Monte Carlo Simulation of SAC

PADME Weekly Meeting INFN-LNF 01/12/2017

Claudia Rella University of Rome La Sapienza

GOAL

- Study of geometry dependence of the properties of the distribution of Cherenkov photons produced in PbF₂ crystals and converted into photo-electrons through PMT photocathode:
 - Distribution of number of collected photons
 - Distribution of arrival times
 - Containment of Cherenkov light
 - Energy resolution

Implementation on Geant4

Construction of PbF₂ crystal:

- Configuration of geometric properties:
 - Rectangular parallelepiped of dimensions 30x30xL mm
 - Different lengths considered: L=10, 12, 14, 16, 18, 20 cm
- Configuration of optical properties:
 - All optical properties are specified as a function of the photon energy
 - Energy spectrum of interest: from 1.6 eV to 5.0 eV with bin width of 0.02 eV
 - Computation of refractive index through the dispersion formula:

$$n^2 - 1 = \frac{0.66959342 \,\lambda^2}{\lambda^2 - 0.00034911^2} + \frac{1.3086319 \,\lambda^2}{\lambda^2 - 0.17144455^2} + \frac{0.01670641 \,\lambda^2}{\lambda^2 - 0.28125513^2} + \frac{2007.8865 \,\lambda^2}{\lambda^2 - 796.67469^2}$$

Reference: I. H. Malitson and M. J. Dodge. Refraction and dispersion of lead fluoride, J. Opt. Soc. Am. **59**, 500A (1969)

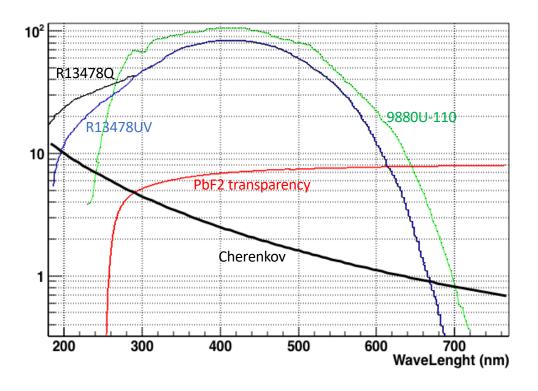
- Computation of absorption length from PbF₂ transparency
- Application of overall scale factor to absorption length spectrum of 1.0, 2.0, 5.0, 10.0

- Configuration of surfaces:
 - Ideally planar surfaces:

```
opPbF2Surface->SetType(dielectric_dielectric);
opPbF2Surface->SetModel(glisur);
opPbF2Surface->SetFinish(polished);
```

Construction of glue cylinder:

- Epoxy EJ-500 (CMS IN1999/026)
- Dimensions:
 - Radius of 1.27 cm
 - Thickness of 0.1 cm
- Refractive index of 1.57

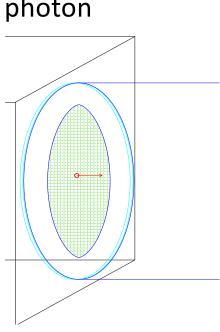


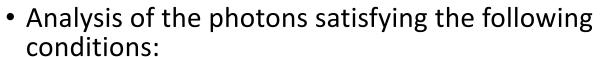
Implementation of PMT R13478UV Quantum Efficiency as function of photon energy

Different energies of the incident photon were considered: 10, 50, 100, 200, 500, 1000 MeV

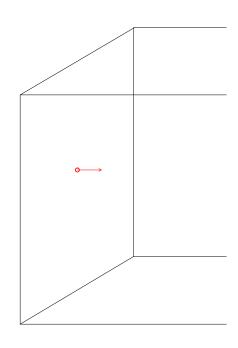
Single optical event

- One beam of a single photon with energy fixed is fired at a distance of 1 mm from the crystal's surface along the direction of the major axis
- Production of about 3000 optical photons by Cherenkov effect inside the crystal
- Simulation step by step of the optical path followed by each photon





- Survived to get to the opposite face of the crystal
- Hit the surface inside the area covered by the glue circle
- Was refracted through the resin and got to PMT surface
- Passed selection due to PMT quantum efficiency



