Analysis of Sales Dataset

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```r
# ==============================================================================
# PROJECT: ANALYSIS OF SALES DATASET
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# DATE: January 2026
# ==============================================================================


# Setup and Data Loading
library(tidyverse)
```

```
## Warning: il pacchetto 'tidyverse' è stato creato con R versione 4.5.2
```

```
## Warning: il pacchetto 'ggplot2' è stato creato con R versione 4.5.2
```

```
## Warning: il pacchetto 'readr' è stato creato con R versione 4.5.2
```

```
## Warning: il pacchetto 'dplyr' è stato creato con R versione 4.5.2
```

```
## Warning: il pacchetto 'forcats' è stato creato con R versione 4.5.2
```

```
## Warning: il pacchetto 'lubridate' è stato creato con R versione 4.5.2
```

```
## -- Attaching core tidyverse packages ------------------------ tidyverse 2.0.0 --
## v dplyr     1.1.4     v readr     2.1.6
## v forcats   1.0.1     v stringr   1.5.2
## v ggplot2   4.0.1     v tibble    3.3.0
## v lubridate 1.9.4     v tidyr     1.3.1
## v purrr     1.1.0
## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
## i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors
```

```r
library(scales)
```

```
## Warning: il pacchetto 'scales' è stato creato con R versione 4.5.2
```

```
##
## Caricamento pacchetto: 'scales'
##
## Il seguente oggetto è mascherato da 'package:purrr':
##
##     discard
##
## Il seguente oggetto è mascherato da 'package:readr':
##
##     col_factor
```

```
data<-read.csv("sales.csv", stringsAsFactors = TRUE)

View(data)
any(is.na(data))
```

```
## [1] FALSE
```

```
# EXPLORATORY DATA ANALYSIS (EDA)

# STATISTICAL SUMMARY  (KPIs)
# We display the summary of data

summary(data)
```

```
##     sale_id       branch           city       customer_type    gender
##  Min.   :   1.0   A:674   Chicago    :330   Member:516    Female:472
##  1st Qu.: 250.8   B:326   Los Angeles:326   Normal:484    Male  :528
##  Median : 500.5           New York   :344
##  Mean   : 500.5
##  3rd Qu.: 750.2
##  Max.   :1000.0
##        product_name      product_category   unit_price        quantity
##  Apple       :185   Beverages    :187     Min.   : 1.020   Min.   : 1.00
##  Detergent   :189   Fruits       :209     1st Qu.: 5.867   1st Qu.: 5.00
##  Notebook    :194   Household    :198     Median :10.615   Median :10.00
##  Orange Juice:208   Personal Care:208     Mean   :10.836   Mean   :10.34
##  Shampoo     :224   Stationery   :198     3rd Qu.:15.883   3rd Qu.:16.00
##                                           Max.   :20.980   Max.   :20.00
##       tax          total_price     reward_points
##  Min.   : 0.080   Min.   :  1.21   Min.   : 0.000
##  1st Qu.: 2.510   1st Qu.: 38.38   1st Qu.: 0.000
##  Median : 5.870   Median : 89.70   Median : 0.000
##  Mean   : 7.758   Mean   :118.58   Mean   : 6.057
##  3rd Qu.:11.523   3rd Qu.:176.07   3rd Qu.:10.000
##  Max.   :28.390   Max.   :433.99   Max.   :43.000
```

```
# We display the main metrics to establish a baseline for our analysis into KPI
# for better visualization

kpi_summary <- data %>%
  summarise(
```

```r
    Total_Revenue = sum(total_price),
    Avg_Order_Value = mean(total_price),
    Total_Units_Sold = sum(quantity),
    Avg_Units_per_Sale = mean(quantity),
    Max_Single_Sale = max(total_price),
    Total_Reward_points = sum(reward_points))

print("GLOBAL KEY PERFORMANCE INDICATORS:")
```

```
## [1] "GLOBAL KEY PERFORMANCE INDICATORS:"
```

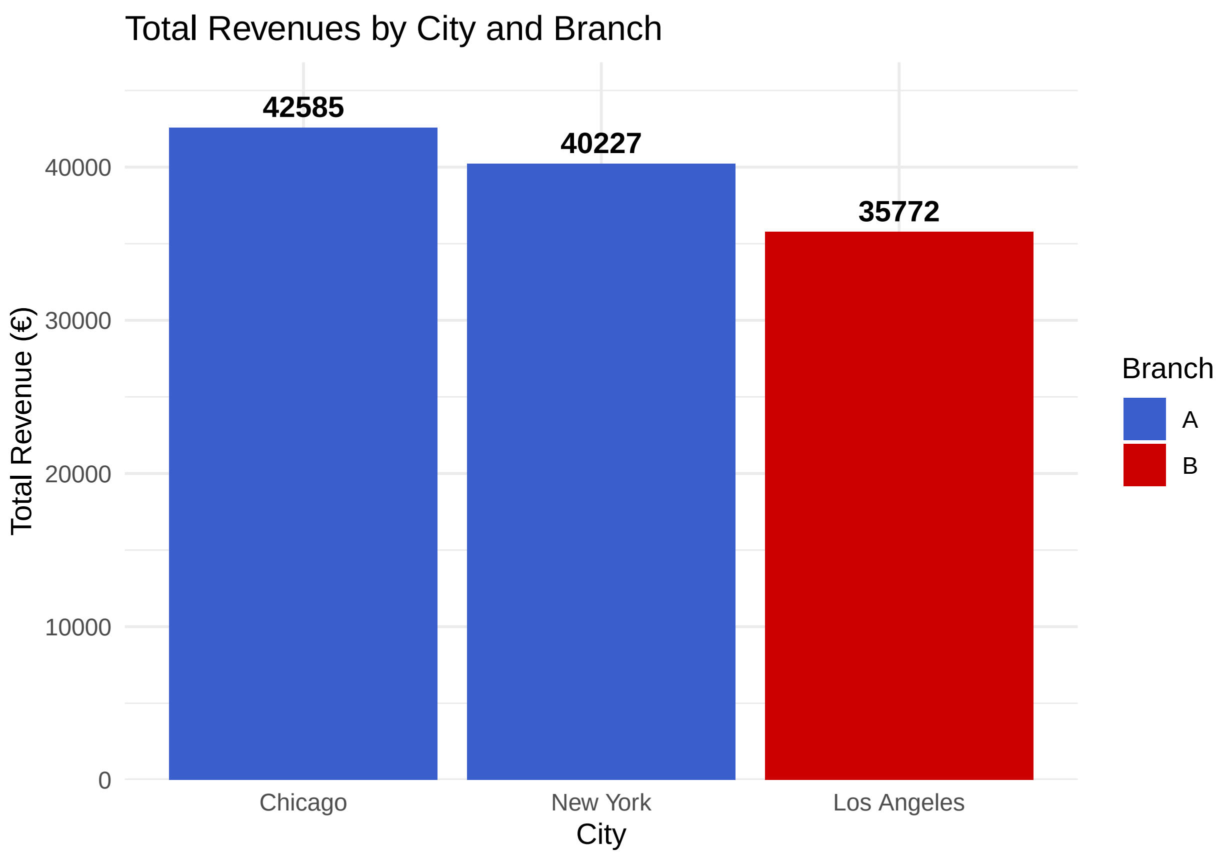
```r
print(kpi_summary)
```

```
##   Total_Revenue Avg_Order_Value Total_Units_Sold Avg_Units_per_Sale
## 1      118583.9        118.5839            10337             10.337
##   Max_Single_Sale Total_Reward_points
## 1          433.99                6057
```

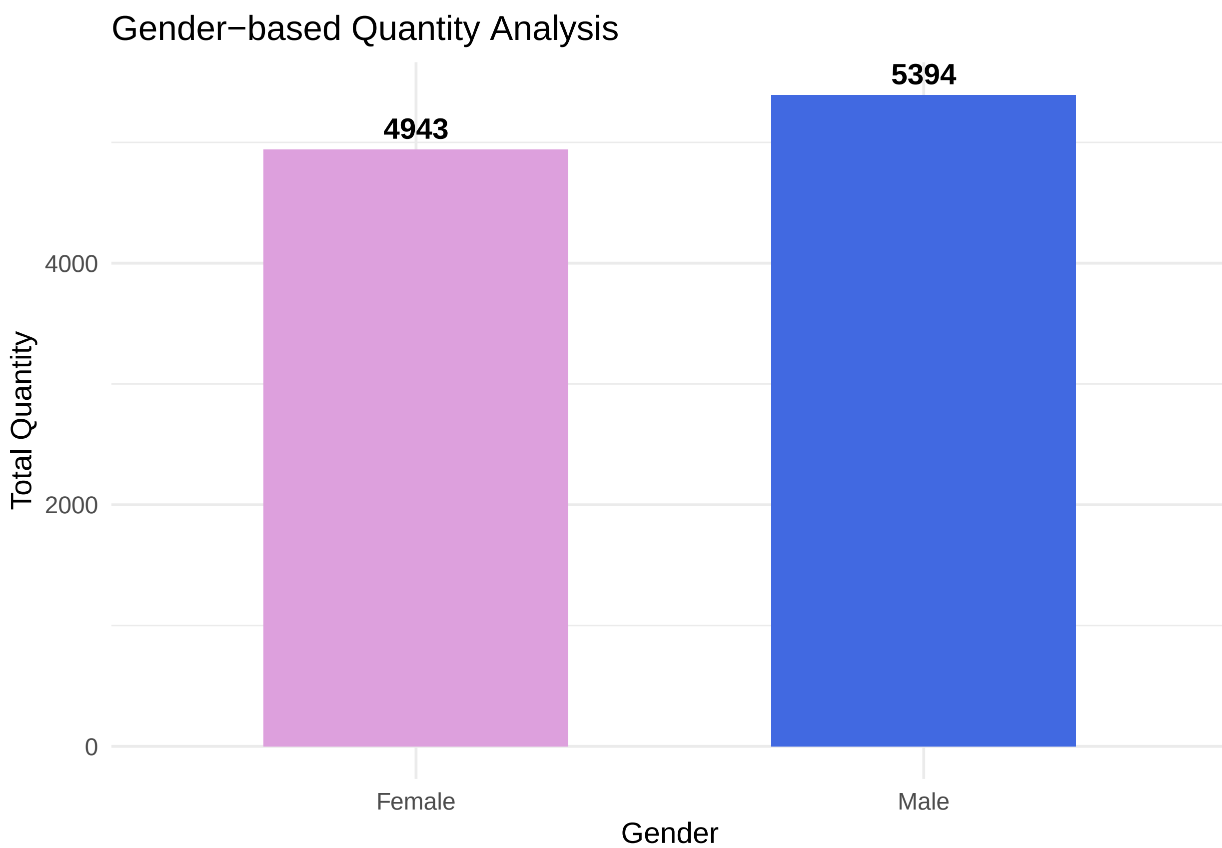
```r
# We verify data integrity, if there are missing values in the dataset
paste("Missing values:", sum(is.na(data)))
```

```
## [1] "Missing values: 0"
```

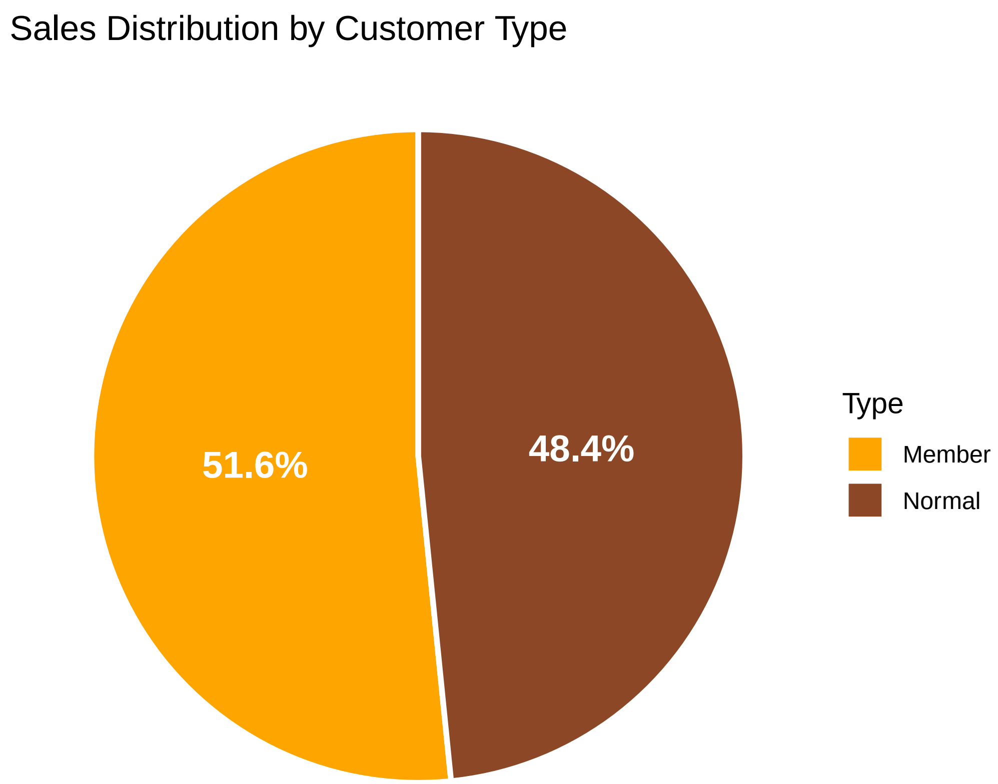
```r
# GEOGRAPHIC PERFORMANCE
# We identify top-performing areas. Note: Each city maps to a single branch.
# Bar plot: Total Revenues by City and Branch
plot_branch <- data %>%
  group_by(city, branch) %>%
  summarise(Total_Revenue = sum(total_price), .groups = 'drop') %>%
  ggplot(aes(x = reorder(city, -Total_Revenue, sum), y = Total_Revenue, fill = branch)) +
  geom_bar(stat = "identity") +
  geom_text(aes(label = paste0(round(Total_Revenue))),
            vjust = -0.5, size = 4, fontface = "bold") +
  scale_y_continuous(expand = expansion(mult = c(0, 0.1))) +
  scale_fill_manual(values = c("A" = "royalblue3", "B" = "red3")) +
  theme_minimal() +
  labs(title = "Total Revenues by City and Branch", x = "City",
  y = "Total Revenue (€)",fill = "Branch")

plot_branch
```



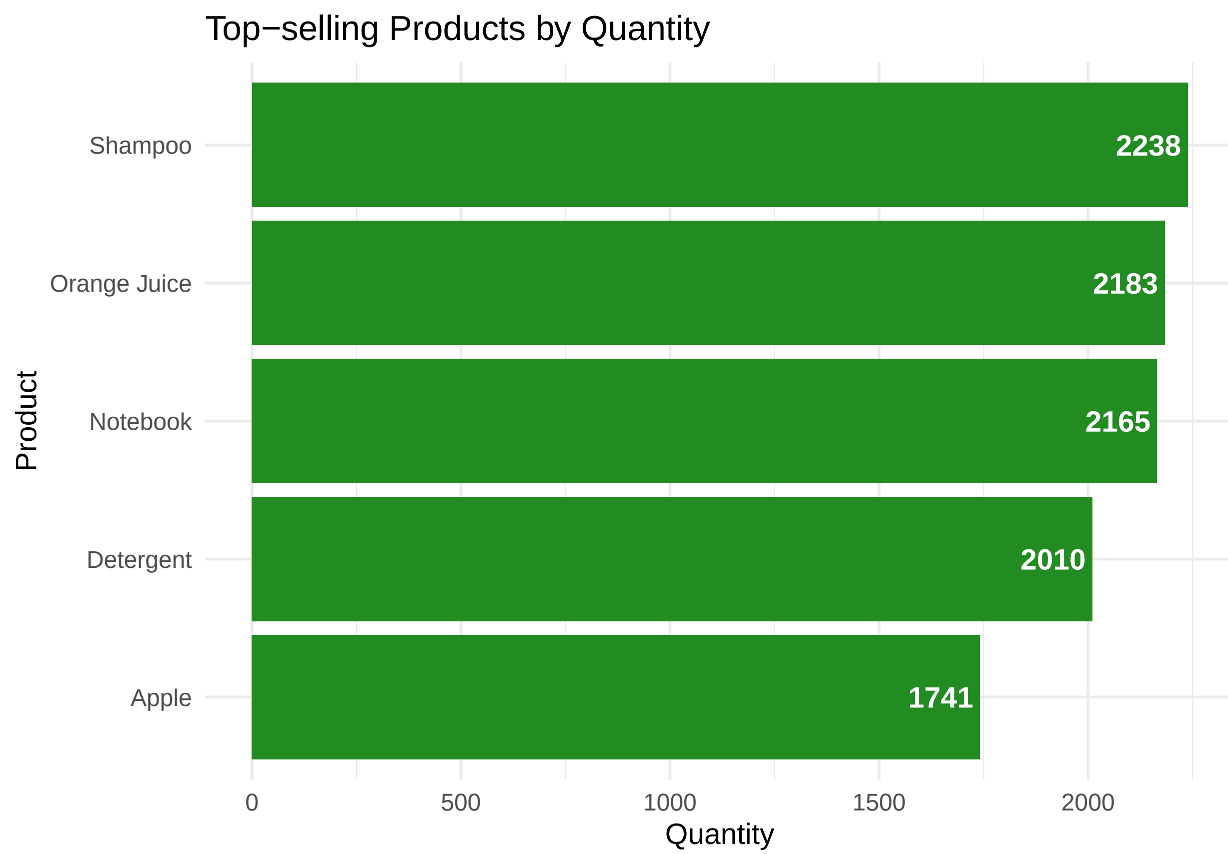

```r
# CUSTOMER DEMOGRAPHIC & LOYALTY

# we perform a gender analysis to understand the purchasing power and volume per gender
# to better tailor future marketing campaigns
# Bar plot: Gender-based Quantity Analysis
plot_gender <- data %>%
  group_by(gender) %>%
  summarise(Total_Qty = sum(quantity)) %>%
  ggplot(aes(x = gender, y = Total_Qty, fill = gender)) +
  geom_bar(stat = "identity", width = 0.6, show.legend = FALSE) +
  # internal labels
  geom_text(aes(label = round(Total_Qty)), vjust = -0.5, fontface = "bold") +
  theme_minimal() +
  scale_fill_manual(values = c("Female" = "plum", "Male" = "royalblue")) +
  labs(title = "Gender-based Quantity Analysis", x = "Gender", y = "Total Quantity")
plot_gender
```



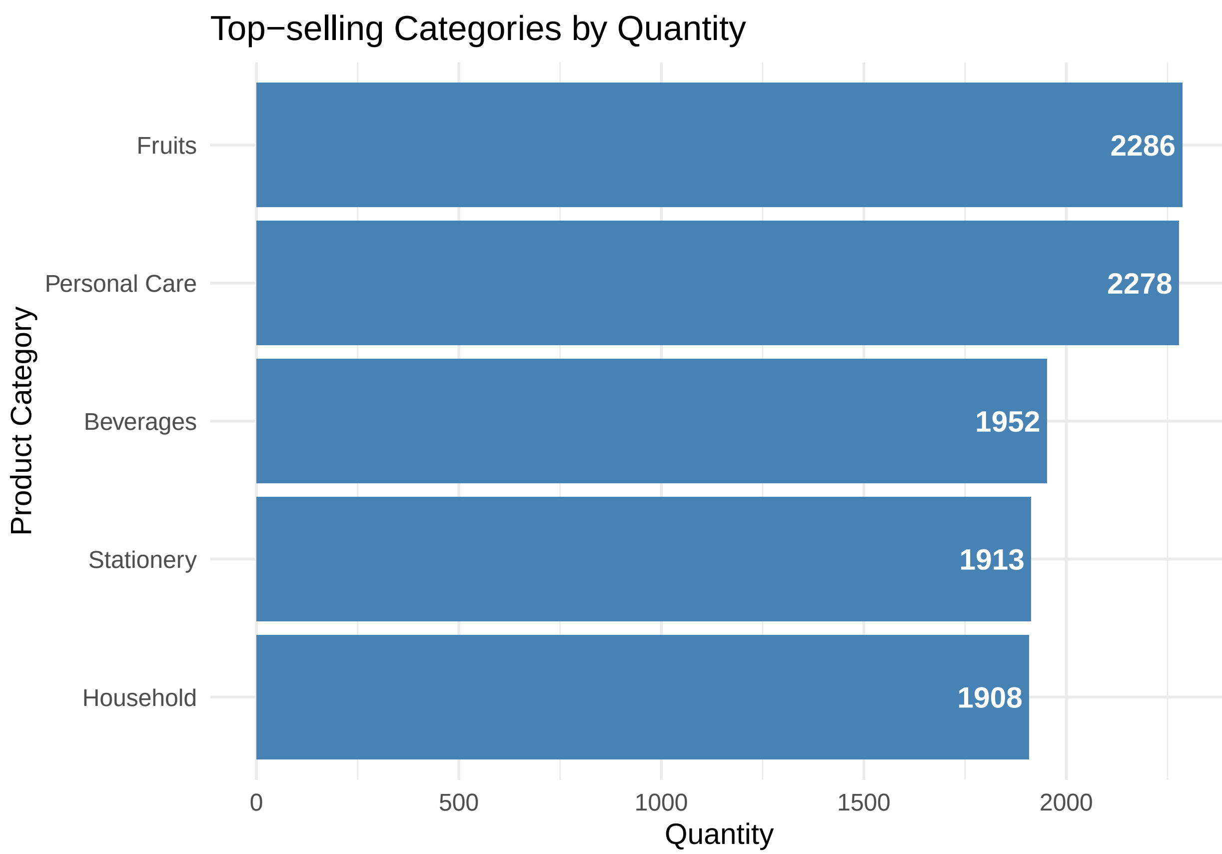

```
# We evaluate the ratio of Members vs. Normal customers to see the loyalty
# program penetration

# Pie chart: Distribution of Sales by Customer Type (Members or not)
plot_cust <- data %>%
  count(customer_type) %>%
  mutate(perc = n / sum(n)) %>%
  ggplot(aes(x = "", y = n, fill = customer_type)) +
  geom_bar(stat = "identity", width = 1, color = "white", linewidth = 1) +
  coord_polar("y", start = 0) +
  geom_text(aes(label = percent(perc, accuracy = 0.1)),
            position = position_stack(vjust = 0.5), color = "white",
            fontface = "bold", size= 5) +
  scale_fill_manual(values = c("Member" = "orange", "Normal" = "sienna4")) +
  theme_void() +
  labs(title = "Sales Distribution by Customer Type", fill = "Type")

plot_cust
```



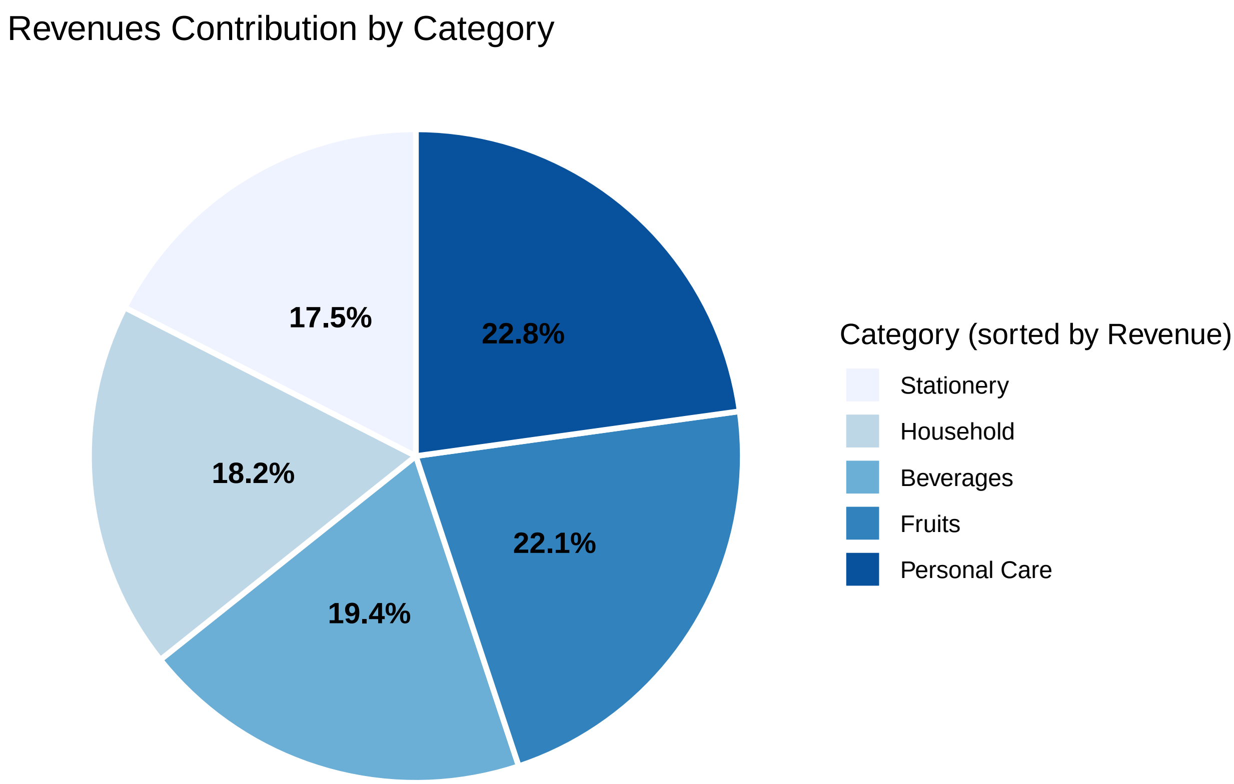

```
# PRODUCT AND CATEGORIES ANALYSIS

# We identify high-rotation products based on total quantity sold (inventory)
# Bar plot: Top-selling Products based on Quantity
plot_prod <- data %>%
  group_by(product_name) %>%
  summarise(Total_Qty = sum(quantity)) %>%
  ggplot(aes(x = reorder(product_name, Total_Qty), y = Total_Qty)) +
  geom_bar(stat = "identity", fill = "forestgreen") +
  # Internal white labels for horizontal bar charts
  geom_text(aes(label = round(Total_Qty)), hjust = 1.1, color = "white",
  fontface = "bold") +
  coord_flip() +  # swapping coordinates x, y for cleaner labeling
  theme_minimal() +
  labs(title = "Top-selling Products by Quantity", x = "Product", y = "Quantity")

plot_prod
```




```r
# Analysis of Sales Volume  to identify which categories have the highest inventory turnover.
# (the most frequently purchased, that usually are "essential" or "high-frequency product"

