Answer Set Solving in Practice

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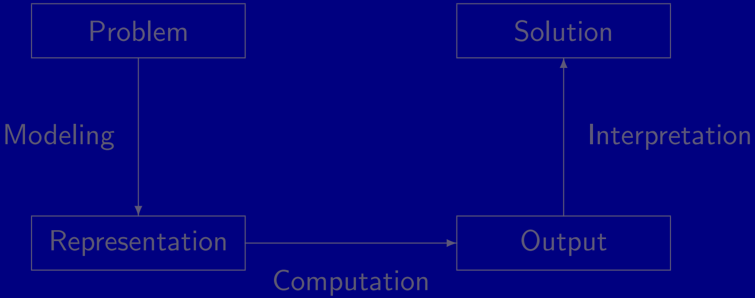
http://www.cs.uni-potsdam.de/~torsten/ijcai11tutorial/asp.pdf

Goal: Declarative problem solving

- *"What is the problem?"*
  instead of
- *"How to solve the problem?"*



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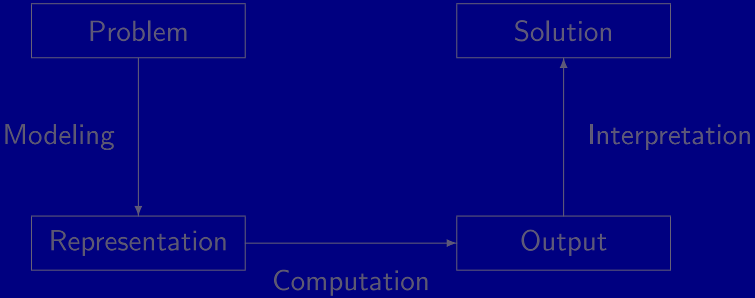

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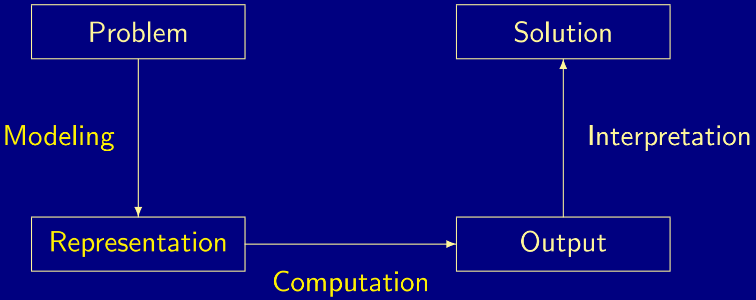

Answer Set Programming (ASP) *in a Nutshell*

- ASP is an approach to declarative problem solving, combining
  - a rich yet simple modeling language
  - with high-performance solving capacities
- ASP has its roots in
  - (logic-based) knowledge representation and (nonmonotonic) reasoning
  - (deductive) databases
  - constraint solving (in particular, SATisfiability testing)
  - logic programming (with negation)
- ASP allows for solving all search problems in NP (and NP^{NP}) in a uniform way (being more compact than SAT)
- The versatility of ASP is reflected by the ASP solver clasp, winning first places at ASP'07/09/11, PB'09/11, and SAT'09/11
- ASP embraces many emerging application areas!

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Logic Programming

- Algorithm = Logic + Control [53]
- Logic as a programming language
  - ➥ Prolog (Colmerauer, Kowalski)
- Features of Prolog
  - Declarative (relational) programming language
  - Based on SLD(NF) Resolution
  - Top-down query evaluation
  - Terms as data structures
  - Parameter passing by unification
  - Solutions are extracted from instantiations of variables occurring in the query

Prolog: Programming in logic

Prolog is great, it's almost declarative!

