STAT 3D03 Mathematical Statistics

Assignment 1 Due at **10pm** on Monday January 20, 2025.

Important Notes - Please read carefully.

- Students are reminded that submitted assignments must be **entirely their own work**. Submission of all or part of someone else's solution (including solutions from the internet or other sources) under your name is academic misconduct and will be dealt with as such. Penalties for academic misconduct can be very severe.
- You are expected to show all details of your solution. Marks will be deducted if all details are not included.
- You must submit a scan of your assignment through Crowdmark.com before the deadline. Follow the instructions on Crowdmark to show which problem is solved where in your scanned file.
- Late assignments will be assessed a penalty of 1 percentage point per hour, or part thereof, that they are late.
- It is **strongly** advised that you submit your assignment early to avoid any last-minute technical issues which would result in late submission and the penalty described above.
- You may update your submission up until the deadline. If you update your submission after the deadline, it will be marked late.
- Students with accommodations through Student Accessibility Services who require accommodations for this assignment must contact their professor **at least 48 hours before the assignment is due** to request any such accommodations.
- No other extensions to the due date and time will be given except in extreme circumstances.

Q. 1 Prove the following results

a) If A and B are two events in a sample space S then

$$P\left(A \bigcup B'\right) \;=\; 1 - P(B) + P\left(A \bigcap B\right)$$

b) If A is a subset of B (that is all outcomes in A are also in B) then $P(A) \leqslant P(B)$.

c) Show that $P(A \bigcap B) \leqslant P(A) \leqslant P(A \bigcup B)$.

Q. 2 The game Yahtzee (dice poker) is played by simultaneously rolling 5 dice and making a poker hand from the top faces of the dice. Assuming that all outcomes are equally likely, find the probability that

a) All five dice have different values.

b) There is exactly one pair in the five values.

Hint: Assume that the dice are distinguishable (e.g. different colours)

Q. 3 Suppose that A and B are two events with $P(A) = 0.3$, $P(B) = 0.5$ and $P(A \bigcup B) = 0.7$.

a) Are A and B independent?

b) What is the probability that either A or B but not both happens?

c) If exactly one of A or B happens, what is the probability that it is A?

Q. 4 There is a 40% probability of being exposed to the influenza virus during the flu season and if a person is exposed to the virus, there is a 90% probability they will actually get sick with the flu. It is impossible to get the flu without exposure to the virus. Getting a flu shot reduces the probability of getting sick if exposed to 40%. Suppose that 20% of the population get a flu shot and getting a flu shot is independent of exposure to the virus.

a) What is the probability that a random person in the population gets sick with the flu?

b) If a person does get sick with the flu, what is the probability that they got did not get a flu shot?