# Bar plot: Top-selling Categories by Quantity
plot_cat <- data %>%
  group_by(product_category) %>%
  summarise(Total_Qty = sum(quantity)) %>%
  ggplot(aes(x = reorder(product_category, Total_Qty), y = Total_Qty)) +
  geom_bar(stat = "identity", fill = "steelblue") +
  # white labels for horizontal bar charts
  geom_text(aes(label = round(Total_Qty)), hjust = 1.1, color = "white",
  fontface = "bold") +
  coord_flip() +
  theme_minimal() +
  labs(title = "Top-selling Categories by Quantity", x = "Product Category", y = "Quantity")

plot_cat
```




```r
# Analysis of Revenue Contribution to understand financial impact
# Pie chart: Product Categories Contribution to Revenues
plot_cat_rev <- data %>%
  group_by(product_category) %>%
  summarise(Revenue = sum(total_price)) %>%
  #reorder categories using a color gradient (darker = higher) to highlight top earners
  mutate(perc = Revenue / sum(Revenue), product_category = reorder(product_category, Revenue)) %>%
  ggplot(aes(x = "", y = Revenue, fill = product_category)) +
  geom_bar(stat = "identity", width = 1, color = "white", linewidth = 1) +
  coord_polar("y", start =0) +
  # Internal labels with percentage
  geom_text(aes(label = percent(perc, accuracy = 0.1)), position = position_stack(vjust = 0.5),
            color = "black", fontface = "bold", size = 4) +
  theme_void() +
  # using a color scale of blues: the darker for the highest % of revenues
  scale_fill_brewer(palette = "Blues") +
  labs(title = "Revenues Contribution by Category", fill = "Category (sorted by Revenue)")
plot_cat_rev
```



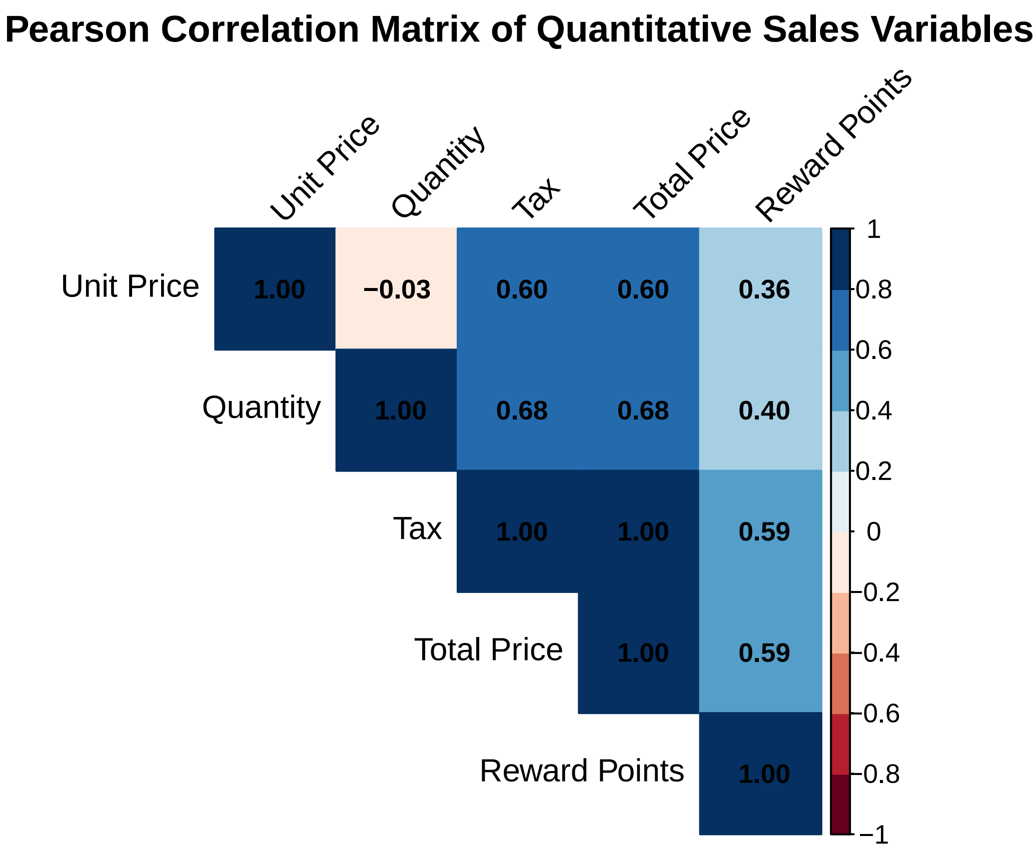

```r
# CORRELATION ANALYSIS
library(corrplot)
```

```
## Warning: il pacchetto 'corrplot' è stato creato con R versione 4.5.2
```

```
## corrplot 0.95 loaded
```

```r
# For correlation analysis we use only numeric variables
# and remove qualitative columns and 'sale_id' (which is just an index).
numeric_data <- data %>%
  select(unit_price, quantity, tax, total_price, reward_points)

# We compute the Pearson correlation coefficient for each pair of variables.
cor_matrix <- cor(numeric_data)

# Rename labels
colnames(cor_matrix) <- c("Unit Price","Quantity", "Tax","Total Price", "Reward Points")
rownames(cor_matrix) <- colnames(cor_matrix)

# We generate the heatmap using a color palette (Red-White-Blue).
# This visualization helps identify strong positive (Blue) or negative (Red) relationships.

corrplot(cor_matrix, method = "color", type = "upper", order = "original",
         addCoef.col = "black", number.cex = 0.8,  # fonts for coefficients
         tl.col = "black", tl.srt = 45,     # text labels
```

```r
         col = COL2('RdBu', 10),   # Red-Blu palette
         title = "Pearson Correlation Matrix of Quantitative Sales Variables",
         mar = c(0,0,2,0))
```




```r
# To deep dive into the correlation between Price and Quantity, we perform
# a PRICE SENSITIVITY ANALYSIS using a Scatter plot to see if the variables
# change respecting to Branches or Cities

plot_price_qty <- data %>%
  ggplot(aes(x = unit_price, y = quantity)) +
  # We map 'branch' to color and 'city' to shape to distinguish NY from Chicago
  # within Branch A
  geom_jitter(aes(color = branch, shape = city), alpha = 0.7, size = 2.5) +
  # We keep the regression lines focused on the Branch level for clarity
  geom_smooth(aes(color = branch), method = "lm", se = FALSE, size = 1) +
  # Colors: Blue for A, Red for B as used at the beginning
  scale_color_manual(values = c("A" = "royalblue3", "B" = "red3")) +
  # Shapes: Circle (19), Triangle (17), Square (15) for maximum distinction
  scale_shape_manual(values = c("Chicago" = 19, "New York" = 17,
                                "Los Angeles" = 15)) +
  theme_minimal() +
  labs(title = "Price Sensitivity Analysis: Unit Price vs. Quantity",
       x = "Unit Price (€)",
       y = "Quantity Purchased",
       color = "Branch",
```

```
       shape = "City")
```

```
## Warning: Using ‘size‘ aesthetic for lines was deprecated in ggplot2 3.4.0.
## i Please use ‘linewidth‘ instead.
## This warning is displayed once every 8 hours.
## Call ‘lifecycle::last_lifecycle_warnings()‘ to see where this warning was
## generated.
```

```
plot_price_qty
```

```
## ‘geom_smooth()‘ using formula = ’y ~ x’
```


```
# ==============================================================
# CLUSTERING ANALYSIS
# ==============================================================
library(cluster)
```

```
## Warning: il pacchetto ’cluster’ è stato creato con R versione 4.5.2
```

```
library(dbscan)
```

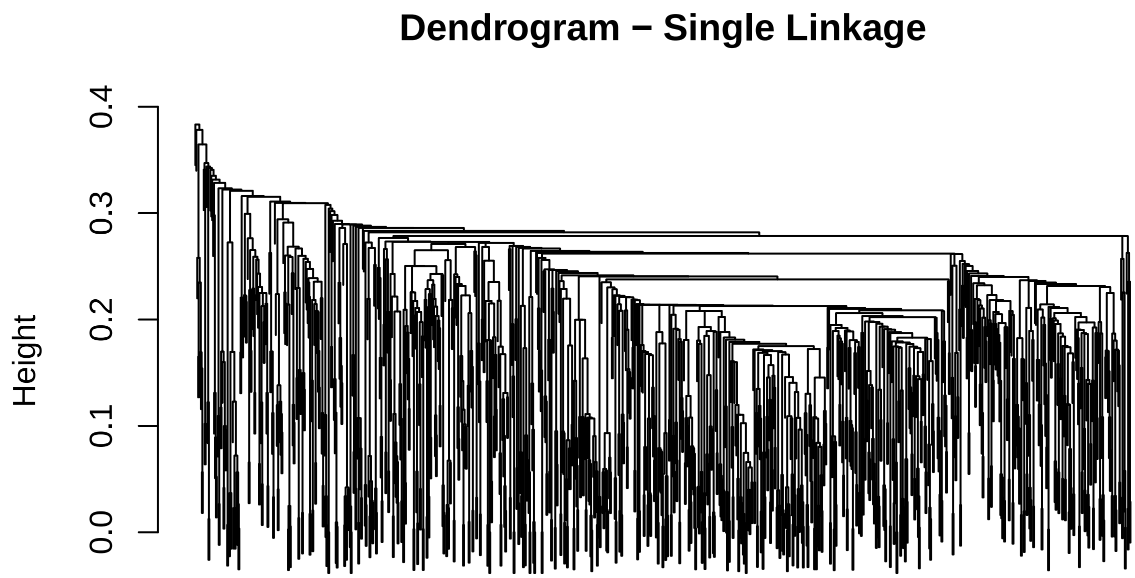
```
## Warning: il pacchetto ’dbscan’ è stato creato con R versione 4.5.2
```

```
##
## Caricamento pacchetto: 'dbscan'
##
## Il seguente oggetto è mascherato da 'package:stats':
##
##     as.dendrogram
```

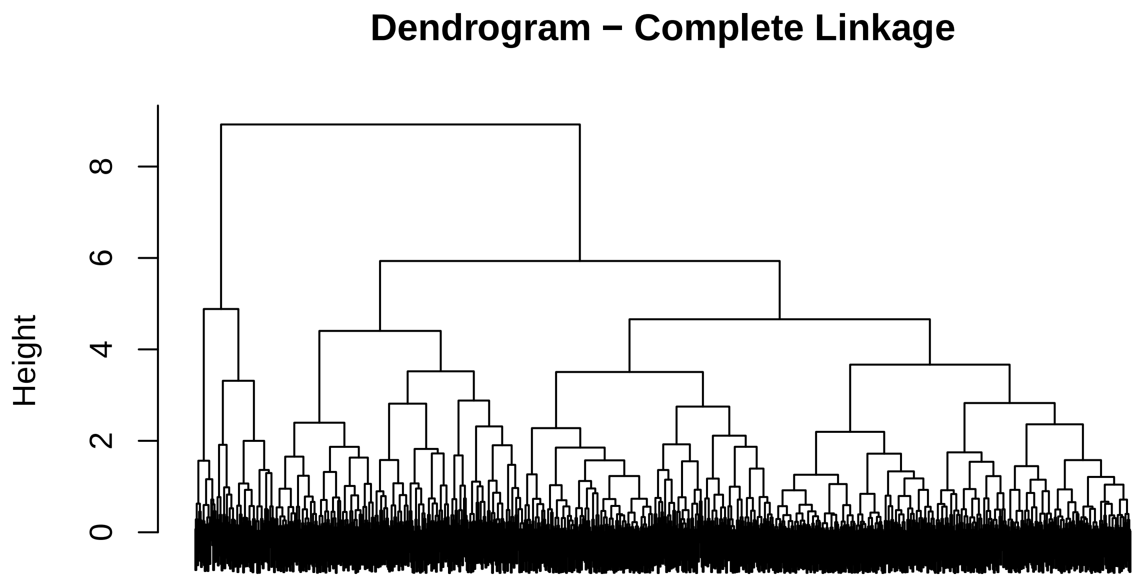
```r
# 1. Setup and Data Loading
View(data)
any(is.na(data))
```

```
## [1] FALSE
```

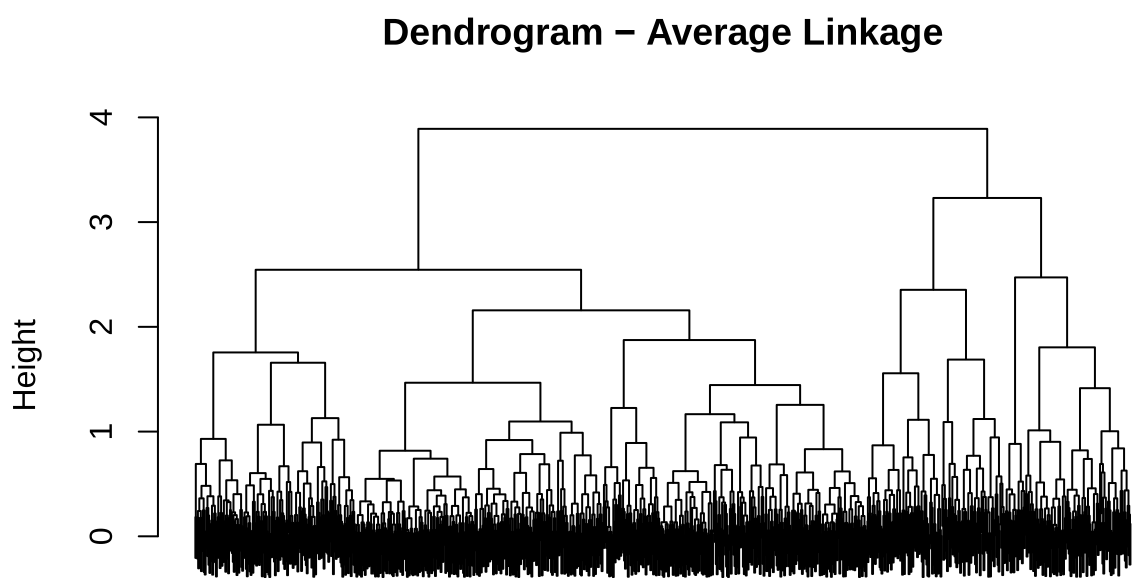
```r
data$customer_type <- as.factor(data$customer_type)
data$product_category <- as.factor(data$product_category)
data$gender <- as.factor(data$gender)
data$city <- as.factor(data$city)
data$branch <- as.factor(data$branch)
data$product_name <- as.factor(data$product_name)


# 2. Preparation
numeric_cols <- data[, c("unit_price", "quantity", "tax", "total_price", "reward_points")]
data_scaled <- scale(numeric_cols)
dist_matrix <- dist(data_scaled, method = "euclidean")


# 3. Hierarchical Clustering

#       A) Models

# -- Single Linkage --
hc_single <- hclust(dist_matrix, method = "single")
plot(hc_single, main = "Dendrogram - Single Linkage", xlab = "", sub = "", labels = FALSE)
```



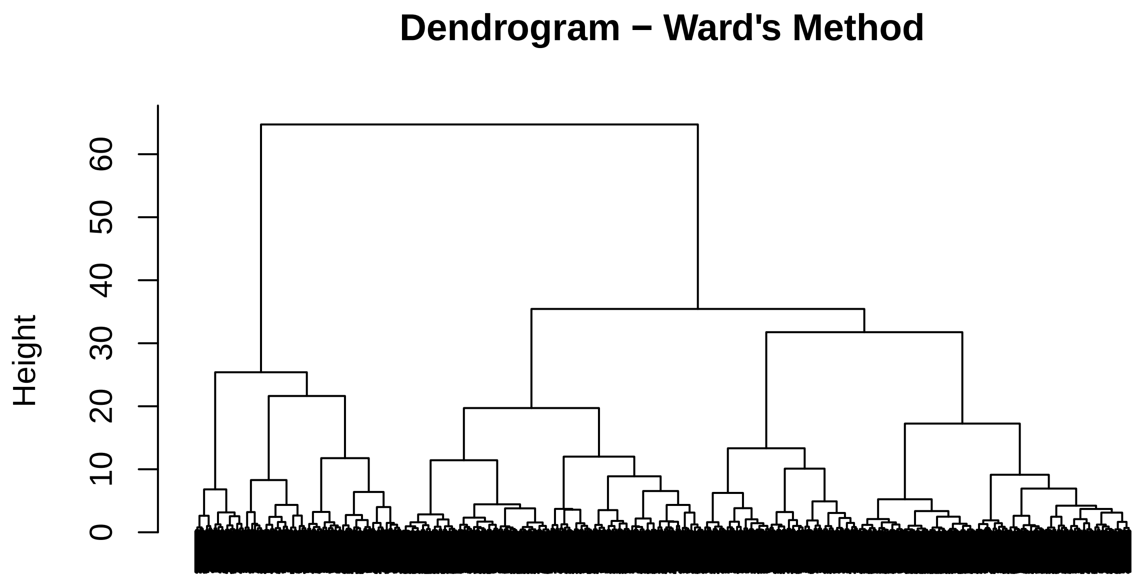

```r
# -- Complete Linkage --
hc_complete <- hclust(dist_matrix, method = "complete")
plot(hc_complete, main = "Dendrogram - Complete Linkage", xlab = "", sub = "", labels = FALSE)
```




```r
# -- Average Linkage --
hc_average <- hclust(dist_matrix, method = "average")
plot(hc_average, main = "Dendrogram - Average Linkage", xlab = "", sub = "", labels = FALSE)
```




```
# -- Ward's Method --
hc_ward <- hclust(dist_matrix, method = "ward.D2")
plot(hc_ward, main = "Dendrogram - Ward's Method", xlab = "", sub = "", labels = FALSE)
```

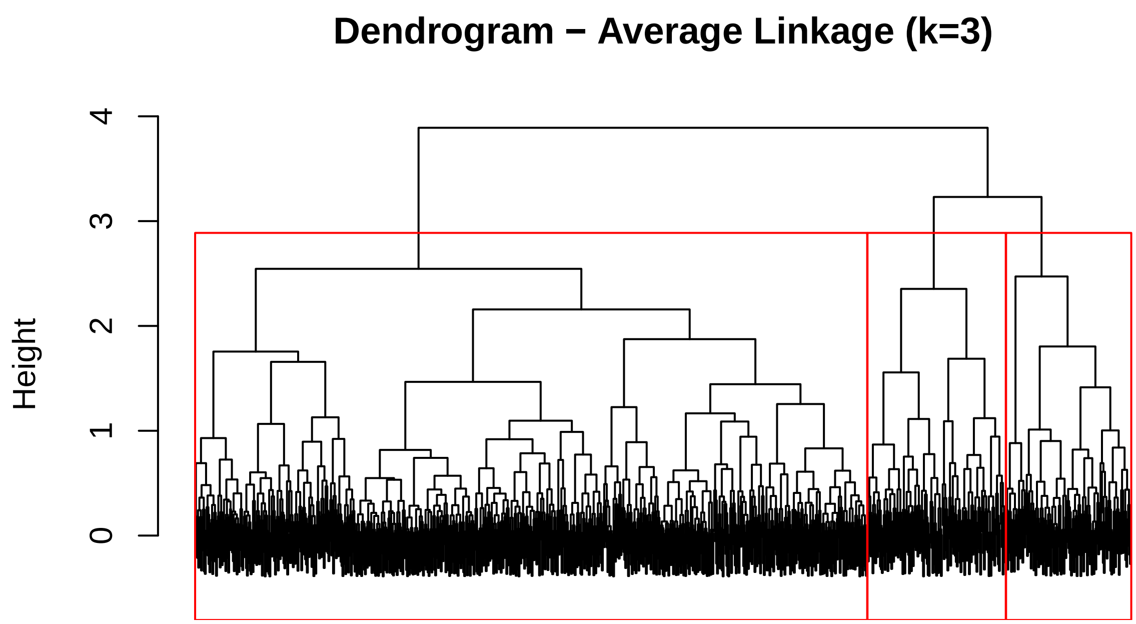



```r
#       B) Cut Tree and Dendrogram (focus on Average Linkage)

# Cut the Average Linkage tree into 3 clusters
clusters_avg <- cutree(hc_average, k = 3)
clusters_avg
```

```
##    [1] 1 1 1 1 1 1 1 2 3 3 1 1 1 1 1 1 1 2 1 1 1 1 1 1 3 2 1 3 3 3 1 1 1 1 1 3 3
##   [38] 1 3 1 1 1 1 1 1 1 1 2 1 2 2 1 1 1 3 3 3 2 3 1 1 3 3 1 1 1 1 1 1 1 1 2 1 1
##   [75] 1 2 1 1 2 3 2 3 1 3 1 1 1 1 1 1 1 1 1 1 2 1 3 1 2 3 1 1 1 2 1 1 1 1 2 1 1
##  [112] 1 1 1 1 1 3 1 1 1 1 1 1 1 1 3 1 1 1 3 1 2 2 3 3 1 1 1 1 1 1 1 1 1 1 3 1 1
##  [149] 3 1 2 2 1 1 1 1 2 1 2 1 3 1 2 2 3 2 1 1 1 1 1 2 3 1 2 1 1 1 1 1 3 1 1 3 3
##  [186] 1 1 1 1 1 1 1 1 1 1 1 3 1 1 1 1 1 1 3 1 1 1 1 1 1 3 3 2 1 1 1 1 3 1 1 1 3
##  [223] 1 1 1 3 1 1 1 2 1 1 1 1 1 3 2 1 3 1 1 1 1 2 2 3 1 3 2 3 1 3 1 1 1 2 1 3 1
##  [260] 1 3 1 1 1 1 1 1 1 3 2 1 3 1 1 1 1 2 3 1 1 1 1 1 1 1 1 1 1 2 1 1 1 3 1 1 1
##  [297] 1 1 2 1 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 2 2 1 1 1 3 1 1 1 1 1 1 3 1
##  [334] 3 3 1 1 1 3 1 1 1 1 2 1 1 3 2 1 1 1 1 1 3 1 1 1 2 1 1 1 2 2 3 2 3 1 1 1 1
##  [371] 1 1 3 1 1 1 2 1 1 1 1 1 3 1 1 2 1 1 3 1 1 1 1 1 1 1 1 1 1 2 1 2 1 1 1 3 1
##  [408] 1 1 1 2 1 1 1 1 1 3 1 2 1 1 1 2 1 1 1 1 1 1 2 1 1 2 1 2 3 3 1 1 1 1 1 1 1
##  [445] 1 2 3 2 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 3 1 1 1 1 1 1 1 3 1 3 1 1 1 1 2 1
##  [482] 2 1 1 2 1 1 3 1 1 1 3 2 2 1 1 1 1 1 2 2 1 2 1 1 2 1 1 1 3 3 1 1 1 1 1 1 1
##  [519] 1 1 1 2 1 1 1 1 3 1 1 2 2 2 1 1 1 1 2 1 2 1 1 3 1 2 1 1 1 1 1 1 1 1 1 1 2
##  [556] 1 1 1 1 2 3 1 1 2 1 1 2 2 1 1 1 1 3 2 1 1 1 3 1 1 1 1 1 3 1 1 2 1 1 1 1 2
##  [593] 1 2 1 1 3 2 1 1 1 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 3 1 1 1 1 2
##  [630] 1 2 2 1 1 1 1 1 1 3 2 1 1 1 2 1 2 1 1 1 1 1 3 2 1 1 1 1 2 1 3 1 3 3 2 1 1
##  [667] 2 3 1 1 1 1 1 1 1 1 1 1 1 3 1 3 3 3 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 2 2 3 2
```

