Modelling a Decentralized Constraint Satisfaction Solver for Collision-Free Channel Access

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***Abstract*—The problem of distributed radio resource assignment in wireless networks is a constraint satisfaction problem that needs to be solved in a distributed fashion. Using as a reference example a protocol to allocate channel slots to contending stations in a multiple access scenario, a particular solver that iteratively uses a stochastic search until it converges to a solution is considered. The convergence process of the solver is modeled by an absorbing Markov chain (MC), and analytical, closed-form expressions for its transition probabilities are derived. The expected number of steps required to reach a solution is found by calculating the MC's expected number of steps to absorption. The analysis is validated by means of simulations and the model is extended to account for the presence of channel errors.**

***Index Terms*—Decentralized constraint satisfaction problem solver, Markov Chain model, collision-free operation, medium access control**

I. Introduction

SINCE the inception of wireless local area networks (WLANs), random medium access mechanisms have played a key role in arbitrating the access to the shared channel. The core principles of the medium access control (MAC) that were introduced in the first release of the IEEE 802.11 standard are still valid today [1]. The contenders for the channels use carrier sense to prevent stations from interrupting other transmissions. Slotted time combined with a backoff counter is used to reduce the chances that two stations simultaneously start a transmission. Until today, this backoff counter has been initialized with a random value. Stations separate two transmission attempts by a random backoff, to give the opportunity to other stations to transmit.

Recently, it has been pointed out that the random choice of the backoff value is not necessary after successful transmissions [2]. In fact, if all the nodes that have successfully transmitted choose a common deterministic backoff value for their next transmission, the chances of collisions are reduced, since in their next transmission they may only collide with the remaining unsuccessful nodes. Furthermore, under certain idealistic conditions, a collision-free operation can be reached.

The idea of using a deterministic backoff after successful transmissions has been explored in more detail in, e.g., [3], [4], [5], [6]. The goal of this class of protocols is to distributively build a collision-free schedule which will then repeat periodically. This translates into the decentralized assignment of stations to slots within one period of the schedule in such a way that there is no slot that contains more than one station.

Similar problems can be found in other areas of networking. Examples are provided in [7], which offers a general framework that encompasses problems such as graph colouring, channel assignment to WLANs cells, the search for feasible inter-flow codes in network coding, and the construction of collision-free schedules in CSMA networks. The same paper also describes a solver and characterizes it by providing a bound that describes the convergence properties of the solver.

We are interested in this convergence process, and in particular in forecasting the expected number of steps required to reach a solution. To this end, an absorbing Markov Chain (MC) model can be used to model the solver. The first contribution of this paper is the derivation of closed-form expressions to compute the transition probabilities of the absorbing MC. The second contribution is the adaptation of the model to an environment in which errors can occur.

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II. Problem Statement and Related Work

The random assignment of channel resources to wireless stations appears repeatedly in the literature (e.g., [8], [9]). This is roughly equivalent to allowing the wireless stations to randomly choose among the existing resources. Obviously, it may happen that the same resource is assigned to more than one station or that more than one station randomly chooses the same resource. This event is called a collision. The stations involved in a collision are called colliding stations while we refer to the remaining stations as successful stations. The probability of obtaining δ successes given N stations and B resources is computed in [8].

Recently, the initial idea of letting the stations randomly select one of the possible resources has been enhanced to contemplate several rounds (See [5] and references therein). In a first round, the stations randomly choose one of the possible resources. Those stations that succeed stick to the chosen resource while the stations that collide randomly choose one of the possible resources again. At some point, if the number of stations is not larger than the number of resources, a solution is reached in which each of the stations has its own resource and there are no collisions. Collisions seriously impair the network performance, specially when they trigger the auto rate fallback (ARF) mechanism [10], [11].

The assignment of channel resources to wireless stations is just an instance of a constraint satisfaction problem (CSP). A CSP with N variables is defined as follows (adopting the notation from [7]). We consider N variables, $\mathbf{x} := (x_1, \dots, x_N)$, with $x_i \in \mathbf{B} = \{1, \dots, B\}, \forall i$, and M clauses, $\{\Phi_1(\mathbf{x}), \dots, \Phi_M(\mathbf{x})\}$, that are boolean functions. The M clauses represent the constraints and take a value equal to 1 if the constraint is satisfied and 0 otherwise. An assignment $\mathbf{x}$ is a solution to the problem if all the constraints are satisfied.

In radio resource allocation problems, it is natural that each of the variables represents one of the wireless stations, and that $\mathbf{B}$ represents the available resources. It is likely that the different stations cannot communicate until the CSP is solved. Therefore, it is necessary to find a solution in a distributed fashion, thus the problem becomes a decentralized CSP [7].