To see this, consider

```
above(X,Y) :- on(X,Y).
above(X,Y) :- on(X,Z),above(Z,Y).
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and compare it to

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An interpretation in classical logic amounts to

$$\forall xy(on(x,y) \vee \exists z(on(x,z) \wedge above(z,y)) \rightarrow above(x,y))$$

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Model-based Problem Solving

Common approach (eg. Prolog)

1. Provide a specification of the problem.
2. A solution is given by a derivation of an appropriate query.

Model-based approach (eg. ASP and SAT)

1. Provide a specification of the problem.
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Automated planning, Kautz and Selman [51]

Represent planning problems as propositional theories so that models not proofs describe solutions (eg. `Satplan`)

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Specification	Associated Structures
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propositional theories	models
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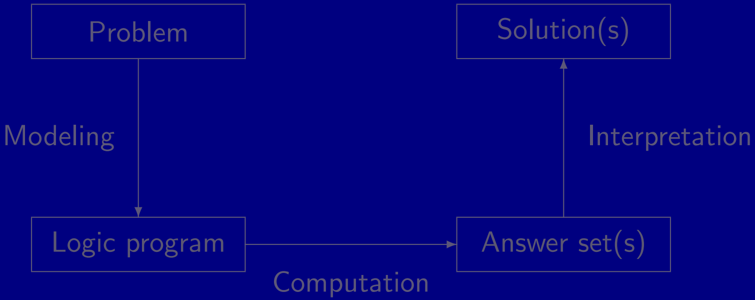

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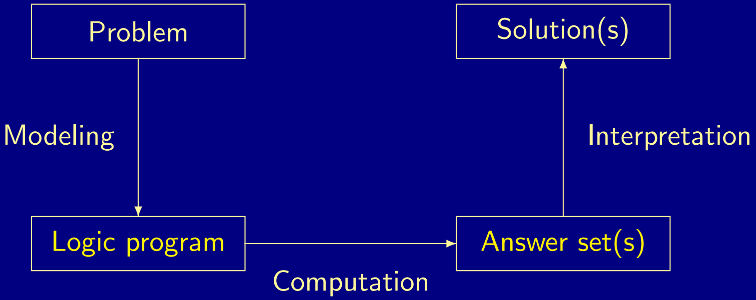

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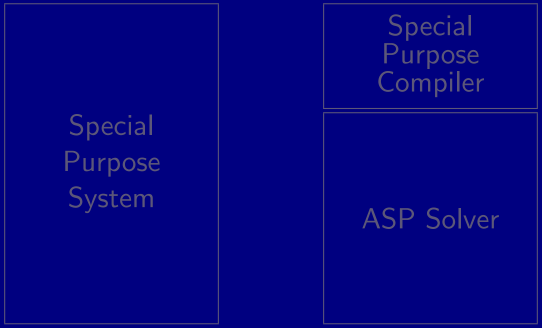

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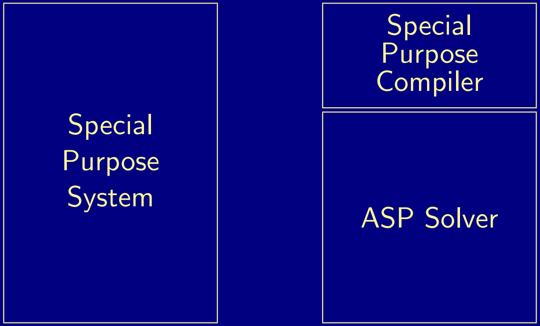

What is ASP good for?

- Combinatorial search problems (some with substantial amount of data):
  - For instance, auctions, bio-informatics, computer-aided verification, configuration, constraint satisfaction, diagnosis, information integration, planning and scheduling, security analysis, semantic web, wire-routing, zoology and linguistics, and many more
- My favorite: Using ASP as a basis for a decision support system for NASA's space shuttle (Gelfond et al., Texas Tech)
- Our own applications:
  - Automatic synthesis of multiprocessor systems
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  - Home monitoring for risk prevention in ambient assisted living
  - General game playing

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What does ASP offer?

- Integration of KR, DB, and search techniques
- Compact, easily maintainable problem representations
- Rapid application development tool
- Easy handling of dynamic, knowledge intensive applications
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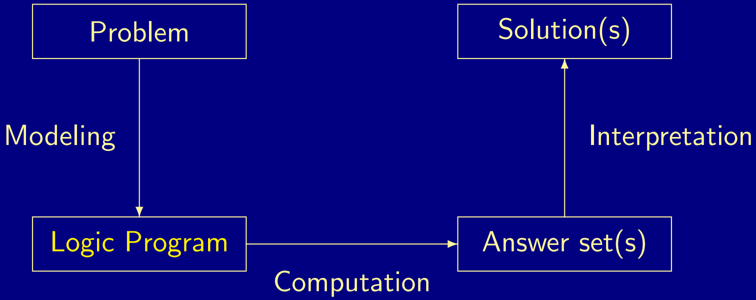
$$\text{ASP} = \text{KR} + \text{DB} + \text{Search}$$

Introduction
Overview

Problem solving in ASP: Syntax



Normal logic programs

- A (normal) rule, r, is an ordered pair of the form

$$A_0 \leftarrow A_1, \dots, A_m, not\ A_{m+1}, \dots, not\ A_n,$$

where $n \geq m \geq 0$, and each A_i $(0 \leq i \leq n)$ is an atom.

- A (normal) logic program is a finite set of rules.
- Notation

