Wireless Communications

Cellular Network Simulation Report

Claudia Hereter
Arnau Gil
Adrià Casals

19/12/2025

Project structure and execution

We have implemented the simulation using several Python files. The core of the project is the file *cellular_network_simulator.py,* which contains the complete simulator class. This class generates the 19-cell hexagonal network, placing users in each sector, modelling path loss and shadow fading, and computing SIR metrics with and without power control.

Additionally, we have used four separate Python scripts for the results of the assignment, one for each question. Each script imports the simulator class and executes the corresponding simulations, giving the necessary numerical results and plots.

This structure keeps the code organized and makes it easy to run and analyze each question independently while reusing the same simulator implementation.

Finally, the folder also includes the figures that have been obtained when executing the exercises. These images are included in the report.

Python Functions

Main Simulator Class:

`CellularNetworkSimulator`

Constructor: `__init__()`

- **Inputs**: `cell_radius (float = 1.00), pathloss_exp (float = 3.8), shadow_sigma_dB (float = 8.00), seed (int, optional)`
- **Output**: Initialized simulator object with 19-cell hexagonal network.
- **Description**: Configures network geometry, path-loss parameters, and random number generator.

Core Simulation Functions:

`generate_user_positions()`

- **Inputs**: None (uses instance parameters)
- **Output**: `position` → array of shape `(num_cells, num_sectors, 2)` with user coordinates.
- **Description**: Generates uniform random user positions within sectors using rejection sampling.

`compute_sir_snapshot()`

- **Inputs**: `reuse_mode` (str: "reuse1", "reuse3" or "reuse9")
- **Output**: `sir` → array of 3 SIR values (linear scale) for central cell sectors.
- **Description**: Computes Signal-to-Interference Ratio for one snapshot considering frequency reuse pattern.

`run_monte_carlo()`

- **Inputs**: `num_snapshots (int), reuse_mode (str)`
- **Output**: `sir_samples` → array of shape `(num_snapshots, 3)` with SIR values under power control.
- **Description**: Monte Carlo simulation driver.

`find_best_eta()`

- **Inputs**: `num_snapshots (int), reuse_mode (str), threshold_db (float), etas` (array of η values to test)
- **Output**: Tuple `(best_eta, best_coverage, etas_tested, coverages)`
- **Description**: Optimization routine that sweeps power control exponent to maximize percentage of users above SIR threshold.

Utility Functions:

`linear_to_dB()`

- **Inputs**: `x` (array of linear values)
- **Output**: `out` (array converted to dB scale (10 log10)
- **Description**: Converts SIR from linear to decibel scale.

`_shadow_fading_linear()`

- **Inputs**: `size` (int or tuple)
- **Output**: Array of log-normal shadow fading samples (linear scale)
- **Description**: Generates shadow fading: X_dB ~ $N(0, \sigma^2)$, returns 10^(X_dB/10).

Function Interoperability:

1. **Geometry Layer**: `_generate_hex_grid()`, `_compute_cell_groups_*()` → establish network topology
2. **User Generation Layer**: `generate_user_positions()` → samples locations
3. **Channel Model Layer**: `_shadow_fading_linear()` → generates fading realizations
4. **SIR Computation Layer**: `compute_sir_snapshot()` / `compute_sir_snapshot_power_control()` → combine geometry + channel
5. **Monte Carlo Layer**: `run_monte_carlo()` / `run_monte_carlo_power_control()` → aggregate statistics
6. **Optimization Layer**: `find_best_eta()` → sweep parameters to optimize coverage