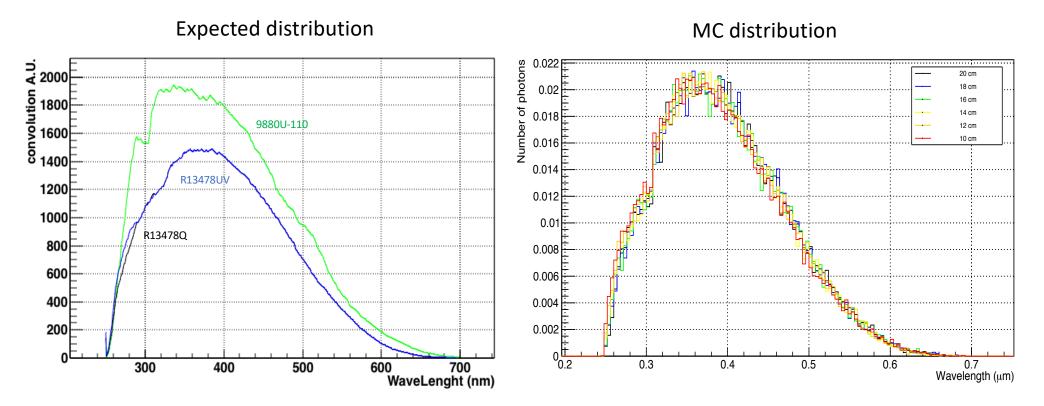
```
### Run 0 start.
OpNoviceEventAction - Event 0 Begin
X(mm)
                 Z(mm) KinE(MeV) dE(MeV) StepLeng TrackLeng NextVolume ProcName
Step#
           Y(mm)
                                                World initStep
  0
        0
                 -71
                        200
                               0
                                     0
                                           1 PbF2 Crystal Transportation
                  -70
                        200
                 -61.1
                                         9.91 PbF2 Crystal conv
                                   8.91
     *************************
               Particle = e-. Track ID = 2, Parent ID = 1
 G4Track Information:
Z(mm) KinE(MeV) dE(MeV) StepLeng TrackLeng NextVolume ProcName
Step#
           Y(mm)
     X(mm)
                 -61.1
                                           0 PbF2 Crystal initStep
                         65
                               0
     0.0118 1.44e-05
                 -60.9
                        64.8
                             0.171
                                  0.222
                                         0.222 PbF2 Crystal Cerenkov
           *************************
Step#
     X(mm)
           Y(mm)
                 Z(mm) KinE(MeV)
                           dE(MeV) StepLeng TrackLeng NextVolume ProcName
          0.0375
                -60.1 4.63e-06
                               0
                                           0 PbF2 Crystal initStep
     0.0562
                                     0
                 -53.2 4.63e-06
                               0
                                   16.5
                                         16.5
                                                World Transportation
  1
       15
            1.6
                                          647 OutOfWorld Transportation
       291
            127
                  500 4.63e-06
                                    631
                               0
```

```
******************************
 G4Track Information:
                      Particle = opticalphoton. Track ID = 107, Parent ID = 2
**************************
                        Z(mm) KinE(MeV) dE(MeV) StepLeng TrackLeng NextVolume ProcName
Step#
       X(mm)
                Y(mm)
   0
       0.0688
               0.0513
                        -59.9 3.86e-06
                                                     0
                                                               0 PbF2 Crystal initStep
   1
         -15
                -4.85
                        -48.2 3.86e-06
                                             0
                                                   19.7
                                                            19.7
                                                                      World Transportation
   2
          -15
                -4.85
                        -48.2 3.86e-06
                                             0
                                                     0
                                                            19.7 PbF2 Crystal Transportation
   3
          15
                -14.6
                        -24.9 3.86e-06
                                             0
                                                   39.2
                                                            58.9
                                                                      World Transportation
   4
          15
                -14.6
                        -24.9 3.86e-06
                                             0
                                                     0
                                                            58.9 PbF2 Crystal Transportation
   5
         13.8
                  -15
                          -24 3.86e-06
                                             0
                                                    1.6
                                                            60.5
                                                                      World Transportation
   6
         13.8
                  -15
                          -24 3.86e-06
                                             0
                                                     0
                                                            60.5 PbF2 Crystal Transportation
   7
         -15
                -5.65
                         -1.7 3.86e-06
                                             0
                                                   37.6
                                                                      World Transportation
   8
                                                              98 PbF2 Crystal Transportation
         -15
                -5.65
                         -1.7 3.86e-06
                                             0
                                                     0
   9
          15
                4.11
                         21.5 3.86e-06
                                             0
                                                   39.2
                                                                      World Transportation
                                                             137
  10
          15
                 4.11
                         21.5 3.86e-06
