

1. For the population in Sampling Survey I slide 4, define the  $k$ -th moment:

$$\mu_k = \frac{1}{N} \sum_{i=1}^N v_i^k$$

where  $k$  is a positive integer. Express  $\mu$  and  $\sigma^2$  in terms of the moments.

2. Let  $X$  be the result of a random draw from a population with mean  $\mu$  and variance  $\sigma^2$ . We know  $E(X) = \mu$ . Define a function  $h$  like  $g$  in Sampling Survey I slide 7 to show  $E(X^2) = \mu_2$ . Deduce that  $\text{var}(X) = \sigma^2$ .
3. Let  $M_1, \dots, M_N$  be as defined in Sampling Survey I slide 9.
  - (a) Calculate the probability of the event  $\{M_i = i, 1 \leq i \leq N\}$ .
  - (b) Let  $\pi$  be a bijective function from  $\{1, \dots, N\}$  to itself, so that  $\pi(1), \dots, \pi(N)$  is a permutation, or rearrangement, of  $1, \dots, N$ . Calculate the probability of the event  $\{M_i = \pi(i), 1 \leq i \leq N\}$ .
  - (c) What can you deduce about the joint distribution of  $M_1, \dots, M_N$ ?
4. At a university with 10,000 undergraduates, the Student Union conducted a survey of 400 undergraduates on a Monday. These students consumed an average of 5.4 bubble teas during the previous week, and the SD is 5.3.
  - (a) Describe a population variable for which the mean  $\mu$  could be estimated from the data.
  - (b) State an assumption about the 400 responses, that enables you to view 5.4 as a realisation of some random variable. Estimate  $\mu$ .
  - (c) What is the SE of your estimate of  $\mu$ ? Calculate an estimated SE.
  - (d) It was discovered later that every respondent was approached near one of several bubble tea outlets on campus. How would you modify your conclusion?
5. A survey organisation took a simple random sample of 1,500 persons from the residents of a large city. Among these sample persons, 1,035 were renters. Let  $\hat{p}$  be the random proportion of sample persons who were renters.
  - (a)  $E(\hat{p})$  is \_\_\_\_\_ 0.69.
  - (b)  $\text{SD}(\hat{p})$  is \_\_\_\_\_ 0.01, to two decimal places.

Fill in the blanks, and explain. Options: (i) exactly equal to (ii) estimated from the data as