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Time Series Zillow Research

Business Problem

Our client is an investor who is looking for buying in Austin, TX. We will build a model to help the client make the decision. The reason that the client chose Austin, TX is because the rapidly expanding economy.

Obtaining Data

Import the Libraries & Load Data

In [192…

```
# importing relevant libraries
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

import statsmodels.api as sm
from statsmodels.tsa.stattools import adfuller
from statsmodels.graphics.tsaplots import plot_acf
from statsmodels.graphics.tsaplots import plot_pacf
from statsmodels.tsa.seasonal import seasonal_decompose

from pmdarima.arima import auto_arima

from sklearn.metrics import mean_squared_error
import math
import warnings
warnings.filterwarnings('ignore')

import itertools
from collections import Counter



import seaborn as sns

from statsmodels.tsa.arima.model import  ARIMA
from statsmodels.tsa.statespace.sarimax import SARIMAX
from statsmodels.tsa.arima_model import ARIMAResults

%matplotlib inline
from matplotlib.pylab import rcParams
#import pmdarima as pm
#from pyramid.arima.utils import ndiffs
#from pmdarima.arima import auto_arima
```

In [193…

```
# Loading the data
df = pd.read_csv('zillow_data.csv')
```

In [194…
```
df.head()
```

Out[194…

	RegionID	RegionName	City	State	Metro	CountyName	SizeRank	1996-04	1996-05
0	84654	60657	Chicago	IL	Chicago	Cook	1	334200.0	335400.0
1	90668	75070	McKinney	TX	Dallas-Fort Worth	Collin	2	235700.0	236900.0
2	91982	77494	Katy	TX	Houston	Harris	3	210400.0	212200.0
3	84616	60614	Chicago	IL	Chicago	Cook	4	498100.0	500900.0
4	93144	79936	El Paso	TX	El Paso	El Paso	5	77300.0	77300.0

5 rows × 272 columns

In [195…
```
df.dtypes
```

Out[195…
```
RegionID       int64
RegionName     int64
City          object
State         object
Metro         object
               ...
2017-12        int64
2018-01        int64
2018-02        int64
2018-03        int64
2018-04        int64
Length: 272, dtype: object
```

In [196…
```
df.iloc[:,0:5].nunique()
```

Out[196…
```
RegionID      14723
RegionName    14723
City           7554
State            51
Metro           701
dtype: int64
```

In [197…
```
# The RegionID and RegionName looks similar but RegionName is the actual zipcode
# Rename colunmn " RegionName" to "Zipcode"

df.rename({'RegionName': 'Zipcode'}, axis='columns', inplace=True)
df.head()
```

Out[197…

	RegionID	Zipcode	City	State	Metro	CountyName	SizeRank	1996-04	1996-05	19
0	84654	60657	Chicago	IL	Chicago	Cook	1	334200.0	335400.0	33
1	90668	75070	McKinney	TX	Dallas-Fort Worth	Collin	2	235700.0	236900.0	23

	RegionID	Zipcode	City	State	Metro	CountyName	SizeRank	1996-04	1996-05	19
2	91982	77494	Katy	TX	Houston	Harris	3	210400.0	212200.0	21
3	84616	60614	Chicago	IL	Chicago	Cook	4	498100.0	500900.0	50
4	93144	79936	El Paso	TX	El Paso	El Paso	5	77300.0	77300.0	7

5 rows × 272 columns

In [198…
```
df.Zipcode
```

Out[198…
```
0         60657
1         75070
2         77494
3         60614
4         79936
          ...
14718      1338
14719      3293
14720     40404
14721     81225
14722     89155
Name: Zipcode, Length: 14723, dtype: int64
```

In [199…
```
# Narrow down the data to the desired state
tx = df.loc[df['State']=='TX']
tx.head()
```

Out[199…

	RegionID	Zipcode	City	State	Metro	CountyName	SizeRank	1996-04	1996-05	19
1	90668	75070	McKinney	TX	Dallas-Fort Worth	Collin	2	235700.0	236900.0	23
2	91982	77494	Katy	TX	Houston	Harris	3	210400.0	212200.0	21
4	93144	79936	El Paso	TX	El Paso	El Paso	5	77300.0	77300.0	7
5	91733	77084	Houston	TX	Houston	Harris	6	95000.0	95200.0	9
8	91940	77449	Katy	TX	Houston	Harris	9	95400.0	95600.0	9

5 rows × 272 columns

In [200…
```
# Find metro area in TX
tx.Metro.value_counts()
```

Out[200…
```
Dallas-Fort Worth    217
Houston              187
San Antonio           78
Austin                73
Corpus Christi        22
Beaumont              21
El Paso               21
Killeen               19
```

```
Waco                 16
Lubbock              15
Tyler                15
Wichita Falls        12
College Station      11
Amarillo             11
Longview             11
McAllen              11
Athens               11
Abilene              11
Brownsville          10
Texarkana             7
Odessa                6
Marshall              6
Victoria              5
San Angelo            5
Huntsville            4
Midland               4
El Campo              4
Laredo                4
Port Lavaca           3
Brownwood             3
Nacogdoches           3
Palestine             3
Bay City              3
Brenham               3
Corsicana             3
Jacksonville          2
Levelland             2
Paris                 2
Plainview             2
Dumas                 2
Kerrville             2
Mineral Wells         2
Rio Grande City       2
Gainesville           2
Lufkin                2
Lamesa                1
Vernon                1
Sweetwater            1
Hereford              1
Del Rio               1
Uvalde                1
Eagle Pass            1
Pampa                 1
Beeville              1
Stephenville          1
Kingsville            1
Mount Pleasant        1
Fredericksburg        1
Alice                 1
Borger                1
Name: Metro, dtype: int64
```

In [201…

```
# Narrow down the data to specific metro "Austin"
tx_aus = tx.loc[tx['City']=='Austin']
tx_aus.head()
```

Out[201…

	RegionID	Zipcode	City	State	Metro	CountyName	SizeRank	1996-04	1996-05	1996-
66	92617	78704	Austin	TX	Austin	Travis	67	221300.0	221100.0	2210C
98	92654	78745	Austin	TX	Austin	Travis	99	135000.0	134200.0	1338C
422	92667	78758	Austin	TX	Austin	Travis	423	129000.0	128300.0	1275C
432	92651	78741	Austin	TX	Austin	Travis	433	93800.0	93600.0	935C
502	92662	78753	Austin	TX	Austin	Travis	503	111300.0	110600.0	1099C

5 rows × 272 columns

In [202…
```
#Check the unique value in the dataset
tx_aus.iloc[:,0:7].nunique()
```

Out[202…
```
RegionID      38
Zipcode       38
City           1
State          1
Metro          1
CountyName     1
SizeRank      38
dtype: int64
```

Data Preprocessing

Reshape from Wide to Long Format

In [203…
```
def melt_data(df):
    '''
        Takes a dataframe with datetime data that is in wide format and melts it
        Transforms data into datetime object with time as index, defaulted as mo


    '''

    melted = pd.melt(df, id_vars=['RegionID', 'Zipcode', 'City', 'State', 'Metro
                     var_name='Month', value_name='MeanValue')
    melted['Month'] = pd.to_datetime(melted['Month'], format='%Y-%m')
    melted = melted.dropna(subset=['MeanValue'])
    return melted
#     return melted.groupby('time').aggregate({'value':'mean'})
```

In [204…
```
tx_aus = melt_data(tx_aus)
tx_aus.head()
```

Out[204…

	RegionID	Zipcode	City	State	Metro	CountyName	SizeRank	Month	MeanValue
0	92617	78704	Austin	TX	Austin	Travis	67	1996-04-01	221300.0
1	92654	78745	Austin	TX	Austin	Travis	99	1996-04-01	135000.0
2	92667	78758	Austin	TX	Austin	Travis	423	1996-04-01	129000.0

	RegionID	Zipcode	City	State	Metro	CountyName	SizeRank	Month	MeanValue
3	92651	78741	Austin	TX	Austin	Travis	433	1996-04-01	93800.0
4	92662	78753	Austin	TX	Austin	Travis	503	1996-04-01	111300.0

Make the datetime as index

In [205…

```
def make_datetime_index(df,col_index = 'Month', drop=False, verbose=True):
    '''
         Transforms data into datetime object with time as index, defaulted as m

         df          Dataframe to transform
    '''

    df[col_index] = pd.to_datetime(df[col_index], errors='coerce')
    df = df.set_index('Month', drop=False)
    if verbose:
        display(df.index)
    return df
```

In [206…

```
tx_aus= make_datetime_index(tx_aus)
```

```
DatetimeIndex(['1996-04-01', '1996-04-01', '1996-04-01', '1996-04-01',
               '1996-04-01', '1996-04-01', '1996-04-01', '1996-04-01',
               '1996-04-01', '1996-04-01',
               ...
               '2018-04-01', '2018-04-01', '2018-04-01', '2018-04-01',
               '2018-04-01', '2018-04-01', '2018-04-01', '2018-04-01',
               '2018-04-01', '2018-04-01'],
              dtype='datetime64[ns]', name='Month', length=10070, freq=None)
```

Use the frequency "Month" and group by zip codes.

In [207…

```
# Use the default frequency "Month"  and group by zipcode
tx_aus_zip = tx_aus.groupby('Zipcode').resample('MS').asfreq()
```

In [208…

```
tx_aus_zip.head(10)
```

Out[208…

		RegionID	Zipcode	City	State	Metro	CountyName	SizeRank	Month	MeanV
Zipcode	**Month**									
78617	**1996-04-01**	92555	78617	Austin	TX	Austin	Travis	6240	1996-04-01	1219
	1996-05-01	92555	78617	Austin	TX	Austin	Travis	6240	1996-05-01	1205
	1996-06-01	92555	78617	Austin	TX	Austin	Travis	6240	1996-06-01	1190
	1996-07-01	92555	78617	Austin	TX	Austin	Travis	6240	1996-07-01	1174
	1996-08-01	92555	78617	Austin	TX	Austin	Travis	6240	1996-08-01	1160

Zipcode	Month	RegionID	Zipcode	City	State	Metro	CountyName	SizeRank	Month	MeanV
	1996-09-01	92555	78617	Austin	TX	Austin	Travis	6240	1996-09-01	1146
	1996-10-01	92555	78617	Austin	TX	Austin	Travis	6240	1996-10-01	1133
	1996-11-01	92555	78617	Austin	TX	Austin	Travis	6240	1996-11-01	1121
	1996-12-01	92555	78617	Austin	TX	Austin	Travis	6240	1996-12-01	1109
	1997-01-01	92555	78617	Austin	TX	Austin	Travis	6240	1997-01-01	1099

In [209…
```
tx_aus_zip.isna().sum()
```

Out[209…
```
RegionID      0
Zipcode       0
City          0
State         0
Metro         0
CountyName    0
SizeRank      0
Month         0
MeanValue     0
dtype: int64
```

In [210…
```
tx_aus_zip['Zipcode'].unique()
```

Out[210…
```
array([78617, 78702, 78703, 78704, 78705, 78717, 78721, 78722, 78723,
       78724, 78725, 78726, 78727, 78728, 78730, 78731, 78732, 78733,
       78735, 78736, 78737, 78739, 78741, 78744, 78745, 78746, 78747,
       78748, 78749, 78750, 78751, 78752, 78753, 78754, 78756, 78757,
       78758, 78759])
```

In [211…
```
# Calculate the ROI% from the month housing bubble bursted to the end of the dat
# Housing bubble bursted in December 2008

df_aus_recession = df.loc[df['City'] == 'Austin'].copy()
#df_aus_recession = df_aus_recession.loc[df_aus_recession['City']=='Austin'].cop
df_aus_recession['ROI'] = ((df_aus_recession['2018-04']- df_aus_recession['2008-
                                    *100)/df_aus_recession['2008-12']
df_aus_recession= df_aus_recession.sort_values(by=['ROI'], ascending=False)
df_aus_recession.head(10)
```

Out[211…

	RegionID	Zipcode	City	State	Metro	CountyName	SizeRank	1996-04	1996-05	199
8350	92632	78721	Austin	TX	Austin	Travis	8351	69200.0	68800.0	684
422	92667	78758	Austin	TX	Austin	Travis	423	129000.0	128300.0	127
3926	92615	78702	Austin	TX	Austin	Travis	3927	55600.0	56700.0	579
2462	92634	78723	Austin	TX	Austin	Travis	2463	97600.0	99000.0	100

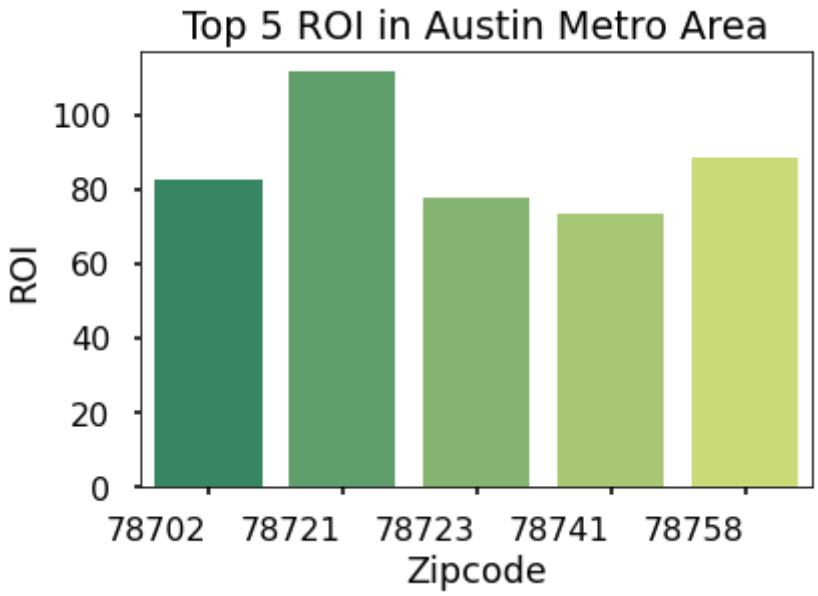
	RegionID	Zipcode	City	State	Metro	CountyName	SizeRank	1996-04	1996-05	199
432	92651	78741	Austin	TX	Austin	Travis	433	93800.0	93600.0	935
502	92662	78753	Austin	TX	Austin	Travis	503	111300.0	110600.0	109
98	92654	78745	Austin	TX	Austin	Travis	99	135000.0	134200.0	133
1671	92653	78744	Austin	TX	Austin	Travis	1672	85400.0	85400.0	853
5307	92661	78752	Austin	TX	Austin	Travis	5308	87200.0	87000.0	868
2644	92638	78727	Austin	TX	Austin	Travis	2645	150300.0	150200.0	150

10 rows × 273 columns

In [212…
```
df_aus_roi=df_aus_recession.head()
```

Plot the top 5 ROI from January 2009 - April 2018

In [213…
```
fig, ax = plt.subplots()
ax=sns.barplot(data=df_aus_roi,x='Zipcode',y='ROI',palette='summer')
ax.set_xlabel('Zipcode')
ax.set_ylabel('ROI')
ax.set_title('Top 5 ROI in Austin Metro Area')
ax.set_xticklabels(ax.get_xticklabels(),rotation=0,ha="right");
```



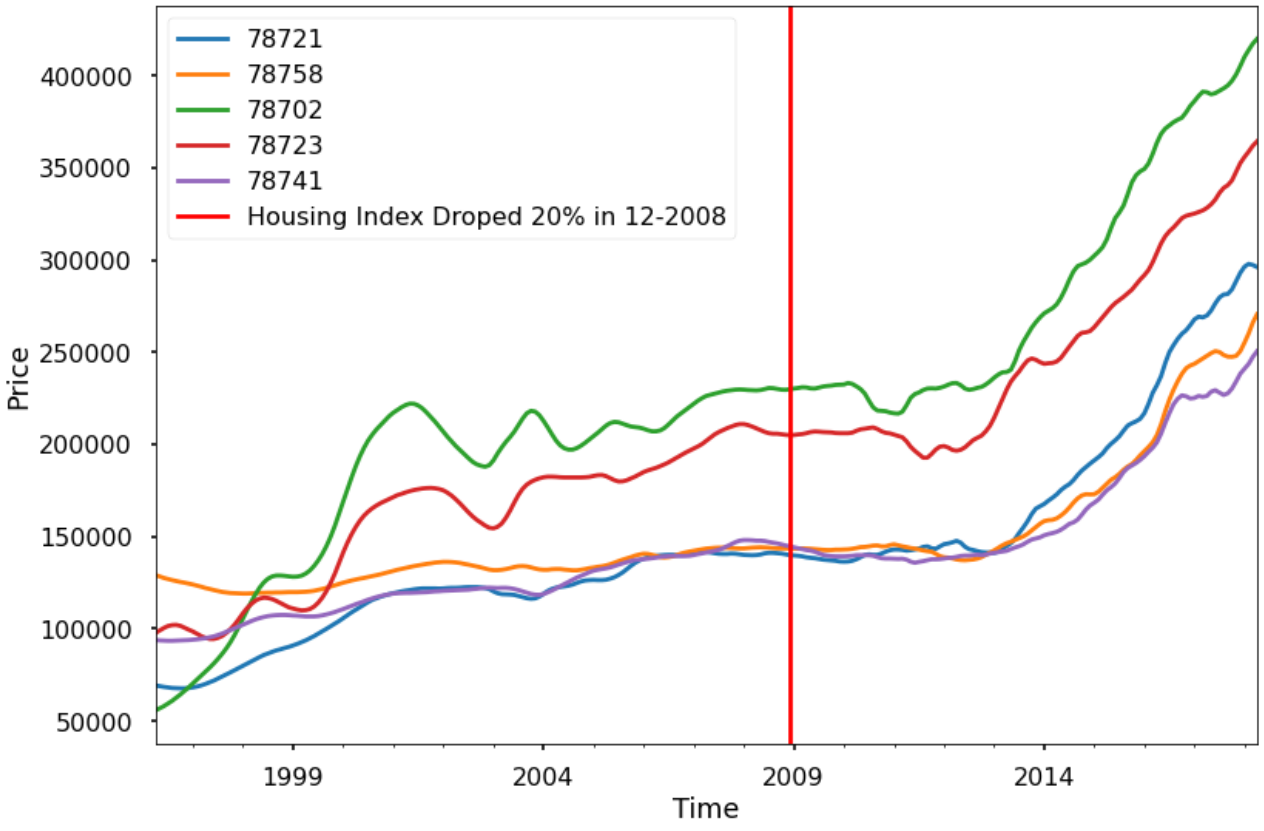
In [214…
```
#Select the top 5 zipcodes with highest ROI
zip_select=list(df_aus_recession.Zipcode.values)[0:5]

zip_select
```

Out[214… `[78721, 78758, 78702, 78723, 78741]`

Plot the price trend for the top 5 ROI zip codes and mark the December 2008 financial crisis

In [215…
```
plt.style.use('seaborn-poster')
#ts = tx_aus_zip['MeanValue'].loc[78721]
def plot_ts(df,zip_select, col='MeanValue'):

    fig, ax = plt.subplots()

    for zc in zip_select:
            ts = df[col].loc[zc]
            ts.plot(label=str(zc), ax=ax)

    crisis = '12-2008'
    #min_ = ts.loc['2008':'2018'].idxmin()
    ax.axvline(crisis,label=f'Housing Index Droped 20% in {crisis}',color='red')
    #ax.axvline(min_,label=f'Min Price between 2008 to 2018 {min_}',color='black
    ax.set_xlabel('Time')
    ax.set_ylabel('Price')
    ax.legend()
    return fig,ax
```

In [216…
```
# Plot the top 5 zipcodes with highest ROI
fig,ax = plot_ts(tx_aus_zip, zip_select)
```



Preprocess the selected zip codes for the modeling

In [217…
```
def create_top5(zip_select,df):
    df_5 = pd.DataFrame()
    for zip_code in zip_select:

        data = df.loc[(df['Zipcode']==zip_code)]
        df_5=pd.concat([data,df_5],axis=0)
```

```
    return df_5
```

In [218…
```
df_aus_5 = create_top5(zip_select, df)
```

In [219…
```
'''
# Create a data frame for top 5 ROI zipcodes
df_aus_5 = df.loc[(df['Zipcode'] == 78721) |
                        (df['Zipcode'] == 78758) |
                           (df['Zipcode'] == 78702) |
                           (df['Zipcode'] == 78723) |
                           (df['Zipcode'] == 78957)].copy()

'''
```

Out[219…
```
"\n# Create a data frame for top 5 ROI zipcodes\ndf_aus_5 = df.loc[(df['Zipcod
e'] == 78721) |\n                        (df['Zipcode'] == 78758) |\n
(df['Zipcode'] == 78702) |\n                           (df['Zipcode'] == 78723)
|\n                        (df['Zipcode'] == 78957)].copy()\n\n"
```

In [220…
```
df_aus_5.head()
```

Out[220…

	RegionID	Zipcode	City	State	Metro	CountyName	SizeRank	1996-04	1996-05	199
432	92651	78741	Austin	TX	Austin	Travis	433	93800.0	93600.0	935
2462	92634	78723	Austin	TX	Austin	Travis	2463	97600.0	99000.0	1003
3926	92615	78702	Austin	TX	Austin	Travis	3927	55600.0	56700.0	579
422	92667	78758	Austin	TX	Austin	Travis	423	129000.0	128300.0	1275
8350	92632	78721	Austin	TX	Austin	Travis	8351	69200.0	68800.0	684

5 rows × 272 columns

In [221…
```
df_aus_5 = melt_data(df_aus_5)
```

In [222…
```
df_aus_5 = make_datetime_index(df_aus_5)
```

```
DatetimeIndex(['1996-04-01', '1996-04-01', '1996-04-01', '1996-04-01',
               '1996-04-01', '1996-05-01', '1996-05-01', '1996-05-01',
               '1996-05-01', '1996-05-01',
               ...
               '2018-03-01', '2018-03-01', '2018-03-01', '2018-03-01',
               '2018-03-01', '2018-04-01', '2018-04-01', '2018-04-01',
               '2018-04-01', '2018-04-01'],
              dtype='datetime64[ns]', name='Month', length=1325, freq=None)
```

In [223…
```
df_aus_5.head()
```

Out[223…

	RegionID	Zipcode	City	State	Metro	CountyName	SizeRank	Month	MeanValue

Month	RegionID	Zipcode	City	State	Metro	CountyName	SizeRank	Month	MeanValue
Month									
1996-04-01	92651	78741	Austin	TX	Austin	Travis	433	1996-04-01	93800.0
1996-04-01	92634	78723	Austin	TX	Austin	Travis	2463	1996-04-01	97600.0
1996-04-01	92615	78702	Austin	TX	Austin	Travis	3927	1996-04-01	55600.0
1996-04-01	92667	78758	Austin	TX	Austin	Travis	423	1996-04-01	129000.0
1996-04-01	92632	78721	Austin	TX	Austin	Travis	8351	1996-04-01	69200.0

In [224…
```
df_aus_5zip = df_aus_5.groupby('Zipcode').resample('MS').asfreq()
```

In [225…
```
df_aus_5zip.isna().sum()
```

Out[225…
```
RegionID      0
Zipcode       0
City          0
State         0
Metro         0
CountyName    0
SizeRank      0
Month         0
MeanValue     0
dtype: int64
```

In [226…
```
df_aus_5zip.head(10)
```

Out[226…

		RegionID	Zipcode	City	State	Metro	CountyName	SizeRank	Month	MeanV
Zipcode	**Month**									
78702	**1996-04-01**	92615	78702	Austin	TX	Austin	Travis	3927	1996-04-01	556
	1996-05-01	92615	78702	Austin	TX	Austin	Travis	3927	1996-05-01	567
	1996-06-01	92615	78702	Austin	TX	Austin	Travis	3927	1996-06-01	579
	1996-07-01	92615	78702	Austin	TX	Austin	Travis	3927	1996-07-01	593
	1996-08-01	92615	78702	Austin	TX	Austin	Travis	3927	1996-08-01	608
	1996-09-01	92615	78702	Austin	TX	Austin	Travis	3927	1996-09-01	625
	1996-10-01	92615	78702	Austin	TX	Austin	Travis	3927	1996-10-01	643

Zipcode	Month	RegionID	Zipcode	City	State	Metro	CountyName	SizeRank	Month	MeanV
	1996-11-01	92615	78702	Austin	TX	Austin	Travis	3927	1996-11-01	662
	1996-12-01	92615	78702	Austin	TX	Austin	Travis	3927	1996-12-01	682
	1997-01-01	92615	78702	Austin	TX	Austin	Travis	3927	1997-01-01	703

In [227…

```
#df_aus_zipgroup.index
#U.S. housing bubble bursted in Dec 2008
#Use data from Jan 2009 to April 2018
df_aus_5zip_index = df_aus_5zip.loc[pd.IndexSlice[:,'2009-01-01':],:].copy()
df_aus_5zip_index.head()
```

Out[227…

Zipcode	Month	RegionID	Zipcode	City	State	Metro	CountyName	SizeRank	Month	MeanV
78702	**2009-01-01**	92615	78702	Austin	TX	Austin	Travis	3927	2009-01-01	2302
	2009-02-01	92615	78702	Austin	TX	Austin	Travis	3927	2009-02-01	2305
	2009-03-01	92615	78702	Austin	TX	Austin	Travis	3927	2009-03-01	2309
	2009-04-01	92615	78702	Austin	TX	Austin	Travis	3927	2009-04-01	2308
	2009-05-01	92615	78702	Austin	TX	Austin	Travis	3927	2009-05-01	2303

In [228…

```
df_aus_5zip_index.reset_index(inplace=True,level=0,drop=True)
df_aus_5zip_index.head()
```

Out[228…

Month	RegionID	Zipcode	City	State	Metro	CountyName	SizeRank	Month	MeanValue
2009-01-01	92615	78702	Austin	TX	Austin	Travis	3927	2009-01-01	230200.0
2009-02-01	92615	78702	Austin	TX	Austin	Travis	3927	2009-02-01	230500.0
2009-03-01	92615	78702	Austin	TX	Austin	Travis	3927	2009-03-01	230900.0
2009-04-01	92615	78702	Austin	TX	Austin	Travis	3927	2009-04-01	230800.0
2009-05-01	92615	78702	Austin	TX	Austin	Travis	3927	2009-05-01	230300.0

In [229…

```
ts_78721 = df_aus_5zip_index[df_aus_5zip_index['Zipcode']==78721]
ts_78721 = ts_78721['MeanValue']
ts_78721 = ts_78721.rename('78721')
ts_78721.head()
```

Out[229…
```
Month
2009-01-01    139700.0
2009-02-01    139500.0
2009-03-01    139200.0
2009-04-01    138700.0
2009-05-01    138400.0
Name: 78721, dtype: float64
```

In [230…
```
ts_78723 = df_aus_5zip_index[df_aus_5zip_index['Zipcode']==78723]
ts_78723 = ts_78723['MeanValue']
ts_78723 = ts_78723.rename('78723')
ts_78723.head()
```

Out[230…
```
Month
2009-01-01    205000.0
2009-02-01    205300.0
2009-03-01    205500.0
2009-04-01    206000.0
2009-05-01    206600.0
Name: 78723, dtype: float64
```

In [231…
```
ts_78741 = df_aus_5zip_index[df_aus_5zip_index['Zipcode']==78741]
ts_78741 = ts_78741['MeanValue']
ts_78741 = ts_78741.rename('78741')
ts_78741.head()
```

Out[231…
```
Month
2009-01-01    144100.0
2009-02-01    143600.0
2009-03-01    143100.0
2009-04-01    142600.0
2009-05-01    142300.0
Name: 78741, dtype: float64
```

In [232…
```
ts_78758 = df_aus_5zip_index[df_aus_5zip_index['Zipcode']==78758]
ts_78758 = ts_78758['MeanValue']
ts_78758 = ts_78758.rename('78758')
```

In [233…
```
ts_78702 = df_aus_5zip_index[df_aus_5zip_index['Zipcode']==78702]
ts_78702 = ts_78702['MeanValue']
ts_78702 = ts_78702.rename('78702')
```

In [234…
```
ts_set = [ts_78721,ts_78723,ts_78758,ts_78702,ts_78741]
```

Check Stationarity

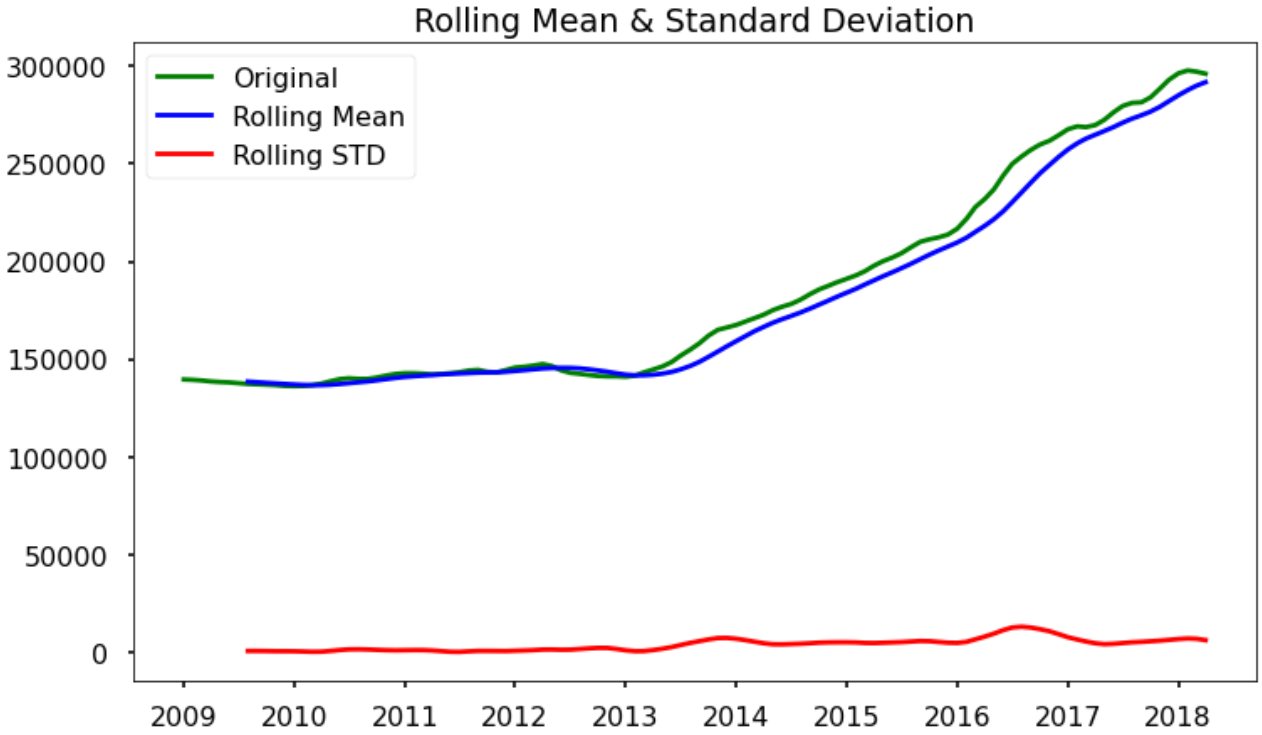
In [235…
```
def check_stationarity(ts):
    roll_mean = ts.rolling(window=8, center=False).mean()
```

```
    roll_std = ts.rolling(window=8, center=False).std()

    fig = plt.figure(figsize=(12,7))
    plt.plot(ts, color='green', label='Original')
    plt.plot(roll_mean, color='blue', label='Rolling Mean')
    plt.plot(roll_std, color='red', label = 'Rolling STD')
    plt.legend(loc='best')
    plt.title('Rolling Mean & Standard Deviation')
    plt.show(block=False)

    dftest = adfuller(ts)

    # Extract and display test results in a user friendly manner
    dfoutput = pd.Series(dftest[0:4], index=['Test Statistic', 'p-value', '#Lags
    for key,value in dftest[4].items():
        dfoutput['Critical Value (%s)'%key] = value

    print ('Results of Dickey-Fuller test: \n')

    print(dfoutput)


    return None
```

In [236…

```
for ts, name in zip(ts_set, zip_select):
    print(f'\n\n Zipcode: {name}\n')
    check_stationarity(ts)
```

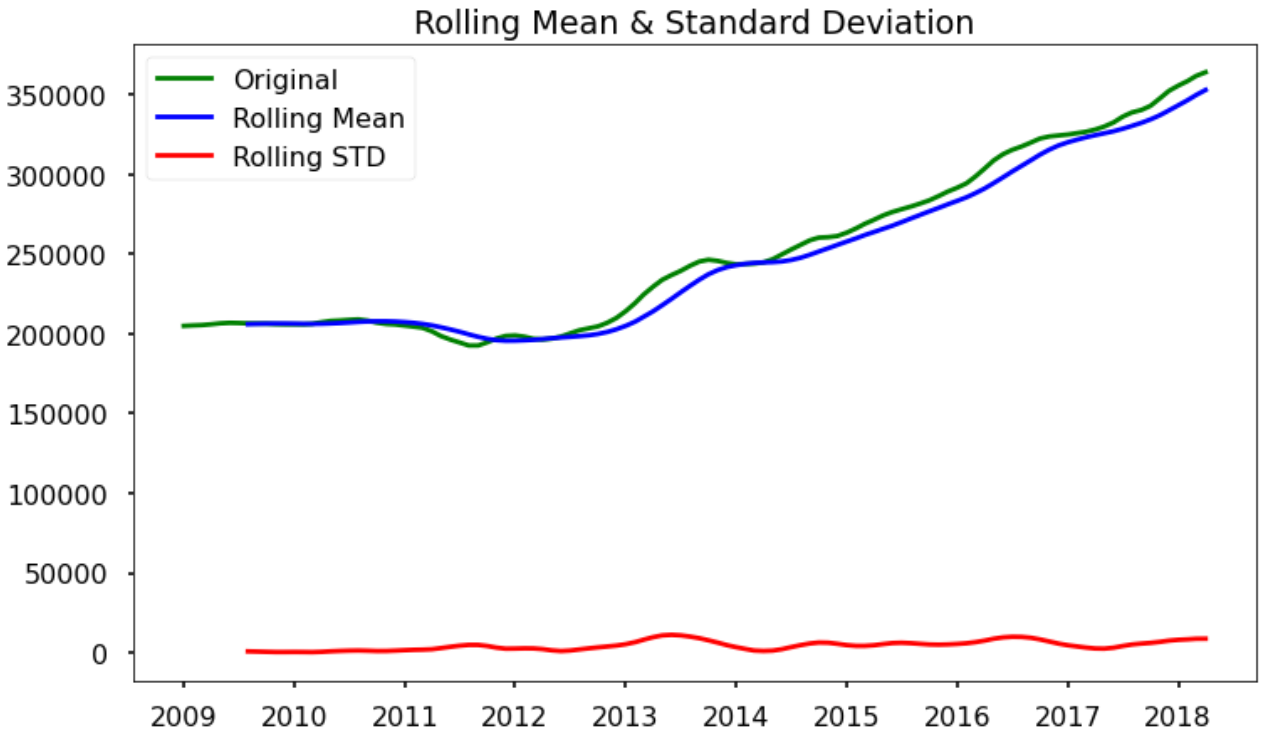
```
Zipcode: 78721
```




```
Results of Dickey-Fuller test:

Test Statistic                  0.586320
p-value                         0.987274
```

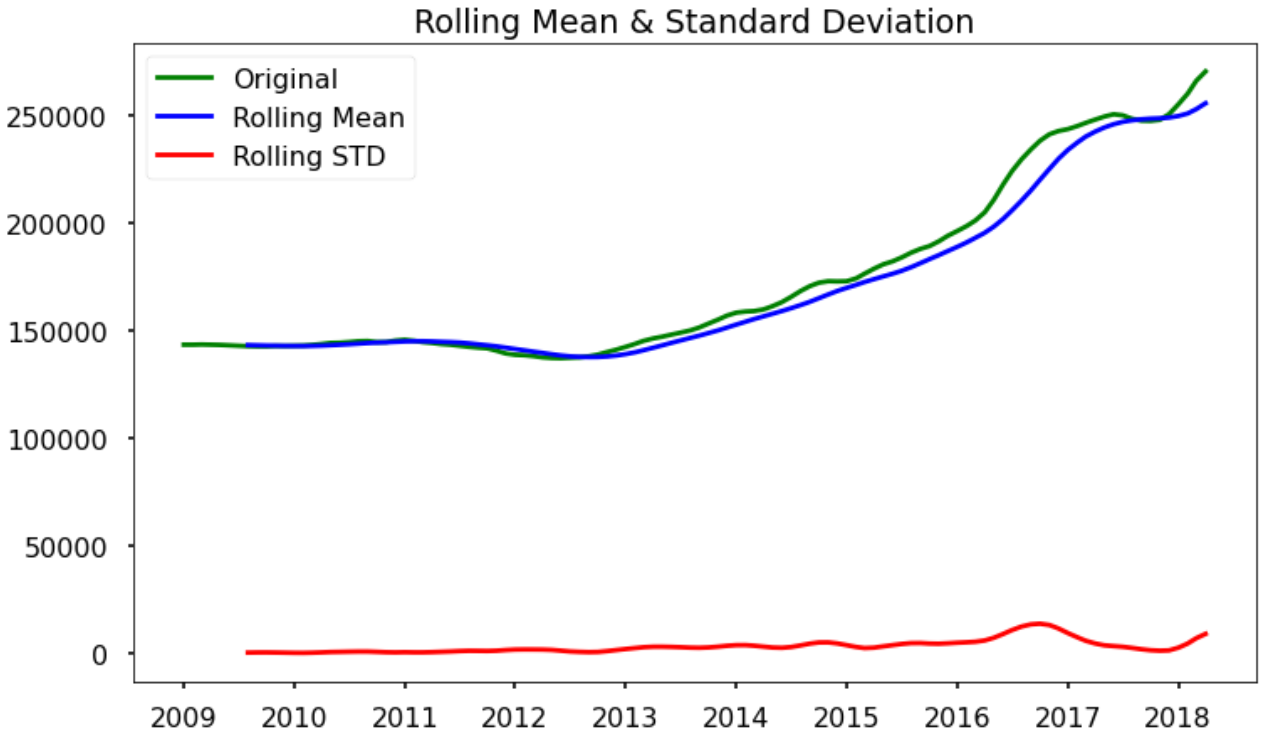
```
#Lags Used                        8.000000
Number of Observations Used     103.000000
Critical Value (1%)              -3.495493
Critical Value (5%)              -2.890037
Critical Value (10%)             -2.581971
dtype: float64
```

```
Zipcode: 78758
```

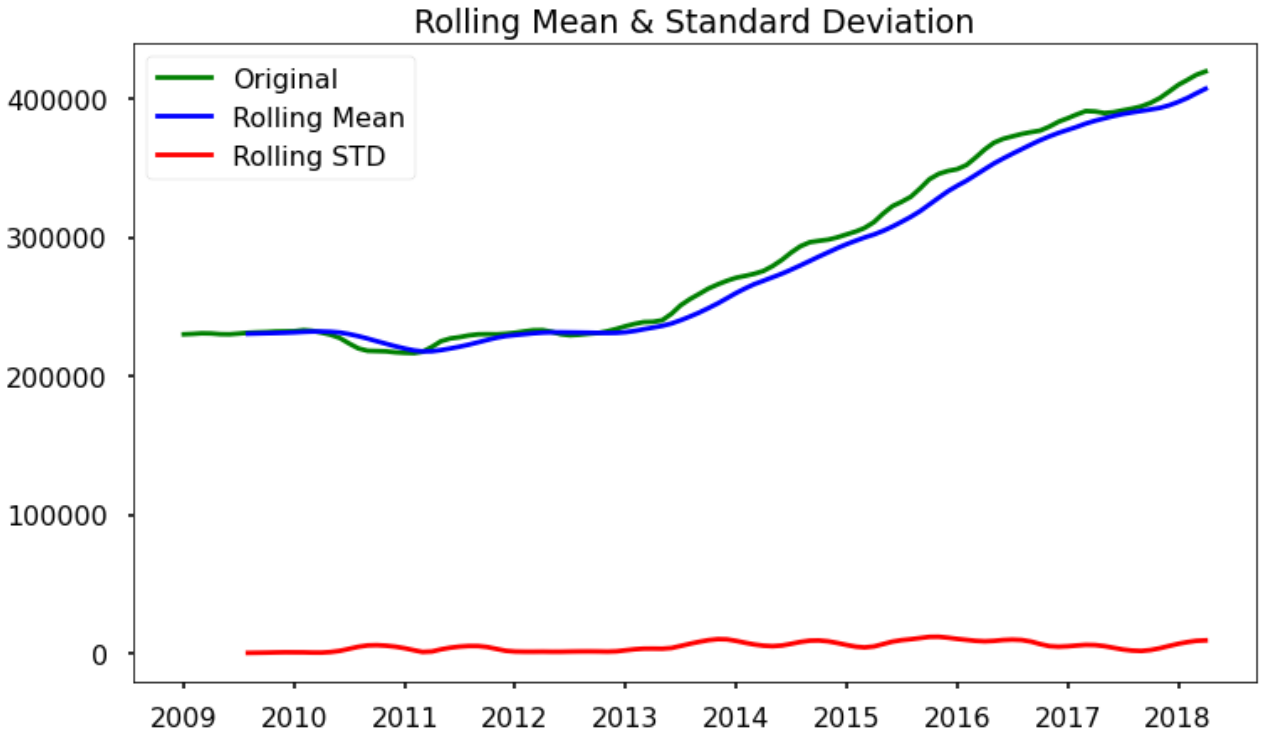



```
Results of Dickey-Fuller test:

Test Statistic                   0.869047
p-value                          0.992673
#Lags Used                      11.000000
Number of Observations Used     100.000000
Critical Value (1%)              -3.497501
Critical Value (5%)              -2.890906
Critical Value (10%)             -2.582435
dtype: float64
```

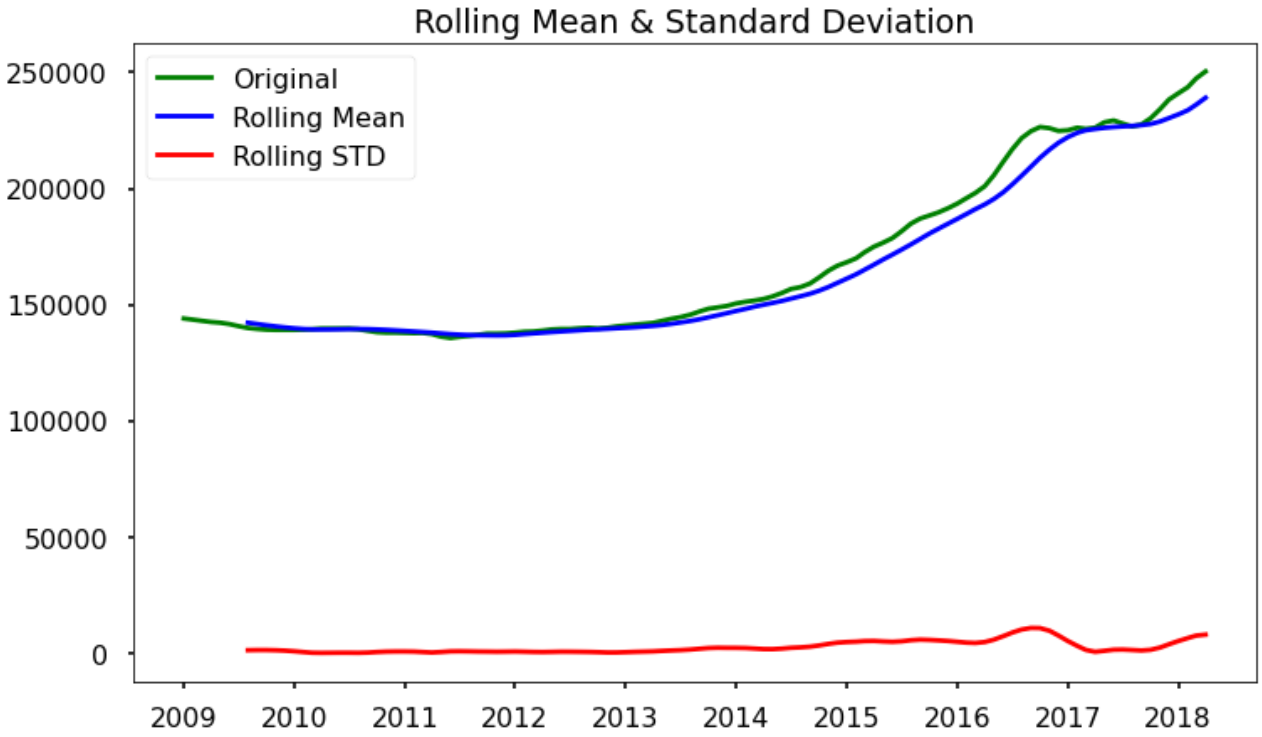
```
Zipcode: 78702
```




```
Results of Dickey-Fuller test:

Test Statistic                   2.183920
p-value                          0.998869
#Lags Used                      10.000000
Number of Observations Used    101.000000
Critical Value (1%)             -3.496818
Critical Value (5%)             -2.890611
Critical Value (10%)            -2.582277
dtype: float64


 Zipcode: 78723
```




```
Results of Dickey-Fuller test:

Test Statistic                   0.346203
p-value                          0.979368
#Lags Used                       8.000000
Number of Observations Used    103.000000
Critical Value (1%)             -3.495493
Critical Value (5%)             -2.890037
Critical Value (10%)            -2.581971
dtype: float64


 Zipcode: 78741
```




```
Results of Dickey-Fuller test:

Test Statistic                   1.904523
p-value                          0.998535
#Lags Used                       4.000000
Number of Observations Used    107.000000
Critical Value (1%)             -3.492996
Critical Value (5%)             -2.888955
Critical Value (10%)            -2.581393
dtype: float64
```

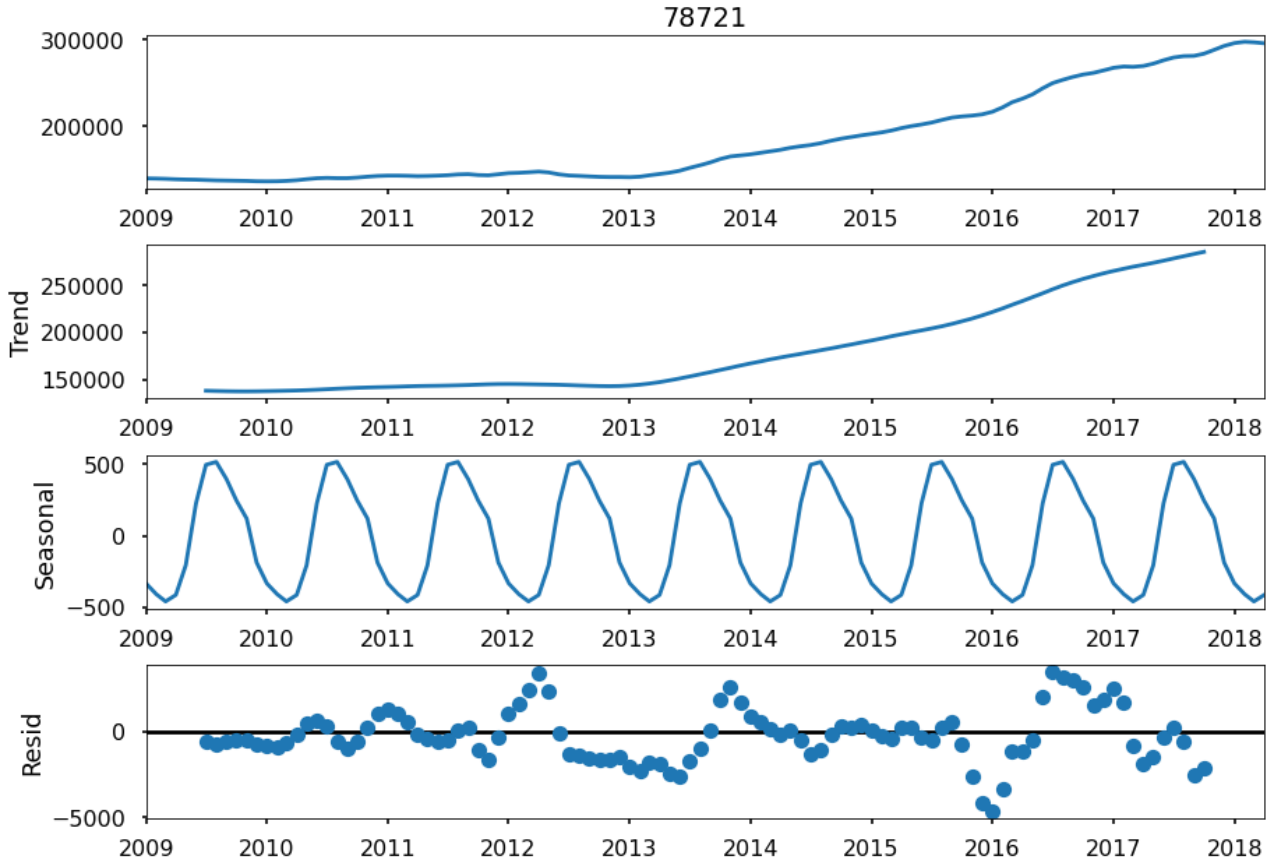
We can know the data is not stationary based on two observation. One thing is from above plots, rolling mean is increasing. The other is that the P-value in each test is larger than 0.05. It means fail to reject the null hypothesis.