                                                             137 PbF2 Crystal Transportation
  11
         13.7
                 4.53
                         22.6 3.86e-06 3.86e-06
                                                   1.69
                                                             139 PbF2 Crystal OpAbsorption
Step#
        X(mm)
                Y(mm)
                        Z(mm) KinE(MeV)
                                       dE(MeV) StepLeng TrackLeng NextVolume ProcName
       0.0595
                0.041
                          -60 1.62e-06
                                                              0 PbF2 Crystal initStep
                                             0
                                                     0
   1
         -15
                -9.99
                          -45 1.62e-06
                                             0
                                                   23.5
                                                            23.5
                                                                      World Transportation
   2
         -15
                -9.99
                          -45 1.62e-06
                                                            23.5 PbF2 Crystal Transportation
                                             0
                                                     0
   3
        -7.48
                  -15
                        -37.4 1.62e-06
                                             0
                                                   11.7
                                                            35.3
                                                                      World Transportation
   4
        -7.48
                  -15
                        -37.4 1.62e-06
                                             0
                                                     0
                                                            35.3 PbF2 Crystal Transportation
   5
          15
              -0.0216
                          -15 1.62e-06
                                             0
                                                   35.1
                                                            70.4
                                                                      World Transportation
   6
                                                            70.4 PbF2 Crystal Transportation
          15
              -0.0216
                          -15 1.62e-06
                                             0
                                                     0
   7
        -7.55
                   15
                         7.56 1.62e-06
                                             0
                                                   35.2
                                                            106
                                                                      World Transportation
   8
        -7.55
                   15
                         7.56 1.62e-06
                                             0
                                                     0
                                                             106 PbF2 Crystal Transportation
   9
         -15
                   10
                           15 1.62e-06
                                             0
                                                  11.6
                                                            117
                                                                      World Transportation
  10
          -15
                   10
                           15 1.62e-06
                                             0
                                                            117 PbF2 Crystal Transportation
                                                     0
                -9.95
  11
          15
                           45 1.62e-06
                                             0
                                                   46.9
                                                            164
                                                                      World Transportation
  12
          15
                -9.95
                           45 1.62e-06
                                             0
                                                             164 PbF2 Crystal Transportation
                                                     0
  13
         7.42
                  -15
                         52.6 1.62e-06
                                             0
                                                  11.8
                                                            176
                                                                      World Transportation
  14
         7.42
                  -15
                         52.6 1.62e-06
                                             0
                                                     0
                                                            176 PbF2 Crystal Transportation
  15
          -10
                -3.38
                           70 1.62e-06
                                             0
                                                   27.3
                                                             203 Epoxy Resin Transportation
*** Killing Photon Event 0 N 1 P = -10.019 -3.380 70.000 mm T = 1.255 ns E = 1.618 eV ***
```

