ST2132 Tutorial 6 2210

1. Let $X_1, \ldots, X_n$ be IID $\mathrm{N}(\mu, \sigma^2)$ RV's.

   (a) State the analogue to the probability statement on Survey Sampling II slide 7, that justifies the CI on slide 14.

   (b) Derive the statement in (a).

   (c) Suppose -1.53, 2.24, 0.73, -0.72, -1.77 are independent realisations from $\mathrm{N}(\mu, \sigma^2)$. Use R to compute an estimate of μ, its estimated SE, and an exact 95%-CI for μ, to two decimal places.

   (d) What can you say about the CI for μ on slide 14, as $n \to \infty$?

2. Let $X_1, \ldots, X_n$ be IID RV's with mean μ and variance σ^2. For a positive integer k, the k-th moment of X_i is $\mu_k = \mathrm{E}(X_i^k)$.

   (a) Is $\hat{\mu}_k = \frac{1}{n}\sum_{i=1}^{n} X_i^k$ an unbiased estimator of μ_k?

   (b) Express $\mathrm{var}(\hat{\mu}_k)$ in terms of the moments.

3. Let $X_1, \ldots, X_n$ be IID random variables with the Bernoulli(p) distribution.

   (a) Derive $\hat{p}$, the MOM estimator of p.

   (b) Write down the distribution of $\hat{p}$.

   (c) Given data 1, 0, 1, 0, 0, 0, 1, 0, 0, 0, find the MOM estimate of p, state the exact SE, and estimate it.

   (d) With the estimate and SE from (c), do you think (estimate 1.96 SE, estimate + 1.96 SE) is a good 95% CI for p?

4. Let $Y \sim \mathrm{Binomial}(n, p)$.

   (a) Find the MOM estimator of p.

   (b) How is this estimator related to that in the previous question?

5. In muon decay, each electron is emitted at a random angle. The cosine of the angle has the density

$$f(x) = \frac{1 + \alpha x}{2}, \qquad -1 \leq x \leq 1.$$

   where $\alpha \in [-1, 1]$ is a parameter. Let $X_1, \ldots, X_n$ are IID RVs having this distribution.

   (a) Find $\hat{\alpha}$, the MOM estimator of α. Is it unbiased?

   (b) For $n = 225$, $\bar{x} = 0.04$. Estimate α, and compute an approximate SE.

   (c) Construct a 95%-CI for α.