

## Overview of Semantic Analysis

### Lecture 9

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## Announcements

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- WA2 due today
  - Electronic hand-in
  - Or after class
  - Or under my office door by 5pm
- PA2 due tonight
  - Electronic hand-in
- Regrades
  - No more than one deduction for each error
  - But you have to prove it to us . . .

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## Midterm Thursday

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- In class
  - SCPD students come to campus for the exam
- Material through lecture 8
- Four sheets hand- or type-written notes

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## Outline

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- The role of semantic analysis in a compiler
  - A laundry list of tasks
- Scope
  - Implementation: symbol tables
- Types

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## The Compiler So Far

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- Lexical analysis
  - Detects inputs with illegal tokens
- Parsing
  - Detects inputs with ill-formed parse trees
- Semantic analysis
  - Last "front end" phase
  - Catches all remaining errors

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## Why a Separate Semantic Analysis?

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- Parsing cannot catch some errors
- Some language constructs not context-free

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## What Does Semantic Analysis Do?

- Checks of many kinds . . . coolc checks:

1. All identifiers are declared
  2. Types
  3. Inheritance relationships
  4. Classes defined only once
  5. Methods in a class defined only once
  6. Reserved identifiers are not misused
- And others . . .

- The requirements depend on the language

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## Scope

- Matching identifier declarations with uses
  - Important static analysis step in most languages
  - Including COOL!

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## What's Wrong?

- Example 1

```
Let y: String ← "abc" in y + 3
```

- Example 2

```
Let y: Int in x + 3
```

*Note: An example a property that is not context free.*

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## Scope (Cont.)

- The *scope* of an identifier is the portion of a program in which that identifier is accessible
- The same identifier may refer to different things in different parts of the program
  - Different scopes for same name don't overlap
- An identifier may have restricted scope

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## Static vs. Dynamic Scope

- Most languages have *static* scope
  - Scope depends only on the program text, not run-time behavior
  - Cool has static scope
- A few languages are *dynamically* scoped
  - Lisp, SNOBOL
  - Lisp has changed to mostly static scoping
  - Scope depends on execution of the program

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## Static Scoping Example

```
let x: Int ← 0 in
{
  x;
  let x: Int ← 1 in
    x;
  x;
}
```

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### Static Scoping Example (Cont.)

```
let x: Int ← 0 in
{
  let x: Int ← 1 in
  {
    x;
  }
}
```

Uses of `x` refer to closest enclosing definition

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### Dynamic Scope

- A dynamically-scoped variable refers to the closest enclosing binding in the execution of the program

- Example

```
g(y) = let a ← 4 in f(3);
f(x) = a;
```

- More about dynamic scope later in the course

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### Scope in Cool

- Cool identifier bindings are introduced by
  - Class declarations (introduce class names)
  - Method definitions (introduce method names)
  - Let expressions (introduce object id's)
  - Formal parameters (introduce object id's)
  - Attribute definitions (introduce object id's)
  - Case expressions (introduce object id's)

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### Scope in Cool (Cont.)

- Not all kinds of identifiers follow the most-closely nested rule
- For example, class definitions in Cool
  - Cannot be nested
  - Are *globally visible* throughout the program
- In other words, a class name can be used before it is defined

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### Example: Use Before Definition

```
Class Foo {
  ... let y: Bar in ...
};

Class Bar {
  ...
};
```

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### More Scope in Cool

Attribute names are global within the class in which they are defined

```
Class Foo {
  f(): Int { a };
  a: Int ← 0;
}
```

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### More Scope (Cont.)

- Method/attribute names have complex rules
- A method need not be defined in the class in which it is used, but in some parent class
- Methods may also be redefined (overridden)

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### Implementing the Most-Closely Nested Rule

- Much of semantic analysis can be expressed as a recursive descent of an AST

- *Before*: Process an AST node  $n$
- *Recurse*: Process the children of  $n$
- *After*: Finish processing the AST node  $n$

- When performing semantic analysis on a portion of the the AST, we need to know which identifiers are defined

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### Implementing . . . (Cont.)

