Map Coloring Problem with *Answer Set Programming – clingo* An Encoding

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Road map of this presentation:

1. About the ASP (clingo)
2. Requisites
3. The problem: map coloring
4. Discussion of this (NP-complete) problem
5. A modelling in ASP
6. A solution in clingo
7. Conclusions

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Attention: some background in logic and declarative language is recommended!

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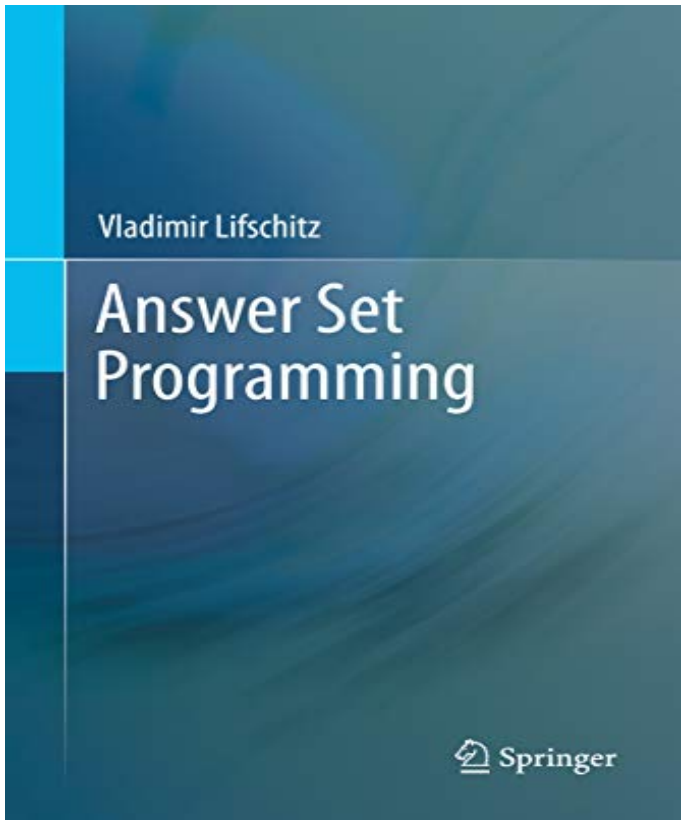
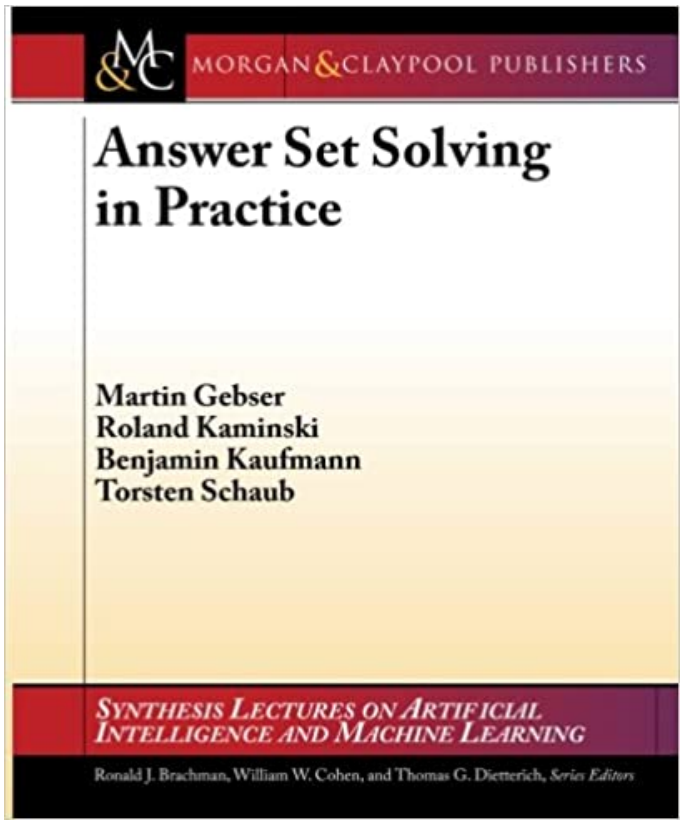
- ASP fundaments in: propositional, first-order, auto-epistemic, default. Many aspects in theory and programs.
- ASP embraces many emerging application areas

