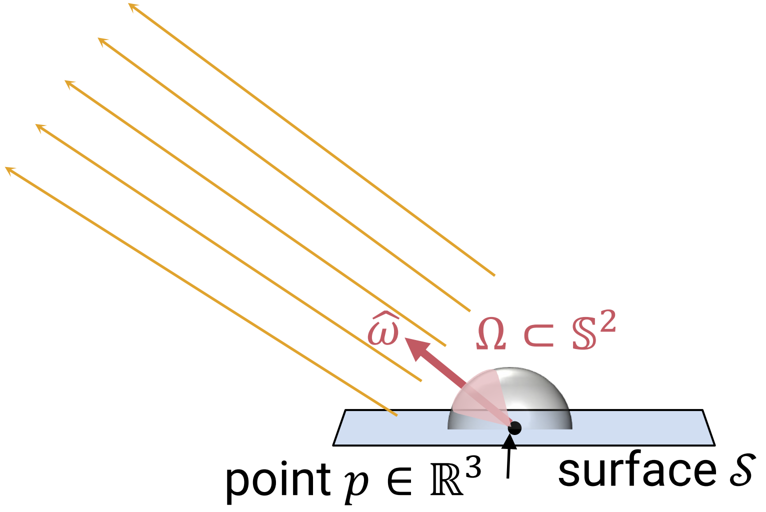
$$L(p,\hat{\omega}) = \lim_{\substack{\text{sol.ang.}(\Omega)\to 0\\ \hat{\omega}\in\Omega}} \frac{1}{|\hat{n}\cdot\hat{\omega}|}\frac{E_{\mathcal{S}}(p,\Omega)}{\text{sol. ang. }(\Omega)}$$

# Radiance

**Irradiance** **of** point $p \in \mathcal{S}$,
solid angle $\Omega \subset \mathbb{S}^2$

per solid angle

**Radiance** **of** point $p \in \mathbb{R}^3$,
direction $\hat{\omega} \in \mathbb{S}^2$



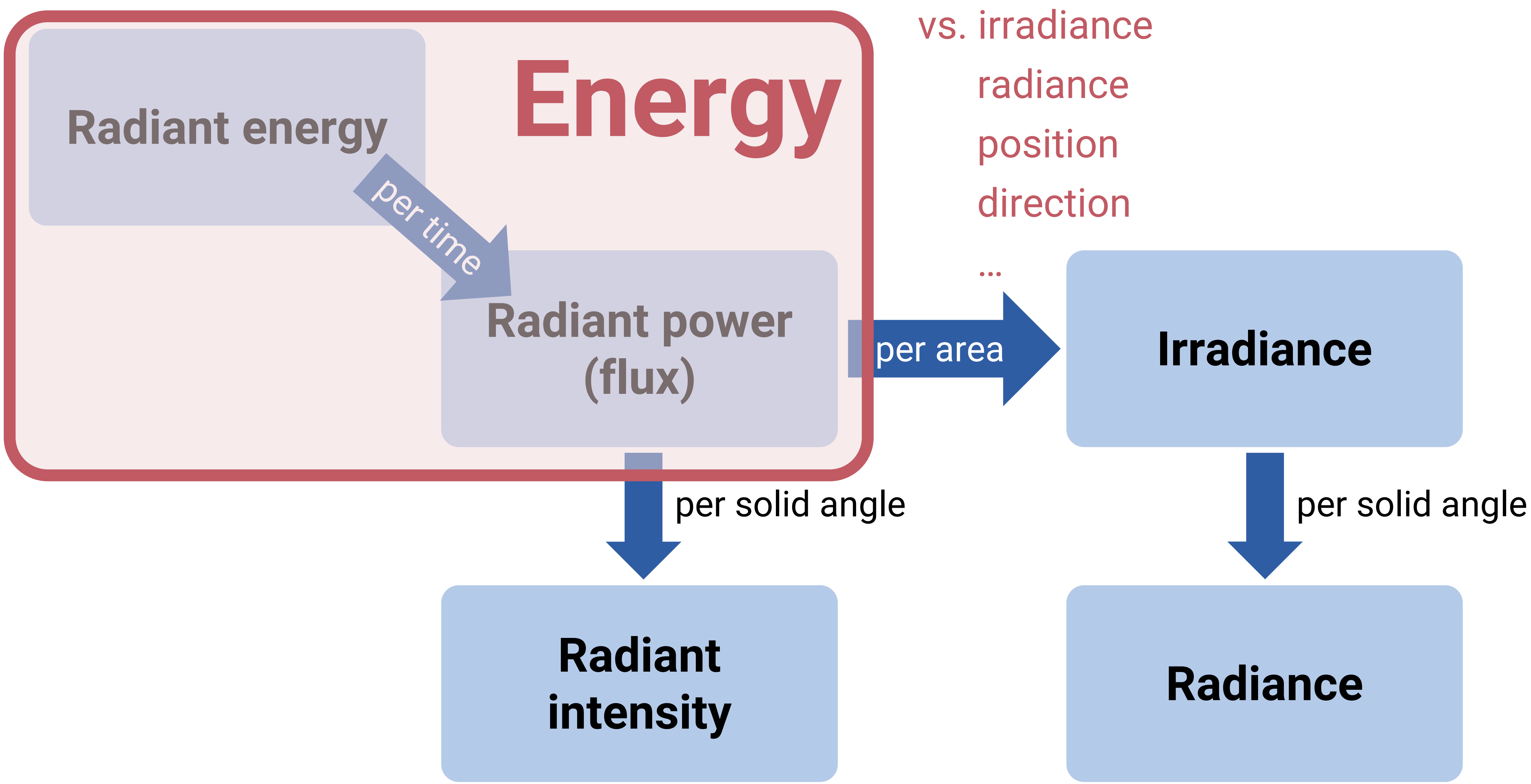

Practice

Radiance is invariant along ray:

