
COHOMOLOGY OF GROUPS — EXERCISE SHEET 1

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EXERCISE 1Let K be a field.

- (a) Prove that

$$\mathrm{GL}_n(K) = \mathrm{SL}_n(K) \rtimes \{\mathrm{diag}(\lambda, 1, \dots, 1) \in \mathrm{GL}_n(K) \mid \lambda \in K^\times\},$$

where $\mathrm{diag}(\lambda, 1, \dots, 1)$ is the diagonal matrix with (ordered) diagonal entries $\lambda, 1, \dots, 1$.
Describe the action.

- (b) Let

$$\begin{aligned} B &:= \left\{ \begin{pmatrix} * & * \\ & \ddots & * \\ 0 & & * \end{pmatrix} \in \mathrm{GL}_n(K) \right\} \quad (= \text{upper triangular matrices}), \\ U &:= \left\{ \begin{pmatrix} 1 & * \\ & \ddots \\ 0 & & 1 \end{pmatrix} \in \mathrm{GL}_n(K) \right\} \quad (= \text{upper unitriangular matrices}), \\ T &:= \left\{ \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix} \in \mathrm{GL}_n(K) \right\} \quad (= \text{diagonal matrices}). \end{aligned}$$

Prove that B is a semi-direct product of U by T , that is, $B = U \rtimes T$. Describe the action.

EXERCISE 2

- (a) Prove that $D_8 \cong V_4 \rtimes C_2$, where V_4 is the Klein-four group. Describe the action of C_2 on V_4 .
- (b) Prove that $S_4 \cong V_4 \rtimes S_3$. Deduce that a Sylow 2-subgroup of S_4 is isomorphic to D_8 .
- (c) Construct all semi-direct products of C_3 by C_3 up to isomorphism.
- (d) Identify the group described in Example 1.6(2) when $m = 7, n = 3$ and $k = 2$. (Hint: is it an abelian group?)