~~the point is that [Fe/H] (or equivalently, [Z/H]) does not map directly to a given $\Delta$ or $Z$.~~

Note $$[Fe/H] = [Z/H] = \log\left(\frac{Z}{Z_{\odot,pro}}\right) - \log\left(\frac{X}{X_{\odot,pro}}\right) \neq \log(Z/Z_\odot) \quad (!)$$

$$= \log\left(\frac{Z}{X}\right) - \log\left(\frac{Z_{\odot,\text{protosolar}}}{X_{\odot,\text{protosolar}}}\right)$$

subscript $\odot$ means protosolar metal and hydrogen mass frac (not present-day photosphere abundances)

where $X = 1 - Y - Z$,

$$Y = Y_p + \underbrace{\left(\frac{Y_{\odot,\text{protosolar}} - Y_p}{Z_{\odot,\text{protosolar}}}\right)}_{=\,\Delta} Z$$

so in fact

$$[Z/H] = \log\left(\frac{Z}{Z_{\odot,\text{protosolar}}}\right) - \log\left(\frac{1-Y-Z}{X_{\odot,\text{protosolar}}}\right)$$

$$= \log\left(\frac{Z}{Z_{\odot,\text{protosolar}}}\right) - \log\left(\frac{1}{X_{\odot,pr}}\left[1 - Y_p - \left(\frac{Y_{\odot,pro} - Y_p}{Z_{\odot pro}}\right)Z - Z\right]\right)$$

$$\boxed{[Z/H] = \log\left(\frac{Z}{Z_{\odot,pro}}\right) - \log\left(\frac{1}{X_{\odot pro}}\left[1 - Y_p - Z\left(1 + \underbrace{\frac{Y_{\odot,pro} - Y_p}{Z_{\odot,pro}}}_{=\,\Delta}\right)\right]\right)}$$

so yes, given $Z$, you do get a definite $[Z/H]$ (1)

(& vice versa: given [Fe/H], get $Z$:)

$$[Z/H] = \log\left(\frac{Z}{Z_{\odot,pro}}\frac{X_{\odot,pro}}{X}\right)$$

$$= \log\left(\frac{Z}{Z_{\odot,pro}}\frac{X_{\odot,pro}}{(1-Y_p - Z(1+\Delta))}\right) \quad \text{so}$$

$$10^{[Fe/H]}\frac{Z_{\odot,pro}}{X_{\odot,pro}}\left(1 - Y_p - Z(1+\Delta)\right) = Z$$

rearranging . . .

$$10^{[Fe/H]} Z_{\odot,pro}(1 - Y_p - Z(1+\Delta)) = X_{\odot,pro} Z$$

$$10^{[Fe/H]} Z_{\odot,pro}(1 - Y_p) = Z\left(X_{\odot,pro} + (1+\Delta) Z_{\odot,pro} 10^{[Fe/H]}\right)$$

$$\boxed{Z = 10^{[Z/H]} \frac{Z_{\odot,pro}(1-Y_p)}{X_{\odot,pro} + (1+\Delta) Z_{\odot,pro} 10^{[Z/H]}}} \quad (2)$$

Limiting case: $[Z/H] = 0$. Then

$$Z = \frac{Z_{\odot,pro}(1-Y_p)}{X_{\odot,pro} + (1+\Delta) Z_{\odot,pro}} = \frac{Z_{\odot,pro}(1-Y_p)}{X_{\odot,pro} + \left(\frac{Y_{\odot,pro} - Y_p}{Z_{\odot,pro}} + 1\right) Z_{\odot,pro}}$$

$$= \frac{Z_{\odot,pro}(1-Y_p)}{X_{\odot,pro} + Y_{\odot,pro} - Y_p + Z_{\odot,pro}}$$

note $X + Y + Z = 1$ regardless of subscript

$\Rightarrow Z = Z_{\odot,pro}$ good

so the grid I did ~~a~~ was

$$Z = 10^{-1} Z_\odot,\ 10^{-0.75} Z_\odot,\ 10^{-0.5} Z_\odot,\ 10^{-0.25} Z_\odot,\ Z_\odot,$$
$$10^{0.25} Z_\odot,\ 10^{0.5} Z_\odot$$