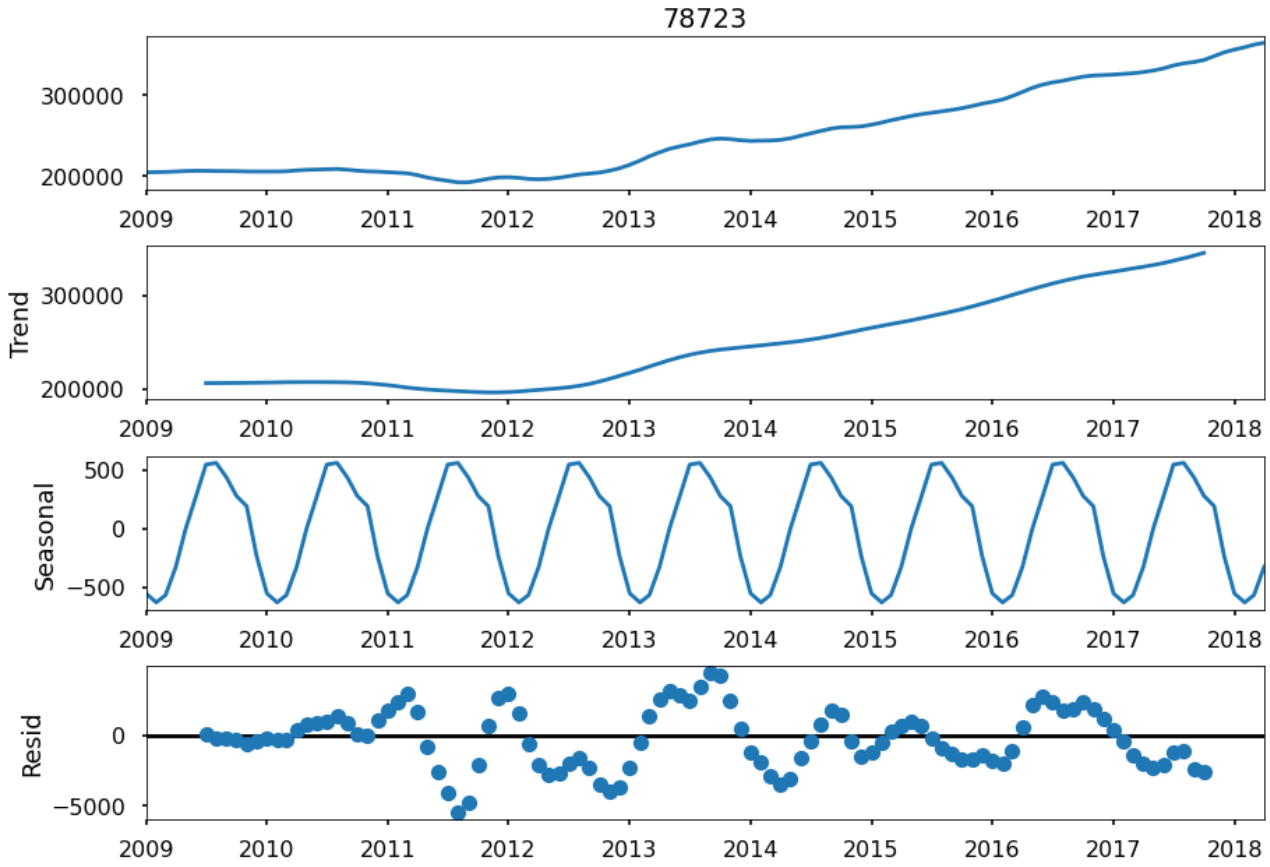
Decomposition

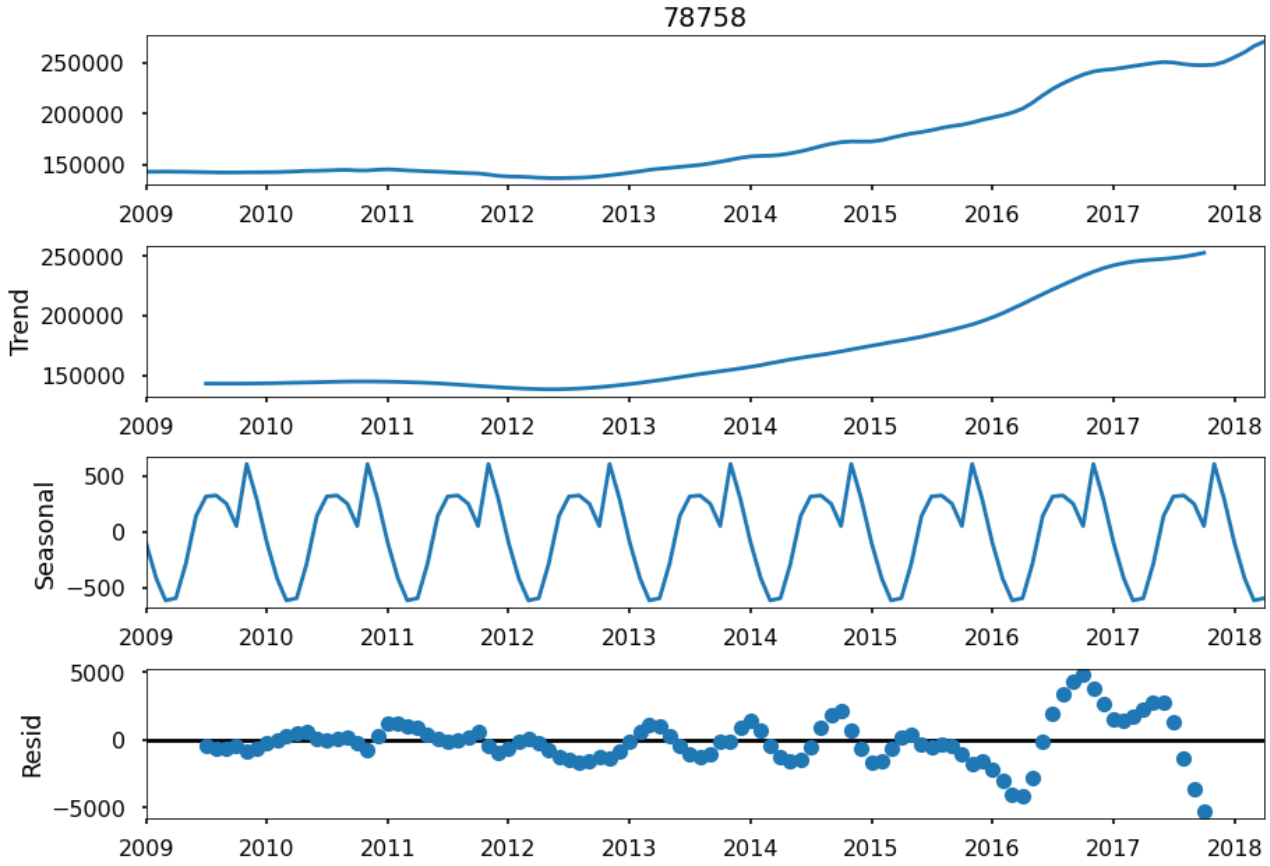
We also want to know if the data is seasonality. So we use seasonal decomposition.

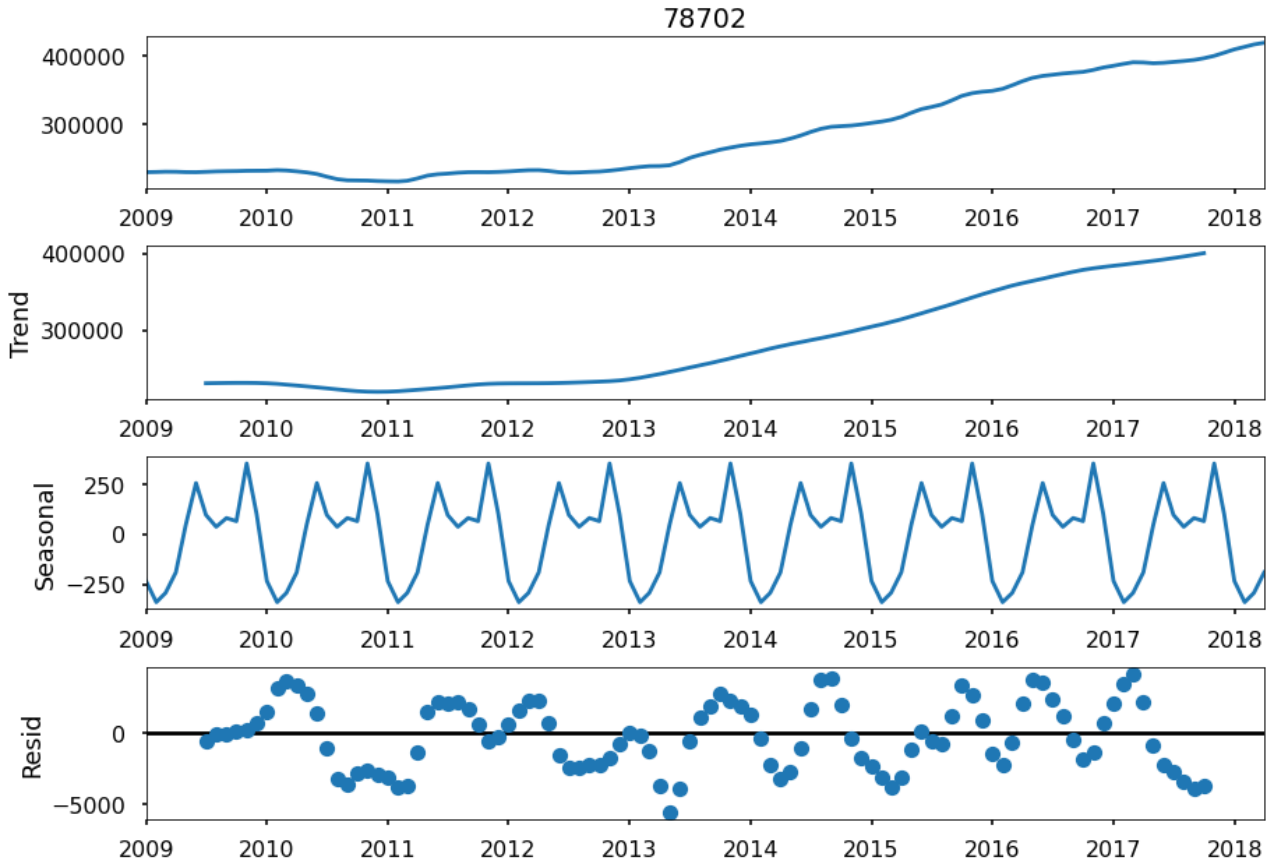
In [237…

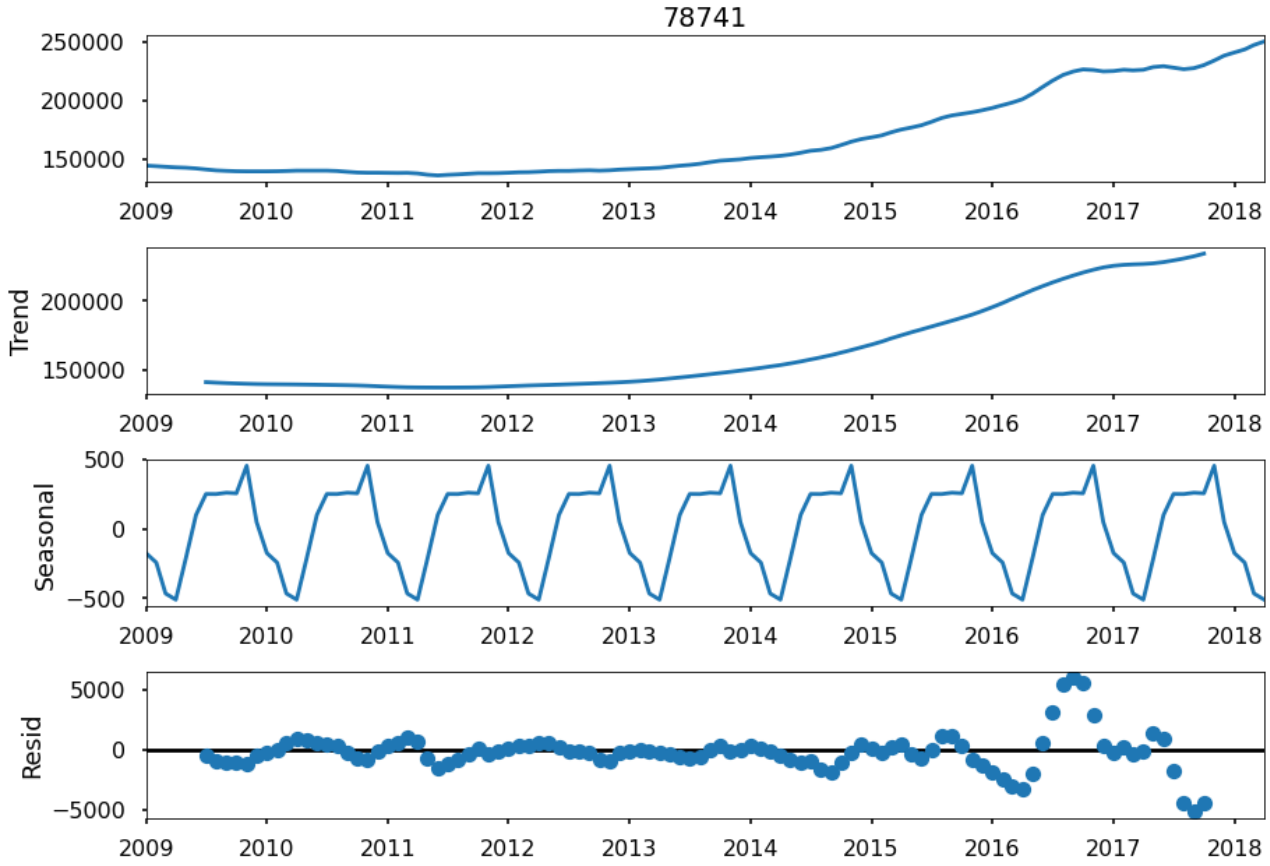
```
# Decompose time series
for ts in ts_set:
    result = seasonal_decompose(ts, model='additive')
    result.plot()
```


78721
300000
200000
2009
2010
2011
2012
2013
2014
2015
2016
2017
2018
Trend
250000
200000
150000
Seasonal
500
0
−500
Resid
0
−5000


78723
300000
200000
2009
2010
2011
2012
2013
2014
2015
2016
2017
2018
Trend
300000
200000
Seasonal
500
0
−500
Resid
0
−5000


78758
250000
200000
150000
2009
2010
2011
2012
2013
2014
2015
2016
2017
2018
Trend
Seasonal
500
0
−500
Resid
5000
−5000


78702
400000
300000
Trend
Seasonal
250
−250
Resid



From above decomposition plots, seasonality is observed in the dataset.

Check Auto Correlation and Partial Auto Correlation

In [238…

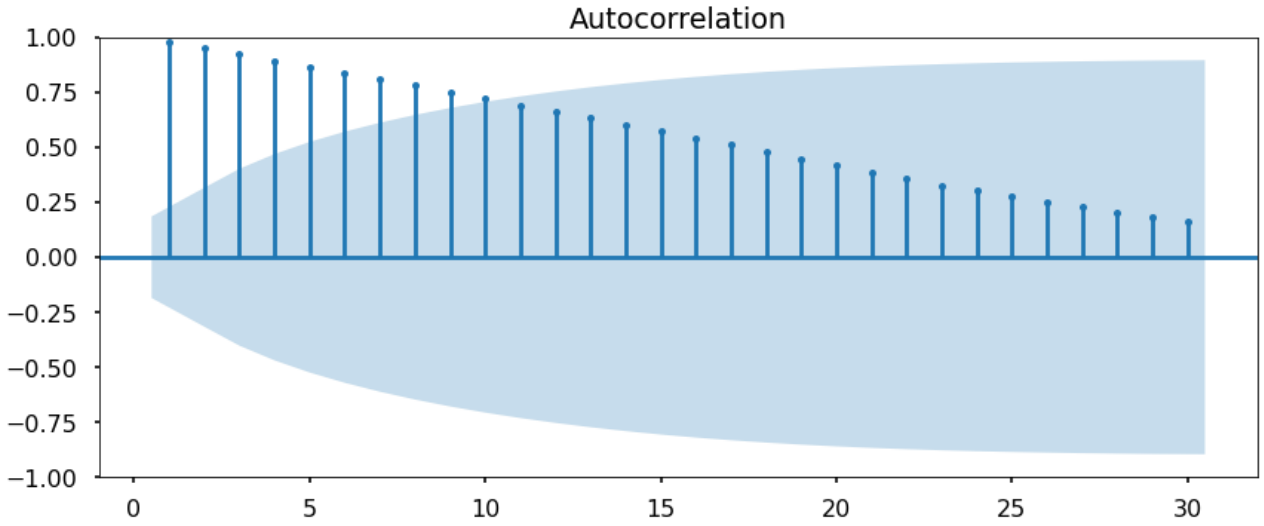
```
def acf_pacf(df,alags=30,plags=30):
    '''
        Performs acf/pacf results plot

        df          Dataframe to Analyse
    '''

    #Create figure
    fig,(ax1,ax2) = plt.subplots(2,1,figsize=(14,12))
    #Make ACF plot
    plot_acf(df,lags=alags, zero=False,ax=ax1)
    #Make PACF plot
    plot_pacf(df,lags=plags, ax=ax2)
    plt.show()
```

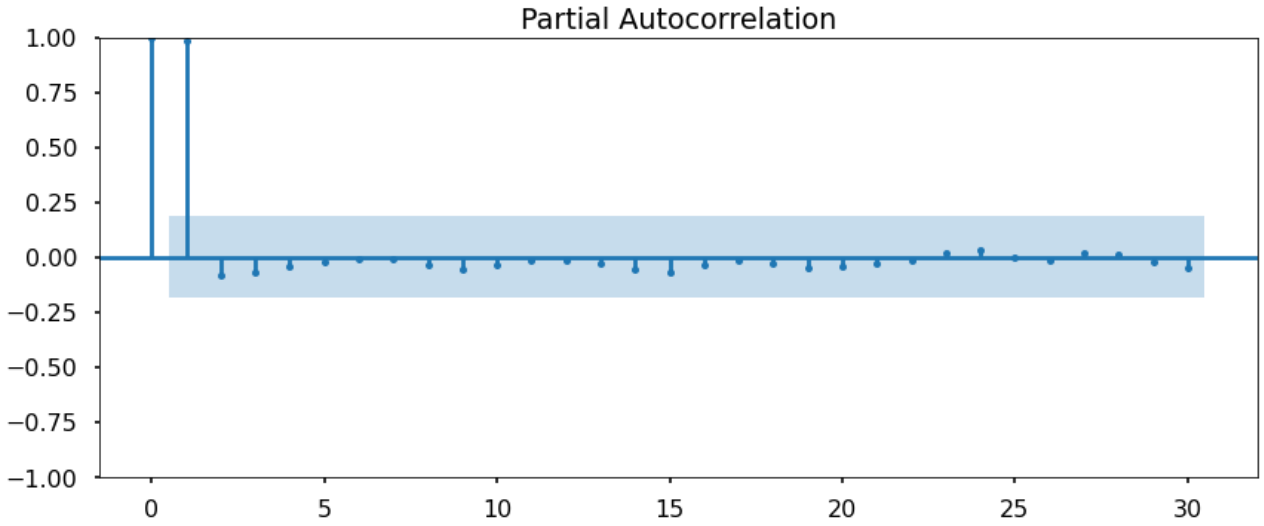
Check ACF and PACF for selected zipcodes

In [239…

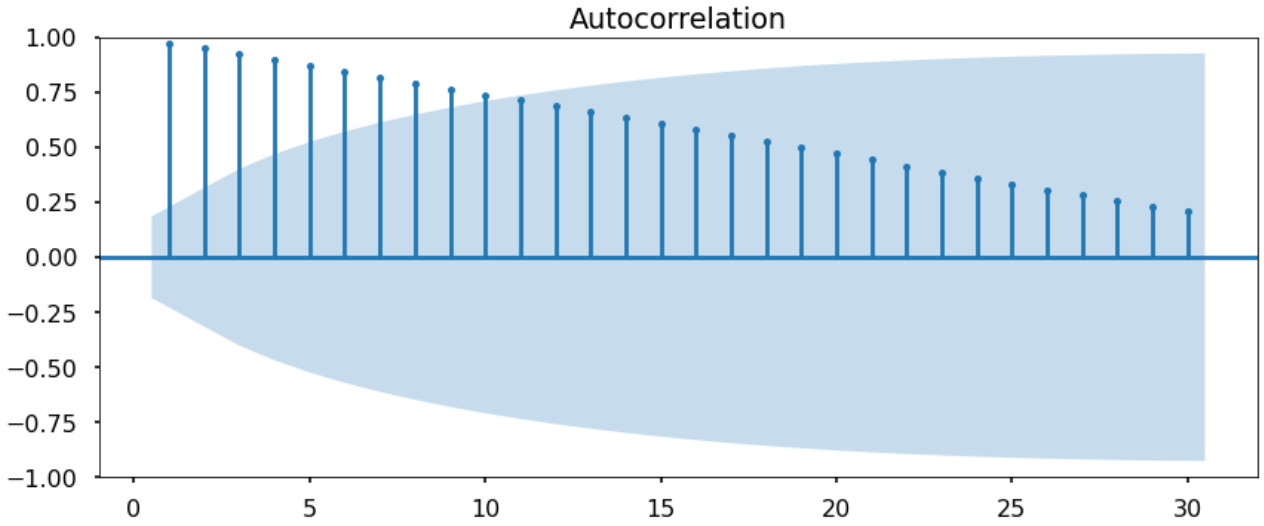
```
# Check ACF and PACF plots  for each zipcode
for ts, name in zip(ts_set, zip_select):
    print(f'\n \n Zipcode {name} \n')
    acf_pacf(ts)
```

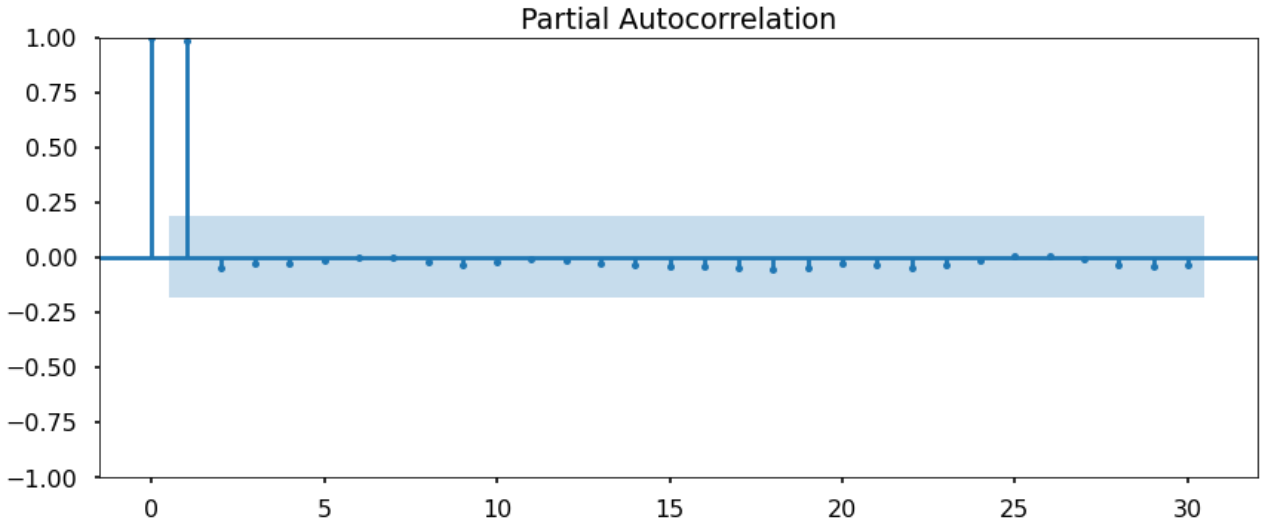
```
Zipcode 78721
```



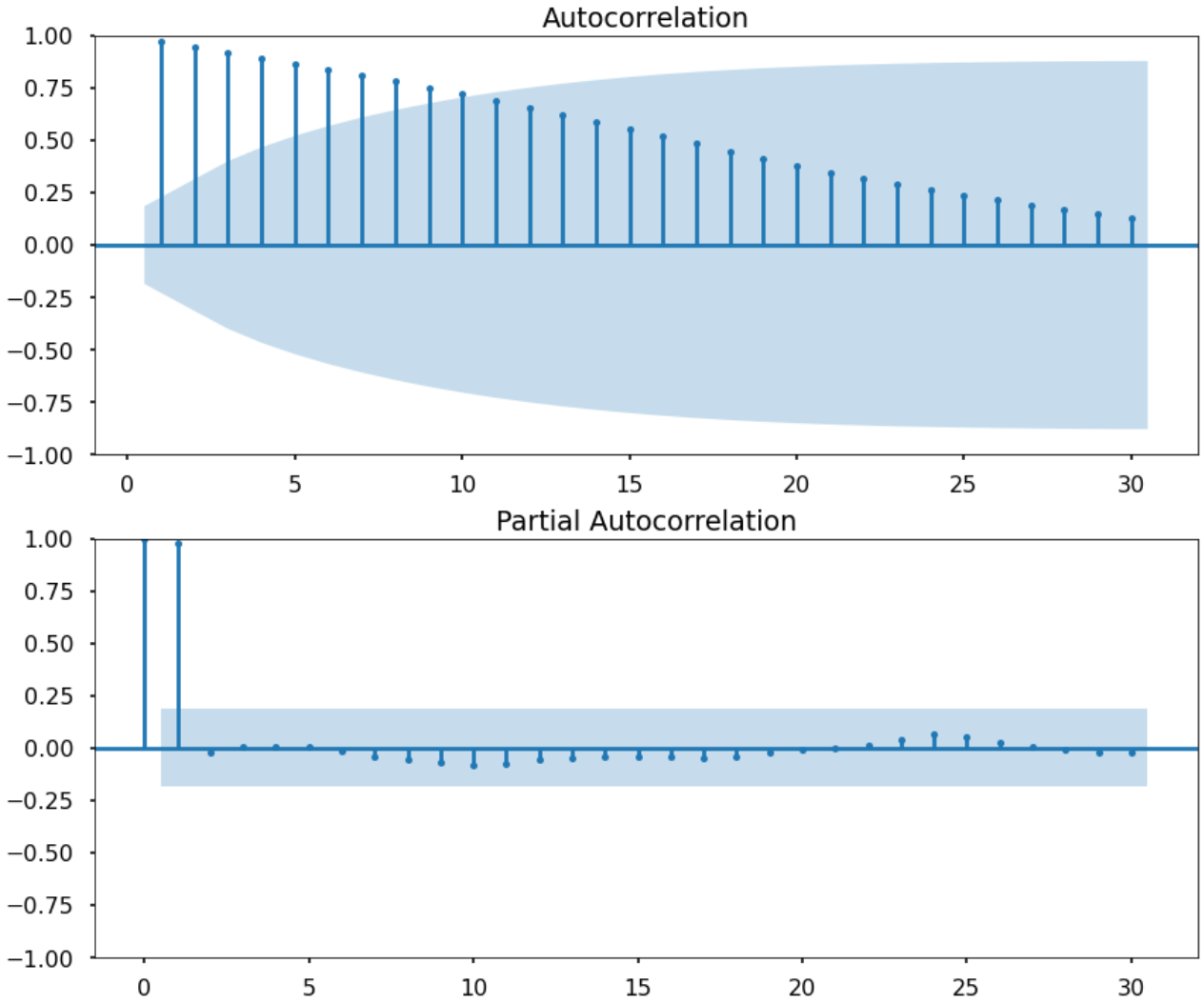



```
Zipcode 78758
```

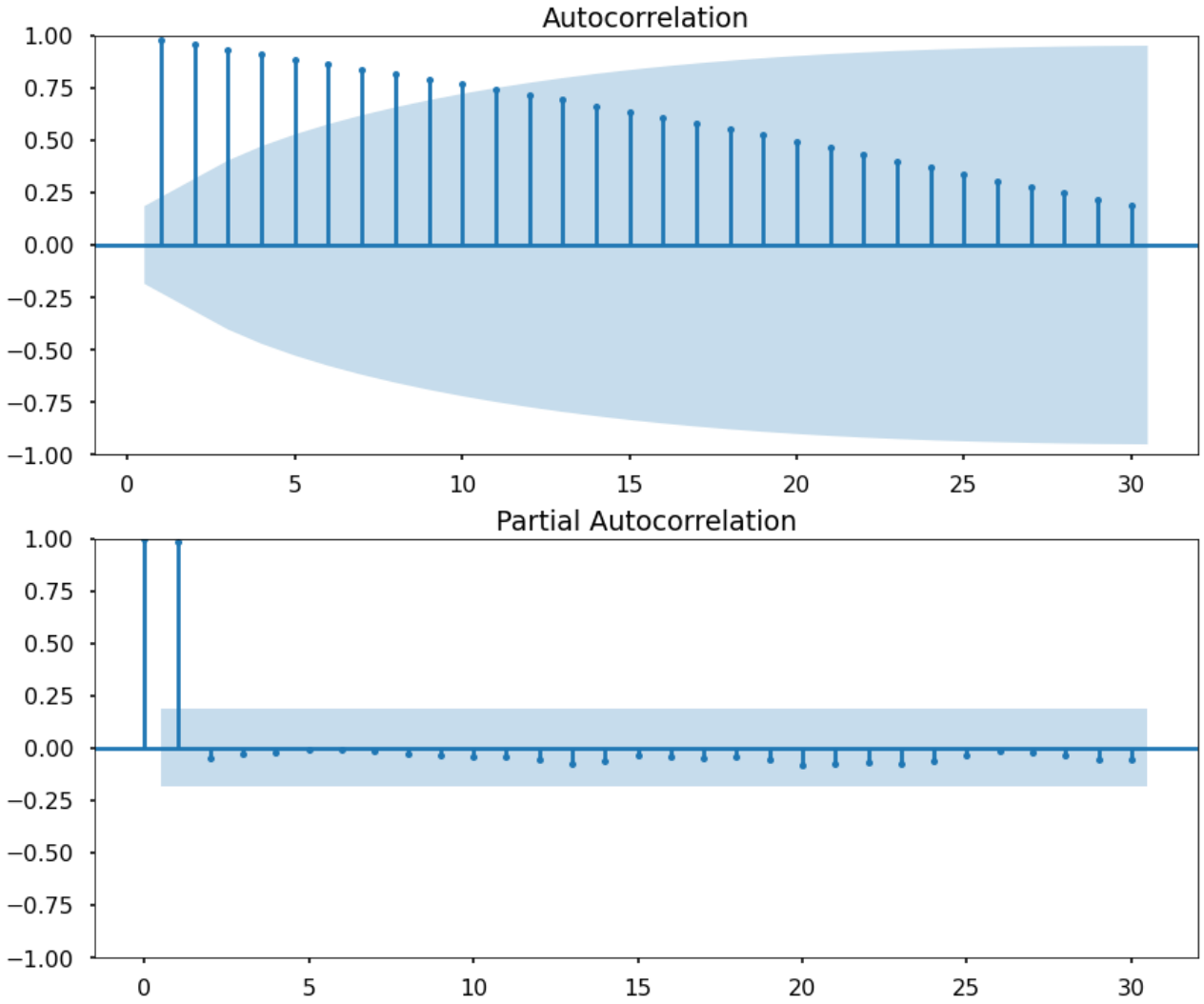




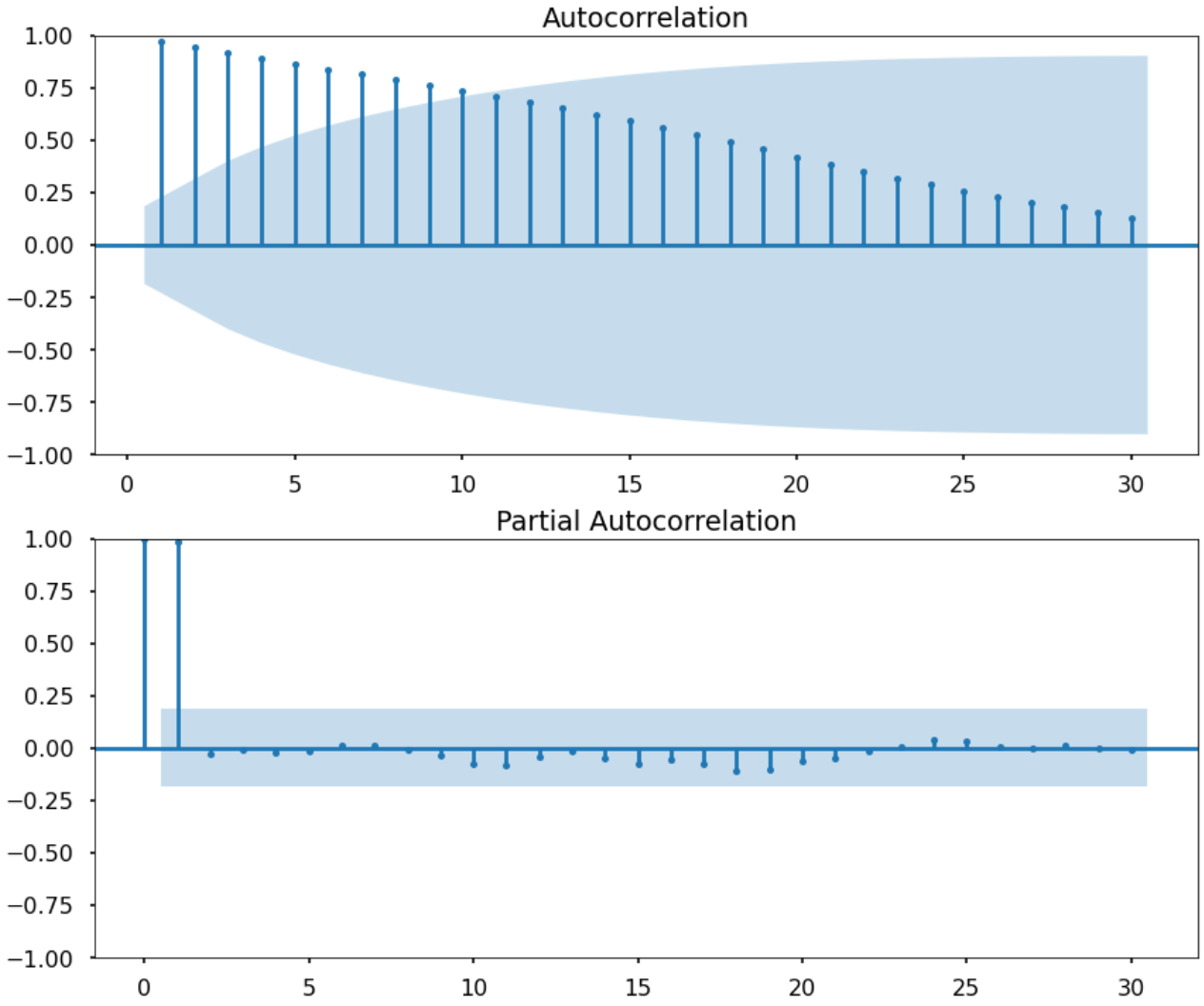

```
Zipcode 78702
```




```
Zipcode 78723
```




```
Zipcode 78741
```



ARIMA Time Series Model

Parameter Selection for the ARIMA Time Series Model

In [240…

```
def AIC_PDQS(df):
    '''
         Runs grid search to return lowest AIC result for permutataions of pdq/s

         df          Dataframe to anlyse for best pdq/s permutation
    '''

    # Define the p, d and q parameters to take any value between 0 and 2
    p = d = q = range(0, 2)

#     Auto-Regressive (p) -> Number of autoregressive terms.
#     Integrated (d) -> Number of nonseasonal differences needed for stationarit
#     Moving Average (q) -> Number of lagged forecast errors in the prediction e

    # Generate all different combinations of p, q and q triplets
    pdq = list(itertools.product(p, d, q))

    # Generate all different combinations of seasonal p, q and q triplets
    pdqs = [(x[0], x[1], x[2], 12) for x in list(itertools.product(p, d, q))]
```

```
    # Run a grid with pdq and seasonal pdq parameters calculated above and get t
    ans = []
    for comb in pdq:
        for combs in pdqs:
            try:
                mod = sm.tsa.statespace.SARIMAX(df,
                                                order=comb,
                                                seasonal_order=combs,
                                                enforce_stationarity=False,
                                                enforce_invertibility=False)

                output = mod.fit()
                ans.append([comb, combs, output.aic])
                print('ARIMA {} x {}12 : AIC Calculated ={}'.format(comb, combs,
            except:
                continue

    # Find the parameters with minimal AIC value
    ans_df = pd.DataFrame(ans, columns=['pdq', 'pdqs', 'aic'])
    print(ans_df.loc[ans_df['aic'].idxmin()])
    return ans_df
```