```
## [704] 3 1 3 3 1 1 2 1 3 1 1 1 3 1 1 1 1 3 1 3 3 1 1 1 3 1 1 1 1 1 3 1 1 1 1 2 1
## [741] 1 1 1 1 3 1 1 1 2 3 2 1 1 2 1 1 1 3 1 1 2 1 3 1 1 1 1 1 2 1 2 1 2 1 1 3 1
## [778] 1 3 1 2 1 1 1 1 1 2 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 2 2 1
## [815] 1 1 3 1 1 1 3 1 1 2 3 2 1 1 1 3 1 1 1 3 1 3 3 2 3 1 1 3 1 1 1 1 3 1 1 1 1
## [852] 2 1 2 1 1 1 1 2 3 1 1 1 1 1 1 3 1 3 1 1 1 1 1 1 1 3 1 1 1 3 1 1 3 1 2 1 3
## [889] 1 1 2 1 1 1 1 1 1 1 1 1 1 2 3 1 1 2 3 1 3 1 1 1 1 1 3 1 1 1 3 2 3 3 1 1 1
## [926] 2 1 1 1 1 3 2 3 1 1 1 1 3 1 1 1 2 1 1 3 1 2 2 1 2 3 1 1 3 3 3 3 3 1 1 1 1
## [963] 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 3 1 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
## [1000] 1
```

```r
# Visualize clusters on the Dendrogram
plot(hc_average, main = "Dendrogram - Average Linkage (k=3)",
labels = FALSE, xlab = "", sub = "")
rect.hclust(hc_average, k = 3, border = "red")
```




```r
#       C) Cluster Visualization
data$cluster=clusters_avg
View(data)

table(data[,c(3,13)])
```

```
##              cluster
## city            1   2   3
##   Chicago     228  47  55
##   Los Angeles 243  43  40
##   New York    247  44  53
```

```r
table(data[,c(5,13)])
```

```
##         cluster
## gender     1   2   3
##   Female 350  54  68
##   Male   368  80  80
```

```r
colors=c("red","blue","green")
as.integer(data$cluster)
```

```
##    [1] 1 1 1 1 1 1 1 2 3 3 1 1 1 1 1 1 1 2 1 1 1 1 1 1 3 2 1 3 3 3 1 1 1 1 1 3 3
##   [38] 1 3 1 1 1 1 1 1 1 1 2 1 2 2 1 1 1 3 3 3 2 3 1 1 3 3 1 1 1 1 1 1 1 1 2 1 1
##   [75] 1 2 1 1 2 3 2 3 1 3 1 1 1 1 1 1 1 1 1 1 2 1 3 1 2 3 1 1 1 2 1 1 1 1 2 1 1
##  [112] 1 1 1 1 1 3 1 1 1 1 1 1 1 1 3 1 1 1 3 1 2 2 3 3 1 1 1 1 1 1 1 1 1 1 3 1 1
##  [149] 3 1 2 2 1 1 1 1 2 1 2 1 3 1 2 2 3 2 1 1 1 1 1 2 3 1 2 1 1 1 1 1 3 1 1 3 3
##  [186] 1 1 1 1 1 1 1 1 1 1 1 3 1 1 1 1 1 1 3 1 1 1 1 1 1 3 3 2 1 1 1 1 3 1 1 1 3
##  [223] 1 1 1 3 1 1 1 2 1 1 1 1 1 3 2 1 3 1 1 1 1 2 2 3 1 3 2 3 1 3 1 1 1 2 1 3 1
##  [260] 1 3 1 1 1 1 1 1 1 3 2 1 3 1 1 1 1 2 3 1 1 1 1 1 1 1 1 1 1 2 1 1 1 3 1 1 1
##  [297] 1 1 2 1 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 2 2 1 1 1 3 1 1 1 1 1 1 3 1
##  [334] 3 3 1 1 1 3 1 1 1 1 2 1 1 3 2 1 1 1 1 1 3 1 1 1 2 1 1 1 2 2 3 2 3 1 1 1 1
##  [371] 1 1 3 1 1 1 2 1 1 1 1 1 3 1 1 2 1 1 3 1 1 1 1 1 1 1 1 1 1 2 1 2 1 1 1 3 1
##  [408] 1 1 1 2 1 1 1 1 1 3 1 2 1 1 1 2 1 1 1 1 1 1 2 1 1 2 1 2 3 3 1 1 1 1 1 1 1
##  [445] 1 2 3 2 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 3 1 1 1 1 1 1 1 3 1 3 1 1 1 1 2 1
##  [482] 2 1 1 2 1 1 3 1 1 1 3 2 2 1 1 1 1 1 2 2 1 2 1 1 2 1 1 1 3 3 1 1 1 1 1 1 1
##  [519] 1 1 1 2 1 1 1 1 3 1 1 2 2 2 1 1 1 1 2 1 2 1 1 3 1 2 1 1 1 1 1 1 1 1 1 1 2
##  [556] 1 1 1 1 2 3 1 1 2 1 1 2 2 1 1 1 1 3 2 1 1 1 3 1 1 1 1 1 3 1 1 2 1 1 1 1 2
##  [593] 1 2 1 1 3 2 1 1 1 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 3 1 1 1 1 2
##  [630] 1 2 2 1 1 1 1 1 1 3 2 1 1 1 2 1 2 1 1 1 1 1 3 2 1 1 1 1 2 1 3 1 3 3 2 1 1
##  [667] 2 3 1 1 1 1 1 1 1 1 1 1 1 3 1 3 3 3 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 2 2 3 2
##  [704] 3 1 3 3 1 1 2 1 3 1 1 1 3 1 1 1 1 3 1 3 3 1 1 1 3 1 1 1 1 1 3 1 1 1 1 2 1
##  [741] 1 1 1 1 3 1 1 1 2 3 2 1 1 2 1 1 1 3 1 1 2 1 3 1 1 1 1 1 2 1 2 1 2 1 1 3 1
##  [778] 1 3 1 2 1 1 1 1 1 2 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 2 2 1
##  [815] 1 1 3 1 1 1 3 1 1 2 3 2 1 1 1 3 1 1 1 3 1 3 3 2 3 1 1 3 1 1 1 1 3 1 1 1 1
##  [852] 2 1 2 1 1 1 1 2 3 1 1 1 1 1 1 3 1 3 1 1 1 1 1 1 1 3 1 1 1 3 1 1 3 1 2 1 3
##  [889] 1 1 2 1 1 1 1 1 1 1 1 1 1 2 3 1 1 2 3 1 3 1 1 1 1 1 3 1 1 1 3 2 3 3 1 1 1
##  [926] 2 1 1 1 1 3 2 3 1 1 1 1 3 1 1 1 2 1 1 3 1 2 2 1 2 3 1 1 3 3 3 3 3 1 1 1 1
##  [963] 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 3 1 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
## [1000] 1
```

```r
point=c(3,4,5)
as.integer(data$city)
```

```
##    [1] 3 2 3 1 2 1 1 2 1 2 1 2 3 2 2 2 2 2 3 1 2 2 3 1 2 2 3 3 2 1 1 3 2 2 3 3 2
##   [38] 2 3 3 3 3 3 1 2 3 3 2 2 1 1 1 2 3 2 1 1 1 1 1 3 1 3 1 3 3 2 3 1 2 1 3 1 3
##   [75] 3 3 2 1 2 3 1 1 2 3 1 2 1 3 1 2 1 2 3 2 2 3 2 2 3 1 2 3 3 3 2 1 3 2 2 3 1
##  [112] 3 1 1 3 3 1 2 1 3 2 3 1 3 3 3 3 1 1 2 3 1 1 3 3 2 2 2 1 2 1 3 3 1 2 3 3 1
##  [149] 3 3 1 1 1 2 1 1 3 3 3 1 1 3 2 2 1 2 2 1 3 2 3 2 3 2 3 2 2 1 3 2 2 3 1 3 1
##  [186] 2 3 1 1 2 3 3 2 1 2 3 3 1 2 1 3 3 2 1 3 3 2 3 2 2 1 1 3 3 2 3 3 1 2 3 2 2
##  [223] 3 1 1 1 3 3 3 3 3 1 3 1 3 1 2 3 1 2 2 3 1 2 1 3 1 3 3 2 1 2 3 1 2 2 3 1 1
##  [260] 2 2 3 3 2 2 2 2 2 2 1 1 3 3 3 2 2 3 2 2 2 2 3 1 2 2 1 1 3 2 3 1 3 3 2 2 3
##  [297] 2 3 1 3 3 1 1 3 1 1 2 2 3 1 1 2 2 2 2 3 1 1 2 3 2 1 2 1 2 2 2 2 2 2 2 3 3
##  [334] 2 1 1 3 2 2 2 3 2 1 3 3 1 2 3 1 3 2 2 2 2 1 1 2 3 2 2 1 1 3 3 2 2 2 3 3 1
```

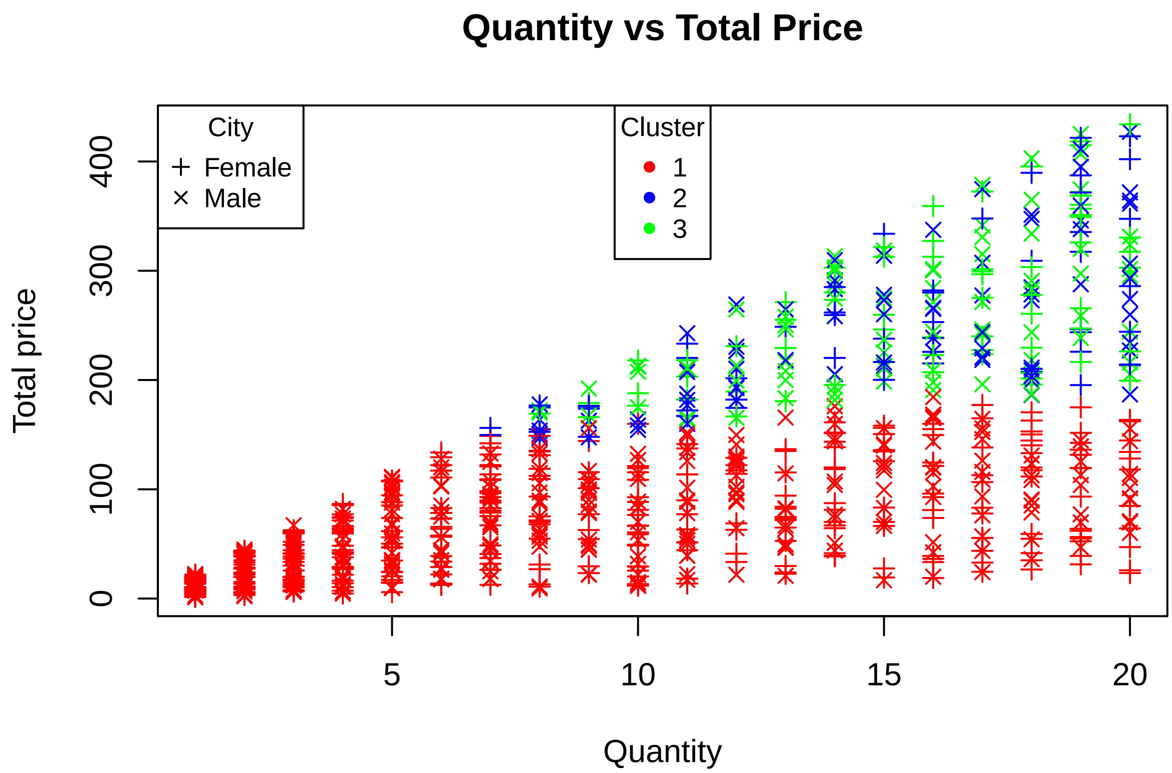
```
##  [371] 2 3 3 1 2 2 3 3 3 2 1 3 1 2 3 2 3 3 1 1 2 2 3 2 1 2 2 3 1 3 2 1 3 2 1 1 2
##  [408] 1 2 2 2 3 1 1 3 1 3 1 3 3 2 3 2 1 2 1 3 1 1 1 1 1 2 2 3 2 2 3 3 1 1 1 1 2
##  [445] 2 2 3 3 2 2 3 3 3 2 1 2 2 3 3 3 3 2 2 2 2 2 2 2 1 2 1 3 1 2 1 2 2 3 2 1 3
##  [482] 1 1 3 1 2 3 3 2 3 3 2 3 2 3 1 1 2 1 1 3 2 3 2 1 1 2 3 1 1 3 1 1 2 3 2 1 3
##  [519] 2 1 1 1 1 2 2 1 3 2 2 1 2 3 3 3 1 1 3 3 1 3 1 3 3 1 2 1 1 3 3 2 1 1 3 3 3
##  [556] 1 1 2 3 3 3 2 2 1 3 1 1 2 2 2 2 2 3 2 2 2 1 1 2 1 2 2 2 2 2 1 3 2 3 2 1 2
##  [593] 1 2 3 3 1 2 2 1 1 2 3 1 3 1 2 2 1 1 3 3 2 3 3 2 1 3 1 2 2 3 2 1 3 3 1 2 2
##  [630] 2 3 1 2 3 3 2 1 1 1 2 1 3 1 3 3 2 1 2 1 1 1 1 3 1 2 1 3 1 1 2 3 3 2 3 2 1
##  [667] 2 3 1 3 3 2 1 1 2 2 3 3 2 1 1 1 3 3 3 1 1 3 3 3 3 2 2 3 3 1 2 1 2 1 1 3 3
##  [704] 3 2 1 1 1 2 1 2 1 2 3 1 3 1 3 1 1 1 3 2 1 3 2 3 3 1 1 1 1 2 2 3 3 3 2 1 3
##  [741] 3 1 1 2 1 1 3 2 1 3 1 1 2 1 1 2 1 2 2 1 2 2 3 2 3 1 1 2 3 3 1 2 3 3 1 1 2
##  [778] 2 2 1 1 3 1 2 3 1 1 1 2 3 2 3 2 3 2 1 3 3 3 1 2 1 1 3 2 1 1 1 2 3 1 2 2 1
##  [815] 1 2 1 2 2 3 2 3 2 1 2 2 1 3 2 3 2 1 3 1 2 3 1 1 3 2 1 1 3 3 3 1 2 3 3 3 2
##  [852] 3 1 1 1 1 2 3 1 3 3 2 1 1 3 1 2 1 2 3 3 1 3 2 1 3 2 3 1 3 3 2 2 3 3 3 1 1
##  [889] 2 3 1 1 2 1 1 2 3 2 1 2 1 3 2 1 2 3 1 2 3 2 1 3 3 3 1 1 3 3 3 2 1 3 3 3 2
##  [926] 3 1 3 2 2 3 2 3 3 1 2 1 3 3 3 1 3 3 2 2 1 1 3 2 1 1 1 1 1 1 1 1 3 1 1 1 1
##  [963] 2 3 2 1 2 3 3 3 1 3 3 3 3 3 1 2 1 2 3 1 1 2 1 3 3 3 2 1 1 3 3 3 3 3 3 3 3
## [1000] 3
```

```r
plot(data$quantity, data$total_price, col = colors[data$cluster],
  pch = point[data$city], xlab = "Quantity", ylab = "Total price",
  main = "Quantity vs Total Price")
legend("top",  legend = unique(data$cluster), col = colors,
  pch = 16, title = "Cluster", cex = 0.8)
legend(  "topleft", legend = levels(data$city), pch = point,
  title = "City", cex = 0.8)
```

```r
as.integer(data$gender)
```

```
##    [1] 2 1 1 2 1 2 2 1 1 2 2 2 1 1 1 2 1 2 2 2 1 2 1 2 2 2 2 1 1 2 2 1 2 2 2 2 2
##   [38] 1 1 1 1 2 1 2 1 2 2 1 1 1 2 2 1 1 2 2 1 2 2 2 2 2 2 1 2 2 2 1 2 1 2 1 2 1
##   [75] 2 2 1 2 1 2 1 2 2 1 1 2 1 1 2 2 1 2 1 1 1 2 2 2 2 2 2 1 2 2 1 2 1 2 2 1 1
##  [112] 2 2 1 2 1 2 1 1 2 1 1 2 1 2 1 2 1 1 1 2 2 2 1 1 1 1 2 2 1 1 1 2 2 2 2 2 2
##  [149] 1 1 2 2 2 2 2 2 1 2 1 1 2 1 2 2 1 2 2 2 2 1 1 1 1 2 2 1 2 2 2 1 1 1 1 1 1
##  [186] 1 2 2 2 1 2 1 2 1 2 1 1 2 2 1 1 1 1 1 1 1 1 1 2 1 1 1 2 2 1 1 1 2 2 2 1 1
##  [223] 2 1 2 2 2 2 1 2 2 2 2 2 1 2 1 2 2 2 2 1 1 1 2 1 2 2 1 1 1 1 2 1 2 2 2 1 2
##  [260] 1 2 2 1 2 1 1 2 2 2 1 2 1 2 2 2 2 1 2 2 1 2 1 1 2 2 2 1 1 1 1 1 1 1 2 2 2
##  [297] 2 1 1 2 2 2 1 2 1 2 1 2 2 2 1 1 1 1 1 1 1 1 1 2 2 1 2 2 1 2 2 2 1 1 1 1 2
##  [334] 2 1 2 1 1 2 1 2 2 2 1 1 2 1 1 2 2 2 1 1 2 1 1 2 2 1 1 1 1 1 2 2 2 1 2 1 2
##  [371] 1 1 2 1 1 1 2 1 1 2 2 1 1 2 2 2 2 2 2 2 2 2 1 2 2 2 1 2 1 2 1 2 1 2 2 2 2
##  [408] 1 1 1 2 2 1 1 2 1 1 1 2 2 1 2 2 1 1 2 1 2 1 1 1 2 1 1 1 2 2 2 1 1 2 1 1 1
##  [445] 1 2 2 2 1 2 1 2 2 2 2 1 2 2 2 1 1 2 1 2 1 2 1 1 2 1 1 1 2 2 2 1 1 2 1 2 1
##  [482] 2 2 2 1 1 2 2 2 1 2 2 2 2 2 2 2 2 1 1 2 2 1 2 2 2 2 1 1 1 1 2 1 2 2 1 2 2
##  [519] 1 2 1 1 2 2 1 1 2 1 1 1 2 2 2 2 2 2 2 1 2 2 1 2 2 2 2 1 2 2 1 1 1 1 2 2 1
##  [556] 2 1 1 1 2 1 1 1 2 2 2 2 2 1 1 2 1 1 1 2 2 2 2 1 2 1 2 2 2 2 2 1 2 2 2 2 2
##  [593] 1 1 1 2 1 2 1 2 1 2 2 2 1 2 2 2 2 1 2 2 2 1 2 1 2 1 1 1 1 1 2 1 2 2 1 2 2
##  [630] 2 1 2 2 1 1 1 2 2 2 2 1 1 1 2 2 2 2 2 1 2 1 1 2 1 2 1 2 1 2 1 1 2 2 2 2 1
##  [667] 1 2 1 1 2 2 2 2 2 2 2 2 2 1 2 1 2 1 1 1 2 1 1 1 2 2 2 1 1 1 1 1 1 2 1 2 1
##  [704] 2 1 2 1 1 2 2 1 2 1 2 2 1 2 2 1 1 1 1 2 2 1 2 2 1 2 2 2 1 2 2 1 2 1 1 2 1
##  [741] 2 2 1 2 2 1 2 1 1 2 2 1 2 2 2 1 1 2 2 2 1 2 1 1 2 1 2 1 2 1 2 2 1 1 1 2 1
##  [778] 2 1 2 2 1 1 2 1 2 2 1 1 1 2 2 2 1 1 2 2 1 1 1 1 2 1 1 1 1 2 2 1 1 1 1 2 1
##  [815] 2 1 1 2 1 1 1 1 1 1 2 2 2 1 2 2 1 1 2 2 1 2 2 1 2 1 1 2 2 2 1 1 2 1 2 2 2
##  [852] 1 1 2 2 1 2 1 2 2 1 1 2 2 1 1 2 2 1 2 1 2 1 2 1 2 2 2 2 1 1 1 1 1 2 1 1 1
##  [889] 1 1 1 2 1 2 2 1 1 1 1 2 2 2 2 1 2 2 1 2 1 2 2 1 2 1 2 2 1 2 1 1 1 1 1 2 1
##  [926] 2 2 1 2 2 2 2 1 1 1 2 1 1 2 1 1 1 2 1 1 2 2 2 1 2 2 2 2 2 2 1 2 1 2 1 2 2
##  [963] 2 1 1 1 2 2 2 2 2 1 1 1 2 2 1 2 1 2 2 2 1 1 2 2 2 1 1 1 1 2 1 1 2 1 2 1 1
## [1000] 2
```

```r
plot(data$quantity,data$total_price,col=colors[as.integer(data$cluster)],
pch=point[as.integer(data$gender)], xlab = "Quantity", ylab = "Total price",
  main = "Quantity vs Total Price")
legend("top", legend = unique(data$cluster), col = colors,
  pch = 16, title = "Cluster", cex = 0.8)
legend(  "topleft", legend = levels(data$gender), pch = point,
  title = "City", cex = 0.8)
```




```r
# 4. K-means Clustering
data_km= kmeans(data_scaled, 3, nstart = 50)
data_km
```

```
## K-means clustering with 3 clusters of sizes 256, 309, 435
##
## Cluster means:
##    unit_price   quantity        tax total_price reward_points
## 1  0.88398731  0.8529929  1.4178263   1.4178297     1.0124029
## 2 -0.81434909  0.6708024 -0.2538953  -0.2538929    -0.2494080
## 3  0.05823705 -0.9784923 -0.6540457  -0.6540494    -0.4186392
##
## Clustering vector:
##    [1] 3 2 2 3 3 3 3 3 1 1 3 2 3 3 3 3 3 1 3 3 2 3 2 3 1 1 3 1 1 1 3 3 2 3 3 1 1
##   [38] 2 1 2 2 2 3 3 3 3 2 2 3 1 1 3 3 2 1 1 1 1 1 3 3 1 1 2 2 3 3 3 2 2 3 2 2 3
##   [75] 2 3 2 2 2 1 1 1 3 1 2 2 3 3 3 2 3 3 2 2 3 3 1 3 1 1 2 3 3 1 2 3 3 2 3 2 1
##  [112] 3 3 2 2 2 1 2 3 2 3 2 2 3 3 1 2 3 3 1 2 1 1 1 1 3 3 2 2 2 3 3 3 1 3 1 2 2
##  [149] 1 2 1 1 2 3 2 2 1 3 1 3 1 3 1 3 1 1 2 2 3 2 3 1 1 3 1 3 2 2 3 3 1 2 2 1 1
##  [186] 3 2 3 3 2 2 2 3 3 3 3 1 2 1 3 2 3 2 1 3 3 2 2 3 2 1 1 1 3 2 3 2 1 2 2 2 1
##  [223] 3 2 2 1 3 3 2 1 3 3 3 3 3 1 3 3 1 2 2 2 3 2 1 1 3 1 2 1 3 1 3 3 3 1 3 1 3
##  [260] 1 1 3 2 3 2 2 3 2 1 1 3 1 2 3 2 2 1 1 2 3 3 3 3 2 3 2 3 2 3 3 2 2 1 2 3 3
##  [297] 3 3 1 2 3 1 1 2 3 3 3 3 3 2 3 3 2 3 3 3 2 1 3 1 1 2 3 2 1 2 2 3 2 2 3 1 3
##  [334] 1 1 2 3 3 1 1 2 3 2 3 3 2 1 3 2 3 2 2 2 1 2 3 2 2 2 2 3 1 1 1 1 1 3 3 3 3
##  [371] 2 3 1 2 3 3 2 2 3 3 2 3 1 2 3 1 3 2 1 2 3 3 2 3 3 2 3 3 2 1 3 3 3 3 2 1 3
##  [408] 2 3 2 1 3 3 3 3 2 1 2 1 2 3 3 1 3 2 2 3 3 2 1 3 3 1 3 1 1 1 3 3 3 3 2 3 3
##  [445] 3 1 1 3 2 1 2 3 3 3 1 3 3 2 2 3 2 3 3 2 1 3 1 3 3 3 2 2 1 3 1 3 3 3 3 3 2
```

