ST2132 Tutorial 10 2210

1. Let $x_1, \ldots, x_n$ be realisations of IID Normal(μ, ν) random variables $X_1, \ldots, X_n$, and $0 < \alpha < 1$.

   (a) For large n, what is the approximate joint distribution of the ML estimators $\hat{\mu}$ and $\hat{\nu}$? Express your answer in terms of $\sigma = \sqrt{\nu}$.

   (b) Construct approximate $(1-\alpha)$-CI for ν, expressing your answer in terms of σ.

2. Refer to Tutorial 8 Question 4, and Tutorial 9 Question 4.

   (a) Write down an approximate distribution for $\hat{\theta}_n$, the ML estimator of θ based on n plants, where n is large.

   (b) Construct a 95%-CI for θ based on the data.

3. Refer to Tutorial 8 Question 3. Write down an approximate distribution for the ML estimators of $(p_1, \ldots, p_5)$ based on n rolls of a die, where n is large. Use slide 15 of Distribution of ML Estimators; there is no need to invert a Fisher information matrix.

4. Let $a > 0$, and define

$$f(x) = x \log\left(\frac{x}{a}\right), \qquad x > 0$$

   Use a Taylor series expansion to show that for x close to a,

$$f(x) \approx (x-a) + \frac{(x-a)^2}{2a}$$