$$\begin{aligned} head(r) &= A_0 \\ body(r) &= \{A_1, \dots, A_m, not\ A_{m+1}, \dots, not\ A_n\} \\ body^+(r) &= \{A_1, \dots, A_m\} \\ body^-(r) &= \{A_{m+1}, \dots, A_n\} \end{aligned}$$

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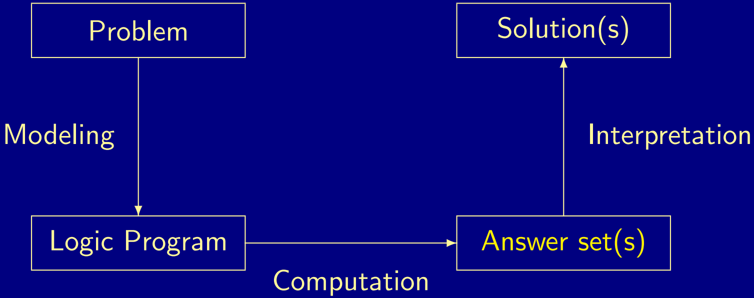

Answer set: Formal Definition

Positive programs

- A set of atoms X is closed under a positive program Π iff for any $r \in \Pi$, $head(r) \in X$ whenever $body^+(r) \subseteq X$.
  - ➥ X corresponds to a model of Π (seen as a formula).
- The smallest set of atoms which is closed under a positive program Π is denoted by $Cn(\Pi)$.
  - ➥ $Cn(\Pi)$ corresponds to the $\subseteq$-smallest model of Π (ditto).
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Some "logical" remarks

- Positive rules are also referred to as definite clauses.
  - Definite clauses are disjunctions with exactly one positive atom:

$$A_0 \vee \neg A_1 \vee \cdots \vee \neg A_m$$

  - A set of definite clauses has a (unique) smallest model.
- Horn clauses are clauses with at most one positive atom.
  - Every definite clause is a Horn clause but not vice versa.
  - A set of Horn clauses has a smallest model or none.
- This smallest model is the intended semantics of a set of Horn clauses.
  - ☞ Given a positive program Π, $Cn(\Pi)$ corresponds to the smallest model of the set of definite clauses corresponding to Π.

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(Rough) notational convention

We sometimes use the following notation interchangeably in order to stress the respective view:

	if	and	or	negation as failure	classical negation
source code	`:-`	`,`	`|`	`not`	`-`
logic program	$\leftarrow$	$,$	$;$	$not/\sim$	$\neg$
formula	$\rightarrow$	$\wedge$	$\vee$	$\sim/(\neg)$	$\neg$

Answer set: Basic idea

Consider the logical formula Φ and its three (classical) models:

Φ $\boxed{q \ \wedge \ (q \wedge \neg r \rightarrow p)}$

$$\{p, q\}, \{q, r\}, \text{ and } \{p, q, r\}$$

Formula Φ has one stable model, called answer set:

$$\Pi_\Phi \quad \begin{array}{|lcl|} \hline q & \leftarrow & \\ p & \leftarrow & q, \ \mathit{not}\ r \\ \hline \end{array}$$

$$\{p, q\}$$

Informally, a set X of atoms is an answer set of a logic program Π

- if X is a (classical) model of Π and
- if all atoms in X are justified by some rule in Π

(rooted in intuitionistic logics HT (Heyting, 1930) and G3 (Gödel, 1932))

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- The reduct, Π^X, of a program Π relative to a set X of atoms is defined by

$$\Pi^X = \{head(r) \leftarrow body^+(r) \mid r \in \Pi \text{ and } body^-(r) \cap X = \emptyset\}.$$

- A set X of atoms is an answer set of a program Π if $Cn(\Pi^X) = X$.
  Recall: $Cn(\Pi^X)$ is the $\subseteq$–smallest (classical) model of Π^X.

Intuition: X is stable under *"applying rules from Π"*

Note: Every atom in X is justified by an *"applying rule from Π"*

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A closer look at Π^X

In other words, given a set X of atoms from Π,

