Graph Optimization
Lab session 2

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Exercise 1

A network with the following data is given:

- A directed graph $G = (N, A)$, where n is the number of nodes, $N = 1 \dots n$
- A capacity u_{ij} and a cost per unit of flow c_{ij} for each arc $(i, j) \in A$
- A set of demands $K = 1 \dots n_d$ to be routed on the graph, each described by three parameters:
  - a source node $s_k \in N$
  - a destination node $t_k \in N$
  - an amount of flow d_k to be sent from the source node to the destination node

N.B. We assume that no 2 demands have the same origin and destination.

Exercise 1

We have to decide on which paths the flow of each demand is routed from its source to its destination, with the goal of minimizing the total flow cost, while guaranteeing that the capacity of the arcs is not exceeded.

Write the model of the problem. Code it in AMPL language and solve instances `istance_1_lab2.1.dat` and `instance_2_lab2.1.dat`. Sets and parameters are defined as described in file `ex-1-parameters.mod`. Analyse the solutions.

How do the model and the solutions change if each demand must be routed on a single path?

Exercise 2

How does the model change if:

- Arcs are not always available.
- Each arc can be activated paying a fixed cost.

Exercise 2

A network with the following data is given:

- A directed graph $G = (N, A)$
- A set of demands K representing different commodities to be routed on the graph, each described by three parameters:
  - a source node $s_k \in N$
  - a destination node $t_k \in N$
  - an amount of flow d_k
- A capacity u_{ij} for each arc $(i, j) \in A$
- A cost f_{ij} of **activating** arc $(i, j) \in A$

We have to route all the demands, and decide which arcs must be activated, with the goal of minimizing the total arc activation cost.

Write the model of the problem. Code it in AMPL language and solve instances `istance_1_lab2.2.dat` and `instance_2_lab2.2.dat`. Sets and parameters are defined as described in file `ex-2-parameters.mod`. Analyse the solutions.

How do the model and the solutions change if each demand must be routed on a single path?

Exercise 3

How does the model change?

- Arc capacity is not given, but it must be decided. The arc capacity is provided by installing transmission channels. The number of channels on each arc must be optimized: **network capacity dimensioning**.
- If no channel is installed on one arc, the arc has no capacity and therefore it does not belong to the network: **topological design**

Exercise 3

Parameters

- A directed graph $G = (N, A)$.
- A set of traffic demands K, each described by three parameters
  - a source node $s_k \in N$
  - a destination node $t_k \in N$
  - an amount of flow d_k to be sent from the source node to the destination node
- The capacity λ of each transmission channel
- The cost c_{ij} of installing one transmission channel on arc $(i, j) \in A$.

Exercise 3

The problem

We have to route all the demands, and decide how many channels must be installed on each arc, with the goal of minimizing the overall cost.

Write the model of the problem. Code it in AMPL language and solve instances `istance_1_lab2.3.dat` and `instance_2_lab2.3.dat`. Sets and parameters are defined as described in file `ex-3-parameters.mod`. Analyse the solution.

Exercise 4

Problem 4

How does the model change if two arc disjoint paths (a nominal and a backup path) must be selected for each demand? Enough capacity must be reserved for the demand on each of such paths.

Write the model of the problem. Code it in AMPL language and solve instances `istance_1_lab2.4.dat` and `instance_2_lab2.4.dat`. Sets and parameters are defined as described in file `ex-3-parameters.mod`. Analyse the solution.

Exercise 5

Problem 5

Let us consider the node dimensioning. Nodes must be equipped with a device that provides the capacity needed to handle the incoming traffic. Different types of devices are available: let δ denote the number of types of device and H the set of types of device. Let u_h be the capacity provided by a device of type h and g_h the cost of a type h device. At most one device can be installed in each node. We have to route all the demands on a single arc (no backup path is considered) and dimension arcs and nodes.
How does the model change?

Write the model of the problem. Code it in AMPL language and solve instances `istance_1_lab2.5.dat` and `instance_2_lab2.5.dat`. Sets and parameters are defined as described in file `ex-5-parameters.mod`.