ST2132 Tutorial 11 2210

1. Use Tutorial 10 Question 4 to show, in Goodness-of-fit,

   (a) slide 16: that $G \approx X^2$.

   (b) slide 25: the approximate formula for G.

2. For the data in Tutorial 8 Question 4, conduct a goodness-of-fit test of

$$H_0 : \mathbf{p} = \left(\frac{9}{16}, \frac{3}{16}, \frac{3}{16}, \frac{1}{16}\right)$$

   using

   (a) Pearson's X^2

   (b) the LR statistic.

   Calculate the test statistic to one decimal place, and report the approximate P values.

3. Let $X_1, \ldots, X_n$ be IID Normal(μ, σ^2) RV's, where σ is known. Consider testing

$$H_0 : \mu = 0 \qquad \text{vs} \qquad H_1 : \mu \neq 0$$

   (a) Show that the maximum loglikelihood under Ω and Ω_0 are as shown in Goodness-of-fit slide 22.

   (b) Show directly that the LR statistic $G = \frac{n\bar{X}^2}{\sigma^2} \sim \chi_1^2$ under H_0.

4. Same assumption as previous question, except that σ is unknown.

   (a) Show that ℓ_1 and ℓ_0 are as shown in slide 23.

   (b) Show that the LR statistic is

$$G = n \log\left(\frac{\hat{\mu}_2}{\hat{\sigma}^2}\right) \approx \frac{n\bar{X}^2}{\hat{\sigma}^2}$$

   (c) Use Tutorial 9 Question 4 to show that for large n, approximately $G \sim \chi_1^2$ under H_0.

5. Let $X_{11}, X_{12}, \ldots, X_{1n_1}, X_{21}, X_{22}, \ldots, X_{2n_2}$ be independent Poisson RV's, with $\mathrm{E}(X_{ij} = \lambda_i$. Consider testing

$$H_0 : \lambda_1 = \lambda_2 \qquad \text{vs} \qquad H_1 : \lambda_1 \neq \lambda_2$$

   (a) Show that

$$\ell_1 = \sum_{i=1}^{2}\left(\sum_{j=1}^{n_i} X_{ij} \log \bar{X}_i - n_i\bar{X}_i\right)$$

where $\bar{X}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} X_{ij}$.

(b) Show that

$$\ell_0 = \sum_{i=1}^{2}\sum_{j=1}^{n_i} X_{ij}\log \bar{X} - (n_1+n_2)\bar{X}$$

where $\bar{X} = \frac{1}{n_1+n_2}\sum_{i=1}^{2}\sum_{j=1}^{n_i} X_{ij}$.

(c) Derive the LR statistic. State its approximate distribution when n_1 and n_2 are both large, under H_0.