Data Flow Analysis

Why:

- Provide information about a program manipulates its data.
- Study function's behavior.
- To help build control flow information.
- Program understanding (a function sorts an array!).
- Generating a model of the original program and verify the model.
- Program validation.
- ...

Reaching Definitions

A particular definition of a variable is said to reach a given point if

- there is an execution path from the defintion to that point
- the variable might *may* have the value assigned by the definition.

In general undecidable.

Different types of analysis

- Intra procedural analysis.
- Whole program (inter-procedural) analysis.
- Generate intra procedural analysis and extend it to whole program.

Iterative Dataflow Analysis

- Build a collection of data flow equations.
- Solve it iteratively.
- Start from an conservative set of initial values.

Example program

```
int f(int n){

        int i, j;
        i=0;
        n=2;
        if (n == 1)
                i = 2;
        while (n > 0){

                j = n+1;
                n = g(n,i);
        }
        return j;
}
```

Definitions

GEN : GEN(b) returns the set of definitions generated in the basic block b; assigned values in the block and not subsequently killed in it.

KILL : KILL(b) returns the set of definitions killed in the basic block b.

IN : IN(b) returns the set of definitions reaching the basic block b.

OUT : OUT(b) returns the set of definitions going out of basic block b.

PRSV : Negation of KILL

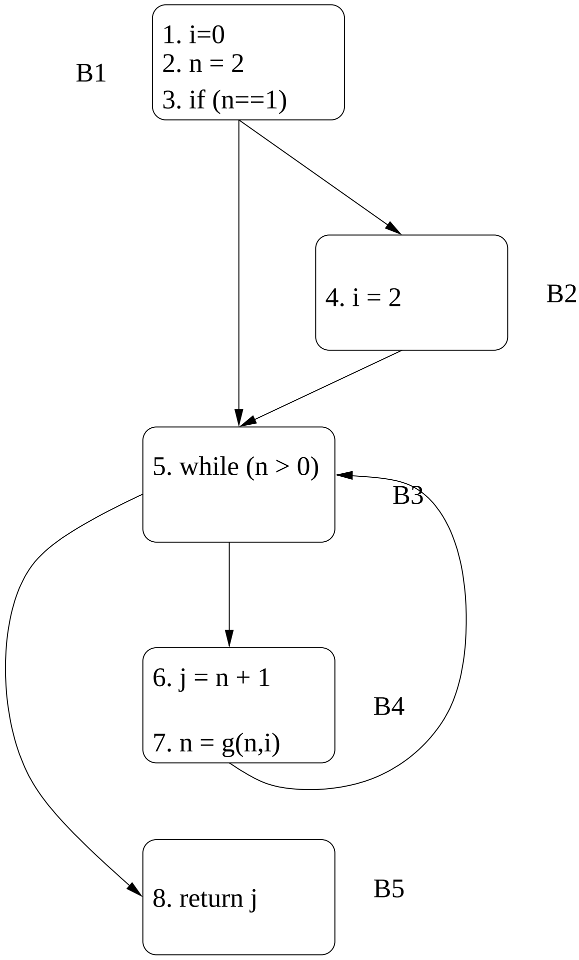
Representation and Initialization

- Set representation. e.g. GEN(B4) = $\{$5,6$\}$
- Bit Vector representation. GEN (B4) = $< 0, 0, 0, 0, 1, 1 >$

Initialization

- GEN, KILL and OUT are created in each basic block. But IN needs to be initialized to something safe.
- IN(entry) = $\{\}$

Reaching definitions



Gen(B1)	=	{1,2}	Kill(B1) = {}
Gen(B2)	=	{4}	Kill(B2) = {1}
Gen(B3)	=	{ }	Kill(B3) = {}
Gen(B4)	=	{6, 7}	Kill(B4) = {2,6,7}
Gen(B5)	=	{ }	Kill(B5) = {}

Computing Reaching Definitions

$$\mathrm{OUT}(b) = \mathrm{GEN}(b) \cup (\mathrm{IN}(b) - \mathrm{KILL}(b))$$

$$\mathrm{IN}(b) = \cup_{p \in pred(b)} \mathrm{OUT}(p)$$

Lattice

What : Lattice is an algebraic structure.

Why : To represent *abstract properties* of variables, expressions, functions, etc etc.

- Values,
- Attributes,

Why "abstract"? Exact interpretation (execution) gives exact values, abstract interpretation gives abstract values.

Lattice contd.

A lattice L consists of a set of *elements* and two n'ary operations.

meet($\sqcap$) : Greatest lower bound.

join($\sqcup$) : Lowest upper bound.

Each lattice has two special elements:

Bottm ($\perp$)

Top ($\top$)

Lattice contd.

Properties of meet and join:

Closure : For all $x, y \in L$, $\exists$ $unique$ z and $w \in L$ such that

$x \sqcap y = z$ and $x \sqcup y = w$.

Commutative : For all $x, y \in L$ $x \sqcup y = y \sqcup x$ and $x \sqcap y = y \sqcap x$.

