#### Workshop: Endo-p-permutation and trivial source modules

**ABSTRACTS** 

SEMESTER - Local representation theory and simple groups

August 29-September 2, 2016

EPFL / CIB, ECUBLENS, CH

### Monday.

15:00-16:00: Registration

16:00-17:00: Jacques Thévenaz [EPF Lausanne]

17:15-17:45: William O'Donovan [Royal Holloway University of London]

## Tuesday.

**09:00 - 10:00**: Jon Carlson [University of Georgia, Athens]

**10:40-11:30**: Eugenio Giannelli [TU Kaiserslautern]

11:40-12:10: Benedetta Lancellotti [Università degli Studi di Milano Bicocca]

**14:30-15:30 & 16:00-17:00**: Jesper Grodal [Københavns Universitet]

### Wednesday.

09:00 - 10:00: Dan Nakano [University of Georgia, Athens]

10:40-11:30: Jürgen Müller [Friedrich-Schiller-Universität Jena]

**11:40-12:10**: Alec Gullon [University of Lancaster]

14:00-17:45: Discussion Session

#### Thursday.

09:00 - 10:00: Nadia Mazza [University of Lancaster]

10:40-11:30: Susanne Danz [KU Eichstätt-Ingolstadt]

11:40-12:10: Deniz Yilmaz [UC Santa Cruz]

14:00-17:45: Discussion Session

17:45 — : Apéro

## Friday.

09:00 - 10:00: Shigeo Koshitani [University of Chiba]

**10:40-11:30**: Rob Carman [UC Santa Cruz]

11:40-12:10: Mehmet Arslan [Boğaziçi University]

## Mehmet Arslan [Boğaziçi University]

Composition Factors of the p<sup>n</sup>-fibered Biset Functor of Complex Characters

**ABSTRACT**. Bisets and biset functors have been significant tools in representation theory of finite groups within last two decades. We study fibered bisets and fibered biset functors, which can be considered as a generalization of the former ones. Burnside Groups, representation rings, the ring of trivial source modules are some of main examples of fibered biset functors under suitable conditions. In this talk, I will introduce the vector space of complex valued class functions a fibered biset functor, and present the structure of this functor when the fiber group is the cyclic group of all  $p^n$ -th roots of unity for a fixed prime number p and a fixed natural number n.

## Jon Carlson [University of Georgia, Athens]

Endotrivial modules for groups of Lie type

**ABSTRACT**. This talk will be a survey of joint work with Nadia Mazza and Dan Nakano on the classification of endotrivial modules over groups of Lie type in a nondefining characteristic.

# Rob Carman [UC Santa Cruz]

Tensor Induction for Trivial Source Rings and their Ghost Rings

**ABSTRACT**. Given two finite groups G, H and a right-free (G, H)-biset, we define a tensor induction construction that gives a multiplicative functor from the category of FH-modules to the category of FG-modules, where F is some field. We show how this induces a multiplicative function between the trivial source rings of H and G over F, and then we show how to extend this function to the level of ghost rings of the trivial source rings.

# Susanne Danz [KU Eichstätt-Ingolstadt]

Simple Specht Modules and Signed Young Modules

**ABSTRACT.** In this talk I shall report on joint work with Kay Jin Lim (NTU Singapore).

Specht modules as well as (signed) Young modules are well known to be of central importance for the representation theory of the symmetric groups. By work of D. Hemmer, every simple Specht module of a finite symmetric group over a field of odd characteristic is a signed Young module. While Specht modules are parametrized by partitions, indecomposable signed Young modules are parametrized by certain pairs of partitions. In this talk we shall establish the signed Young module labels of simple Specht modules. From this we, in particular, obtain the Green vertices, Green correspondents, cohomological varieties, and complexities of simple Specht modules of symmetric groups over fields of positive characteristics.

#### Eugenio Giannelli [TU Kaiserslautern]

Foulkes modules and decomposition numbers of symmetric groups.

**ABSTRACT**. The family of trivial source modules of the symmetric group known as Foulkes modules has been studied for some decades, in connection with a long standing open conjecture of H. Foulkes. In this talk I will present new results on the modular structure of Foulkes modules. I will also show how to use these information to determine entire columns of the decomposition matrix of symmetric groups. This talk is based on my PhD thesis and on more recent joint works with Mark Wildon and Susanne Danz.

## Jesper Grodal [Københavns Universitet]

Endotrivial modules via homotopy theory

**Abstract**. The purpose of my talks will be to explain my recent preprint arXiv:1608.00499. In the preprint I show how the group of "Sylow-trivial" kG-modules can be effectively calculated using homotopy theory, for G a finite group, where a module is Sylow-trivial if it restricts to trivial plus projective on the Sylow p-subgroup. The group of Sylow-trivial modules is a subgroup of the group of endotrivial modules, and is often denoted  $T_k(G,S)$  or K(G). It is the remaining unknown part in a classification of endotrivial modules for all finite groups. I prove that this group is isomorphic to the first cohomology group on the orbit category of non-trivial p-subgroups with values in  $k^x$ . With this as outset, I get a number of structural formulas for this group: I prove the Carlson-Thevenaz conjecture, giving a formula for the group of Sylow-trivial modules only in terms of the p-local structure of normalizers of p-subgroups in the group. I also give other exact sequence descriptions of this group, which e.g., show that if the p-subgroup complex is simply connected then all Sylow-trivial modules are of dimension one, and provide other formulas in terms of the p-fusion system of G. The results can effectively be used for calculations for specific groups, as I will e.g., demonstrate with the Monster.