- Example: the scope of `let` bindings is one subtree of the AST:

`let x: Int ← 0 in e`

- `x` is defined in subtree `e`

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### Symbol Tables

- Consider again: `let x: Int ← 0 in e`
- Idea:
  - *Before* processing `e`, add definition of `x` to current definitions, overriding any other definition of `x`
  - *Recurse*
  - *After* processing `e`, remove definition of `x` and restore old definition of `x`
- A *symbol table* is a data structure that tracks the current bindings of identifiers

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### A Simple Symbol Table Implementation

- Structure is a stack
- Operations
  - `add_symbol(x)` push `x` and associated info, such as `x`'s type, on the stack
  - `find_symbol(x)` search stack, starting from top, for `x`. Return first `x` found or NULL if none found
  - `remove_symbol()` pop the stack
- Why does this work?

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### Limitations

- The simple symbol table works for `let`
  - Symbols added one at a time
  - Declarations are perfectly nested
- What doesn't it work for?

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## A Fancier Symbol Table

- `enter_scope()` start a new nested scope
- `find_symbol(x)` finds current `x` (or null)
- `add_symbol(x)` add a symbol `x` to the table
- `check_scope(x)` true if `x` defined in current scope
- `exit_scope()` exit current scope

We will supply a symbol table manager for your project

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## Class Definitions

- Class names can be used before being defined
- We can't check class names
  - using a symbol table
  - or even in one pass
- Solution
  - Pass 1: Gather all class names
  - Pass 2: Do the checking
- Semantic analysis requires multiple passes
  - Probably more than two

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## Types

- What is a type?
  - The notion varies from language to language
- Consensus
  - A set of values
  - A set of operations on those values
- Classes are one instantiation of the modern notion of type

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## Why Do We Need Type Systems?

Consider the assembly language fragment

```
add $r1, $r2, $r3
```

What are the types of `$r1`, `$r2`, `$r3`?

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## Types and Operations

- Certain operations are legal for values of each type
  - It doesn't make sense to add a function pointer and an integer in C
  - It does make sense to add two integers
  - But both have the same assembly language implementation!

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## Type Systems

- A language's type system specifies which operations are valid for which types
- The goal of type checking is to ensure that operations are used with the correct types
  - Enforces intended interpretation of values, because nothing else will!

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## Type Checking Overview

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- Three kinds of languages:
  - *Statically typed*: All or almost all checking of types is done as part of compilation (C, Java, Cool)
  - *Dynamically typed*: Almost all checking of types is done as part of program execution (Scheme)
  - *Untyped*: No type checking (machine code)

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## The Type Wars

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- Competing views on static vs. dynamic typing
- Static typing proponents say:
  - Static checking catches many programming errors at compile time
  - Avoids overhead of runtime type checks
- Dynamic typing proponents say:
  - Static type systems are restrictive
  - Rapid prototyping difficult within a static type system

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## The Type Wars (Cont.)

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- In practice, most code is written in statically typed languages with an “escape” mechanism
  - Unsafe casts in C, Java
- It's debatable whether this compromise represents the best or worst of both worlds

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## Types Outline

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- Type concepts in COOL
- Notation for type rules
  - Logical rules of inference
- COOL type rules
- General properties of type systems

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## Cool Types

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- The types are:
  - Class Names
  - `SELF_TYPE`
- The user declares types for identifiers
- The compiler infers types for expressions
  - Infers a type for *every* expression

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## Type Checking and Type Inference

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- *Type Checking* is the process of verifying fully typed programs
- *Type Inference* is the process of filling in missing type information
- The two are different, but the terms are often used interchangeably

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## Rules of Inference

- We have seen two examples of formal notation specifying parts of a compiler
  - Regular expressions
  - Context-free grammars
- The appropriate formalism for type checking is logical rules of inference

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## Why Rules of Inference?