Historic and references:

- This programming language has its origin at the University of Potsdam (Universität Potsdam) – 1999
- **Potassco**, the **Po**tsdam **A**nswer **S**et **S**olving **Co**llection – `https://potassco.org/`
- Official repository with a full-course: `https://github.com/potassco-asp-course/`
- Support to start: an active forum and a course covered by videos in the Youtube
- This presentation and its code: `https://github.com/claudiosa/CCS/tree/master/asp_Answer_Set_Programming`
- Books:

Some References:

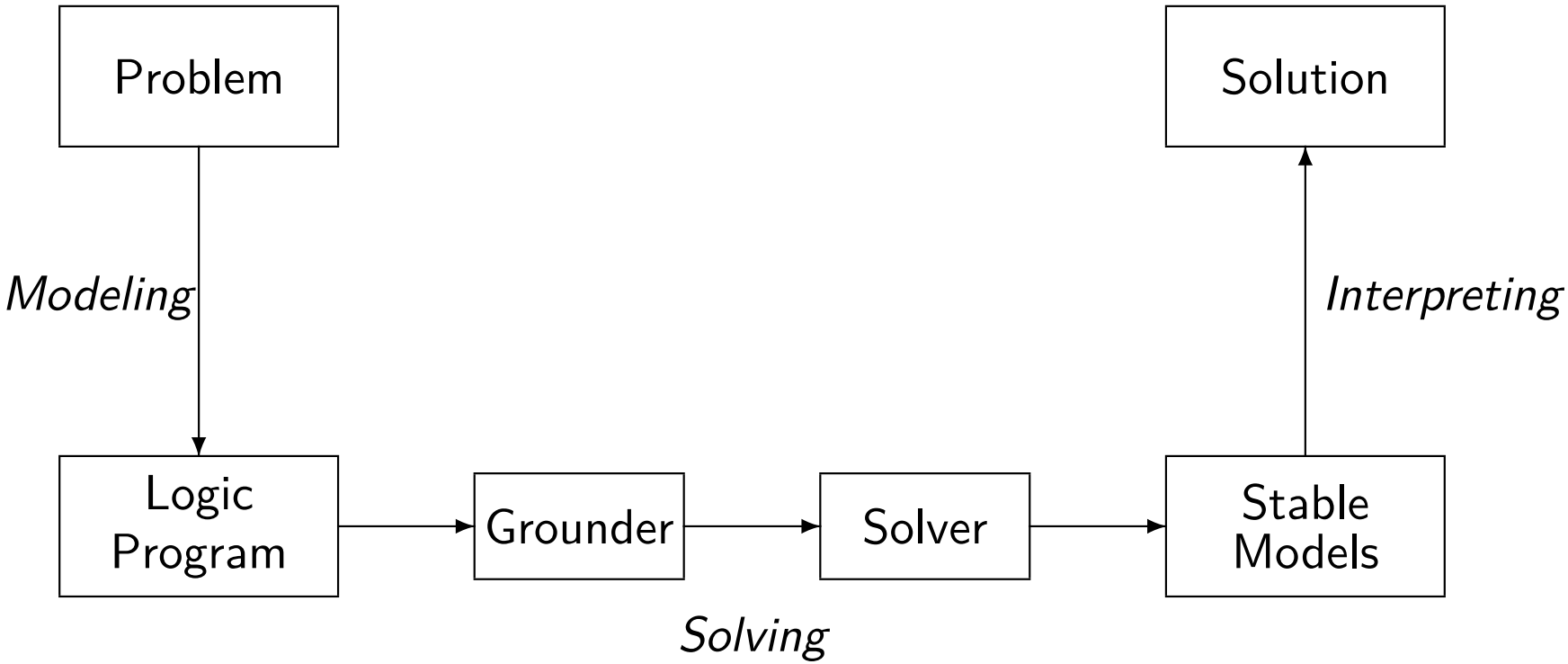




Characteristics:

- The theoretical background has the roots from many logics: propositional, first-order, circumscription (world closed supposition), default and the negation as a fail.
- The concept of **stable model** to deduce one or more answers.
- The combinatorial problems are well modelled with ASP (*wear as glove*)
- The enconding declare **rules**, **ground term** and **constraints**

