Character Theory of Finite Groups

SS23

CHARACTER THEORY OF FINITE GROUPS SS23

A. ORGANISATION

- **LECTURES:** → Mondays 10:00 – 11:30 (12×) Start: 24th April
  + Fr. 28th of April, 11:45 - 13:15

- **EXERCISES:** → Fridays 11:45 – 13:15 (6×) Start: 12th May
  (fortnightly)

  → Assistants : Annika Bartelt & Marie Roth

  → EXERCISE SHEETS: to be downloaded from my webpage

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A. ORGANISATION

- **SCHEINE:** → To obtain an Uebungsschein you need:

  - 40% of the possible points on each Sheet (1-6)
  - attendence to the Exercise Classes
  - present your solutions to 2 exercises at the black board

  → Please note:

  - handing-in exercises (paper or online) is possible:
    1. until the deadline written on the sheets
    2. in groups of maximum 2 students
  - only handwritten solutions are accepted

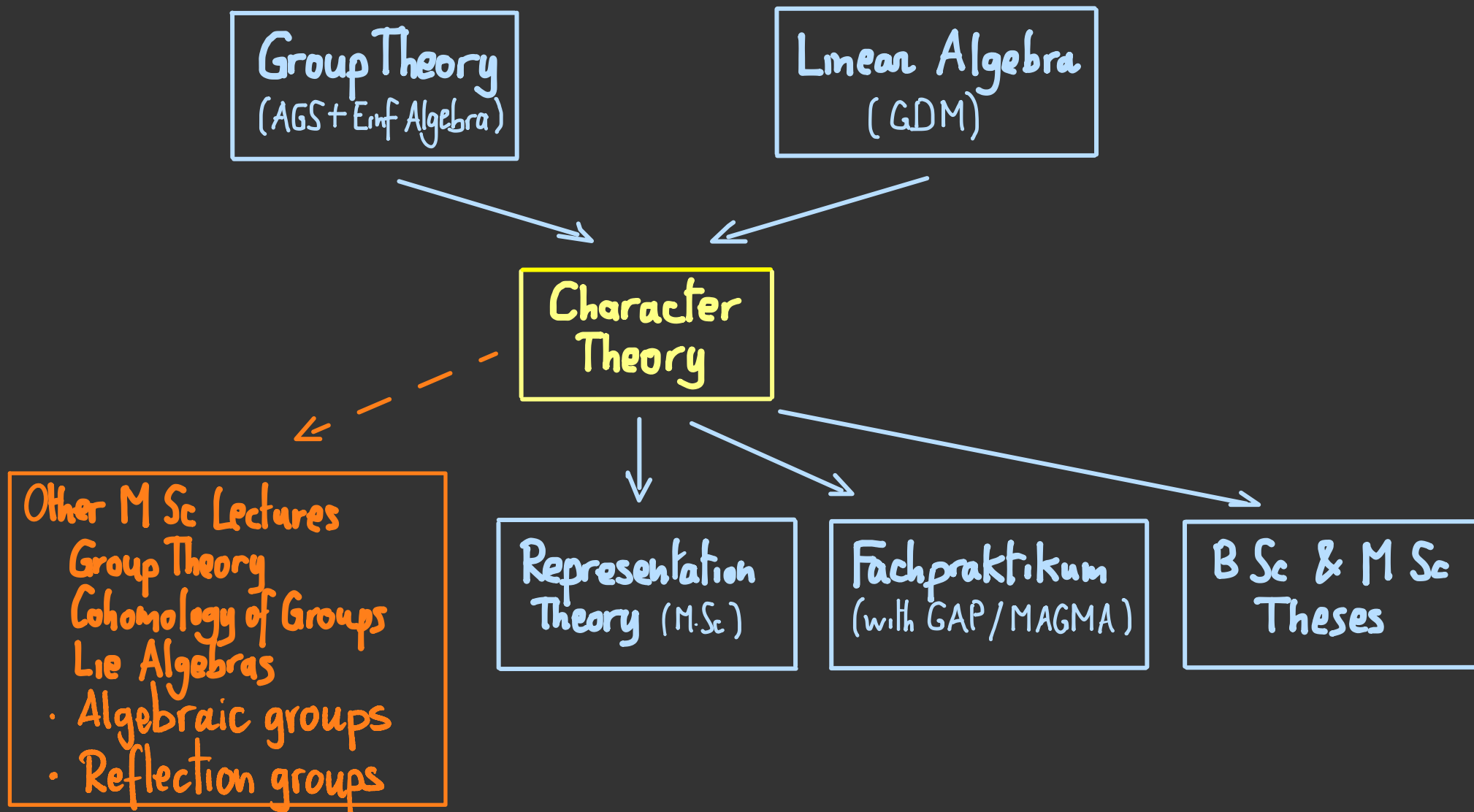# CHARACTER THEORY OF FINITE GROUPS SS23

A. ORGANISATION

- LECTURE NOTES: → We use the LaTeXed lecture notes of the

  SS 2022

  up to minor changes.

  All changes will be updated / incorporated
  in the notes after the last lecture.

C INTRODUCTION

(1) Character Theory in the 2020 Mathematics Subject Classification

www.ams.org/msc/msc2020.pdf

go to

https://mathscinet.ams.org/mathscinet/freeTooils.html?version=2

CHARACTER THEORY OF FINITE GROUPS SS23
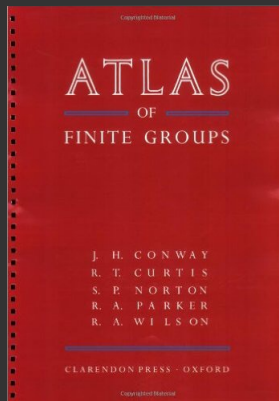
C INTRODUCTION

(1) Character Theory in the 2020 Mathematics Subject Classification

www.ams.org/msc/msc2020.pdf

(2) The ATLAS OF FINITE GROUPS

J.H. CONWAY, R.T. CURTIS, S.P. NORTON, R. PARKER, R.A. WILSON,

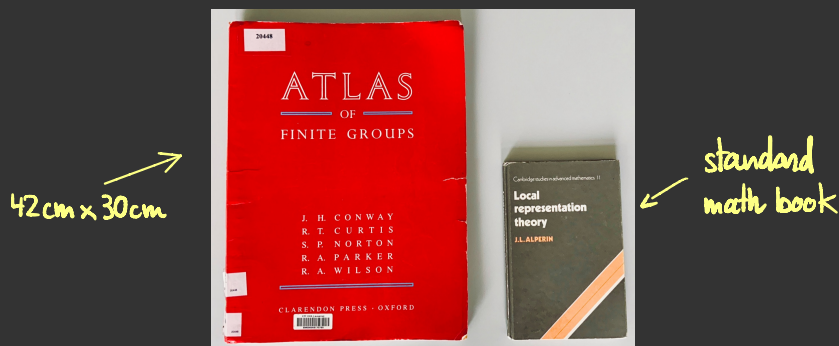*Atlas of Finite Groups.* Clarendon Press, Oxford, 1985.

The Atlas of Finite Groups :

The `Atlas of finite groups´ (often simply called "the Atlas")
is a book on group theory.

You find it easily in any math library because of its size and
color :

42 cm x 30 cm

standard
math book

It was written by J.H. Conway, R.T. Curtis, S.P. Norton, R.A. Parker
and R.A. Wilson, who are all reknown group or group representation
theorists.

J.H. Conway unfortunately passed away on April 11 2020, reportedly
from Covid-19.

The book itself contains a plethora of information about
the 93 smallest non-abelian finite simple groups and other
series of related finite groups, the information being generally:
order, maximal subgroups, extensions, automorphism
group, ..., character table.

—> Show the ATLAS:

    —>   Book

    —>   pdf

$$\boxed{\text{CHARACTER THEORY OF FINITE GROUPS SS23}}$$

C INTRODUCTION

(3) Aims of the lecture

$\longrightarrow$ Character Tables of finite groups

- What are they?
- Elementary methods to compute them
- What information about finite groups do they contain?

$\longrightarrow$ Give a proof of

Burnside's $p^a q^b$ - Theorem (1911)

Let p, q be prime numbers and let $a, b \in \mathbb{N}_0$. Then any finite group of order $p^a q^b$ is soluble.

Example of a character table

	$[1]$	$[g_2]$	$[g_3]$	$[g_4]$	$[g_5]$	$[g_6]$
χ_1	1	1	1	1	1	1
χ_2	3	-1	1	0	α	$\bar{\alpha}$
χ_3	3	-1	1	0	$\bar{\alpha}$	α
χ_4	6	2	0	0	-1	-1
χ_5	7	-1	-1	1	0	0
χ_6	8	0	0	-1	1	1

$$\alpha := \frac{-1 + i\sqrt{7}}{2}$$

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Example of a character table

	[1]	[q_2]	[q_3]	[q_4]	[q_5]	[q_6]
χ_1	1	1	1	1	1	1
χ_2	3	-1	1	0	α	$\bar{\alpha}$
χ_3	3	-1	1	0	$\bar{\alpha}$	α
χ_4	6	2	0	0	-1	-1
χ_5	7	-1	-1	1	0	0
χ_6	8	0	0	-1	1	1

$$\alpha := \frac{-1 + i\sqrt{7}}{2}$$

Conclusions (E.g.)

* $|G| = 168$

* G non abelian

* $Z(G) = \{1\}$

* G simple

...

$$\left(\Rightarrow G \cong PSL_2(\mathbb{F}_7) \right)$$

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D PROGRAMME

   I   Linear representations of finite groups

   II Characters of representations

   III The character table

   IV Burnside's $p^a q^b$-Theorem

   V Induction and restriction

   VI Brauer's Characterization of characters

Please, do register in the **URM** system

https://urm.mathematik.uni-kl.de:4443

by Friday, 24th of April, <u>noon</u>.