Control Check: distribution of wavelengths

Simulation of 100 events of single 200 MeV photon, absorption length scale of 10.0 Normalized histogram of wavelengths of collected photo-electrons

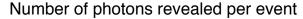
- Invariance under change of crystal's length
- Consistency with expected distribution profile

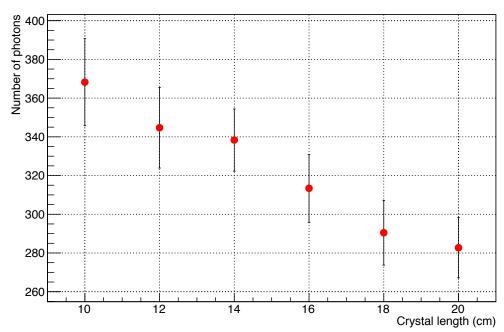


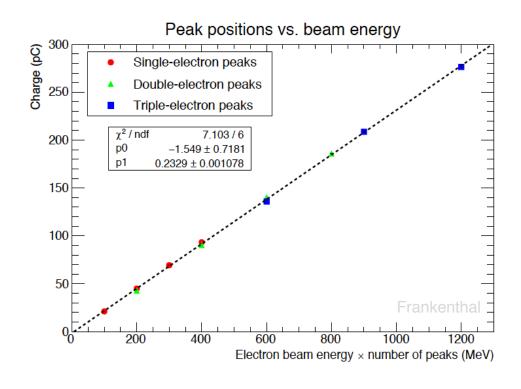
Control Check: number of p.e. per MeV

Simulation of 100 events of single 200 MeV photon, absorption length scale of 10.0 Graph of number of p.e. collected per event as function of crystal's length

- Crystal length of 14 cm: $338.34 \pm 32.12 \ p.e.$ 1.69 $\pm 0.16 \ \frac{p.e.}{MeV}$
- Expected value from experimental data (Test Beam): $\frac{Q}{e \cdot G} = \frac{0.23 \ pC}{(1.602 \cdot 10^{-19} \ C) \ (8 \cdot 10^5)} \approx 1.64 \ \frac{p.e.}{MeV}$





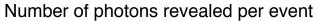


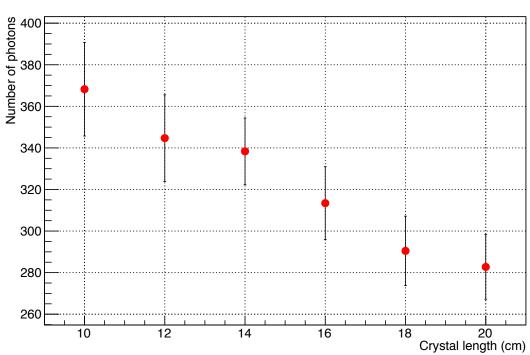
Number of p.e. with different crystal's geometries

Simulation of 100 events of single 200 MeV photon, absorption length scale of 10.0

Graph of number of p.e. collected per event as function of crystal's length

- Crystal length of 14 cm: $338.34 \pm 32.12 \ p.e.$
- Crystal length of 18 cm: $290.45 \pm 33.42 \ p.e.$
- Drop of about 14.2% of collected light
- Advantage of shorter lengths



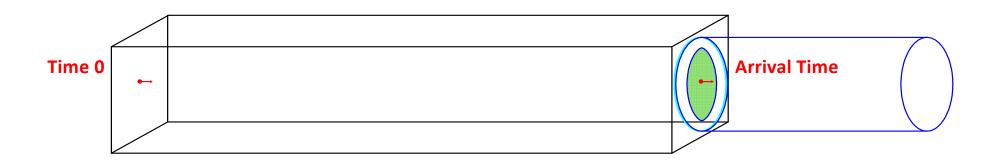


Arrival Times

Simulation of 100 events of single 200 MeV photon, absorption length scale of 10.0

Analysis of the distribution of arrival times of successful optical photons:

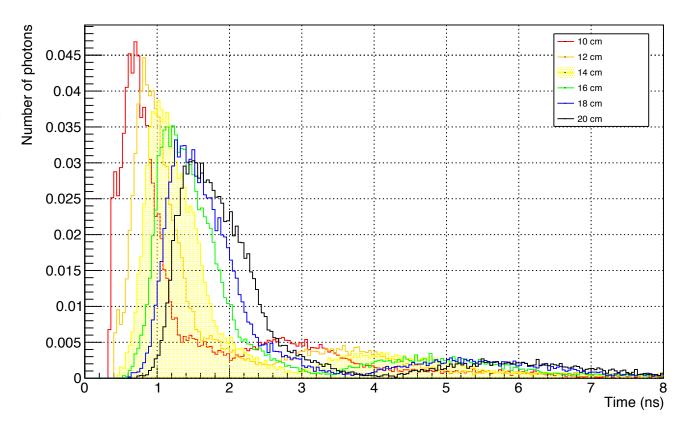
- Time 0: incident 200 MeV photon hits the surface of crystal and refracts inside ≈ start of the simulation
 - Negligibility of time taken to get to the surface (order of $10^{-3} ns$) compared to typical times of simulation (order of ns)
- Arrival Time: optical photon survives to PMT photocathode's quantum efficiency selection and is converted into photo-electron