Modelling, grounding and resolution:



Source: `https://github.com/potassco-asp-course/`

A case-study: Map Coloring Problem (a practical problem from graph coloring)

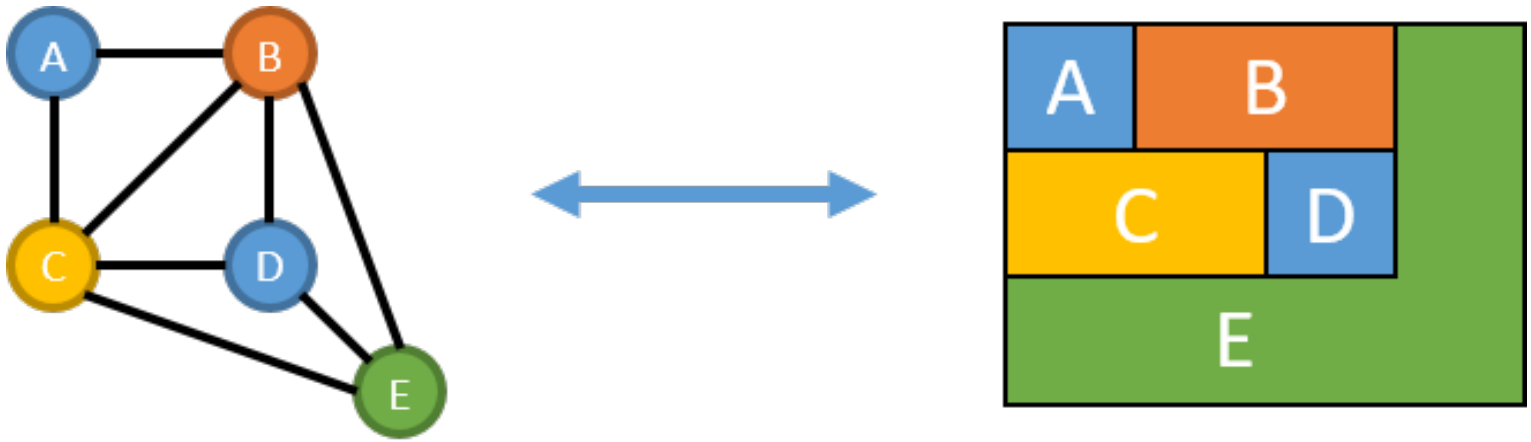


Figura: Let's consider a planar map for readgibility

- Input: a graph (map) G with n vertices (countries) and integer k (colors)
- Output: does G admit a proper vertex coloring with k colors?
- Complexity: NP-Complete
- Optimization: NP-Hard (lesser chromatic number – k)

Borrowed and adapted from:

https://github.com/potassco-asp-course/modeling/

- Problem instance: A map consisting of countries and neighborhood (frontiers)
  - facts formed by predicates `country/1` and `neighbour/2`
  - facts formed by predicate `color/1`
- Problem class: Assign each country one color such that no two countries connected or neighbour have the same color
- In other words,
  1. Each country has one color
  2. Two connected (neighbour) countries must not have the same color

Let's paint this map with **k** colors:

Figura: South America map, **the author had the input data – ☺**

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Finally, let's find a minimal color number to paint this map (without any neighbour country has the same color)!

Some comments:

- Theoretical details can be found in:
  `https://en.wikipedia.org/wiki/Graph_coloring`
- The modelling was almost immediate with an old Prolog code from the author:
  `https://github.com/claudiosa/CCS/......`
- Many approaches for this problem can be taken: Simulated Annealing, Ant Colony, Depth-First Search, ...,
  meta-heuristics in general presents a good efficiency
- The full code discussed here is found in:
  `https://github.com/claudiosa/CCS/tree/master/asp_Answer_Set_Programming/map_coloring.lp`
- We are commenting it in parts

Colors available (k) and countries (n):

```
%% Colors
color(red).
color(blue).
color(green).
color(yellow).
%% Countries
country(antilles).          country(argentina).
country(bolivia).           country(brazil).
country(columbia).          country(chile).
country(ecuador).           country(french_guiana).
country(guyana).            country(paraguay).
country(peru).              country(surinam).
country(uruguay).           country(venezuela).
```

Ground terms, exactly written like Prolog syntax.