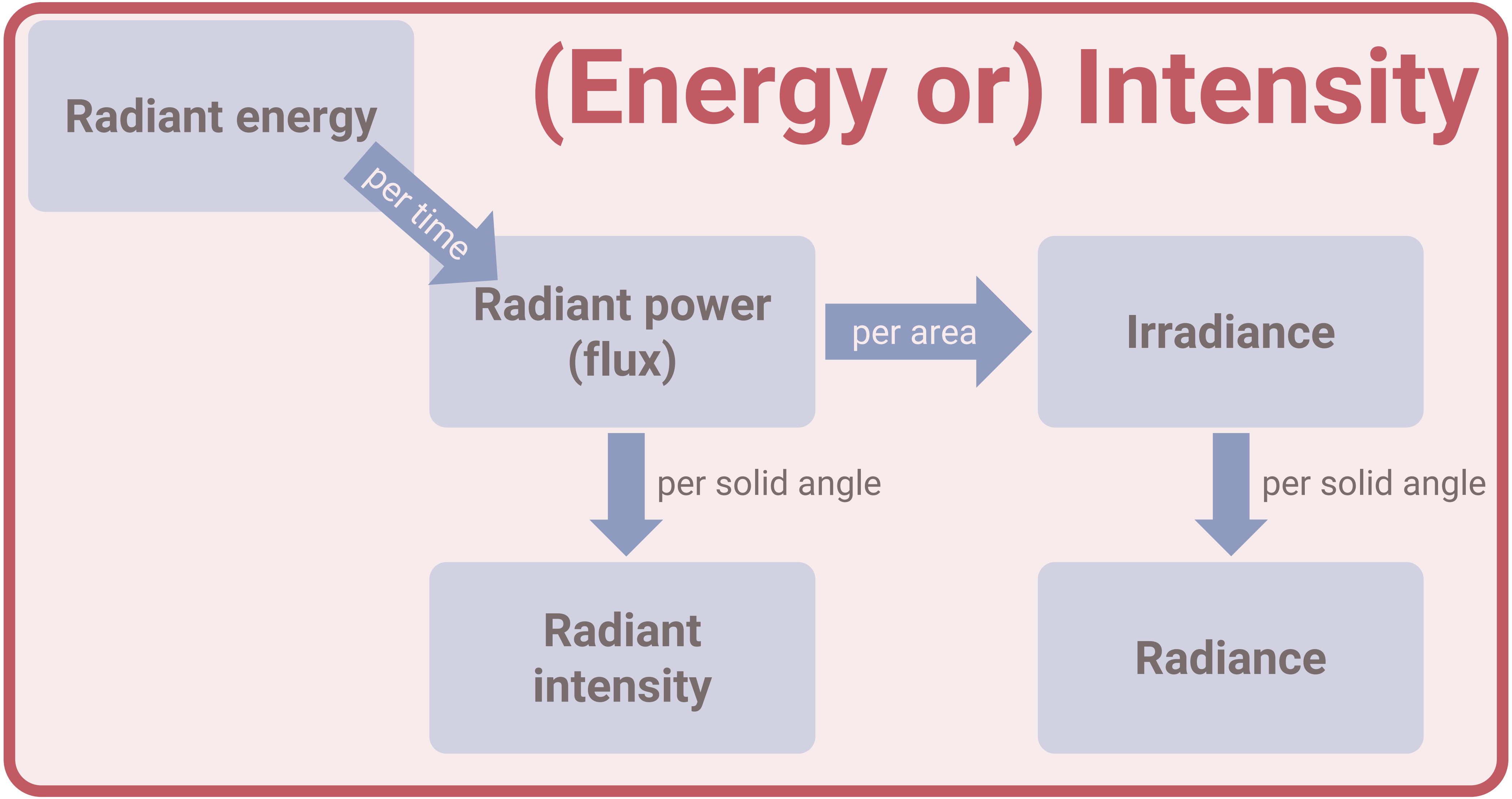
$$L(p, \hat{\omega}) = L(p + t\hat{\omega}, \hat{\omega}) \ \forall t \in \mathbb{R}$$

