
COHOMOLOGY OF GROUPS — EXERCISE SHEET 8**TU KAISERSLAUTERN****JUN.-PROF. DR. CAROLINE LASSUEUR****BERNHARD BÖHMLER**Due date: **Monday, 14th of June 2021, 18:00****SS 2021**

Throughout these exercises (G, \cdot) denotes a group.

EXERCISE 1

Assume $G = \langle g \rangle$ is an infinite cyclic group. Prove that $0 \longrightarrow \mathbb{Z}G \xrightarrow{m_{g^{-1}}} \mathbb{Z}G$ is a free resolution of the trivial $\mathbb{Z}G$ -module, and

$$H^n(G, A) = \begin{cases} A^G & \text{if } n = 0, \\ A/\text{Im}(m_{g^{-1}}) & \text{if } n = 1, \\ 0 & \text{if } n \geq 2. \end{cases}$$

EXERCISE 2

Let A be a $\mathbb{Z}G$ -module. Prove that $\text{Der}(G, A) \cong \text{Hom}_{\mathbb{Z}G}(IG, A)$ via the map sending a derivation d to the homomorphism \tilde{d} such that $\tilde{d}(g - 1) = d(g)$, $\forall g \in G \setminus \{1\}$.

EXERCISE 3

Let A be a left $\mathbb{Z}G$ -module, and let $A \rtimes G$ be the semi-direct product of $(A, +)$ by (G, \cdot) , that is, the group of all pairs $(a, g) \in A \times G$, with group law

$$(a, g) \cdot (b, h) := (a + g \cdot b, gh).$$

Let $\pi : A \rtimes G \longrightarrow G : (a, g) \mapsto g$ and let $\text{Hom}'(G, A \rtimes G)$ be the set of all group homomorphisms $f : G \longrightarrow A \rtimes G$ such that $\pi \circ f = \text{Id}_G$. Prove that $\text{Der}(G, A)$ is in bijection with $\text{Hom}'(G, A \rtimes G)$.