Arrival Times with different crystal's geometries

Normalized histograms of the distribution with two distinguishable peaks

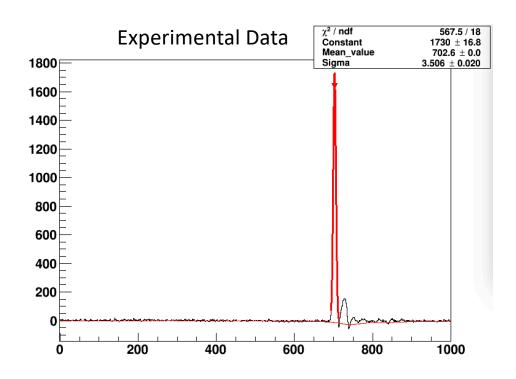
- First peak significantly higher than second one
- Changes in function of crystal length of the absolute and relative proportions of peaks
 - Global translation to right
 - Absolute lowering and widening of peaks
 - Relative departure of peaks

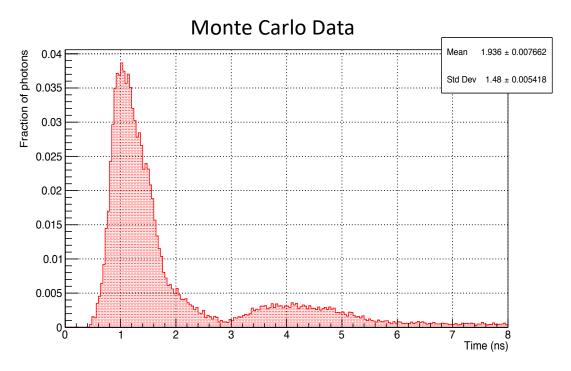


Double Peak on Arrival Times distribution

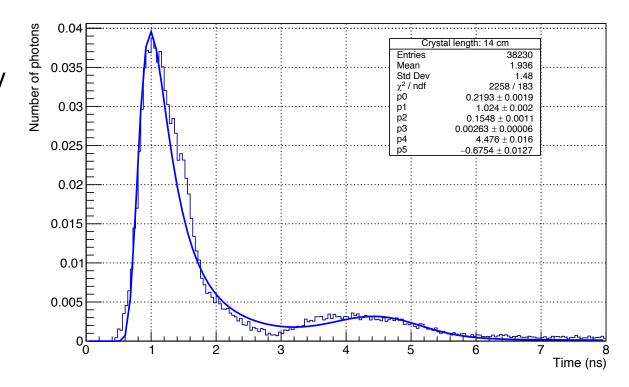
Explanation of the doubly-peaked distribution profile experimentally observed:

- Unavoidable optical effect
- Evident electronic contribution

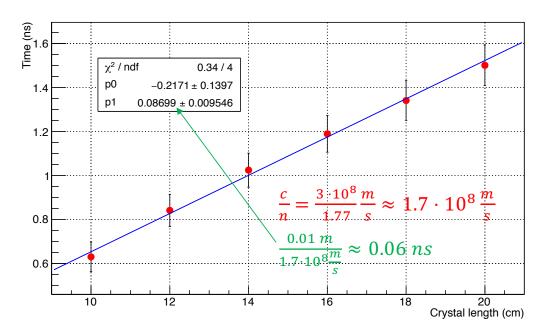




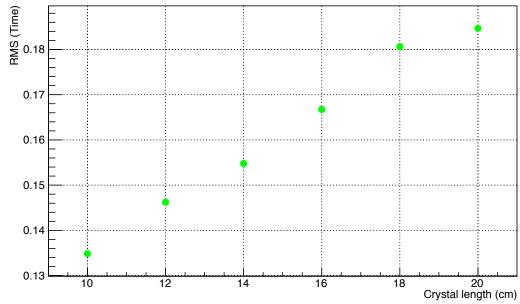
- Fit of the distribution performed using a 6 parameter function given by a Landau function convoluted with a Gaussian function
 - Landau centred at higher peak
 - Gaussian centred at lower peak
- Aim of realizing a tight and high first peak localized at small delay times and a broad, low and distant second peak: quick process and small dispersion in time (purpose of SAC)
 - Selection of the first peak from relative fitting via Landau
 - Graph of Mean Arrival Time as function of crystal length
 - Graph of Arrival Time RMS as function of crystal length