- Inference rules have the form  
*If Hypothesis is true, then Conclusion is true*
- Type checking computes via reasoning  
*If  $E_1$  and  $E_2$  have certain types, then  $E_3$  has a certain type*
- Rules of inference are a compact notation for "If-Then" statements

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## From English to an Inference Rule

- The notation is easy to read with practice
- Start with a simplified system and gradually add features
- Building blocks
  - Symbol  $\wedge$  is "and"
  - Symbol  $\Rightarrow$  is "if-then"
  - $x:T$  is " $x$  has type  $T$ "

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## From English to an Inference Rule (2)

If  $e_1$  has type  $\text{Int}$  and  $e_2$  has type  $\text{Int}$ ,  
then  $e_1 + e_2$  has type  $\text{Int}$

$(e_1 \text{ has type } \text{Int} \wedge e_2 \text{ has type } \text{Int}) \Rightarrow$   
 $e_1 + e_2 \text{ has type } \text{Int}$

$(e_1: \text{Int} \wedge e_2: \text{Int}) \Rightarrow e_1 + e_2: \text{Int}$

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## From English to an Inference Rule (3)

The statement

$(e_1: \text{Int} \wedge e_2: \text{Int}) \Rightarrow e_1 + e_2: \text{Int}$

is a special case of

$\text{Hypothesis}_1 \wedge \dots \wedge \text{Hypothesis}_n \Rightarrow \text{Conclusion}$

This is an inference rule

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## Notation for Inference Rules

- By tradition inference rules are written

$$\frac{\text{Hypothesis}_1 \dots \text{Hypothesis}_n}{\text{Conclusion}}$$

- Cool type rules have hypotheses and conclusions

$\vdash e:T$

- $\vdash$  means "it is provable that ..."

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## Two Rules

$$\frac{i \text{ is an integer}}{\vdash i : \text{Int}} \quad [\text{Int}]$$
$$\frac{\vdash e_1 : \text{Int} \quad \vdash e_2 : \text{Int}}{\vdash e_1 + e_2 : \text{Int}} \quad [\text{Add}]$$

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## Two Rules (Cont.)

- These rules give templates describing how to type integers and + expressions
- By filling in the templates, we can produce complete typings for expressions

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## Example: 1 + 2

$$\frac{\frac{1 \text{ is an integer}}{\vdash 1 : \text{Int}} \quad \frac{2 \text{ is an integer}}{\vdash 2 : \text{Int}}}{\vdash 1+2 : \text{Int}}$$

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## Soundness

- A type system is *sound* if
  - Whenever  $\vdash e : T$
  - Then  $e$  evaluates to a value of type  $T$
- We only want sound rules
  - But some sound rules are better than others:

$$\frac{i \text{ is an integer}}{\vdash i : \text{Object}}$$

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## Type Checking Proofs

- Type checking proves facts  $\vdash e : T$ 
  - Proof is on the structure of the AST
  - Proof has the shape of the AST
  - One type rule is used for each AST node
- In the type rule used for a node  $e$ :
  - Hypotheses are the proofs of types of  $e$ 's subexpressions
  - Conclusion is the type of  $e$
- Types are computed in a bottom-up pass over the AST

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## Rules for Constants

$$\frac{}{\vdash \text{false} : \text{Bool}} \quad [\text{Bool}]$$
$$\frac{s \text{ is a string constant}}{\vdash s : \text{String}} \quad [\text{String}]$$

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### Rule for New

`new T` produces an object of type `T`  
 - Ignore `SELF_TYPE` for now ...

$$\frac{}{\vdash \text{new } T : T} \quad [\text{New}]$$

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### Two More Rules

$$\frac{\vdash e : \text{Bool}}{\vdash \neg e : \text{Bool}} \quad [\text{Not}]$$

$$\frac{\vdash e_1 : \text{Bool} \quad \vdash e_2 : T}{\vdash \text{while } e_1 \text{ loop } e_2 \text{ pool} : \text{Object}} \quad [\text{Loop}]$$

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### A Problem

- What is the type of a variable reference?

$$\frac{\text{x is an identifier}}{\vdash x : ?} \quad [\text{Var}]$$

- The local, structural rule does not carry enough information to give `x` a type.