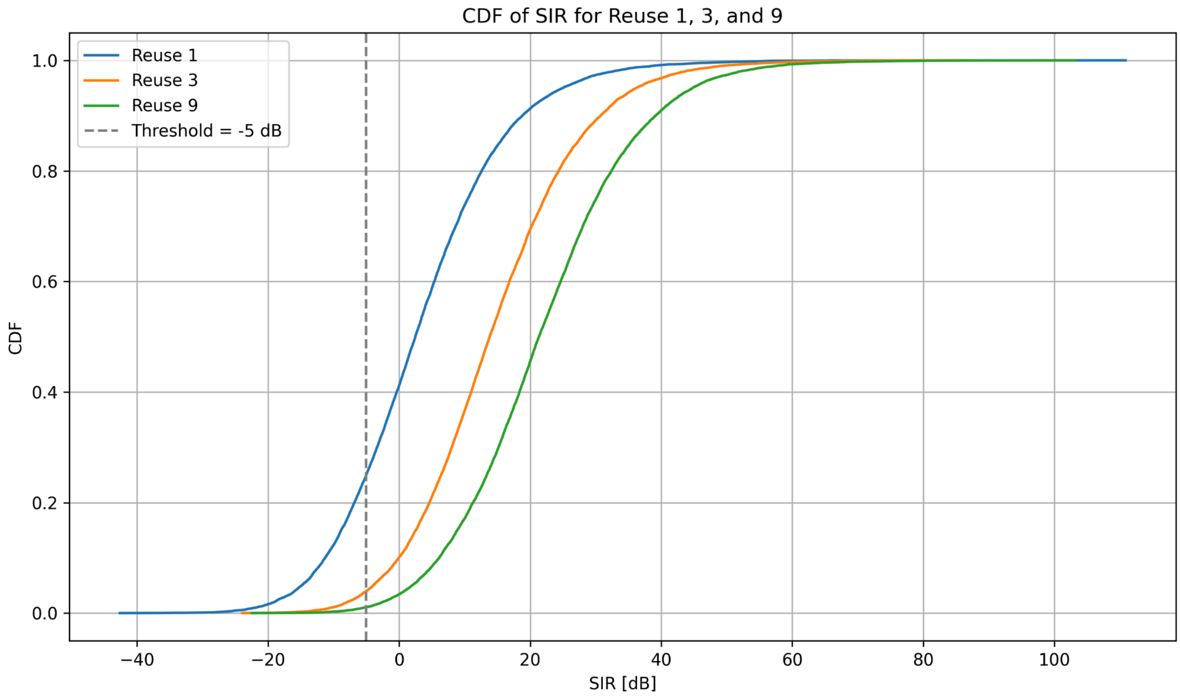
Results

Question 1

```
Reuse 1:
percentage of users with SIR ≥ -5 dB: 75.01%
SIR at 97% probability = 29.04 dB

Reuse 3:
percentage of users with SIR ≥ -5 dB: 96.01%
SIR at 97% probability = 40.54 dB

Reuse 9:
percentage of users with SIR ≥ -5 dB: 98.91%
SIR at 97% probability = 48.78 dB
```



With reuse factor 1 (blue curve), we observe the worst SIR performance, since the CDF curve is farthest to the left. This means that many users have very low SIR. This is expected because with reuse 1, we use the same frequency in all sectors of all cells, which creates strongest inter-cell interference. As a result, only 75.01% of users achieve an SIR above -5dB, so the coverage requirement is not met. However, the SIR at the 97% probability level is 29.04dB, which means that while many users suffer from interference, the strongest users can still achieve very high SIR values. Here we interpret ‘SIR at 97% probability’ as the SIR that 97% of users achieve or exceed (3rd percentile)

With reuse factor 3 (orange curve), the SIR distribution is clearly shifted to the right. In this case, only sectors with the same sector index reuse the frequency, which reduces interference. This leads to a clear improvement in coverage: 96.01% of users now achieve an SIR above -5dB, which is very close to the 97% target but still below it. The SIR at 97% probability increases to 40.54dB, showing a strong overall improvement compared to reuse 1.