whenever there is no material between $p$ and $p + t\hat{\omega}$

# Slight abuse of terminology

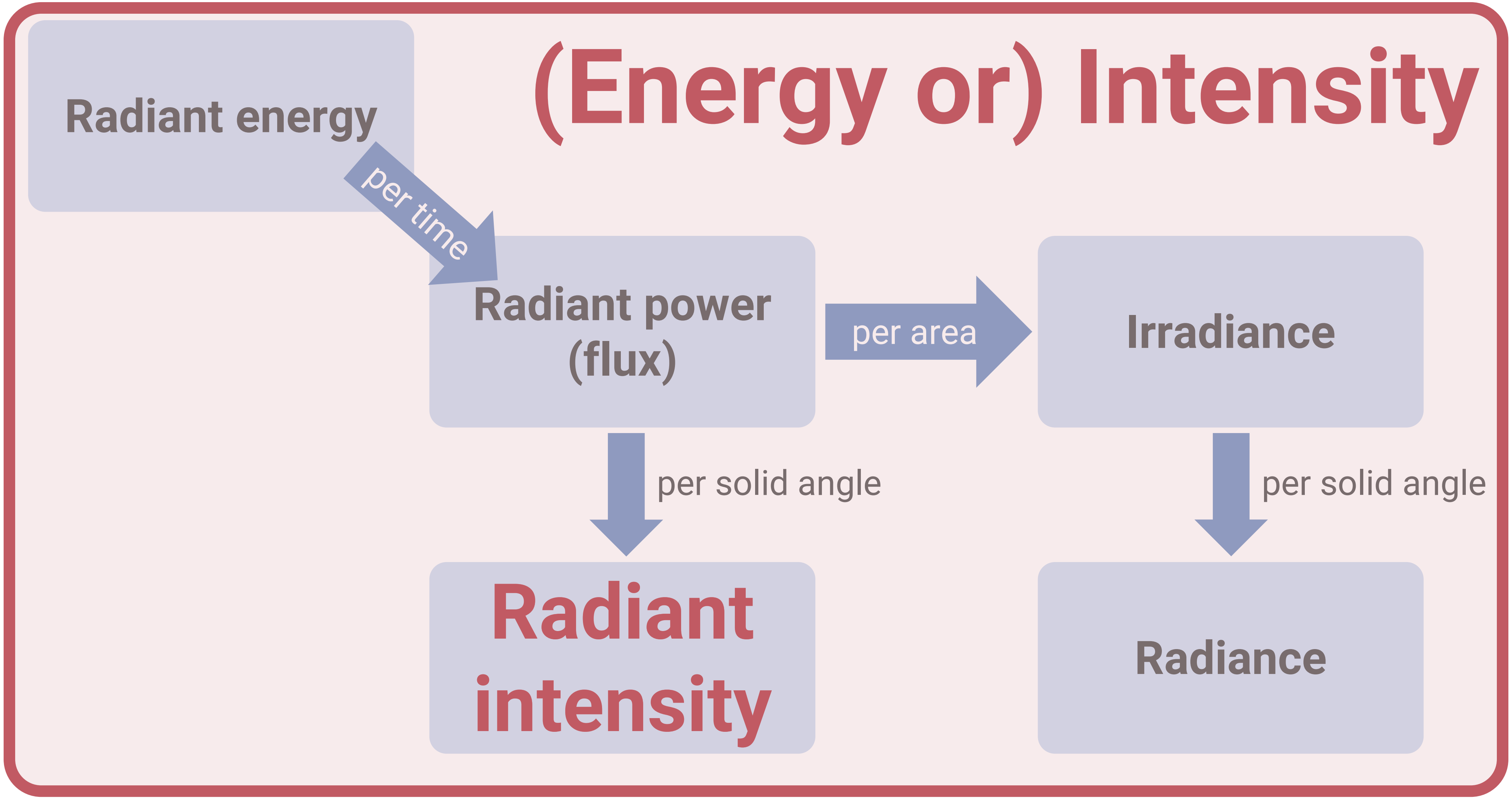


# Slight abuse of terminology




vs. position
direction
…

# Unfortunate ambiguity

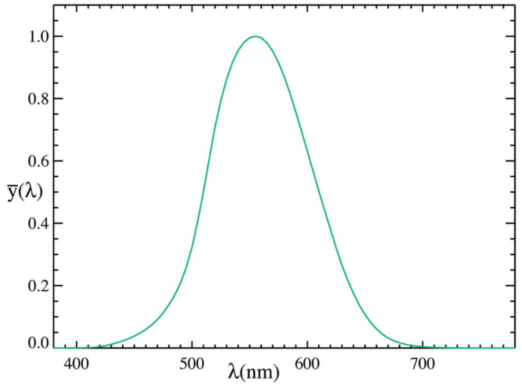


# Photometry?

**Radiometry: physical energy**

**Photometry: how bright human perceive**

$$\int_{380\mathrm{nm}}^{700\mathrm{nm}} (\text{radiometric quantity per wavelength})\,(\text{luminous efficiency function})\,\mathrm{d}\lambda$$