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### A Solution

- Put more information in the rules!
- A *type environment* gives types for *free* variables
  - A type environment is a function from `Object Identifiers` to `Types`
  - A variable is free in an expression if it is not defined within the expression

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### Type Environments

Let  $\mathcal{O}$  be a function from `Object Identifiers` to `Types`

The sentence

$$\mathcal{O} \vdash e : T$$

is read: Under the assumption that variables have the types given by  $\mathcal{O}$ , it is provable that the expression `e` has the type `T`

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### Modified Rules

The type environment is added to the earlier rules:

$$\frac{i \text{ is an integer}}{\mathcal{O} \vdash i : \text{Int}} \quad [\text{Int}]$$

$$\frac{\mathcal{O} \vdash e_1 : \text{Int} \quad \mathcal{O} \vdash e_2 : \text{Int}}{\mathcal{O} \vdash e_1 + e_2 : \text{Int}} \quad [\text{Add}]$$

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## New Rules

And we can write new rules:

$$\frac{O(x) = T}{O \vdash x : T} \quad [\text{Var}]$$

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## Let

$$\frac{O[T_0/x] \vdash e_1 : T_1}{O \vdash \text{let } x:T_0 \text{ in } e_1 : T_1} \quad [\text{Let-No-Init}]$$

$O[T/y]$  means  $O$  modified to return  $T$  on argument  $y$

Note that the **let**-rule enforces variable scope

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## Notes

- The type environment gives types to the free identifiers in the current scope
- The type environment is passed down the AST from the root towards the leaves
- Types are computed up the AST from the leaves towards the root

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## Let with Initialization

Now consider **let** with initialization:

$$\frac{O \vdash e_0 : T_0 \quad O[T_0/x] \vdash e_1 : T_1}{O \vdash \text{let } x:T_0 \leftarrow e_0 \text{ in } e_1 : T_1} \quad [\text{Let-Init}]$$

This rule is weak. Why?

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## Subtyping

- Define a relation  $\leq$  on classes
  - $X \leq X$
  - $X \leq Y$  if  $X$  inherits from  $Y$
  - $X \leq Z$  if  $X \leq Y$  and  $Y \leq Z$
- An improvement

$$\frac{O \vdash e_0 : T \quad T \leq T_0 \quad O[T_0/x] \vdash e_1 : T_1}{O \vdash \text{let } x:T_0 \leftarrow e_0 \text{ in } e_1 : T_1} \quad [\text{Let-Init}]$$

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## Assignment

- Both **let** rules are sound, but more programs typecheck with the second one
- More uses of subtyping:

$$\frac{O(l d) = T_0 \quad O \vdash e_1 : T_1 \quad T_1 \leq T_0}{O \vdash l d \leftarrow e_1 : T_1} \quad [\text{Assign}]$$

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### Initialized Attributes

- Let  $O_C(x) = T$  for all attributes  $x:T$  in class  $C$
- Attribute initialization is similar to `let`, except for the scope of names

$$\frac{O_C(l d) = T_0 \quad O_C \setminus e_1 : T_1 \quad T_1 \leq T_0}{O_C \setminus l d : T_0 \leftarrow e_1;} \quad [\text{Attr-Init}]$$

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### If-Then-Else

- Consider:  
`if  $e_0$  then  $e_1$  else  $e_2$  fi`
- The result can be either  $e_1$  or  $e_2$
- The type is either  $e_1$ 's type or  $e_2$ 's type
- The best we can do is the smallest supertype larger than the type of  $e_1$  or  $e_2$

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### Least Upper Bounds

- $\text{lub}(X, Y)$ , the least upper bound of  $X$  and  $Y$ , is  $Z$  if
  - $X \leq Z \wedge Y \leq Z$   
 $Z$  is an upper bound
  - $X \leq Z' \wedge Y \leq Z' \Rightarrow Z \leq Z'$   
 $Z$  is least among upper bounds
- In COOL, the least upper bound of two types is their least common ancestor in the inheritance tree

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### If-Then-Else Revisited

$$\frac{O \setminus e_0 : \text{Bool} \quad O \setminus e_1 : T_1 \quad O \setminus e_2 : T_2}{O \setminus \text{if } e_0 \text{ then } e_1 \text{ else } e_2 \text{ fi} : \text{lub}(T_1, T_2)} \quad [\text{If-Then-Else}]$$