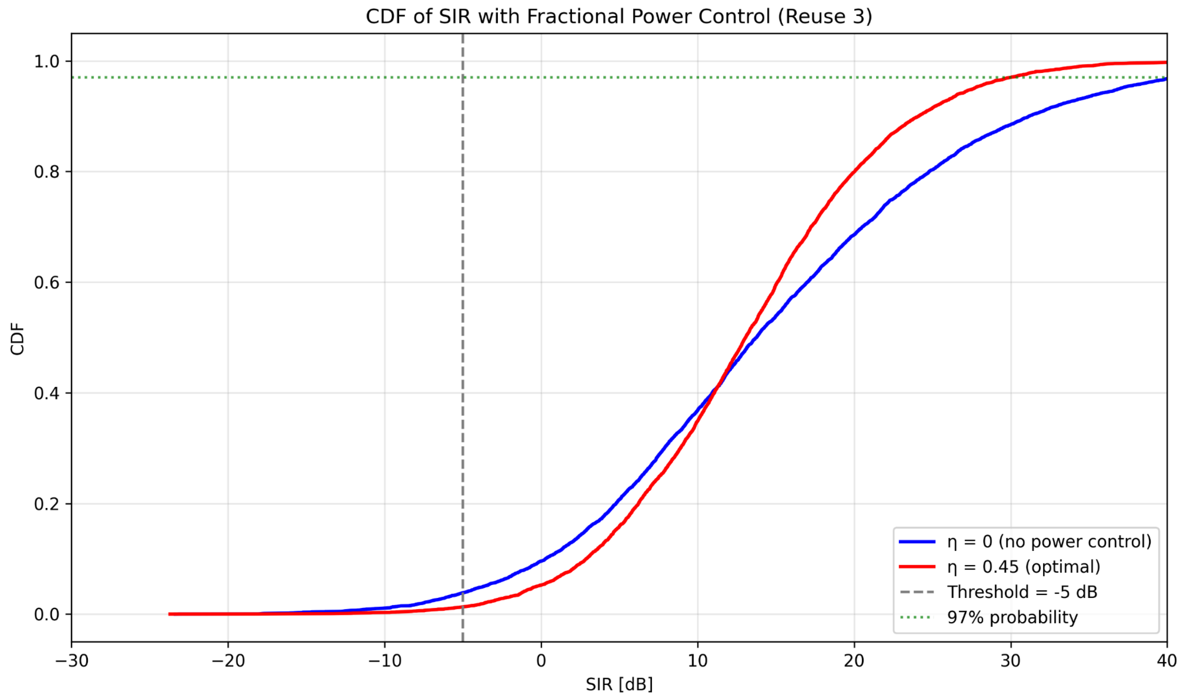
Finally, reuse factor 9 (green curve) provides the best SIR performance. The curve is the most shifted to the right, indicating that almost all users experience high SIR values. This configuration uses a 3x3 frequency pattern, meaning that frequencies are separated across sector index and cell group, which minimizes interference. As a result, 98.91% of users achieve an SIR above -5dB, and this is the only reuse factor that satisfies the 97% requirement. The SIR at the 97% probability level further increases to 48.78dB, confirming the strong interference reduction.

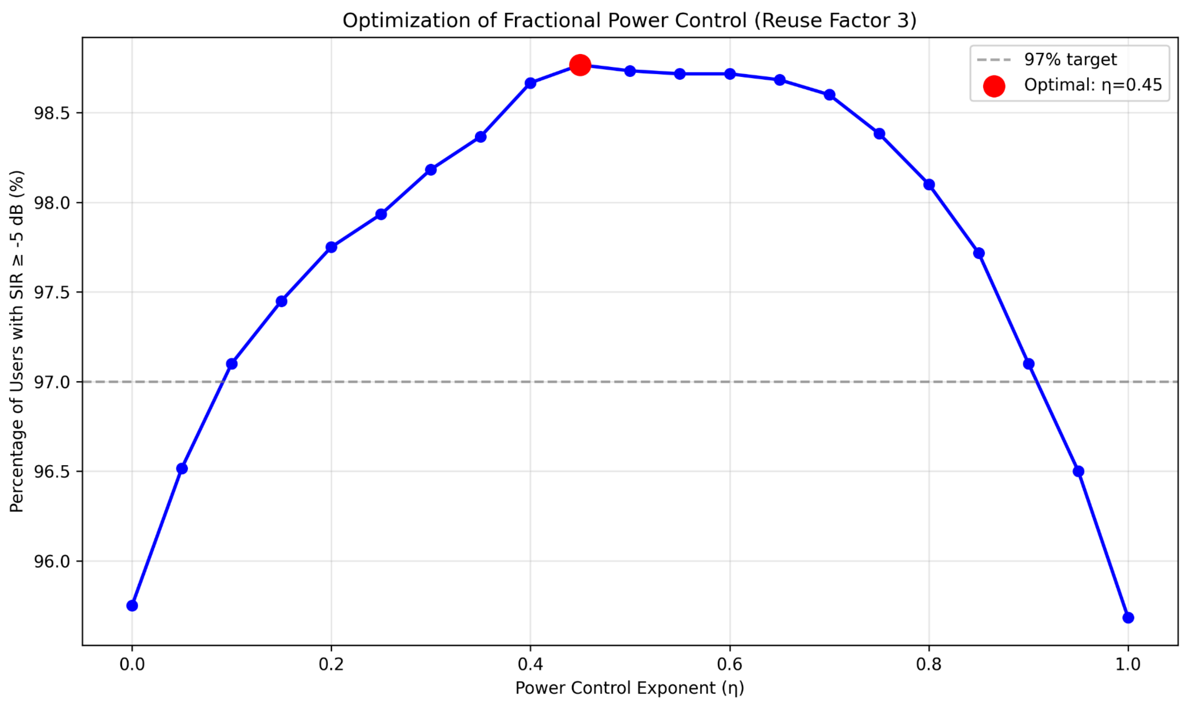
In summary, increasing the reuse factor reduces interference and shifts the SIR distribution to the right. While reuse factors 1 and 3 significantly improve SIR values, only reuse factor 9 guarantees a SIR of at least -5 dB for 97% of users.