```
##   [482] 1 2 2 1 3 3 1 3 3 2 1 1 3 3 2 2 3 2 1 1 3 1 3 2 2 2 3 3 1 1 3 3 3 3 2 3 3
##   [519] 2 3 3 1 3 2 2 3 1 2 2 2 2 3 2 3 1 3 2 1 1 3 3 1 2 1 3 3 2 2 3 2 2 3 2 3 1
##   [556] 3 2 3 2 1 1 3 3 2 2 2 3 1 2 2 3 3 1 1 2 3 3 1 2 3 2 3 2 1 2 2 3 3 2 3 3 1
##   [593] 2 1 2 3 1 2 3 2 2 1 2 3 3 3 3 2 3 3 3 3 3 3 3 3 3 3 2 2 2 1 3 1 3 3 3 3 1
##   [630] 2 1 1 3 3 3 3 3 2 1 1 3 3 3 1 3 1 2 2 2 2 3 1 1 2 3 3 3 1 3 1 3 1 1 1 3 3
##   [667] 1 1 3 1 3 2 3 3 3 3 3 3 2 1 3 1 1 1 3 3 3 3 3 2 2 3 3 3 2 2 1 3 3 1 1 1 1
##   [704] 1 2 1 1 3 3 1 3 1 3 3 3 1 2 2 2 2 1 3 1 1 2 3 2 1 2 1 2 2 2 1 3 1 2 3 1 2
##   [741] 3 3 2 3 1 2 3 3 2 1 1 3 3 1 3 2 2 1 3 3 1 3 1 2 2 3 3 3 1 2 2 3 2 3 2 1 3
##   [778] 3 1 3 1 3 3 2 3 2 1 2 2 3 3 2 2 2 3 3 2 2 3 2 2 3 2 3 1 2 3 3 3 3 2 1 1 2
##   [815] 3 2 1 3 2 3 1 2 3 3 1 1 2 3 2 1 2 2 3 1 1 1 1 1 1 3 3 1 3 3 2 3 1 2 3 2 2
##   [852] 1 3 3 2 2 2 3 3 1 2 2 3 3 3 2 1 2 1 2 2 2 2 2 2 2 1 2 3 3 1 3 1 1 3 3 2 1
##   [889] 2 3 3 3 2 2 2 2 3 2 3 3 3 1 1 3 3 1 1 3 1 2 3 3 3 3 1 3 3 2 1 2 1 1 3 3 3
##   [926] 1 2 3 2 2 1 1 1 3 2 3 2 1 3 3 3 2 3 2 1 3 1 1 3 2 1 3 2 1 1 1 1 1 3 3 3 3
##   [963] 1 3 2 2 2 3 2 3 2 3 2 3 3 2 3 2 1 2 1 3 3 3 3 2 2 3 3 2 3 3 2 2 3 2 3 3 3
## [1000] 3
## 
## Within cluster sum of squares by cluster:
## [1] 896.8464 442.9013 707.9212
##  (between_SS / total_SS =  59.0 %)
## 
## Available components:
## 
## [1] "cluster"      "centers"      "totss"        "withinss"     "tot.withinss"
## [6] "betweenss"    "size"         "iter"         "ifault"
```

```r
data_km$cluster
```

```
##    [1] 3 2 2 3 3 3 3 3 1 1 3 2 3 3 3 3 3 1 3 3 2 3 2 3 1 1 3 1 1 1 3 3 2 3 3 1 1
##   [38] 2 1 2 2 2 3 3 3 3 2 2 3 1 1 3 3 2 1 1 1 1 1 3 3 1 1 2 2 3 3 3 2 2 3 2 2 3
##   [75] 2 3 2 2 2 1 1 1 3 1 2 2 3 3 3 2 3 3 2 2 3 3 1 3 1 1 2 3 3 1 2 3 3 2 3 2 1
##  [112] 3 3 2 2 2 1 2 3 2 3 2 2 3 3 1 2 3 3 1 2 1 1 1 1 3 3 2 2 2 3 3 3 1 3 1 2 2
##  [149] 1 2 1 1 2 3 2 2 1 3 1 3 1 3 1 3 1 1 2 2 3 2 3 1 1 3 1 3 2 2 3 3 1 2 2 1 1
##  [186] 3 2 3 3 2 2 2 3 3 3 3 1 2 1 3 2 3 2 1 3 3 2 2 3 2 1 1 1 3 2 3 2 1 2 2 2 1
##  [223] 3 2 2 1 3 3 2 1 3 3 3 3 3 1 3 3 1 2 2 2 3 2 1 1 3 1 2 1 3 1 3 3 3 1 3 1 3
##  [260] 1 1 3 2 3 2 2 3 2 1 1 3 1 2 3 2 2 1 1 2 3 3 3 3 2 3 2 3 2 3 3 2 2 1 2 3 3
##  [297] 3 3 1 2 3 1 1 2 3 3 3 3 3 2 3 3 2 3 3 3 2 1 3 1 1 2 3 2 1 2 2 3 2 2 3 1 3
##  [334] 1 1 2 3 3 1 1 2 3 2 3 3 2 1 3 2 3 2 2 2 1 2 3 2 2 2 2 3 1 1 1 1 1 3 3 3 3
##  [371] 2 3 1 2 3 3 2 2 3 3 2 3 1 2 3 1 3 2 1 2 3 3 2 3 3 2 3 3 2 1 3 3 3 3 2 1 3
##  [408] 2 3 2 1 3 3 3 3 2 1 2 1 2 3 3 1 3 2 2 3 3 2 1 3 3 1 3 1 1 1 3 3 3 3 2 3 3
##  [445] 3 1 1 3 2 1 2 3 3 3 1 3 3 2 2 3 2 3 3 2 1 3 1 3 3 3 2 2 1 3 1 3 3 3 3 3 2
##  [482] 1 2 2 1 3 3 1 3 3 2 1 1 3 3 2 2 3 2 1 1 3 1 3 2 2 2 3 3 1 1 3 3 3 3 2 3 3
##  [519] 2 3 3 1 3 2 2 3 1 2 2 2 2 3 2 3 1 3 2 1 1 3 3 1 2 1 3 3 2 2 3 2 2 3 2 3 1
##  [556] 3 2 3 2 1 1 3 3 2 2 2 3 1 2 2 3 3 1 1 2 3 3 1 2 3 2 3 2 1 2 2 3 3 2 3 3 1
##  [593] 2 1 2 3 1 2 3 2 2 1 2 3 3 3 3 2 3 3 3 3 3 3 3 3 3 3 2 2 2 1 3 1 3 3 3 3 1
##  [630] 2 1 1 3 3 3 3 3 2 1 1 3 3 3 1 3 1 2 2 2 2 3 1 1 2 3 3 3 1 3 1 3 1 1 1 3 3
##  [667] 1 1 3 1 3 2 3 3 3 3 3 3 2 1 3 1 1 1 3 3 3 3 3 2 2 3 3 3 2 2 1 3 3 1 1 1 1
##  [704] 1 2 1 1 3 3 1 3 1 3 3 3 1 2 2 2 2 1 3 1 1 2 3 2 1 2 1 2 2 2 1 3 1 2 3 1 2
##  [741] 3 3 2 3 1 2 3 3 2 1 1 3 3 1 3 2 2 1 3 3 1 3 1 2 2 3 3 3 1 2 2 3 2 3 2 1 3
##  [778] 3 1 3 1 3 3 2 3 2 1 2 2 3 3 2 2 2 3 3 2 2 3 2 2 3 2 3 1 2 3 3 3 3 2 1 1 2
##  [815] 3 2 1 3 2 3 1 2 3 3 1 1 2 3 2 1 2 2 3 1 1 1 1 1 1 3 3 1 3 3 2 3 1 2 3 2 2
##  [852] 1 3 3 2 2 2 3 3 1 2 2 3 3 3 2 1 2 1 2 2 2 2 2 2 2 1 2 3 3 1 3 1 1 3 3 2 1
##  [889] 2 3 3 3 2 2 2 2 3 2 3 3 3 1 1 3 3 1 1 3 1 2 3 3 3 3 1 3 3 2 1 2 1 1 3 3 3
##  [926] 1 2 3 2 2 1 1 1 3 2 3 2 1 3 3 3 2 3 2 1 3 1 1 3 2 1 3 2 1 1 1 1 1 3 3 3 3
##  [963] 1 3 2 2 2 3 2 3 2 3 2 3 3 2 3 2 1 2 1 3 3 3 3 2 2 3 3 2 3 3 2 2 3 2 3 3 3
```

```
## [1000] 3
```

```
centroids=data_km$centers
centroids
```

```
##    unit_price   quantity        tax total_price reward_points
## 1  0.88398731  0.8529929  1.4178263   1.4178297     1.0124029
## 2 -0.81434909  0.6708024 -0.2538953  -0.2538929    -0.2494080
## 3  0.05823705 -0.9784923 -0.6540457  -0.6540494    -0.4186392
```

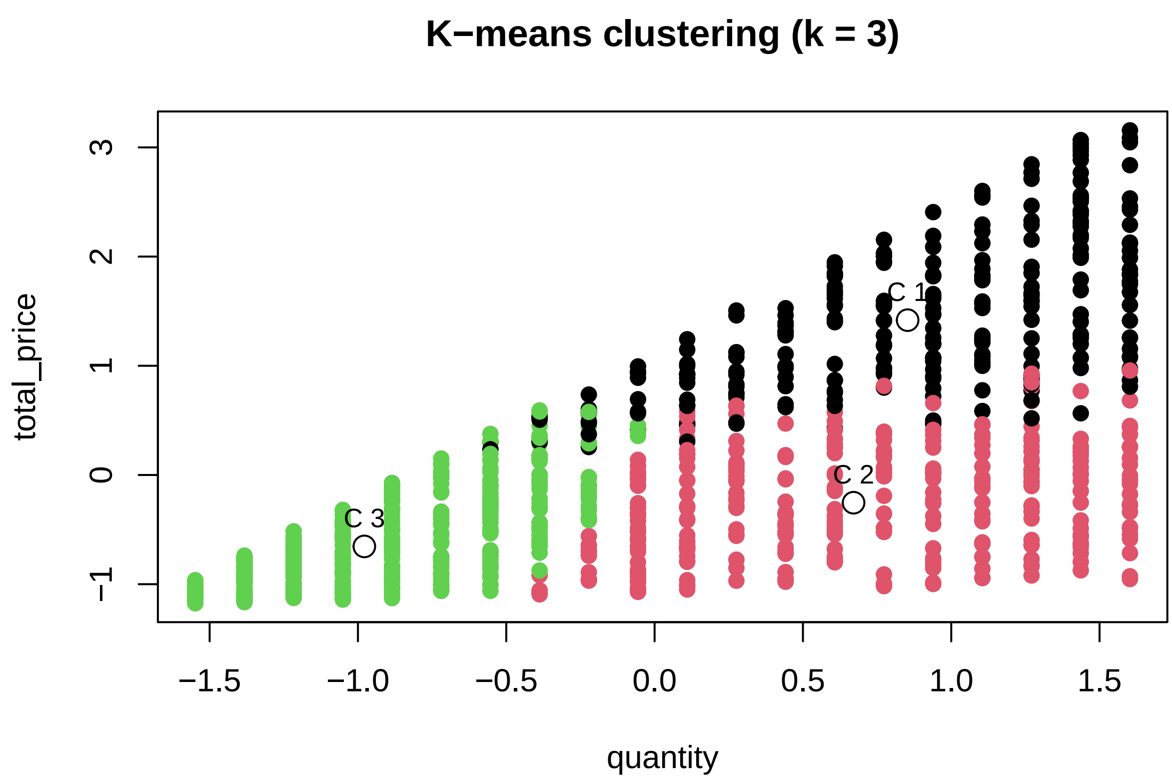
```
data_km$iter
```

```
## [1] 4
```

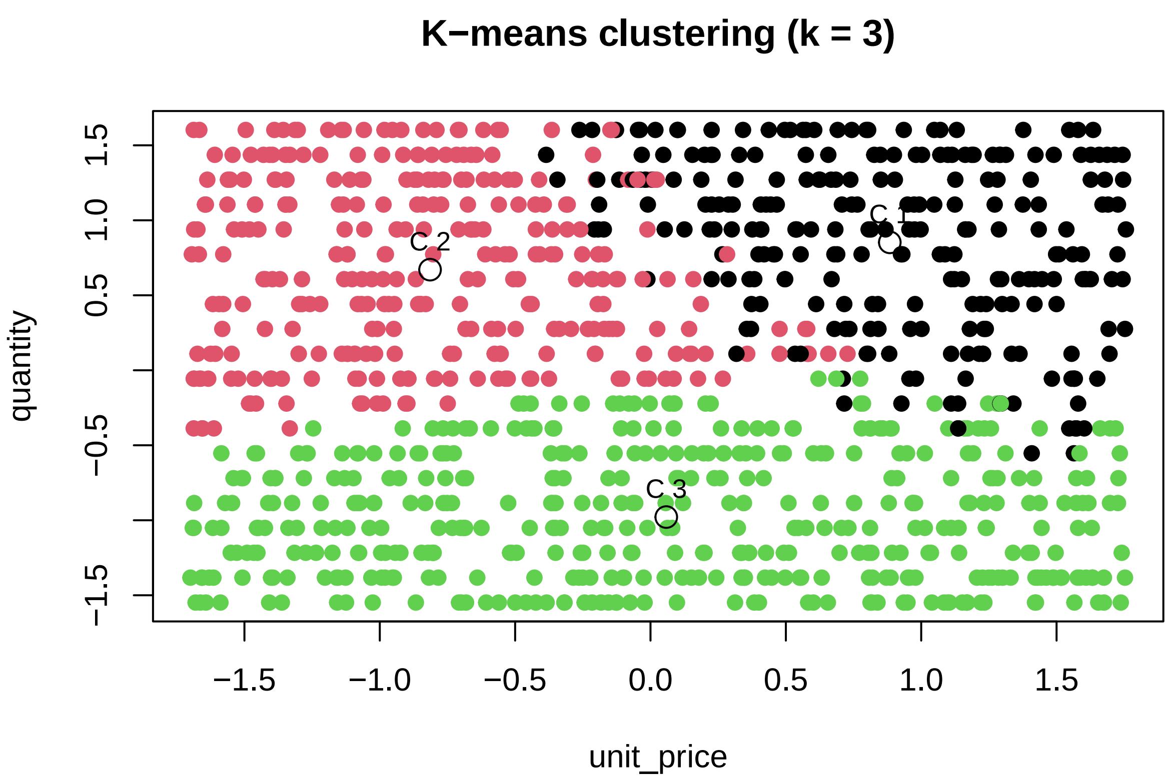
```
data_km$ifault
```

```
## [1] 0
```

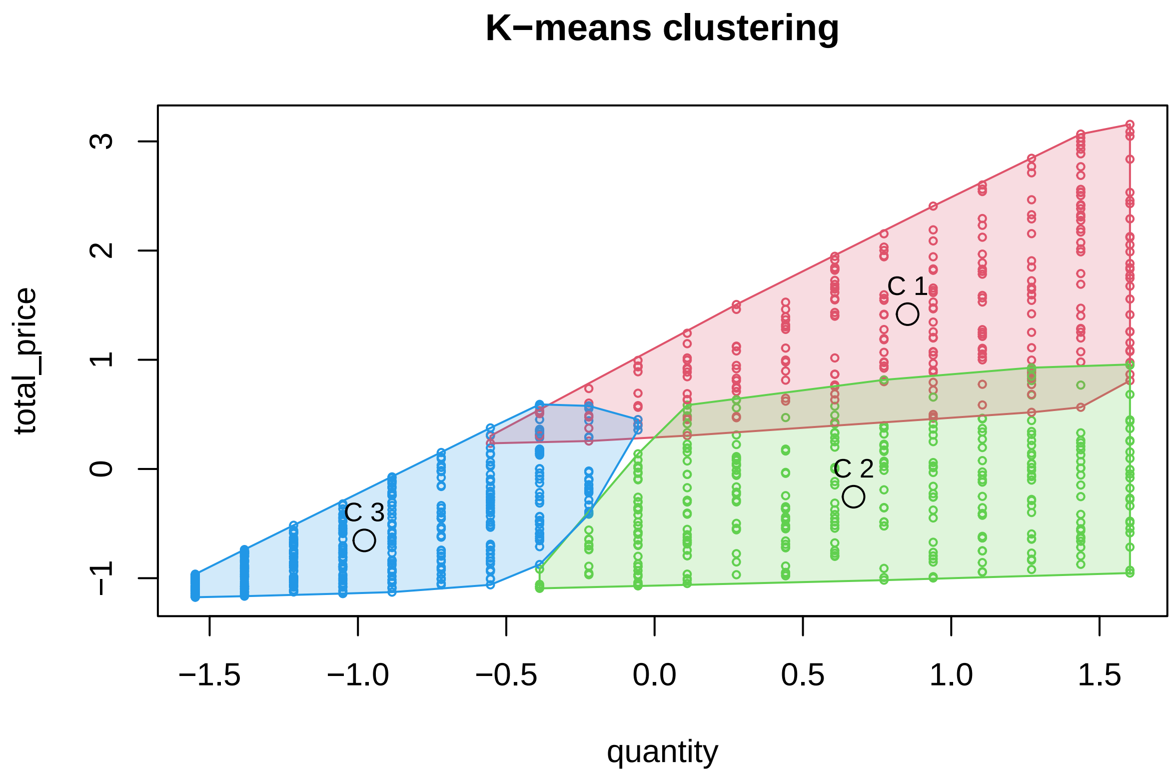
```
View(data_scaled)

plot(data_scaled[, 2], data_scaled[, 4], col = data_km$cluster,
pch = 19,xlab = colnames(data_scaled)[2],  ylab = colnames(data_scaled)[4],
main = "K-means clustering (k = 3)")
points(centroids[, 2], centroids[, 4],pch = 1,cex = 1.5,  col = "black")
text(centroids[, 2], centroids[, 4], labels = paste("C", 1:nrow(centroids)),
pos = 3, cex = 0.8)
```



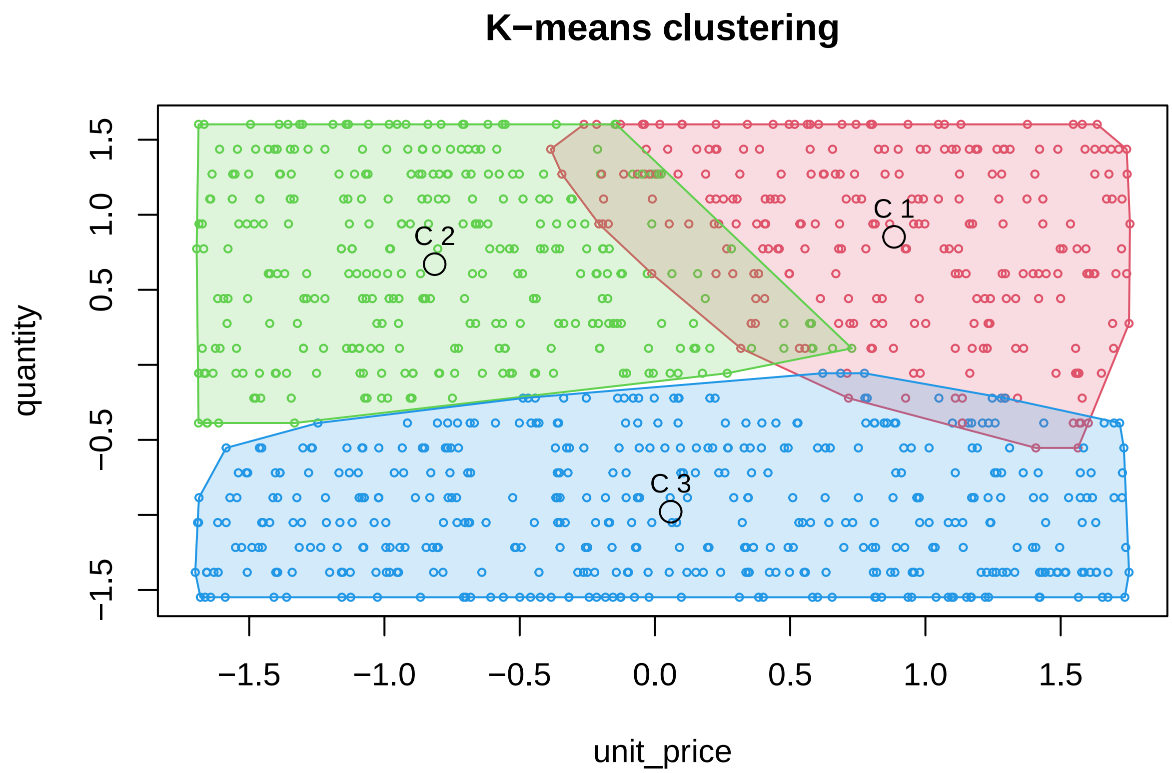
```r
plot(data_scaled[, 1], data_scaled[, 2], col = data_km$cluster,
pch = 19,xlab = colnames(data_scaled)[1], ylab = colnames(data_scaled)[2],
main = "K-means clustering (k = 3)")
points(centroids[, 1], centroids[, 2],pch = 1,cex = 1.5,  col = "black")
text(centroids[, 1], centroids[, 2], labels = paste("C", 1:nrow(centroids)),
pos = 3, cex = 0.8)
```




```r
hullplot(data_scaled[, c(2, 4)], data_km$cluster,main = "K-means clustering")
points(centroids[, 2], centroids[,4],pch = 1,cex = 1.5,col = "black")
text(centroids[, 2], centroids[, 4],labels = paste("C", 1:nrow(centroids)),pos = 3,cex = 0.8)
```




```r
hullplot(data_scaled[, c(1, 2)], data_km$cluster,main = "K-means clustering")
points(centroids[, 1], centroids[, 2],pch = 1,cex = 1.5,col = "black")
text(centroids[, 1], centroids[, 2],labels = paste("C", 1:nrow(centroids)),pos = 3,cex = 0.8)
```




```r
# =================================================================
# PRINCIPAL COMPONENT ANALYSIS
# =================================================================

# 1. DATA PREPARATION FOR PCA
# Selecting only numeric columns for PCA
library(factoextra)
```

```
## Warning: il pacchetto 'factoextra' è stato creato con R versione 4.5.2

## Welcome! Want to learn more? See two factoextra-related books at https://goo.gl/ve3WBa
```

```r
numeric_cols <- data[, c("unit_price", "quantity", "tax", "total_price", "reward_points")]

#Scaling the data (normalization)
data_scaled <- scale(numeric_cols)