Associative : For all $x, y, z \in L$ $x \sqcup (y \sqcup z) = (x \sqcup y) \sqcup z$ and

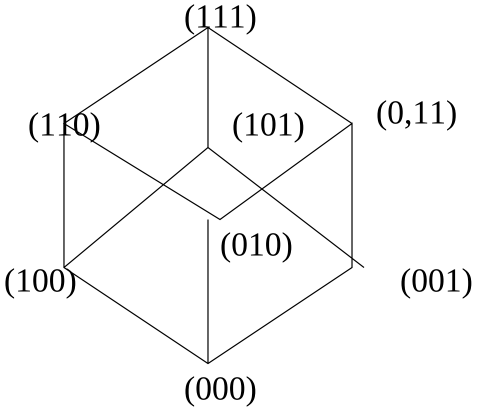
$x \sqcap (y \sqcap z) = (x \sqcap y) \sqcap z$.

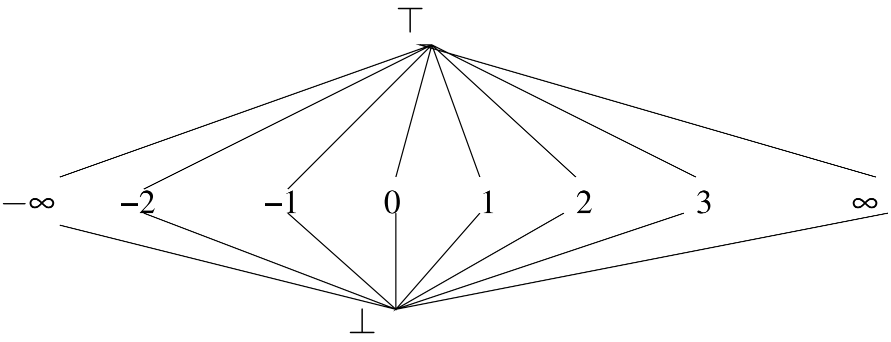
For all $x \in L$, $x \sqcap \bot = \bot$ and $x \sqcup \top = \top$.

Distributive : (Only some lattices are.) For all $x, y, z \in L$

$x \sqcup (y \sqcap z) = (x \sqcup y) \sqcap (x \sqcup z)$ and $x \sqcap (y \sqcup z) = (x \sqcap y) \sqcup (x \sqcap z)$.

Examples of Lattices.





Lattice properties contd..

Meet and join induce a partial order ($\sqsubseteq$) :

$$x \sqsubseteq y \text{ if and only if } x \sqcap y = x$$

Transitivity : For all, x, y, z if $x \sqsubseteq y$ and $y \sqsubseteq z$ then $x \sqsubseteq z$.

Antisymmetry : For all, x, y if $x \sqsubseteq y$ and $y \sqsubseteq x$ then $x = y$.

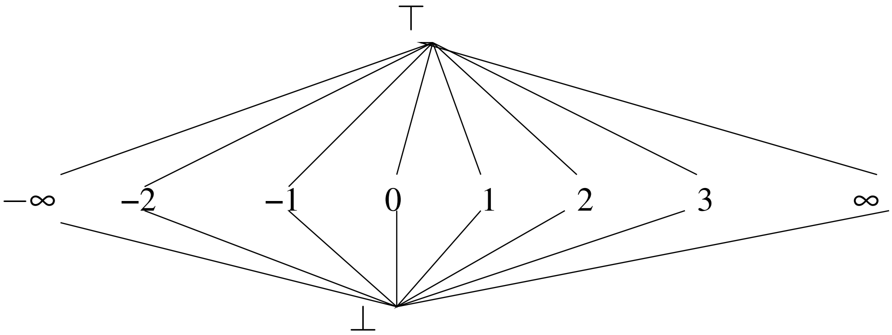
Reflexivity : For all, x if $x \sqsubseteq x$.

Constant Propagation.

Goal: Discover values that are constants on *all possible executions* of a program and to propagate these constant values as far *forward* through the program as possible

Conservative Can discover only a subset of all the possible constants.

Constant Propagation Lattice.



Constant Propagation Lattice Meet rules.

$\perp$ = Constant value cannot be guarranteed.

$\top$ = May be a constant, but not yet determined.

$$x \sqcap \top = x$$
$$x \sqcap \perp = \perp$$
$$c_1 \sqcap c_1 = c_1$$
$$c_1 \sqcap c_2 = \perp$$

Simple Constant Propagation.

Gary A. Kildall: A Unified Approach to Global Program Optimization - POPL 1973.

Reif, Lewis: Symbolic evaluation and the global value graph - POPL 1977.

Simple constant Constants that can be proved to be constant provided: No information is assumed about which direction branches will take. Only one value of each variable is maintainedalong each path in the program.

Kildall’s algorithm

- Start with an entry node in the program graph.
- Process the entry node, and produce the constant propagation information. Send it to all the immediate successors of the entry node.
- At a merge point, get an intersection of the information.
- If at any successor node, if for any variable the value is ”reduced”, the process the successor, similar to the processing done for entry node.