Question 2

```
Best eta = 0.45
Max coverage = 98.77 %

Without power control: 96.13%
With optimal η=0.45: 98.70%
Improvement: +2.57 pp
```





In this second part we activated the fractional power control and evaluated the power control exponent η in the range [0, 1] for reuse factor 3. To do so, we added a method that computes the SIR with power control following the formula seen in class. For each value η, we ran the Monte Carlo simulation, computed the SIR and evaluated the percentage of users whose SIR was above -5dB. Then, we selected the exponent that maximizes this metric. This is done with the method *find_best_eta*().

The results show that when no power control is applied (η=0), 96.13% of users meet the SIR requirement. As η increases, the performance improves regularly too. This is because users that are farther away from their base station can transmit with more power, which helps overcome the signal loss caused by distance.

The optimal η is found to be 0.45, providing the maximum coverage of 98.7%. This represents an improvement of almost 2.6% compared to the first case without power control.

However, increasing η more becomes counterproductive. When users increase their transmit power too much, it creates more interference for users in nearby cells. This ends up degrading the overall network performance and lowers the coverage.

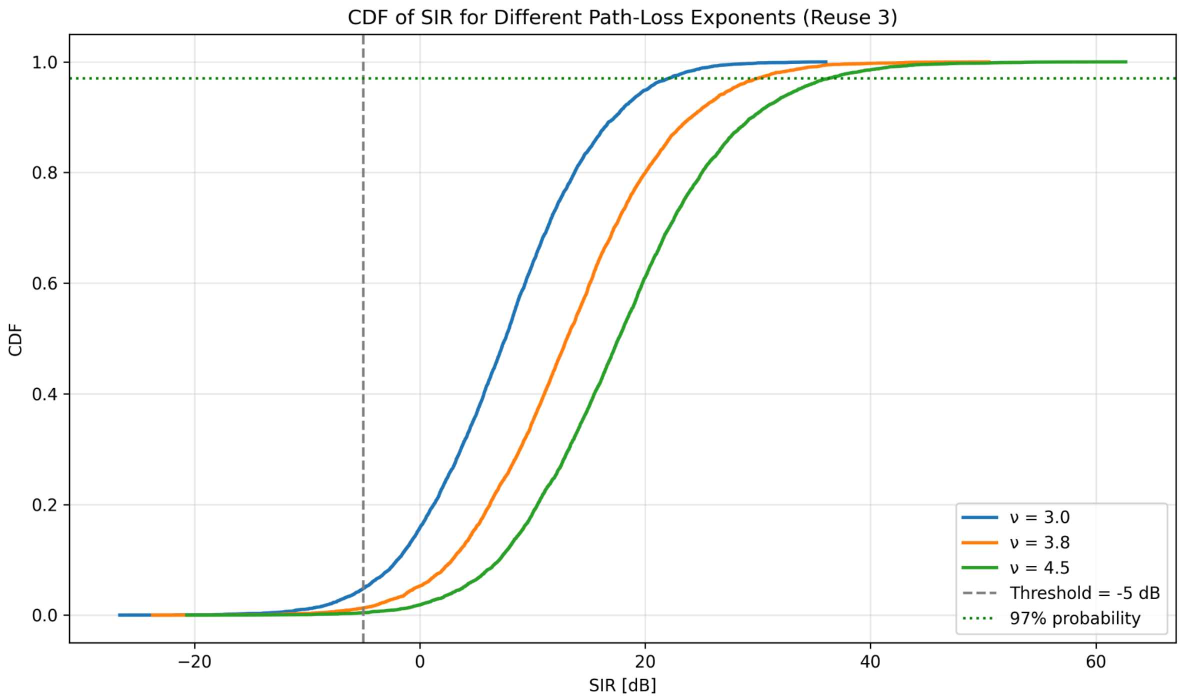
We have plotted the CDF of the SIR for this power control exponent and the CDF of the SIR without power control to compare its effect. The curve for the optimal exponent is shifted to the right in the lower tail, which indicates that fewer users have very low SIR values when power control is applied. Contrarily, on the right side of the plot, the red curve is above the blue one. This means that very high SIR values become a bit less frequent with power control. Also, the median SIR remains practically the same. Therefore, we can say that power control improves mainly fairness, reliability, and protects users in the cell edge (weakest users) rather than maximizing SIR.