In [241…

```
AIC_PDQS(ts_78702)
```

```
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            1     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.38870D+01    |proj g|=  1.11910D-08

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    1      0      1      0     0     0   1.119D-08   1.389D+01
  F =   13.887027823194225

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 0) x (0, 0, 0, 12)12 : AIC Calculated =3112.6942323955063
RUNNING THE L-BFGS-B CODE

           * * *
```

```
Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  2.51846D+03    |proj g|=  2.34191D+01

At iterate    5    f=  1.21549D+01    |proj g|=  5.88021D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      7     14      1     0     0   3.693D-06   1.215D+01
  F =   12.154937906452842

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 0) x (0, 0, 1, 12)12 : AIC Calculated =2726.7060910454366
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            1     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.03084D+01    |proj g|=  4.41087D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    1      0      1      0     0     0   4.411D-06   1.031D+01
  F =   10.308358131071902

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 0) x (0, 1, 0, 12)12 : AIC Calculated =2311.072221360106
RUNNING THE L-BFGS-B CODE

           * * *
```

```
Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.25917D+03    |proj g|=  2.59978D+01

 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
At iterate    5    f=  4.65149D+01    |proj g|=  4.41651D-01

At iterate   10    f=  1.81728D+01    |proj g|=  2.14923D-02

At iterate   15    f=  1.49598D+01    |proj g|=  8.56521D-04

At iterate   20    f=  1.48139D+01    |proj g|=  4.58344D-05
  ys=-3.442E+01  -gs= 7.103E-01 BFGS update SKIPPED

At iterate   25    f=  9.47499D+00    |proj g|=  3.01888D-02
  ys=-1.744E+01  -gs= 7.413E-01 BFGS update SKIPPED

At iterate   30    f=  9.09954D+00    |proj g|=  3.09015D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2     31     65      1     2     0   1.103D-07   9.100D+00
  F =   9.0995370427191684

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 0) x (0, 1, 1, 12)12 : AIC Calculated =2042.2962975690937
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.97002D+00    |proj g|=  1.18060D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
```

```
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     12      1     0     0   1.180D-03   9.970D+00
  F =   9.9700235063512217

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 0) x (1, 0, 0, 12)12 : AIC Calculated =2237.2852654226735
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  2.71040D+03    |proj g|=  7.25824D+02

At iterate    5    f=  1.57682D+03    |proj g|=  6.71004D+02

At iterate   10    f=  2.80753D+01    |proj g|=  5.76595D+01

At iterate   15    f=  1.01838D+01    |proj g|=  4.65778D+00

At iterate   20    f=  9.78676D+00    |proj g|=  2.85990D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     21     30      1     0     0   3.142D-04   9.787D+00
  F =   9.7867552430937401

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 0) x (1, 0, 1, 12)12 : AIC Calculated =2198.233174452998
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.77832D+00    |proj g|=  1.07889D-05
 This problem is unconstrained.
```

```
 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
 This problem is unconstrained.
 This problem is unconstrained.
          * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

          * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     13      1     0     0   1.073D-05   8.778D+00
  F =   8.7783191603250064

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 0) x (1, 1, 0, 12)12 : AIC Calculated =1970.3434919128013
RUNNING THE L-BFGS-B CODE

          * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.21866D+03    |proj g|=  1.52213D+02

At iterate    5    f=  1.20031D+01    |proj g|=  2.08763D-02

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
 This problem is unconstrained.
At iterate   10    f=  1.20030D+01    |proj g|=  1.07653D-02

At iterate   15    f=  1.20028D+01    |proj g|=  5.46772D-04

          * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

          * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     17     19      1     0     0   6.608D-05   1.200D+01
  F =   12.002821720212539
```

```
CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 0) x (1, 1, 1, 12)12 : AIC Calculated =2694.632065327609
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.82863D+02    |proj g|=  1.01830D+01

At iterate    5    f=  1.34103D+01    |proj g|=  3.02631D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      8     15      1     0     0   2.410D-06   1.341D+01
  F =   13.410267337964964

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 1) x (0, 0, 0, 12)12 : AIC Calculated =3007.8998837041518
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  2.53516D+03    |proj g|=  2.30902D+01

At iterate    5    f=  1.18796D+01    |proj g|=  5.84570D-02

At iterate   10    f=  1.18717D+01    |proj g|=  1.68010D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *
```

```
   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     11     46      1     0     0   1.881D-05   1.187D+01
  F =   11.871734414744626

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 1) x (0, 0, 1, 12)12 : AIC Calculated =2665.2685089027964
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.04258D+01    |proj g|=  3.84890D-01

At iterate    5    f=  1.03349D+01    |proj g|=  1.74486D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      7     13      1     0     0   2.837D-05   1.033D+01
  F =   10.334882104645692

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 1) x (0, 1, 0, 12)12 : AIC Calculated =2319.013591440635
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.26755D+03    |proj g|=  1.61966D+01
  ys=-9.727E+02  -gs= 1.452E+02 BFGS update SKIPPED

At iterate    5    f=  3.63756D+01    |proj g|=  1.14433D+02

At iterate   10    f=  1.61942D+01    |proj g|=  1.39741D+01
 This problem is unconstrained.
At iterate   15    f=  1.34726D+01    |proj g|=  4.24632D-01
```

```
At iterate   20    f=  1.34486D+01    |proj g|=  1.58027D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     21     34      1     1     0   8.261D-05   1.345D+01
  F =   13.448570391374520

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 1) x (0, 1, 1, 12)12 : AIC Calculated =3018.4797676678927
 This problem is unconstrained.
 This problem is unconstrained.
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.35619D+01    |proj g|=  1.67722D-01

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      4      7      1     0     0   3.008D-06   1.207D+01
  F =   12.070113942662877

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 1) x (1, 0, 0, 12)12 : AIC Calculated =2709.7055231564846
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds
```

```
At iterate    0    f=  2.56425D+03    |proj g|=  6.90554D+02
  ys=-1.364E-01  -gs= 5.443E-01 BFGS update SKIPPED

At iterate    5    f=  1.25773D+01    |proj g|=  4.84722D-02

At iterate   10    f=  1.25764D+01    |proj g|=  2.93093D-02

At iterate   15    f=  1.25512D+01    |proj g|=  1.31003D-02

At iterate   20    f=  1.25428D+01    |proj g|=  2.10234D-02

At iterate   25    f=  1.25422D+01    |proj g|=  2.16122D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     25     49      1     1     0   2.161D-05   1.254D+01
  F =   12.542233407049796

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 1) x (1, 0, 1, 12)12 : AIC Calculated =2817.4602831791544
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.33859D+00    |proj g|=  3.69223D-01
  ys=-9.059E-01  -gs= 3.347E-02 BFGS update SKIPPED

At iterate    5    f=  8.81372D+00    |proj g|=  2.34383D-01

At iterate   10    f=  8.81309D+00    |proj g|=  4.92522D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
```

```
    3     13     30      1     1     0   7.846D-05   8.813D+00
  F =   8.8130831983530609

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (0, 0, 1) x (1, 1, 0, 12)12 : AIC Calculated =1980.1306364310858
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.48428D+03    |proj g|=  2.47987D+02

At iterate    5    f=  6.93516D+01    |proj g|=  3.35900D+01

At iterate   10    f=  2.36103D+01    |proj g|=  1.05824D+01

At iterate   15    f=  1.43108D+01    |proj g|=  2.81580D+00

At iterate   20    f=  1.15856D+01    |proj g|=  2.75764D-02
 This problem is unconstrained.
At iterate   25    f=  1.15653D+01    |proj g|=  8.10178D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     27     43      1     0     0   2.879D-05   1.157D+01
  F =   11.565312817301054

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 1) x (1, 1, 1, 12)12 : AIC Calculated =2598.6300710754363
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            1     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.16124D+00    |proj g|=  6.58034D-06

           * * *

Tit   = total number of iterations
```

```
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    1      0      1      0     0     0   6.580D-06   9.161D+00
  F =   9.1612351695382070

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 0) x (0, 0, 0, 12)12 : AIC Calculated =2054.1166779765585
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.33508D+00    |proj g|=  1.14208D+00

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      4      6      1     0     0   8.278D-05   8.149D+00
  F =   8.1492191379362726

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 1, 0) x (0, 0, 1, 12)12 : AIC Calculated =1829.4250868977251
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            1     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.12892D+00    |proj g|=  5.10612D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
```

```
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    1      2      5      1     0     0   5.733D-06   8.125D+00
  F =   8.1249272383663982

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 0) x (0, 1, 0, 12)12 : AIC Calculated =1821.983701394073
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.22840D+00    |proj g|=  7.26698D-01

At iterate    5    f=  7.09933D+00    |proj g|=  2.34776D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      5      8      1     0     0   2.348D-05   7.099D+00
  F =   7.0993326681474889

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (0, 1, 0) x (0, 1, 1, 12)12 : AIC Calculated =1594.2505176650375
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.12198D+00    |proj g|=  5.66498D-06

           * * *
```

```
Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      0      1      0     0     0   5.665D-06   8.122D+00
  F =   8.1219796784875129

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 0) x (1, 0, 0, 12)12 : AIC Calculated =1823.323447981203
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.05157D+00    |proj g|=  3.01158D-01

At iterate    5    f=  8.04298D+00    |proj g|=  1.23186D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      7      9      1     0     0   4.479D-06   8.043D+00
  F =   8.0429780011822238

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 0) x (1, 0, 1, 12)12 : AIC Calculated =1807.6270722648183
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.12520D+00    |proj g|=  4.99591D-06

           * * *
```

```
Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      0      1      0     0     0   4.996D-06   7.125D+00
  F =   7.1252036936909189

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (0, 1, 0) x (1, 1, 0, 12)12 : AIC Calculated =1600.0456273867658
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.35728D+00    |proj g|=  1.28020D+00

At iterate    5    f=  7.11296D+00    |proj g|=  3.80622D-01

At iterate   10    f=  7.04695D+00    |proj g|=  2.66682D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     14     22      1     0     0   7.802D-06   7.047D+00
  F =   7.0468726919418359

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
 This problem is unconstrained.
ARIMA (0, 1, 0) x (1, 1, 1, 12)12 : AIC Calculated =1584.4994829949712
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
```

```
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.98186D+00    |proj g|=  5.21054D-01

At iterate    5    f=  8.62503D+00    |proj g|=  1.15289D-02

At iterate   10    f=  8.62501D+00    |proj g|=  6.79690D-04

At iterate   15    f=  8.62500D+00    |proj g|=  1.30125D-02

At iterate   20    f=  8.62257D+00    |proj g|=  2.29938D-01

At iterate   25    f=  8.51243D+00    |proj g|=  3.73886D-01

At iterate   30    f=  8.47993D+00    |proj g|=  5.25642D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2     31     39      1     0     0   6.096D-06   8.480D+00
  F =   8.4799293202117578

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 1) x (0, 0, 0, 12)12 : AIC Calculated =1903.5041677274337
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.05805D+00    |proj g|=  4.24969D-01

At iterate    5    f=  7.66674D+00    |proj g|=  5.86730D-03

At iterate   10    f=  7.66673D+00    |proj g|=  1.14493D-03

At iterate   15    f=  7.66645D+00    |proj g|=  1.80414D-02

At iterate   20    f=  7.65233D+00    |proj g|=  1.32999D-01

At iterate   25    f=  7.52220D+00    |proj g|=  1.10779D-02

           * * *

Tit   = total number of iterations
```

```
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     29     34      1     0     0   3.999D-06   7.522D+00
  F =   7.5219235047507640

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 1) x (0, 0, 1, 12)12 : AIC Calculated =1690.910865064171
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.39613D+00    |proj g|=  7.80064D+00
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
At iterate    5    f=  8.10604D+00    |proj g|=  2.09135D-03

At iterate   10    f=  8.10603D+00    |proj g|=  2.65364D-03

At iterate   15    f=  8.10550D+00    |proj g|=  3.78629D-02

At iterate   20    f=  8.03130D+00    |proj g|=  6.31158D-01

At iterate   25    f=  7.86139D+00    |proj g|=  2.39795D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2     29     32      1     0     0   3.437D-06   7.860D+00
  F =   7.8601703905723088

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 1) x (0, 1, 0, 12)12 : AIC Calculated =1764.6781674881972
RUNNING THE L-BFGS-B CODE

           * * *
```

```
Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  6.86774D+00    |proj g|=  5.41189D-01

At iterate    5    f=  6.73612D+00    |proj g|=  8.21783D-02

At iterate   10    f=  6.73567D+00    |proj g|=  1.92554D-04
 This problem is unconstrained.
At iterate   15    f=  6.73567D+00    |proj g|=  7.72317D-03

At iterate   20    f=  6.73451D+00    |proj g|=  8.69982D-02

At iterate   25    f=  6.70332D+00    |proj g|=  3.88473D-01

At iterate   30    f=  6.68520D+00    |proj g|=  4.50370D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     32     38      1     0     0   7.257D-06   6.685D+00
  F =   6.6852020421659928

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 1) x (0, 1, 1, 12)12 : AIC Calculated =1503.4852574451825
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.07723D+00    |proj g|=  2.95920D+00

At iterate    5    f=  7.90245D+00    |proj g|=  1.43125D+00

At iterate   10    f=  7.85351D+00    |proj g|=  2.10753D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
```

```
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     11     22      1     0     0   2.108D-04   7.854D+00
  F =   7.8535071971271506

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 1, 1) x (1, 0, 0, 12)12 : AIC Calculated =1765.1856121564817
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.09206D+00    |proj g|=  3.43982D-01

At iterate    5    f=  7.82082D+00    |proj g|=  6.77174D-01

At iterate   10    f=  7.66787D+00    |proj g|=  8.62312D-02

At iterate   15    f=  7.66666D+00    |proj g|=  2.30852D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     15     24      1     0     0   2.309D-04   7.667D+00
  F =   7.6666564170531890

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (0, 1, 1) x (1, 0, 1, 12)12 : AIC Calculated =1725.3310374199143
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.07709D+00    |proj g|=  8.70886D+00

At iterate    5    f=  7.11893D+00    |proj g|=  3.18219D-01
```

```
At iterate   10    f=  6.86854D+00    |proj g|=  1.11774D-02

At iterate   15    f=  6.86803D+00    |proj g|=  4.40842D-01
 This problem is unconstrained.
At iterate   20    f=  6.85782D+00    |proj g|=  6.27958D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     23     50      1     0     0   2.227D-04   6.858D+00
  F =   6.8577472173500977

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 1, 1) x (1, 1, 0, 12)12 : AIC Calculated =1542.135376686422
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.10671D+00    |proj g|=  2.72483D+00

At iterate    5    f=  6.88448D+00    |proj g|=  3.51274D-01

At iterate   10    f=  6.64835D+00    |proj g|=  1.04389D+00

At iterate   15    f=  6.64152D+00    |proj g|=  4.82602D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     18     33      1     0     0   1.371D-04   6.642D+00
  F =   6.6415186464535338

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
```

```
ARIMA (0, 1, 1) x (1, 1, 1, 12)12 : AIC Calculated =1495.7001768055916
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.91227D+00    |proj g|=  1.12053D-01

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     17      1     0     0   1.121D-01   8.912D+00
  F =   8.9122719068139897

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 0) x (0, 0, 0, 12)12 : AIC Calculated =2000.3489071263336
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.80272D+00    |proj g|=  1.88638D+01

At iterate    5    f=  9.34196D+00    |proj g|=  6.26548D+01

At iterate   10    f=  8.22537D+00    |proj g|=  1.68503D+01

At iterate   15    f=  8.18085D+00    |proj g|=  9.39126D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
```

```
    3     17     47      1     0     0   1.016D-03   8.181D+00
  F =   8.1808545059689468

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 0) x (0, 0, 1, 12)12 : AIC Calculated =1838.5114093370441
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.20339D+00    |proj g|=  5.07810D-04
 This problem is unconstrained.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
 This problem is unconstrained.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
 This problem is unconstrained.
           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     21      1     0     0   5.078D-04   8.203D+00
  F =   8.2033930054436190

ABNORMAL_TERMINATION_IN_LNSRCH
ARIMA (1, 0, 0) x (0, 1, 0, 12)12 : AIC Calculated =1841.5600332193706
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  2.83993D+01    |proj g|=  2.85643D+01

At iterate    5    f=  2.59491D+01    |proj g|=  3.12223D+01

At iterate   10    f=  1.24047D+01    |proj g|=  8.99277D+00

At iterate   15    f=  1.02933D+01    |proj g|=  1.90307D+00
```

```
At iterate   20    f=  1.01664D+01    |proj g|=  4.34173D-03

At iterate   25    f=  1.01657D+01    |proj g|=  2.16486D-03

 Line search cannot locate an adequate point after MAXLS
  function and gradient evaluations.
  Previous x, f and g restored.
 Possible causes: 1 error in function or gradient evaluation;
                  2 rounding error dominate computation.
 This problem is unconstrained.
At iterate   30    f=  1.00911D+01    |proj g|=  2.85395D-02
  ys=-2.153E+01  -gs= 6.728E-01 BFGS update SKIPPED

At iterate   35    f=  7.26476D+00    |proj g|=  1.37525D+00

At iterate   40    f=  7.23121D+00    |proj g|=  7.76493D-04

At iterate   45    f=  7.23074D+00    |proj g|=  5.95580D-02

At iterate   50    f=  7.23032D+00    |proj g|=  1.70721D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     50     76      1     1     0   1.707D-03   7.230D+00
  F =   7.2303155772025267

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT
ARIMA (1, 0, 0) x (0, 1, 1, 12)12 : AIC Calculated =1625.590689293366
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.35976D+00    |proj g|=  1.58477D+00

At iterate    5    f=  7.96739D+00    |proj g|=  6.92561D+00

At iterate   10    f=  7.96505D+00    |proj g|=  1.67164D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
```

```
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     10     18      1     0     0   1.672D-05   7.965D+00
  F =   7.9650499193865807

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 0) x (1, 0, 0, 12)12 : AIC Calculated =1790.171181942594
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  2.36540D+02    |proj g|=  4.46137D+02

At iterate    5    f=  9.11782D+00    |proj g|=  2.91381D-01

At iterate   10    f=  9.11619D+00    |proj g|=  1.91985D+00

At iterate   15    f=  8.63386D+00    |proj g|=  2.88278D+01

At iterate   20    f=  8.17597D+00    |proj g|=  6.04849D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     22     34      1     0     0   3.008D-04   8.176D+00
  F =   8.1759650104781230

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (1, 0, 0) x (1, 0, 1, 12)12 : AIC Calculated =1839.4161623470995
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds
```

```
At iterate    0    f=  7.84182D+00    |proj g|=  1.23526D+00

At iterate    5    f=  7.20062D+00    |proj g|=  1.43133D+00

At iterate   10    f=  7.13229D+00    |proj g|=  2.98179D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     11     14      1     0     0   5.913D-05   7.132D+00
  F =   7.1322916680570518

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (1, 0, 0) x (1, 1, 0, 12)12 : AIC Calculated =1603.6333336447797
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  5.44354D+01    |proj g|=  2.96764D+01

At iterate    5    f=  1.10003D+01    |proj g|=  3.58622D+00

At iterate   10    f=  9.41947D+00    |proj g|=  7.97849D-01

At iterate   15    f=  9.37028D+00    |proj g|=  2.88545D+00

At iterate   20    f=  9.25577D+00    |proj g|=  3.08753D-02

At iterate   25    f=  9.25574D+00    |proj g|=  8.94515D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
```

```
    4     25     35      1     0     0   8.945D-04   9.256D+00
  F =   9.2557356773060118

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (1, 0, 0) x (1, 1, 1, 12)12 : AIC Calculated =2081.284791716547
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.47457D+00    |proj g|=  5.55747D-01

At iterate    5    f=  8.33194D+00    |proj g|=  1.33925D-02
ARIMA (1, 0, 1) x (0, 0, 0, 12)12 : AIC Calculated =1872.3549907993686
           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      8     17      1     0     0   2.734D-04   8.332D+00
  F =   8.3319419232114669

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH

RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.62701D+00    |proj g|=  2.04465D+01

At iterate    5    f=  9.40084D+00    |proj g|=  3.77455D-01

At iterate   10    f=  9.39569D+00    |proj g|=  4.87028D+00

At iterate   15    f=  9.38451D+00    |proj g|=  3.44709D+00
  ys=-1.553E-01  -gs= 5.571E-01 BFGS update SKIPPED

At iterate   20    f=  7.96868D+00    |proj g|=  8.70468D+00

At iterate   25    f=  7.74883D+00    |proj g|=  1.60671D-02

           * * *
```

```
Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     27     58      1     1     0   3.751D-04   7.749D+00
  F =   7.7488275863117648

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (1, 0, 1) x (0, 0, 1, 12)12 : AIC Calculated =1743.7373793338354
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.84208D+01    |proj g|=  3.29012D+01

At iterate    5    f=  8.53861D+00    |proj g|=  6.99722D-01

At iterate   10    f=  8.48350D+00    |proj g|=  6.52683D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     13     14      1     0     0   2.819D-04   8.484D+00
  F =   8.4835032571328473

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 1) x (0, 1, 0, 12)12 : AIC Calculated =1906.3047295977578
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds
```

```
At iterate    0    f=  3.33246D+01    |proj g|=  6.91391D+01

At iterate    5    f=  2.81174D+01    |proj g|=  2.38111D+01
 This problem is unconstrained.
 This problem is unconstrained.
At iterate   10    f=  9.57033D+00    |proj g|=  7.27497D+00

At iterate   15    f=  7.67956D+00    |proj g|=  5.66717D+00

At iterate   20    f=  6.88654D+00    |proj g|=  1.40944D+00

At iterate   25    f=  6.78022D+00    |proj g|=  2.04075D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     29     49      1     0     0   3.282D-04   6.780D+00
  F =   6.7800958429203630

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 1) x (0, 1, 1, 12)12 : AIC Calculated =1526.7414688141614
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  4.65530D+01    |proj g|=  8.66630D+01

At iterate    5    f=  9.83607D+00    |proj g|=  1.09675D+01

At iterate   10    f=  8.30418D+00    |proj g|=  5.29764D-01

At iterate   15    f=  8.19350D+00    |proj g|=  2.00662D+00

At iterate   20    f=  7.80413D+00    |proj g|=  1.09137D+01

At iterate   25    f=  7.51701D+00    |proj g|=  4.21131D+00

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
```

```
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     29     55      1     0     0   1.970D-02   7.516D+00
  F =   7.5155833787530515

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
ARIMA (1, 0, 1) x (1, 0, 0, 12)12 : AIC Calculated =1691.4906768406836
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  2.34803D+02    |proj g|=  4.33750D+02

At iterate    5    f=  8.88486D+00    |proj g|=  6.82925D-01

At iterate   10    f=  8.87134D+00    |proj g|=  6.64150D+00

At iterate   15    f=  8.75353D+00    |proj g|=  9.35108D-01

At iterate   20    f=  8.75236D+00    |proj g|=  4.96603D-01

At iterate   25    f=  8.56015D+00    |proj g|=  1.21008D+01

At iterate   30    f=  7.95033D+00    |proj g|=  1.65882D+01

At iterate   35    f=  7.83175D+00    |proj g|=  4.05192D-01

At iterate   40    f=  7.81389D+00    |proj g|=  7.97291D+00

At iterate   45    f=  7.75733D+00    |proj g|=  6.41899D+00

At iterate   50    f=  7.74207D+00    |proj g|=  2.69650D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
```

```
    5     50     63      1     0     0   2.696D-02   7.742D+00
  F =   7.7420713393042293

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT
 This problem is unconstrained.
ARIMA (1, 0, 1) x (1, 0, 1, 12)12 : AIC Calculated =1744.2239800041473
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.47141D+01    |proj g|=  1.57279D+01

At iterate    5    f=  7.15498D+00    |proj g|=  3.19404D-01

At iterate   10    f=  6.89194D+00    |proj g|=  2.32113D-01

At iterate   15    f=  6.85845D+00    |proj g|=  6.93403D-01

At iterate   20    f=  6.85484D+00    |proj g|=  1.35781D-01

At iterate   25    f=  6.85473D+00    |proj g|=  1.08940D-02
 This problem is unconstrained.
At iterate   30    f=  6.85422D+00    |proj g|=  2.83239D-01

At iterate   35    f=  6.85363D+00    |proj g|=  8.90600D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     35     48      1     0     0   8.906D-05   6.854D+00
  F =   6.8536316677911149

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 1) x (1, 1, 0, 12)12 : AIC Calculated =1543.2134935852098
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.05095D+01    |proj g|=  9.18178D+01
```

```
At iterate    5    f=  9.93828D+00    |proj g|=  1.74596D+01

At iterate   10    f=  7.90925D+00    |proj g|=  6.65989D-01

At iterate   15    f=  7.89513D+00    |proj g|=  1.16652D-01

At iterate   20    f=  7.79455D+00    |proj g|=  2.79820D+00
 This problem is unconstrained.
At iterate   25    f=  7.60278D+00    |proj g|=  1.13772D+01

At iterate   30    f=  7.28889D+00    |proj g|=  3.96205D-01

At iterate   35    f=  7.28790D+00    |proj g|=  3.18258D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     35     49      1     0     0   3.183D-04   7.288D+00
  F =   7.2879017559984218

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 1) x (1, 1, 1, 12)12 : AIC Calculated =1642.4899933436466
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.25449D+00    |proj g|=  3.08805D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     14      1     0     0   3.080D-05   8.254D+00
  F =   8.2544921498921404
```

```
CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 0) x (0, 0, 0, 12)12 : AIC Calculated =1853.0062415758393
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.13791D+01    |proj g|=  1.94634D+01

At iterate    5    f=  7.39679D+00    |proj g|=  1.30437D-02

At iterate   10    f=  7.39673D+00    |proj g|=  7.69730D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     10     11      1     0     0   7.697D-05   7.397D+00
  F =   7.3967305435503450

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 0) x (0, 0, 1, 12)12 : AIC Calculated =1662.8676417552772
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.69721D+00    |proj g|=  1.14956D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     10      1     0     0   1.142D-05   7.697D+00
```

```
  F =   7.6972055915772870

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 0) x (0, 1, 0, 12)12 : AIC Calculated =1728.1740525133123
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  6.85732D+00    |proj g|=  8.39130D-01
 This problem is unconstrained.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
At iterate    5    f=  6.74750D+00    |proj g|=  2.61279D-03

At iterate   10    f=  6.74743D+00    |proj g|=  1.84482D-02

At iterate   15    f=  6.74724D+00    |proj g|=  7.41434D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     15     24      1     0     0   7.414D-06   6.747D+00
  F =   6.7472400669671364

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (1, 1, 0) x (0, 1, 1, 12)12 : AIC Calculated =1517.3817750006385
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.60294D+00    |proj g|=  6.07133D-01

At iterate    5    f=  7.39927D+00    |proj g|=  5.19588D-03

           * * *
```

```
Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      7      9      1     0     0   8.373D-05   7.399D+00
  F =   7.3992590871831334

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 0) x (1, 0, 0, 12)12 : AIC Calculated =1663.434035529022
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.66959D+00    |proj g|=  1.07914D+00

At iterate    5    f=  7.40261D+00    |proj g|=  2.06511D-01

At iterate   10    f=  7.39859D+00    |proj g|=  2.49853D-02

At iterate   15    f=  7.39387D+00    |proj g|=  2.40155D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     19     26      1     0     0   6.229D-05   7.394D+00
  F =   7.3937895115219519

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 0) x (1, 0, 1, 12)12 : AIC Calculated =1664.2088505809172
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
```

```
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  6.71445D+00    |proj g|=  5.87897D-02

At iterate    5    f=  6.71168D+00    |proj g|=  3.56876D-04

At iterate   10    f=  6.71163D+00    |proj g|=  8.31467D-03

At iterate   15    f=  6.71004D+00    |proj g|=  3.44441D-02

At iterate   20    f=  6.70908D+00    |proj g|=  2.06981D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     20     24      1     0     0   2.070D-06   6.709D+00
  F =   6.7090792548981408

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (1, 1, 0) x (1, 1, 0, 12)12 : AIC Calculated =1508.8337530971835
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.18582D+00    |proj g|=  3.89940D+00

At iterate    5    f=  6.81363D+00    |proj g|=  2.94004D-01

At iterate   10    f=  6.72687D+00    |proj g|=  1.03597D-01

At iterate   15    f=  6.71098D+00    |proj g|=  1.94676D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *
```

```
   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     19     24      1     0     0   5.495D-05   6.711D+00
  F =   6.7109465668459460

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (1, 1, 0) x (1, 1, 1, 12)12 : AIC Calculated =1511.2520309734919
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.00580D+01    |proj g|=  1.06365D+01

At iterate    5    f=  8.14586D+00    |proj g|=  6.18089D-02

At iterate   10    f=  8.14538D+00    |proj g|=  9.00706D-04

At iterate   15    f=  8.14535D+00    |proj g|=  1.93403D-02

At iterate   20    f=  8.14126D+00    |proj g|=  1.99595D-01

At iterate   25    f=  8.12205D+00    |proj g|=  4.75847D-02

ARIMA (1, 1, 1) x (0, 0, 0, 12)12 : AIC Calculated =1825.2764495329543
At iterate   30    f=  8.12177D+00    |proj g|=  3.20878D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     30     33      1     0     0   3.209D-05   8.122D+00
  F =   8.1217698639864029

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.65000D+00    |proj g|=  6.18248D+00
```

```
At iterate    5    f=  7.08161D+00    |proj g|=  2.08052D-01

At iterate   10    f=  7.07263D+00    |proj g|=  4.31448D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     12     14      1     0     0   6.050D-06   7.073D+00
  F =   7.0726316465433365

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (1, 1, 1) x (0, 0, 1, 12)12 : AIC Calculated =1592.2694888257074
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  2.28339D+01    |proj g|=  5.36208D+01

At iterate    5    f=  7.61548D+00    |proj g|=  1.35583D-02

At iterate   10    f=  7.61543D+00    |proj g|=  1.33010D-03

At iterate   15    f=  7.61541D+00    |proj g|=  2.25337D-02
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
At iterate   20    f=  7.61247D+00    |proj g|=  2.21708D-01

At iterate   25    f=  7.57943D+00    |proj g|=  3.12683D-01

At iterate   30    f=  7.56853D+00    |proj g|=  2.12569D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
```

```
    3     31     34      1     0     0   1.413D-05   7.569D+00
  F =   7.5685265555365708

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 1) x (0, 1, 0, 12)12 : AIC Calculated =1701.3499484401918
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  6.62940D+00    |proj g|=  3.25692D+00

At iterate    5    f=  6.47304D+00    |proj g|=  8.53995D-02

At iterate   10    f=  6.47075D+00    |proj g|=  6.02389D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     10     12      1     0     0   6.024D-06   6.471D+00
  F =   6.4707540139155864

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
 This problem is unconstrained.
ARIMA (1, 1, 1) x (0, 1, 1, 12)12 : AIC Calculated =1457.4488991170913
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.53977D+01    |proj g|=  2.08538D+01

At iterate    5    f=  7.33898D+00    |proj g|=  5.25287D-01

At iterate   10    f=  7.14708D+00    |proj g|=  1.70222D-01

At iterate   15    f=  7.14261D+00    |proj g|=  5.90461D-07

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
```

```
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     15     20      1     0     0   5.905D-07   7.143D+00
  F =   7.1426119971939128

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (1, 1, 1) x (1, 0, 0, 12)12 : AIC Calculated =1607.9450873714366
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.17068D+00    |proj g|=  3.82225D+00

At iterate    5    f=  7.07279D+00    |proj g|=  1.64576D-01

At iterate   10    f=  7.07022D+00    |proj g|=  5.72618D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     13     16      1     0     0   2.007D-05   7.070D+00
  F =   7.0702078779259852

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (1, 1, 1) x (1, 0, 1, 12)12 : AIC Calculated =1593.7265646554206
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  2.11227D+01    |proj g|=  5.50051D+01
```

```
At iterate    5    f=  6.66553D+00    |proj g|=  2.74057D-01

At iterate   10    f=  6.56908D+00    |proj g|=  1.94745D-01

At iterate   15    f=  6.55634D+00    |proj g|=  1.31264D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     16     21      1     0     0   1.517D-04   6.556D+00
  F =   6.5563409164076054

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (1, 1, 1) x (1, 1, 0, 12)12 : AIC Calculated =1476.6203652753036
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.80890D+01    |proj g|=  6.04498D+01

At iterate    5    f=  6.76296D+00    |proj g|=  6.37963D-01

At iterate   10    f=  6.51475D+00    |proj g|=  4.14947D-01

At iterate   15    f=  6.42433D+00    |proj g|=  8.65967D-02
 This problem is unconstrained.
At iterate   20    f=  6.42011D+00    |proj g|=  9.69051D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     23     45      1     0     0   2.069D-04   6.420D+00
  F =   6.4200881372037246
```

```
CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
ARIMA (1, 1, 1) x (1, 1, 1, 12)12 : AIC Calculated =1448.0997427336342
pdq         (1, 1, 1)
pdqs    (1, 1, 1, 12)
aic       1448.099743
Name: 63, dtype: object
```

Out[241…

	pdq	pdqs	aic
0	(0, 0, 0)	(0, 0, 0, 12)	3112.694232
1	(0, 0, 0)	(0, 0, 1, 12)	2726.706091
2	(0, 0, 0)	(0, 1, 0, 12)	2311.072221
3	(0, 0, 0)	(0, 1, 1, 12)	2042.296298
4	(0, 0, 0)	(1, 0, 0, 12)	2237.285265
...	...	...	...
59	(1, 1, 1)	(0, 1, 1, 12)	1457.448899
60	(1, 1, 1)	(1, 0, 0, 12)	1607.945087
61	(1, 1, 1)	(1, 0, 1, 12)	1593.726565
62	(1, 1, 1)	(1, 1, 0, 12)	1476.620365
63	(1, 1, 1)	(1, 1, 1, 12)	1448.099743

64 rows × 3 columns

Train Test Split Function

In [242…

```
# Train, test split
def train_test(df):

    #Use data before 2017-05 as training data set
    train = df['2009-01':'2017-05']
    #Use 12 month prior to 2018-04 as test data set test, which starts from 2017
    test = df['2017-05':]
    return train, test
```

Fitting an ARIMA Time Series Model

Use the best parameters from grid search and fit the model

In [243…

```
def model_fit(df,pdq=(1,0,1),pdqs=(0,0,0,1)):


    train, test = train_test(df)
    model = SARIMAX(train,order=pdq,seasonal_order=pdqs,
                                            )
    results = model.fit()
    results.summary
```

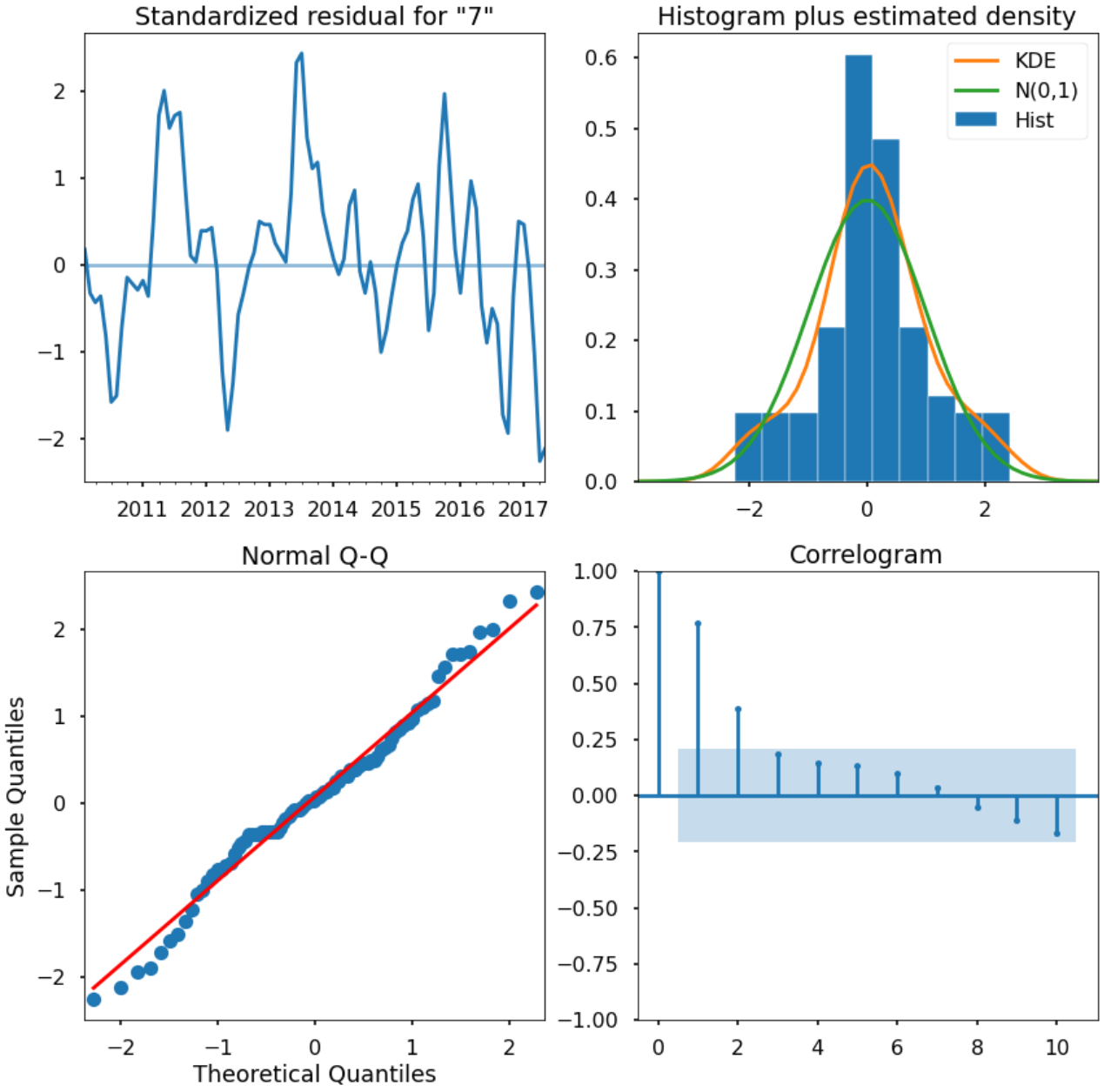
```
    residuals = results.resid
    print(results.summary())
    results.plot_diagnostics(figsize=(14,14))
    plt.show();
    return train, test, results


#print(output.summary().tables[1])
```

In [244…

```
#Fit the SARIMA model and get results.
pdq = (1, 1, 1)
pdqs = (1, 1, 1, 12)

train, test, results = model_fit(ts_78702,pdq=pdq,pdqs=pdqs)
```

```
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.44442D+01    |proj g|=  3.25356D+01

At iterate    5    f=  9.21474D+00    |proj g|=  1.25322D+00

At iterate   10    f=  9.17903D+00    |proj g|=  2.92406D-03

At iterate   15    f=  9.17890D+00    |proj g|=  3.45532D-02
 This problem is unconstrained.
At iterate   20    f=  9.14020D+00    |proj g|=  3.18868D-01

At iterate   25    f=  8.12189D+00    |proj g|=  5.52481D-04
  ys=-1.223E-01  -gs= 4.653E-02 BFGS update SKIPPED

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     26     51      1     1     0   4.175D-04   8.122D+00
  F =   8.1218911053922529

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
                                     SARIMAX Results
```

```
==========================================================================================
Dep. Variable:                               78702   No. Observations:
101
Model:             SARIMAX(1, 1, 1)x(1, 1, 1, 12)   Log Likelihood
-820.311
Date:                            Wed, 19 Oct 2022   AIC
1650.622
Time:                                    13:08:22   BIC
1663.009
Sample:                                01-01-2009   HQIC
1655.612
                                     - 05-01-2017
Covariance Type:                              opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ar.L1          1.0000      0.570      1.753      0.080      -0.118       2.118
ma.L1         -1.0000      0.580     -1.724      0.085      -2.137       0.137
ar.S.L12      -1.0000      0.629     -1.589      0.112      -2.234       0.234
ma.S.L12       1.0000      0.629      1.591      0.112      -0.232       2.232
sigma2      7.787e+06   5.62e-08   1.39e+14      0.000    7.79e+06    7.79e+06
==============================================================================
===
Ljung-Box (L1) (Q):                   54.26   Jarque-Bera (JB):
0.17
Prob(Q):                              0.00   Prob(JB):
0.92
Heteroskedasticity (H):               0.98   Skew:
0.00
Prob(H) (two-sided):                  0.95   Kurtosis:
3.21
==============================================================================
===

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-s
tep).
[2] Covariance matrix is singular or near-singular, with condition number 4.66e+
30. Standard errors may be unstable.
```



In [245…

```
def test_model(df,pdq=(1,0,1),pdqs=(0,0,0,1), display=True):
    '''
            Predicts for 12 months on test model with SARIMA model

    '''

    X = df.values
    train, test = X[:-12],X[-12:]
    history = [x for x in train]

    predictions = []
    for t in range(len(test)):
        model = SARIMAX(history, order=pdq,seasonal_order=pdqs, enforce_stationa
                                        enforce_invertibility=False)
        model_fit = model.fit(disp=0)
        output = model_fit.forecast()
        yhat = output[0]
        predictions.append(yhat)
```

```
        history.append(test[t])

    mse= mean_squared_error(test,predictions)
    rmse = math.sqrt(mse)
    print(f'SARIMA model RMSE on train data: %.5f' % rmse)

    if display:
        plt.figure(figsize=(13,6))
        plt.title('Actual Test Data vs. Predictions')
        #plt.plot(history[-12],label='Actual', color='b')
        plt.plot(history[-12:],label='Actual',color='b')
        plt.plot(predictions,label='Predictions',color='g')
        plt.xlabel('Month')
        plt.legend(loc='best')
        plt.show()
```

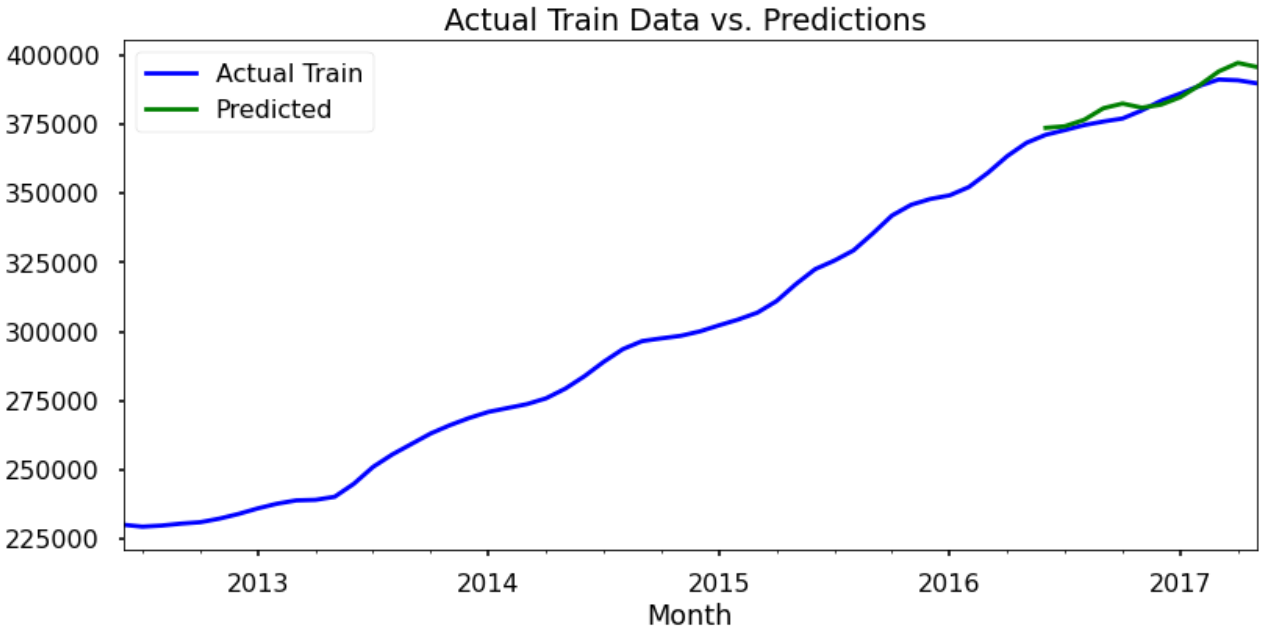
In [246…

```
def train_model(train, results, display = True):
    '''
        Returns train  v.s. predictions pefromed on on SARIMA model

        results        Results from SARIMA model
    '''

    train_pred = results.predict(-12)
    mse= mean_squared_error(train[-12:],train_pred)
    rmse = math.sqrt(mse)
    print(f'SARIMA model RMSE on train data: %.5f' % rmse)

    if display:
        plt.figure(figsize=(13,6))
        train[-60:].plot(label='Actual Train',color='b')
        train_pred.plot(label='Predicted',color='g')
        plt.legend(loc='best')
        plt.title('Actual Train Data vs. Predictions ')
        plt.show()
```