Π^X is obtained from Π by deleting

1. each rule having a *not* A in its body with $A \in X$
   and then
2. all negative atoms of the form *not* A
   in the bodies of the remaining rules.

A first example

$\Pi = \{p \leftarrow p,\ q \leftarrow not\ p\}$

X	Π^X	$Cn(\Pi^X)$
$\emptyset$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$
$\{p\}$	$p \leftarrow p$	$\emptyset$
$\{q\}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$
$\{p, q\}$	$p \leftarrow p$	$\emptyset$

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$\{p\}$	$p \leftarrow p$	$\emptyset$	
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$\{p\}$	$p \leftarrow p$	$\emptyset$	✗
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$\{p, q\}$	$p \leftarrow p$	$\emptyset$	✗

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$\{p, q\}$	$p \leftarrow p$	$\emptyset$	✗

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A third example

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Answer set: Some properties

- A program may have zero, one, or multiple answer sets!
- If X is an answer set of a logic program Π,
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- If X and Y are answer sets of a *normal* program Π,
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A closer look at Cn

Inductive characterization

Let Π be a positive program and X a set of atoms.

- The immediate consequence operator T_Π is defined as follows:

$$T_\Pi X = \{head(r) \mid r \in \Pi \text{ and } body(r) \subseteq X\}$$

- Iterated applications of T_Π are written as T^j_Π for $j \geq 0$, where $T^0_\Pi X = X$ and $T^i_\Pi X = T_\Pi T^{i-1}_\Pi X$ for $i \geq 1$.

Theorem

For any positive program Π, we have

- $Cn(\Pi) = \bigcup_{i \geq 0} T^i_\Pi \emptyset$,
- $X \subseteq Y$ *implies* $T_\Pi X \subseteq T_\Pi Y$,
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Let's iterate T_Π

$$\Pi = \{p \leftarrow,\ q \leftarrow,\ r \leftarrow p,\ s \leftarrow q, t,\ t \leftarrow r,\ u \leftarrow v\}$$

$$\begin{array}{lclclcl}
T_\Pi^0\emptyset & = & \emptyset & & & & \\
T_\Pi^1\emptyset & = & \{p,q\} & = & T_\Pi T_\Pi^0\emptyset & = & T_\Pi\emptyset \\
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To see that $Cn(\Pi) = \{p,q,r,t,s\}$ is the smallest fixpoint of T_Π, note that $T_\Pi\{p,q,r,t,s\} = \{p,q,r,t,s\}$ and $T_\Pi X \neq X$ for every $X \subseteq \{p,q,r,t,s\}$.

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Programs with Variables

Let Π be a logic program.

- Herbranduniverse U^{Π}: Set of constants in Π
- Herbrandbase B^{Π}: Set of (variable-free) atoms constructible from U^{Π}
  ☞ We usually denote this as $\mathcal{A}$, and call it alphabet.
- Ground Instances of $r \in \Pi$: Set of variable-free rules obtained by replacing all variables in r by elements from U^{Π}:

  $$ground(r) = \{r\theta \mid \theta : var(r) \rightarrow U^{\Pi}\}$$