Question 3

```
Path-loss exponent ν = 3.0
Optimal η = 0.50
Coverage = 95.55 %

Path-loss exponent ν = 3.8
Optimal η = 0.45
Coverage = 98.77 %

Path-loss exponent ν = 4.5
Optimal η = 0.45
Coverage = 99.70 %
```



The blue curve represents the SIR with ν = 3. This CDF is the most shifted to the left, meaning that users experience a lower SIR overall. When the pathloss exponent is small, distance doesn't weaken the signal as much, it remains quite strong even if distance increases. As a result, interferers that are far away can still cause a lot of interference instead of fading away quickly, which lowers the SIR.

That is why the coverage at -5dB is the worst, with 95.55%, even after optimizing the power control. Also, the power control exponent η is slightly higher (η=0.5), because with weaker pathloss you need more compensation for users that are farther from their base station.

With ν = 3.8 (the orange curve), the whole distribution is shifted to the right compared to the previous case, so fewer users fall below -5dB and the coverage increases to 98.77%. In this case, the optimal η = 0.45. This provides enough compensation for weaker users but not so much that it creates unnecessary interference.

Finally, with ν = 4.5 (the green curve), the CDF is the most shifted to the right, and coverage becomes very high, increasing to 99.7%. A large exponent means that the wanted signal decays very fast with distance, but interference from other cells decays even faster, so only nearby users contribute significantly to interference. This makes it easier to maintain high SIR values. Also, the optimal η is still 0.45, which suggests that increasing η further doesn't help, since the network already suppresses enough interference.

Overall, in scenarios where the performance is limited by interference, it has been seen that a larger ν is preferable, because it reduces the impact of interferers that are far away and also improves SIR and coverage.

A system designer can influence the pathloss exponent ν through the network design. To make signals fade faster with distance, you can use higher carrier frequencies, lower the base station height, deploy smaller and denser cells, or operate in environments that have more buildings and obstacles, which will make the receiver power drop faster. Also, the use of directional antennas have a similar effect, since they focus energy in a specific direction and thus reduce interference from far users.