### **Alec Gullon** [University of Lancaster]

Bounding the dimension of the Scott module

**ABSTRACT**. Given a finite group G and a subgroup H containing a Sylow p-subgroup P, the Scott Module is a p-permutation module which is defined to be the unique summand of the permutation module k[G/H] containing a copy of the trivial module in its socle. Although this module is known to exist, actually constructing it in specific cases is an extremely difficult problem. We shall present a result which allows us to provide an upper bound on the dimension of the Scott module in terms of so-called 'set decompositions' for the group G and show how this links to examples involving classical groups.

## Shigeo Koshitani [University of Chiba]

Yet small 1-PIMs

**ABSTRACT**. We will discuss improvements of a few results in [Koshitani-Kuelshammer-Sambale, Math. Proc. Cambridge Phil. Soc.156 (2014), 555-570]. This is joint work with Juergen Mueller.

#### Benedetta Lancellotti [Università degli Studi di Milano Bicocca]

Canonical sections in the representation theory of finite groups

**ABSTRACT.** In a paper of Robert Boltje, the author introduced a canonical induction formula for the Grothendieck rings and the representation rings of a finite group G. In a preprint, Shigeo Koshitani and Thomas Weigel tried to link McKay's conjecture for a finite group G and a prime number p with indecomposable linear source OG-modules with maximal vertex, where O is a suitable discrete valuation ring of characteristic O with residue field F of characteristic O and quotient field O. In this context the canonical maps O are the O-representation and the O-representation rings respectively, while O and O are the O-representation and the O-representation rings respectively, while O and O are the O-representation rings of trivial and linear source O-representations and their link with the species of the representation rings and the O-representation rings.

### Nadia Mazza [University of Lancaster]

*Invertible modules for discrete groups.* 

**ABSTRACT.** Given a commutative noetherian ring k of finite global dimension and a discrete group G, subject to certain conditions, we introduce the concept of *invertible* kG-module in an attempt to generalise the endotrivial modules for finite groups. In the first part of the talk we will present the categorical framework which we (want to) use, before giving the definition and a few results. (Joint with Peter Symonds)

### Jürgen Müller [Friedrich-Schiller-Universität Jena]

Computing with Trivial-Source Modules

**ABSTRACT**. Trivial-source modules feature prominently in modular representation theory of finite groups. They are a rich class of modules, but at the same time sufficiently well-behaved to allow for efficient computational treatment. We discuss ideas, how these modules can be explicitly handled, both using character-theoretic and module-theoretic methods.

## Daniel K. Nakano [University of Georgia, Athens]

Endotrivial Modules for Finite Group Schemes

**ABSTRACT**. It is well known that if *G* is a finite group then the group of endotrivial modules is finitely generated. Much less is known when one considers endotrivial modules over arbitrary finite group schemes. In this talk I will present results on endotrivial modules over arbitrary finite group schemes. Our results can be applied to computing the endotrivial group for several classes of infinitesimal group schemes which include the Frobenius kernels of parabolic subgroups, and their unipotent radicals (for reductive algebraic groups).

For G reductive, we also present a classification of simple, induced/Weyl and tilting modules (G-modules) which are endotrivial over the Frobenius kernel  $G_r$  of G.

At the end of the talk, open conjectures will be presented. This is joint work with Jon Carlson.

## William O'Donovan [Royal Holloway University of London]

Conjugation Modules for Symmetric Groups

**ABSTRACT**. The structure of Foulkes modules for the symmetric group over a field of characteristic p has been an active area of recent research, with surprising applications to determining decomposition numbers of the symmetric group. In this talk, I will introduce a related family of trivial source modules for the symmetric group  $S_{pn}$ , the so-called conjugation modules, obtained by inducing the trivial module for the wreath product  $C_p \wr S_n$  to  $S_{pn}$ . I will discuss my work on identifying the vertices of summands of conjugation modules, as well as possible applications.

## Jacques Thévenaz [EPFL]

Endo-trivial modules and weak homomorphisms

**ABSTRACT**. Endo-trivial modules play an important role among representations of a finite group G over a field k of characteristic p. The equivalence classes of endo-trivial modules for G form an abelian group T(G) which one hopes to understand. If P is a Sylow p-subgroup of G, the structure of T(P) is known and one defines K(G) to be the kernel of the restriction map from T(G) to T(P). The abelian group T(G) is finite, but its structure remains unknown in general. By a recent result of Balmer, T(G) is isomorphic to the group T(G) of all weak T(G) homomorphisms from T(G) to T(F). The talk will give an introduction to the subject and present some results about T(G) and T(G). This is joint work with Jon Carlson.

# Deniz Yilmaz [UC Santa Cruz]

Fibered p-biset functor of the fibered Burnside rings

**ABSTRACT.** In representation theory, group actions on sets plays a crucial role. Given finite groups G and H and an abelian group A, Boltje and Coşkun combined the notions of (G,H)-bisets and A-fibered G-sets and introduced A-fibered (G,H)-bisets. In this talk, our aim is to extend the results of Bouc and Thévnaz on Burnside functors and to determine the composition factors of the A-fibered Burnside functor  $kB^A$  of p-groups over a field k of characteristic q with  $q \neq p$ . We show that if A is a cyclic p-group, then  $kB^A$  is uniserial. Moreover, we also show that the parametrization of simple composition factors of  $kB^A$  depends only on the prime p and the characteristic q, and not on the particular fiber group A. This is joint work with Coşkun.