The map, countries and their relations with neighbours:

```
neighbour(antilles,venezuela).    neighbour(argentina,bolivia).
neighbour(argentina,brazil).      neighbour(argentina,chile).
neighbour(argentina,paraguay).    neighbour(argentina,uruguay).
neighbour(bolivia,brazil).        neighbour(bolivia,chile).
neighbour(bolivia,paraguay).      neighbour(bolivia,peru).
neighbour(brazil,columbia).       neighbour(brazil,french_guiana).
neighbour(brazil,guyana).         neighbour(brazil,paraguay).
neighbour(brazil,peru).           neighbour(brazil,surinam).
neighbour(brazil,uruguay).        neighbour(brazil,venezuela).
neighbour(chile,peru).            neighbour(columbia,ecuador).
% To avoid duplication of the facts above
neighbour(X,Y) :- neighbour(Y,X).
% To obtain more symmetric results at the end
```

Another representation is possible, but until now, everything was reused – kept simple as possible!

Modelling the problem under its requisites:

```
%%  Country X Colors - Assign any color for each country
1 { country_color(P, C) : color(C) } 1 :- country(P).

%% Brazil must be green
:- not country_color(brazil,green).
%% OR
%% country_color(brazil,green).

%% Finally: none adjacent countries receive at the same color
% C != C1 :- country_color(P, C), country_color(P1, C1),
             neighbour(P,P1).
%% OR -- by Susana - ASP Community
:- country_color(P, C), country_color(P1, C), neighbour(P,P1).
```

Basically, that's all!

Preparing for a optimization:

```
%% number of colors used
n_colors(N) :-   N = #count{C : country_color(P,C)}.

%% A minimization on this value
#minimize{ N : n_colors(N) }.

%% OUTPUT
#show country_color/2.
#show n_colors/1.
```

That's all!

An output:

```
clingo ../map_coloring.lp 0 --out-ifs='\n' --out-atomf=%s.
clingo version 5.3.0
Reading from ../map_coloring.lp
Solving...
Answer: 1
country_color(argentina,red).
country_color(columbia,red).
country_color(surinam,red).
country_color(guyana,blue).
country_color(paraguay,blue).
.........................
country_color(french_guiana,yellow).
country_color(venezuela,yellow).
n_colors(4).
Optimization: 4
OPTIMUM FOUND
Models       : 1
  Optimum    : yes
Optimization : 4
Calls        : 1
```

For future explorations:

A cool suggestion from Adam Smith: *express preferences for country colors at a lower priority, searching for a min colors number. In the sequence, try to find the max number with the countries satisfy with their preferences. Here's a little snippet that shows mixing minimization and maximization at different priorities*:

```
% minimize number of colors used at higher priority
color_used(C) :- country_color(P,C).
#minimize {1,C@2:color_used(C)}.

% maximize number of color preferences satisfied at lower priori
color_preference(brazil,green).
preference_satisfied(P) :- color_preference(P,C),
                           country_color(P,C)
#maximize {1,P@1:preference_satisfied(P,C) }.
```

Not tested yet, but, surely it works!

Conclusions:

- ASP is strongly declarative (roots from the logic to attack the problems representation)
- Generate and test methodology
- ASP's workflow, modeling, grounding, solving (and optimizing)
- clingo = gringo+clasp + ...
- Allows you to embed a Python coding in order to minimize the difficulties (☹) of input and output data
- An encoding in ASP is excellent exercise to keep your mind very active!
- Finally, a huge gratitude for the **potassco-users list**, always reactive for my silly doubts, where I had been learning much.

Contact and comments (are must welcome ☺):

- `https://claudiocesar.wordpress.com/`
- This presentation and the code discussed: **`https://github.com/claudiosa/CCS/tree/master/asp_Answer_Set_Programming`**
- There is a directory to Youtube!
- ✉: `ccs1664@gmail.com`
- This material has a partial support from WhatsTV Inc. `https://en.whatstv.com.br/`, here our gratitude!
- *Thank you so much*!