  where $var(r)$ stands for the set of all variables occurring in r; θ is a (ground) substitution.
- Ground Instantiation of Π:

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An example

$$\Pi = \{\ r(a,b) \leftarrow,\ \ r(b,c) \leftarrow,\ \ t(X,Y) \leftarrow r(X,Y)\ \}$$

$$U^\Pi = \{a,b,c\}$$

$$B^\Pi = \left\{ \begin{array}{l} r(a,a), r(a,b), r(a,c), r(b,a), r(b,b), r(b,c), r(c,a), r(c,b), r(c,c), \\ t(a,a), t(a,b), t(a,c), t(b,a), t(b,b), t(b,c), t(c,a), t(c,b), t(c,c) \end{array} \right\}$$

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Answer sets of programs with Variables

Let Π be a normal logic program with variables.

We define a set X of (ground) atoms as an answer set of Π
if $Cn(ground(\Pi)^X) = X$.

Language Constructs

- Variables (over the Herbrand Universe)
  - `p(X) :- q(X)` over constants $\{a, b, c\}$ stands for
    `p(a) :- q(a), p(b) :- q(b), p(c) :- q(c)`
- Conditional Literals
  - `p :- q(X) : r(X)` given `r(a), r(b), r(c)` stands for
    `p :- q(a), q(b), q(c)`
- Disjunction
  - `p(X) | q(X) :- r(X)`
- Integrity Constraints
  - `:- q(X), p(X)`
- Choice
  - `2 { p(X,Y) : q(X) } 7 :- r(Y)`
- Aggregates
  - `s(Y) :- r(Y), 2 #count { p(X,Y) : q(X) } 7`
    also: `#sum, #avg, #min, #max, #even, #odd`

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  - `2 { p(X,Y) : q(X) } 7 :- r(Y)`
- Aggregates
  - `s(Y) :- r(Y), 2 #count { p(X,Y) : q(X) } 7`
  - also: `#sum`, `#avg`, `#min`, `#max`, `#even`, `#odd`

Language Constructs

- Variables (over the Herbrand Universe)
  - `p(X) :- q(X)` over constants $\{a, b, c\}$ stands for
    `p(a) :- q(a), p(b) :- q(b), p(c) :- q(c)`
- Conditional Literals
  - `p :- q(X) : r(X)` given `r(a), r(b), r(c)` stands for
    `p :- q(a), q(b), q(c)`
- Disjunction
  - `p(X) | q(X) :- r(X)`
- Integrity Constraints
  - `:- q(X), p(X)`
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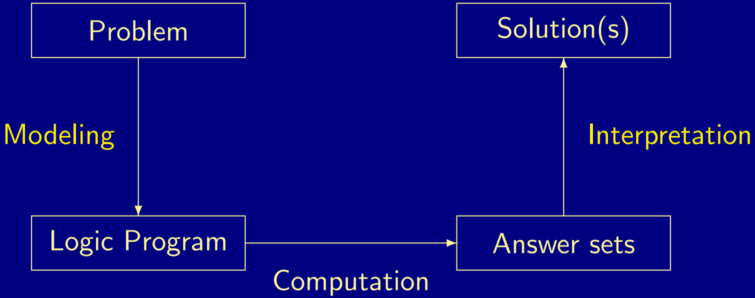
Reasoning Modes

- Satisfiability
- Enumeration[†]
- Projection[†]
- Intersection[‡]
- Union[‡]
- Optimization
- Sampling

[†] without solution recording

[‡] without solution enumeration

Modeling and Interpreting



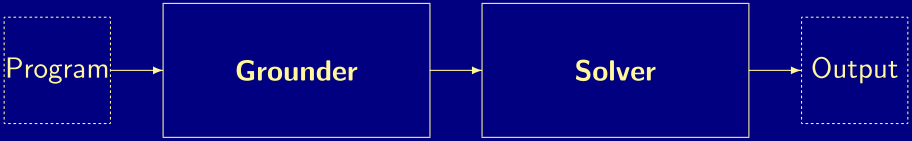
Modeling

For solving a problem class P for a problem instance I,
encode

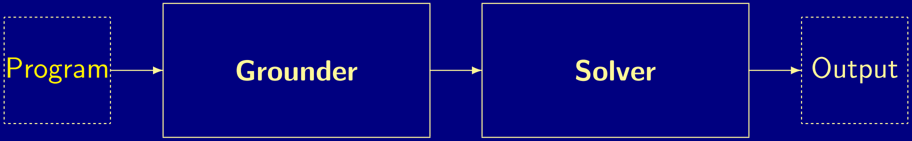
1. the problem instance I as a set $C(I)$ of facts and
2. the problem class P as a set $C(P)$ of rules

such that the solutions to P for I can be (polynomially) extracted from the answer sets of $C(I) \cup C(P)$.

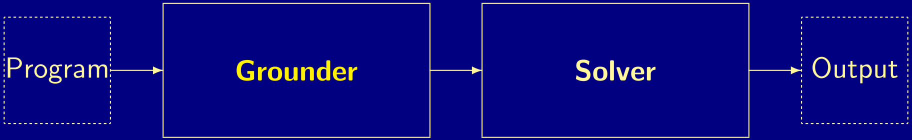
ASP Solving Process



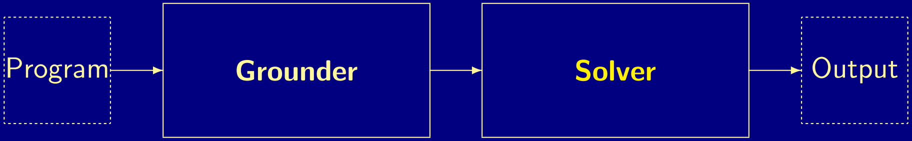
ASP Solving Process



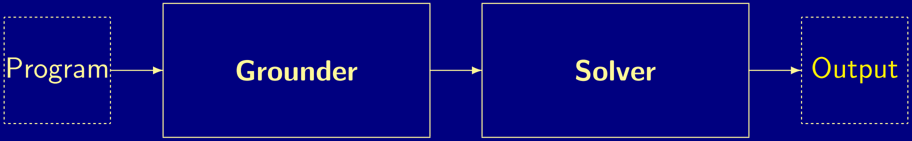
ASP Solving Process



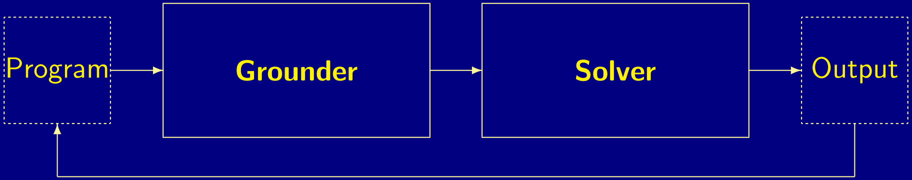
ASP Solving Process



ASP Solving Process



ASP Solving Process



Graph Coloring

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).   col(b).   col(g).

1 {color(X,C) : col(C)} 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
```