# 2. IMPLEMENTATION OF PCA
data.pca=prcomp(data_scaled)

#View PCA results (standard deviations, rotations/loadings)
data.pca
```

```
## Standard deviations (1, .., p=5):
```

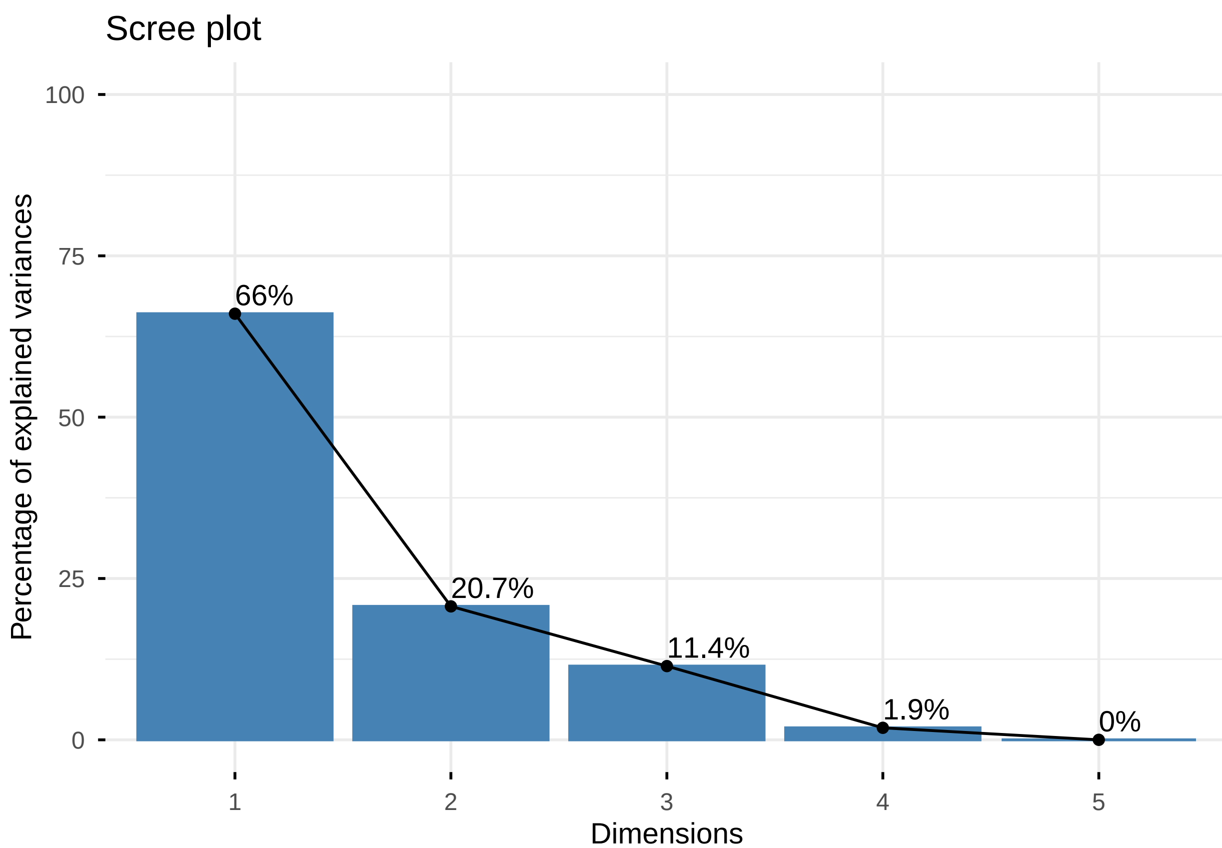
```
## [1] 1.8170424936 1.0168631527 0.7558783845 0.3049486613 0.0002950077
##
## Rotation (n x k) = (5 x 5):
##                    PC1          PC2        PC3         PC4           PC5
## unit_price    0.3386671  0.745248905  0.1848882  0.54380608 -2.108033e-05
## quantity      0.3839801 -0.666564068  0.1818530  0.61252027 -2.621510e-05
## tax           0.5387977 -0.002857421  0.2145163 -0.40459271 -7.070901e-01
## total_price   0.5387992 -0.002859670  0.2145124 -0.40453440  7.071235e-01
## reward_points 0.3965556  0.016735284 -0.9169041  0.04183928 -4.677485e-06
```

```
# 3. ANALYZING VARIANCE EXPLAINED
# Extract eigenvalues and explained variance from the PCA results
eig.values=get_eigenvalue(data.pca)
eig.values
```

```
##         eigenvalue variance.percent cumulative.variance.percent
## Dim.1 3.301643e+00     6.603287e+01                    66.03287
## Dim.2 1.034011e+00     2.068021e+01                    86.71308
## Dim.3 5.713521e-01     1.142704e+01                    98.14012
## Dim.4 9.299369e-02     1.859874e+00                   100.00000
## Dim.5 8.702953e-08     1.740591e-06                   100.00000
```

```
# Scree plot shows the variance explained by each principal component.
fviz_eig(data.pca, addlabels = TRUE, ylim = c(0, 100))
```

```
## Warning in geom_bar(stat = "identity", fill = barfill, color = barcolor, :
## Ignoring empty aesthetic: ‘width‘.
```




```
# 4. PCA PROJECTION AND CLUSTER ANALYSIS
data.pca$x
```

```
##              PC1         PC2          PC3          PC4           PC5
##  [1,] -2.08242303  0.119273378 -0.329309568 -0.4534867104 -4.835642e-04
##  [2,] -1.71341045 -1.011814347 -0.057640090 -0.1011061791 -4.973866e-04
##  [3,] -1.55353195 -1.762147220 -0.098335771  0.3479656290 -1.384251e-05
##  [4,] -1.60346182  0.191784324  0.005868754 -0.2328434806  1.763940e-05
##  [5,] -1.81026445 -0.579652083 -0.334007857 -0.3002961083 -4.897382e-04
##  [6,] -0.42997898  0.189660217  0.522105671 -0.0410606754  1.433029e-04
##  [7,] -2.01392191  1.011155780 -0.151554420 -0.1201289563 -2.153278e-05
##  [8,]  0.70537203  1.087708452  1.034702324  0.0723024630  5.264992e-04
##  [9,]  3.81150571 -0.608282446 -1.159475270 -0.0908306823  4.896093e-05
## [10,]  5.06796936  0.061632861 -1.431649189 -0.3394101726 -7.030010e-05
## [11,] -1.08395498  0.165211403  0.235485636 -0.1141035278 -3.055524e-04
## [12,] -0.36761456 -0.687787562 -0.395902298  0.0497782066 -2.319349e-04
## [13,] -2.22981527 -0.278001279 -0.287392679 -0.5351101384  1.454691e-04
## [14,] -1.31664324  0.328953025 -0.330965274 -0.1633708777  3.684995e-04
## [15,] -0.82175390  1.918713700  0.408699363  0.4953165586  7.529844e-05
## [16,] -1.48187317 -0.430486408  0.049196058 -0.1868533565 -1.863579e-04
## [17,] -2.52760966 -0.206042998 -0.425618450 -0.7705820540 -1.497381e-04
## [18,]  1.19805229 -0.218945528  1.216619772 -0.0880027789  3.457428e-04
## [19,] -0.93171210  2.051788524 -0.098710069  0.5889601294  2.485429e-04
## [20,] -1.26610773 -0.246677973  0.147491876 -0.1512707483 -1.044015e-04
## [21,] -1.31946650 -0.915875551  0.117960195 -0.0250445665 -2.739510e-04
## [22,] -2.28796855  0.506898624 -0.294545231 -0.4546288021 -3.833951e-04
```

```
## [23,] -1.20911304 -1.773146675 -0.179795183  0.4150194023  3.896562e-04
## [24,] -1.14498443  1.975018431  0.271478205  0.5292074783 -3.140960e-04
## [25,]  2.22532244 -0.762322669 -0.790535994  0.0848572851 -3.957111e-04
## [26,]  1.58013499 -0.243228025  1.379438472 -0.1315394050  3.320268e-04
## [27,] -1.83640142  0.754006451 -0.316013590 -0.1916750748 -2.909147e-04
## [28,]  3.67704192  0.019802082 -1.092463930 -0.1551816066 -2.104752e-04
## [29,]  2.32035667 -0.525149196 -0.864044042  0.0344552214 -2.712663e-04
## [30,]  1.79803259 -0.298958464 -0.736169722  0.0285751581 -1.227174e-04
## [31,] -2.25044266 -0.112643382 -0.411802479 -0.5760452492 -2.778282e-04
## [32,] -2.09328599  0.105103843 -0.334587731 -0.4605295363  2.140657e-04
## [33,]  0.02161637 -1.250529708  0.703083049  0.2208511198 -1.292893e-04
## [34,] -0.93532685 -0.209899493  0.293958477 -0.0913436653  2.099515e-04
## [35,] -1.52034482  1.252867212 -0.156643787  0.1057539827 -1.188324e-04
## [36,]  2.06139100 -0.196735956 -0.854534393  0.0098629964 -2.717621e-04
## [37,]  1.70540205 -0.567757752 -0.778902526  0.0775652627 -2.984376e-04
## [38,] -0.40525441 -1.098124835 -0.414015621  0.1737102054  1.652767e-04
## [39,]  2.69835373  0.090472373 -0.926706842 -0.0554630181 -2.630723e-04
## [40,] -1.91168456 -1.325321409 -0.145466300 -0.0211541145  9.229150e-05
## [41,]  0.15343573  0.041613786 -0.509016119  0.0089793040  1.754631e-05
## [42,]  0.09868495 -0.177387848  0.744748727 -0.0257563223  3.502521e-04
## [43,] -2.36141455  0.056924530 -0.341772152 -0.6134910909  4.110836e-04
## [44,] -0.88720281  2.121394837 -0.076309832  0.6301552046 -1.975572e-04
## [45,]  0.60589851  0.901288865 -0.525653141  0.1243016430 -1.041911e-04
## [46,]  0.63882304  0.929568519 -0.510638267  0.1299787536  2.194145e-04
## [47,] -2.12625617 -0.979340413 -0.244358188 -0.2930251432 -3.109229e-04
## [48,]  1.33229227 -1.194911683  1.265669372  0.1049553562  2.352887e-04
## [49,] -2.18652171  0.332779702 -0.253596403 -0.4491132723  3.652067e-04
## [50,]  2.96564106 -0.250089183  1.967657690 -0.3699472355 -4.287915e-04
## [51,]  3.10998062 -0.183599269  2.029630495 -0.4064744705 -2.635160e-04
## [52,] -1.00021865  0.633233052 -0.299565734 -0.0399280142 -3.921452e-04
## [53,] -1.42838299  2.088020500  0.153928496  0.5939878817  4.074355e-04
## [54,] -0.90699749 -1.207291187  0.299368204  0.1576487064 -3.593254e-04
## [55,]  2.13762844  0.337560899 -0.812734844 -0.0104007682 -9.717832e-05
## [56,]  2.15388414  1.222949572 -0.783503269  0.1219875762  2.316477e-04
## [57,]  5.79081414  0.323239382 -1.586393570 -0.4619200892 -3.922325e-04
## [58,]  4.22016602  0.080558945  2.501916859 -0.6844365782  5.112157e-04
## [59,]  4.32344121 -0.432934747 -1.288964656 -0.1824922174  2.594405e-04
## [60,] -0.89622231  1.108912637 -0.238442500  0.1236305557 -2.767491e-04
## [61,] -1.85014218  0.733385291 -0.322870796 -0.2034659047  4.776550e-04
## [62,]  3.47501259  0.702566979 -1.048713135 -0.1107658406 -3.343465e-04
## [63,]  2.97789925 -0.687388382 -0.933750284  0.0078931922  1.292502e-04
## [64,]  0.28798295  0.111463815 -0.565800054  0.0175053750  1.191120e-04
## [65,] -0.33419729 -0.459634551  0.553338510 -0.0047102121 -4.648865e-04
## [66,] -1.77902937  0.844238581 -0.287105715 -0.1380298641 -4.948445e-04
## [67,] -0.40045217  0.216236827 -0.397174779 -0.0284713687  3.389999e-04
## [68,] -2.29204784  0.500453372 -0.296602632 -0.4584746257  2.737408e-04
## [69,] -1.44092959 -1.874815119 -0.163903994  0.4449850219  2.224635e-05
## [70,] -1.32822090 -1.452091765  0.115112851  0.2100119746  4.616887e-04
## [71,] -1.65314549  1.173232860  0.015260291  0.0513070390 -3.791225e-04
## [72,]  0.50777100  0.348408262  0.930805074 -0.0360074757  2.047547e-04
## [73,]  0.54841364 -1.541258524  0.930095757  0.3190813620 -1.890415e-04
## [74,] -1.04536944  2.132282138  0.321710685  0.6229859764 -2.363422e-04
## [75,] -0.52905616 -0.369089659  0.469566331 -0.0299010131 -2.434115e-04
## [76,]  0.56928534  1.316333060  0.983601812  0.1508309518 -1.355012e-04
```

```
##    [77,] -0.42461848 -0.060258273  0.519686370 -0.0449218455  2.239992e-04
##    [78,]  0.39179524 -1.610270052  0.863061000  0.3620648086  2.112624e-04
##    [79,]  0.50247678  0.344556214  0.928437292 -0.0361775672 -1.086474e-04
##    [80,]  3.41469446  0.147491605 -1.085561183 -0.1245635503  2.509509e-04
##    [81,]  2.31023358 -0.762938049  1.684312165 -0.1623066133  4.624239e-04
##    [82,]  2.31188896 -0.079646080 -0.861947967 -0.0175485767  2.372765e-04
##    [83,] -0.22311838  0.641513249  0.622622885  0.0203891443 -1.395293e-04
##    [84,]  1.62643269  0.922226454 -0.668597161  0.0912679914  2.288440e-04
##    [85,] -1.21056467 -1.210586201  0.165989430  0.1132138647 -1.080166e-04
##    [86,] -1.32232924 -0.722878978  0.117560984 -0.0838566419  1.219428e-05
##    [87,] -2.47824116 -0.127408816 -0.400676994 -0.7233631522  3.945087e-04
##    [88,] -0.22603170  0.638941948  0.621289675  0.0198131064 -2.608335e-04
##    [89,] -1.09908679  2.047204135  0.294604504  0.5721236468 -4.772398e-04
##    [90,] -0.32847453  0.010367476  0.562730808 -0.0411155265 -2.803415e-04
##    [91,] -0.69446050  1.810414815 -0.123230266  0.4555900597 -3.657023e-05
##    [92,] -0.97187254  1.988626034 -0.118945581  0.5514084819  3.912938e-04
##    [93,]  0.42108074 -0.277791850 -0.514463617  0.0367268975 -4.671697e-04
##    [94,] -1.12599418 -0.519571927 -0.378167673 -0.1092629097 -3.839941e-05
##    [95,]  0.67535411  1.063293515  1.021104648  0.0685881660 -4.541693e-04
##    [96,] -2.28156598  0.519798442 -0.291129567 -0.4456185440  3.238132e-04
##    [97,]  4.48009237  0.325679362 -1.327360668 -0.2559977556 -1.188089e-04
##    [98,] -1.18774802  1.412357606  0.228927757  0.2270930869 -4.525600e-04
##    [99,]  2.13886235 -0.186964833  1.618044998 -0.2206605363 -1.928018e-04
##   [100,]  1.99335884 -0.854936684 -0.774096747  0.1246057309  1.461240e-04
##   [101,]  0.05344896 -1.912393227 -0.327726832  0.5989257295 -4.180915e-04
##   [102,] -0.82588319  1.447677793  0.387683294  0.2622986526  3.283048e-04
##   [103,] -0.81174314 -0.112258371  0.349747457 -0.0791236992  8.966538e-05
##   [104,]  3.17432103  0.084056730  2.061013357 -0.4356043246  3.253356e-04
##   [105,]  0.44269397 -1.432636101 -0.395010545  0.3554762347 -1.848374e-05
##   [106,] -2.61871076 -0.349124710 -0.471508018 -0.8555538444  3.180277e-04
##   [107,] -1.78690478 -0.340232996 -0.320847324 -0.3290539624  1.770996e-05
##   [108,] -0.17988245 -0.981195368  0.616454583  0.1244849319 -4.398732e-04
##   [109,]  0.48997855  0.916831832  0.937421177  0.0502582169 -2.737402e-04
##   [110,] -0.42893498 -0.924418495 -0.424003007  0.1098029228 -1.853987e-04
##   [111,]  0.76979218  0.693932196 -0.577126565  0.0819392814  1.311134e-04
##   [112,] -0.24272566  0.051594377 -0.448243838 -0.0206047547 -4.692265e-04
##   [113,] -0.43547209  2.111119428 -0.114958037  0.6180143150 -3.247022e-04
##   [114,]  0.93557558 -0.689162159 -0.645675624  0.1177378778 -8.402937e-05
##   [115,] -1.24389292 -0.863247972  0.151601641 -0.0251374360  3.031645e-04
##   [116,]  0.32815159 -1.463512645  0.835343107  0.2991471977  3.452057e-04
##   [117,]  2.70379982 -0.588090840 -0.933432878  0.0169364377 -6.547423e-05
##   [118,] -0.11031405 -1.318085122 -0.401923217  0.2873588759  4.077756e-05
##   [119,]  0.07145821  1.231292436  0.767335297  0.1638254774 -9.151343e-05
##   [120,] -1.50246658 -0.658279764  0.037916793 -0.1477366383  3.367430e-04
##   [121,] -2.39844752 -0.485340592 -0.368482988 -0.6309551109  6.693022e-05
##   [122,] -1.43417146 -1.175504074  0.067045408  0.0501927729 -1.984839e-04
##   [123,] -1.51118518 -1.048076416  0.032704551 -0.0234122463  4.209633e-04
##   [124,] -0.75807791  1.229185352 -0.290414608  0.1833446116  4.895871e-04
##   [125,] -1.40313981 -0.035588833 -0.377668637 -0.2212373475  2.989281e-04
##   [126,]  2.24289212  1.288439664 -0.743646537  0.1256249584 -7.061986e-05
##   [127,] -2.00296778 -1.224978998 -0.187308155 -0.1130618496  3.080987e-04
##   [128,] -2.14524953 -0.452602612 -0.368282846 -0.4833639429  3.248244e-04
##   [129,] -0.69335181  1.307738058 -0.259358752  0.2190320003 -4.411728e-04
##   [130,]  2.32840837 -0.301169439 -0.858018041  0.0006103349  1.957927e-04
```

```
## [131,]  0.09539292 -1.570921578  0.735394529  0.3578484362 -3.882731e-04
## [132,]  1.31301705 -0.601638098  1.261170113 -0.0482569895  2.333381e-04
## [133,]  1.86052246  0.403803060  1.510143140 -0.1834962439 -4.495437e-04
## [134,]  1.91996718  0.774398816 -0.779501192  0.0500615776  2.829768e-04
## [135,]  5.83751242  0.344979067 -1.566328182 -0.4734939956  2.254818e-04
## [136,] -2.42126921  0.263157256 -0.363995239 -0.6156924045  4.007291e-04
## [137,] -1.09946287  1.851483401 -0.063898088  0.4658067928 -3.130512e-04
## [138,] -0.10279960 -1.638846851 -0.280961587  0.4391728576  1.550773e-04
## [139,] -0.56440155 -1.167844352 -0.367025768  0.1836552998  2.242589e-04
## [140,]  0.57605324 -0.428192524 -0.565675906  0.0618608783 -3.766420e-04
## [141,] -2.00767739 -0.480189424 -0.188040859 -0.3823891295  5.042228e-04
## [142,]  0.07272386  0.859539811 -0.407865533  0.1073879713  4.385245e-04
## [143,]  0.45220672  1.134458983 -0.468384834  0.1866391596 -1.785735e-04
## [144,]  1.23281437 -0.754954616 -0.634527331  0.1272460887 -3.447280e-04
## [145,] -1.39811562  1.576707365  0.143920526  0.2923080047  4.417171e-04
## [146,]  2.76720224 -0.967923093 -0.909237975  0.0981619911 -1.703677e-04
## [147,] -1.07748467 -0.943294892  0.224739911  0.0324699934 -1.526682e-04
## [148,] -1.80243136 -0.959189519 -0.331937031 -0.1773456064  1.928621e-05
## [149,]  3.11458406 -0.642123889 -0.991329674 -0.0114393586  3.534385e-04
## [150,] -0.65442939 -1.059892610  0.409890006  0.1269081085  3.528846e-04
## [151,]  2.47883524  0.763962082  1.780665837 -0.2594873896 -5.399780e-05
## [152,]  1.26840115  1.189217076  1.278054565  0.0283131857  3.432829e-04
## [153,] -0.07485715 -1.648914436  0.662325925  0.4013950349  4.868979e-04
## [154,] -1.55824461  1.849434403  0.086194322  0.4358690771  4.637030e-04
## [155,]  0.53392888  0.008435504 -0.577586318  0.0254562312  2.823593e-04
## [156,]  0.45317091 -1.769506065 -0.389027390  0.5220398932  1.954100e-04
## [157,]  3.88228949 -0.068973793  2.357257500 -0.5924046197 -3.060258e-04
## [158,] -1.42966350  2.085440537  0.153245363  0.5921858301  2.659938e-04
## [159,]  1.54871296  1.068471110  1.393855384 -0.0424400809  1.880290e-04
## [160,] -1.89898382  0.721282601 -0.228064879 -0.2141655400  3.570701e-04
## [161,]  5.97170851  0.388963385 -1.624985074 -0.4923396418  1.160575e-04
## [162,] -0.74165788  1.248498919 -0.282577350  0.1917419089 -3.192211e-04
## [163,]  1.13880455  0.784809141  1.211505184 -0.0400258980  1.084697e-05
## [164,]  0.81404433  1.524561869  1.095087474  0.1908969611 -4.172655e-04
## [165,]  2.42455635 -0.685055557 -0.821045970  0.0531031669 -4.884952e-05
## [166,]  2.09028634 -0.436597169  1.594151698 -0.1840227718  4.816748e-04
## [167,] -1.05105082 -0.522551138  0.239084998 -0.0719936608  2.172618e-04
## [168,]  0.34189596 -0.329106893 -0.549456148  0.0409130842 -6.406400e-05
## [169,] -0.39226517  1.627620003 -0.233025886  0.3705728043 -2.835145e-04
## [170,] -1.00885649 -0.898390737  0.255096200  0.0293015592  2.526693e-04
## [171,] -0.91472549  0.682649274 -0.377266321 -0.0166773792 -4.435114e-04
## [172,]  1.55967419 -0.019096997  1.374112659 -0.1439753486  1.508208e-04
## [173,]  4.05052128 -0.519585983 -1.173366859 -0.1400194621  2.556357e-04
## [174,] -2.12099659  0.066449497 -0.348220027 -0.4811757279 -8.855833e-05
## [175,]  2.13935406  0.301172249  1.626567849 -0.2337933164 -3.429689e-04
## [176,] -1.71906899  1.477222594 -0.117911039  0.1922322679  1.689637e-04
## [177,] -1.93914963 -1.207782249 -0.275804770 -0.1080804714 -2.986209e-04
## [178,]  0.37899443  0.178236658 -0.525059004  0.0210201650 -4.863589e-04
## [179,] -2.16643284  0.003314998 -0.370556024 -0.5147659798 -3.806962e-04
## [180,] -1.77905221  0.175025988 -0.307574036 -0.3186966173 -1.435685e-04
## [181,]  3.25273465 -0.153995737 -1.043031443 -0.0929979391  5.207362e-04
## [182,] -1.84565933 -0.740260650 -0.116524442 -0.2467059854 -1.477774e-04
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## [571,] -1.49615114  1.887334361 -0.001602418  0.4642537566  4.219058e-04
## [572,] -0.74993001  0.473850160  0.389270511 -0.0332860721  3.014080e-04
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## [574,]  1.71240530 -0.826754065  1.429618260 -0.0521167220 -2.878166e-04
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## [926,]  3.29604102 -0.097930020  2.109513684 -0.4536018026 -1.477514e-04
## [927,]  1.02432484 -1.039381177 -0.609396129  0.2117121159  2.353338e-04
## [928,] -1.63473065  0.625345462 -0.347160490 -0.1781150005  4.173273e-04
## [929,] -1.49447470 -0.651855271  0.041531926 -0.1468285923  4.884200e-04
## [930,] -0.04733731 -1.630659423 -0.373391564  0.4407559923  4.581135e-04
## [931,]  4.06838822 -0.291778163 -1.163173099 -0.1770889443  4.803830e-04
## [932,]  0.73830517  0.507564148  1.033334333 -0.0377677663 -3.178248e-04
## [933,]  2.20232064  0.941761458 -0.770852319  0.0526198035 -3.915927e-04
## [934,] -2.29363466 -0.171909654 -0.432984987 -0.6071752987  6.674034e-05
## [935,]  0.43116521 -1.233569806  0.879610030  0.2009077923  4.627126e-04
## [936,] -1.38861348 -0.264861891 -0.375052173 -0.2098725212  9.601120e-05
## [937,]  0.32833925 -1.487644688 -0.444265425  0.3819248908 -2.031210e-04
## [938,]  1.17141696  0.107833174 -0.649943521  0.0262244367  3.481998e-04
## [939,] -2.28544254  0.176805690 -0.303465852 -0.5420312238  4.289311e-04
## [940,]  0.49385241  1.545800428 -0.436418466  0.3279261107 -1.539038e-04
```

```
## [941,] -0.88095601  0.128368106 -0.376568170 -0.0997402040 -4.400296e-04
## [942,]  1.22522187 -0.852355737  1.221575647  0.0164471707 -4.413266e-05
## [943,] -1.27589707  0.692444230 -0.302563383 -0.0725339570 -4.333266e-04
## [944,] -0.74648994 -1.958348803  0.373954357  0.5713110931 -3.988927e-04
## [945,]  2.28709374 -0.322937687 -0.875940015  0.0081418874  1.544818e-04
## [946,] -1.19836301 -0.188835344  0.178363014 -0.1399581608  9.809176e-05
## [947,]  2.40897524 -0.719487036  1.726571022 -0.1894679397  6.064145e-05
## [948,]  1.95749861 -0.044756389  1.543112025 -0.2007136177 -6.423367e-05
## [949,] -0.94526903  2.029873011 -0.105580738  0.5756494473 -1.351056e-04
## [950,]  1.12966680 -0.898410412  1.180412169  0.0384522375 -4.102629e-04
## [951,]  3.76223879  0.824063540 -1.156275338 -0.1266729059 -1.698750e-04
## [952,] -1.37061388  2.193771068  0.184034351  0.6639161657  1.421031e-04
## [953,] -0.18951480 -1.867155090  0.614203034  0.5208133939 -2.078929e-04
## [954,]  1.51134473  0.025721717 -0.737957212  0.0206532014  4.113682e-04
## [955,]  5.12400840  0.590863361 -1.398237962 -0.3500347636  4.674569e-04
## [956,]  4.59613901 -0.102287204 -1.285412627 -0.2654261741  3.577520e-06
## [957,]  3.76445794  0.825345145 -1.155305160 -0.1269574148 -1.900490e-04
## [958,]  6.13658558  0.198415422 -1.674940416 -0.5174774251  2.899230e-04
## [959,] -2.28846450 -0.566381532 -0.317896220 -0.5323618423 -1.287348e-04
## [960,] -0.69737903 -0.002783645 -0.415092155 -0.0753577308 -3.086701e-04
## [961,] -0.18980368  0.093995102 -0.424314049 -0.0147435122  3.296935e-04
## [962,] -1.43128868  1.076077838  0.109347993  0.0493172750 -2.553168e-04
## [963,]  2.06713309  0.261460669  1.595127773 -0.2224000779  1.104281e-04
## [964,] -0.13246467  1.043541034  0.673494437  0.1152054522  3.731129e-04
## [965,] -0.26345780 -0.413448219  0.584622047 -0.0080821968 -2.212376e-04
## [966,]  0.68710122 -1.683369378 -0.405116915  0.4739058810 -4.066550e-04
## [967,] -0.30120081 -2.034825459 -0.247094950  0.6640768974  3.846258e-04
## [968,] -1.55649101  1.326632137  0.064054109  0.1431632026  2.849368e-04
## [969,] -0.71254521 -0.496160669  0.387933822 -0.0289209781 -4.782811e-04
## [970,] -0.05647730  1.483154614  0.721039896  0.2611054168  4.182214e-04
## [971,]  0.35903954 -1.449449352  0.848593910  0.2911540246 -2.801559e-05
## [972,] -1.61663975  1.231241224  0.033694110  0.0860758054 -7.704592e-05
## [973,] -0.85592792 -0.555472513 -0.375844570 -0.0475711767  4.157721e-04
## [974,] -1.35221038 -0.319943751  0.108271913 -0.1653802572 -2.397965e-05
## [975,] -1.79920151  0.813304512 -0.297216740 -0.1560457618  1.526400e-04
## [976,] -0.39619599 -0.497241299 -0.406714330  0.0075138075  2.855885e-04
## [977,]  0.19929886  0.938485789 -0.467216873  0.1315724237 -1.682907e-04
## [978,] -1.75239972 -1.099407814 -0.309205257 -0.0972984159 -1.964938e-04
## [979,]  2.88508930  0.699920362 -0.950919376 -0.0436160751 -1.865533e-05
## [980,] -2.15524972 -1.003760787 -0.257548032 -0.2975085444  5.279772e-05
## [981,]  3.73344089  0.273994454 -1.180227457 -0.1586109274 -2.302280e-04
## [982,] -2.67883392 -0.211439631 -0.498430886 -0.9307872676 -3.560342e-04
## [983,] -0.95326018 -0.224031516  0.285865358 -0.0930776443  2.805581e-04
## [984,] -1.70671206  0.958965615 -0.250601110 -0.0693528780 -3.216953e-05
## [985,] -0.81992852  2.228388353 -0.042332982  0.6943282162  4.269888e-04
## [986,]  0.83527541 -1.460705001 -0.458421341  0.3684035782  5.460092e-05
## [987,] -0.97763790 -2.064472904  0.274733714  0.6301847324  2.119811e-04
## [988,]  0.01205694  0.221690050 -0.450398566  0.0054189795  6.769384e-05
## [989,] -0.12751052  0.111199941 -0.513417811 -0.0086170786 -1.153740e-04
## [990,] -0.95119386 -0.626102918 -0.418539440 -0.0520368012 -9.062727e-05
## [991,] -2.19171104  0.684874203 -0.244262952 -0.3362209889  2.794187e-04
## [992,] -0.41506680  0.201221511  0.528822651 -0.0398217759 -1.901225e-04
## [993,] -0.34435921 -1.772942355  0.546621970  0.4697225613  4.701552e-04
## [994,] -0.92446361 -1.034911196  0.291768349  0.0876934678 -4.367276e-04
```

```
## [995,] -1.61581699  0.042688779 -0.354283439 -0.2804220460  1.720560e-05
## [996,] -1.79899493 -1.274756098 -0.212252372 -0.0169425079  3.555046e-04
## [997,] -2.00048776 -0.717755801 -0.303983385 -0.3402507779 -4.288216e-04
## [998,] -1.11024310  1.834726671 -0.069316632  0.4559382973 -1.307871e-04
## [999,] -0.45370726  1.552930010 -0.262514290  0.3365648750  4.854838e-04
## [1000,] -2.14385714 -0.172400266 -0.246063799 -0.4863479223  1.038577e-04
```

```r
# Select the first three principal components because they explain approximately 98% of the total varia
data.pca.3=data.pca$x[,c(1,2,3)]
data.pca.3
```

```
##             PC1          PC2          PC3
##    [1,] -2.08242303  0.119273378 -0.329309568
##    [2,] -1.71341045 -1.011814347 -0.057640090
##    [3,] -1.55353195 -1.762147220 -0.098335771
##    [4,] -1.60346182  0.191784324  0.005868754
##    [5,] -1.81026445 -0.579652083 -0.334007857
##    [6,] -0.42997898  0.189660217  0.522105671
##    [7,] -2.01392191  1.011155780 -0.151554420
##    [8,]  0.70537203  1.087708452  1.034702324
##    [9,]  3.81150571 -0.608282446 -1.159475270
##   [10,]  5.06796936  0.061632861 -1.431649189
##   [11,] -1.08395498  0.165211403  0.235485636
##   [12,] -0.36761456 -0.687787562 -0.395902298
##   [13,] -2.22981527 -0.278001279 -0.287392679
##   [14,] -1.31664324  0.328953025 -0.330965274
##   [15,] -0.82175390  1.918713700  0.408699363
##   [16,] -1.48187317 -0.430486408  0.049196058
##   [17,] -2.52760966 -0.206042998 -0.425618450
##   [18,]  1.19805229 -0.218945528  1.216619772
##   [19,] -0.93171210  2.051788524 -0.098710069
##   [20,] -1.26610773 -0.246677973  0.147491876
##   [21,] -1.31946650 -0.915875551  0.117960195
##   [22,] -2.28796855  0.506898624 -0.294545231
##   [23,] -1.20911304 -1.773146675 -0.179795183
##   [24,] -1.14498443  1.975018431  0.271478205
##   [25,]  2.22532244 -0.762322669 -0.790535994
##   [26,]  1.58013499 -0.243228025  1.379438472
##   [27,] -1.83640142  0.754006451 -0.316013590
##   [28,]  3.67704192  0.019802082 -1.092463930
##   [29,]  2.32035667 -0.525149196 -0.864044042
##   [30,]  1.79803259 -0.298958464 -0.736169722
##   [31,] -2.25044266 -0.112643382 -0.411802479
##   [32,] -2.09328599  0.105103843 -0.334587731
##   [33,]  0.02161637 -1.250529708  0.703083049
##   [34,] -0.93532685 -0.209899493  0.293958477
##   [35,] -1.52034482  1.252867212 -0.156643787
##   [36,]  2.06139100 -0.196735956 -0.854534393
##   [37,]  1.70540205 -0.567757752 -0.778902526
##   [38,] -0.40525441 -1.098124835 -0.414015621
##   [39,]  2.69835373  0.090472373 -0.926706842
##   [40,] -1.91168456 -1.325321409 -0.145466300
##   [41,]  0.15343573  0.041613786 -0.509016119
##   [42,]  0.09868495 -0.177387848  0.744748727
```

```
##   [43,] -2.36141455  0.056924530 -0.341772152
##   [44,] -0.88720281  2.121394837 -0.076309832
##   [45,]  0.60589851  0.901288865 -0.525653141
##   [46,]  0.63882304  0.929568519 -0.510638267
##   [47,] -2.12625617 -0.979340413 -0.244358188
##   [48,]  1.33229227 -1.194911683  1.265669372
##   [49,] -2.18652171  0.332779702 -0.253596403
##   [50,]  2.96564106 -0.250089183  1.967657690
##   [51,]  3.10998062 -0.183599269  2.029630495
##   [52,] -1.00021865  0.633233052 -0.299565734
##   [53,] -1.42838299  2.088020500  0.153928496
##   [54,] -0.90699749 -1.207291187  0.299368204
##   [55,]  2.13762844  0.337560899 -0.812734844
##   [56,]  2.15388414  1.222949572 -0.783503269
##   [57,]  5.79081414  0.323239382 -1.586393570
##   [58,]  4.22016602  0.080558945  2.501916859
##   [59,]  4.32344121 -0.432934747 -1.288964656
##   [60,] -0.89622231  1.108912637 -0.238442500
##   [61,] -1.85014218  0.733385291 -0.322870796
##   [62,]  3.47501259  0.702566979 -1.048713135
##   [63,]  2.97789925 -0.687388382 -0.933750284
##   [64,]  0.28798295  0.111463815 -0.565800054
##   [65,] -0.33419729 -0.459634551  0.553338510
##   [66,] -1.77902937  0.844238581 -0.287105715
##   [67,] -0.40045217  0.216236827 -0.397174779
##   [68,] -2.29204784  0.500453372 -0.296602632
##   [69,] -1.44092959 -1.874815119 -0.163903994
##   [70,] -1.32822090 -1.452091765  0.115112851
##   [71,] -1.65314549  1.173232860  0.015260291
##   [72,]  0.50777100  0.348408262  0.930805074
##   [73,]  0.54841364 -1.541258524  0.930095757
##   [74,] -1.04536944  2.132282138  0.321710685
##   [75,] -0.52905616 -0.369089659  0.469566331
##   [76,]  0.56928534  1.316333060  0.983601812
##   [77,] -0.42461848 -0.060258273  0.519686370
##   [78,]  0.39179524 -1.610270052  0.863061000
##   [79,]  0.50247678  0.344556214  0.928437292
##   [80,]  3.41469446  0.147491605 -1.085561183
##   [81,]  2.31023358 -0.762938049  1.684312165
##   [82,]  2.31188896 -0.079646080 -0.861947967
##   [83,] -0.22311838  0.641513249  0.622622885
##   [84,]  1.62643269  0.922226454 -0.668597161
##   [85,] -1.21056467 -1.210586201  0.165989430
##   [86,] -1.32232924 -0.722878978  0.117560984
##   [87,] -2.47824116 -0.127408816 -0.400676994
##   [88,] -0.22603170  0.638941948  0.621289675
##   [89,] -1.09908679  2.047204135  0.294604504
##   [90,] -0.32847453  0.010367476  0.562730808
##   [91,] -0.69446050  1.810414815 -0.123230266
##   [92,] -0.97187254  1.988626034 -0.118945581
##   [93,]  0.42108074 -0.277791850 -0.514463617
##   [94,] -1.12599418 -0.519571927 -0.378167673
##   [95,]  0.67535411  1.063293515  1.021104648
##   [96,] -2.28156598  0.519798442 -0.291129567
```

```
##   [97,]  4.48009237  0.325679362 -1.327360668
##   [98,] -1.18774802  1.412357606  0.228927757
##   [99,]  2.13886235 -0.186964833  1.618044998
##  [100,]  1.99335884 -0.854936684 -0.774096747
##  [101,]  0.05344896 -1.912393227 -0.327726832
##  [102,] -0.82588319  1.447677793  0.387683294
##  [103,] -0.81174314 -0.112258371  0.349747457
##  [104,]  3.17432103  0.084056730  2.061013357
##  [105,]  0.44269397 -1.432636101 -0.395010545
##  [106,] -2.61871076 -0.349124710 -0.471508018
##  [107,] -1.78690478 -0.340232996 -0.320847324
##  [108,] -0.17988245 -0.981195368  0.616454583
##  [109,]  0.48997855  0.916831832  0.937421177
##  [110,] -0.42893498 -0.924418495 -0.424003007
##  [111,]  0.76979218  0.693932196 -0.577126565
##  [112,] -0.24272566  0.051594377 -0.448243838
##  [113,] -0.43547209  2.111119428 -0.114958037
##  [114,]  0.93557558 -0.689162159 -0.645675624
##  [115,] -1.24389292 -0.863247972  0.151601641
##  [116,]  0.32815159 -1.463512645  0.835343107
##  [117,]  2.70379982 -0.588090840 -0.933432878
##  [118,] -0.11031405 -1.318085122 -0.401923217
##  [119,]  0.07145821  1.231292436  0.767335297
##  [120,] -1.50246658 -0.658279764  0.037916793
##  [121,] -2.39844752 -0.485340592 -0.368482988
##  [122,] -1.43417146 -1.175504074  0.067045408
##  [123,] -1.51118518 -1.048076416  0.032704551
##  [124,] -0.75807791  1.229185352 -0.290414608
##  [125,] -1.40313981 -0.035588833 -0.377668637
##  [126,]  2.24289212  1.288439664 -0.743646537
##  [127,] -2.00296778 -1.224978998 -0.187308155
##  [128,] -2.14524953 -0.452602612 -0.368282846
##  [129,] -0.69335181  1.307738058 -0.259358752
##  [130,]  2.32840837 -0.301169439 -0.858018041
##  [131,]  0.09539292 -1.570921578  0.735394529
##  [132,]  1.31301705 -0.601638098  1.261170113
##  [133,]  1.86052246  0.403803060  1.510143140
##  [134,]  1.91996718  0.774398816 -0.779501192
##  [135,]  5.83751242  0.344979067 -1.566328182
##  [136,] -2.42126921  0.263157256 -0.363995239
##  [137,] -1.09946287  1.851483401 -0.063898088
##  [138,] -0.10279960 -1.638846851 -0.280961587
##  [139,] -0.56440155 -1.167844352 -0.367025768
##  [140,]  0.57605324 -0.428192524 -0.565675906
##  [141,] -2.00767739 -0.480189424 -0.188040859
##  [142,]  0.07272386  0.859539811 -0.407865533
##  [143,]  0.45220672  1.134458983 -0.468384834
##  [144,]  1.23281437 -0.754954616 -0.634527331
##  [145,] -1.39811562  1.576707365  0.143920526
##  [146,]  2.76720224 -0.967923093 -0.909237975
##  [147,] -1.07748467 -0.943294892  0.224739911
##  [148,] -1.80243136 -0.959189519 -0.331937031
##  [149,]  3.11458406 -0.642123889 -0.991329674
##  [150,] -0.65442939 -1.059892610  0.409890006
```

```
##  [151,]  2.47883524  0.763962082  1.780665837
##  [152,]  1.26840115  1.189217076  1.278054565
##  [153,] -0.07485715 -1.648914436  0.662325925
##  [154,] -1.55824461  1.849434403  0.086194322
##  [155,]  0.53392888  0.008435504 -0.577586318
##  [156,]  0.45317091 -1.769506065 -0.389027390
##  [157,]  3.88228949 -0.068973793  2.357257500
##  [158,] -1.42966350  2.085440537  0.153245363
##  [159,]  1.54871296  1.068471110  1.393855384
##  [160,] -1.89898382  0.721282601 -0.228064879
##  [161,]  5.97170851  0.388963385 -1.624985074
##  [162,] -0.74165788  1.248498919 -0.282577350
##  [163,]  1.13880455  0.784809141  1.211505184
##  [164,]  0.81404433  1.524561869  1.095087474
##  [165,]  2.42455635 -0.685055557 -0.821045970
##  [166,]  2.09028634 -0.436597169  1.594151698
##  [167,] -1.05105082 -0.522551138  0.239084998
##  [168,]  0.34189596 -0.329106893 -0.549456148
##  [169,] -0.39226517  1.627620003 -0.233025886
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## [757,] -1.63259692 -1.010800285 -0.255528198
## [758,]  2.10106448  0.583764799 -0.822998695
## [759,] -1.35430050  1.645023982  0.165957732
## [760,]  0.10017808  1.640115700  0.793982135
## [761,]  4.14290131  0.292057553  2.473108862
## [762,] -0.41323710  0.796274239 -0.387785087
## [763,]  1.95088992 -0.875405428 -0.792391626
## [764,] -0.92648266 -1.885990084  0.294887693
## [765,]  0.55672387 -0.981759137  0.934326110
## [766,] -1.37331121  0.572676829 -0.349406540
## [767,] -0.43920334  0.182814571 -0.414856072
## [768,] -1.29413410  0.534181957  0.152073917
## [769,]  0.82740259  0.569173629  1.072966696
## [770,]  0.28834227 -0.511328299  0.822329779
## [771,]  1.23827391 -0.845958090  1.227205405
## [772,] -1.92294389  1.101931022 -0.224298231
## [773,]  0.58627999  0.402313647  0.965701872
## [774,] -0.59361450  1.309079414  0.483143270
## [775,] -1.47510891 -1.697322376  0.052338117
## [776,]  3.23809180  0.323874612 -1.041227443
## [777,] -1.52378963  0.438242384 -0.303298277
## [778,] -2.20623104 -0.299392435 -0.394507556
## [779,]  2.43640558 -1.080134844 -0.818416078
## [780,] -1.45813483 -0.179428922  0.063287274
## [781,]  3.47118903 -0.250452518  2.181278698
## [782,] -1.56761482 -0.392678116 -0.339866582
## [783,] -2.16493667  0.005893817 -0.369787031
## [784,] -0.41235005 -0.051269710  0.525177464
## [785,] -1.17659325  0.356097514  0.199170355
## [786,] -0.43679635 -1.480905814  0.504974588
## [787,]  1.41386692  0.678318192  1.326169258
## [788,]  1.34079818 -1.191076675  1.269315843
## [789,] -1.07130826 -2.011102002 -0.116445723
## [790,] -0.11002073  1.064116019  0.683816676
## [791,] -1.64725750  1.053066831 -0.220604141
## [792,] -1.67547968 -0.983561254 -0.040632055
## [793,]  0.34382609 -1.479966580 -0.437579630
## [794,] -1.25604154 -1.736260556  0.149214864
## [795,] -2.23729508  0.187618684 -0.398634800
## [796,] -1.11583383  0.211937016 -0.361675183
## [797,] -1.05737821 -2.004709012 -0.110466397
## [798,]  0.69366363  0.091085691 -0.623475640
```

```
##  [799,] -2.26052388 -0.069336427 -0.297838399
##  [800,]  0.81036332 -1.645793945 -0.468560305
##  [801,]  0.76534902 -0.554201101 -0.601576402
##  [802,] -1.85321871  0.184323689 -0.106346249
##  [803,] -1.50050670 -1.885567475 -0.073285584
##  [804,] -1.66212352 -0.480126463 -0.383386528
##  [805,]  1.46793574 -0.939274383  1.324660535
##  [806,]  0.65566121 -1.493972502  0.976001085
##  [807,] -1.37340840  0.446652785  0.114619311
##  [808,] -1.63956439  0.447039428 -0.003604794
##  [809,] -2.13172893 -0.207957809 -0.358692952
##  [810,] -0.47005129  1.532322663 -0.270407941
##  [811,]  0.50850502 -1.745225438 -0.365350458
##  [812,]  3.67401725  0.075964170  2.271784636
##  [813,]  4.00192848  0.226843157  2.412563127
##  [814,] -1.10638250 -0.563645700  0.214282481
##  [815,]  0.64363202  0.933423142 -0.508463668
##  [816,] -0.75747166 -1.739446732 -0.215532578
##  [817,]  3.30481435 -0.355013699 -1.023557199
##  [818,] -2.13394193  0.790624771 -0.214157096
##  [819,] -0.07069114 -1.949964004 -0.264633011
##  [820,] -0.20736091  0.637843228 -0.419250802
##  [821,]  3.78443028  0.836879593 -1.146573555
##  [822,]  0.40840748 -1.449268120 -0.409787742
##  [823,] -0.61510277  1.863743724 -0.203113608
##  [824,]  0.79568640  1.509138720  1.086738648
##  [825,]  2.52147179  0.014380052 -0.887211118
##  [826,]  1.88177601  0.158973766  1.514398043
##  [827,]  0.48351134 -0.169350432  0.910826243
##  [828,] -1.32091913  0.637067804 -0.324214529
##  [829,] -1.40801054 -1.968530425  0.085283791
##  [830,]  1.71910744 -0.113318622 -0.767371965
##  [831,]  0.56386655 -0.977916897  0.937429828
##  [832,] -0.53262729 -0.375965299 -0.465144673
##  [833,]  0.11396167  1.641057692 -0.363448840
##  [834,]  3.09934018  1.104600578 -0.965697580
##  [835,]  0.85509727  1.431241406 -0.518111540
##  [836,]  4.33453292  0.027985145 -1.278041397
##  [837,]  1.80576817 -0.725057017 -0.737257411
##  [838,]  1.74593536 -1.012153070  1.442747142
##  [839,]  3.67571153 -0.437289461 -1.099240895
##  [840,] -1.32588766  2.275016639  0.207300877
##  [841,] -0.65454330  0.561293849  0.433140026
##  [842,]  2.03160535  1.162088105 -0.721228240
##  [843,] -0.87341754  0.136088383 -0.373046899
##  [844,] -0.11024334  1.844560485  0.711639882
##  [845,] -1.95855402 -1.221906468 -0.284483572
##  [846,] -0.55626536  0.652602334  0.478420607
##  [847,]  2.67595653 -0.159298851 -0.940177449
##  [848,] -1.86327626 -0.943495601 -0.124972162
##  [849,] -2.06687303 -0.078392049 -0.209087705
##  [850,] -1.69159951 -1.228042834 -0.281406226
##  [851,]  0.23071412 -0.953893994  0.794091968
##  [852,]  1.75075941 -0.808850248  1.446101572
```

```
## [853,] -1.45506727 -0.326591505 -0.405669321
## [854,]  0.47196680  0.601075026  0.921277006
## [855,]  0.14676550 -0.800561967  0.758723253
## [856,] -0.51706088 -1.959117717 -0.225150286
## [857,] -1.63091616 -1.783152131 -0.015500252
## [858,] -1.31005972 -0.192849440 -0.338924338
## [859,]  0.38295882  0.535584934  0.881420274
## [860,]  3.56857098 -0.265727090 -1.142629902
## [861,]  1.09758894 -0.805400368 -0.576713544
## [862,] -2.12815183 -0.980623735 -0.245199577
## [863,] -0.94388068  2.032452403 -0.104854675
## [864,] -2.04271681 -0.556884842 -0.322222146
## [865,] -1.12995264  1.492240459  0.257310909
## [866,] -0.77258652 -1.087003816 -0.341678749
## [867,]  3.41667561 -0.319964536 -1.091712971
## [868,] -0.77471277 -0.942582418  0.357628737
## [869,]  1.98086021 -0.443040028 -0.775858632
## [870,] -0.91449156 -1.387299026  0.296510593
## [871,]  1.04643862 -0.627668154 -0.597377613
## [872,] -0.93223028 -1.888546348  0.292426048
## [873,] -0.90585451 -1.023369517  0.299957202
## [874,]  0.02683246 -1.428018223  0.705459108
## [875,] -0.73686358 -1.245253501 -0.325856985
## [876,] -0.73754652 -1.729205414 -0.206906425
## [877,]  1.37521165  0.766339051 -0.664071216
## [878,]  0.59567724  0.050789403 -0.550142436
## [879,] -1.66498398  1.652113754  0.030438978
## [880,] -2.11836943  0.439768562 -0.219269985
## [881,]  4.77731672  0.720865718 -1.309710864
## [882,] -1.53686553  0.760631015 -0.299097887
## [883,]  1.49385558 -1.228505241 -0.640744833
## [884,]  2.77694981 -0.762059636 -0.903750005
## [885,] -2.20216115  0.306995102 -0.261556166
## [886,]  0.37099982  1.147952430  0.893305251
## [887,]  0.30203205 -1.324271443 -0.455991827
## [888,]  2.23002196 -0.960502504 -0.789752133
## [889,] -0.86667501 -2.013326459  0.322378213
## [890,] -1.77764103  0.846817972 -0.286379653
## [891,]  0.71921264  1.097982053  1.040906019
## [892,] -2.04085712  0.177254896 -0.308861125
## [893,] -0.04219944 -1.097304495  0.675729904
## [894,]  0.22398695 -0.548494852  0.794194607
## [895,]  0.18435732 -0.620270661 -0.504472920
## [896,] -1.23255673 -0.774791540 -0.310462665
## [897,] -0.45747430  1.459682447  0.547484410
## [898,] -1.19121951 -2.162929929  0.183026793
## [899,] -2.25140091  0.575253690 -0.275393506
## [900,] -2.26388206  0.210319150 -0.292628765
## [901,]  0.03536724  1.166536718 -0.414372031
## [902,]  1.66766717 -0.423740016  1.414414104
## [903,]  5.58471986  0.498485473 -1.553888130
## [904,]  0.73413324  1.357439175 -0.456179640
## [905,] -0.87642257  2.138151567 -0.070891288
## [906,]  2.61554447  0.843426266  1.840467324
```

