
Foreword

This text constitutes a faithful transcript of the lecture **Modular Representation Theory** held at the TU Kaiserslautern during the Winter Semester 2019/20 together with **Niamh Farrell** (14 Weeks, 4SWS Lecture + 2SWS Exercises).

Together with the necessary theoretical foundations the main aims of this lecture are to:

- provide students with a modern approach to **finite group theory**;
- learn about the **representation theory of finite-dimensional algebras** and in particular of the **group algebra of a finite group**;
- establish connections between the representation theory of a finite group over a field of **positive characteristic** and that over a field of characteristic **zero**;
- consistently work with **universal properties** and get acquainted with the **language of category theory**.

We assume as pre-requisites bachelor-level algebra courses dealing with *linear algebra* and *elementary group theory*, such as the standard lectures *Grundlagen der Mathematik*, *Algebraische Strukturen*, and *Einführung in die Algebra*. The lecture is built, so that you don't need to have attended *Commutative Algebra* and *Character Theory of Finite Groups* prior to this lecture. However, both these lectures share common ideas with *Representation Theory*. Therefore, in order to complement these pre-requisites, but avoid repetitions, the first chapter will deal formally with some background material on module theory, but some proofs will be omitted.

Sections marked with a star symbol (*) are presented in this Skript, for the sake of completeness, under a much more detailed version than in the lecture. The two main reasons are the following. Firstly, these notions are dealt with in details in the *Commutative Algebra* lecture, where the commutativity of rings is most of the time indeed not needed. Secondly these notions are partly well-known from either group theory or linear algebra and easily pass over to modules with the same arguments.
The proofs of the results in these sections are not subject to direct exam questions.

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