Examples of channel resources that can be assigned using decentralized CSP solvers include channel time slots [2], frequency channels [7] and code division multiple access scramble codes [13]. The solution described above is an instance of a decentralized CSP, which provably finds a solution in finite time and exhibits a performance comparable to well known CSP solvers such as WalkSAT [12].

Even though the literature on CSP is vast, and comprises different families of solvers as explained in [7], the concept of decentralized CSP solvers is very recent and it is yet to be explored in depth.

The solver presented in [7] accepts two parameters to adjust the learning process in the search for a solution. For a particular value of these parameters, it is easy to model the solving process as a MC (see, e.g., [6], [3], [5]). The computation of the transition probabilities from the starting state is the well-known problem of randomly assigning stations to resources mentioned above. However, the transition probabilities from the other states need to take into account that only a fraction of the stations randomly select the resource, while the others make a deterministic decision.

Compared to previous work, the contribution of this paper is the derivation of closed form expressions for those transition probabilities. Furthermore, we extend the Markov Chain to account for the presence of errors, taking into account that the contending stations, after an unsuccessful transmission, cannot differentiate between a collision or a channel error. In the following section, we detail the reference scenario and the reference protocol that are the CSP problem and decentralized CSP solver which we consider in the remainder of the paper.

III. The Reference Scenario and Reference Protocol

Our reference scenario is the distributed assignment of N contending wireless stations to B channel time slots. In each slot a single transmission can be completed and acknowledged. From a station's perspective, there are two possible outcomes for every transmission. The station succeeds if no other station is assigned the same slot and its transmission is acknowledged in the same slot. If at least one other station picks the same slot, all stations involved will suffer a collision, and the transmission is unsuccessful, in which case no acknowledgment will be received.

For this example, the variables $\{x_i\}_{i=1}^{N}$ in the corresponding CSP are the slots chosen by the N contending wireless stations from the set of available slots $\mathbf{B}$. There is one clause per pair of variables evaluating if they have the same value or not. The clause will return 0 if the two participating stations have selected the same slot (i.e., a collision) and returns 1 otherwise.

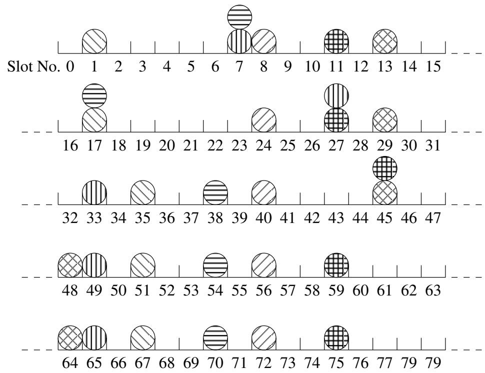

Fig. 1. CSMA/ECA contention

A. The Reference Protocol

In this section, we describe a protocol that distributively solves the problem posed in the previous section, whenever a solution exists. The contention is organized in transmission rounds that contain B transmission slots. In every round, each of the N stations transmit exactly once. A solution to the problem is an assignment in which there is no slot containing more than one wireless stations. The protocol describes how the stations pick their transmission slots in each of the transmission rounds, taking into account that each station is only aware of the outcome of its own transmission in the previous round.

The protocol works as follows. In the first round, each station randomly and independently picks one of the B possible slots in the round. If the transmission is successful, the station will pick exactly the same slot in the next round. Otherwise, it will pick one of the B slots in the next time randomly. This process repeats until all stations successfully transmit in the same round, from which point on, all the stations will transmit periodically and no collisions will occur. The operation of this protocol is illustrated in Fig. 1 for five consecutive rounds. In this example, a collision-free solution is reached in the fourth round, after which the same schedule can be repeated endlessly without further collisions.

This protocol is a simplified version of a variant of CSMA/CA, called CSMA with enhanced collision avoidance (CSMA/ECA). A detailed explanation of CSMA/ECA can be found in, e.g., [2], [14], [3], [5].

The reference protocol can be viewed as a distributed solver for the corresponding CSP defined in the previous section. In the first round, the variables x_i are assigned a value in $\{1, \cdots, B\}$ randomly, with probability $1/B$. Then the constraints are evaluated. Those variables that are involved in constraints that are not satisfied take a random value again in the next round, while the other variables keep the same value as in the previous round. When a solution is reached, all the variables keep the same value. This solver is, in fact, an instance of the parametrized solver proposed in [7] with the parameter values set to $a = b = 1$. An attractive property of this decentralized solver is that it reaches the solution in finite time, if it exists, and its performance is comparable to the known centralized solvers.