In [247…

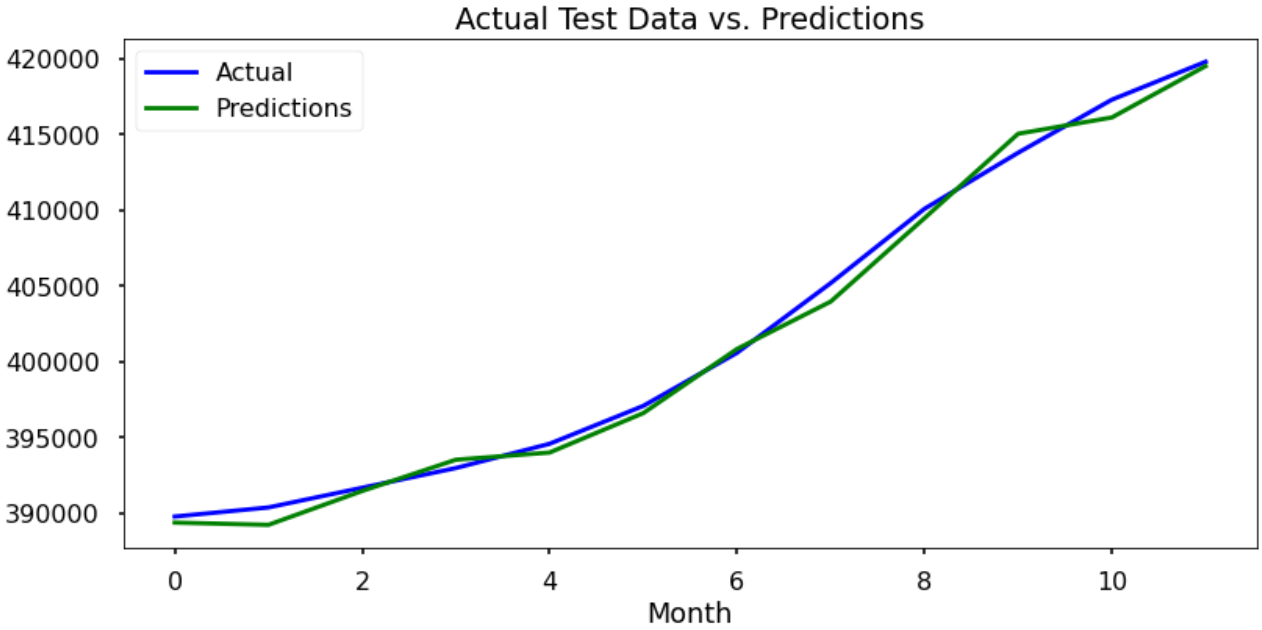
```
train_model(train,results)
```

```
SARIMA model RMSE on train data: 3554.57164
```



In [248…
```
test_model(ts_78702,pdq=pdq,pdqs=pdqs,display=True)
```

```
SARIMA model RMSE on train data: 782.62593
```



Validating the Model

In [249…
```
ARIMA_MODEL = sm.tsa.statespace.SARIMAX(ts_78721,
                                        order=(1,1,1),
                                        seasonal_order=(1,1,1,12),
                                        enforce_stationarity=False,
                                        enforce_invertibility=False)
output= ARIMA_MODEL.fit()
```

```
RUNNING THE L-BFGS-B CODE

           * * *
```

```
Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.57025D+00    |proj g|=  6.49412D+00

At iterate    5    f=  6.40126D+00    |proj g|=  2.44188D-01

At iterate   10    f=  6.33497D+00    |proj g|=  1.56754D-01
 This problem is unconstrained.
At iterate   15    f=  6.32700D+00    |proj g|=  5.49380D-03

At iterate   20    f=  6.32698D+00    |proj g|=  1.17922D-03

At iterate   25    f=  6.32688D+00    |proj g|=  8.53403D-02

At iterate   30    f=  6.32029D+00    |proj g|=  2.42813D-01

At iterate   35    f=  6.31483D+00    |proj g|=  4.10234D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     38     45      1     0     0   9.909D-06   6.315D+00
  F =   6.3148169261707769

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
```

Use SARIMA model to perform 5 year price projection and get ROI

In [250…

```
# Perform predictions on SARIMA model for 5 year projection

def forecast_model(df,pdq=(1,0,1),pdqs=(0,0,0,1), display=True,zc='enter zipcode


    #model = SARIMAX(df, order=pdq,seasonal_order=pdqs, enforce_stationarity=Fal
                                          #enforce_invertibility=False)
    model = SARIMAX(df, order=pdq,seasonal_order=pdqs)

    output = model.fit()
    pred_dynamic = output.get_prediction(start='2017-05',end='2022-06', dynamic=
    pred_dynamic_ci = pred_dynamic.conf_int()


    if display:
        fig, ax = plt.subplots(figsize=(13,6))
        pred_dynamic.predicted_mean.plot(label='Forecast')
```

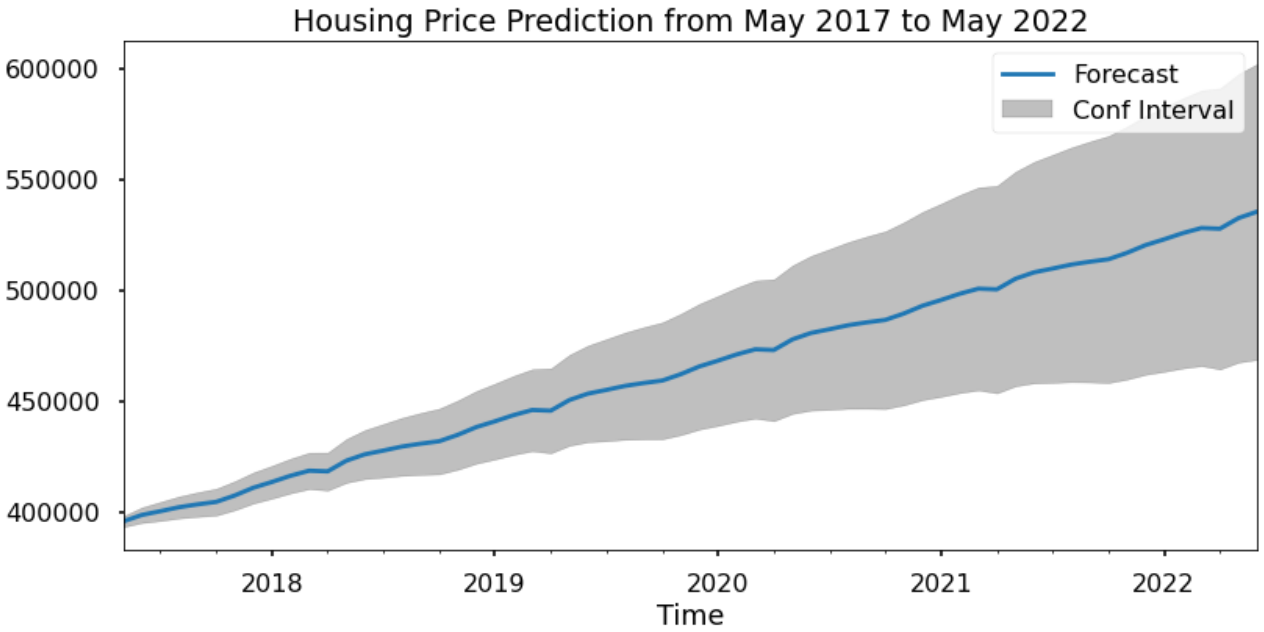
```
        ax.fill_between(pred_dynamic_ci.index,pred_dynamic_ci.iloc[:, 0],pred_dy
                        color='k', alpha=.25,label='Conf Interval')
        plt.title('Housing Price Prediction from May 2017 to May 2022')
        plt.xlabel('Time')
        plt.legend(loc='best')
        plt.show()

        #Used May 2017 average price as a calculation based to get 1 yr, 3 yr an


        year_0 = pred_dynamic.predicted_mean['2017-05-01']
        year_1= pred_dynamic.predicted_mean['2018-05-01']
        year_1_ROI_100K = ((year_1 - year_0)*100)/year_0
        year_3= pred_dynamic.predicted_mean['2020-05-01']
        year_3_ROI_100K = ((year_3 - year_0)*100)/year_0
        year_5= pred_dynamic.predicted_mean['2022-05-01']
        year_5_ROI_100K = ((year_5 - year_0)*100)/year_0

        print(f'Housing Price Projection for 2017, 2018,2020 and 2022:')
        print(f'Housing price at enter point 2017-05:$ {round(year_0,2)}')
        print(f'Housing price in year 1:$ {round(year_1,2)} with {round(year_1_R
        print(f'Housing price in year 3:$ {round(year_3,2)} with {round(year_3_R
        print(f'Housing price in year 5:$ {round(year_5,2)} with  {round(year_5_


        tot_ret = [zc,year_1,year_3,year_5]
    return tot_ret
```

In [251…

```
print('Zipcode 78702 Property Mean Value Forecast:')
pred_78702 = forecast_model(ts_78702,pdq=pdq,pdqs=pdqs,zc=78702)
```

```
Zipcode 78702 Property Mean Value Forecast:
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.58734D+01    |proj g|=  3.58720D+01

At iterate    5    f=  9.43539D+00    |proj g|=  1.52701D+00

At iterate   10    f=  9.14177D+00    |proj g|=  1.10285D-02

At iterate   15    f=  9.14156D+00    |proj g|=  1.12393D-01

At iterate   20    f=  9.13200D+00    |proj g|=  7.80560D-01
 This problem is unconstrained.
At iterate   25    f=  9.09247D+00    |proj g|=  4.33692D-01

At iterate   30    f=  9.00050D+00    |proj g|=  1.15057D+00
  ys=-2.898E-03  -gs= 5.040E-04 BFGS update SKIPPED
 Bad direction in the line search;
   refresh the lbfgs memory and restart the iteration.
```

```
 Line search cannot locate an adequate point after MAXLS
  function and gradient evaluations.
  Previous x, f and g restored.
 Possible causes: 1 error in function or gradient evaluation;
                  2 rounding error dominate computation.
           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     33     110      2     1     0   2.523D+00   9.000D+00
  F =   9.0004903294765324

ABNORMAL_TERMINATION_IN_LNSRCH
```




```
Housing Price Projection for 2017, 2018,2020 and 2022:
Housing price at enter point 2017-05:$ 395699.6
Housing price in year 1:$ 423098.16 with 6.92 % ROI
Housing price in year 3:$ 477892.21 with 20.77 % ROI
Housing price in year 5:$ 532682.26 with  34.62 % ROI
```

In [252…

```
#Initialise collection of 5yr ROI data for each of the 5 top zipcodes
zip_roi= {'Zipcode': [78702, 78758,78721,78723,78741], '5_YR_PRED_ROI': [0,0,0,0
df_zip_5YR_ROI = pd.DataFrame(data=zip_roi)
```

In [253…

```
# Add the 5 yr roi to the results summary df
df_zip_5YR_ROI.loc[df_zip_5YR_ROI['Zipcode'] == 78702, '5YR_PRED_ROI'] = 34.83
df_zip_5YR_ROI
```

Out[253…

	Zipcode	5_YR_PRED_ROI	5YR_PRED_ROI

	Zipcode	5_YR_PRED_ROI	5YR_PRED_ROI
0	78702	0	34.83
1	78758	0	NaN
2	78721	0	NaN
3	78723	0	NaN
4	78741	0	NaN

Zipcode 78758

In [254…

```
AIC_PDQS(ts_78758)
```

```
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            1     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.33918D+01    |proj g|=  1.77636D-08

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    1      0      1      0     0     0   1.776D-08   1.339D+01
  F =   13.391819788263229

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 0) x (0, 0, 0, 12)12 : AIC Calculated =3001.767632570963
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.05796D+03    |proj g|=  1.23312D+00

At iterate    5    f=  1.17053D+01    |proj g|=  7.27724D-02
```

```
           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      9     23      1     0     0   6.478D-06   1.170D+01
  F =   11.702867677723580

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 0) x (0, 0, 1, 12)12 : AIC Calculated =2625.442359810082
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            1     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.00176D+01    |proj g|=  6.13145D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    1      0      1      0     0     0   6.131D-06   1.002D+01
  F =   10.017550511103925

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 0) x (0, 1, 0, 12)12 : AIC Calculated =2245.9313144872795
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  2.81771D+02    |proj g|=  1.58554D+01

At iterate    5    f=  4.01583D+01    |proj g|=  7.60660D-01
```

```
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
At iterate   10    f=  1.65190D+01    |proj g|=  3.62560D-02

At iterate   15    f=  1.39954D+01    |proj g|=  1.36275D-03

At iterate   20    f=  1.39079D+01    |proj g|=  7.38938D-05

At iterate   25    f=  1.12127D+01    |proj g|=  1.57424D-03
  ys=-2.927E+01  -gs= 6.304E-01 BFGS update SKIPPED
  ys=-6.628E+00  -gs= 7.006E-01 BFGS update SKIPPED

At iterate   30    f=  8.84969D+00    |proj g|=  5.46141D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2     31     61      1     2     0   8.837D-07   8.850D+00
  F =   8.8496907802727591

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 0) x (0, 1, 1, 12)12 : AIC Calculated =1986.330734781098
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.77507D+00    |proj g|=  6.59097D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     14      1     0     0   6.589D-04   9.775D+00
  F =   9.7750721010277335
```

```
CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 0) x (1, 0, 0, 12)12 : AIC Calculated =2193.6161506302124
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.02489D+03    |proj g|=  1.74682D+02
  ys=-2.755E+02  -gs= 8.645E+01 BFGS update SKIPPED

At iterate    5    f=  5.25951D+02    |proj g|=  2.98230D+02
 This problem is unconstrained.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
 This problem is unconstrained.
 This problem is unconstrained.
At iterate   10    f=  1.54018D+01    |proj g|=  2.71046D+01

At iterate   15    f=  9.87926D+00    |proj g|=  2.25803D+00

At iterate   20    f=  9.66082D+00    |proj g|=  1.29249D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     21     32      1     1     0   8.122D-06   9.661D+00
  F =   9.6608156201944571

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 0) x (1, 0, 1, 12)12 : AIC Calculated =2170.0226989235584
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.67101D+00    |proj g|=  6.91696D-06

           * * *
```

```
Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      0      1      0     0     0   6.917D-06   8.671D+00
  F =   8.6710084579045805

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 0) x (1, 1, 0, 12)12 : AIC Calculated =1946.305894570626
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  2.84972D+02    |proj g|=  2.44596D+01

At iterate    5    f=  5.39700D+01    |proj g|=  1.97236D+00

At iterate   10    f=  1.76190D+01    |proj g|=  9.47219D-02

At iterate   15    f=  1.35511D+01    |proj g|=  3.96248D-03

At iterate   20    f=  1.33437D+01    |proj g|=  9.28861D-05

At iterate   25    f=  1.32022D+01    |proj g|=  8.61577D-04
  ys=-3.695E+00  -gs= 6.124E-01 BFGS update SKIPPED
 This problem is unconstrained.
At iterate   30    f=  8.94341D+00    |proj g|=  3.07035D-01

At iterate   35    f=  8.57672D+00    |proj g|=  2.25626D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     37     72      1     1     0   2.416D-07   8.577D+00
  F =   8.5767217095842945

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 0) x (1, 1, 1, 12)12 : AIC Calculated =1927.185662946882
```

```
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.19311D+01    |proj g|=  1.30360D+00

ARIMA (0, 0, 1) x (0, 0, 0, 12)12 : AIC Calculated =2897.894430767661At iterate
5    f=  1.29204D+01    |proj g|=  1.79730D-01

At iterate   10    f=  1.29192D+01    |proj g|=  5.22344D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2     11     25      1     0     0   1.310D-05   1.292D+01
  F =   12.919171565927059

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH

RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.06010D+03    |proj g|=  1.43477D+00

At iterate    5    f=  1.27977D+01    |proj g|=  8.01108D-01

At iterate   10    f=  1.21055D+01    |proj g|=  3.21070D-01

At iterate   15    f=  1.20543D+01    |proj g|=  6.69845D-02

At iterate   20    f=  1.20494D+01    |proj g|=  3.17530D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
```

```
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     22     61      1     0     0   8.640D-06   1.205D+01
  F =   12.049427003784157

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (0, 0, 1) x (0, 0, 1, 12)12 : AIC Calculated =2705.071648847651
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.03888D+01    |proj g|=  3.04540D-01

At iterate    5    f=  9.65196D+00    |proj g|=  4.80671D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      7     15      1     0     0   2.639D-05   9.652D+00
  F =   9.6519579233906931

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 1) x (0, 1, 0, 12)12 : AIC Calculated =2166.038574839515
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  4.18016D+02    |proj g|=  2.17056D+01
  ys=-4.309E+02  -gs= 8.811E+01 BFGS update SKIPPED

At iterate    5    f=  2.93864D+01    |proj g|=  1.66141D+00

At iterate   10    f=  2.90648D+01    |proj g|=  2.92740D+01
```

```
At iterate   15    f=  2.18125D+01    |proj g|=  3.60557D+01
 This problem is unconstrained.
 This problem is unconstrained.
At iterate   20    f=  1.34334D+01    |proj g|=  1.34453D+00

At iterate   25    f=  1.28052D+01    |proj g|=  2.95696D-02

At iterate   30    f=  1.28028D+01    |proj g|=  1.16379D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     30     55      2     1     0   1.164D-02   1.280D+01
  F =   12.802765664805280

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
ARIMA (0, 0, 1) x (0, 1, 1, 12)12 : AIC Calculated =2873.819508916383
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.37444D+01    |proj g|=  8.28753D-02

At iterate    5    f=  1.16238D+01    |proj g|=  9.18618D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      8     16      1     0     0   1.518D-05   1.162D+01
  F =   11.623766667170264
```

```
CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 1) x (1, 0, 0, 12)12 : AIC Calculated =2609.7237334461392
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.57934D+02    |proj g|=  1.75871D+02
 This problem is unconstrained.
 This problem is unconstrained.
At iterate    5    f=  1.22111D+01    |proj g|=  3.24753D-01
  ys=-2.310E+00  -gs= 7.090E-01 BFGS update SKIPPED

At iterate   10    f=  1.20457D+01    |proj g|=  5.39381D-02

At iterate   15    f=  1.20271D+01    |proj g|=  4.85286D-02

At iterate   20    f=  1.20252D+01    |proj g|=  1.63529D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     22     48      1     1     0   5.354D-06   1.203D+01
  F =   12.025223129725504

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 1) x (1, 0, 1, 12)12 : AIC Calculated =2701.649981058513
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.31758D+00    |proj g|=  2.78311D-01
 This problem is unconstrained.
At iterate    5    f=  9.28743D+00    |proj g|=  6.90164D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
```

```
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      7     11      1     0     0   1.943D-05   9.287D+00
  F =   9.2874344658328987

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 1) x (1, 1, 0, 12)12 : AIC Calculated =2086.385320346569
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  4.83300D+02    |proj g|=  6.05106D+01
 This problem is unconstrained.
At iterate    5    f=  6.56361D+01    |proj g|=  6.00719D+00

At iterate   10    f=  1.45133D+01    |proj g|=  8.01642D-02

At iterate   15    f=  1.43665D+01    |proj g|=  2.43961D-02

At iterate   20    f=  1.43570D+01    |proj g|=  5.80595D-05

At iterate   25    f=  1.43570D+01    |proj g|=  1.59181D-03

At iterate   30    f=  1.43521D+01    |proj g|=  2.38084D-02

At iterate   35    f=  1.25773D+01    |proj g|=  2.76103D-03

At iterate   40    f=  1.17363D+01    |proj g|=  6.74528D-02

At iterate   45    f=  1.14033D+01    |proj g|=  3.30564D-02

At iterate   50    f=  1.13935D+01    |proj g|=  4.08546D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     50     81      1     0     0   4.085D-02   1.139D+01
  F =   11.393514715001038
```

```
STOP: TOTAL NO. of ITERATIONS REACHED LIMIT
ARIMA (0, 0, 1) x (1, 1, 1, 12)12 : AIC Calculated =2560.1472961602326
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            1     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.88243D+00    |proj g|=  8.79581D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    1      0      1      0     0     0   8.796D-06   8.882D+00
  F =   8.8824292551758042

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 0) x (0, 0, 0, 12)12 : AIC Calculated =1991.6641531593802
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.02363D+00    |proj g|=  2.86701D-01

At iterate    5    f=  7.93519D+00    |proj g|=  1.23744D-04

At iterate   10    f=  7.93515D+00    |proj g|=  6.61666D-03

At iterate   15    f=  7.93049D+00    |proj g|=  7.59048D-02

At iterate   20    f=  7.90825D+00    |proj g|=  2.05133D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value
```

```
           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2     21     24      1     0     0   5.332D-07   7.908D+00
  F =   7.9082507579699470

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 0) x (0, 0, 1, 12)12 : AIC Calculated =1775.4481697852682
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            1     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.96037D+00    |proj g|=  6.20958D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    1      2      5      1     0     0   6.778D-06   7.956D+00
  F =   7.9563519806730030

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 0) x (0, 1, 0, 12)12 : AIC Calculated =1784.2228436707526
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.01382D+00    |proj g|=  2.43877D-01

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *
```

```
   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      4      7      1     0     0   9.630D-05   7.006D+00
  F =   7.0061056521963820

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (0, 1, 0) x (0, 1, 1, 12)12 : AIC Calculated =1573.3676660919896
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.94365D+00    |proj g|=  4.14371D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      0      1      0     0     0   4.144D-06   7.944D+00
  F =   7.9436469209270513

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 0) x (1, 0, 0, 12)12 : AIC Calculated =1783.3769102876595
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.92750D+00    |proj g|=  4.96745D-01

At iterate    5    f=  7.87791D+00    |proj g|=  1.60346D-04

At iterate   10    f=  7.87788D+00    |proj g|=  4.78962D-03

At iterate   15    f=  7.87501D+00    |proj g|=  6.35773D-02

At iterate   20    f=  7.86379D+00    |proj g|=  5.84111D-03

           * * *
```

```
Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     22     27      1     0     0   6.876D-06   7.864D+00
  F =   7.8637780123258745

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 0) x (1, 0, 1, 12)12 : AIC Calculated =1767.486274760996
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  6.89284D+00    |proj g|=  2.88543D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      0      1      0     0     0   2.885D-06   6.893D+00
  F =   6.8928375389150540

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (0, 1, 0) x (1, 1, 0, 12)12 : AIC Calculated =1547.9956087169721
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.41732D+00    |proj g|=  9.00745D+00
```

```
At iterate    5    f=  6.86974D+00    |proj g|=  2.55237D-01

At iterate   10    f=  6.82759D+00    |proj g|=  2.70271D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     11     12      1     0     0   8.538D-05   6.828D+00
  F =   6.8275918637380455

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (0, 1, 0) x (1, 1, 1, 12)12 : AIC Calculated =1535.3805774773223
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.91023D+00    |proj g|=  4.51774D-01

At iterate    5    f=  8.38261D+00    |proj g|=  3.35131D-04

At iterate   10    f=  8.38261D+00    |proj g|=  5.38217D-03

At iterate   15    f=  8.38192D+00    |proj g|=  7.92442D-02

At iterate   20    f=  8.25642D+00    |proj g|=  4.62031D-01

At iterate   25    f=  8.20543D+00    |proj g|=  1.66712D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2     28     34      1     0     0   1.296D-06   8.205D+00
  F =   8.2051595335595220
```

```
CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 1) x (0, 0, 0, 12)12 : AIC Calculated =1841.9557355173329
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.96420D+00    |proj g|=  3.76034D-01

At iterate    5    f=  7.45983D+00    |proj g|=  4.30236D-02

At iterate   10    f=  7.45909D+00    |proj g|=  3.17518D-04

At iterate   15    f=  7.45904D+00    |proj g|=  9.65712D-03

At iterate   20    f=  7.45457D+00    |proj g|=  3.08771D-01

At iterate   25    f=  7.30838D+00    |proj g|=  2.91023D-01

At iterate   30    f=  7.29504D+00    |proj g|=  8.58321D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     31     40      1     0     0   2.919D-05   7.295D+00
  F =   7.2950391683541644

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 1, 1) x (0, 0, 1, 12)12 : AIC Calculated =1640.0887737113328
RUNNING THE L-BFGS-B CODE

           * * *


 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.05846D+00    |proj g|=  6.95724D-01

At iterate    5    f=  7.99919D+00    |proj g|=  3.08799D-04

At iterate   10    f=  7.99917D+00    |proj g|=  6.98280D-03
```

```
At iterate   15    f=  7.99681D+00    |proj g|=  9.60510D-02

At iterate   20    f=  7.80663D+00    |proj g|=  6.76801D-01

At iterate   25    f=  7.65986D+00    |proj g|=  1.75219D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2     29     32      1     0     0   9.113D-08   7.659D+00
  F =   7.6593702810992061

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 1) x (0, 1, 0, 12)12 : AIC Calculated =1719.6989429662221
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  6.87590D+00    |proj g|=  3.60630D-01

At iterate    5    f=  6.66244D+00    |proj g|=  6.35898D-02

At iterate   10    f=  6.66058D+00    |proj g|=  3.33934D-04
 This problem is unconstrained.
At iterate   15    f=  6.66052D+00    |proj g|=  1.48201D-02

At iterate   20    f=  6.65605D+00    |proj g|=  2.03813D-01

At iterate   25    f=  6.51803D+00    |proj g|=  4.34578D-02

At iterate   30    f=  6.51766D+00    |proj g|=  2.58115D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *
```

```
   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     31     40      1     0     0   5.889D-06   6.518D+00
  F =   6.5176570092034192

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 1) x (0, 1, 1, 12)12 : AIC Calculated =1465.955170061566
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.25209D+00    |proj g|=  6.32167D-01

At iterate    5    f=  7.66049D+00    |proj g|=  3.65675D-01

At iterate   10    f=  7.62558D+00    |proj g|=  2.31804D-02

At iterate   15    f=  7.62552D+00    |proj g|=  2.95068D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     15     26      1     0     0   2.951D-04   7.626D+00
  F =   7.6255151811027293

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 1, 1) x (1, 0, 0, 12)12 : AIC Calculated =1714.1154005670114
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.95140D+00    |proj g|=  3.59940D-01

At iterate    5    f=  7.47287D+00    |proj g|=  1.02456D-01

At iterate   10    f=  7.45823D+00    |proj g|=  2.63484D-03

At iterate   15    f=  7.45823D+00    |proj g|=  1.23313D-03

At iterate   20    f=  7.45815D+00    |proj g|=  1.93622D-02
```

```
 This problem is unconstrained.
 This problem is unconstrained.
At iterate   25    f=  7.44896D+00    |proj g|=  1.98357D-01

At iterate   30    f=  7.30016D+00    |proj g|=  2.44756D-01

At iterate   35    f=  7.29217D+00    |proj g|=  8.42299D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     38     42      1     0     0   3.824D-05   7.292D+00
  F =   7.2921413603724377

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 1, 1) x (1, 0, 1, 12)12 : AIC Calculated =1641.439664723426
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.79268D+00    |proj g|=  4.00277D+00

At iterate    5    f=  6.77713D+00    |proj g|=  1.53456D+00
 This problem is unconstrained.
At iterate   10    f=  6.75615D+00    |proj g|=  1.94432D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     14     27      1     0     0   3.020D-04   6.756D+00
  F =   6.7561258720526638

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 1, 1) x (1, 1, 0, 12)12 : AIC Calculated =1519.3721953397967
RUNNING THE L-BFGS-B CODE
```

```
           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.72227D+00    |proj g|=  5.81027D+00

At iterate    5    f=  6.79160D+00    |proj g|=  7.88600D-01

At iterate   10    f=  6.65243D+00    |proj g|=  9.31451D-01

At iterate   15    f=  6.60580D+00    |proj g|=  1.43294D+00

At iterate   20    f=  6.56004D+00    |proj g|=  2.15531D-03

At iterate   25    f=  6.56004D+00    |proj g|=  3.72465D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     25     34      1     0     0   3.725D-04   6.560D+00
  F =   6.5600412416726162

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (0, 1, 1) x (1, 1, 1, 12)12 : AIC Calculated =1477.449238134666
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.68804D+00    |proj g|=  6.47207D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value
```

```
           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     21      1     0     0   6.472D-02   8.688D+00
  F =   8.6880423412729613

ABNORMAL_TERMINATION_IN_LNSRCH
ARIMA (1, 0, 0) x (0, 0, 0, 12)12 : AIC Calculated =1950.1214844451433
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.05738D+01    |proj g|=  1.13120D+01

At iterate    5    f=  1.05084D+01    |proj g|=  6.58685D+00
  ys=-2.022E+00  -gs= 4.469E-01 BFGS update SKIPPED

At iterate   10    f=  8.18345D+00    |proj g|=  3.10377D+01

At iterate   15    f=  7.97854D+00    |proj g|=  8.09981D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     17     51      1     1     0   1.617D-04   7.979D+00
  F =   7.9785442149859440

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 0) x (0, 0, 1, 12)12 : AIC Calculated =1793.1939041568514
 This problem is unconstrained.

 Line search cannot locate an adequate point after MAXLS
  function and gradient evaluations.
  Previous x, f and g restored.
 Possible causes: 1 error in function or gradient evaluation;
                  2 rounding error dominate computation.
 This problem is unconstrained.
 This problem is unconstrained.
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10
```

```
At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.03299D+00    |proj g|=  3.86013D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     13      1     0     0   3.858D-04   8.033D+00
  F =   8.0329882218998598

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 0) x (0, 1, 0, 12)12 : AIC Calculated =1803.3893617055685
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.68271D+00    |proj g|=  1.30162D+00

At iterate    5    f=  9.64424D+00    |proj g|=  5.29819D-01

At iterate   10    f=  9.50515D+00    |proj g|=  3.66882D-01

At iterate   15    f=  9.50007D+00    |proj g|=  8.78164D-04

At iterate   20    f=  9.50004D+00    |proj g|=  2.13587D-02

At iterate   25    f=  9.49652D+00    |proj g|=  2.86181D-01
 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
 This problem is unconstrained.
At iterate   30    f=  8.87037D+00    |proj g|=  4.54864D+00

At iterate   35    f=  7.16505D+00    |proj g|=  5.68354D-01

At iterate   40    f=  7.13082D+00    |proj g|=  1.30326D-02

At iterate   45    f=  7.12088D+00    |proj g|=  4.65412D-02

At iterate   50    f=  7.11699D+00    |proj g|=  3.59509D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
```

```
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     50     69      1     0     0   3.595D-02   7.117D+00
  F =   7.1169900360786462

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT
ARIMA (1, 0, 0) x (0, 1, 1, 12)12 : AIC Calculated =1600.2057680816167
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.21831D+00    |proj g|=  1.73551D+00

At iterate    5    f=  7.81157D+00    |proj g|=  1.53405D+01

At iterate   10    f=  7.77252D+00    |proj g|=  6.76453D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     11     21      1     0     0   3.057D-05   7.773D+00
  F =   7.7725168249234162

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 0) x (1, 0, 0, 12)12 : AIC Calculated =1747.0437687828453
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.96273D+01    |proj g|=  1.56709D+02

At iterate    5    f=  9.43199D+00    |proj g|=  7.36652D-02
```

```
At iterate   10    f=  9.43080D+00    |proj g|=  1.30827D+00

At iterate   15    f=  8.81291D+00    |proj g|=  3.66346D+01
  ys=-3.021E-01  -gs= 3.619E-01 BFGS update SKIPPED

At iterate   20    f=  7.99308D+00    |proj g|=  9.84756D-01

At iterate   25    f=  7.97481D+00    |proj g|=  1.03319D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     26     52      1     1     0   1.033D-03   7.975D+00
  F =   7.9748114319349712

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
 This problem is unconstrained.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
ARIMA (1, 0, 0) x (1, 0, 1, 12)12 : AIC Calculated =1794.3577607534335
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.50537D+00    |proj g|=  7.61595D-01

At iterate    5    f=  6.95740D+00    |proj g|=  2.68434D+00

At iterate   10    f=  6.88695D+00    |proj g|=  1.61014D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *
```

```
   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     10     13      1     0     0   1.610D-04   6.887D+00
  F =   6.8869514498391071

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (1, 0, 0) x (1, 1, 0, 12)12 : AIC Calculated =1548.67712476396
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.17183D+01    |proj g|=  2.09712D+00

At iterate    5    f=  8.96803D+00    |proj g|=  1.37522D+00

At iterate   10    f=  8.74680D+00    |proj g|=  1.18830D-01

At iterate   15    f=  8.50817D+00    |proj g|=  4.61691D-01

At iterate   20    f=  8.48544D+00    |proj g|=  5.11422D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     23     36      1     0     0   8.902D-04   8.485D+00
  F =   8.4854391836812884

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (1, 0, 0) x (1, 1, 1, 12)12 : AIC Calculated =1908.7383771446084
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.44626D+00    |proj g|=  5.22553D-01

At iterate    5    f=  8.17999D+00    |proj g|=  1.98242D+01

At iterate   10    f=  8.13873D+00    |proj g|=  1.55333D-03
```

```
           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     10     32      1     0     0   1.553D-03   8.139D+00
  F =   8.1387307847631938

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 1) x (0, 0, 0, 12)12 : AIC Calculated =1829.0756957869555
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.06097D+01    |proj g|=  1.36470D+01

At iterate    5    f=  1.02371D+01    |proj g|=  1.04083D-01

At iterate   10    f=  1.02346D+01    |proj g|=  5.10845D-01

At iterate   15    f=  1.02115D+01    |proj g|=  4.55980D+00

At iterate   20    f=  8.15988D+00    |proj g|=  1.13396D+01

At iterate   25    f=  7.53006D+00    |proj g|=  4.28384D+00

At iterate   30    f=  7.51551D+00    |proj g|=  3.88168D-02
 This problem is unconstrained.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
 This problem is unconstrained.
           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
```

```
    4     33     66      1     0     0   2.838D-04   7.516D+00
  F =   7.5155125672489742

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 1) x (0, 0, 1, 12)12 : AIC Calculated =1691.4748150637702
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.67344D+00    |proj g|=  5.29185D+00

At iterate    5    f=  8.28126D+00    |proj g|=  5.90165D-02

At iterate   10    f=  8.28058D+00    |proj g|=  3.18439D-04

At iterate   15    f=  8.28052D+00    |proj g|=  1.35815D-02

At iterate   20    f=  8.27333D+00    |proj g|=  1.39237D-01

At iterate   25    f=  7.85024D+00    |proj g|=  1.17033D+00

At iterate   30    f=  7.83566D+00    |proj g|=  6.55488D-03

At iterate   35    f=  7.83487D+00    |proj g|=  1.23874D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     36     49      1     0     0   5.586D-06   7.835D+00
  F =   7.8348668566308692

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (1, 0, 1) x (0, 1, 0, 12)12 : AIC Calculated =1761.0101758853148
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.07693D+01    |proj g|=  5.61549D+00
```

```
At iterate    5    f=  9.63417D+00    |proj g|=  4.82152D+00

At iterate   10    f=  8.90126D+00    |proj g|=  1.31440D-03

At iterate   15    f=  8.90117D+00    |proj g|=  9.42763D-03

At iterate   20    f=  8.90084D+00    |proj g|=  7.42742D-03

At iterate   25    f=  8.89571D+00    |proj g|=  1.30609D-01

At iterate   30    f=  8.21404D+00    |proj g|=  2.39772D+00

At iterate   35    f=  6.72728D+00    |proj g|=  1.16637D+00

At iterate   40    f=  6.65651D+00    |proj g|=  2.27606D-01

At iterate   45    f=  6.61498D+00    |proj g|=  3.42317D-01

At iterate   50    f=  6.59987D+00    |proj g|=  2.11604D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     50     76      1     0     0   2.116D-02   6.600D+00
  F =   6.5998686170398893

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT
ARIMA (1, 0, 1) x (0, 1, 1, 12)12 : AIC Calculated =1486.3705702169352
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.70660D+01    |proj g|=  1.83348D+01

At iterate    5    f=  8.42673D+00    |proj g|=  1.88235D+00

At iterate   10    f=  8.09079D+00    |proj g|=  3.06036D-01

At iterate   15    f=  7.80483D+00    |proj g|=  1.70281D+00

At iterate   20    f=  7.32922D+00    |proj g|=  2.95008D+00

           * * *
```

```
Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     24     49      1     0     0   4.109D-04   7.328D+00
  F =   7.3281855783348631

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 1) x (1, 0, 0, 12)12 : AIC Calculated =1649.5135695470094
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.98707D+01    |proj g|=  1.76926D+02
 This problem is unconstrained.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
 This problem is unconstrained.
At iterate    5    f=  9.42685D+00    |proj g|=  5.59951D-01

At iterate   10    f=  9.37976D+00    |proj g|=  1.12392D+01

At iterate   15    f=  9.13447D+00    |proj g|=  1.72146D+00

At iterate   20    f=  9.13170D+00    |proj g|=  1.39800D-01

At iterate   25    f=  9.13031D+00    |proj g|=  2.98453D-01

At iterate   30    f=  8.59369D+00    |proj g|=  3.74324D+01

At iterate   35    f=  7.52125D+00    |proj g|=  4.62646D-01

At iterate   40    f=  7.51469D+00    |proj g|=  4.01601D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *
```

```
   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     42     57      1     0     0   2.906D-04   7.515D+00
  F =   7.5146931085944377

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 1) x (1, 0, 1, 12)12 : AIC Calculated =1693.291256325154
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.68780D+00    |proj g|=  3.34378D+00

At iterate    5    f=  6.88400D+00    |proj g|=  1.89116D+00

At iterate   10    f=  6.72261D+00    |proj g|=  3.44500D-02

At iterate   15    f=  6.72250D+00    |proj g|=  1.94001D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     15     28      1     0     0   1.940D-04   6.723D+00
  F =   6.7225030333424325

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (1, 0, 1) x (1, 1, 0, 12)12 : AIC Calculated =1513.8406794687048
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.59702D+01    |proj g|=  1.21061D+01

At iterate    5    f=  8.51083D+00    |proj g|=  6.87377D-01

At iterate   10    f=  8.24812D+00    |proj g|=  2.66577D-01

At iterate   15    f=  8.15405D+00    |proj g|=  2.37128D+00

At iterate   20    f=  8.04568D+00    |proj g|=  8.99025D-01
```

```
 This problem is unconstrained.
At iterate   25    f=  7.70200D+00    |proj g|=  3.98319D+00

At iterate   30    f=  7.18486D+00    |proj g|=  4.79481D-01

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     34     60      1     0     0   2.603D-04   7.183D+00
  F =   7.1825336560408939

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 1) x (1, 1, 1, 12)12 : AIC Calculated =1618.8875389531602
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.74010D+00    |proj g|=  4.81633D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     11      1     0     0   4.809D-05   7.740D+00
  F =   7.7401030066323058

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 0) x (0, 0, 0, 12)12 : AIC Calculated =1737.7830734856366
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds
```

```
At iterate    0    f=  8.65823D+00    |proj g|=  3.47909D+00

At iterate    5    f=  6.93000D+00    |proj g|=  7.34612D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      8      9      1     0     0   8.126D-05   6.930D+00
  F =   6.9297456758946385

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 0) x (0, 0, 1, 12)12 : AIC Calculated =1558.263031400399
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.25355D+00    |proj g|=  2.14023D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     12      1     0     0   2.136D-05   7.254D+00
  F =   7.2535506617520946

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 0) x (0, 1, 0, 12)12 : AIC Calculated =1628.7953482324692
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds
```

```
At iterate    0    f=  6.40854D+00    |proj g|=  8.71617D-01
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
 This problem is unconstrained.
At iterate    5    f=  6.36361D+00    |proj g|=  7.52417D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      7      9      1     0     0   2.434D-05   6.364D+00
  F =   6.3636080714102174

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 0) x (0, 1, 1, 12)12 : AIC Calculated =1431.4482079958887
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.13930D+00    |proj g|=  5.31194D-01

At iterate    5    f=  6.90645D+00    |proj g|=  1.51909D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      8     11      1     0     0   5.290D-05   6.906D+00
  F =   6.9064164159376844

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
```

```
ARIMA (1, 1, 0) x (1, 0, 0, 12)12 : AIC Calculated =1553.0372771700413
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.44756D+00    |proj g|=  3.07788D+00

At iterate    5    f=  6.93494D+00    |proj g|=  2.08716D-01

At iterate   10    f=  6.91734D+00    |proj g|=  3.93597D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     14     15      1     0     0   1.359D-05   6.917D+00
  F =   6.9173395471568853

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 0) x (1, 0, 1, 12)12 : AIC Calculated =1557.4840585631423
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  6.23660D+00    |proj g|=  9.13708D-02

At iterate    5    f=  6.22865D+00    |proj g|=  2.83710D-04

At iterate   10    f=  6.22860D+00    |proj g|=  9.21543D-03

At iterate   15    f=  6.22354D+00    |proj g|=  1.04472D-01

At iterate   20    f=  6.19656D+00    |proj g|=  6.69836D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
```

```
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     24     27      1     0     0   1.341D-06   6.196D+00
  F =   6.1963566214915939

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (1, 1, 0) x (1, 1, 0, 12)12 : AIC Calculated =1393.983883214117
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.02573D+01    |proj g|=  1.21888D+01

At iterate    5    f=  6.22951D+00    |proj g|=  2.98316D-02

At iterate   10    f=  6.22865D+00    |proj g|=  2.83717D-04

At iterate   15    f=  6.22863D+00    |proj g|=  4.47163D-03

At iterate   20    f=  6.22620D+00    |proj g|=  7.78112D-02

At iterate   25    f=  6.19660D+00    |proj g|=  7.33203D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     29     33      1     0     0   2.057D-06   6.196D+00
  F =   6.1963538220712673

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
 This problem is unconstrained.
ARIMA (1, 1, 0) x (1, 1, 1, 12)12 : AIC Calculated =1395.983256143964
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10
```

```
At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.99757D+00    |proj g|=  2.70670D+00

At iterate    5    f=  7.57632D+00    |proj g|=  1.45223D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      6      8      1     0     0   1.213D-04   7.576D+00
  F =   7.5763184184592678

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 1) x (0, 0, 0, 12)12 : AIC Calculated =1703.095325734876
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.63919D+00    |proj g|=  2.33403D+00

At iterate    5    f=  6.68794D+00    |proj g|=  1.50513D-01

At iterate   10    f=  6.67374D+00    |proj g|=  1.65238D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     12     14      1     0     0   3.947D-05   6.674D+00
  F =   6.6737374252389063

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 1) x (0, 0, 1, 12)12 : AIC Calculated =1502.917183253515
RUNNING THE L-BFGS-B CODE
```

```
           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.00262D+01    |proj g|=  1.31509D+01

At iterate    5    f=  7.12581D+00    |proj g|=  7.23065D-02

At iterate   10    f=  7.12543D+00    |proj g|=  1.98996D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     10     12      1     0     0   1.990D-04   7.125D+00
  F =   7.1254271067611716

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (1, 1, 1) x (0, 1, 0, 12)12 : AIC Calculated =1602.0956719145024
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  6.23928D+00    |proj g|=  1.09529D+00

At iterate    5    f=  6.13478D+00    |proj g|=  1.55026D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
```

```
    4      8     10      1     0     0   1.129D-04   6.135D+00
  F =   6.1347789594643700

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (1, 1, 1) x (0, 1, 1, 12)12 : AIC Calculated =1382.1904869200189
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.67125D+00    |proj g|=  6.58777D+00

At iterate    5    f=  6.79159D+00    |proj g|=  3.58537D-01

At iterate   10    f=  6.76488D+00    |proj g|=  1.78907D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     13     18      1     0     0   9.674D-05   6.765D+00
  F =   6.7648757656041454

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 1) x (1, 0, 0, 12)12 : AIC Calculated =1523.3321714953286
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.02227D+00    |proj g|=  8.49766D-01

At iterate    5    f=  6.66838D+00    |proj g|=  3.14125D-02

At iterate   10    f=  6.66795D+00    |proj g|=  5.09564D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
```

```
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     10     12      1     0     0   5.096D-05   6.668D+00
  F =   6.6679509920376647

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (1, 1, 1) x (1, 0, 1, 12)12 : AIC Calculated =1503.6210222164368
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.23267D+01    |proj g|=  1.74118D+01

At iterate    5    f=  6.19917D+00    |proj g|=  8.51591D-02

At iterate   10    f=  6.19601D+00    |proj g|=  6.98079D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     10     12      1     0     0   6.981D-05   6.196D+00
  F =   6.1960063373415704

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (1, 1, 1) x (1, 1, 0, 12)12 : AIC Calculated =1395.9054195645117
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.21579D+01    |proj g|=  1.25489D+01
```

```
At iterate    5    f=  6.11963D+00    |proj g|=  5.07362D-01

At iterate   10    f=  6.09801D+00    |proj g|=  2.59513D-02

At iterate   15    f=  6.09797D+00    |proj g|=  1.48476D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     15     16      1     0     0   1.485D-04   6.098D+00
  F =   6.0979663159543973

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (1, 1, 1) x (1, 1, 1, 12)12 : AIC Calculated =1375.944454773785
pdq         (1, 1, 1)
pdqs    (1, 1, 1, 12)
aic       1375.944455
Name: 63, dtype: object
```