```
##   [907,]  3.74739800  0.035670765 -1.178444226
##   [908,] -2.26596458  0.206450063 -0.293717859
##   [909,]  3.64107542  0.001885596 -1.107996657
##   [910,]  0.43832661 -0.266242939 -0.506817517
##   [911,] -1.07604721  0.549565735 -0.335388494
##   [912,] -1.91217880  0.112207287 -0.134673163
##   [913,]  0.18000049  0.322403989 -0.491806467
##   [914,] -1.74321780  0.900957250 -0.269034928
##   [915,]  3.18985884  0.049666260 -1.066630670
##   [916,] -1.74253895 -0.441769304 -0.067622132
##   [917,] -2.15491807  0.076255089 -0.246000689
##   [918,] -0.60004045 -0.610698555 -0.380466404
##   [919,]  4.19175143 -0.252909887 -1.226489571
##   [920,]  0.72952912  0.233146462  1.023640274
##   [921,]  2.45581975  0.763105134 -0.899775620
##   [922,]  1.47196129  0.001356466 -0.755283630
##   [923,] -0.90527233  1.098613620 -0.242738894
##   [924,] -2.43003968 -0.303832006 -0.381100302
##   [925,] -0.14357060  1.883110972 -0.231885030
##   [926,]  3.29604102 -0.097930020  2.109513684
##   [927,]  1.02432484 -1.039381177 -0.609396129
##   [928,] -1.63473065  0.625345462 -0.347160490
##   [929,] -1.49447470 -0.651855271  0.041531926
##   [930,] -0.04733731 -1.630659423 -0.373391564
##   [931,]  4.06838822 -0.291778163 -1.163173099
##   [932,]  0.73830517  0.507564148  1.033334333
##   [933,]  2.20232064  0.941761458 -0.770852319
##   [934,] -2.29363466 -0.171909654 -0.432984987
##   [935,]  0.43116521 -1.233569806  0.879610030
##   [936,] -1.38861348 -0.264861891 -0.375052173
##   [937,]  0.32833925 -1.487644688 -0.444265425
##   [938,]  1.17141696  0.107833174 -0.649943521
##   [939,] -2.28544254  0.176805690 -0.303465852
##   [940,]  0.49385241  1.545800428 -0.436418466
##   [941,] -0.88095601  0.128368106 -0.376568170
##   [942,]  1.22522187 -0.852355737  1.221575647
##   [943,] -1.27589707  0.692444230 -0.302563383
##   [944,] -0.74648994 -1.958348803  0.373954357
##   [945,]  2.28709374 -0.322937687 -0.875940015
##   [946,] -1.19836301 -0.188835344  0.178363014
##   [947,]  2.40897524 -0.719487036  1.726571022
##   [948,]  1.95749861 -0.044756389  1.543112025
##   [949,] -0.94526903  2.029873011 -0.105580738
##   [950,]  1.12966680 -0.898410412  1.180412169
##   [951,]  3.76223879  0.824063540 -1.156275338
##   [952,] -1.37061388  2.193771068  0.184034351
##   [953,] -0.18951480 -1.867155090  0.614203034
##   [954,]  1.51134473  0.025721717 -0.737957212
##   [955,]  5.12400840  0.590863361 -1.398237962
##   [956,]  4.59613901 -0.102287204 -1.285412627
##   [957,]  3.76445794  0.825345145 -1.155305160
##   [958,]  6.13658558  0.198415422 -1.674940416
##   [959,] -2.28846450 -0.566381532 -0.317896220
##   [960,] -0.69737903 -0.002783645 -0.415092155
```

```
##  [961,] -0.18980368  0.093995102 -0.424314049
##  [962,] -1.43128868  1.076077838  0.109347993
##  [963,]  2.06713309  0.261460669  1.595127773
##  [964,] -0.13246467  1.043541034  0.673494437
##  [965,] -0.26345780 -0.413448219  0.584622047
##  [966,]  0.68710122 -1.683369378 -0.405116915
##  [967,] -0.30120081 -2.034825459 -0.247094950
##  [968,] -1.55649101  1.326632137  0.064054109
##  [969,] -0.71254521 -0.496160669  0.387933822
##  [970,] -0.05647730  1.483154614  0.721039896
##  [971,]  0.35903954 -1.449449352  0.848593910
##  [972,] -1.61663975  1.231241224  0.033694110
##  [973,] -0.85592792 -0.555472513 -0.375844570
##  [974,] -1.35221038 -0.319943751  0.108271913
##  [975,] -1.79920151  0.813304512 -0.297216740
##  [976,] -0.39619599 -0.497241299 -0.406714330
##  [977,]  0.19929886  0.938485789 -0.467216873
##  [978,] -1.75239972 -1.099407814 -0.309205257
##  [979,]  2.88508930  0.699920362 -0.950919376
##  [980,] -2.15524972 -1.003760787 -0.257548032
##  [981,]  3.73344089  0.273994454 -1.180227457
##  [982,] -2.67883392 -0.211439631 -0.498430886
##  [983,] -0.95326018 -0.224031516  0.285865358
##  [984,] -1.70671206  0.958965615 -0.250601110
##  [985,] -0.81992852  2.228388353 -0.042332982
##  [986,]  0.83527541 -1.460705001 -0.458421341
##  [987,] -0.97763790 -2.064472904  0.274733714
##  [988,]  0.01205694  0.221690050 -0.450398566
##  [989,] -0.12751052  0.111199941 -0.513417811
##  [990,] -0.95119386 -0.626102918 -0.418539440
##  [991,] -2.19171104  0.684874203 -0.244262952
##  [992,] -0.41506680  0.201221511  0.528822651
##  [993,] -0.34435921 -1.772942355  0.546621970
##  [994,] -0.92446361 -1.034911196  0.291768349
##  [995,] -1.61581699  0.042688779 -0.354283439
##  [996,] -1.79899493 -1.274756098 -0.212252372
##  [997,] -2.00048776 -0.717755801 -0.303983385
##  [998,] -1.11024310  1.834726671 -0.069316632
##  [999,] -0.45370726  1.552930010 -0.262514290
## [1000,] -2.14385714 -0.172400266 -0.246063799
```

```
# Assign clusters based on hierarchical clustering (3 groups)
dist_pca=dist(data.pca.3, method = "euclidean")
data.pca.3.hc=hclust(dist_pca)

data$pca.3=cutree(data.pca.3.hc,3) #add cluster membership to each observation (1-3)
data$pca.3
```

```
##    [1] 1 1 1 1 1 1 1 1 2 2 1 1 1 1 1 1 1 3 1 1 1 1 1 1 3 3 1 2 3 3 1 1 1 1 1 3 3
##   [38] 1 3 1 1 1 1 1 1 1 1 1 3 1 3 3 1 1 1 3 3 2 3 2 1 1 2 3 1 1 1 1 1 1 1 1 1 1
##   [75] 1 1 1 1 1 2 3 3 1 3 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 3 3 1 1 1 3 1 1 1 1 1 1
##  [112] 1 1 1 1 1 3 1 1 1 1 1 1 1 1 1 3 1 1 1 3 1 3 3 3 2 1 1 1 1 1 1 1 1 1 1 3 1 1
##  [149] 3 1 3 3 1 1 1 1 3 1 3 1 2 1 3 1 3 3 1 1 1 1 1 1 3 2 1 3 1 1 1 1 1 2 1 1 2 2
##  [186] 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 3 1 1 1 1 1 1 1 2 3 3 1 1 1 1 3 1 1 1 2
```

```
## [223] 1 1 1 2 1 1 1 1 1 1 1 1 1 3 1 1 2 1 1 1 1 3 3 3 1 3 1 3 1 2 1 1 1 3 1 2 1
## [260] 1 2 1 1 1 1 1 1 1 1 2 3 1 3 1 1 1 1 3 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 3 1 1 1
## [297] 1 1 3 1 1 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 3 1 3 3 1 1 1 2 1 1 1 1 1 1 2 1
## [334] 3 2 1 1 1 3 1 1 1 1 1 1 1 3 1 1 1 1 1 1 3 1 1 1 1 1 1 1 3 3 2 3 3 1 1 1 1
## [371] 1 1 3 1 1 1 1 1 1 1 1 1 1 3 1 1 3 1 1 3 1 1 1 1 1 1 1 1 1 1 1 3 1 1 1 1 1 3 1
## [408] 1 1 1 3 1 1 1 1 1 3 1 3 1 1 1 3 1 1 1 1 1 1 3 1 1 3 1 3 3 1 1 1 1 1 1 1 1
## [445] 1 3 2 1 1 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1
## [482] 3 1 1 3 1 1 3 1 1 1 1 3 1 1 1 1 1 1 3 3 1 3 1 1 1 1 1 1 2 2 1 1 1 1 1 1 1
## [519] 1 1 1 3 1 1 1 1 3 1 1 1 1 1 1 1 1 1 1 1 3 1 1 3 1 3 1 1 1 1 1 1 1 1 1 1 3
## [556] 1 1 1 1 3 3 1 1 1 1 1 1 1 1 1 1 1 3 3 1 1 1 3 1 1 1 1 1 1 2 1 1 1 1 1 1 1 3
## [593] 1 3 1 1 3 1 1 1 1 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 3 1 2 1 1 1 1 3
## [630] 1 3 3 1 1 1 1 1 1 2 3 1 1 1 3 1 3 1 1 1 1 1 3 3 1 1 1 1 3 1 3 1 3 2 3 1 1
## [667] 3 3 1 1 1 1 1 1 1 1 1 1 1 2 1 3 3 3 1 1 1 1 1 1 1 1 1 1 1 1 1 3 1 1 3 3 3 3
## [704] 3 1 3 3 1 1 3 1 3 1 1 1 2 1 1 1 1 2 1 3 3 1 1 1 3 1 1 1 1 1 2 1 1 1 1 3 1
## [741] 1 1 1 1 2 1 1 1 1 2 3 1 1 3 1 1 1 3 1 1 3 1 3 1 1 1 1 1 1 1 1 1 1 1 1 2 1
## [778] 1 3 1 3 1 1 1 1 1 3 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 3 1 1 1 1 1 1 3 3 1
## [815] 1 1 2 1 1 1 2 1 1 1 3 3 1 1 1 3 1 1 1 2 1 2 3 3 2 1 1 3 1 1 1 1 3 1 1 1 1
## [852] 3 1 1 1 1 1 1 1 2 1 1 1 1 1 1 2 1 3 1 1 1 1 1 1 1 3 1 1 1 2 1 1 3 1 1 1 3
## [889] 1 1 1 1 1 1 1 1 1 1 1 1 1 3 2 1 1 3 2 1 2 1 1 1 1 1 2 1 1 1 2 1 3 3 1 1 1
## [926] 3 1 1 1 1 2 1 3 1 1 1 1 1 1 1 1 1 1 1 3 1 3 3 1 1 2 1 1 3 2 2 2 2 1 1 1 1
## [963] 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 3 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
## [1000] 1
```

```r
View(data)

# Summarize numeric and categorical variables for each cluster
summary(data[data$pca.3 == 1, ])
```

```
##     sale_id       branch          city      customer_type    gender
##  Min.   :   1.0   A:506   Chicago    :242   Member:372    Female:369
##  1st Qu.: 262.8   B:258   Los Angeles:258   Normal:392    Male  :395
##  Median : 500.5           New York   :264
##  Mean   : 503.1
##  3rd Qu.: 753.5
##  Max.   :1000.0
##        product_name     product_category   unit_price        quantity
##  Apple        :147   Beverages    :138    Min.   : 1.020   Min.   : 1.000
##  Detergent    :141   Fruits       :152    1st Qu.: 4.728   1st Qu.: 4.000
##  Notebook     :143   Household    :158    Median : 8.425   Median : 8.000
##  Orange Juice:160   Personal Care:149    Mean   : 9.211   Mean   : 8.634
##  Shampoo      :173   Stationery   :167    3rd Qu.:13.113   3rd Qu.:13.000
##                                           Max.   :20.960   Max.   :20.000
##       tax          total_price      reward_points      cluster          pca.3
##  Min.   : 0.08   Min.   :  1.21   Min.   : 0.00   Min.   :1.000   Min.   :1
##  1st Qu.: 1.80   1st Qu.: 27.49   1st Qu.: 0.00   1st Qu.:1.000   1st Qu.:1
##  Median : 3.97   Median : 60.67   Median : 0.00   Median :1.000   Median :1
##  Mean   : 4.72   Mean   : 72.15   Mean   : 3.11   Mean   :1.065   Mean   :1
##  3rd Qu.: 7.38   3rd Qu.:112.77   3rd Qu.: 5.00   3rd Qu.:1.000   3rd Qu.:1
##  Max.   :13.82   Max.   :211.28   Max.   :17.00   Max.   :3.000   Max.   :1
```

```r
summary(data[data$pca.3 == 2, ])
```

```
##     sale_id      branch          city     customer_type    gender
```

```
##  Min.   :  9.0   A:48   Chicago    :27   Member:63     Female:31
##  1st Qu.:216.5   B:15   Los Angeles:15   Normal: 0     Male  :32
##  Median :511.0          New York   :21
##  Mean   :523.0
##  3rd Qu.:837.5
##  Max.   :981.0
##        product_name      product_category   unit_price       quantity
##  Apple        : 6    Beverages    :10      Min.   :13.82   Min.   :12.00
##  Detergent    :11    Fruits       :14      1st Qu.:15.94   1st Qu.:15.50
##  Notebook     :15    Household    :10      Median :18.14   Median :17.00
##  Orange Juice:15    Personal Care:17      Mean   :17.84   Mean   :17.11
##  Shampoo      :16    Stationery   :12      3rd Qu.:20.03   3rd Qu.:19.00
##                                            Max.   :20.98   Max.   :20.00
##       tax          total_price    reward_points    cluster      pca.3
##  Min.   :16.99   Min.   :259.7   Min.   :25    Min.   :3   Min.   :2
##  1st Qu.:19.39   1st Qu.:296.3   1st Qu.:29    1st Qu.:3   1st Qu.:2
##  Median :20.49   Median :313.2   Median :31    Median :3   Median :2
##  Mean   :21.22   Mean   :324.3   Mean   :32    Mean   :3   Mean   :2
##  3rd Qu.:23.16   3rd Qu.:353.9   3rd Qu.:35    3rd Qu.:3   3rd Qu.:2
##  Max.   :28.39   Max.   :434.0   Max.   :43    Max.   :3   Max.   :2
```

```r
summary(data[data$pca.3 == 3, ])
```

```
##     sale_id       branch           city     customer_type    gender
##  Min.   : 18.0   A:120   Chicago    :61   Member:81     Female: 72
##  1st Qu.:244.0   B: 53   Los Angeles:53   Normal:92     Male  :101
##  Median :493.0           New York   :59
##  Mean   :480.6
##  3rd Qu.:706.0
##  Max.   :979.0
##        product_name      product_category   unit_price       quantity
##  Apple        :32    Beverages    :39      Min.   : 9.32   Min.   : 8.00
##  Detergent    :37    Fruits       :43      1st Qu.:12.95   1st Qu.:13.00
##  Notebook     :36    Household    :30      Median :15.53   Median :16.00
##  Orange Juice:33    Personal Care:42      Mean   :15.46   Mean   :15.39
##  Shampoo      :35    Stationery   :19      3rd Qu.:18.00   3rd Qu.:18.00
##                                            Max.   :20.96   Max.   :20.00
##       tax          total_price    reward_points       cluster         pca.3
##  Min.   :10.87   Min.   :166.1   Min.   : 0.000   Min.   :2.000   Min.   :3
##  1st Qu.:13.66   1st Qu.:208.9   1st Qu.: 0.000   1st Qu.:2.000   1st Qu.:3
##  Median :15.08   Median :230.5   Median : 0.000   Median :2.000   Median :3
##  Mean   :16.27   Mean   :248.7   Mean   : 9.624   Mean   :2.468   Mean   :3
##  3rd Qu.:17.93   3rd Qu.:274.1   3rd Qu.:20.000   3rd Qu.:3.000   3rd Qu.:3
##  Max.   :27.94   Max.   :427.1   Max.   :26.000   Max.   :3.000   Max.   :3
```

```r
# Explore cluster distribution by categorical and numeric variables
table(data$city,data$pca.3)
```

```
##
##                  1   2   3
##   Chicago      242  27  61
##   Los Angeles 258  15  53
##   New York     264  21  59
```

```r
table(data$gender,data$pca.3)
```

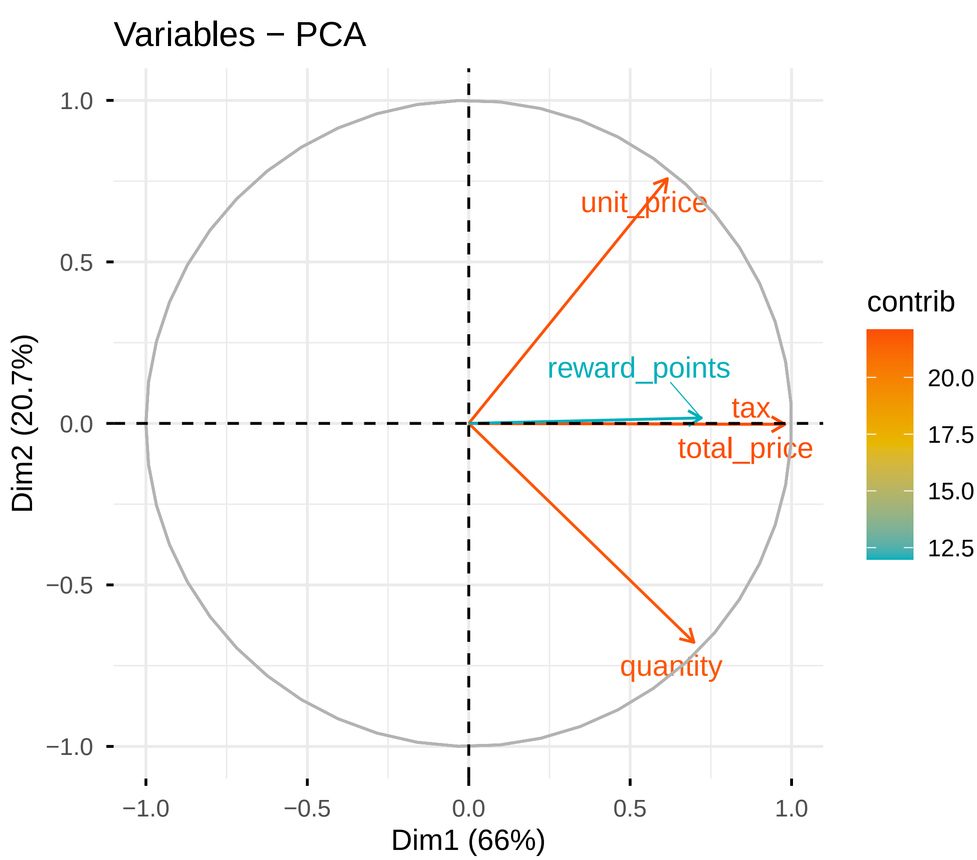
```
##
##            1   2   3
##   Female 369  31  72
##   Male   395  32 101
```

```r
# 5. ADVANCED PCA USING FactoMineR for a more detailed analysis after prcomp
library(FactoMineR)
```

```
## Warning: il pacchetto 'FactoMineR' è stato creato con R versione 4.5.2
```

```r
data.pc=PCA(data[,c(8:12)],graph=FALSE)
fviz_pca_var(data.pc, col.var = "contrib",
gradient.cols = c("#00AFBB", "#E7B800", "#FC4E07"),
repel = TRUE,ggtheme = theme_minimal())
```

```
## Warning: `aes_string()` was deprecated in ggplot2 3.0.0.
## i Please use tidy evaluation idioms with `aes()`.
## i See also `vignette("ggplot2-in-packages")` for more information.
## i The deprecated feature was likely used in the factoextra package.
##   Please report the issue at <https://github.com/kassambara/factoextra/issues>.
## This warning is displayed once every 8 hours.
## Call `lifecycle::last_lifecycle_warnings()` to see where this warning was
## generated.
```



```r
summary(data.pc)
```

```
##
## Call:
## PCA(X = data[, c(8:12)], graph = FALSE)
##
##
## Eigenvalues
##                        Dim.1   Dim.2   Dim.3   Dim.4   Dim.5
## Variance               3.302   1.034   0.571   0.093   0.000
## % of var.             66.033  20.680  11.427   1.860   0.000
## Cumulative % of var.  66.033  86.713  98.140 100.000 100.000
##
## Individuals (the 10 first)
##                   Dist    Dim.1    ctr   cos2    Dim.2    ctr   cos2    Dim.3
## 1             |  2.161 | -2.083  0.131  0.930 |  0.119  0.001  0.003 |  0.329
## 2             |  1.994 | -1.714  0.089  0.739 | -1.012  0.099  0.258 |  0.058
## 3             |  2.378 | -1.554  0.073  0.427 | -1.763  0.301  0.550 |  0.098
## 4             |  1.632 | -1.604  0.078  0.966 |  0.192  0.004  0.014 | -0.006
## 5             |  1.954 | -1.811  0.099  0.859 | -0.580  0.033  0.088 |  0.334
## 6             |  0.704 | -0.430  0.006  0.373 |  0.190  0.003  0.073 | -0.522
## 7             |  2.263 | -2.015  0.123  0.793 |  1.012  0.099  0.200 |  0.152
## 8             |  1.661 |  0.706  0.015  0.181 |  1.088  0.115  0.429 | -1.035
## 9             |  4.033 |  3.813  0.440  0.894 | -0.609  0.036  0.023 |  1.160
## 10            |  5.280 |  5.071  0.779  0.922 |  0.062  0.000  0.000 |  1.432
##                  ctr   cos2
## 1              0.019  0.023 |
## 2              0.001  0.001 |
## 3              0.002  0.002 |
## 4              0.000  0.000 |
## 5              0.020  0.029 |
## 6              0.048  0.551 |
## 7              0.004  0.004 |
## 8              0.188  0.388 |
## 9              0.236  0.083 |
## 10             0.359  0.074 |
##
## Variables
##                  Dim.1    ctr   cos2    Dim.2    ctr   cos2    Dim.3    ctr
## unit_price    |  0.615 11.470  0.379 |  0.758 55.540  0.574 | -0.140  3.418
## quantity      |  0.698 14.744  0.487 | -0.678 44.431  0.459 | -0.137  3.307
## tax           |  0.979 29.030  0.958 | -0.003  0.001  0.000 | -0.162  4.602
## total_price   |  0.979 29.030  0.958 | -0.003  0.001  0.000 | -0.162  4.602
## reward_points |  0.721 15.726  0.519 |  0.017  0.028  0.000 |  0.693 84.071
##                 cos2
## unit_price     0.020 |
## quantity       0.019 |
## tax            0.026 |
## total_price    0.026 |
## reward_points  0.480 |
```

```r
# Extract variable information from PCA: contributions of variables to the first five PCs
var=get_pca_var(data.pc)
var
```