# Photometry?

**radiant → luminous**

**radiance → luminance**

**Radiant energy**

per time

**Radiant power (flux)**

per area

**Irradiance**

per solid angle

**Radiant intensity**

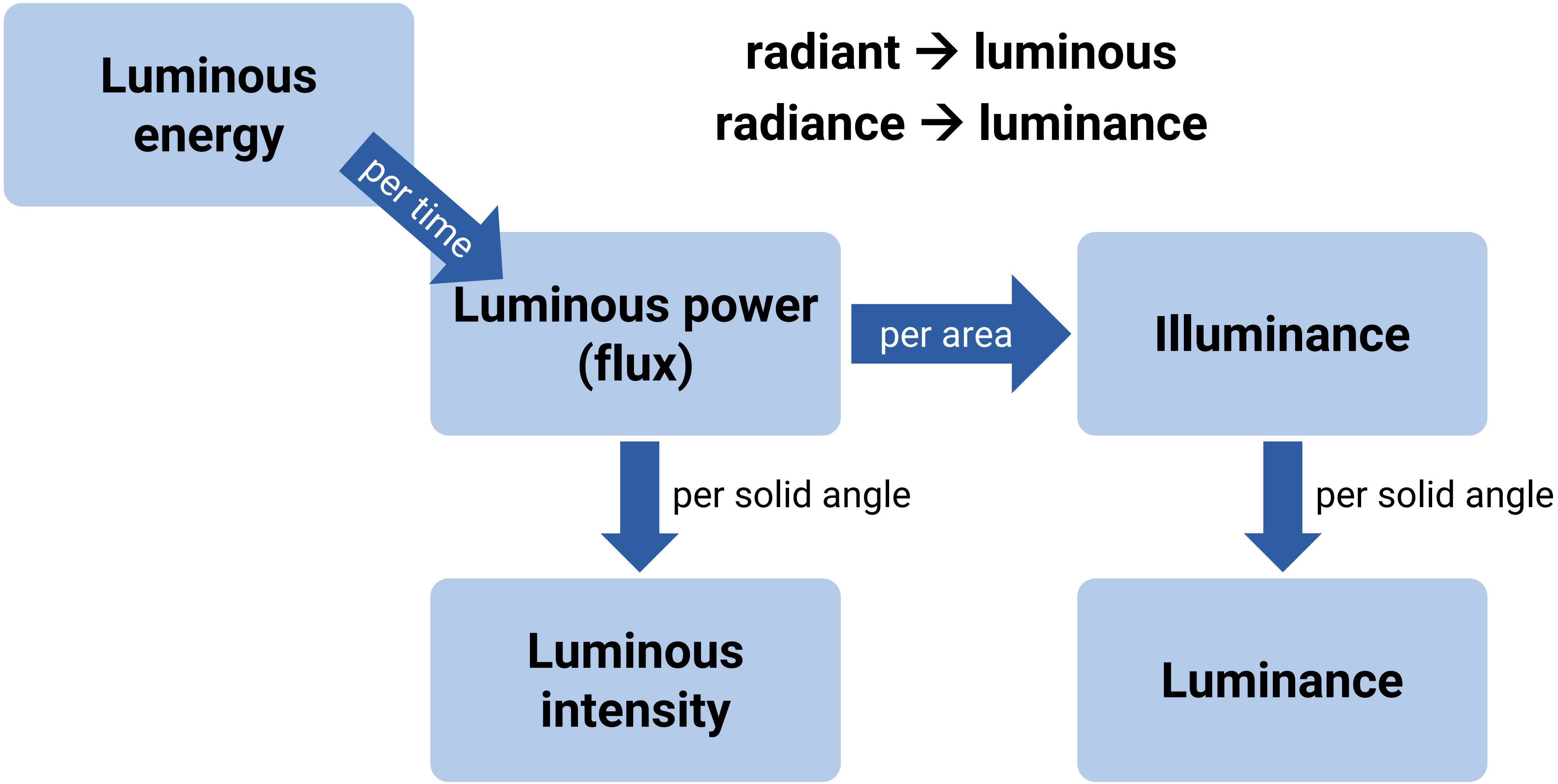
per solid angle

**Radiance**

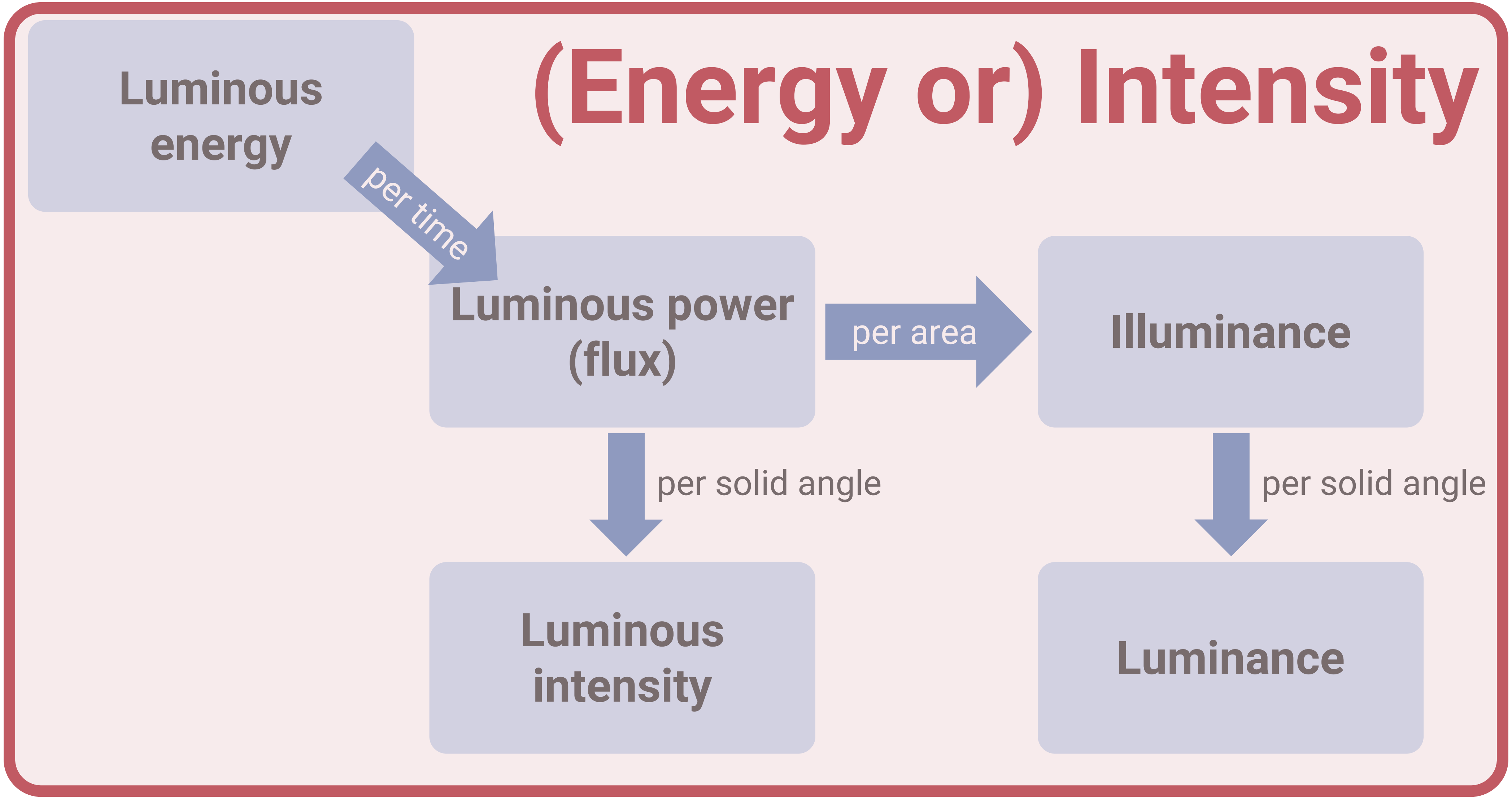
# Photometry

**radiant → luminous**

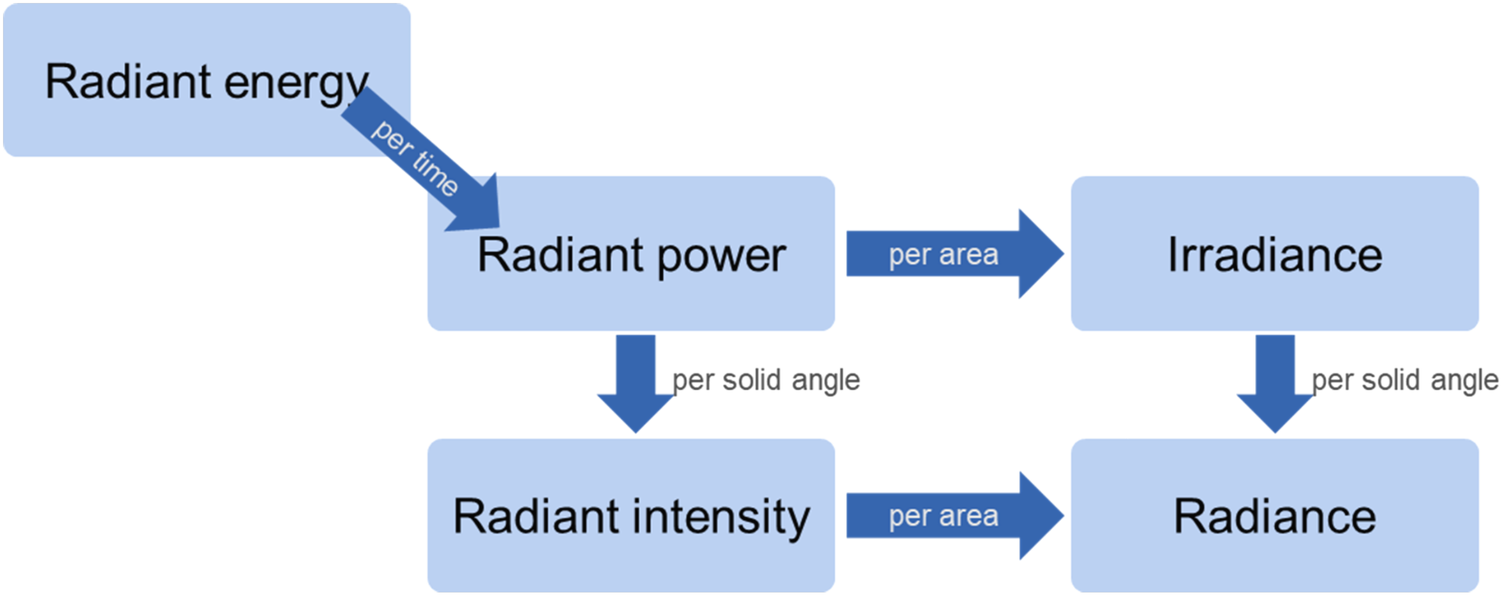
**radiance → luminance**



# Slight abuse of terminology



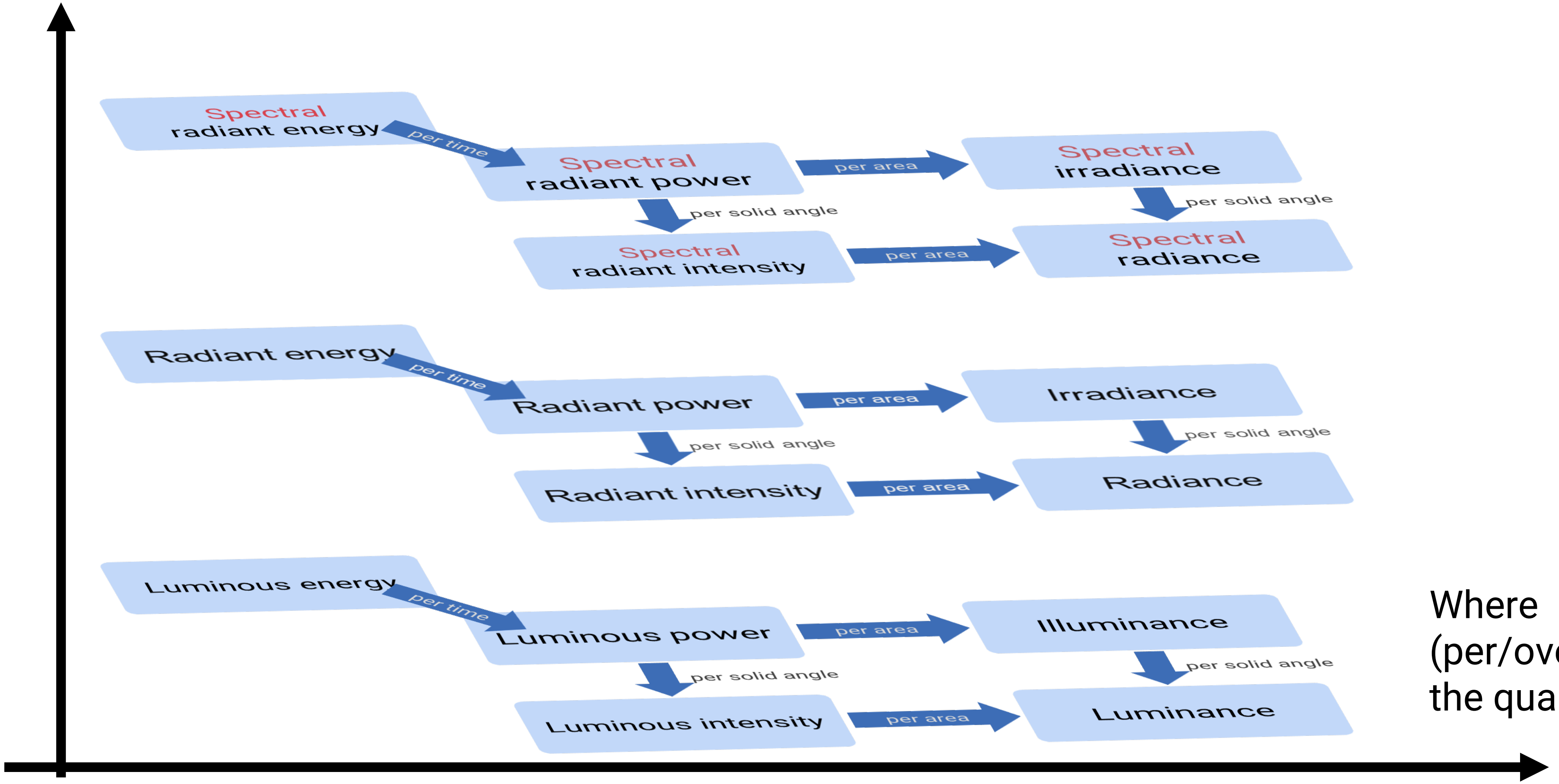
# Radiometric quantities



# Radiometric quantities

Physical energy? (per wavelength?)

Visible brightness for human?




Where
(per/over position? direction?)
the quantity is defined

# Bidirectional reflectance distribution function

***We roughly say….***

$$\text{BRDF } f_s\text{:} \frac{\text{outgoing radiance along } \omega_o}{\text{incident irradiance at } \omega_i}$$