Out[254…

	pdq	pdqs	aic
0	(0, 0, 0)	(0, 0, 0, 12)	3001.767633
1	(0, 0, 0)	(0, 0, 1, 12)	2625.442360
2	(0, 0, 0)	(0, 1, 0, 12)	2245.931314
3	(0, 0, 0)	(0, 1, 1, 12)	1986.330735
4	(0, 0, 0)	(1, 0, 0, 12)	2193.616151
...	...	...	...
59	(1, 1, 1)	(0, 1, 1, 12)	1382.190487
60	(1, 1, 1)	(1, 0, 0, 12)	1523.332171
61	(1, 1, 1)	(1, 0, 1, 12)	1503.621022
62	(1, 1, 1)	(1, 1, 0, 12)	1395.905420
63	(1, 1, 1)	(1, 1, 1, 12)	1375.944455

64 rows × 3 columns

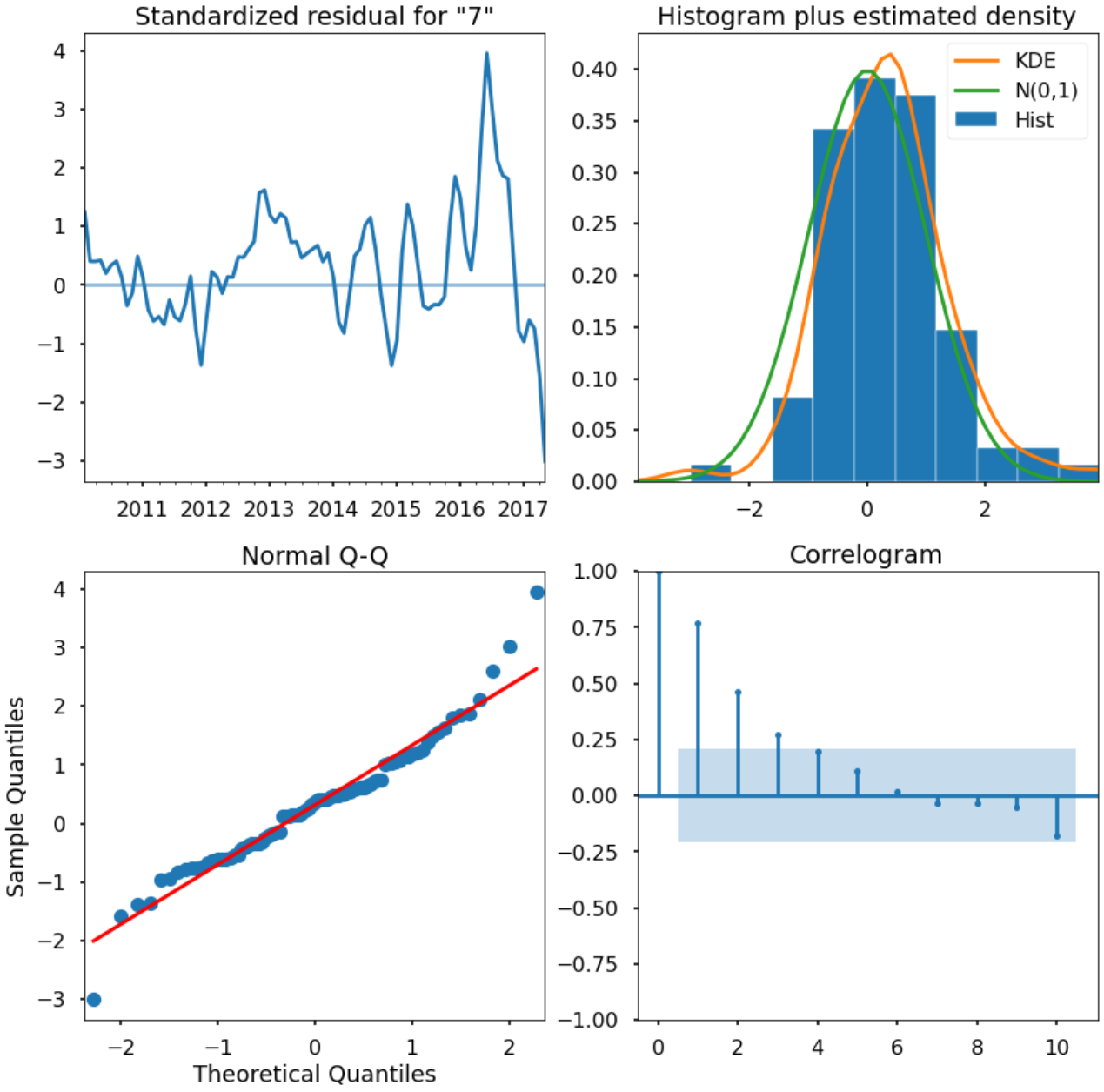
In [255…

```
pdq = (1, 1, 1)
pdqs = (1, 1, 1, 12)

train,test,results = model_fit(ts_78758,pdq=pdq,pdqs=pdqs)
```

```
RUNNING THE L-BFGS-B CODE
```

```
           * * *

Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  3.63264D+01    |proj g|=  1.01094D+01

At iterate    5    f=  8.88368D+00    |proj g|=  2.59045D-01

At iterate   10    f=  8.86981D+00    |proj g|=  3.43447D-01

At iterate   15    f=  8.67003D+00    |proj g|=  9.60970D-01

At iterate   20    f=  8.63663D+00    |proj g|=  4.70368D-02

At iterate   25    f=  8.63614D+00    |proj g|=  8.33944D-02
 This problem is unconstrained.
At iterate   30    f=  8.62152D+00    |proj g|=  3.88815D-01

At iterate   35    f=  8.61477D+00    |proj g|=  4.42994D-03

At iterate   40    f=  8.61346D+00    |proj g|=  5.00433D-02

At iterate   45    f=  8.41707D+00    |proj g|=  1.21657D+00

At iterate   50    f=  7.63294D+00    |proj g|=  1.68200D-01

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     50     58      1     0     0   1.682D-01   7.633D+00
  F =   7.6329389368977569

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT
                                      SARIMAX Results
==========================================================================
==========
Dep. Variable:                                78758   No. Observations:
101
Model:             SARIMAX(1, 1, 1)x(1, 1, 1, 12)   Log Likelihood
-770.927
Date:                            Wed, 19 Oct 2022   AIC
1551.854
Time:                                    13:08:48   BIC
1564.240
Sample:                                01-01-2009   HQIC
1556.844
```

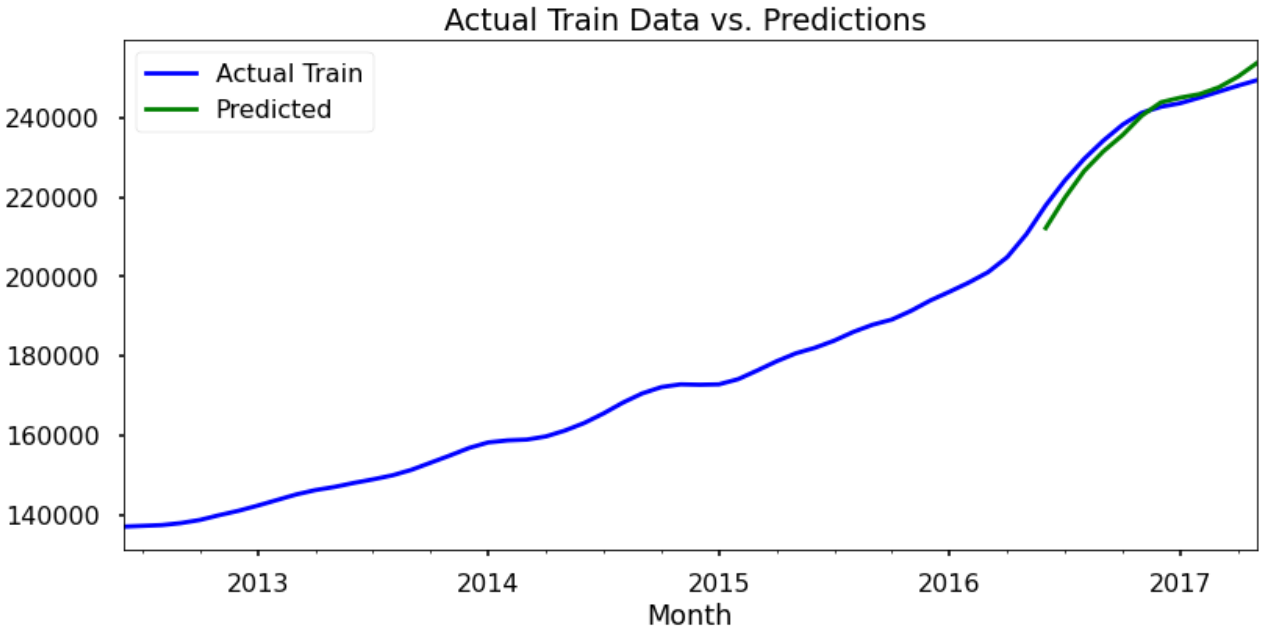
```
                                      - 05-01-2017
Covariance Type:                               opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ar.L1          0.2491      1.443      0.173      0.863      -2.579       3.077
ma.L1         -0.2277      1.446     -0.157      0.875      -3.063       2.607
ar.S.L12      -0.9962      2.979     -0.334      0.738      -6.835       4.843
ma.S.L12       0.9954      3.038      0.328      0.743      -4.960       6.951
sigma2       2.08e+06   3.24e-05   6.41e+10      0.000    2.08e+06    2.08e+06
==============================================================================
===
Ljung-Box (L1) (Q):                  53.54   Jarque-Bera (JB):                 1
6.47
Prob(Q):                              0.00   Prob(JB):
0.00
Heteroskedasticity (H):               8.90   Skew:
0.40
Prob(H) (two-sided):                  0.00   Kurtosis:
4.97
==============================================================================
===

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-s
tep).
[2] Covariance matrix is singular or near-singular, with condition number 6.91e+
25. Standard errors may be unstable.
```



In [256…

```
train_model(train,results)
```

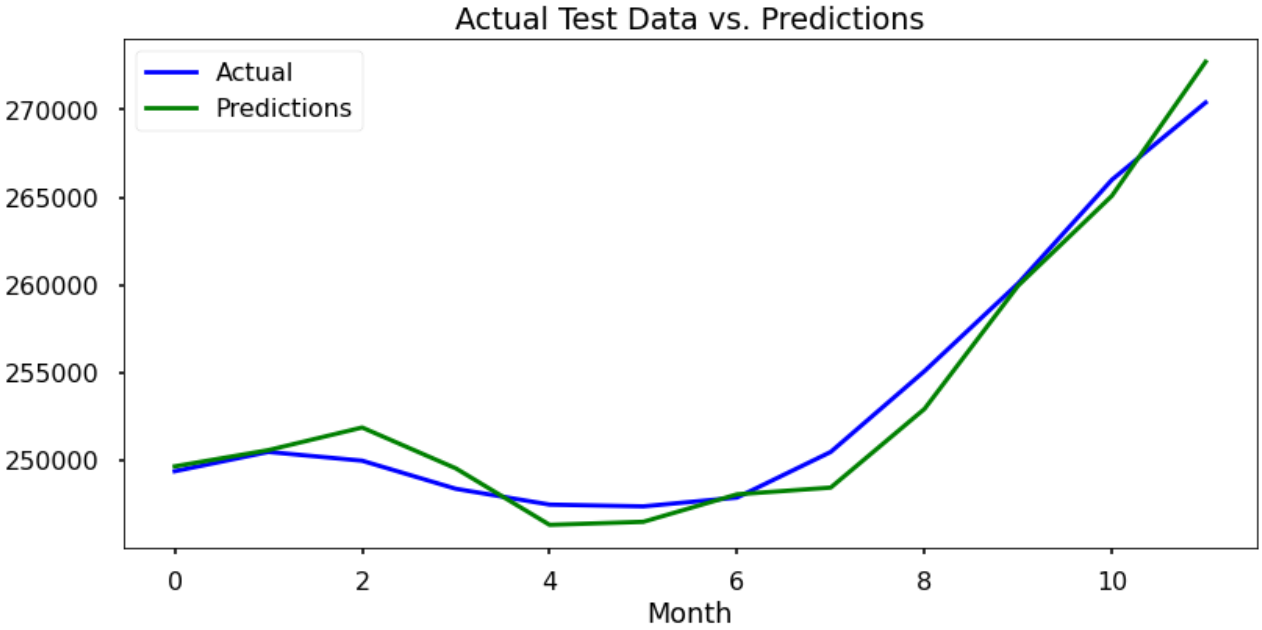
```
SARIMA model RMSE on train data: 2954.67443
```



In [257…

```
test_model(ts_78758,pdq=pdq,pdqs=pdqs)
```

```
SARIMA model RMSE on train data: 1358.95471
```

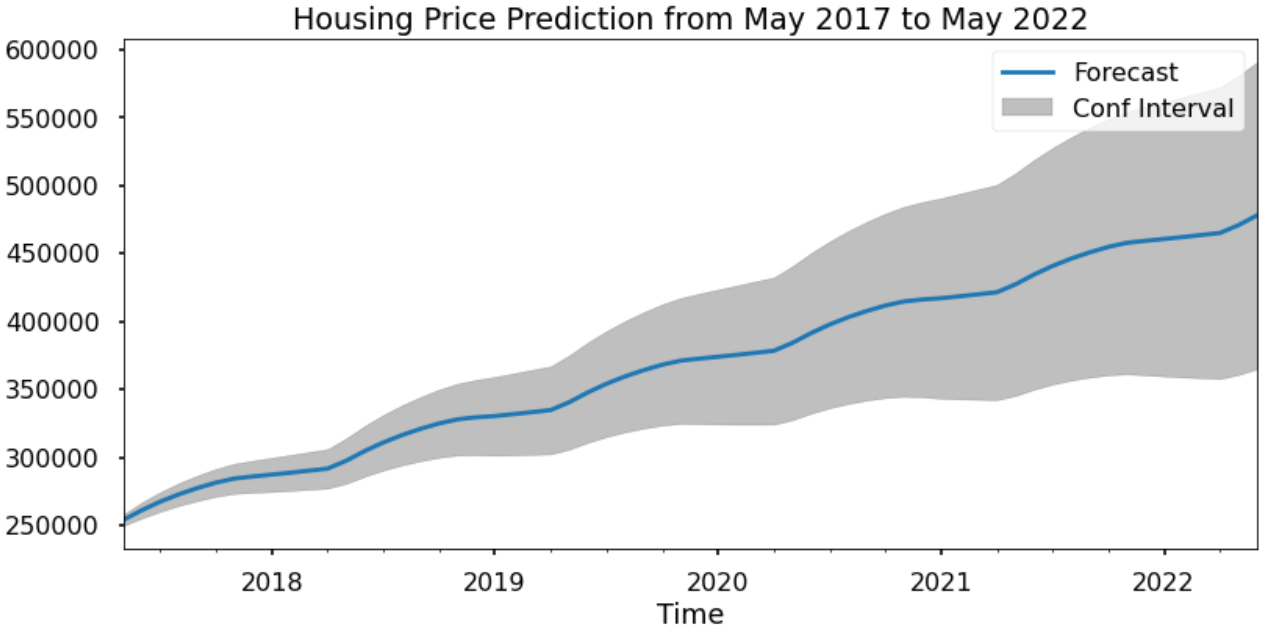


In [258…

```
pdq = (1, 1, 1)
pdqs = (1, 1, 1, 12)
pred_78758=forecast_model(ts_78758,pdq=pdq,pdqs=pdqs,zc=78758)
```

```
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds
```

```
At iterate    0    f=  3.78633D+01    |proj g|=  1.18473D+01

At iterate    5    f=  9.57297D+00    |proj g|=  2.60268D-01

At iterate   10    f=  9.47534D+00    |proj g|=  2.57474D+00

At iterate   15    f=  9.25442D+00    |proj g|=  2.17343D-02

At iterate   20    f=  9.25428D+00    |proj g|=  5.05260D-02
 This problem is unconstrained.
At iterate   25    f=  9.25383D+00    |proj g|=  4.09520D-03

At iterate   30    f=  9.25371D+00    |proj g|=  6.29053D-02

At iterate   35    f=  9.23905D+00    |proj g|=  6.63290D-01

At iterate   40    f=  8.10410D+00    |proj g|=  3.28532D-02

At iterate   45    f=  8.03288D+00    |proj g|=  1.25452D-03

At iterate   50    f=  8.03130D+00    |proj g|=  1.22708D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     50     58      1     0     0   1.227D-04   8.031D+00
  F =   8.0312965237224692

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT
```



```
Housing Price Projection for 2017, 2018,2020 and 2022:
Housing price at enter point 2017-05:$ 253812.17
Housing price in year 1:$ 297436.54 with 17.19 % ROI
Housing price in year 3:$ 384152.59 with 51.35 % ROI
Housing price in year 5:$ 470848.71 with  85.51 % ROI
```

In [259…

```
# Add the 5 yr roi to the results summary df
df_zip_5YR_ROI.loc[df_zip_5YR_ROI['Zipcode'] == 78758, '5YR_PRED_ROI'] = 84.89
df_zip_5YR_ROI
```

Out[259…

	Zipcode	5_YR_PRED_ROI	5YR_PRED_ROI
0	78702	0	34.83
1	78758	0	84.89
2	78721	0	NaN
3	78723	0	NaN
4	78741	0	NaN

Zipcode 78721

AIC_PDQS

In [260…

```
AIC_PDQS(ts_78721)
```

```
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            1     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.34625D+01    |proj g|=  2.18492D-08

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    1      0      1      0     0     0   2.185D-08   1.346D+01
  F =   13.462543713814764

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 0) x (0, 0, 0, 12)12 : AIC Calculated =3017.609791894507
```

```
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.72273D+02    |proj g|=  1.67648D+00

At iterate    5    f=  1.18048D+01    |proj g|=  1.66715D-01

At iterate   10    f=  1.17816D+01    |proj g|=  2.94094D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2     10     20      1     0     0   2.941D-06   1.178D+01
  F =   11.781614787305704

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 0) x (0, 0, 1, 12)12 : AIC Calculated =2643.081712356478
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            1     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.01713D+01    |proj g|=  5.14344D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    1      0      1      0     0     0   5.143D-06   1.017D+01
  F =   10.171298610784964
```

```
CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (0, 0, 0) x (0, 1, 0, 12)12 : AIC Calculated =2280.370888815832
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  3.59444D+02    |proj g|=  1.47029D+01

At iterate    5    f=  3.41273D+01    |proj g|=  4.46983D-01

At iterate   10    f=  1.60459D+01    |proj g|=  2.12061D-02

At iterate   15    f=  1.42133D+01    |proj g|=  7.20299D-04

At iterate   20    f=  1.41652D+01    |proj g|=  1.04624D-04
  ys=-3.221E+02  -gs= 7.255E-01 BFGS update SKIPPED

At iterate   25    f=  8.99185D+00    |proj g|=  9.30844D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2     29     56      1     1     0   2.981D-07   8.984D+00
  F =   8.9841243031843572

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
 This problem is unconstrained.
ARIMA (0, 0, 0) x (0, 1, 1, 12)12 : AIC Calculated =2016.4438439132962
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.73043D+00    |proj g|=  8.13389D-04

           * * *
```

```
Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     14      1     0     0   8.130D-04   9.730D+00
  F =   9.7304289419152621

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 0) x (1, 0, 0, 12)12 : AIC Calculated =2183.6160829890187
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.66637D+02    |proj g|=  2.53585D+02

At iterate    5    f=  5.59047D+02    |proj g|=  4.18189D+02

At iterate   10    f=  1.17531D+01    |proj g|=  2.16813D+01

At iterate   15    f=  9.75659D+00    |proj g|=  5.48708D+00

At iterate   20    f=  9.59174D+00    |proj g|=  3.64773D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     23     43      1     0     0   5.457D-04   9.592D+00
  F =   9.5917059993775631

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 0) x (1, 0, 1, 12)12 : AIC Calculated =2154.542143860574
 This problem is unconstrained.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
 This problem is unconstrained.
```

```
 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.62656D+00    |proj g|=  1.04341D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     12      1     0     0   1.038D-05   8.627D+00
  F =   8.6265554031380169

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 0) x (1, 1, 0, 12)12 : AIC Calculated =1936.348410302916
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  3.26902D+02    |proj g|=  3.10448D+01

At iterate    5    f=  1.15881D+01    |proj g|=  3.93222D-01

 This problem is unconstrained.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
 This problem is unconstrained.
At iterate   10    f=  1.15623D+01    |proj g|=  9.16311D-02

At iterate   15    f=  1.15460D+01    |proj g|=  1.06690D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
```

```
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     19     20      1     0     0   4.355D-05   1.155D+01
  F =   11.545918865739106

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 0) x (1, 1, 1, 12)12 : AIC Calculated =2592.2858259255595
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.66323D+01    |proj g|=  3.67928D+00

At iterate    5    f=  1.29899D+01    |proj g|=  2.94857D-03
ARIMA (0, 0, 1) x (0, 0, 0, 12)12 : AIC Calculated =2913.7420214047875
           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      8     19      1     0     0   3.702D-06   1.299D+01
  F =   12.989919738414230

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL

RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.73178D+02    |proj g|=  1.62513D+00
  ys=-5.817E+00  -gs= 7.768E-01 BFGS update SKIPPED

At iterate    5    f=  1.15113D+01    |proj g|=  6.17399D-02

At iterate   10    f=  1.14981D+01    |proj g|=  5.99016D-04
```

```
           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     12     46      1     1     0   7.030D-06   1.150D+01
  F =   11.498071000813249

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (0, 0, 1) x (0, 0, 1, 12)12 : AIC Calculated =2581.5679041821677
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.04371D+01    |proj g|=  3.42388D-01

At iterate    5    f=  1.01469D+01    |proj g|=  3.28445D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      5     10      1     0     0   3.284D-05   1.015D+01
  F =   10.146876356118140

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 1) x (0, 1, 0, 12)12 : AIC Calculated =2276.9003037704633
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds
```

```
At iterate    0    f=  4.14248D+02    |proj g|=  1.79668D+01
  ys=-2.449E+00  -gs= 6.951E+01 BFGS update SKIPPED

At iterate    5    f=  4.59890D+01    |proj g|=  1.00324D+01

At iterate   10    f=  4.14251D+01    |proj g|=  2.12221D+02
 This problem is unconstrained.
 This problem is unconstrained.
At iterate   15    f=  1.74203D+01    |proj g|=  7.32652D+00

At iterate   20    f=  1.34750D+01    |proj g|=  1.34351D+00

At iterate   25    f=  1.31300D+01    |proj g|=  2.87481D-01

At iterate   30    f=  1.31273D+01    |proj g|=  9.48570D-03

At iterate   35    f=  1.31273D+01    |proj g|=  1.80476D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     35     52      2     1     0   1.805D-03   1.313D+01
  F =   13.127339933962134

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 1) x (0, 1, 1, 12)12 : AIC Calculated =2946.524145207518
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.32053D+01    |proj g|=  1.63106D-01

At iterate    5    f=  1.16889D+01    |proj g|=  4.60841D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value
```

```
           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      5      8      1     0     0   4.608D-06   1.169D+01
  F =   11.688928620454410

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 1) x (1, 0, 0, 12)12 : AIC Calculated =2624.320010981788
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.14745D+02    |proj g|=  2.50699D+02

At iterate    5    f=  1.21880D+01    |proj g|=  7.57822D-01

At iterate   10    f=  1.21114D+01    |proj g|=  4.11373D-02

At iterate   15    f=  1.20975D+01    |proj g|=  1.52625D-02

At iterate   20    f=  1.20934D+01    |proj g|=  2.77292D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
 This problem is unconstrained.
 This problem is unconstrained.
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     22     45      1     0     0   6.686D-05   1.209D+01
  F =   12.093432667508887

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 1) x (1, 0, 1, 12)12 : AIC Calculated =2716.9289175219906
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.35258D+00    |proj g|=  3.16990D-01

At iterate    5    f=  8.72892D+00    |proj g|=  1.00102D-03
```

```
 This problem is unconstrained.
           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      8     12      1     0     0   2.615D-05   8.729D+00
  F =   8.7289193709240021

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 1) x (1, 1, 0, 12)12 : AIC Calculated =1961.2779390869764
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  4.93863D+02    |proj g|=  6.98831D+01

At iterate    5    f=  6.26780D+01    |proj g|=  1.09054D+01

At iterate   10    f=  1.78952D+01    |proj g|=  3.80631D-01

At iterate   15    f=  1.46638D+01    |proj g|=  6.61826D-02

At iterate   20    f=  1.45220D+01    |proj g|=  1.69924D-03

At iterate   25    f=  1.45218D+01    |proj g|=  1.61727D-04

At iterate   30    f=  1.45218D+01    |proj g|=  3.40970D-03
 This problem is unconstrained.
At iterate   35    f=  1.43554D+01    |proj g|=  7.77314D-02
  ys=-1.735E-01  -gs= 5.751E-01 BFGS update SKIPPED

At iterate   40    f=  1.26068D+01    |proj g|=  2.65201D-02

At iterate   45    f=  1.24279D+01    |proj g|=  2.72558D-02

At iterate   50    f=  1.16909D+01    |proj g|=  7.16002D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value
```

```
           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     50     90      1     1     0   7.160D-02   1.169D+01
  F =   11.690924327342387

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT
ARIMA (0, 0, 1) x (1, 1, 1, 12)12 : AIC Calculated =2626.7670493246947
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            1     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.96405D+00    |proj g|=  8.06040D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    1      0      1      0     0     0   8.060D-06   8.964D+00
  F =   8.9640547826751433

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 0) x (0, 0, 0, 12)12 : AIC Calculated =2009.948271319232
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.94953D+00    |proj g|=  1.36079D-01

At iterate    5    f=  7.93947D+00    |proj g|=  4.20296D-04

At iterate   10    f=  7.93921D+00    |proj g|=  1.67276D-02

At iterate   15    f=  7.91723D+00    |proj g|=  1.58872D-01

At iterate   20    f=  7.87344D+00    |proj g|=  9.42821D-05

           * * *

Tit   = total number of iterations
```

```
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2     21     25      1     0     0   2.311D-07   7.873D+00
  F =   7.8734362693267466

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 0) x (0, 0, 1, 12)12 : AIC Calculated =1767.6497243291913
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            1     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.86702D+00    |proj g|=  6.89546D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    1      2      5      1     0     0   7.366D-06   7.863D+00
  F =   7.8630134920584380

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 0) x (0, 1, 0, 12)12 : AIC Calculated =1763.3150222210902
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  6.96324D+00    |proj g|=  2.27161D-01

At iterate    5    f=  6.91816D+00    |proj g|=  1.00059D-04

           * * *

Tit   = total number of iterations
```

```
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      5      8      1     0     0   1.001D-04   6.918D+00
  F =   6.9181587317147271

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (0, 1, 0) x (0, 1, 1, 12)12 : AIC Calculated =1553.667555904099
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.88651D+00    |proj g|=  6.08358D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      0      1      0     0     0   6.084D-06   7.887D+00
  F =   7.8865077764516354

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 0) x (1, 0, 0, 12)12 : AIC Calculated =1770.5777419251663
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.82243D+00    |proj g|=  1.44799D-01

At iterate    5    f=  7.81949D+00    |proj g|=  8.98089D-05
```

```
           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      6      8      1     0     0   8.981D-05   7.819D+00
  F =   7.8194858461345911

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 1, 0) x (1, 0, 1, 12)12 : AIC Calculated =1757.5648295341484
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  6.95830D+00    |proj g|=  3.98090D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      0      1      0     0     0   3.981D-06   6.958D+00
  F =   6.9583006598104689

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (0, 1, 0) x (1, 1, 0, 12)12 : AIC Calculated =1562.659347797545
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds
```

```
At iterate    0    f=  7.01478D+00    |proj g|=  8.15588D-01

At iterate    5    f=  6.90578D+00    |proj g|=  5.88502D-02

At iterate   10    f=  6.88710D+00    |proj g|=  1.26228D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     14     25      1     0     0   7.195D-05   6.887D+00
  F =   6.8868656576039582

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (0, 1, 0) x (1, 1, 1, 12)12 : AIC Calculated =1548.6579073032867
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.71235D+00    |proj g|=  5.71132D-01

At iterate    5    f=  8.45642D+00    |proj g|=  2.08500D-03

At iterate   10    f=  8.45630D+00    |proj g|=  2.34781D-02

At iterate   15    f=  8.43399D+00    |proj g|=  3.80131D-01

At iterate   20    f=  8.32050D+00    |proj g|=  5.20698D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2     24     31      1     0     0   2.448D-07   8.320D+00
  F =   8.3201229124703335
```

```
CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 1) x (0, 0, 0, 12)12 : AIC Calculated =1867.7075323933545
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.75036D+00    |proj g|=  5.01829D-01

At iterate    5    f=  7.50552D+00    |proj g|=  2.81276D-03

At iterate   10    f=  7.50550D+00    |proj g|=  1.40123D-03

At iterate   15    f=  7.50474D+00    |proj g|=  2.12676D-02

At iterate   20    f=  7.45104D+00    |proj g|=  2.96987D-01

At iterate   25    f=  7.35098D+00    |proj g|=  1.51710D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     29     35      1     0     0   5.413D-06   7.351D+00
  F =   7.3505451779061959

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (0, 1, 1) x (0, 0, 1, 12)12 : AIC Calculated =1652.5221198509878
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.06976D+01    |proj g|=  1.46838D+01

At iterate    5    f=  7.87721D+00    |proj g|=  2.41104D-03

At iterate   10    f=  7.87720D+00    |proj g|=  3.69578D-03

At iterate   15    f=  7.87627D+00    |proj g|=  5.31304D-02
```

```
At iterate   20    f=  7.78270D+00    |proj g|=  8.35717D-01

At iterate   25    f=  7.60972D+00    |proj g|=  2.69641D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2     29     32      1     0     0   1.990D-06   7.608D+00
  F =   7.6079154083166838

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 1) x (0, 1, 0, 12)12 : AIC Calculated =1708.173051462937
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  6.56933D+00    |proj g|=  2.05551D-01
 This problem is unconstrained.
 This problem is unconstrained.
At iterate    5    f=  6.50647D+00    |proj g|=  1.66904D-04

At iterate   10    f=  6.50647D+00    |proj g|=  1.80566D-03

At iterate   15    f=  6.50633D+00    |proj g|=  2.31002D-02

At iterate   20    f=  6.49646D+00    |proj g|=  1.58060D-01

At iterate   25    f=  6.48114D+00    |proj g|=  8.00894D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     27     32      1     0     0   1.637D-05   6.481D+00
  F =   6.4811350906832974
```

```
CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 1, 1) x (0, 1, 1, 12)12 : AIC Calculated =1457.7742603130587
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.88010D+00    |proj g|=  2.63744D+00

At iterate    5    f=  7.85516D+00    |proj g|=  9.52364D-05

At iterate   10    f=  7.85516D+00    |proj g|=  1.05034D-03

At iterate   15    f=  7.85507D+00    |proj g|=  1.14647D-02

At iterate   20    f=  7.72259D+00    |proj g|=  4.64839D-01

At iterate   25    f=  7.62421D+00    |proj g|=  1.08243D-01

At iterate   30    f=  7.58574D+00    |proj g|=  2.65414D-01

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     34     56      1     0     0   1.620D-06   7.584D+00
  F =   7.5844605328842905

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 1) x (1, 0, 0, 12)12 : AIC Calculated =1704.919159366081
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.84488D+00    |proj g|=  3.14265D-01
 This problem is unconstrained.
 This problem is unconstrained.
At iterate    5    f=  7.53907D+00    |proj g|=  2.19623D-01

At iterate   10    f=  7.50516D+00    |proj g|=  4.54877D-04

At iterate   15    f=  7.50512D+00    |proj g|=  7.68367D-03
```

```
At iterate   20    f=  7.50062D+00    |proj g|=  8.03310D-02

At iterate   25    f=  7.38408D+00    |proj g|=  2.55458D-01

At iterate   30    f=  7.34837D+00    |proj g|=  2.09629D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     33     41      1     0     0   2.244D-05   7.348D+00
  F =   7.3483734863845545

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 1, 1) x (1, 0, 1, 12)12 : AIC Calculated =1654.0356609501403
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.84852D+00    |proj g|=  1.04042D+01

At iterate    5    f=  6.89465D+00    |proj g|=  5.35752D-01

At iterate   10    f=  6.68493D+00    |proj g|=  4.58359D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     13     36      1     0     0   9.456D-05   6.685D+00
  F =   6.6849245140749085

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.

 Warning:  more than 10 function and gradient
```

```
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
ARIMA (0, 1, 1) x (1, 1, 0, 12)12 : AIC Calculated =1503.4230911527795
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.07996D+00    |proj g|=  1.04008D+01

At iterate    5    f=  6.74395D+00    |proj g|=  3.17084D-01
  ys=-3.229E-02  -gs= 1.274E-01 BFGS update SKIPPED

At iterate   10    f=  6.48715D+00    |proj g|=  4.81913D-01

At iterate   15    f=  6.48085D+00    |proj g|=  1.80133D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     16     25      1     1     0   1.801D-04   6.481D+00
  F =   6.4808534791967647

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (0, 1, 1) x (1, 1, 1, 12)12 : AIC Calculated =1459.7111793400752
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.67585D+00    |proj g|=  7.63655D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value
```

```
           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     20      1     0     0   7.637D-02   8.676D+00
  F =   8.6758495319687814

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 0) x (0, 0, 0, 12)12 : AIC Calculated =1947.390295161007
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.04200D+01    |proj g|=  1.41124D+01

At iterate    5    f=  1.03102D+01    |proj g|=  8.09847D+00
  ys=-5.369E-01  -gs= 3.787E-01 BFGS update SKIPPED

At iterate   10    f=  7.73773D+00    |proj g|=  5.54572D+00

At iterate   15    f=  7.72359D+00    |proj g|=  3.71232D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     16     35      1     1     0   1.728D-05   7.724D+00
  F =   7.7235945929131518

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 0) x (0, 0, 1, 12)12 : AIC Calculated =1736.085188812546
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.93828D+00    |proj g|=  6.77619D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
```

```
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     12      1     0     0   6.774D-04   7.938D+00
  F =   7.9382848289945178

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
 This problem is unconstrained.
 This problem is unconstrained.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
ARIMA (1, 0, 0) x (0, 1, 0, 12)12 : AIC Calculated =1782.175801694772
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.67005D+00    |proj g|=  1.83447D+00

At iterate    5    f=  9.61478D+00    |proj g|=  7.00599D-01

At iterate   10    f=  9.33441D+00    |proj g|=  7.18192D-01

At iterate   15    f=  9.31104D+00    |proj g|=  2.71849D-03

At iterate   20    f=  9.31094D+00    |proj g|=  4.20142D-02

At iterate   25    f=  9.29770D+00    |proj g|=  6.03234D-01

At iterate   30    f=  8.30276D+00    |proj g|=  2.05544D+00
 This problem is unconstrained.
At iterate   35    f=  7.02987D+00    |proj g|=  3.80849D-01

At iterate   40    f=  7.02443D+00    |proj g|=  2.61952D-02

At iterate   45    f=  7.02196D+00    |proj g|=  8.37755D-02

At iterate   50    f=  7.02002D+00    |proj g|=  5.23927D-04

           * * *

Tit   = total number of iterations
```

```
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     50     75      1     0     0   5.239D-04   7.020D+00
  F =   7.0200181171738683

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT
ARIMA (1, 0, 0) x (0, 1, 1, 12)12 : AIC Calculated =1578.4840582469465
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.07547D+00    |proj g|=  1.97661D+00
 This problem is unconstrained.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
 This problem is unconstrained.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
At iterate    5    f=  7.96062D+00    |proj g|=  3.39506D+00

At iterate   10    f=  7.72497D+00    |proj g|=  3.01466D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     11     38      1     0     0   3.015D-03   7.725D+00
  F =   7.7249723173854594

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 0) x (1, 0, 0, 12)12 : AIC Calculated =1736.393799094343
RUNNING THE L-BFGS-B CODE

           * * *
```

```
Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.81145D+01    |proj g|=  1.53942D+02

At iterate    5    f=  8.99506D+00    |proj g|=  1.48704D-01

At iterate   10    f=  8.99252D+00    |proj g|=  1.17442D+00

At iterate   15    f=  7.93563D+00    |proj g|=  4.63001D+01

At iterate   20    f=  7.72394D+00    |proj g|=  3.71284D-01

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     23     49      1     0     0   6.611D-03   7.724D+00
  F =   7.7239222433359016

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 0) x (1, 0, 1, 12)12 : AIC Calculated =1738.158582507242
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.60641D+00    |proj g|=  1.16903D+00

At iterate    5    f=  7.06523D+00    |proj g|=  3.24135D+00

At iterate   10    f=  6.95266D+00    |proj g|=  6.92849D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *
```

```
   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     11     14      1     0     0   2.435D-05   6.953D+00
  F =   6.9526564555557888

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (1, 0, 0) x (1, 1, 0, 12)12 : AIC Calculated =1563.3950460444967
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.29110D+01    |proj g|=  3.00597D+00

At iterate    5    f=  8.62981D+00    |proj g|=  5.50008D-01

At iterate   10    f=  8.53125D+00    |proj g|=  5.26017D-02

At iterate   15    f=  8.46044D+00    |proj g|=  7.77469D-01
 This problem is unconstrained.
At iterate   20    f=  8.10886D+00    |proj g|=  2.84915D+00

At iterate   25    f=  6.94989D+00    |proj g|=  3.04881D-02

At iterate   30    f=  6.94971D+00    |proj g|=  1.46733D-01

At iterate   35    f=  6.94917D+00    |proj g|=  1.00941D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     36     64      1     0     0   1.438D-04   6.949D+00
  F =   6.9491652886780226

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 0) x (1, 1, 1, 12)12 : AIC Calculated =1564.613024663877
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds
```

```
At iterate    0    f=  8.20899D+00    |proj g|=  4.94539D-01

At iterate    5    f=  8.14299D+00    |proj g|=  6.62218D-01

At iterate   10    f=  8.14278D+00    |proj g|=  3.25248D-03

At iterate   15    f=  8.14276D+00    |proj g|=  1.65981D-01

At iterate   20    f=  8.14180D+00    |proj g|=  4.71246D-01

At iterate   25    f=  8.10520D+00    |proj g|=  1.08131D-01

At iterate   30    f=  8.10213D+00    |proj g|=  2.80122D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     31     39      1     0     0   7.331D-05   8.102D+00
  F =   8.1021322939723603

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 1) x (0, 0, 0, 12)12 : AIC Calculated =1820.8776338498087
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.01819D+01    |proj g|=  1.62136D+01

At iterate    5    f=  9.99561D+00    |proj g|=  1.89273D-01

At iterate   10    f=  9.99033D+00    |proj g|=  3.35826D+00
  ys=-3.110E-01  -gs= 4.123E-01 BFGS update SKIPPED

At iterate   15    f=  8.74088D+00    |proj g|=  2.95325D+01

At iterate   20    f=  8.40430D+00    |proj g|=  1.95670D+00

At iterate   25    f=  8.13435D+00    |proj g|=  3.18551D+00

At iterate   30    f=  8.10316D+00    |proj g|=  2.02127D-01
 This problem is unconstrained.
 This problem is unconstrained.
At iterate   35    f=  8.03009D+00    |proj g|=  3.02809D-01
```

```
           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     38     56      1     1     0   2.611D-04   8.030D+00
  F =   8.0300749740221136

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 1) x (0, 0, 1, 12)12 : AIC Calculated =1806.7367941809534
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.48478D+01    |proj g|=  2.25139D+01

At iterate    5    f=  8.21828D+00    |proj g|=  8.75883D-02

At iterate   10    f=  8.21509D+00    |proj g|=  5.96413D-04

At iterate   15    f=  8.21508D+00    |proj g|=  1.21976D-02

At iterate   20    f=  8.21451D+00    |proj g|=  1.44090D-01

At iterate   25    f=  8.14935D+00    |proj g|=  2.09406D+00

 This problem is unconstrained.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
At iterate   30    f=  7.81795D+00    |proj g|=  2.90397D-01

At iterate   35    f=  7.75897D+00    |proj g|=  2.09648D-01

At iterate   40    f=  7.75799D+00    |proj g|=  2.94627D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value
```

```
           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     41     58      1     0     0   2.946D-04   7.758D+00
  F =   7.7579869611287222

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 1) x (0, 1, 0, 12)12 : AIC Calculated =1743.7890792928338
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.00874D+01    |proj g|=  5.72392D+00

At iterate    5    f=  8.95845D+00    |proj g|=  2.35217D+00

 This problem is unconstrained.
At iterate   10    f=  8.75340D+00    |proj g|=  2.02732D-02

At iterate   15    f=  8.74229D+00    |proj g|=  9.12065D-02

At iterate   20    f=  8.73670D+00    |proj g|=  9.96887D-04

At iterate   25    f=  8.73636D+00    |proj g|=  5.73752D-02

At iterate   30    f=  8.68663D+00    |proj g|=  9.28530D-01

At iterate   35    f=  6.77857D+00    |proj g|=  4.83463D+00

At iterate   40    f=  6.62846D+00    |proj g|=  2.52836D-01

At iterate   45    f=  6.62350D+00    |proj g|=  2.14515D-01

At iterate   50    f=  6.60266D+00    |proj g|=  1.73094D-01

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     50     67      1     0     0   1.731D-01   6.603D+00
  F =   6.6026579022550811

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT
ARIMA (1, 0, 1) x (0, 1, 1, 12)12 : AIC Calculated =1486.9953701051381
RUNNING THE L-BFGS-B CODE
```

```
           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  2.79014D+01    |proj g|=  3.70966D+01

At iterate    5    f=  8.93099D+00    |proj g|=  3.36212D+00

At iterate   10    f=  7.99398D+00    |proj g|=  3.23382D-01

At iterate   15    f=  7.74081D+00    |proj g|=  2.29369D+01

At iterate   20    f=  7.33558D+00    |proj g|=  1.72689D+00

At iterate   25    f=  7.33452D+00    |proj g|=  2.64886D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     25     32      1     0     0   2.649D-04   7.335D+00
  F =   7.3345200502777290

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 1) x (1, 0, 0, 12)12 : AIC Calculated =1650.9324912622112
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.74372D+01    |proj g|=  1.56191D+02
 This problem is unconstrained.
 This problem is unconstrained.
At iterate    5    f=  8.66400D+00    |proj g|=  6.31943D-01

At iterate   10    f=  8.65198D+00    |proj g|=  1.77227D+00

At iterate   15    f=  8.64787D+00    |proj g|=  1.38141D-01

At iterate   20    f=  8.56155D+00    |proj g|=  2.33439D+00

At iterate   25    f=  8.48546D+00    |proj g|=  6.35718D+00

At iterate   30    f=  8.03594D+00    |proj g|=  2.87565D+01
```

```
At iterate   35    f=  7.45102D+00    |proj g|=  2.89277D-01

At iterate   40    f=  7.44960D+00    |proj g|=  1.27100D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     42     58      1     0     0   2.827D-04   7.450D+00
  F =   7.4496003653115084

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 1) x (1, 0, 1, 12)12 : AIC Calculated =1678.7104818297778
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.25608D+01    |proj g|=  1.40311D+01

At iterate    5    f=  7.02375D+00    |proj g|=  3.24171D-01

At iterate   10    f=  6.75590D+00    |proj g|=  1.50484D+00

At iterate   15    f=  6.70728D+00    |proj g|=  5.24220D-01

At iterate   20    f=  6.69222D+00    |proj g|=  1.79997D-01

At iterate   25    f=  6.68968D+00    |proj g|=  1.43364D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     25     36      1     0     0   1.434D-04   6.690D+00
  F =   6.6896803608716642
```

```
CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (1, 0, 1) x (1, 1, 0, 12)12 : AIC Calculated =1506.4884008352528
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.65214D+01    |proj g|=  1.04258D+01

At iterate    5    f=  1.35006D+01    |proj g|=  6.91000D+00

At iterate   10    f=  8.05021D+00    |proj g|=  3.56699D-01

At iterate   15    f=  8.02693D+00    |proj g|=  1.10458D+00

At iterate   20    f=  7.46359D+00    |proj g|=  1.07485D+00
 This problem is unconstrained.
At iterate   25    f=  7.04735D+00    |proj g|=  2.11918D-01

At iterate   30    f=  7.03929D+00    |proj g|=  6.84166D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     31     41      1     0     0   1.393D-04   7.039D+00
  F =   7.0392877774977682

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 1) x (1, 1, 1, 12)12 : AIC Calculated =1586.8004621595
ARIMA (1, 1, 0) x (0, 0, 0, 12)12 : AIC Calculated =1810.3332612069871RUNNING TH
E L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.06399D+00    |proj g|=  3.06301D-05


           * * *
```

```
Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     15      1     0     0   3.061D-05   8.064D+00
  F =   8.0639877732454774

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.34369D+00    |proj g|=  5.52568D+00

At iterate    5    f=  7.23620D+00    |proj g|=  1.33719D-02

At iterate   10    f=  7.23596D+00    |proj g|=  2.85075D-03

At iterate   15    f=  7.23581D+00    |proj g|=  3.97105D-02

At iterate   20    f=  7.23320D+00    |proj g|=  2.56014D-02

At iterate   25    f=  7.23313D+00    |proj g|=  2.83989D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     26     28      1     0     0   2.857D-05   7.233D+00
  F =   7.2331323966676653

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 0) x (0, 0, 1, 12)12 : AIC Calculated =1626.221656853557
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
```

```
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.44547D+00    |proj g|=  1.12271D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     21      1     0     0   1.123D-05   7.445D+00
  F =   7.4454712594841164

ABNORMAL_TERMINATION_IN_LNSRCH
ARIMA (1, 1, 0) x (0, 1, 0, 12)12 : AIC Calculated =1671.785562124442
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  6.59923D+00    |proj g|=  3.10396D-01
 This problem is unconstrained.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
 This problem is unconstrained.
 This problem is unconstrained.

 Line search cannot locate an adequate point after MAXLS
  function and gradient evaluations.
  Previous x, f and g restored.
 Possible causes: 1 error in function or gradient evaluation;
                  2 rounding error dominate computation.
 This problem is unconstrained.
At iterate    5    f=  6.53774D+00    |proj g|=  9.13241D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value
```

```
           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      7     12      1     0     0   4.703D-06   6.538D+00
  F =   6.5377387652941854

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (1, 1, 0) x (0, 1, 1, 12)12 : AIC Calculated =1470.4534834258975
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.39978D+00    |proj g|=  4.76473D-01

At iterate    5    f=  7.23598D+00    |proj g|=  3.81522D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      8     10      1     0     0   1.161D-04   7.236D+00
  F =   7.2359750230424513

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 0) x (1, 0, 0, 12)12 : AIC Calculated =1626.8584051615092
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.40871D+00    |proj g|=  5.21504D-01

At iterate    5    f=  7.23934D+00    |proj g|=  1.04560D-01

At iterate   10    f=  7.23600D+00    |proj g|=  1.69708D-03

At iterate   15    f=  7.23593D+00    |proj g|=  2.41431D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
```

```
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     16     25      1     0     0   1.171D-04   7.236D+00
  F =   7.2359284712329552

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 0) x (1, 0, 1, 12)12 : AIC Calculated =1628.847977556182
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  6.51841D+00    |proj g|=  1.59276D-02
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      4      6      1     0     0   2.586D-05   6.518D+00
  F =   6.5182633432634178

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 0) x (1, 1, 0, 12)12 : AIC Calculated =1466.0909888910055
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  6.81903D+00    |proj g|=  1.69064D+00

At iterate    5    f=  6.58082D+00    |proj g|=  2.46840D-01

At iterate   10    f=  6.51969D+00    |proj g|=  1.19913D-01
```

```
At iterate   15    f=  6.51336D+00    |proj g|=  4.19247D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     17     22      1     0     0   3.636D-05   6.513D+00
  F =   6.5133618109395881

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (1, 1, 0) x (1, 1, 1, 12)12 : AIC Calculated =1466.9930456504678
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.11163D+00    |proj g|=  1.64262D+00

At iterate    5    f=  7.92851D+00    |proj g|=  6.22544D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      7     10      1     0     0   9.803D-05   7.929D+00
  F =   7.9285059940996820

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 1) x (0, 0, 0, 12)12 : AIC Calculated =1781.9853426783288
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds
```

```
At iterate    0    f=  7.44444D+00    |proj g|=  1.79934D+00

At iterate    5    f=  6.99951D+00    |proj g|=  3.96090D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4      9     11      1     0     0   1.745D-05   6.999D+00
  F =   6.9992374615103570

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 1) x (0, 0, 1, 12)12 : AIC Calculated =1575.82919137832
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.19056D+00    |proj g|=  6.34203D+00

At iterate    5    f=  7.35260D+00    |proj g|=  4.21269D-02

At iterate   10    f=  7.35172D+00    |proj g|=  9.97699D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     11     13      1     0     0   9.976D-05   7.352D+00
  F =   7.3517213144489304

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 1) x (0, 1, 0, 12)12 : AIC Calculated =1652.7855744365604
RUNNING THE L-BFGS-B CODE
```

```
           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  6.42649D+00    |proj g|=  4.13809D-01
 This problem is unconstrained.
At iterate    5    f=  6.34875D+00    |proj g|=  1.41183D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4      7     10      1     0     0   1.197D-04   6.349D+00
  F =   6.3487508298961535

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 1) x (0, 1, 1, 12)12 : AIC Calculated =1430.1201858967383
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.16486D+00    |proj g|=  6.57987D+00

At iterate    5    f=  7.16293D+00    |proj g|=  3.52989D-01

At iterate   10    f=  7.07040D+00    |proj g|=  7.74545D-02

At iterate   15    f=  7.07027D+00    |proj g|=  1.73153D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
```

```
    4     15     21      1     0     0   1.732D-05   7.070D+00
  F =   7.0702713068734075

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 1) x (1, 0, 0, 12)12 : AIC Calculated =1591.7407727396433
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.34148D+00    |proj g|=  7.47811D-01

At iterate    5    f=  6.99764D+00    |proj g|=  3.28394D-02

At iterate   10    f=  6.99746D+00    |proj g|=  5.58286D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     11     15      1     0     0   1.837D-05   6.997D+00
  F =   6.9974600316829081

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (1, 1, 1) x (1, 0, 1, 12)12 : AIC Calculated =1577.4310470969715
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.26205D+00    |proj g|=  4.17600D+00

At iterate    5    f=  6.51110D+00    |proj g|=  8.61292D-02

At iterate   10    f=  6.46173D+00    |proj g|=  2.49545D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
```

```
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     13     21      1     0     0   6.274D-06   6.461D+00
  F =   6.4614281874067450

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
 This problem is unconstrained.
ARIMA (1, 1, 1) x (1, 1, 0, 12)12 : AIC Calculated =1455.3599139791108
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.57025D+00    |proj g|=  6.49412D+00

At iterate    5    f=  6.40126D+00    |proj g|=  2.44188D-01

At iterate   10    f=  6.33497D+00    |proj g|=  1.56754D-01

At iterate   15    f=  6.32700D+00    |proj g|=  5.49380D-03

At iterate   20    f=  6.32698D+00    |proj g|=  1.17922D-03
 This problem is unconstrained.
At iterate   25    f=  6.32688D+00    |proj g|=  8.53403D-02

At iterate   30    f=  6.32029D+00    |proj g|=  2.42813D-01

At iterate   35    f=  6.31483D+00    |proj g|=  4.10234D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     38     45      1     0     0   9.909D-06   6.315D+00
  F =   6.3148169261707769

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (1, 1, 1) x (1, 1, 1, 12)12 : AIC Calculated =1424.518991462254
pdq         (1, 1, 1)
pdqs    (1, 1, 1, 12)
```

```
aic       1424.518991
Name: 63, dtype: object
```

