

# Coloring Map Problem with *Answer Set Programming – Clingo* An Encoding

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## Roteiro

1. About the ASP (Clingo)
2. Requisites
3. The problem: map coloring
4. Discussion of this NP-complete problem
5. A solution in Clingo
- 6.
7. Conclusions

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**Attention: some background in logic and declarative language is recommended!**

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- ▶ ASP has its roots in
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  - ▶ logic programming (with negation)
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- ▶ ASP embraces many emerging application areas

# Hystorical and references

- ▶ This programming language has its root at the Universität Potsdam – 1999
- ▶ **Potassco**, the **P**otsdam **A**nswer **S**et **S**olving **C**ollection – <https://potassco.org/>
- ▶ Official repository with a full-courese: <https://github.com/potassco-asp-course/>
- ▶ Support to start: an active forum and a course covered by videos in the Youtube
- ▶ This presentation and its code: [https://github.com/claudiosa/CCS/tree/master/asp\\_Answer\\_Set\\_Programming](https://github.com/claudiosa/CCS/tree/master/asp_Answer_Set_Programming)
- ▶ Books:

## Some References:

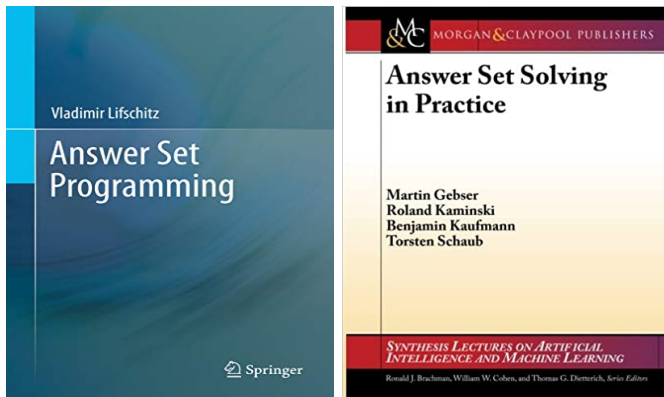


Figure: Estou usando o do Vladimir

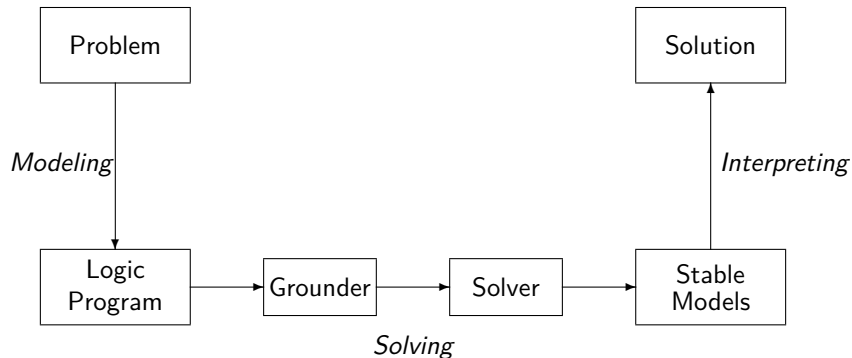
# Características

- ▶ Mais declarativa que Prolog e seus predecessores (**apenas, e quase nada mais, tem uma sintaxe que lembra Prolog**)
- ▶ Raízes em várias lógicas, incluindo as que tratam de informações incompletas: LP, LPO, *default*, circunscrição (suposição do mundo-fechado) e negação como falha, (auto-epistêmica)
- ▶ Usa o conceito de **modelo estável**: *semântica bem-fundamentada e ramificação*
- ▶ Uso: problemas combinatoriais baseados em conhecimento declarativo – faremos um exemplo
- ▶ Consistem de **decisões e restrições**
- ▶ Tudo isto na ordem de milhões!
- ▶ Na indústria: desde gerenciador de pacotes do Debian a sistemas da NASA

# Nesta apresentação

- ▶ A linguagem ASP com o sistema *clingo*
- ▶ *clingo* = *gringo* + *clasp*
- ▶ Há outras ramificações: *clingocon*, *aspcud* e *asprin*
- ▶ Alguns elementos da linguagem e um exemplo