IV. The Markov Chain Model

We are interested in calculating the expected number of rounds required to reach a solution. To this end, we construct a Markov chain to model the behavior of the DSC solver and use the reference protocol to explain it in a more tangible manner. By the pigeonhole principle, a solution exists only when $N \leq B$, i.e., when there are at least as many slots in a round as the total number of stations. Considering $N \leq B$ contending stations in the reference protocol, the associated MC model has $N+1$ different states, $S_0, \ldots, S_N$. The system is in state S_d if exactly d stations ($0 \leq d \leq N$) were successful in the previous round and, therefore, will deterministically choose their transmission slot in the current round. From the CSP perspective, this is equivalent to saying that there are exactly d variables that were not involved in any constraint that was not satisfied in the previous round.

We are interested in the computation of the transition probability, $p_{d,\delta}^{B,N}$, from one state S_d to another state S_δ, $0 \leq \delta \leq N$. In other words, $p_{d,\delta}^{B,N}$ is the probability of obtaining δ successful transmissions given N stations and B slots when d of the stations use a deterministically chosen slot while the remaining $N-d$ stations transmit in a randomly chosen slot.

Note that the considered MC is an absorbing MC, as $p_{N,N}^{B,N} = 1$. This is because, once a collision-free schedule is found, the same collision-free schedule is repeated in every subsequent step. Since the MC is absorbing, the expected number of steps before convergence can be computed if the values of $p_{d,\delta}^{B,N}$ are known [15].

A. Calculating the Transition Probabilities

To calculate the transition probabilities, we number the stations from 1 to N for convenience and define A_i to be the event that station i succeeds, and the set $\mathbf{A} = \{A_i\}_{i=1}^N$ to be the collection of all such events. These events are partially overlapping since more than one station may successfully transmit. For a given d, the transition probability $p_{d,\delta}^{B,N}$ is the probability that exactly δ out of the N events in $\mathbf{A}$ happen. As mentioned before, when $d = N$, the system is in the absorbing state and $p_{d,\delta}^{B,N} = 1$ when $\delta = d$, and it is zero otherwise. When $d < N$, this probability can be calculated applying a generalized version of the inclusion-exclusion principle (see, e.g., the theorem in Sec. IV.3 of [16]) as follows:

$$p_{d,\delta}^{B,N} = \sum_{j=\delta}^{N} (-1)^{j+\delta} \binom{j}{\delta} S(j), \tag{1}$$

where $S(j)$ is given by

$$S(j) = \sum_{\forall \mathbf{A}^j \subseteq \mathbf{A}} \Pr\left\{ \bigcap_{A_i \in \mathbf{A}^j} A_i \right\}. \tag{2}$$

Here $\mathbf{A}^j$ denotes a subset of $\mathbf{A}$ that has exactly j elements, i.e., $|\mathbf{A}^j| = j$. Therefore, $\Pr\left\{\bigcap_{A_i \in \mathbf{A}^j} A_i\right\}$ is the probability that all the j stations represented in $\mathbf{A}^j$ successfully transmit. Note that $S(j)$ is a sum of probabilities, but it is not itself a probability.

For a given set of j tagged stations in $\mathbf{A}^j$, the probability $\Pr\left\{\bigcap_{A_i \in \mathbf{A}^j} A_i\right\}$ depends on k, the number of deterministic stations within the j tagged stations. The j tagged stations succeed if the $j-k$ random stations among them choose unoccupied slots, and the remaining $N-d-(j-k)$ untagged random stations choose slots that are different from the ones selected by the j tagged stations. The first event occurs with probability

$$\binom{B-d}{j-k} \frac{(j-k)!}{B^{j-k}},$$

and the second with probability

$$\left(\frac{B-j}{B}\right)^{N-d-(j-k)}.$$

When $j = N$, we have $k = d$, and therefore, there are no untagged random stations, hence the second probability is 1. Consequently, the probability that all of the j stations of $\mathbf{A}^j$ succeed, given that k of them are deterministic, after some simplification is

$$\Pr\left\{ \bigcap_{A_i \in \mathbf{A}^j} A_i \right\} \tag{3}$$

$$= \begin{cases} \dfrac{(B-d)!(B-j)^{N-d-(j-k)}}{(B-d-(j-k))!\ B^{N-d}}, & j < N \\[2ex] \dfrac{(B-d)!}{(B-N)!\ B^{N-d}}, & j = N \end{cases}$$

For any given j, there are $\binom{d}{k}\binom{N-d}{j-k}$ sets $\mathbf{A}^j$ with k deterministic stations. Furthermore, the number of deterministic stations, k, among the j tagged stations is bounded by

$$\max(0, j+d-N) \leq k \leq \min(d, j), \tag{4}$$

since in a set of j nodes, there cannot be more deterministic stations than the total number of deterministic stations ($k \leq d$), or more random stations than the total number of random stations ($j-k \leq N-d$).

Using (2), (3), and (4), $S(j)$ can be calculated as

$$S(j) = \sum_{k=\max(0,j+d-N)}^{\min(d,j)} \binom{d}{k}\binom{N-d}{j-k} \times \frac{(B-d)!(B-j)^{N-d-(j-k)}}{(B-d-(j-k))!\ B^{N-d}}, \quad j < N \tag{5}$$

and for $j = N$,

$$S(N) = \frac{(B-d)!}{(B-N)!\ B^{N-d}}. \tag{6}$$

Finally, the transition probabilities for $d < N$ can be calculated by replacing $S(j)$ in (1).

B. Calculating the Number of Steps until Absorption

To compute the expected number of rounds needed for the solver to reach a solution, we calculate the expected number of transitions that the MC takes to reach the absorbing state S_N (see, e.g., [15] for the theory behind this calculation).

Let $\mathbf{P}^{B,N}$ be the transition probability matrix of the MC. This matrix is a square matrix of size $N+1$. If we number the rows and columns of $\mathbf{P}^{B,N}$ starting with zero, the element in row d and column δ is simply $\left[\mathbf{P}^{B,N}\right]_{d,\delta} = p^{B,N}_{d,\delta}$ as in (1). In this matrix, rows 0 to $N-1$ represent transitions from the transient states and row N the transitions from the absorbing state. Therefore, $\mathbf{P}^{B,N}$ has the following general form:

$$\mathbf{P}^{B,N} = \begin{array}{c c} & \begin{array}{cc} \text{TR} & \text{ABS} \end{array} \\ \begin{array}{c} \text{TR} \\ \\ \text{ABS} \end{array} & \left(\begin{array}{c|c} \mathbf{Q}_{(N\times N)} & \mathbf{c}_{(N\times 1)} \\ \hline \mathbf{0}_{(1\times N)} & 1 \end{array}\right) \end{array} \tag{7}$$

where $\mathbf{Q}$ is a matrix containing the first N rows and columns of $\mathbf{P}^{B,N}$, from which we calculate the fundamental matrix of the absorbing MC as $\mathbf{N} = (\mathbf{I}_{N\times N} - \mathbf{Q})^{-1}$, where $\mathbf{I}_{N\times N}$ is the $N \times N$ identity matrix. The expected number of steps to absorption, if the system starts in state S_0, is the sum of all the elements in the first row of $\mathbf{N}$.

C. The Markov Chain in the Presence of Channel Errors

So far we have not considered the possibility that the channel introduced errors. In the case of channel error, a transmission may be unsuccessful even if it has not suffered a collision. In fact, after an unsuccessful transmission, a wireless station does not know whether it as suffered a collision or a channel error, and the response of the protocol is exactly the same: move the station back to the random behaviour.

The probability of moving from the state S_d to the state S_δ when a channel error happens with probability ϵ is

$$p^{B,N,\epsilon}_{d,\delta} = p^{B,N}_{d,\delta}(1-\epsilon)^\delta + \sum_{i=\delta+1}^{N} \binom{i}{\delta} \epsilon^{i-\delta}(1-\epsilon)^\delta p^{B,N}_{d,i}. \tag{8}$$

In other words, the probability that there are exactly δ successful stations is the probability that there are δ stations that do not collide and none of them suffers a collision plus the probability that there are i ($\delta < i \leq N$) stations that do not collide and exactly $i-\delta$ of those stations suffer a collision. Note that the resulting MC is no longer absorbing.

D. Validation

The expressions presented in this section have been validated by means of a custom simulator that captures the reference scenario and protocol presented in Section III. We do not include the plots in this letter to conserve space. They are available in the technical report [].

V. Conclusion

In this paper we have studied a decentralized CSP solver to assign channel slots to contending stations which eventually converges to collision-free operation under ideal channel conditions.

We have modeled the convergence process as a MC and have derived closed expressions for the transition probabilities. These values can be used to compute the expected number of steps required for the system to converge to a solution.

We have also considered the possibility of the presence of channel errors and constructed the MC that accounts for channel errors, which is no longer an absorbing MC.

The presented results have been validated by means of simulation.

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