Out[260…

	pdq	pdqs	aic
0	(0, 0, 0)	(0, 0, 0, 12)	3017.609792
1	(0, 0, 0)	(0, 0, 1, 12)	2643.081712
2	(0, 0, 0)	(0, 1, 0, 12)	2280.370889
3	(0, 0, 0)	(0, 1, 1, 12)	2016.443844
4	(0, 0, 0)	(1, 0, 0, 12)	2183.616083
…	…	…	…
59	(1, 1, 1)	(0, 1, 1, 12)	1430.120186
60	(1, 1, 1)	(1, 0, 0, 12)	1591.740773
61	(1, 1, 1)	(1, 0, 1, 12)	1577.431047
62	(1, 1, 1)	(1, 1, 0, 12)	1455.359914
63	(1, 1, 1)	(1, 1, 1, 12)	1424.518991

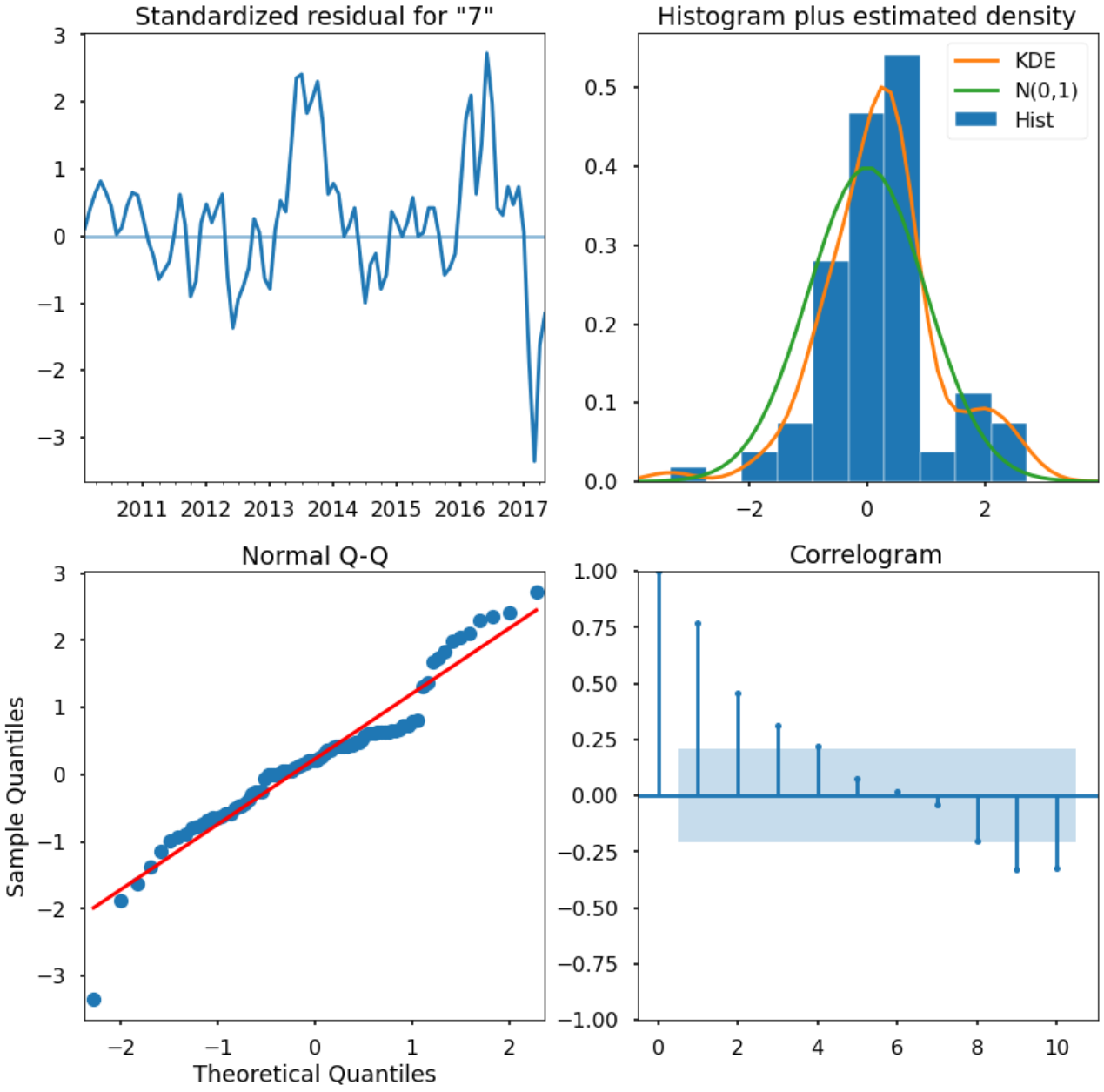
64 rows × 3 columns

In [261…

```
pdq= (1,1,1)
pdqs=(1,1,1,12)
train,test,results = model_fit(ts_78721,pdq=pdq,pdqs=pdqs)
```

```
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  2.63060D+01    |proj g|=  1.34624D+01

At iterate    5    f=  8.20676D+00    |proj g|=  2.07185D-02

At iterate   10    f=  8.20426D+00    |proj g|=  3.24111D-01

At iterate   15    f=  8.19846D+00    |proj g|=  4.33220D-02

At iterate   20    f=  8.19255D+00    |proj g|=  6.20316D-01

At iterate   25    f=  8.17689D+00    |proj g|=  4.31699D-02

 This problem is unconstrained.
At iterate   30    f=  8.17673D+00    |proj g|=  1.49277D-03

At iterate   35    f=  8.17669D+00    |proj g|=  8.62341D-03

At iterate   40    f=  8.17534D+00    |proj g|=  2.08387D-01

At iterate   45    f=  8.04561D+00    |proj g|=  2.02324D-01
```

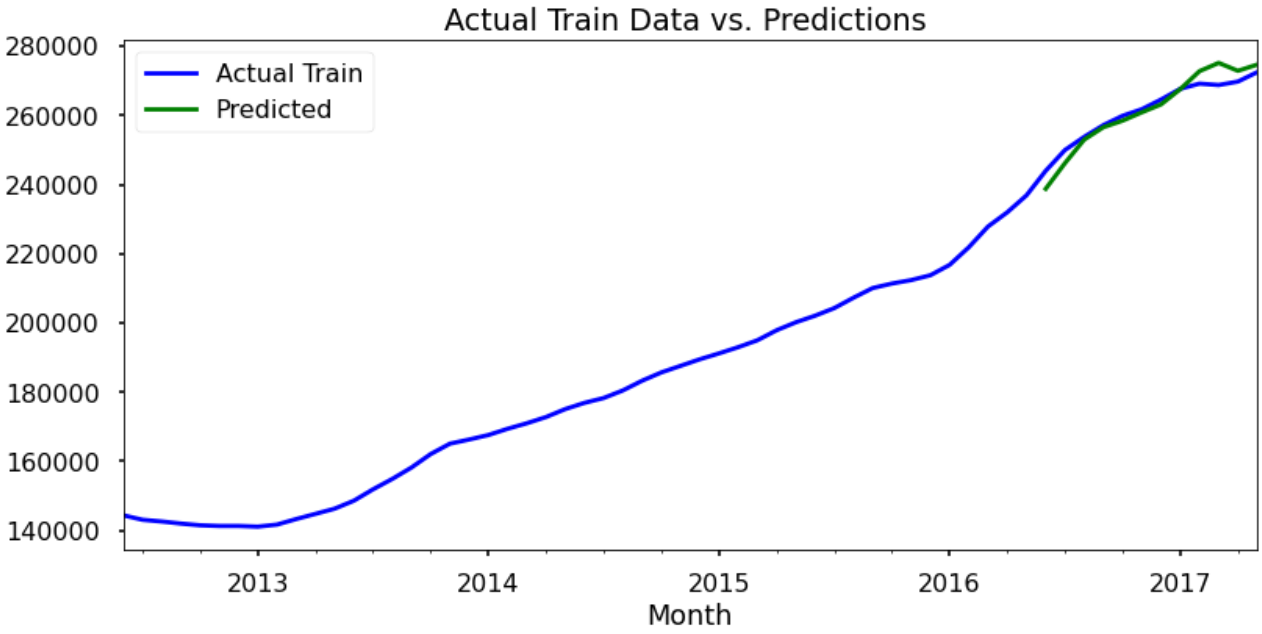
```
At iterate   50    f=  7.81773D+00    |proj g|=  2.96362D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     50     57      1     0     0   2.964D-02   7.818D+00
  F =   7.8177253829004787

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT
                                     SARIMAX Results
==========================================================================
==========
Dep. Variable:                                78721   No. Observations:
101
Model:             SARIMAX(1, 1, 1)x(1, 1, 1, 12)   Log Likelihood
-789.590
Date:                            Wed, 19 Oct 2022   AIC
1589.181
Time:                                   13:09:13   BIC
1601.567
Sample:                               01-01-2009   HQIC
1594.171
                                    - 05-01-2017
Covariance Type:                              opg
==========================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
--------------------------------------------------------------------------
ar.L1         -0.9226      2.135     -0.432      0.666      -5.107       3.262
ma.L1          0.9214      2.139      0.431      0.667      -3.271       5.114
ar.S.L12       0.9997      0.196      5.095      0.000       0.615       1.384
ma.S.L12      -0.9999      0.266     -3.756      0.000      -1.522      -0.478
sigma2      3.649e+06   1.82e-07      2e+13      0.000    3.65e+06    3.65e+06
==========================================================================
===
Ljung-Box (L1) (Q):                  54.00   Jarque-Bera (JB):                1
1.33
Prob(Q):                              0.00   Prob(JB):
0.00
Heteroskedasticity (H):               4.97   Skew:                            -
0.07
Prob(H) (two-sided):                  0.00   Kurtosis:
4.75
==========================================================================
===

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-s
tep).
[2] Covariance matrix is singular or near-singular, with condition number 4.98e+
30. Standard errors may be unstable.
```



In [262…

```
train_model(train,results)
```

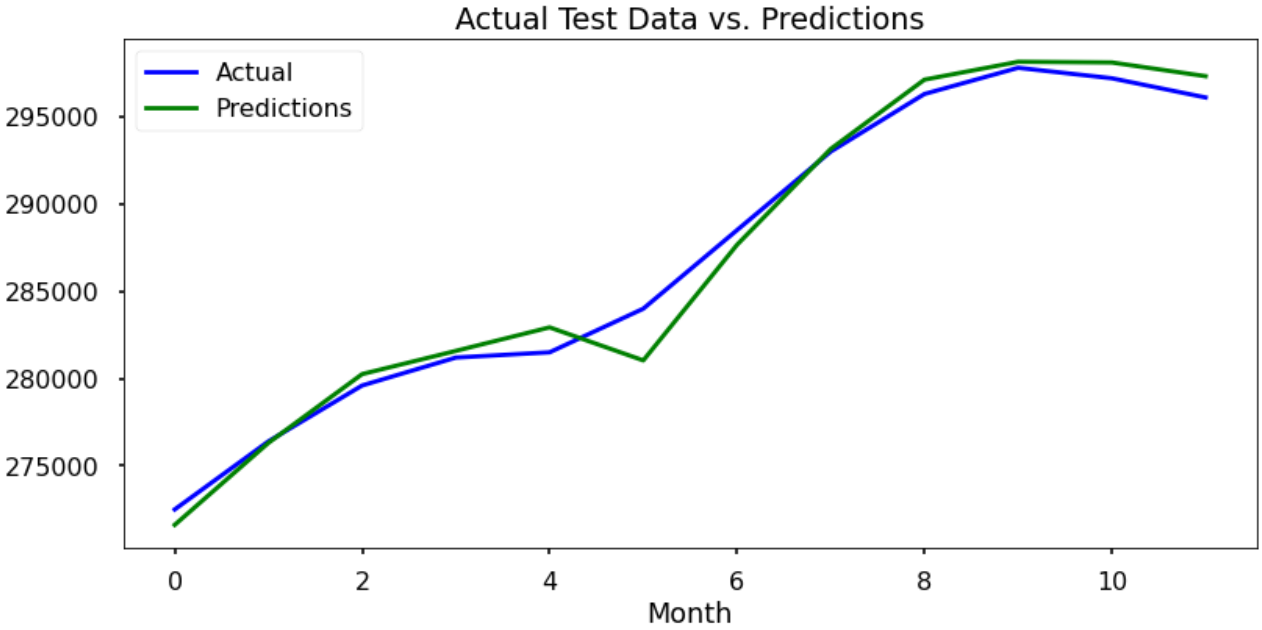
```
SARIMA model RMSE on train data: 3105.44484
```



In [263…

```
test_model(ts_78721,pdq=pdq, pdqs=pdqs)
```

```
SARIMA model RMSE on train data: 1155.21395
```

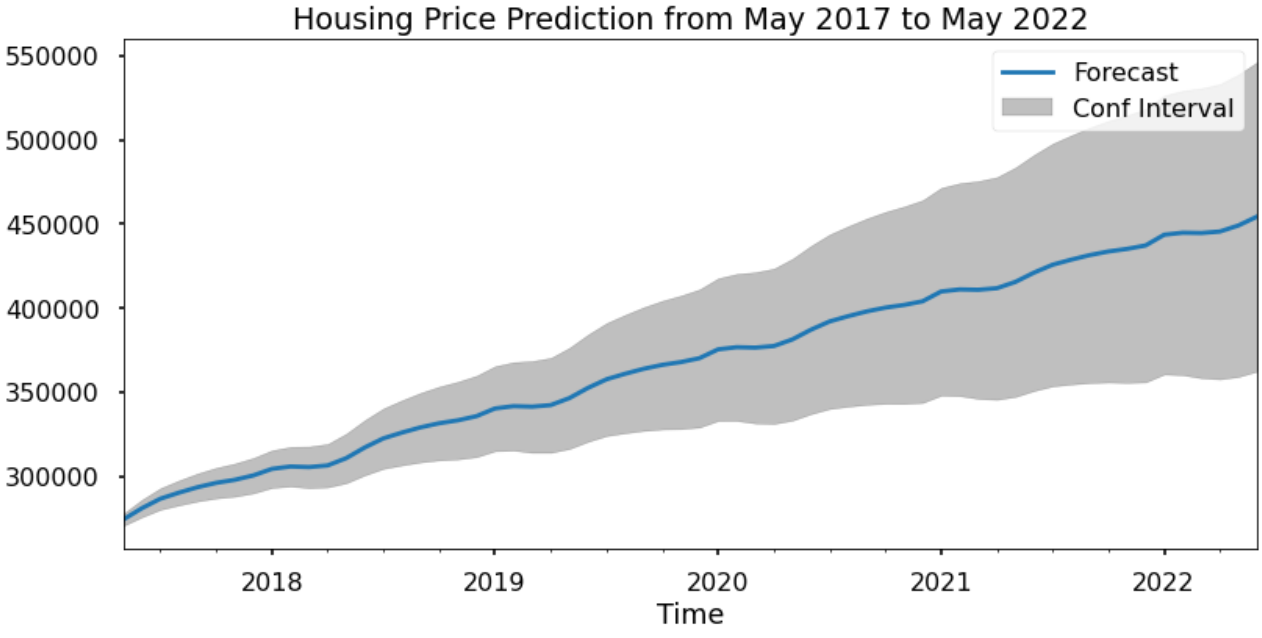


In [264…

```
pdq = (1, 1, 1)
pdqs =(1, 1, 1, 12)
pred_78721 = forecast_model(ts_78721,pdq=pdq, pdqs=pdqs,zc=78721)
```

```
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds
```

```
At iterate    0    f=  2.36122D+01    |proj g|=  1.24015D+01

At iterate    5    f=  8.30328D+00    |proj g|=  3.61061D-02

At iterate   10    f=  8.29418D+00    |proj g|=  4.22355D-01

At iterate   15    f=  8.28936D+00    |proj g|=  1.87379D-02
 This problem is unconstrained.
At iterate   20    f=  8.28643D+00    |proj g|=  7.47973D-01

At iterate   25    f=  8.26137D+00    |proj g|=  8.16602D-02

At iterate   30    f=  8.25800D+00    |proj g|=  8.67972D-03

At iterate   35    f=  8.25498D+00    |proj g|=  9.60794D-02

At iterate   40    f=  8.23870D+00    |proj g|=  1.40736D-01

At iterate   45    f=  8.10679D+00    |proj g|=  2.47785D-01

At iterate   50    f=  7.91697D+00    |proj g|=  3.62634D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     50     59      1     0     0   3.626D-03   7.917D+00
  F =   7.9169708104980643

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT
```



```
Housing Price Projection for 2017, 2018,2020 and 2022:
Housing price at enter point 2017-05:$ 274360.65
Housing price in year 1:$ 310882.48 with 13.31 % ROI
Housing price in year 3:$ 381456.8 with 39.03 % ROI
Housing price in year 5:$ 449211.77 with  63.73 % ROI
```

In [265…

```
# Add the 5 yr roi to the results summary df
df_zip_5YR_ROI.loc[df_zip_5YR_ROI['Zipcode'] == 78721, '5YR_PRED_ROI'] = 63.65
df_zip_5YR_ROI
```

Out[265…

	Zipcode	5_YR_PRED_ROI	5YR_PRED_ROI
0	78702	0	34.83
1	78758	0	84.89
2	78721	0	63.65
3	78723	0	NaN
4	78741	0	NaN

Zipcode 78723

In [266…

```
AIC_PDQS(ts_78723)
```

```
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            1     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.37514D+01    |proj g|=  1.26121D-08

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    1      0      1      0     0     0   1.261D-08   1.375D+01
  F =   13.751424702840371

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 0) x (0, 0, 0, 12)12 : AIC Calculated =3082.319133436243
RUNNING THE L-BFGS-B CODE

           * * *
```

```
Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.57093D+03    |proj g|=  1.12365D+02

At iterate    5    f=  1.20284D+01    |proj g|=  1.41740D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      7     10      1     0     0   5.950D-06   1.203D+01
  F =   12.028317434865803

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 0) x (0, 0, 1, 12)12 : AIC Calculated =2698.34310540994
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            1     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.01510D+01    |proj g|=  5.27187D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    1      0      1      0     0     0   5.272D-06   1.015D+01
  F =   10.151045908687690

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 0) x (0, 1, 0, 12)12 : AIC Calculated =2275.8342835460426
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
```

```
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.68589D+02    |proj g|=  3.22264D+01

At iterate    5    f=  5.90022D+01    |proj g|=  9.64532D-01

At iterate   10    f=  1.93396D+01    |proj g|=  4.67040D-02

At iterate   15    f=  1.47086D+01    |proj g|=  1.99300D-03

At iterate   20    f=  1.44425D+01    |proj g|=  5.49818D-05

At iterate   25    f=  1.09119D+01    |proj g|=  3.91857D-03
  ys=-9.425E+01  -gs= 7.267E-01 BFGS update SKIPPED

At iterate   30    f=  8.96735D+00    |proj g|=  9.57101D-07

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2     30     56      1     1     0   9.571D-07   8.967D+00
  F =   8.9673457996458925

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 0) x (0, 1, 1, 12)12 : AIC Calculated =2012.68545912068
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.83196D+00    |proj g|=  1.23005D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
```

```
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     16      1     0     0   1.230D-03   9.832D+00
  F =   9.8319560469103120

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 0) x (1, 0, 0, 12)12 : AIC Calculated =2206.35815450791
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  2.12098D+03    |proj g|=  5.14444D+02

At iterate    5    f=  1.38786D+03    |proj g|=  6.60593D+02

At iterate   10    f=  7.89255D+01    |proj g|=  1.21927D+02

At iterate   15    f=  1.20816D+01    |proj g|=  1.38946D+01

At iterate   20    f=  9.76854D+00    |proj g|=  3.36734D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     22     78      2     0     0   2.834D-04   9.769D+00
  F =   9.7685398953894538

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
 This problem is unconstrained.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
 This problem is unconstrained.

 Bad direction in the line search;
   refresh the lbfgs memory and restart the iteration.

 Warning:  more than 10 function and gradient
```

```
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
ARIMA (0, 0, 0) x (1, 0, 1, 12)12 : AIC Calculated =2194.1529365672377
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.77028D+00    |proj g|=  7.04414D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      0      1      0     0     0   7.044D-06   8.770D+00
  F =   8.7702777562637042

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 0) x (1, 1, 0, 12)12 : AIC Calculated =1968.5422174030696
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.41013D+02    |proj g|=  8.85388D+01
 This problem is unconstrained.
 This problem is unconstrained.
At iterate    5    f=  1.19341D+01    |proj g|=  8.32226D-01

At iterate   10    f=  1.18074D+01    |proj g|=  7.14145D-01

At iterate   15    f=  1.15933D+01    |proj g|=  7.75213D-02

At iterate   20    f=  1.15921D+01    |proj g|=  6.04038D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
```

```
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     20     21      1     0     0   6.040D-05   1.159D+01
  F =   11.592110331866149

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 0) x (1, 1, 1, 12)12 : AIC Calculated =2602.632714338017
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.58980D+02    |proj g|=  5.29900D+00

At iterate    5    f=  1.32758D+01    |proj g|=  6.66231D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      8     24      1     0     0   4.411D-06   1.328D+01
  F =   13.275748818037362

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 1) x (0, 0, 0, 12)12 : AIC Calculated =2977.7677352403693
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.58167D+03    |proj g|=  1.11855D+02

At iterate    5    f=  1.17516D+01    |proj g|=  2.32164D-02

At iterate   10    f=  1.17513D+01    |proj g|=  7.83822D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
```

```
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     13     38      1     0     0   1.199D-05   1.175D+01
  F =   11.751275595425117

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 1) x (0, 0, 1, 12)12 : AIC Calculated =2638.2857333752263
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.04827D+01    |proj g|=  3.17457D-01
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
At iterate    5    f=  9.79154D+00    |proj g|=  1.00037D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      6     11      1     0     0   2.201D-05   9.792D+00
  F =   9.7915448625038568

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 1) x (0, 1, 0, 12)12 : AIC Calculated =2197.306049200864
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.56769D+02    |proj g|=  2.80944D+01
  ys=-1.082E+02  -gs= 9.517E+01 BFGS update SKIPPED

At iterate    5    f=  6.15088D+01    |proj g|=  1.86422D+02
```

```
At iterate   10    f=  2.84229D+01    |proj g|=  3.65463D+01
 This problem is unconstrained.
At iterate   15    f=  1.45555D+01    |proj g|=  6.41191D+00

At iterate   20    f=  1.31287D+01    |proj g|=  2.04665D-01

At iterate   25    f=  1.31281D+01    |proj g|=  2.37226D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     27     60      1     1     0   1.412D-02   1.313D+01
  F =   13.128083405935596

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 1) x (0, 1, 1, 12)12 : AIC Calculated =2946.6906829295735
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.36824D+01    |proj g|=  1.27753D-01

At iterate    5    f=  1.28092D+01    |proj g|=  6.73386D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      9     19      1     0     0   4.902D-06   1.281D+01
  F =   12.808816056962675

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 1) x (1, 0, 0, 12)12 : AIC Calculated =2875.1747967596393
RUNNING THE L-BFGS-B CODE
```

```
           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  2.00016D+03    |proj g|=  4.96206D+02

At iterate    5    f=  1.42022D+01    |proj g|=  6.61653D+00
  ys=-6.508E-01  -gs= 6.154E-01 BFGS update SKIPPED

At iterate   10    f=  1.24114D+01    |proj g|=  2.52868D-02

At iterate   15    f=  1.24102D+01    |proj g|=  3.65784D-02

At iterate   20    f=  1.24018D+01    |proj g|=  3.37496D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     22     45      1     1     0   5.274D-06   1.240D+01
  F =   12.401835022381315

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (0, 0, 1) x (1, 0, 1, 12)12 : AIC Calculated =2786.0110450134143
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.40121D+00    |proj g|=  2.89434D-01

At iterate    5    f=  8.71945D+00    |proj g|=  1.09388D-01

At iterate   10    f=  8.68755D+00    |proj g|=  4.85478D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
```

```
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     11     27      1     0     0   3.189D-03   8.688D+00
  F =   8.6875542760782185

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (0, 0, 1) x (1, 1, 0, 12)12 : AIC Calculated =1952.012157841521
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.10304D+03    |proj g|=  1.67824D+02

At iterate    5    f=  2.67325D+01    |proj g|=  6.89337D+00

At iterate   10    f=  1.22161D+01    |proj g|=  1.06287D+00

At iterate   15    f=  1.13492D+01    |proj g|=  1.18068D-01

At iterate   20    f=  1.13288D+01    |proj g|=  4.42359D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     22     29      1     0     0   3.413D-05   1.133D+01
  F =   11.328790644971008

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (0, 0, 1) x (1, 1, 1, 12)12 : AIC Calculated =2545.6491044735058
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            1     M =           10

At X0         0 variables are exactly at the bounds
```

```
At iterate    0    f=  8.99872D+00    |proj g|=  7.74563D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    1      0      1      0     0     0   7.746D-06   8.999D+00
  F =   8.9987182112835136

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 0) x (0, 0, 0, 12)12 : AIC Calculated =2017.7128793275072
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.03553D+00    |proj g|=  1.33741D-01

At iterate    5    f=  8.02043D+00    |proj g|=  2.32594D-04

At iterate   10    f=  8.02025D+00    |proj g|=  1.21579D-02

At iterate   15    f=  8.00524D+00    |proj g|=  1.04793D-01

At iterate   20    f=  7.99452D+00    |proj g|=  5.48983D-07

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2     20     24      1     0     0   5.490D-07   7.995D+00
  F =   7.9945216510458863

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 0) x (0, 0, 1, 12)12 : AIC Calculated =1794.7728498342785
RUNNING THE L-BFGS-B CODE

           * * *
```

```
Machine precision = 2.220D-16
 N =            1     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.05871D+00    |proj g|=  5.54305D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    1      2      5      1     0     0   6.158D-06   8.055D+00
  F =   8.0546961173443172

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 0) x (0, 1, 0, 12)12 : AIC Calculated =1806.251930285127
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.01115D+01    |proj g|=  2.25599D+01
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
At iterate    5    f=  7.05950D+00    |proj g|=  1.65441D-01

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      9     12      1     0     0   2.854D-05   7.056D+00
  F =   7.0560074140186577

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 1, 0) x (0, 1, 1, 12)12 : AIC Calculated =1584.5456607401793
```

```
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.03849D+00    |proj g|=  4.48779D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      0      1      0     0     0   4.488D-06   8.038D+00
  F =   8.0384852274950820

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 0) x (1, 0, 0, 12)12 : AIC Calculated =1804.6206909588982
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.48211D+00    |proj g|=  2.79548D+00

At iterate    5    f=  7.99514D+00    |proj g|=  2.19319D-03

At iterate   10    f=  7.99512D+00    |proj g|=  5.62382D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     11     12      1     0     0   2.329D-05   7.995D+00
  F =   7.9951191257264327
```

```
CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 1, 0) x (1, 0, 1, 12)12 : AIC Calculated =1796.906684162721
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.09841D+00    |proj g|=  4.47278D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      0      1      0     0     0   4.473D-06   7.098D+00
  F =   7.0984120717333132

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (0, 1, 0) x (1, 1, 0, 12)12 : AIC Calculated =1594.0443040682621
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.06465D+00    |proj g|=  1.04609D+00

At iterate    5    f=  7.01699D+00    |proj g|=  1.45989D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
```

```
    3      8     11      1     0     0   2.440D-05   7.017D+00
  F =   7.0169771497498994

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (0, 1, 0) x (1, 1, 1, 12)12 : AIC Calculated =1577.8028815439775
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.90501D+00    |proj g|=  5.09009D-01

At iterate    5    f=  8.49413D+00    |proj g|=  1.09899D-01

At iterate   10    f=  8.49151D+00    |proj g|=  3.03446D-04

At iterate   15    f=  8.49151D+00    |proj g|=  5.76295D-03

At iterate   20    f=  8.49107D+00    |proj g|=  8.39560D-02

At iterate   25    f=  8.36625D+00    |proj g|=  4.78961D-01

At iterate   30    f=  8.31191D+00    |proj g|=  1.39911D-01

At iterate   35    f=  8.30422D+00    |proj g|=  4.04773D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2     36     46      1     0     0   5.459D-06   8.304D+00
  F =   8.3042177035175637

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 1) x (0, 0, 0, 12)12 : AIC Calculated =1864.1447655879344
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.92781D+00    |proj g|=  4.51015D-01
```

```
At iterate    5    f=  7.55934D+00    |proj g|=  1.00801D-01

At iterate   10    f=  7.55591D+00    |proj g|=  9.63125D-03

At iterate   15    f=  7.55576D+00    |proj g|=  7.27457D-04

At iterate   20    f=  7.55569D+00    |proj g|=  1.65650D-02

At iterate   25    f=  7.54729D+00    |proj g|=  1.92579D-01

At iterate   30    f=  7.42420D+00    |proj g|=  2.23970D-01

At iterate   35    f=  7.38424D+00    |proj g|=  3.56863D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     38     50      1     0     0   2.129D-06   7.384D+00
  F =   7.3841961000836580

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (0, 1, 1) x (0, 0, 1, 12)12 : AIC Calculated =1660.0599264187395
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.58943D+00    |proj g|=  3.32021D+00

At iterate    5    f=  8.11444D+00    |proj g|=  2.86854D-04

At iterate   10    f=  8.11443D+00    |proj g|=  4.36038D-03

At iterate   15    f=  8.11334D+00    |proj g|=  5.90494D-02

At iterate   20    f=  8.00331D+00    |proj g|=  7.87646D-01

At iterate   25    f=  7.78393D+00    |proj g|=  1.48989D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
```

```
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2     29     34      1     0     0   2.662D-06   7.783D+00
  F =   7.7832399145034916

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 1) x (0, 1, 0, 12)12 : AIC Calculated =1747.4457408487822
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.51204D+00    |proj g|=  1.80996D+01

At iterate    5    f=  6.74118D+00    |proj g|=  2.69397D-01
 This problem is unconstrained.
 This problem is unconstrained.
At iterate   10    f=  6.71967D+00    |proj g|=  2.08907D-04

At iterate   15    f=  6.71967D+00    |proj g|=  4.14418D-03

At iterate   20    f=  6.71938D+00    |proj g|=  5.33563D-02

At iterate   25    f=  6.68079D+00    |proj g|=  4.38633D-01

At iterate   30    f=  6.61518D+00    |proj g|=  1.92480D-01

At iterate   35    f=  6.59363D+00    |proj g|=  8.36712D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     36     45      1     0     0   7.921D-06   6.594D+00
  F =   6.5936114351050232

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 1) x (0, 1, 1, 12)12 : AIC Calculated =1482.9689614635251
RUNNING THE L-BFGS-B CODE

           * * *
```

```
Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.59331D+00    |proj g|=  1.92685D+00

At iterate    5    f=  8.05963D+00    |proj g|=  1.62860D+00

At iterate   10    f=  7.72949D+00    |proj g|=  1.13216D-01

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     14     24      1     0     0   2.620D-04   7.729D+00
  F =   7.7293104200875771

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 1, 1) x (1, 0, 0, 12)12 : AIC Calculated =1737.3655340996172
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.30590D+00    |proj g|=  1.76767D+00

At iterate    5    f=  7.56176D+00    |proj g|=  1.07320D-01
 This problem is unconstrained.
 This problem is unconstrained.
At iterate   10    f=  7.55573D+00    |proj g|=  1.23598D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     14     21      1     0     0   2.728D-04   7.556D+00
```

```
  F =   7.5556062574662066

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 1, 1) x (1, 0, 1, 12)12 : AIC Calculated =1700.4558016724302
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.20573D+00    |proj g|=  4.80544D+00

At iterate    5    f=  6.99229D+00    |proj g|=  4.07351D-01
 This problem is unconstrained.
At iterate   10    f=  6.84828D+00    |proj g|=  5.25335D-01

At iterate   15    f=  6.83609D+00    |proj g|=  1.18224D-01

At iterate   20    f=  6.83584D+00    |proj g|=  4.37793D-03

At iterate   25    f=  6.83553D+00    |proj g|=  2.74380D-01

At iterate   30    f=  6.82763D+00    |proj g|=  5.69557D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     34     64      1     0     0   1.380D-04   6.828D+00
  F =   6.8275403348900685

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 1, 1) x (1, 1, 0, 12)12 : AIC Calculated =1535.3690350153754
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.10456D+00    |proj g|=  1.37498D+00

At iterate    5    f=  6.71716D+00    |proj g|=  4.96278D-01

At iterate   10    f=  6.65563D+00    |proj g|=  4.36907D-01
```

```
At iterate   15    f=  6.62550D+00    |proj g|=  2.37011D-01

 This problem is unconstrained.
At iterate   20    f=  6.62080D+00    |proj g|=  1.52386D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     22     36      1     0     0   3.881D-04   6.621D+00
  F =   6.6207951008521588

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 1, 1) x (1, 1, 1, 12)12 : AIC Calculated =1491.0581025908837
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.75316D+00    |proj g|=  1.17549D-01

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     20      1     0     0   1.175D-01   8.753D+00
  F =   8.7531584690129378

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 0) x (0, 0, 0, 12)12 : AIC Calculated =1964.707497058898
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds
```

```
At iterate    0    f=  8.74795D+00    |proj g|=  1.07780D+01

At iterate    5    f=  7.84176D+00    |proj g|=  4.64909D+00

At iterate   10    f=  7.82873D+00    |proj g|=  2.36636D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     11     43      1     0     0   2.366D-04   7.829D+00
  F =   7.8287325085339035

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 0) x (0, 0, 1, 12)12 : AIC Calculated =1759.6360819115944
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.13046D+00    |proj g|=  4.13972D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     14      1     0     0   4.138D-04   8.130D+00
  F =   8.1304644220130466

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
 This problem is unconstrained.
```

```
 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
 This problem is unconstrained.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
ARIMA (1, 0, 0) x (0, 1, 0, 12)12 : AIC Calculated =1825.2240305309226
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.46828D+01    |proj g|=  1.02615D+01

At iterate    5    f=  1.42461D+01    |proj g|=  5.89460D+00

At iterate   10    f=  1.07599D+01    |proj g|=  4.55382D+00

At iterate   15    f=  9.95211D+00    |proj g|=  5.26078D-02

At iterate   20    f=  9.94766D+00    |proj g|=  9.29796D-04

At iterate   25    f=  9.94759D+00    |proj g|=  3.95626D-02

At iterate   30    f=  9.93961D+00    |proj g|=  4.52061D-01

At iterate   35    f=  8.53119D+00    |proj g|=  1.03997D+00
 This problem is unconstrained.
At iterate   40    f=  7.24480D+00    |proj g|=  1.27860D+00

At iterate   45    f=  7.19716D+00    |proj g|=  2.24096D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     47     68      1     0     0   2.132D-04   7.197D+00
  F =   7.1971606557477559

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 0) x (0, 1, 1, 12)12 : AIC Calculated =1618.1639868874972
 This problem is unconstrained.
 This problem is unconstrained.
RUNNING THE L-BFGS-B CODE
```

```
           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.35488D+00    |proj g|=  1.52638D+00

At iterate    5    f=  8.09782D+00    |proj g|=  3.25929D+00

At iterate   10    f=  7.82867D+00    |proj g|=  1.22318D+00

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     14     27      1     0     0   2.623D-05   7.829D+00
  F =   7.8286115423811911

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 0) x (1, 0, 0, 12)12 : AIC Calculated =1759.6089854933869
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.75508D+02    |proj g|=  3.26466D+02

At iterate    5    f=  9.10425D+00    |proj g|=  1.94829D-01

At iterate   10    f=  9.10320D+00    |proj g|=  1.22271D+00

At iterate   15    f=  8.29540D+00    |proj g|=  1.13185D+02

At iterate   20    f=  7.83028D+00    |proj g|=  7.34066D-02

At iterate   25    f=  7.83007D+00    |proj g|=  2.08816D+00

At iterate   30    f=  7.82864D+00    |proj g|=  5.97804D-01

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
```

```
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     33     59      1     0     0   1.080D-02   7.829D+00
  F =   7.8286088811884502

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 0) x (1, 0, 1, 12)12 : AIC Calculated =1761.6083893862128
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.76251D+00    |proj g|=  1.05520D+00

At iterate    5    f=  7.11370D+00    |proj g|=  2.38257D+00

At iterate   10    f=  7.09416D+00    |proj g|=  4.77610D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     10     21      1     0     0   4.776D-05   7.094D+00
  F =   7.0941554601190573

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
 This problem is unconstrained.
ARIMA (1, 0, 0) x (1, 1, 0, 12)12 : AIC Calculated =1595.090823066669
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  2.65217D+01    |proj g|=  1.34319D+01
```

```
At iterate    5    f=  1.07066D+01    |proj g|=  1.11508D+01

At iterate   10    f=  9.87230D+00    |proj g|=  1.29865D+01

At iterate   15    f=  9.03131D+00    |proj g|=  1.51868D-01

At iterate   20    f=  9.02367D+00    |proj g|=  1.77603D-01
 This problem is unconstrained.
At iterate   25    f=  9.02255D+00    |proj g|=  8.15790D-04

At iterate   30    f=  9.02131D+00    |proj g|=  3.25865D-02

At iterate   35    f=  8.90699D+00    |proj g|=  1.42270D+00
  ys=-1.503E+00  -gs= 4.935E-01 BFGS update SKIPPED

At iterate   40    f=  7.10589D+00    |proj g|=  1.00365D-01

At iterate   45    f=  7.10550D+00    |proj g|=  1.27181D-01

At iterate   50    f=  7.10186D+00    |proj g|=  4.72547D-01

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     50     80      1     1     0   4.725D-01   7.102D+00
  F =   7.1018564570932119

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT
ARIMA (1, 0, 0) x (1, 1, 1, 12)12 : AIC Calculated =1598.8158463888794
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.40978D+00    |proj g|=  5.65089D-01

At iterate    5    f=  8.19136D+00    |proj g|=  1.30662D-01

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
```

```
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      8     17      1     0     0   3.832D-04   8.191D+00
  F =   8.1913483803904921

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 1) x (0, 0, 0, 12)12 : AIC Calculated =1840.8620372074702
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.70371D+00    |proj g|=  1.51465D+01

At iterate    5    f=  8.38681D+00    |proj g|=  5.70292D-01

At iterate   10    f=  8.37308D+00    |proj g|=  5.13981D-01

At iterate   15    f=  8.02737D+00    |proj g|=  4.81656D+01

At iterate   20    f=  7.60547D+00    |proj g|=  2.76417D+00

At iterate   25    f=  7.60358D+00    |proj g|=  3.50802D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     25     40      1     0     0   3.508D-04   7.604D+00
  F =   7.6035782968141286

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (1, 0, 1) x (0, 0, 1, 12)12 : AIC Calculated =1711.2015384863648
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds
```

```
At iterate    0    f=  1.31657D+01    |proj g|=  1.64533D+01

At iterate    5    f=  8.44926D+00    |proj g|=  3.10529D-01

At iterate   10    f=  8.43572D+00    |proj g|=  2.92100D-04

At iterate   15    f=  8.43568D+00    |proj g|=  5.83600D-03

At iterate   20    f=  8.43011D+00    |proj g|=  9.01642D-02

At iterate   25    f=  8.23622D+00    |proj g|=  9.74767D-01

At iterate   30    f=  7.96929D+00    |proj g|=  9.48144D-02

At iterate   35    f=  7.95436D+00    |proj g|=  1.67771D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     37     43      1     0     0   8.535D-06   7.954D+00
  F =   7.9543605726636288

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
 This problem is unconstrained.
ARIMA (1, 0, 1) x (0, 1, 0, 12)12 : AIC Calculated =1787.7767682766528
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.97261D+01    |proj g|=  3.37682D+01

At iterate    5    f=  1.47297D+01    |proj g|=  2.77501D+01

At iterate   10    f=  1.06861D+01    |proj g|=  2.93289D+00

At iterate   15    f=  1.00661D+01    |proj g|=  2.13788D+00

At iterate   20    f=  9.38663D+00    |proj g|=  7.36012D-01

At iterate   25    f=  9.29113D+00    |proj g|=  1.00043D-01

At iterate   30    f=  9.28794D+00    |proj g|=  8.53689D-04
 This problem is unconstrained.
```

```
At iterate   35    f=  9.28785D+00    |proj g|=  2.29400D-02

At iterate   40    f=  9.27646D+00    |proj g|=  3.17823D-01
  ys=-3.383E-01  -gs= 7.827E-01 BFGS update SKIPPED

At iterate   45    f=  6.76533D+00    |proj g|=  2.30374D+00

At iterate   50    f=  6.70112D+00    |proj g|=  2.12884D-01

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     50     71      1     1     0   2.129D-01   6.701D+00
  F =   6.7011187349760055

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT
ARIMA (1, 0, 1) x (0, 1, 1, 12)12 : AIC Calculated =1509.0505966346252
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  3.32388D+01    |proj g|=  5.35242D+01

At iterate    5    f=  9.53473D+00    |proj g|=  7.32743D+00

At iterate   10    f=  8.07043D+00    |proj g|=  3.29892D-01

At iterate   15    f=  7.53224D+00    |proj g|=  3.83894D+00

At iterate   20    f=  7.38639D+00    |proj g|=  1.84994D+00

At iterate   25    f=  7.38444D+00    |proj g|=  3.32074D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *
```

```
   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     25     37      1     0     0   3.321D-04   7.384D+00
  F =   7.3844362902096403

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (1, 0, 1) x (1, 0, 0, 12)12 : AIC Calculated =1662.1137290069594
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.85810D+02    |proj g|=  3.42677D+02

At iterate    5    f=  8.98739D+00    |proj g|=  6.35566D-01

At iterate   10    f=  8.97157D+00    |proj g|=  7.04603D+00

At iterate   15    f=  8.79963D+00    |proj g|=  3.05121D+00

At iterate   20    f=  8.79329D+00    |proj g|=  1.14145D-01

At iterate   25    f=  8.16562D+00    |proj g|=  1.34204D+00

At iterate   30    f=  8.16443D+00    |proj g|=  4.28305D-01

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     34     52      1     0     0   8.947D-03   8.164D+00
  F =   8.1642549231960118

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (1, 0, 1) x (1, 0, 1, 12)12 : AIC Calculated =1838.7931027959066
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.12263D+01    |proj g|=  7.76683D+00
```

```
At iterate    5    f=  7.09506D+00    |proj g|=  1.20509D+00

At iterate   10    f=  6.81492D+00    |proj g|=  3.34681D-01

At iterate   15    f=  6.81163D+00    |proj g|=  2.22247D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     16     23      1     0     0   1.634D-03   6.812D+00
  F =   6.8116286949583005

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (1, 0, 1) x (1, 1, 0, 12)12 : AIC Calculated =1533.8048276706593
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  4.17880D+01    |proj g|=  5.63780D+01

At iterate    5    f=  8.91698D+00    |proj g|=  5.46862D+00

At iterate   10    f=  8.07313D+00    |proj g|=  1.03100D-01

At iterate   15    f=  8.05223D+00    |proj g|=  1.29668D-01
 This problem is unconstrained.
At iterate   20    f=  7.85797D+00    |proj g|=  4.95861D-01

At iterate   25    f=  7.55276D+00    |proj g|=  4.98759D+00

At iterate   30    f=  7.41512D+00    |proj g|=  1.28789D-01

At iterate   35    f=  7.41430D+00    |proj g|=  4.33446D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
```

```
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     35     63      1     0     0   4.334D-04   7.414D+00
  F =   7.4142961004421801

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 1) x (1, 1, 1, 12)12 : AIC Calculated =1670.8023264990484
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.93272D+00    |proj g|=  4.26520D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     11      1     0     0   4.259D-05   7.933D+00
  F =   7.9327245770438495

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 0) x (0, 0, 0, 12)12 : AIC Calculated =1780.9303052578223
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.17645D+00    |proj g|=  4.86179D-01

At iterate    5    f=  7.10002D+00    |proj g|=  5.76426D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
```

```
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      7      9      1     0     0   7.008D-05   7.100D+00
  F =   7.1000196280873666

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 0) x (0, 0, 1, 12)12 : AIC Calculated =1596.4043966915701
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.45483D+00    |proj g|=  1.72120D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     11      1     0     0   1.719D-05   7.455D+00
  F =   7.4548336800457360

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (1, 1, 0) x (0, 1, 0, 12)12 : AIC Calculated =1673.8827443302448
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.07733D+01    |proj g|=  2.12299D+01

At iterate    5    f=  6.54094D+00    |proj g|=  2.34742D-01

At iterate   10    f=  6.53536D+00    |proj g|=  3.15553D-05

           * * *

Tit   = total number of iterations
```

```
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     10     13      1     0     0   3.156D-05   6.535D+00
  F =   6.5353589760494026

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (1, 1, 0) x (0, 1, 1, 12)12 : AIC Calculated =1469.9204106350662
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.32842D+00    |proj g|=  6.14099D-01

At iterate    5    f=  7.09101D+00    |proj g|=  8.01621D-03

At iterate   10    f=  7.09100D+00    |proj g|=  1.37554D-03

At iterate   15    f=  7.09091D+00    |proj g|=  1.91752D-02

At iterate   20    f=  7.09039D+00    |proj g|=  6.03666D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     23     28      1     0     0   4.725D-06   7.090D+00
  F =   7.0903792115397053

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (1, 1, 0) x (1, 0, 0, 12)12 : AIC Calculated =1594.244943384894
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10
```

```
At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.52521D+00    |proj g|=  4.83391D+00

At iterate    5    f=  7.09483D+00    |proj g|=  5.55575D-02

At iterate   10    f=  7.09082D+00    |proj g|=  1.61776D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     13     16      1     0     0   6.062D-05   7.091D+00
  F =   7.0907949758546440

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 0) x (1, 0, 1, 12)12 : AIC Calculated =1596.3380745914403
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  6.45543D+00    |proj g|=  8.70312D-02
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      4      6      1     0     0   1.525D-04   6.449D+00
  F =   6.4494977895814083

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 0) x (1, 1, 0, 12)12 : AIC Calculated =1450.6875048662355
RUNNING THE L-BFGS-B CODE

           * * *
```

```
Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  6.63001D+00    |proj g|=  2.91877D+00

At iterate    5    f=  6.47498D+00    |proj g|=  1.57832D-01

At iterate   10    f=  6.44947D+00    |proj g|=  1.21957D-01

At iterate   15    f=  6.44918D+00    |proj g|=  1.53720D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     16     20      1     0     0   1.537D-04   6.449D+00
  F =   6.4491771865459073

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (1, 1, 0) x (1, 1, 1, 12)12 : AIC Calculated =1452.6156897862832
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.70288D+00    |proj g|=  5.26802D+00

At iterate    5    f=  7.77316D+00    |proj g|=  3.58673D-04

At iterate   10    f=  7.77316D+00    |proj g|=  2.76019D-03

At iterate   15    f=  7.77303D+00    |proj g|=  3.52795D-02

At iterate   20    f=  7.76750D+00    |proj g|=  1.04760D-01

At iterate   25    f=  7.76610D+00    |proj g|=  1.57417D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
```

```
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     26     28      1     0     0   1.329D-05   7.766D+00
  F =   7.7660978493658188

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 1) x (0, 0, 0, 12)12 : AIC Calculated =1745.6059182579434
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  6.87983D+00    |proj g|=  4.75456D-01

At iterate    5    f=  6.81750D+00    |proj g|=  1.19275D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4      8     10      1     0     0   5.805D-05   6.818D+00
  F =   6.8175006009665173

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 1) x (0, 0, 1, 12)12 : AIC Calculated =1535.1201346164999
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.48250D+01    |proj g|=  3.14285D+01

At iterate    5    f=  7.35838D+00    |proj g|=  1.30784D-01

At iterate   10    f=  7.35673D+00    |proj g|=  3.04548D-04

At iterate   15    f=  7.35672D+00    |proj g|=  6.89767D-03
 This problem is unconstrained.
```

```
 This problem is unconstrained.
 This problem is unconstrained.
At iterate   20    f=  7.35632D+00    |proj g|=  7.88677D-02

At iterate   25    f=  7.34050D+00    |proj g|=  2.21006D-01

At iterate   30    f=  7.33478D+00    |proj g|=  1.55329D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     31     33      1     0     0   1.455D-05   7.335D+00
  F =   7.3347754280275925

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 1) x (0, 1, 0, 12)12 : AIC Calculated =1648.9896958781808
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.28508D+00    |proj g|=  9.41642D+00

At iterate    5    f=  6.26809D+00    |proj g|=  3.19908D-02

At iterate   10    f=  6.26770D+00    |proj g|=  4.78958D-04
 This problem is unconstrained.
At iterate   15    f=  6.26767D+00    |proj g|=  1.01166D-02

At iterate   20    f=  6.26526D+00    |proj g|=  7.84676D-02

At iterate   25    f=  6.26144D+00    |proj g|=  1.23643D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
```

```
    4     26     30      1     0     0   4.879D-05   6.261D+00
  F =   6.2614430627130755

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 1) x (0, 1, 1, 12)12 : AIC Calculated =1410.5632460477289
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.29429D+01    |proj g|=  1.44975D+01

At iterate    5    f=  6.90364D+00    |proj g|=  3.84823D-01

At iterate   10    f=  6.88774D+00    |proj g|=  8.82958D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     13     19      1     0     0   1.101D-04   6.888D+00
  F =   6.8877169549761250

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 1) x (1, 0, 0, 12)12 : AIC Calculated =1550.848597914652
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.01246D+00    |proj g|=  3.67174D+00

At iterate    5    f=  6.78310D+00    |proj g|=  1.74633D-01

At iterate   10    f=  6.77497D+00    |proj g|=  4.44140D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
```

```
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     12     13      1     0     0   7.057D-05   6.775D+00
  F =   6.7749637425929787

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (1, 1, 1) x (1, 0, 1, 12)12 : AIC Calculated =1527.5918783408272
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.44712D+01    |proj g|=  2.90046D+01

At iterate    5    f=  6.39936D+00    |proj g|=  3.54097D-01

At iterate   10    f=  6.38931D+00    |proj g|=  5.68658D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     12     37      1     0     0   4.680D-05   6.389D+00
  F =   6.3893132649318991

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
ARIMA (1, 1, 1) x (1, 1, 0, 12)12 : AIC Calculated =1439.2061713447454
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds
```

```
At iterate    0    f=  1.19399D+01    |proj g|=  3.15431D+01

At iterate    5    f=  6.31075D+00    |proj g|=  3.19721D-01

At iterate   10    f=  6.27257D+00    |proj g|=  6.68790D-02

At iterate   15    f=  6.27081D+00    |proj g|=  7.75197D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     16     19      1     0     0   7.752D-05   6.271D+00
  F =   6.2708121769165714

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (1, 1, 1) x (1, 1, 1, 12)12 : AIC Calculated =1414.661927629312
pdq         (1, 1, 1)
pdqs    (0, 1, 1, 12)
aic       1410.563246
Name: 59, dtype: object
```

