COHOMOLOGY OF GROUPS
SS 2021

A. ORGANISATION

- **LECTURES:**
  - → Tu. 12:00 – 13:30 / Video → OpenOLAT + Panopto ⎫
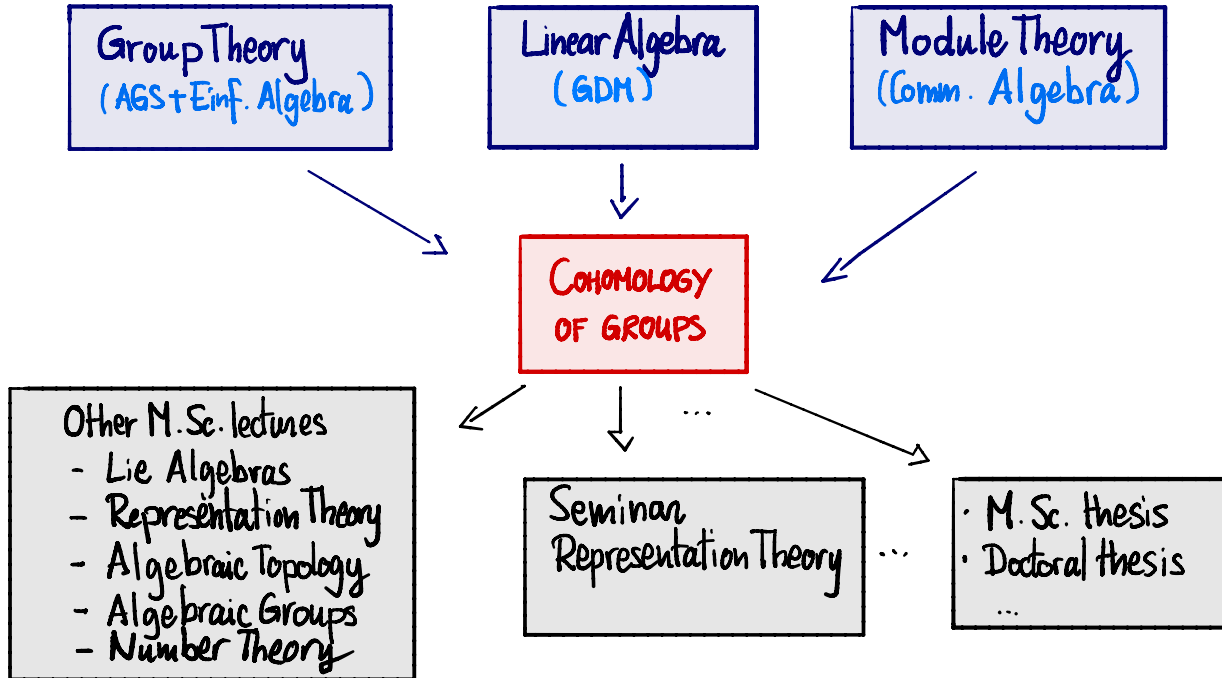 - → Th. 12:00 – 13:30 / Video → OpenOLAT + Panopto ⎭ available 14 days.

  (Later: Hybrid? Room 48-562 + live stream)

- **EXERCISES:**

  `START: WEEK 2`

  - → Wed. 14:00 – 15:30 / In OpenOlat – BigBlueButton
  - → <u>Assistant</u> :   Bernhard Böhmler
  - → <u>Registration</u> :   in the URM system by Fr. 23rd April, noon.
    - —→ also gives you access to the videos on Panopto.
  - → <u>Übungsschein</u>: let us know this week if you need to obtain one.

Cohomology of Groups
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B. Before and After

Group Theory
(AGS + Einf. Algebra)

Linear Algebra
(GDM)

Module Theory
(Comm. Algebra)

Cohomology
of Groups

Other M.Sc. lectures
- Lie Algebras
- Representation Theory
- Algebraic Topology
- Algebraic Groups
- Number Theory

Seminar
Representation Theory
...

· M.Sc. thesis
· Doctoral thesis
...

C. BACKGROUND GROUP THEORY + MODULE THEORY

The following elementary notions of group theory are assumed as known:

→ Actions of groups on sets (G-set theory)
→ Sylow theory
→ Galois theory

The following elementary notions of module theory are assumed as known:

→ Modules/submodules/homomorphisms/quotient modules
→ Free modules / projective modules
→ Direct sums/ direct products
→ Exact sequences/short exact sequences
→ Tensor products
→ Algebras

> ## COHOMOLOGY OF GROUPS
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D. AIMS OF THE LECTURE

① Introduction to : homological algebra / homology + cohomology   (module theory)

② Advanced group theory: Two problems we will solve using "cohomological" methods:

Problem 1: Classify group extensions $1 \to N \to E \to G \to 1$

Solution uses two cohomology groups when N is abelian: $H^1(G,N)$ & $H^2(G,N)$

Problem 2: Classify finite groups of a given order with "elegant" methods.

E.g. for $|G| = 8$: $C_8, C_4 \times C_2, C_2 \times C_2 \times C_2, D_8, Q_8$

D. AIMS OF THE LECTURE

② Advanced group theory:

Further problem: Understand the "Schur multiplier" of a finite group G : $H^2(G, \mathbb{C}^\times)$
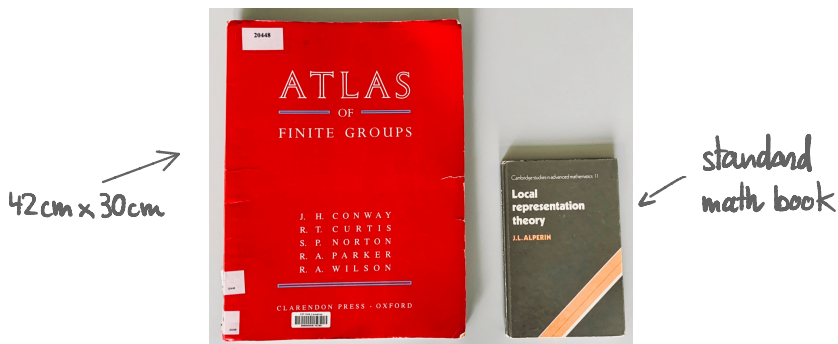
→ importance for the theory of finite groups

(reductions to quasi-simple groups ...)

→ classification of the quasi-simple groups

+ reading the ATLAS OF FINITE GROUPS .

THE ATLAS OF FINITE GROUPS :

The `ATLAS of finite groups' (often simply called "the ATLAS") is a book on group theory.

You find it easily in any math library because of its size and color :

42cm x 30cm

standard math book

It was written by J.H. Conway, R.T. Curtis, S.P. Norton, R.A. Parker and R.A. Wilson, who are all reknown group or group representation theorists.

J.H. Conway unfortunately passed away on April 11 2020, reportedly from Covid-19.

The book itself contains a plethora of information about the 93 smallest non-abelian finite simple groups and other series of related finite groups such as central extensions (= quasi-simple groups).