```
## Principal Component Analysis Results for variables
##  ===================================================
##   Name       Description
## 1 "$coord"   "Coordinates for the variables"
## 2 "$cor"     "Correlations between variables and dimensions"
## 3 "$cos2"    "Cos2 for the variables"
## 4 "$contrib" "contributions of the variables"
```

```r
var$contrib
```

```
##                  Dim.1        Dim.2     Dim.3      Dim.4        Dim.5
## unit_price    11.46954 5.553959e+01  3.418364 29.5725054 4.443803e-08
## quantity      14.74407 4.443077e+01  3.307051 37.5181086 6.872317e-08
## tax           29.03030 8.164857e-04  4.601725 16.3695257 4.999764e+01
## total_price   29.03046 8.177714e-04  4.601555 16.3648078 5.000236e+01
## reward_points 15.72564 2.800697e-02 84.071304  0.1750526 2.187887e-09
```

```r
round((var$contrib[,1:5]/100),3)
```

```
##               Dim.1 Dim.2 Dim.3 Dim.4 Dim.5
## unit_price    0.115 0.555 0.034 0.296   0.0
## quantity      0.147 0.444 0.033 0.375   0.0
## tax           0.290 0.000 0.046 0.164   0.5
## total_price   0.290 0.000 0.046 0.164   0.5
## reward_points 0.157 0.000 0.841 0.002   0.0
```

```r
# ================================================================
# DECISION TREE MODELLING
# ================================================================
# Install and load required packages
library(rpart)
```

```
## Warning: il pacchetto 'rpart' è stato creato con R versione 4.5.2
```

```r
library(rpart.plot)
```

```
## Warning: il pacchetto 'rpart.plot' è stato creato con R versione 4.5.2
```

```r
# Split dataset into training (13%) and test set
length(data)
```

```
## [1] 14
```

```r
nrow(data)
```

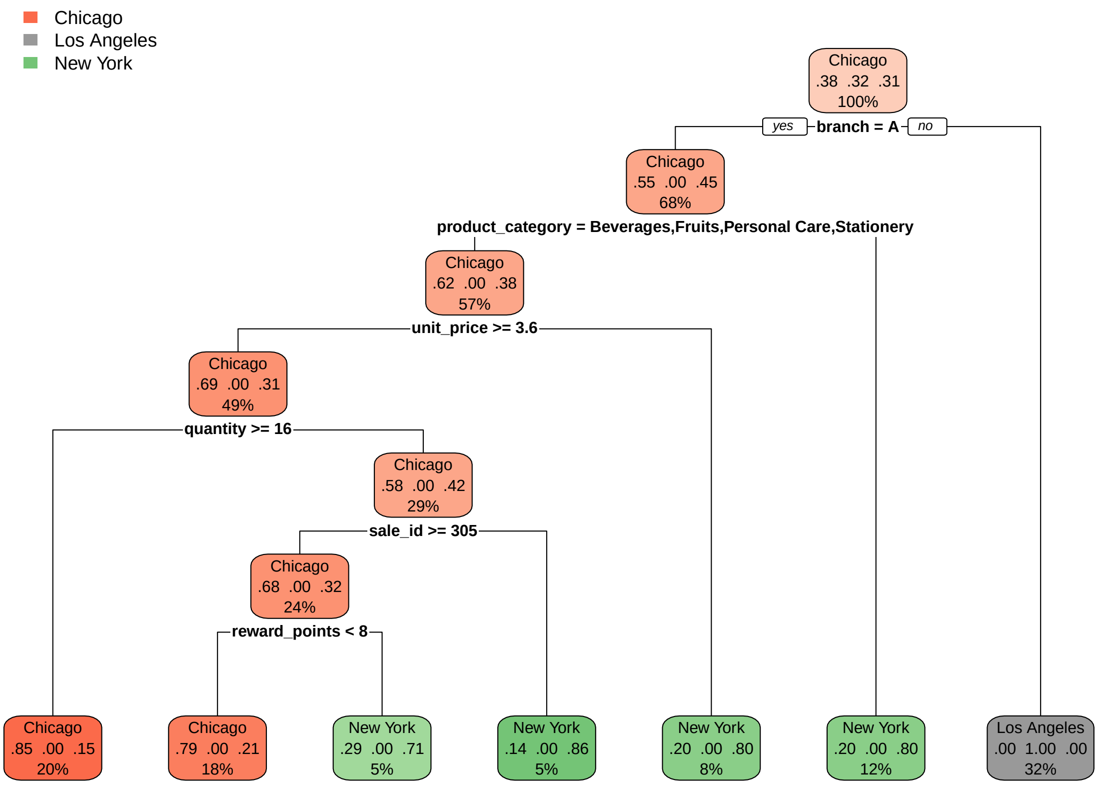
```
## [1] 1000
```

```r
set.seed(2025)
data.idx=sample(1000,1000*.13)
data.train=data[data.idx,]
data.test=data[-data.idx,]

# Build a decision tree to classify data based on selected features
data.dc=rpart(city~.,data=data.train)
data.dc
```

```
## n= 130
##
## node), split, n, loss, yval, (yprob)
##       * denotes terminal node
##
##  1) root 130 81 Chicago (0.3769231 0.3153846 0.3076923)
##    2) branch=A 89 40 Chicago (0.5505618 0.0000000 0.4494382)
##      4) product_category=Beverages,Fruits,Personal Care,Stationery 74 28 Chicago (0.6216216 0.0000000
##        8) unit_price>=3.615 64 20 Chicago (0.6875000 0.0000000 0.3125000)
##         16) quantity>=15.5 26  4 Chicago (0.8461538 0.0000000 0.1538462) *
##         17) quantity< 15.5 38 16 Chicago (0.5789474 0.0000000 0.4210526)
##           34) sale_id>=305 31 10 Chicago (0.6774194 0.0000000 0.3225806)
##             68) reward_points< 8 24  5 Chicago (0.7916667 0.0000000 0.2083333) *
##             69) reward_points>=8 7  2 New York (0.2857143 0.0000000 0.7142857) *
##           35) sale_id< 305 7  1 New York (0.1428571 0.0000000 0.8571429) *
##        9) unit_price< 3.615 10  2 New York (0.2000000 0.0000000 0.8000000) *
##      5) product_category=Household 15  3 New York (0.2000000 0.0000000 0.8000000) *
##    3) branch=B 41  0 Los Angeles (0.0000000 1.0000000 0.0000000) *
```

```r
rpart.plot(data.dc)
```




```r
# Predict city for test set and valuate results with confusion matrix
data.dc.pred=predict(data.dc,data.test,type='class')
conf.matrix=table(data.test$city,data.dc.pred)
conf.matrix
```

```
##              data.dc.pred
##               Chicago Los Angeles New York
##   Chicago         117           0      164
##   Los Angeles       0         285        0
##   New York        147           0      157
```

```r
# Compute overall accuracy
accuracy=sum(diag(conf.matrix)) /sum(conf.matrix)
accuracy
```

```
## [1] 0.6425287
```

```r
# =================================================================
# LINEAR REGRESSION
# =================================================================

# 1. INSTALL AND LOAD THE NECCESSARY VISUALIZATION PACKAGE
library(ggplot2)
```

```r
# 2. MODEL CONSTRUCTION
# Model 1: simple linear regression using total_price as the only predictor
model1=lm(reward_points ~ total_price, data = data)

# Model 2: Multiple linear regression adding customer_type
model2=lm(reward_points ~ total_price + customer_type, data = data)

# Model 3: Full model including product_category
model3=lm(reward_points ~ total_price + customer_type + product_category, data = data)

# 3.MODEL EVALUATION AND COMPARATIVE ANALYSIS
summary(model1)
```

```
##
## Call:
## lm(formula = reward_points ~ total_price, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -23.1318  -3.8322   0.3499   4.4898  19.4892
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.505151   0.370279  -1.364    0.173
## total_price  0.055338   0.002388  23.171   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.544 on 998 degrees of freedom
## Multiple R-squared:  0.3498, Adjusted R-squared:  0.3492
## F-statistic: 536.9 on 1 and 998 DF,  p-value: < 2.2e-16
```

```r
summary(model2)
```

```
##
## Call:
## lm(formula = reward_points ~ total_price + customer_type, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -16.5887  -3.6358   0.0761   3.8452  14.7396
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)           5.240206   0.293344   17.86   <2e-16 ***
## total_price           0.053043   0.001584   33.49   <2e-16 ***
## customer_typeNormal -11.308422   0.316529  -35.73   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.998 on 997 degrees of freedom
## Multiple R-squared:  0.7149, Adjusted R-squared:  0.7143
## F-statistic:  1250 on 2 and 997 DF,  p-value: < 2.2e-16
```

```r
summary(model3)
```

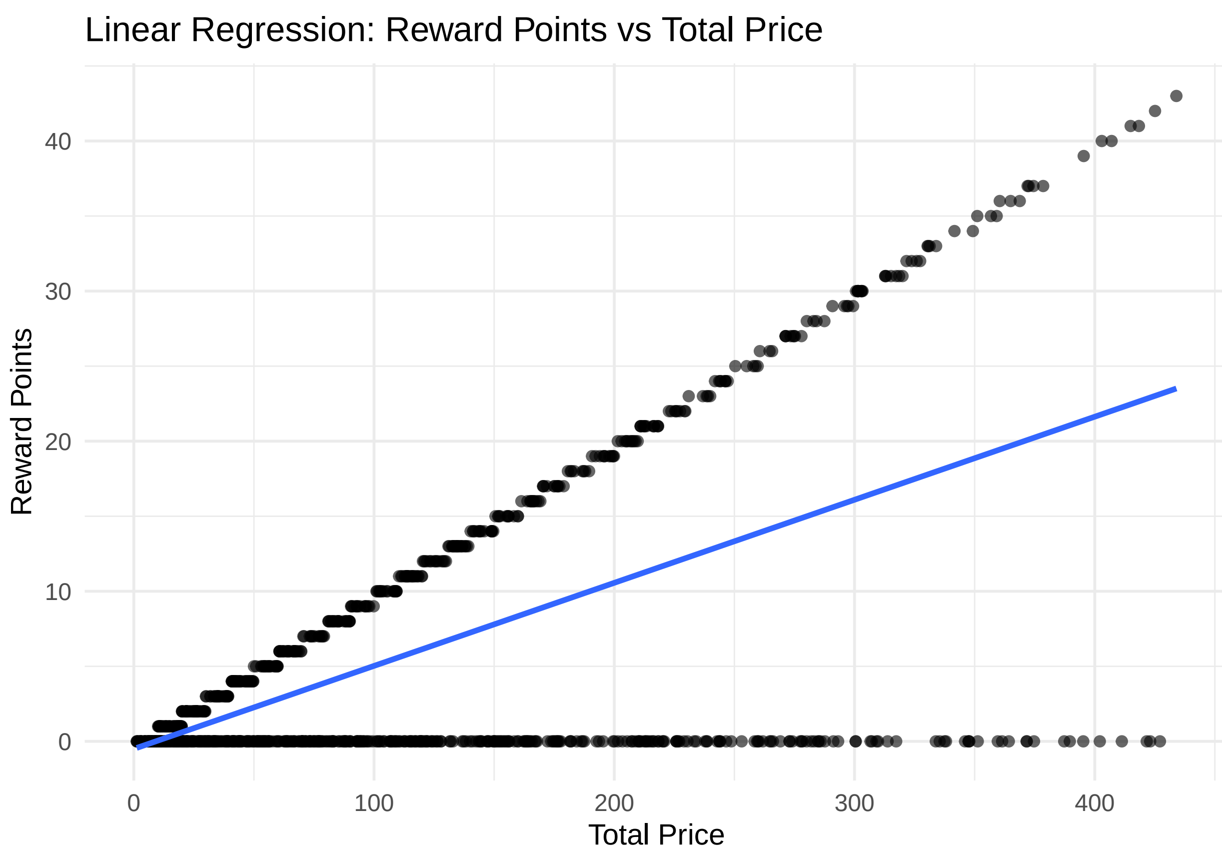
```
##
## Call:
## lm(formula = reward_points ~ total_price + customer_type + product_category,
##     data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -16.5955  -3.6993  -0.0002   3.8398  15.0249
##
## Coefficients:
##                                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)                      5.032167   0.445384  11.298   <2e-16 ***
## total_price                      0.052923   0.001593  33.226   <2e-16 ***
## customer_typeNormal            -11.311686   0.317033 -35.680   <2e-16 ***
## product_categoryFruits           0.269414   0.503612   0.535    0.593
## product_categoryHousehold       -0.025165   0.510715  -0.049    0.961
## product_categoryPersonal Care    0.598977   0.504304   1.188    0.235
## product_categoryStationery       0.242177   0.511039   0.474    0.636
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.003 on 993 degrees of freedom
## Multiple R-squared:  0.7154, Adjusted R-squared:  0.7137
## F-statistic: 416.1 on 6 and 993 DF,  p-value: < 2.2e-16
```

```r
# Perform an Analysis of Variance (ANOVA) to compare the nested models and
# determine if the increased complexity is statistically justified
anova(model1,model2,model3)
```

```
## Analysis of Variance Table
##
## Model 1: reward_points ~ total_price
## Model 2: reward_points ~ total_price + customer_type
## Model 3: reward_points ~ total_price + customer_type + product_category
##   Res.Df   RSS Df Sum of Sq         F Pr(>F)
## 1    998 56791
## 2    997 24906  1     31885 1273.8745 <2e-16 ***
## 3    993 24855  4        51    0.5121 0.7269
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

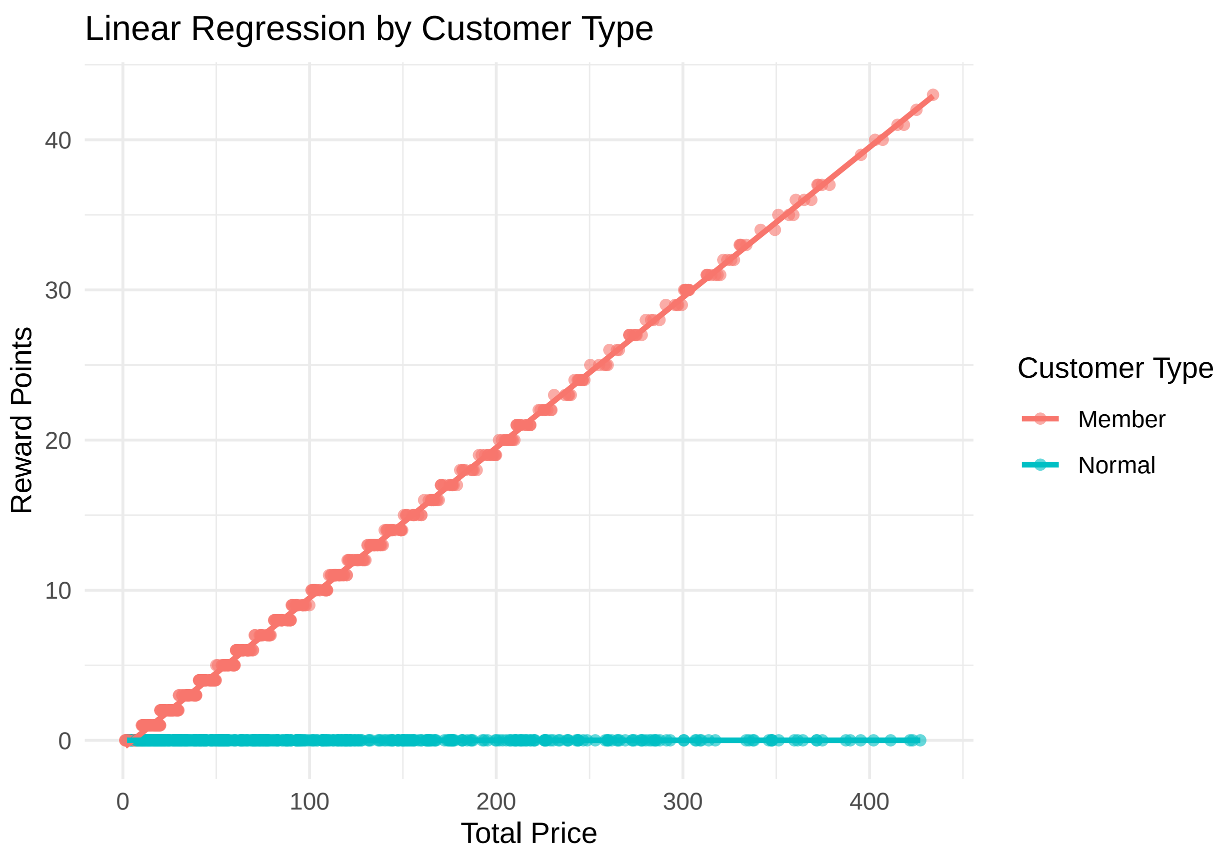
```r
# 4. GRAPHS OF LINEAR REGRESSION MODELS
# Model 1
ggplot(data, aes(x = total_price, y = reward_points)) +
geom_point(alpha = 0.6) +
geom_smooth(method = "lm", se = FALSE) +
  labs(title = "Linear Regression: Reward Points vs Total Price",
  x = "Total Price",
  y = "Reward Points") +
  theme_minimal()
```

```
## `geom_smooth()` using formula = 'y ~ x'
```



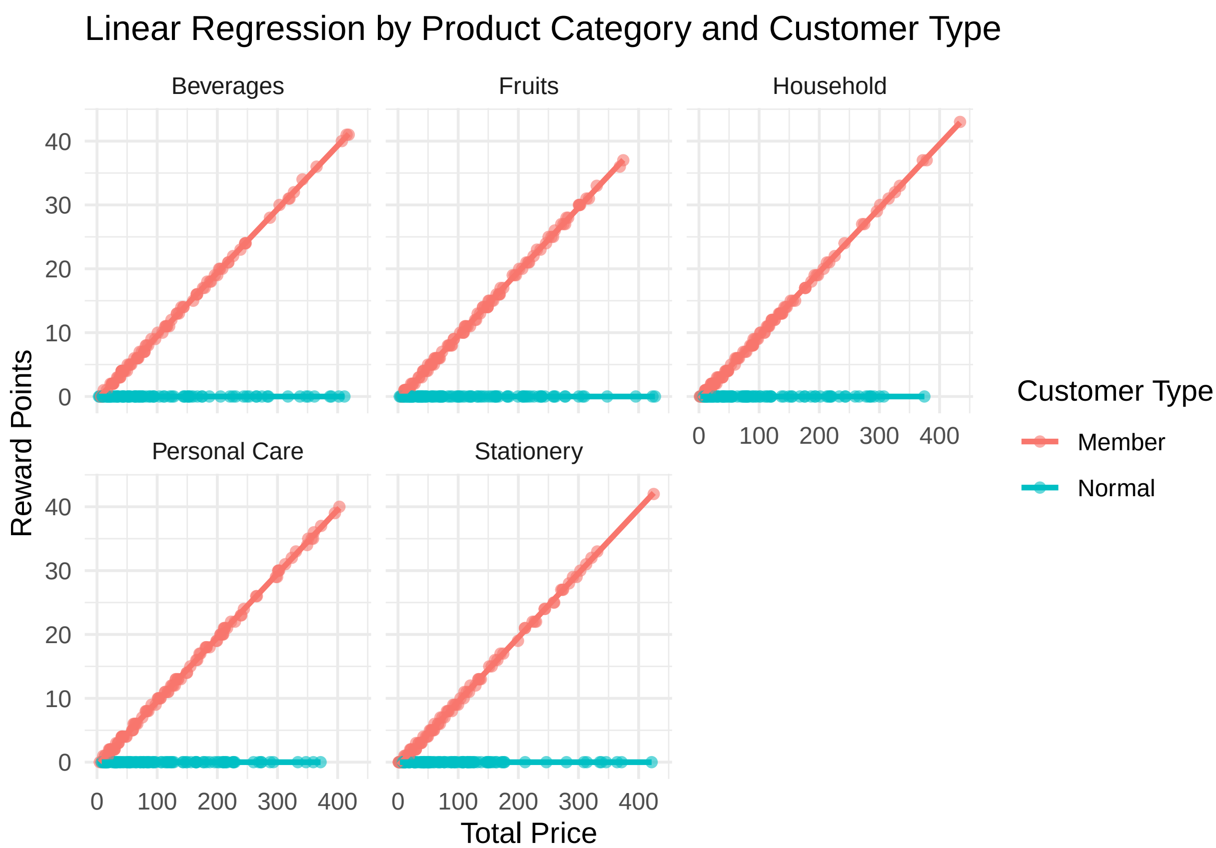

```r
# Model 2
ggplot(data, aes(x = total_price, y = reward_points, color = customer_type)) +
  geom_point(alpha = 0.6) +
  geom_smooth(method = "lm", se = FALSE) +
  labs(
    title = "Linear Regression by Customer Type",
    x = "Total Price",
    y = "Reward Points",
    color = "Customer Type"
  ) +
  theme_minimal()
```

```
## `geom_smooth()` using formula = 'y ~ x'
```

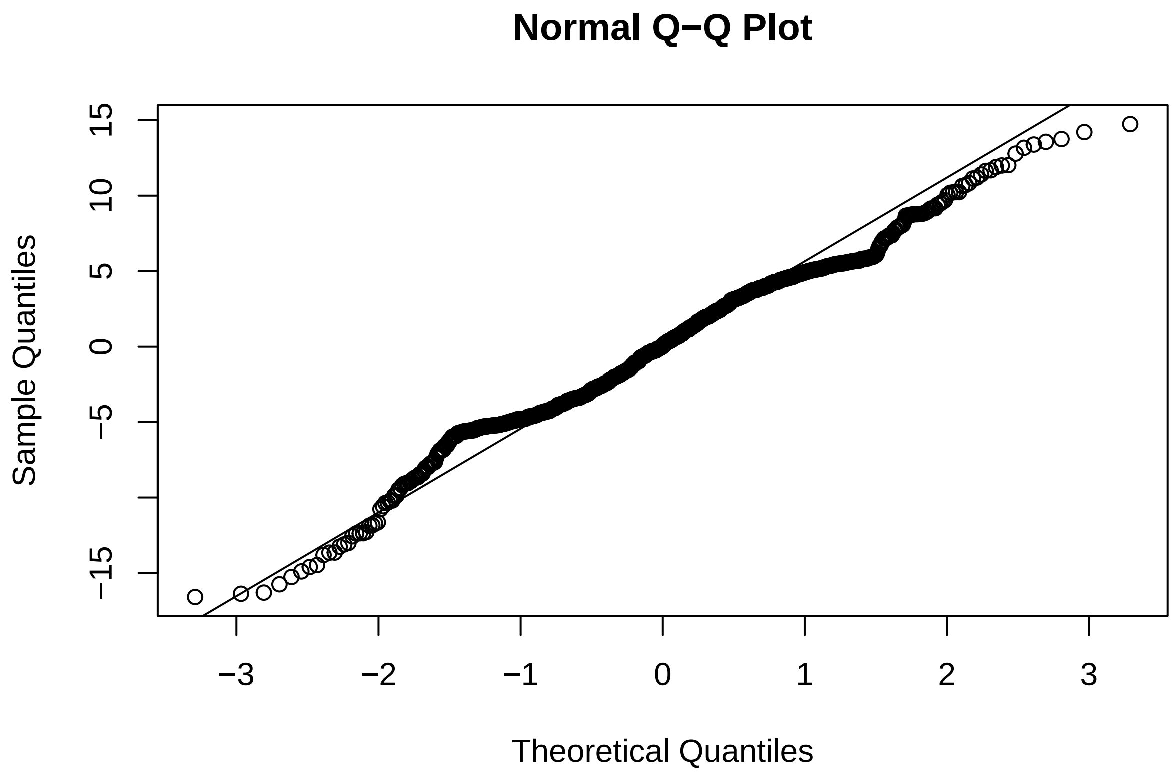



```r
# Model 3
ggplot(data, aes(x = total_price, y = reward_points, color = customer_type)) +
geom_point(alpha = 0.6) +
  geom_smooth(method = "lm", se = FALSE) +
  facet_wrap(~ product_category) +
  labs(title = "Linear Regression by Product Category and Customer Type",
  x = "Total Price",
  y = "Reward Points",
  color = "Customer Type") +
  theme_minimal()
```

```
## `geom_smooth()` using formula = 'y ~ x'
```




```r
# 5. NORMAL Q-Q PLOT
par(mfrow = c(1,1))
qqnorm(resid(model2))
qqline(resid(model2))
```


Normal Q−Q Plot
Sample Quantiles
Theoretical Quantiles
15
10
5
0
−5
−15
−3
−2
−1
0
1
2
3