Out[266…

	pdq	pdqs	aic
0	(0, 0, 0)	(0, 0, 0, 12)	3082.319133
1	(0, 0, 0)	(0, 0, 1, 12)	2698.343105
2	(0, 0, 0)	(0, 1, 0, 12)	2275.834284
3	(0, 0, 0)	(0, 1, 1, 12)	2012.685459
4	(0, 0, 0)	(1, 0, 0, 12)	2206.358155
...	...	...	...
59	(1, 1, 1)	(0, 1, 1, 12)	1410.563246
60	(1, 1, 1)	(1, 0, 0, 12)	1550.848598
61	(1, 1, 1)	(1, 0, 1, 12)	1527.591878
62	(1, 1, 1)	(1, 1, 0, 12)	1439.206171
63	(1, 1, 1)	(1, 1, 1, 12)	1414.661928

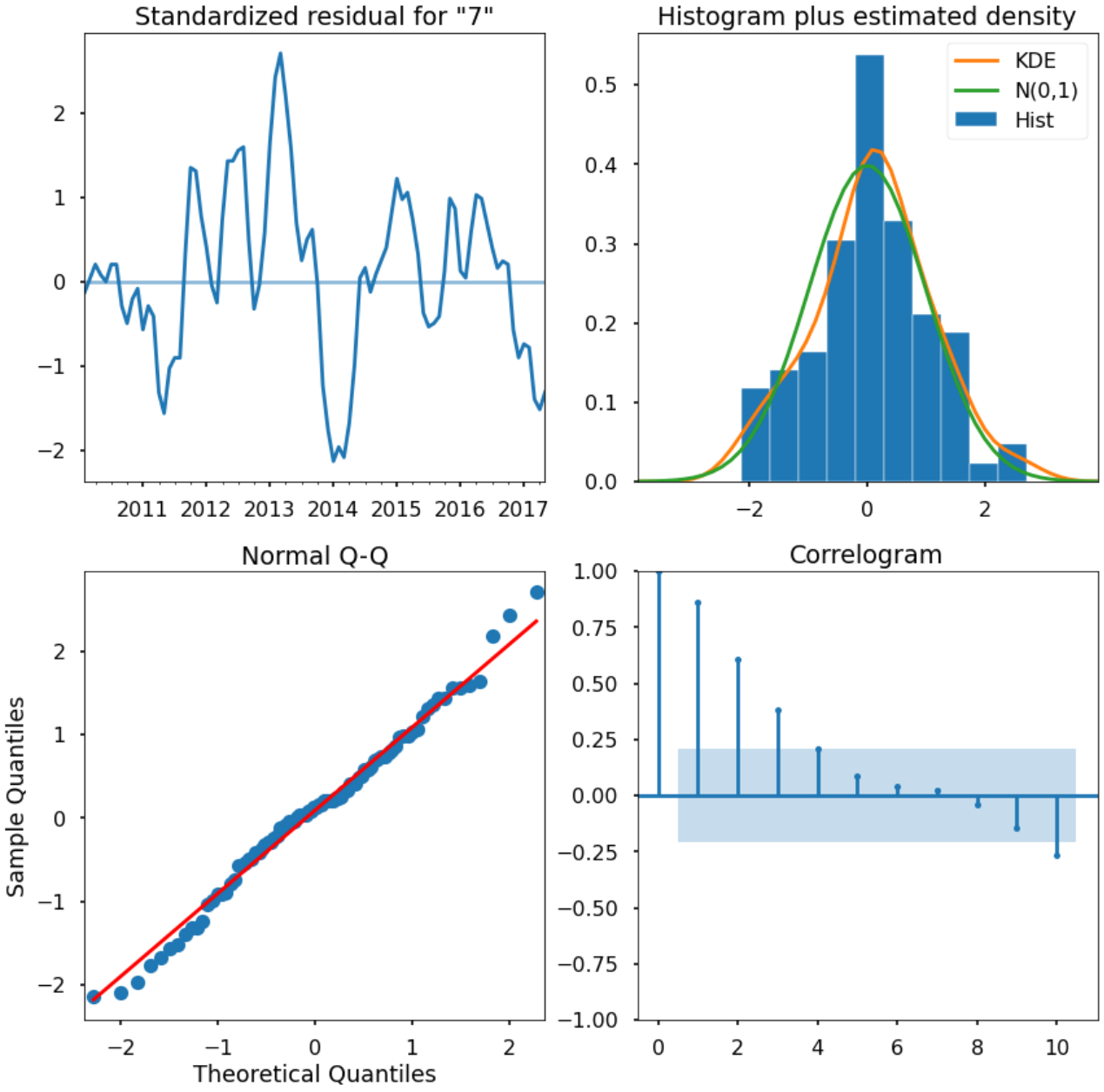
64 rows × 3 columns

In [267…

```
pdq =(1, 1, 1)
pdqs =(0, 1, 1, 12)
train,test,results= model_fit(ts_78723,pdq=pdq,pdqs=pdqs)
```

```
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.04431D+01    |proj g|=  1.05202D+01

At iterate    5    f=  9.44569D+00    |proj g|=  1.05304D-01

At iterate   10    f=  9.39262D+00    |proj g|=  1.42885D+00

At iterate   15    f=  9.35281D+00    |proj g|=  5.67666D-03

At iterate   20    f=  9.35222D+00    |proj g|=  7.63955D-02

At iterate   25    f=  9.10796D+00    |proj g|=  3.69612D+00
 This problem is unconstrained.
At iterate   30    f=  8.04025D+00    |proj g|=  7.13872D-02

At iterate   35    f=  8.02713D+00    |proj g|=  2.29903D-04

At iterate   40    f=  8.02711D+00    |proj g|=  1.15894D-02

At iterate   45    f=  8.02685D+00    |proj g|=  2.33932D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     49     68      1     0     0   1.064D-03   8.027D+00
  F =   8.0268183517932457

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
                                      SARIMAX Results
==========================================================================
==========
Dep. Variable:                               78723   No. Observations:
101
Model:             SARIMAX(1, 1, 1)x(0, 1, 1, 12)   Log Likelihood
-810.709
Date:                            Wed, 19 Oct 2022   AIC
1629.417
Time:                                   13:09:38   BIC
1639.327
Sample:                                 01-01-2009   HQIC
1633.410
```

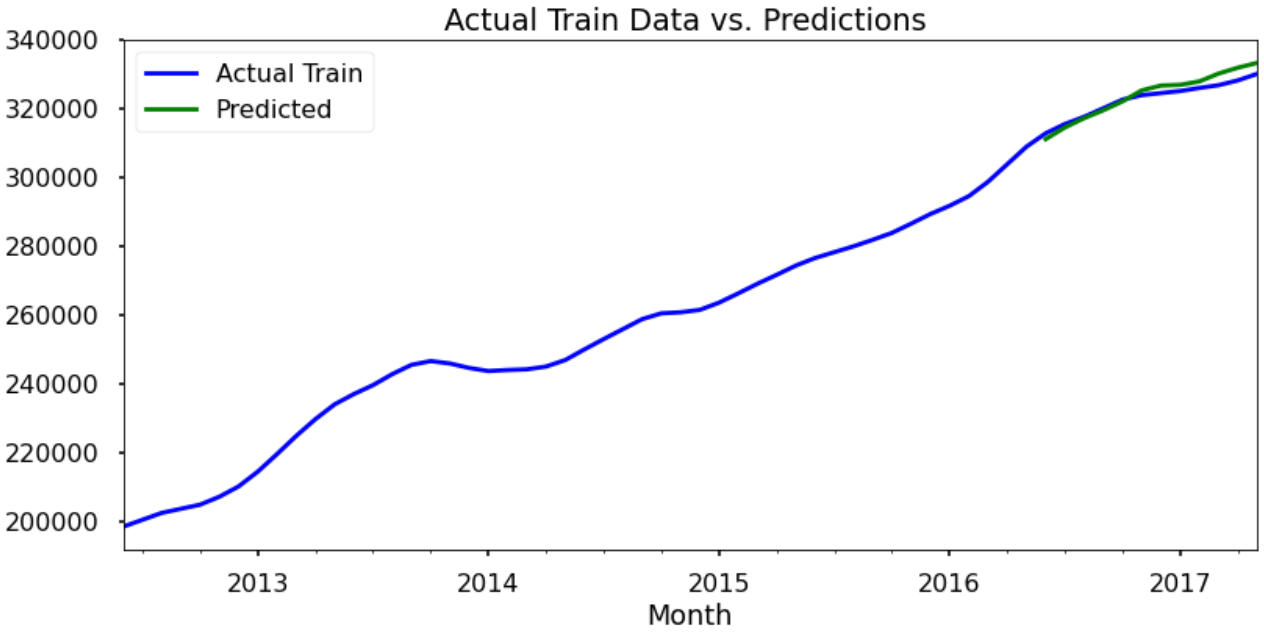
```
                                      - 05-01-2017
Covariance Type:                              opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ar.L1          0.9999     22.929      0.044      0.965     -43.940      45.939
ma.L1         -0.9999     23.024     -0.043      0.965     -46.126      44.126
ma.S.L12      -0.0044      0.027     -0.164      0.869      -0.057       0.048
sigma2      5.898e+06      0.001    8.1e+09      0.000     5.9e+06     5.9e+06
==============================================================================
===
Ljung-Box (L1) (Q):                  68.04   Jarque-Bera (JB):
0.00
Prob(Q):                              0.00   Prob(JB):
1.00
Heteroskedasticity (H):               1.07   Skew:                             -
0.00
Prob(H) (two-sided):                  0.87   Kurtosis:
3.03
==============================================================================
===

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-s
tep).
[2] Covariance matrix is singular or near-singular, with condition number 3.45e+
25. Standard errors may be unstable.
```



In [268…

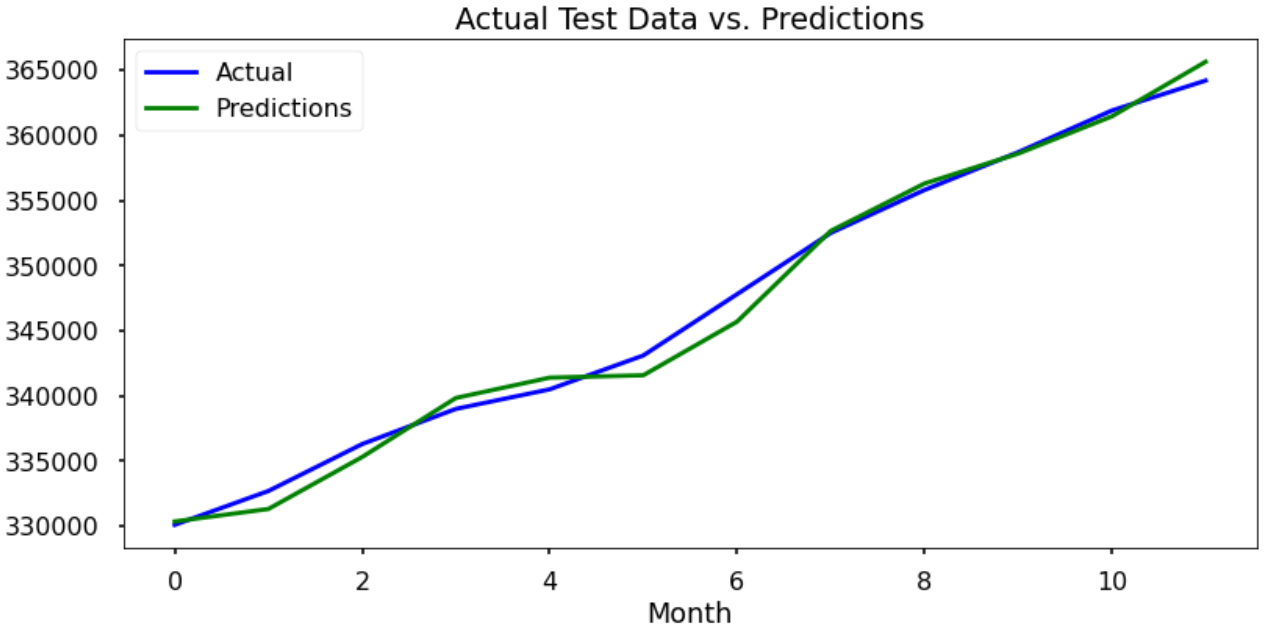
```
train_model(train,results)
```

```
SARIMA model RMSE on train data: 2111.51308
```



In [269…
```
test_model(ts_78723,pdq=pdq,pdqs=pdqs)
```

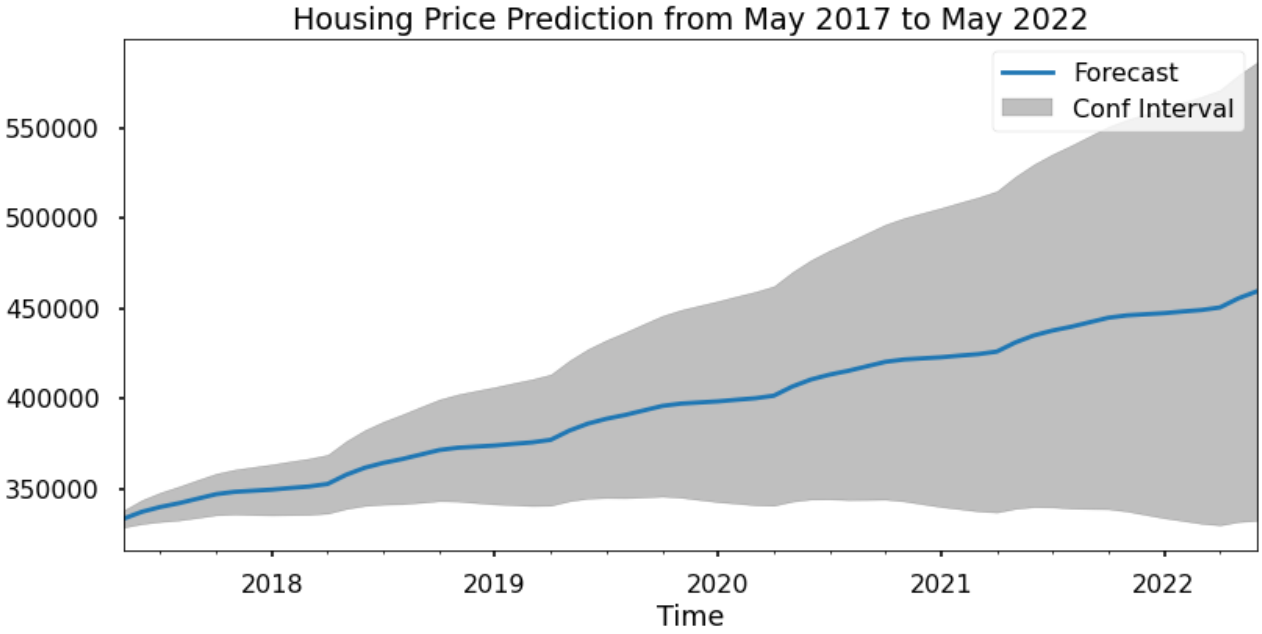
```
SARIMA model RMSE on train data: 1071.15522
```



In [270…
```
pred_78723 = forecast_model(ts_78723,pdq=pdq,pdqs=pdqs,zc=78723)
```

```
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  6.64169D+01    |proj g|=  1.03347D+01
```

```
At iterate    5    f=  9.43871D+00    |proj g|=  8.86867D-02

At iterate   10    f=  9.38803D+00    |proj g|=  2.65350D+00
 This problem is unconstrained.
At iterate   15    f=  9.31244D+00    |proj g|=  3.63308D-03

At iterate   20    f=  9.31225D+00    |proj g|=  7.38875D-02

At iterate   25    f=  9.28875D+00    |proj g|=  5.29279D-01

At iterate   30    f=  8.13239D+00    |proj g|=  7.93081D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     34     51      1     0     0   4.606D-05   8.132D+00
  F =   8.1318980270379662

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
```




```
Housing Price Projection for 2017, 2018,2020 and 2022:
Housing price at enter point 2017-05:$ 333285.63
Housing price in year 1:$ 357732.69 with 7.34 % ROI
Housing price in year 3:$ 406628.24 with 22.01 % ROI
Housing price in year 5:$ 455525.69 with  36.68 % ROI
```

In [271…

```
# Add the 5 yr roi to the results summary df
df_zip_5YR_ROI.loc[df_zip_5YR_ROI['Zipcode'] == 78723, '5YR_PRED_ROI'] = 36.68
df_zip_5YR_ROI
```

Out[271…

	Zipcode	5_YR_PRED_ROI	5YR_PRED_ROI
0	78702	0	34.83
1	78758	0	84.89
2	78721	0	63.65
3	78723	0	36.68
4	78741	0	NaN