# Modelagem, aterramento, e resolução



Fonte: <https://github.com/potassco-asp-course/>

## Graph coloring – a set of problems related

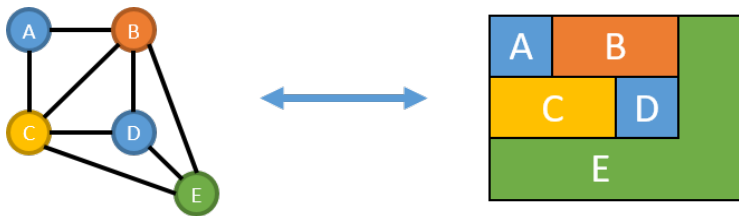


Figure: Let's consider a planar map for readability

- ▶ Input: a graph (map)  $G$  with  $n$  vertices (countries) and integer  $k$  (colors)
- ▶ Output: does  $G$  admit a proper vertex coloring with  $k$  colors?
- ▶ Complexity: NP-Complete
- ▶ Optimization: NP-Hard (lesser chromatic number –  $k$ )
- ▶ More details:

[https://en.wikipedia.org/wiki/Graph\\_coloring](https://en.wikipedia.org/wiki/Graph_coloring)

# The map to be colorized!



Figure: South America map, the author had the input data – 😊



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So, let's find a minimal color number to paint this map!

## Some comments:

- ▶ The modelling is immediate with an old Prolog code
- ▶ Many approaches for this problem can be taken: Simulated Annealing, Ant Colony, Depth-First Search, ..., meta-heuristics in general presents a good efficiency
- ▶ The full code discussed here is found in:  
`https://github.com/claudiosa/CCS/tree/master/asp\_Answer\_Set\_Programming/map\_coloring.lp`
- ▶ We are commenting it in parts

## Colors available (k) and countries (n):

```
color(red).  
color(blue).  
color(green).  
color(yellow).
```

country(antilles).	country(argentina).
country(bolivia).	country(brazil).
country(columbia).	country(chile).
country(ecuador).	country(french_guiana).
country(guyana).	country(paraguay).
country(peru).	country(surinam).
country(uruguay).	country(venezuela).

Ground terms, exactly written like Prolog syntax.

# The map, countries and their relations with neighbours:

```
neighbour(antilles,venezuela).
neighbour(argentina,brazil).
neighbour(argentina,paraguay).
neighbour(bolivia,brazil).
neighbour(bolivia,paraguay).
neighbour(brazil,columbia).
neighbour(brazil,guyana).
neighbour(brazil,peru).
neighbour(brazil,uruguay).
neighbour(chile,peru).
neighbour(argentina,bolivia).
neighbour(argentina,chile).
neighbour(argentina,uruguay).
neighbour(bolivia,chile).
neighbour(bolivia,peru).
neighbour(brazil,french_guiana).
neighbour(brazil,paraguay).
neighbour(brazil,surinam).
neighbour(brazil,venezuela).
neighbour(columbia,ecuador).
```

Another representation is possible, but until now, everything was reused!

## Modelling the problem under its requisites:

```
%% Country X Colors - Assign any color for each country
1 { country_color(P, C) : color(C) } 1 :- country(P).

%% Brazil must be green
:- not country_color(brazil,green).
%% OR
%% country_color(brazil,green).

%% Finally: none adjacents countries receive at the same color
% C != C1 :- country_color(P, C), country_color(P1, C1),
            neighbour(P,P1).
%% OR -- by Susana - ASP Community
:- country_color(P, C), country_color(P1, C), neighbour(P,P1).
```

Basically, that's all!

# Preparing for a optimization

```
%% number of colors used
n_colors(N) :-    N = #count{C : country_color(P,C)}.

%% A minimizations on this value
#minimize{ N : n_colors(N) }.

%% OUTPUT
#show country_color/2.
#show n_colors/1.
```

---

That's all!

## An output

```
clingo ../map_coloring.lp 0 --out-ifs='\n' --out-atomf=%s.
clingo version 5.3.0
Reading from ../map_coloring.lp
Solving...
Answer: 1
country_color(argentina,red).
country_color(columbia,red).
country_color(surinam,red).
country_color(guyana,blue).
country_color(paraguay,blue).
.....
country_color(french_guiana,yellow).
country_color(venezuela,yellow).
n_colors(4).
Optimization: 4
OPTIMUM FOUND
Models          : 1
  Optimum       : yes
Optimization    : 4
Calls           : 1
```

# Conclusions





## Contact and Comments:

- ▶ <https://claudiocesars.wordpress.com/>
- ▶ <https://github.com/claudiosa>
- ▶ Neste git, repostiório CCS  $\Rightarrow$  asp...
- ▶ Email: [ccs1664@gmail.com](mailto:ccs1664@gmail.com)
- ▶ *Thank you so much!*