Previous slide

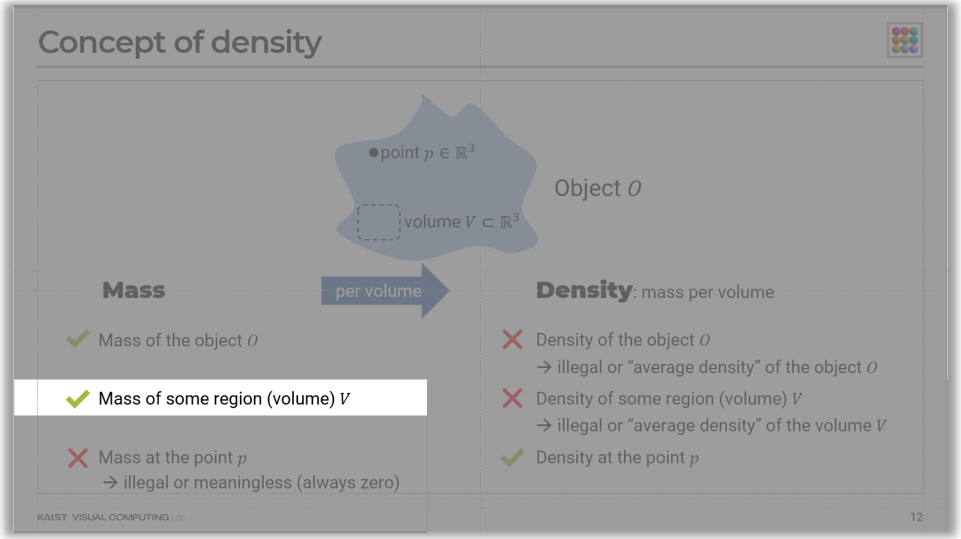
**Irradiance**

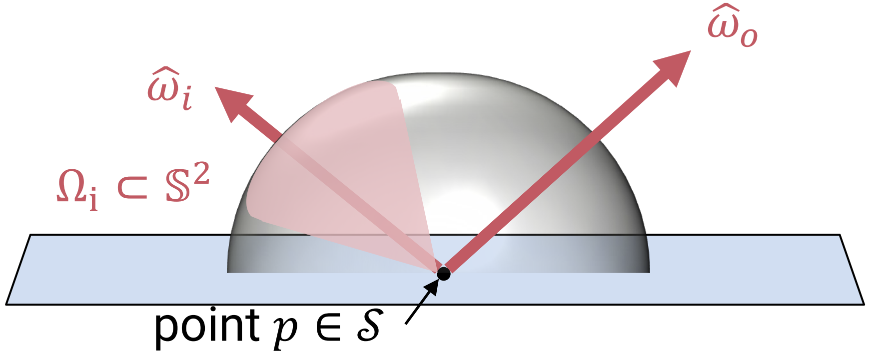
**at a point *on a surface***

***(dosen't depend on direction)***

**????**

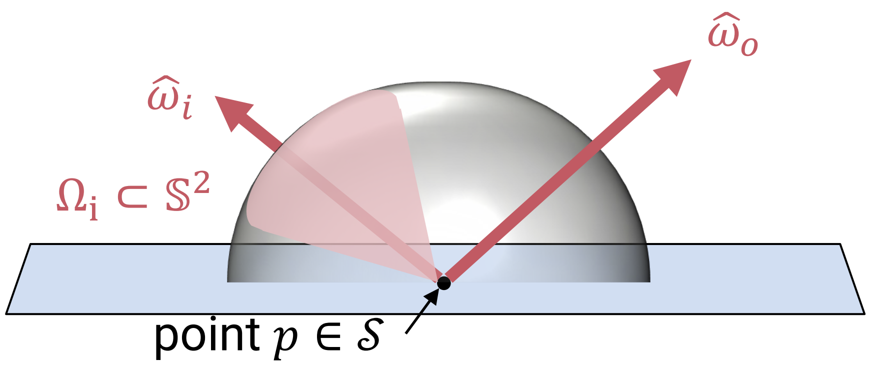
# Bidirectional reflectance distribution function




$$\text{BRDF } f_s(p, \widehat{\omega}_i, \widehat{\omega}_o) = \lim_{\substack{\text{sol.ang.}(\Omega_i) \to 0 \\ \omega_i \in \Omega_i}} \frac{L(p, \widehat{\omega}_o)}{E_{\mathcal{S}}^{(\text{in})}(p, \Omega_i)}$$

# Bidirectional reflectance distribution function




$$\text{BRDF } f_s(p, \widehat{\omega}_i, \widehat{\omega}_o) = \lim_{\substack{\text{sol.ang.}(\Omega_i)\to 0 \\ \omega_i \in \Omega_i}} \frac{L(p, \widehat{\omega}_o)}{E_{\mathcal{S}}^{(\text{in})}(p, \Omega_i)}$$

$$= \lim_{\substack{\text{sol.ang.}(\Omega_i)\to 0 \\ \omega_i \in \Omega_i}} \frac{L(p, \widehat{\omega}_o)}{\int_{\Omega_i} L^{(\text{in})}(p, \widehat{\omega}_i)\, |\hat{n} \cdot \widehat{\omega}_i| \mathrm{d}\widehat{\omega}_i} = \lim_{\substack{\text{sol.ang.}(\Omega_i)\to 0 \\ \omega_i \in \Omega_i}} \frac{L(p, \widehat{\omega}_o)}{L^{(\text{in})}(p, \widehat{\omega}_i) |\hat{n} \cdot \widehat{\omega}_i| \text{sol. ang.}(\Omega_i)}$$

# Rendering equation

$+L_e(p, \widehat{\omega}_o) \Rightarrow$**, then we get the rendering equation**

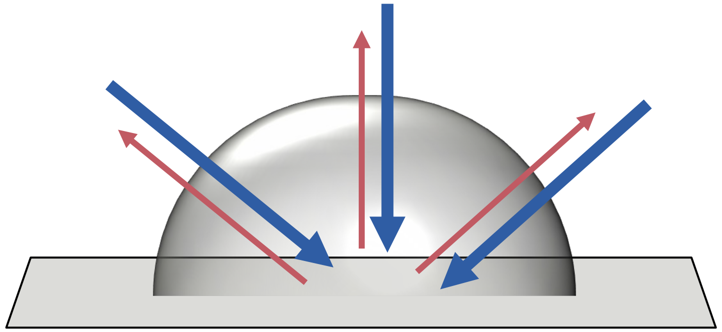
$$L^{(\text{out})}(p, \widehat{\omega}_o) = \int_{\mathbb{S}^2} L^{(\text{in})}(p, \widehat{\omega}_i) f_s(p, \widehat{\omega}_i, \widehat{\omega}_o) |\hat{n} \cdot \widehat{\omega}_i| \mathrm{d}\widehat{\omega}_i$$

per solid angle →

← over solid angle

$$f_s(p, \widehat{\omega}_i, \widehat{\omega}_o) = \lim_{\substack{\text{sol.ang.}(\Omega_i) \to 0 \\ \omega_i \in \Omega_i}} \frac{L(p, \widehat{\omega}_o)}{L^{(\text{in})}(p, \widehat{\omega}_i) |\hat{n} \cdot \widehat{\omega}_i| \, \text{sol. ang.} \, (\Omega_i)}$$

# Properties of BRDF: energy conservation



$$\frac{\text{outgoing irradiance}}{\text{incident irradiance}} \leq 1$$, for any illumination condition

$$\frac{\int_{\mathbb{S}^2} L^{(\text{out})}(p, \widehat{\omega})|\hat{n} \cdot \widehat{\omega}| \mathrm{d}\widehat{\omega}}{\int_{\mathbb{S}^2} L^{(\text{in})}(p, \widehat{\omega})|\hat{n} \cdot \widehat{\omega}| \mathrm{d}\widehat{\omega}} =$$

rendering equation

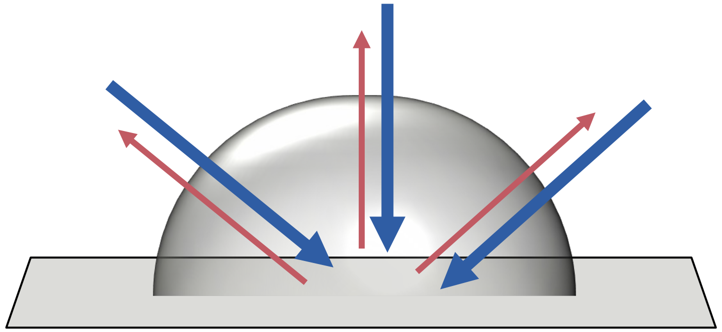
$$\frac{\int_{\mathbb{S}^2} \int_{\mathbb{S}^2} L^{(\text{in})}(p, \widehat{\omega}_i) f_s(p, \widehat{\omega}_i, \widehat{\omega}_o)|\hat{n} \cdot \widehat{\omega}_i| \mathrm{d}\widehat{\omega} \, |\hat{n} \cdot \widehat{\omega}_o| \mathrm{d}\widehat{\omega}_o}{\int_{\mathbb{S}^2} L^{(\text{in})}(p, \widehat{\omega})|\hat{n} \cdot \widehat{\omega}| \mathrm{d}\widehat{\omega}} \leq 1,$$

for **any positive function** $L^{(\text{in})}(p,\cdot)$.

Taking $L^{(\text{in})}(p,\cdot)$ as a Dirac delta function centered at $\widehat{\omega}_i$,

$$\therefore \int_{\mathbb{S}^2} f_s(p, \widehat{\omega}_i, \widehat{\omega}_o)|\hat{n} \cdot \omega_o| \mathrm{d}\widehat{\omega}_o \leq 1, \forall \widehat{\omega}_i$$

# Properties of BRDF: energy conservation



$$\frac{\text{outgoing irradiance}}{\text{incident irradiance}} \leq 1, \text{ for any illumination condition}$$

$< 1$: energy losses

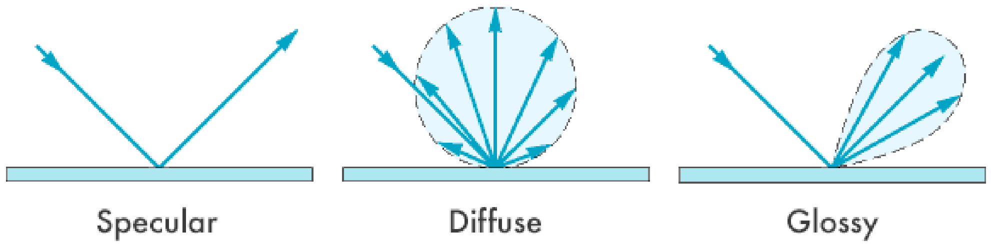
$= 1$: energy conserves

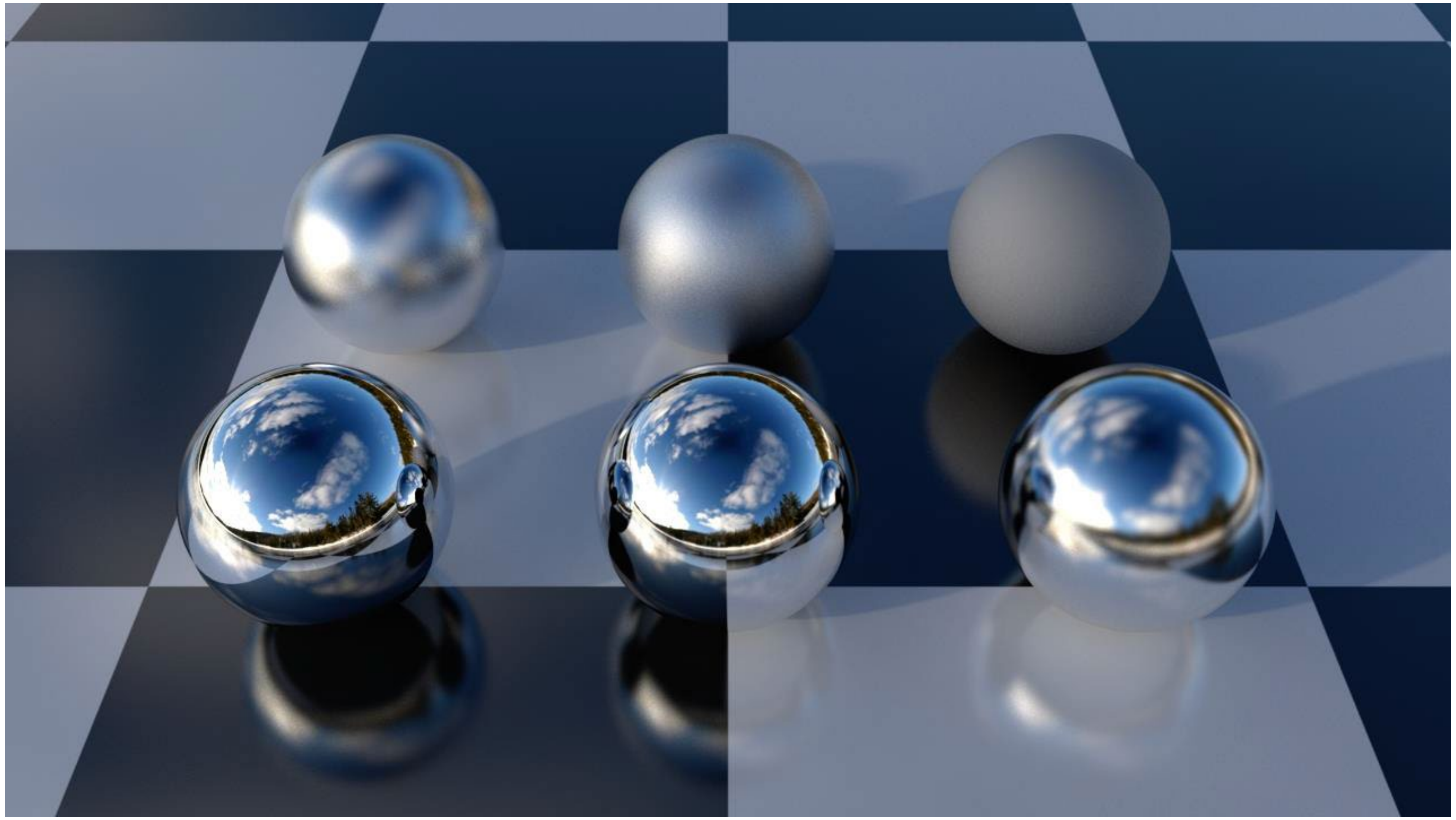
$> 1$: impossible!