Zipcode 78741

In [272…

```
AIC_PDQS(ts_78741)
```

```
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            1     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.33503D+01    |proj g|=  1.54543D-08

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    1      0      1      0     0     0   1.545D-08   1.335D+01
  F =   13.350329206414203

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 0) x (0, 0, 0, 12)12 : AIC Calculated =2992.4737422367816
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.41043D+02    |proj g|=  8.82318D+00

           * * *
```

```
Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      4     11      1     0     0   4.708D-06   1.166D+01
  F =   11.664645627066021

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 0) x (0, 0, 1, 12)12 : AIC Calculated =2616.8806204627886
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            1     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.85535D+00    |proj g|=  7.30918D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    1      0      1      0     0     0   7.309D-06   9.855D+00
  F =   9.8553539910548942

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 0) x (0, 1, 0, 12)12 : AIC Calculated =2209.599293996296
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  3.53724D+02    |proj g|=  2.17860D+01

At iterate    5    f=  3.71952D+01    |proj g|=  7.81985D-01
 This problem is unconstrained.
 This problem is unconstrained.
```

```
 This problem is unconstrained.
 This problem is unconstrained.
At iterate   10    f=  1.59229D+01    |proj g|=  3.69737D-02

At iterate   15    f=  1.37075D+01    |proj g|=  1.33844D-03

At iterate   20    f=  1.36390D+01    |proj g|=  1.11015D-04

At iterate   25    f=  1.05836D+01    |proj g|=  2.96446D-03
  ys=-4.499E+01  -gs= 5.942E-01 BFGS update SKIPPED

At iterate   30    f=  8.70663D+00    |proj g|=  1.26350D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2     31     55      1     1     0   2.630D-06   8.707D+00
  F =   8.7066322935733584

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 0) x (0, 1, 1, 12)12 : AIC Calculated =1954.2856337604321
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.59536D+00    |proj g|=  9.22960D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     15      1     0     0   9.229D-04   9.595D+00
  F =   9.5953590496974837

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 0) x (1, 0, 0, 12)12 : AIC Calculated =2153.3604271322365
```

```
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.00855D+03    |proj g|=  2.99901D+02

At iterate    5    f=  5.69145D+02    |proj g|=  2.91183D+02

At iterate   10    f=  1.21248D+01    |proj g|=  1.31293D+01

At iterate   15    f=  9.46106D+00    |proj g|=  4.13049D-01

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     18     22      1     0     0   1.835D-05   9.461D+00
  F =   9.4607217872874987

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 0) x (1, 0, 1, 12)12 : AIC Calculated =2125.2016803523998
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.36377D+00    |proj g|=  1.08022D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     11      1     0     0   1.075D-05   8.364D+00
```

```
  F =   8.3637730241766501

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (0, 0, 0) x (1, 1, 0, 12)12 : AIC Calculated =1877.4851574155698
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  2.80996D+02    |proj g|=  3.74154D+01

At iterate    5    f=  1.11667D+01    |proj g|=  2.26018D-01

At iterate   10    f=  1.11585D+01    |proj g|=  2.91222D-02

At iterate   15    f=  1.11575D+01    |proj g|=  4.19094D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     17     19      1     0     0   5.892D-05   1.116D+01
  F =   11.157496755782875

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (0, 0, 0) x (1, 1, 1, 12)12 : AIC Calculated =2505.279273295364
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.57578D+01    |proj g|=  5.22733D+00

At iterate    5    f=  1.29825D+01    |proj g|=  3.34457D-01
```

```
At iterate   10    f=  1.28782D+01    |proj g|=  1.13207D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2     12     22      1     0     0   3.201D-05   1.288D+01
  F =   12.878220717290633

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 1) x (0, 0, 0, 12)12 : AIC Calculated =2888.721440673102
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.43365D+02    |proj g|=  8.69765D+00

At iterate    5    f=  1.14057D+01    |proj g|=  5.55663D-02

At iterate   10    f=  1.13965D+01    |proj g|=  5.85757D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     14     40      1     0     0   4.578D-06   1.140D+01
  F =   11.396420079540174

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (0, 0, 1) x (0, 0, 1, 12)12 : AIC Calculated =2558.798097816999
RUNNING THE L-BFGS-B CODE

           * * *
```

```
Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.01071D+01    |proj g|=  3.46718D-01

At iterate    5    f=  9.49756D+00    |proj g|=  3.06924D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      5     14      1     0     0   3.069D-05   9.498D+00
  F =   9.4975583845648508

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 1) x (0, 1, 0, 12)12 : AIC Calculated =2131.4530781425265
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  4.30458D+02    |proj g|=  2.25557D+01
 This problem is unconstrained.
 This problem is unconstrained.
At iterate    5    f=  3.62399D+01    |proj g|=  2.11126D+01

At iterate   10    f=  2.11023D+01    |proj g|=  3.16899D+01

At iterate   15    f=  1.42605D+01    |proj g|=  5.10558D+00

At iterate   20    f=  1.23684D+01    |proj g|=  7.98927D-02

At iterate   25    f=  1.23277D+01    |proj g|=  2.91378D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *
```

```
   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     28     48      1     0     0   8.978D-05   1.233D+01
  F =   12.327739934980974

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 0, 1) x (0, 1, 1, 12)12 : AIC Calculated =2767.4137454357383
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.29224D+01    |proj g|=  1.99492D-01

At iterate    5    f=  1.15859D+01    |proj g|=  7.68795D-04

At iterate   10    f=  1.15859D+01    |proj g|=  5.47171D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     10     13      1     0     0   5.472D-06   1.159D+01
  F =   11.585864412031066

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 1) x (1, 0, 0, 12)12 : AIC Calculated =2601.2336282949586
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.61460D+02    |proj g|=  2.90001D+02

At iterate    5    f=  1.18912D+01    |proj g|=  6.91235D-01

At iterate   10    f=  1.17817D+01    |proj g|=  2.64945D-01

At iterate   15    f=  1.13679D+01    |proj g|=  6.29265D-02

At iterate   20    f=  1.13553D+01    |proj g|=  2.35805D-02

At iterate   25    f=  1.13536D+01    |proj g|=  8.69934D-04
```

```
 This problem is unconstrained.
 This problem is unconstrained.
           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     29     57      1     0     0   8.113D-06   1.135D+01
  F =   11.353553367320908

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 0, 1) x (1, 0, 1, 12)12 : AIC Calculated =2551.195954279883
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.05822D+00    |proj g|=  3.17733D-01

At iterate    5    f=  8.46079D+00    |proj g|=  8.43272D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      8     12      1     0     0   3.561D-05   8.461D+00
  F =   8.4607815876014900

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (0, 0, 1) x (1, 1, 0, 12)12 : AIC Calculated =1901.2150756227338
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds
```

```
At iterate    0    f=  5.08101D+02    |proj g|=  7.54710D+01

At iterate    5    f=  5.28162D+01    |proj g|=  8.81578D+00

At iterate   10    f=  1.80419D+01    |proj g|=  7.08400D-01

At iterate   15    f=  1.43264D+01    |proj g|=  1.93370D-01

At iterate   20    f=  1.41361D+01    |proj g|=  9.74839D-04

At iterate   25    f=  1.41357D+01    |proj g|=  3.26351D-04

At iterate   30    f=  1.41296D+01    |proj g|=  7.29867D-03
 This problem is unconstrained.
At iterate   35    f=  1.29371D+01    |proj g|=  5.41637D-02

At iterate   40    f=  1.26793D+01    |proj g|=  2.26473D-02
  ys=-1.398E-01  -gs= 4.354E-01 BFGS update SKIPPED

At iterate   45    f=  1.13712D+01    |proj g|=  3.29275D-02

At iterate   50    f=  1.12063D+01    |proj g|=  7.69529D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     50     90      1     1     0   7.695D-03   1.121D+01
  F =   11.206252923167893

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT
ARIMA (0, 0, 1) x (1, 1, 1, 12)12 : AIC Calculated =2518.200654789608
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            1     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.72721D+00    |proj g|=  9.86962D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
```

```
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    1      0      1      0     0     0   9.870D-06   8.727D+00
  F =   8.7272106248080696

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 0) x (0, 0, 0, 12)12 : AIC Calculated =1956.8951799570077
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.78282D+00    |proj g|=  3.84136D-02

At iterate    5    f=  7.78049D+00    |proj g|=  8.54391D-04

At iterate   10    f=  7.78019D+00    |proj g|=  1.34272D-02

At iterate   15    f=  7.75303D+00    |proj g|=  1.04707D-01

At iterate   20    f=  7.74289D+00    |proj g|=  2.48690D-09

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2     20     22      1     0     0   2.487D-09   7.743D+00
  F =   7.7428872499495744

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 0) x (0, 0, 1, 12)12 : AIC Calculated =1738.4067439887046
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            1     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.80204D+00    |proj g|=  7.37014D-05

           * * *
```

```
Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    1      2      5      1     0     0   7.702D-06   7.798D+00
  F =   7.7980827363435470

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 0) x (0, 1, 0, 12)12 : AIC Calculated =1748.7705329409546
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.95768D+00    |proj g|=  8.14543D+00

At iterate    5    f=  6.87373D+00    |proj g|=  3.90776D-03

At iterate   10    f=  6.87373D+00    |proj g|=  1.87642D-03

At iterate   15    f=  6.87359D+00    |proj g|=  2.69068D-02
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
At iterate   20    f=  6.85937D+00    |proj g|=  2.56148D-01

At iterate   25    f=  6.84870D+00    |proj g|=  1.82165D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2     25     30      1     0     0   1.822D-06   6.849D+00
  F =   6.8487045645363311

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 0) x (0, 1, 1, 12)12 : AIC Calculated =1538.109822456138
RUNNING THE L-BFGS-B CODE
```

```
           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.79101D+00    |proj g|=  4.20357D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      0      1      0     0     0   4.204D-06   7.791D+00
  F =   7.7910140264167627

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 0) x (1, 0, 0, 12)12 : AIC Calculated =1749.187141917355
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.78771D+00    |proj g|=  6.19832D-01

At iterate    5    f=  7.72921D+00    |proj g|=  2.93342D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      7      9      1     0     0   1.070D-04   7.729D+00
  F =   7.7292081067823402

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 1, 0) x (1, 0, 1, 12)12 : AIC Calculated =1737.3426159192443
RUNNING THE L-BFGS-B CODE
```

```
           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  6.86034D+00    |proj g|=  1.90470D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      0      1      0     0     0   1.905D-06   6.860D+00
  F =   6.8603416886423316

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (0, 1, 0) x (1, 1, 0, 12)12 : AIC Calculated =1540.7165382558824
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  6.80702D+00    |proj g|=  4.01794D-01

At iterate    5    f=  6.80010D+00    |proj g|=  8.38891D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      8     10      1     0     0   1.021D-04   6.800D+00
  F =   6.8000868858516776
```

```
CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (0, 1, 0) x (1, 1, 1, 12)12 : AIC Calculated =1529.2194624307758
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.47422D+00    |proj g|=  5.57289D-01

At iterate    5    f=  8.21779D+00    |proj g|=  3.53914D-04

At iterate   10    f=  8.21778D+00    |proj g|=  7.97686D-03

At iterate   15    f=  8.21615D+00    |proj g|=  8.75956D-02

At iterate   20    f=  8.15224D+00    |proj g|=  4.18428D-01

At iterate   25    f=  8.10619D+00    |proj g|=  6.15913D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2     27     32      1     0     0   1.061D-06   8.106D+00
  F =   8.1061839192256606

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 1) x (0, 0, 0, 12)12 : AIC Calculated =1819.785197906548
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.54057D+00    |proj g|=  4.96008D-01

At iterate    5    f=  7.30807D+00    |proj g|=  8.05273D-03

At iterate   10    f=  7.30790D+00    |proj g|=  3.15187D-03

At iterate   15    f=  7.30517D+00    |proj g|=  4.73672D-02
```

```
At iterate   20    f=  7.20221D+00    |proj g|=  4.32173D-02

At iterate   25    f=  7.19712D+00    |proj g|=  5.35341D-06

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     25     27      1     0     0   5.353D-06   7.197D+00
  F =   7.1971153697737691

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 1) x (0, 0, 1, 12)12 : AIC Calculated =1618.1538428293243
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.07796D+00    |proj g|=  7.00279D+00

At iterate    5    f=  7.80316D+00    |proj g|=  5.54922D-04
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
At iterate   10    f=  7.80315D+00    |proj g|=  4.10763D-03

At iterate   15    f=  7.80200D+00    |proj g|=  5.71551D-02

At iterate   20    f=  7.70282D+00    |proj g|=  6.28705D-01

At iterate   25    f=  7.54204D+00    |proj g|=  3.84288D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2     29     34      1     0     0   9.400D-06   7.540D+00
  F =   7.5398980172988743
```

```
CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 1) x (0, 1, 0, 12)12 : AIC Calculated =1692.9371558749478
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.50044D+00    |proj g|=  6.48274D+00

At iterate    5    f=  6.48600D+00    |proj g|=  8.93946D-02

At iterate   10    f=  6.47899D+00    |proj g|=  5.34830D-04

At iterate   15    f=  6.47883D+00    |proj g|=  4.26721D-02

At iterate   20    f=  6.46141D+00    |proj g|=  4.05343D-01
 This problem is unconstrained.
At iterate   25    f=  6.41654D+00    |proj g|=  9.16874D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     28     32      1     0     0   8.461D-06   6.417D+00
  F =   6.4165228943584420

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (0, 1, 1) x (0, 1, 1, 12)12 : AIC Calculated =1443.301128336291
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.61986D+00    |proj g|=  3.22064D+00

At iterate    5    f=  7.56735D+00    |proj g|=  5.37161D-01

At iterate   10    f=  7.48363D+00    |proj g|=  2.96066D-04

           * * *

Tit   = total number of iterations
```

```
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     10     16      1     0     0   2.961D-04   7.484D+00
  F =   7.4836252743300582

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 1, 1) x (1, 0, 0, 12)12 : AIC Calculated =1682.332061449933
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.65993D+00    |proj g|=  4.70631D-01

At iterate    5    f=  7.30829D+00    |proj g|=  1.10375D-02

At iterate   10    f=  7.30792D+00    |proj g|=  4.13833D-04

At iterate   15    f=  7.30785D+00    |proj g|=  1.62948D-02

At iterate   20    f=  7.30015D+00    |proj g|=  1.28866D-01
 This problem is unconstrained.
 This problem is unconstrained.
At iterate   25    f=  7.25939D+00    |proj g|=  1.57982D-01

At iterate   30    f=  7.20473D+00    |proj g|=  1.10660D-01

At iterate   35    f=  7.19749D+00    |proj g|=  1.06642D-02

At iterate   40    f=  7.19699D+00    |proj g|=  3.55197D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     41     53      1     0     0   3.035D-06   7.197D+00
  F =   7.1969920482233052

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
```

```
ARIMA (0, 1, 1) x (1, 0, 1, 12)12 : AIC Calculated =1620.1262188020203
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.10987D+01    |proj g|=  1.16678D+01
 This problem is unconstrained.
At iterate    5    f=  6.90500D+00    |proj g|=  3.12175D-01

At iterate   10    f=  6.61081D+00    |proj g|=  1.47691D-01

At iterate   15    f=  6.60837D+00    |proj g|=  8.06731D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     16     27      1     0     0   8.067D-05   6.608D+00
  F =   6.6083703537254346

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (0, 1, 1) x (1, 1, 0, 12)12 : AIC Calculated =1486.2749592344974
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.79318D+00    |proj g|=  6.33426D+00

At iterate    5    f=  6.47505D+00    |proj g|=  3.03251D-01

At iterate   10    f=  6.42229D+00    |proj g|=  1.57741D-01

At iterate   15    f=  6.41195D+00    |proj g|=  7.52416D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
```

```
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     18     29      1     0     0   1.637D-04   6.412D+00
  F =   6.4119446321616689

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (0, 1, 1) x (1, 1, 1, 12)12 : AIC Calculated =1444.2755976042138
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.54996D+00    |proj g|=  7.88069D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     16      1     0     0   7.881D-02   8.550D+00
  F =   8.5499576543279669

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 0) x (0, 0, 0, 12)12 : AIC Calculated =1919.1905145694648
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.51573D+00    |proj g|=  1.06212D+01

At iterate    5    f=  8.89569D+00    |proj g|=  4.12412D+01

At iterate   10    f=  7.62054D+00    |proj g|=  1.03498D-01

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
```

```
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     12     30      1     0     0   1.178D-05   7.621D+00
  F =   7.6205414593929062

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 0) x (0, 0, 1, 12)12 : AIC Calculated =1713.001286904011
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.87285D+00    |proj g|=  3.81269D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     15      1     0     0   3.811D-04   7.873D+00
  F =   7.8728452954209649

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
 This problem is unconstrained.
 This problem is unconstrained.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
ARIMA (1, 0, 0) x (0, 1, 0, 12)12 : AIC Calculated =1767.517346174296
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10
```

```
At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.95072D+00    |proj g|=  2.67196D+00

At iterate    5    f=  9.85158D+00    |proj g|=  1.30854D+00

At iterate   10    f=  9.26335D+00    |proj g|=  8.08877D-01

At iterate   15    f=  9.21811D+00    |proj g|=  3.32649D-02

At iterate   20    f=  9.21806D+00    |proj g|=  8.32319D-03

At iterate   25    f=  9.21753D+00    |proj g|=  1.15102D-01

At iterate   30    f=  9.15249D+00    |proj g|=  9.33625D-01
 This problem is unconstrained.
At iterate   35    f=  8.19383D+00    |proj g|=  2.17447D+00

At iterate   40    f=  6.97039D+00    |proj g|=  4.34432D-01

At iterate   45    f=  6.96826D+00    |proj g|=  2.75829D-02

At iterate   50    f=  6.96753D+00    |proj g|=  8.53753D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     50     79      1     0     0   8.538D-03   6.968D+00
  F =   6.9675280941712234

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT
ARIMA (1, 0, 0) x (0, 1, 1, 12)12 : AIC Calculated =1566.726293094354
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.01489D+00    |proj g|=  1.71892D+00

At iterate    5    f=  7.69733D+00    |proj g|=  8.09910D+00

At iterate   10    f=  7.61840D+00    |proj g|=  6.00237D-03

           * * *
```

```
Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     12     63      2     0     0   4.311D-05   7.618D+00
  F =   7.6183983788507925

ABNORMAL_TERMINATION_IN_LNSRCH
ARIMA (1, 0, 0) x (1, 0, 0, 12)12 : AIC Calculated =1712.5212368625776
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.15167D+01    |proj g|=  1.44045D+02

At iterate    5    f=  8.74921D+00    |proj g|=  2.83765D-01

At iterate   10    f=  8.74155D+00    |proj g|=  1.73570D+00

At iterate   15    f=  8.24764D+00    |proj g|=  1.60709D+01

At iterate   20    f=  7.75909D+00    |proj g|=  2.41930D-01

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     23     34      1     0     0   1.882D-04   7.759D+00
  F =   7.7590827761894170

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.

 Bad direction in the line search;
   refresh the lbfgs memory and restart the iteration.

 Line search cannot locate an adequate point after MAXLS
  function and gradient evaluations.
  Previous x, f and g restored.
```

```
 Possible causes: 1 error in function or gradient evaluation;
                  2 rounding error dominate computation.
 This problem is unconstrained.
ARIMA (1, 0, 0) x (1, 0, 1, 12)12 : AIC Calculated =1746.0345418664294
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.32069D+00    |proj g|=  9.05151D-01

At iterate    5    f=  6.99353D+00    |proj g|=  2.02433D+00

At iterate   10    f=  6.85068D+00    |proj g|=  6.38236D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     13     17      1     0     0   7.603D-05   6.851D+00
  F =   6.8506674856898888

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (1, 0, 0) x (1, 1, 0, 12)12 : AIC Calculated =1540.549516794535
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.33818D+01    |proj g|=  3.58103D+00

At iterate    5    f=  8.75410D+00    |proj g|=  2.23442D-01

At iterate   10    f=  8.44352D+00    |proj g|=  6.19732D-01

At iterate   15    f=  8.27463D+00    |proj g|=  2.96951D+00

At iterate   20    f=  7.99342D+00    |proj g|=  4.73079D+00
 This problem is unconstrained.
At iterate   25    f=  6.93212D+00    |proj g|=  2.80795D+00
```

```
At iterate   30    f=  6.89745D+00    |proj g|=  1.63139D-01

At iterate   35    f=  6.85017D+00    |proj g|=  2.82595D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     39     60      1     0     0   7.685D-05   6.850D+00
  F =   6.8501273474662980

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 0) x (1, 1, 1, 12)12 : AIC Calculated =1542.4285258324508
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.10951D+00    |proj g|=  5.30195D-01

At iterate    5    f=  8.02232D+00    |proj g|=  7.07359D-01

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      8     31      1     0     0   5.800D-04   8.022D+00
  F =   8.0222746610076836

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 1) x (0, 0, 0, 12)12 : AIC Calculated =1802.989524065721
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10
```

```
At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.33884D+00    |proj g|=  1.31856D+01

At iterate    5    f=  9.15329D+00    |proj g|=  3.73133D-01

At iterate   10    f=  9.14266D+00    |proj g|=  1.36953D+00

At iterate   15    f=  8.37127D+00    |proj g|=  4.30730D+01

At iterate   20    f=  7.23082D+00    |proj g|=  6.31191D-01

At iterate   25    f=  7.15413D+00    |proj g|=  5.90689D-01

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     28     60      1     0     0   3.251D-04   7.154D+00
  F =   7.1540902711759156

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.

 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
 This problem is unconstrained.
ARIMA (1, 0, 1) x (0, 0, 1, 12)12 : AIC Calculated =1610.516220743405
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.30261D+01    |proj g|=  1.53810D+01

At iterate    5    f=  8.13624D+00    |proj g|=  1.96253D-01

At iterate   10    f=  8.13107D+00    |proj g|=  4.07740D-04

At iterate   15    f=  8.13102D+00    |proj g|=  1.16108D-02

At iterate   20    f=  8.12491D+00    |proj g|=  3.17721D-01

At iterate   25    f=  7.88529D+00    |proj g|=  4.09955D+00

At iterate   30    f=  7.70184D+00    |proj g|=  3.33724D-02
```

```
At iterate   35    f=  7.69276D+00    |proj g|=  1.21076D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     39     48      1     0     0   5.267D-06   7.693D+00
  F =   7.6927526349777278

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
ARIMA (1, 0, 1) x (0, 1, 0, 12)12 : AIC Calculated =1729.176590235011
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.05097D+01    |proj g|=  8.00498D+00
 This problem is unconstrained.
 This problem is unconstrained.
At iterate    5    f=  8.80396D+00    |proj g|=  1.98443D+00

At iterate   10    f=  8.56677D+00    |proj g|=  4.03772D-02

At iterate   15    f=  8.54878D+00    |proj g|=  3.38638D-01

At iterate   20    f=  8.52607D+00    |proj g|=  2.23980D-03

At iterate   25    f=  8.52605D+00    |proj g|=  6.88191D-03

At iterate   30    f=  8.51956D+00    |proj g|=  7.70470D-02

At iterate   35    f=  7.60972D+00    |proj g|=  1.20661D+00

At iterate   40    f=  6.62283D+00    |proj g|=  2.85599D-01

At iterate   45    f=  6.53074D+00    |proj g|=  1.61608D-01

At iterate   50    f=  6.52262D+00    |proj g|=  5.58853D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
```

```
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     50     67      1     0     0   5.589D-02   6.523D+00
  F =   6.5226168450025694

STOP: TOTAL NO. of ITERATIONS REACHED LIMIT
ARIMA (1, 0, 1) x (0, 1, 1, 12)12 : AIC Calculated =1469.0661732805756
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  2.68654D+01    |proj g|=  3.64213D+01

At iterate    5    f=  8.41069D+00    |proj g|=  6.82921D+00

At iterate   10    f=  7.90296D+00    |proj g|=  3.72147D-01

At iterate   15    f=  7.28502D+00    |proj g|=  9.57850D-01

At iterate   20    f=  7.22513D+00    |proj g|=  2.23160D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     22     31      1     0     0   3.331D-04   7.225D+00
  F =   7.2251256439378322

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 1) x (1, 0, 0, 12)12 : AIC Calculated =1626.4281442420745
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.33210D+01    |proj g|=  1.50560D+02

At iterate    5    f=  8.45881D+00    |proj g|=  7.10315D-01
```

```
 This problem is unconstrained.
 This problem is unconstrained.
At iterate   10    f=  8.43808D+00    |proj g|=  4.72897D+00

At iterate   15    f=  8.40602D+00    |proj g|=  2.01766D-01

At iterate   20    f=  8.06931D+00    |proj g|=  1.42258D+01

At iterate   25    f=  7.22919D+00    |proj g|=  5.54058D+00

At iterate   30    f=  7.15820D+00    |proj g|=  3.26886D+00

At iterate   35    f=  7.15438D+00    |proj g|=  1.99344D+00

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     39     50      1     0     0   3.257D-04   7.154D+00
  F =   7.1537578485001907

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 1) x (1, 0, 1, 12)12 : AIC Calculated =1612.4417580640427
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.14315D+01    |proj g|=  1.04467D+01

At iterate    5    f=  6.92283D+00    |proj g|=  3.09203D-01

At iterate   10    f=  6.71254D+00    |proj g|=  1.89166D+00

At iterate   15    f=  6.60344D+00    |proj g|=  3.30209D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *
```

```
   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     19     32      1     0     0   9.766D-04   6.603D+00
  F =   6.6033690876871782

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (1, 0, 1) x (1, 1, 0, 12)12 : AIC Calculated =1487.154675641928
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.61569D+01    |proj g|=  1.25472D+01

At iterate    5    f=  1.24850D+01    |proj g|=  1.80347D+01

At iterate   10    f=  8.10159D+00    |proj g|=  3.13108D+00

At iterate   15    f=  7.88331D+00    |proj g|=  8.64599D-02

At iterate   20    f=  7.78376D+00    |proj g|=  1.53494D+00
 This problem is unconstrained.
At iterate   25    f=  7.24538D+00    |proj g|=  5.76408D+00

At iterate   30    f=  6.99286D+00    |proj g|=  1.08798D+00

At iterate   35    f=  6.98737D+00    |proj g|=  1.48144D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     35     48      1     0     0   1.481D-04   6.987D+00
  F =   6.9873681786793700

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 0, 1) x (1, 1, 1, 12)12 : AIC Calculated =1575.170472024179
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds
```

```
At iterate    0    f=  7.87225D+00    |proj g|=  2.72741D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     11      1     0     0   2.723D-05   7.872D+00
  F =   7.8722519154531581

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 0) x (0, 0, 0, 12)12 : AIC Calculated =1767.3844290615075
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.22286D+00    |proj g|=  9.01547D-01

At iterate    5    f=  7.03195D+00    |proj g|=  6.55422D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      8     10      1     0     0   1.491D-05   7.032D+00
  F =   7.0319429201066521

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 0) x (0, 0, 1, 12)12 : AIC Calculated =1581.15521410389
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            2     M =           10

At X0         0 variables are exactly at the bounds
```

```
At iterate    0    f=  7.35285D+00    |proj g|=  1.18776D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    2      1     10      1     0     0   1.179D-05   7.353D+00
  F =   7.3528471416459951

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (1, 1, 0) x (0, 1, 0, 12)12 : AIC Calculated =1651.037759728703
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.92761D+00    |proj g|=  8.28669D+00

At iterate    5    f=  6.47304D+00    |proj g|=  1.97544D-01

At iterate   10    f=  6.46533D+00    |proj g|=  7.87073D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3     11     14      1     0     0   1.035D-05   6.465D+00
  F =   6.4653309925251694

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (1, 1, 0) x (0, 1, 1, 12)12 : AIC Calculated =1454.234142325638
RUNNING THE L-BFGS-B CODE
```

```
           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.25452D+00    |proj g|=  5.17067D-01

At iterate    5    f=  7.02825D+00    |proj g|=  2.27247D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      8     10      1     0     0   4.007D-05   7.028D+00
  F =   7.0281364335219525

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 0) x (1, 0, 0, 12)12 : AIC Calculated =1580.3025611089174
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.38435D+00    |proj g|=  1.29753D+00

At iterate    5    f=  7.03571D+00    |proj g|=  7.02507D-02

At iterate   10    f=  7.02576D+00    |proj g|=  6.01928D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     14     25      1     0     0   4.837D-05   7.025D+00
  F =   7.0254067017931563
```

```
CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 0) x (1, 0, 1, 12)12 : AIC Calculated =1581.691101201667
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  6.40993D+00    |proj g|=  3.96178D-02
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      4      6      1     0     0   8.281D-05   6.408D+00
  F =   6.4078002269525145

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 0) x (1, 1, 0, 12)12 : AIC Calculated =1441.3472508373632
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  6.42249D+00    |proj g|=  7.33903D-01

At iterate    5    f=  6.40759D+00    |proj g|=  8.29178D-02

At iterate   10    f=  6.40662D+00    |proj g|=  8.61604D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *
```

```
   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     10     12      1     0     0   8.616D-05   6.407D+00
  F =   6.4066218141226390

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
ARIMA (1, 1, 0) x (1, 1, 1, 12)12 : AIC Calculated =1443.0832863634712
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.87443D+00    |proj g|=  1.27462D+00

At iterate    5    f=  7.73857D+00    |proj g|=  7.83078D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      6      8      1     0     0   5.451D-05   7.739D+00
  F =   7.7385725320175762

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 1) x (0, 0, 0, 12)12 : AIC Calculated =1739.440247171937
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  6.94218D+00    |proj g|=  3.15215D-01

At iterate    5    f=  6.82902D+00    |proj g|=  9.05821D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
```

```
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4      9     12      1     0     0   6.957D-05   6.829D+00
  F =   6.8289888835754127

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 1) x (0, 0, 1, 12)12 : AIC Calculated =1537.6935099208924
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            3     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  9.36266D+00    |proj g|=  1.18451D+01

At iterate    5    f=  7.25164D+00    |proj g|=  6.61027D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    3      9     11      1     0     0   1.595D-04   7.250D+00
  F =   7.2502020291665730

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (1, 1, 1) x (0, 1, 0, 12)12 : AIC Calculated =1630.0452545333123
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  6.89444D+00    |proj g|=  3.81557D+00

At iterate    5    f=  6.26211D+00    |proj g|=  3.36092D-02

At iterate   10    f=  6.26177D+00    |proj g|=  2.09539D-04

At iterate   15    f=  6.26173D+00    |proj g|=  7.94819D-03
```

```
At iterate   20    f=  6.25964D+00    |proj g|=  3.81342D-02

At iterate   25    f=  6.25885D+00    |proj g|=  3.27399D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     26     30      1     0     0   9.773D-07   6.259D+00
  F =   6.2588505723164189

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
 This problem is unconstrained.
ARIMA (1, 1, 1) x (0, 1, 1, 12)12 : AIC Calculated =1409.9825281988778
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.77344D+00    |proj g|=  5.80340D+00

At iterate    5    f=  6.92083D+00    |proj g|=  7.90496D-01

At iterate   10    f=  6.90989D+00    |proj g|=  3.44178D-03

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     13     20      1     0     0   2.580D-05   6.910D+00
  F =   6.9098816371489713

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
ARIMA (1, 1, 1) x (1, 0, 0, 12)12 : AIC Calculated =1555.8134867213696
RUNNING THE L-BFGS-B CODE

           * * *
```

```
Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  7.10740D+00    |proj g|=  7.04452D-01

At iterate    5    f=  6.81484D+00    |proj g|=  2.12543D-02

At iterate   10    f=  6.81457D+00    |proj g|=  9.40024D-05

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     10     13      1     0     0   9.400D-05   6.815D+00
  F =   6.8145735178655737

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
 This problem is unconstrained.
ARIMA (1, 1, 1) x (1, 0, 1, 12)12 : AIC Calculated =1536.4644680018885
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.06734D+01    |proj g|=  1.59410D+01

At iterate    5    f=  6.40952D+00    |proj g|=  1.39670D-01

At iterate   10    f=  6.40237D+00    |proj g|=  3.15605D-04

At iterate   15    f=  6.40236D+00    |proj g|=  1.90152D-03

At iterate   20    f=  6.40227D+00    |proj g|=  2.44805D-02

At iterate   25    f=  6.40010D+00    |proj g|=  7.08021D-02

At iterate   30    f=  6.39939D+00    |proj g|=  2.28881D-04

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
```

```
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     31     35      1     0     0   2.890D-06   6.399D+00
  F =   6.3993891459519405

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
 This problem is unconstrained.
ARIMA (1, 1, 1) x (1, 1, 0, 12)12 : AIC Calculated =1441.4631686932346
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            5     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  8.22935D+00    |proj g|=  1.18402D+01

At iterate    5    f=  6.31131D+00    |proj g|=  2.01995D-01

At iterate   10    f=  6.28824D+00    |proj g|=  1.52048D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    5     14     16      1     0     0   9.095D-06   6.288D+00
  F =   6.2879243376315372

CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL
 This problem is unconstrained.
ARIMA (1, 1, 1) x (1, 1, 1, 12)12 : AIC Calculated =1418.4950516294643
pdq         (1, 1, 1)
pdqs    (0, 1, 1, 12)
aic       1409.982528
Name: 59, dtype: object
```

Out[272…

	pdq	pdqs	aic
0	(0, 0, 0)	(0, 0, 0, 12)	2992.473742
1	(0, 0, 0)	(0, 0, 1, 12)	2616.880620
2	(0, 0, 0)	(0, 1, 0, 12)	2209.599294

	pdq	pdqs	aic
3	(0, 0, 0)	(0, 1, 1, 12)	1954.285634
4	(0, 0, 0)	(1, 0, 0, 12)	2153.360427
...	...	...	...
59	(1, 1, 1)	(0, 1, 1, 12)	1409.982528
60	(1, 1, 1)	(1, 0, 0, 12)	1555.813487
61	(1, 1, 1)	(1, 0, 1, 12)	1536.464468
62	(1, 1, 1)	(1, 1, 0, 12)	1441.463169
63	(1, 1, 1)	(1, 1, 1, 12)	1418.495052

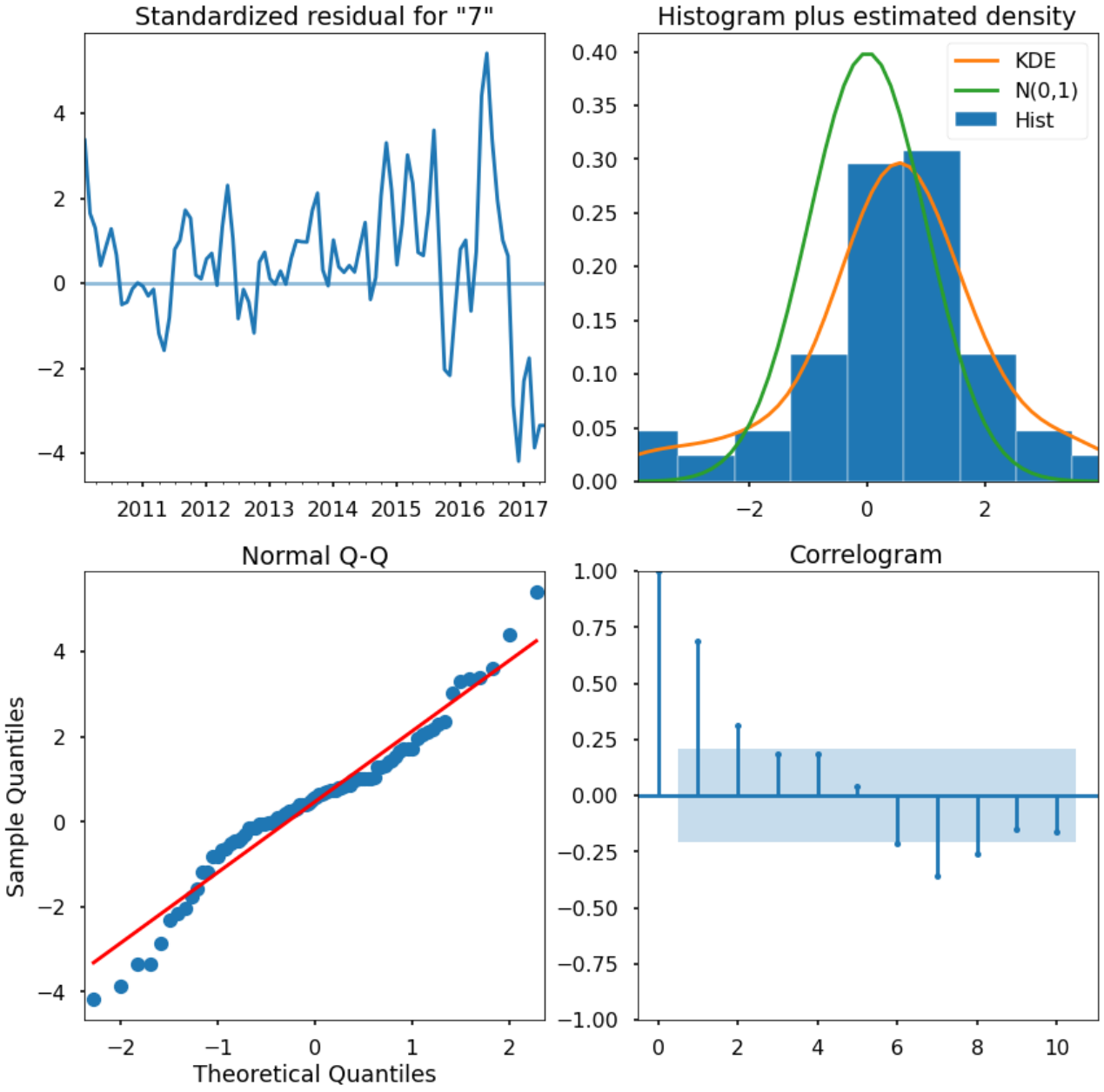
64 rows × 3 columns

In [273…

```
pdq =(1, 1, 1)
pdqs =(0, 1, 1, 12)
train,test,results= model_fit(ts_78741,pdq=pdq,pdqs=pdqs)
```

```
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  2.45479D+01    |proj g|=  1.62137D+01

At iterate    5    f=  7.78487D+00    |proj g|=  1.75494D-02

At iterate   10    f=  7.78137D+00    |proj g|=  3.34570D-01

At iterate   15    f=  7.77355D+00    |proj g|=  1.23192D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     17     20      1     0     0   2.371D-03   7.774D+00
  F =   7.7735454441724583

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 This problem is unconstrained.
                                     SARIMAX Results
```

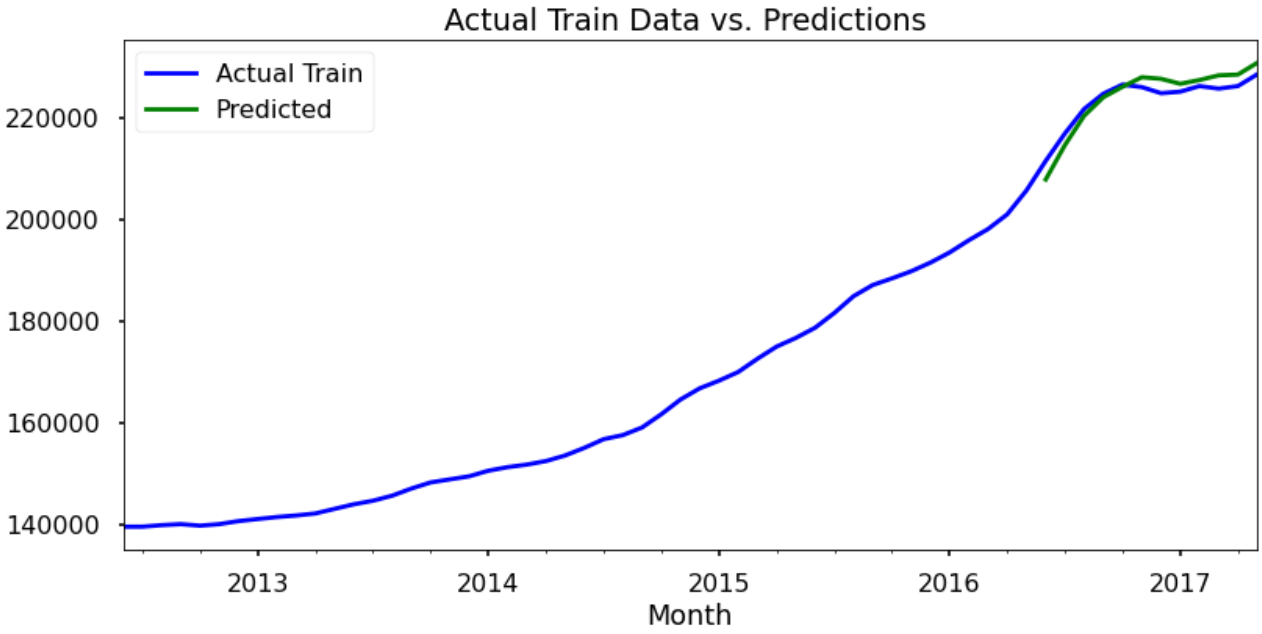
```
==========================================================================
==========
Dep. Variable:                               78741   No. Observations:
101
Model:             SARIMAX(1, 1, 1)x(0, 1, 1, 12)   Log Likelihood
-785.128
Date:                            Wed, 19 Oct 2022   AIC
1578.256
Time:                                    13:10:00   BIC
1588.166
Sample:                                01-01-2009   HQIC
1582.248
                                     - 05-01-2017
Covariance Type:                              opg
==========================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
--------------------------------------------------------------------------
ar.L1          0.3090      0.207      1.491      0.136      -0.097       0.715
ma.L1         -0.2643      0.208     -1.268      0.205      -0.673       0.144
ma.S.L12       0.0015      0.043      0.034      0.973      -0.082       0.085
sigma2      4.564e+05   1.91e+04     23.925      0.000    4.19e+05    4.94e+05
==========================================================================
===
Ljung-Box (L1) (Q):                  43.41   Jarque-Bera (JB):
6.42
Prob(Q):                              0.00   Prob(JB):
0.04
Heteroskedasticity (H):               4.59   Skew:                            -
0.20
Prob(H) (two-sided):                  0.00   Kurtosis:
4.26
==========================================================================
===

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-s
tep).
```



In [274…

```
train_model(train,results)
```

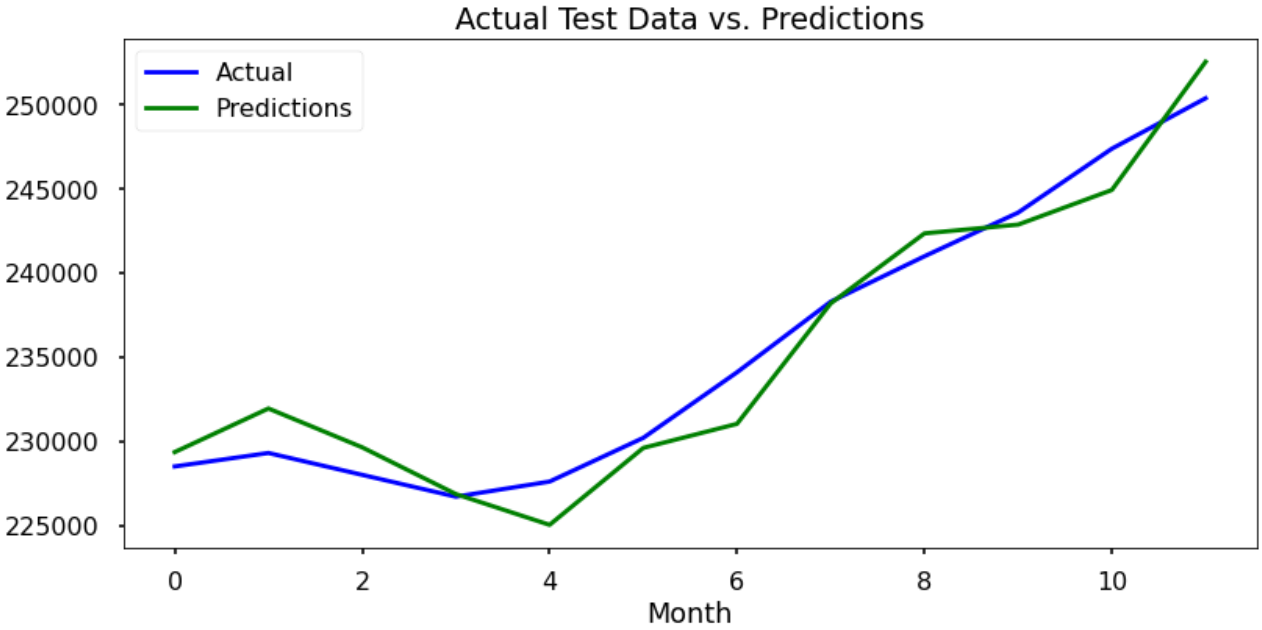
```
SARIMA model RMSE on train data: 2114.13075
```



In [275…

```
test_model(ts_78741,pdq=pdq,pdqs=pdqs)
```

```
SARIMA model RMSE on train data: 1824.55694
```

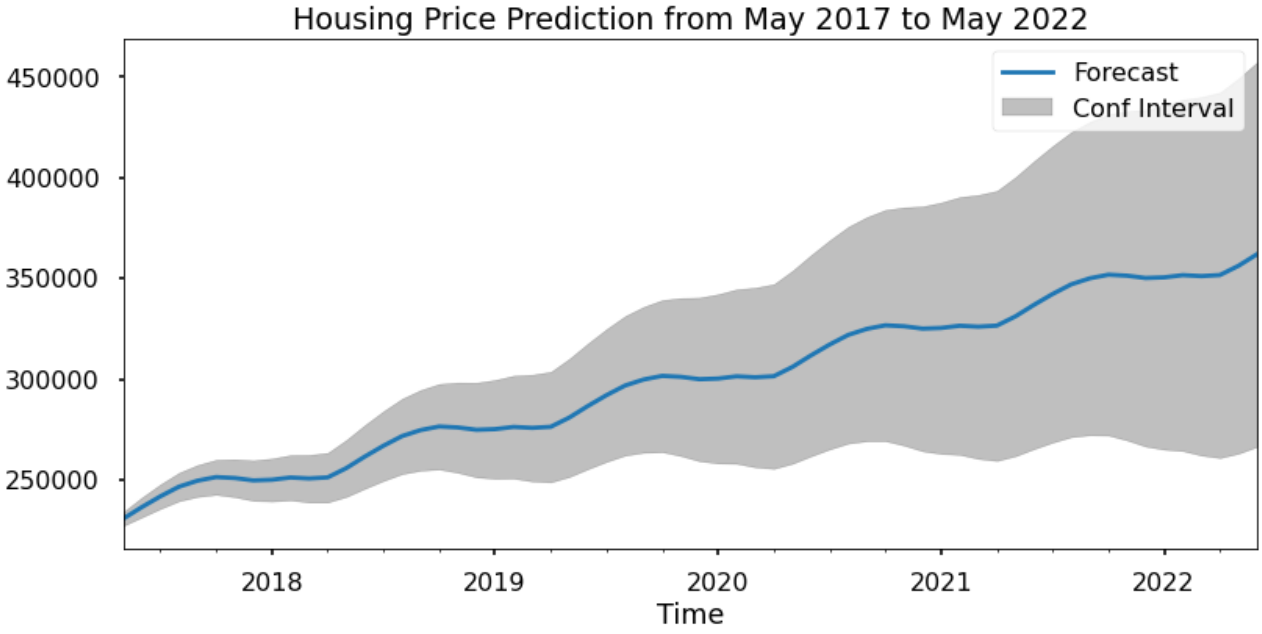


In [276…

```
pred_78741 = forecast_model(ts_78741,pdq=pdq,pdqs=pdqs,zc=78741)
```

```
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =            4     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  3.12006D+01    |proj g|=  9.25939D+00
```

```
At iterate    5    f=  8.38287D+00    |proj g|=  8.48742D-02

At iterate   10    f=  8.38027D+00    |proj g|=  3.66610D-01

At iterate   15    f=  8.35874D+00    |proj g|=  8.04270D-03

At iterate   20    f=  8.35864D+00    |proj g|=  5.48286D-02

At iterate   25    f=  8.34740D+00    |proj g|=  7.45747D-01
 This problem is unconstrained.
At iterate   30    f=  8.01149D+00    |proj g|=  2.10220D+00

At iterate   35    f=  7.86615D+00    |proj g|=  3.38132D-02

At iterate   40    f=  7.86610D+00    |proj g|=  2.00919D-02

At iterate   45    f=  7.86568D+00    |proj g|=  7.67766D-02

           * * *

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

           * * *

   N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
    4     49     70      1     0     0   2.907D-04   7.865D+00
  F =   7.8654919131303691

CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
 Warning:  more than 10 function and gradient
   evaluations in the last line search.  Termination
   may possibly be caused by a bad search direction.
```




```
Housing Price Projection for 2017, 2018,2020 and 2022:
```

```
Housing price at enter point 2017-05:$ 230792.82
Housing price in year 1:$ 255889.44 with 10.87 % ROI
Housing price in year 3:$ 306083.43 with 32.62 % ROI
Housing price in year 5:$ 356277.42 with  54.37 % ROI
```

We have 5 year ROI for selected 5 zip codes in Austin

In [277…

```
# Add the 5 yr roi to the results summary df
df_zip_5YR_ROI.loc[df_zip_5YR_ROI['Zipcode'] == 78741, '5YR_PRED_ROI'] = 54.37
df_zip_5YR_ROI
```

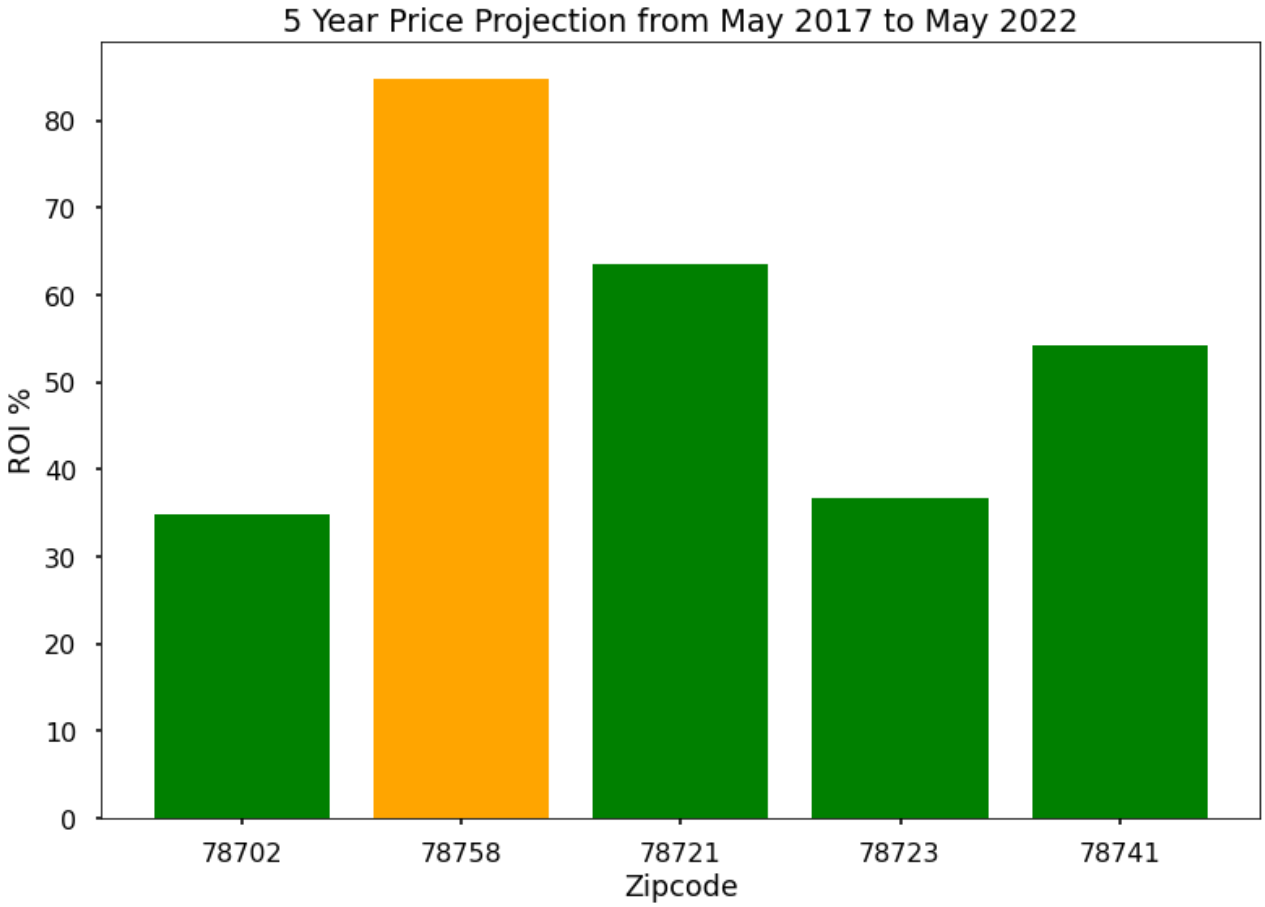
Out[277…

	Zipcode	5_YR_PRED_ROI	5YR_PRED_ROI
0	78702	0	34.83
1	78758	0	84.89
2	78721	0	63.65
3	78723	0	36.68
4	78741	0	54.37

Now we can see the ROI for 5 zip codes through below bar plot

In [278…

```
zipcode = ['78702','78758','78721','78723','78741']
zipselect_5yr_ROI = [34.83,84.89,63.65,36.68,54.33]
clrs=['green' if (value < max(zipselect_5yr_ROI)) else 'orange' for value in zip
plt.bar(zipcode, zipselect_5yr_ROI,color=clrs)
plt.title('5 Year Price Projection from May 2017 to May 2022')
plt.xlabel('Zipcode')
plt.ylabel('ROI %')
plt.show()
```



Conclusion

From the 5 year price projection, zipcode 78758 has the highest ROI which is 84.89%.

Next Step

Use the model to predict other interested state, metro area to find the property with highest ROI to invest in.

In [ ]: