ST2132 Tutorial 7 2210

1. Let $(X_1, \dots, X_r) \sim \text{Multinomial}(n, \mathbf{p})$.

   (a) Derive the MOM estimator of p_i, where $i = 1. \dots, r$.

   (b) Given $r = 3$ and $(x_1, x_2, x_3) = (2, 7, 1)$, find the MOM estimate of p_i, state the exact SE, and estimate it to two decimal places, for $i = 1, 2, 3$.

2. Let $X_1, \dots, X_n$ be IID Poisson(λ) random variables. Define

$$\hat{\lambda}_1 = \frac{1}{n}\sum_{i=1}^{n} X_i, \qquad \hat{\lambda}_2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \hat{\lambda}_1)^2$$

   (a) Are $\hat{\lambda}_1$ and $\hat{\lambda}_2$ unbiased estimators of λ?

   (b) Calculate $\text{SD}(\hat{\lambda}_1)$.

   (c) Let $\lambda = 0.8$ and $n = 100$. In R, perform a Monte Carlo approximation of the expectations and SDs of the two estimators to two decimal places, using 10,000 iterations. Are your results consistent with your answers to (a) and (b)?

   (d) Which estimator seems to have a smaller SE?

3. Let $X_1, \dots, X_n$ be IID Exponential(λ) random variables. You are given that $Y = X_1 + \cdots + X_n \sim \text{Gamma}(n, \lambda)$.

   (a) Obtain $\hat{\lambda}$, the MOM estimator of λ.

   (b) Is $\hat{\lambda}$ unbiased for λ?

4. Let $x_1, \dots, x_n$ be realisations of IID Bernoulli(p) random variables $X_1, \dots, X_n$, and let $y = \sum_{i=1}^{n} x_i$.

   (a) Write down $\Pr(X_i = x_i)$ as a single expression.

   (b) Write down the joint probability of $x_1, \dots, x_n$. Find the value of p that maximises the logarithm of the joint probability.

   (c) Write down the probability of y. Find the value of p that maximises the logarithm of the probability.

5. Let $\hat{\theta}$ be an estimator of θ. I.e., $\hat{\theta}$ is an RV and θ is an unknown constant. The MSE of $\hat{\theta}$ is

$$\text{E}\{(\hat{\theta} - \theta)^2\}$$

   Show that the MSE is equal to "SE^2 + bias^2".