**Energy Conservation**

$$\int_{\mathbb{S}^2} f_s(p, \hat{\omega}_i, \hat{\omega}_o) |\hat{n} \cdot \omega_o| \mathrm{d}\hat{\omega}_o \leq 1, \forall \hat{\omega}_i$$

# Example BRDFs



source: Keenan Crane []

# Example BRDFs

- Pure diffuse (Lambertian reflection)
  - Albedo $\rho_d$: ratio of energy conservation
  - $f_s$ is a constant function on $\mathbb{S}^2_{\hat{z}}$

$$\int_{\mathbb{S}^2_{\hat{z}}} f_s |\hat{n} \cdot \widehat{\omega}_o| \mathrm{d}\omega_o = \pi f_s = \rho_d$$

$$\therefore f_s = \frac{\rho_d}{\pi}$$

# Example BRDFs

- Pure specular
  - A Dirac delta function centered at $\mathrm{refl}_{\hat{n}}(\hat{\omega}_i)$....
  - Be careful when you treat Dirac delta functions

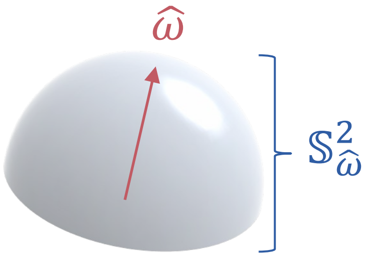
$$f_s(\widehat{\omega}_i, \widehat{\omega}_o) = a \cdot \delta_{\mathbb{S}^2}\big(\widehat{\omega}_o, \mathrm{refl}_{\hat{n}}(\widehat{\omega}_i)\big), \int_{\mathbb{S}^2_{\hat{n}}} f_s(\widehat{\omega}_i, \widehat{\omega}_o) |\hat{n} \cdot \widehat{\omega}_o| \mathrm{d}\widehat{\omega}_o = 1$$

$$\therefore\ f_s(\widehat{\omega}_i, \widehat{\omega}_o) = \frac{\delta_{\mathbb{S}^2}\big(\widehat{\omega}_o, \mathrm{refl}_{\hat{n}}(\widehat{\omega}_i)\big)}{\hat{n} \cdot \widehat{\omega}_o}$$

# Notation table

## Sets

| | |
|---|---|
| $\mathbb{R}^n$ | Euclidean space |
| $\mathbb{S}^2$ | the unit sphere (the set of all unit vectors) |
| $\mathbb{S}^2_{\hat{\omega}}$ | the hemisphere facing a direction $\hat{\omega} \in \mathbb{S}^2$ |




## Convention of variables

| | |
|---|---|
| $p \in \mathbb{R}^3$ | point in the space (or a surface) |
| $\hat{\omega} \in \mathbb{S}^2$ | direction (unit vector) |
| • $\hat{\omega}_{p_1 p_2} := \frac{p_2 - p_1}{\lVert p_2 - p_1 \rVert}$ for any $p_1, p_2 \in \mathbb{R}^3$ | |
| $\hat{n} \in \mathbb{S}^2$ | surface normal,<br>where a point and a surface are given in context |
| $\mathcal{S} \subset \mathbb{R}^3$ | surface |
| $\mathcal{V} \subset \mathbb{R}^3$ | volume |
| $\Omega \subset \mathbb{S}^2$ | solid angle (region on the unit sphere $\mathbb{S}^2$) |

## Radiometric quantities

* time dependency is omitted for simplicity
* $(\cdot\ ",\Omega")$ is usually omitted and assumed as an entire $\mathbb{S}^2$ or hemisphere

| | |
|---|---|
| $\Phi(\mathcal{S}, \Omega)$ [W] | radiant power (flux) at a surface $\mathcal{S} \subset \mathbb{R}^3$ and a solid angle $\Omega \subset \mathbb{S}^2$ |
| $I(\hat{\omega})$ [W/sr] | radiant intensity at a direction $\hat{\omega} \in \mathbb{S}^2$,<br>where a point source is given in context |
| $E(p, \Omega)$ [W/m$^2$] | irradiance at $p \in \mathcal{S}$ and $\Omega \subset \mathbb{S}^2$,<br>where the surface $\mathcal{S} \subset \mathbb{R}^3$ is given in context |
| $L(p, \omega)$ [W/m$^2$sr] | radiance at $p \in \mathbb{R}^3$ and $\hat{\omega} \in \mathbb{S}^2$ |
| $f_s(p, \omega_i, \omega_o)$ [sr$^{-1}$] | BSDF at $p \in \mathcal{S}$ from $\hat{\omega}_i \in \mathbb{S}^2$ to $\hat{\omega}_o \in \mathbb{S}^2$,<br>where the surface $\mathcal{S}$ is given in context |