Mean Arrival Time vs. Crystal length



Arrival Time RMS vs. Crystal length



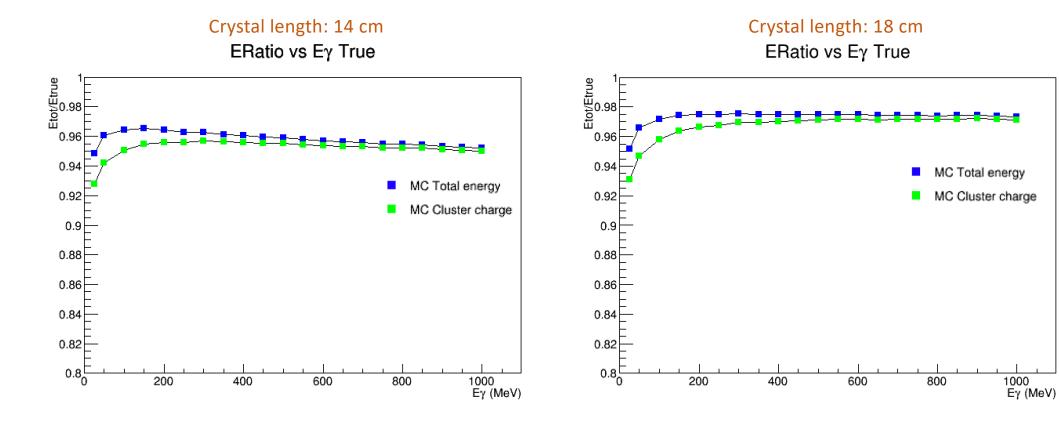
- Linear interpolation partially satisfying
- Absolute delay can easily be managed by a translation of the origin of time
- No real contribution to our previous knowledge

- Quantification of the broadening of time distribution as a consequence of crystal's length increasing
- Bigger lengths involve a lost in time resolution

Containment of Cherenkov light

Quantitative analysis of the containment of optical photons by crystals of different lengths:

- Graph of $E_{\text{ratio}} = \frac{E_{tot}}{E_{true}}$ as function of E_{γ}
- At higher energies, bigger lengths show a greater stability

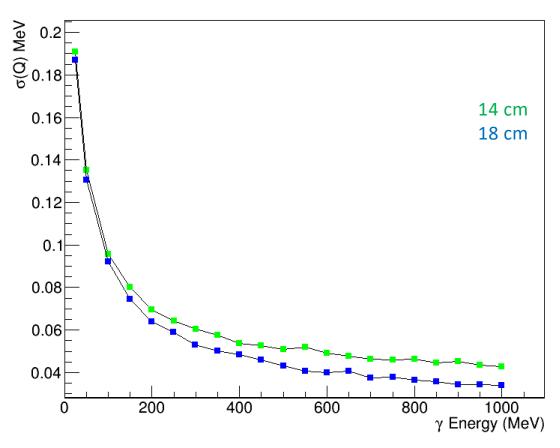


Energy resolution

Analysis of energy resolution as function of crystal's length:

- Graph of $\sigma(Q)$ as function of E_{γ}
- Difference in resolution is quite moderate if $E_{\gamma} < 500~MeV$ (region of interest for PADME)
- This analysis does not consider the decreasing collected light as the crystal's length increases
- Advantage of bigger lengths seems unsubstantial

Charge resolution vs Ey



Conclusions

- MC simulation provides a valid estimates of the PbF₂ photo-electron yield, given the reasonable configuration of the optical system
- Study of different crystal's geometries shows a drop of the number of p.e. collected per event of about 15% going from 14 cm to 18 cm of length
- Study of arrival times:
 - Partially explain double peak behaviour observed in experimental data
 - Time arrival distribution RMS increases with crystal length
- Energy resolution is only slightly dependent on crystal's length
- 14 cm seems to be an adequate choice