Question 4

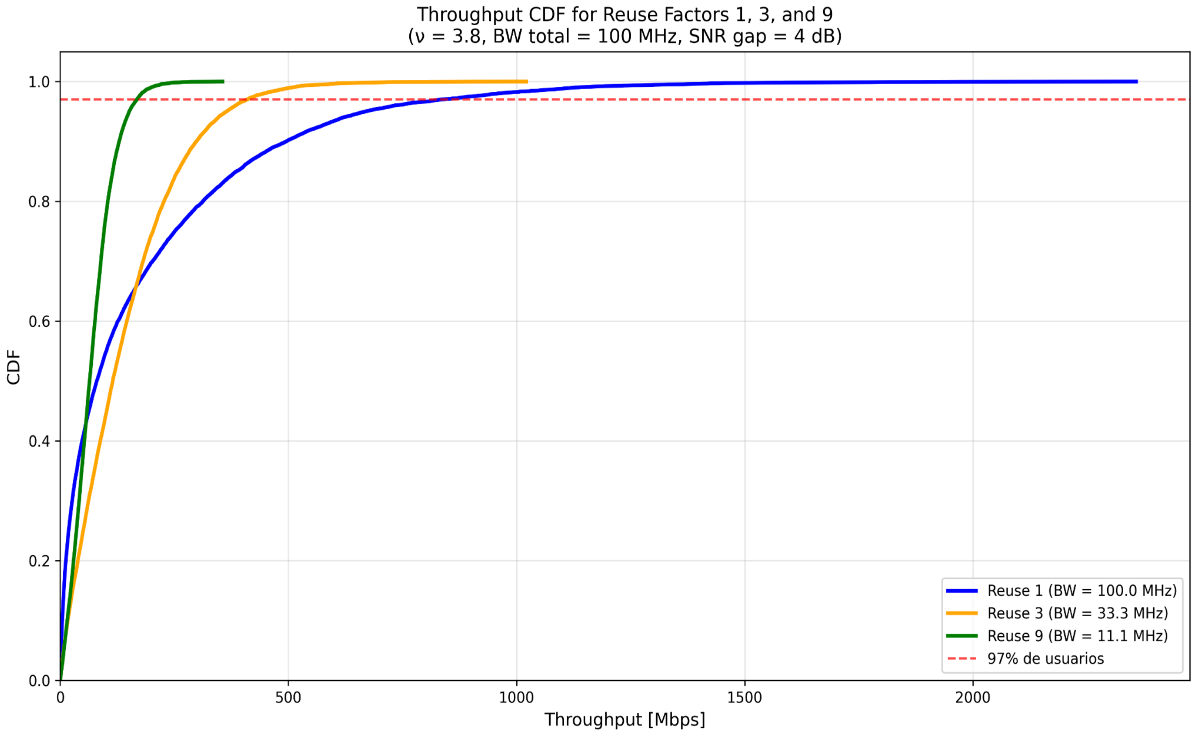
Reuse Factor: 1
Bandwidth per sector: 100.00 MHz
Average throughput: 180.50 Mbps
Throughput achieved by 97% of users: 1.05 Mbps

Reuse Factor: 3
Bandwidth per sector: 33.33 MHz
Average throughput: 140.89 Mbps
Throughput achieved by 97% of users: 4.31 Mbps

Reuse Factor: 9
Bandwidth per sector: 11.11 MHz
Average throughput: 69.69 Mbps
Throughput achieved by 97% of users: 5.12 Mbps

Table of Throughput Results:

R.F	BW (MHz)	Average Throughput (Mbps)	Throughput 97% (Mbps)
1	100.00	180.50	1.05
3	33.33	140.89	4.31
9	11.11	69.69	5.12



For the fourth question, we compute the user throughput by simulating SIR values without power control and using the Shannon capacity formula with a 4 dB gap and fixed ν = 3.8. Since the total bandwidth is shared differently depending on the reuse factor, each sector effectively has less bandwidth when the reuse factor increases (100MHz for reuse 1, 33.3MHz for reuse 3, and 11.1MHz for reuse 9. We consider this when computing the throughput.

The results show that reuse factor 1 provides the highest average throughput, around 180.5Mbps, while for reuse 3 and reuse 9 the mean throughput decreases to 140.9Mbps and 69.7Mbps respectively. This is expected, since reuse 1 maximizes the available bandwidth per sector despite suffering from higher interference.

Furthermore, we interpret the "rate attained by 97% of users" as the rate that almost all users (97%) can achieve or exceed. This corresponds to the lower percentile of the throughput distribution, because only a small fraction of users (3%) are allowed to be worse than this value. Thus, we have computed the 3rd percentile (np.percentile(R, 3).

When looking at this metric, reuse 1 performs the worst, providing only about 1.05 Mbps to 97% of users, due to strong inter-cell interference. Reuse 3 improves this value to 4.31 Mbps, while reuse 9 offers the best fairness, with 5.12 Mbps guaranteed to 97% of users.

We can observe this behavior in the throughput CDFs. The curve for reuse 1 extends far to the right, indicating very high peak throughputs for some users, but it rises slowly at low rates. This indicates poor performance for users at the edge of the cell. However, reuse 9 shows a much steeper CDF, meaning that most users achieve similar and more reliable throughputs, although the maximum rates are limited by the reduced bandwidth. Reuse factor 3 provides the best balance between bandwidth and interference, offering good average throughput while significantly improving the performance of the weakest users.