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### Case

- The rule for `case` expressions takes a lub over all branches

$$\frac{O \setminus e_0 : T_0 \quad O[T_1 / x_1] \setminus e_1 : T_1' \quad \dots \quad O[T_n / x_n] \setminus e_n : T_n'}{O \setminus \text{case } e_0 \text{ of } x_1 : T_1 \mid e_1; \dots; x_n : T_n \mid e_n; \text{esac} : \text{lub}(T_1', \dots, T_n')} \quad [\text{Case}]$$

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### Method Dispatch

- There is a problem with type checking method calls:

$$\frac{O \setminus e_0 : T_0 \quad O \setminus e_1 : T_1 \quad \vdots \quad O \setminus e_n : T_n}{O \setminus e_0.f(e_1, \dots, e_n) : ?} \quad [\text{Dispatch}]$$

- We need information about the formal parameters and return type of  $f$

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## Notes on Dispatch

- In Cool, method and object identifiers live in different name spaces
  - A method `foo` and an object `foo` can coexist in the same scope
- In the type rules, this is reflected by a separate mapping  $M$  for method signatures

$$M(C, f) = (T_1, \dots, T_n, T_{n+1})$$

means in class  $C$  there is a method  $f$

$$f(x_1:T_1, \dots, x_n:T_n): T_{n+1}$$

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## The Dispatch Rule Revisited

$$O, M \vdash e_0 : T_0$$

$$O, M \vdash e_1 : T_1$$

$\vdots$

$$O, M \vdash e_n : T_n$$

$$M(T_0, f) = (T'_1, \dots, T'_n, T'_{n+1}) \quad [\text{Dispatch}]$$

$$T_i \leq T'_i \text{ for } 1 \leq i \leq n$$

$$O, M \vdash e_0.f(e_1, \dots, e_n) : T'_{n+1}$$

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## Static Dispatch

- Static dispatch is a variation on normal dispatch
- The method is found in the class explicitly named by the programmer
- The inferred type of the dispatch expression must conform to the specified type

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## Static Dispatch (Cont.)

$$O, M \vdash e_0 : T_0$$

$$O, M \vdash e_1 : T_1$$

$\vdots$

$$O, M \vdash e_n : T_n$$

$$T_0 \leq T$$

[StaticDispatch]

$$M(T, f) = (T'_1, \dots, T'_n, T'_{n+1})$$

$$T_i \leq T'_i \text{ for } 1 \leq i \leq n$$

$$O, M \vdash e_0.T.f(e_1, \dots, e_n) : T'_{n+1}$$

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## The Method Environment

- The method environment must be added to all rules
- In most cases,  $M$  is passed down but not actually used
  - Only the dispatch rules use  $M$

$$O, M \vdash e_1 : \text{Int}$$

$$O, M \vdash e_2 : \text{Int}$$

[Add]

$$O, M \vdash e_1 + e_2 : \text{Int}$$

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## More Environments

- For some cases involving `SELF_TYPE`, we need to know the class in which an expression appears
- The full type environment for COOL:
  - A mapping  $O$  giving types to object id's
  - A mapping  $M$  giving types to methods
  - The current class  $C$

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## Sentences

The form of a *sentence* in the logic is

$$O, M, C \vdash e : T$$

Example:

$$\frac{O, M, C \vdash e_1 : \text{int} \quad O, M, C \vdash e_2 : \text{int}}{O, M, C \vdash e_1 + e_2 : \text{int}} \quad [\text{Add}]$$

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## Type Systems

- The rules in this lecture are COOL-specific
  - More info on rules for [self](#) next time
  - Other languages have very different rules
- General themes
  - Type rules are defined on the structure of expressions
  - Types of variables are modeled by an environment
- Warning: Type rules are very compact!

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## One-Pass Type Checking

- COOL type checking can be implemented in a single traversal over the AST
- Type environment is passed down the tree
  - From parent to child
- Types are passed up the tree
  - From child to parent

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## Implementing Type Systems

$$\frac{O, M, C \vdash e_1 : \text{Int} \quad O, M, C \vdash e_2 : \text{Int}}{O, M, C \vdash e_1 + e_2 : \text{Int}} \quad [\text{Add}]$$

```
TypeCheck(Environment, e1 + e2) = {  
  T1 = TypeCheck(Environment, e1);  
  T2 = TypeCheck(Environment, e2);  
  Check T1 == T2 == Int;  
  return Int; }
```

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