Graph Optimization
Lab session 4

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Exercise 1: column generation for multicommodity flow problem

Consider the multicommodity flow problem. Write a .mod and a .run files to solve the multicommodity flow problem with the column generation applied to the path-based formulation. Solve the two instances available online. Use the parameters as defined in the file `ex4-parameters.mod` and the scheme for the column generation in the file `column-generation-scheme.run`.

Exercise 2: arc based formulation for multicommodity flow problem

Consider the multicommodity flow problem. Write a .mod and a .run files to solve the multicommodity flow problem with the arc-based formulation. Solve the two instances available online. Use the parameters name as defined in the file `ex4-parameters.mod`. (Exercise 1 Lab session 2)

Column generation for multicommodity flow problem

(Restricted) Master problem

$$\begin{aligned}
\text{min} \quad & \sum_{p \in P} c_p x_p \\
\text{s. t.} \quad & \sum_{p \in P_k} x_p = d_k, \forall k \in K \\
& \sum_{p \in P_{ij}} x_p \leq u_{ij}, \forall (i,j) \in A \\
& x_p \geq 0, \forall p \in P
\end{aligned}$$

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& x_p \geq 0, \forall p \in P
\end{aligned}
$$

Pricing problem

$$
\min \sum_{(i,j) \in A} g_{ij} z_{ij}
$$

$$
\begin{aligned}
\sum_{(j,i) \in A} z_{ji} - \sum_{(i,j) \in A} z_{ij} \quad & = b_i, \forall i \in N \\
z_{ij} \in \{0,1\} \quad & \forall (i,j) \in A
\end{aligned}
$$

where $g_{ij} = c_{ij} - \mu_{ij}$ and

$$
b_i = \begin{cases} -1, i = s_k \\ 1, i = t_k \\ 0, \text{otherwise} \end{cases}
$$

Column generation scheme

1. Define an initial set of paths P
2. Solve the restricted master problem on P
3. For each commodity k:
    - Build the pricing problem ($g_{ij} = c_{ij} - \mu_{ij}$)
    - Solve the pricing problem
    - Compare the optimal solution of the pricing with σ_k
    - If the optimal solution of the pricing is lesser than σ_k then a new path must be added
4. If at least one path has been added, go to 1

Useful commands

Defining and solving more than one problem

```
problem ProblemName:variables, objective function,
constraints;
```

to define a problem with the list of variables, objective fuction and constraints

```
problem ProblemName;
option solver gurobi;
solve ProblemName;
```

to solve the problem

Useful commands

Defining the set of paths

```
set N;
param n_p;
set P := 1..n_p;
set Path {P} within N cross N;
param or {P} within N; #Origin
param dest {P} within N; # Destination
param cp {P}; # Cost
```

Useful commands

Updating the set of paths

```
let n_p := n_p + 1;
let Path[n_p] := {(i,j) in A: z[i,j] = 1};
let cp[n_p] := sum {(i,j) in Path[n_p]} c[i,j];
```

Useful commands

- `ConstraintName.dual;` to access dual variable associated with ConstraintName
- `param` ParameterName `symbolic in` N;
- `let :=` : assign a value to a parameter that can be changed
- `for` : for statement
- `if` : if statement