Graph Coloring

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

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```

Graph Coloring

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edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).   col(b).   col(g).

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edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).   col(b).   col(g).

1 {color(X,C) : col(C)} 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
```

Graph Coloring: Grounding

```
$ gringo -t color.lp
```

```
node(1).  node(2).  node(3).  node(4).  node(5).  node(6).

edge(1,2).  edge(1,3).  edge(1,4).  edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).  edge(4,1).  edge(4,2).  edge(5,3).
edge(5,4).  edge(5,6).  edge(6,2).  edge(6,3).  edge(6,5).

col(r).  col(b).  col(g).

1 {color(1,r), color(1,b), color(1,g)} 1.
1 {color(2,r), color(2,b), color(2,g)} 1.
1 {color(3,r), color(3,b), color(3,g)} 1.
1 {color(4,r), color(4,b), color(4,g)} 1.
1 {color(5,r), color(5,b), color(5,g)} 1.
1 {color(6,r), color(6,b), color(6,g)} 1.

 :- color(1,r), color(2,r).  :- color(2,g), color(5,g). ...  :- color(6,r), color(2,r).
 :- color(1,b), color(2,b).  :- color(2,r), color(6,r).      :- color(6,b), color(2,b).
 :- color(1,g), color(2,g).  :- color(2,b), color(6,b).      :- color(6,g), color(2,g).
 :- color(1,r), color(3,r).  :- color(2,g), color(6,g).      :- color(6,r), color(3,r).
 :- color(1,b), color(3,b).  :- color(3,r), color(1,r).      :- color(6,b), color(3,b).
 :- color(1,g), color(3,g).  :- color(3,b), color(1,b).      :- color(6,g), color(3,g).
 :- color(1,r), color(4,r).  :- color(3,g), color(1,g).      :- color(6,r), color(5,r).
 :- color(1,b), color(4,b).  :- color(3,r), color(4,r).      :- color(6,b), color(5,b).
 :- color(1,g), color(4,g).  :- color(3,b), color(4,b).      :- color(6,g), color(5,g).
 :- color(2,r), color(4,r).  :- color(3,g), color(4,g).
 :- color(2,b), color(4,b).  :- color(3,r), color(5,r).
 :- color(2,g), color(4,g).  :- color(3,b), color(5,b).
```

Graph Coloring: Grounding

```
$ gringo -t color.lp
```

```
node(1).  node(2).  node(3).  node(4).  node(5).  node(6).

edge(1,2).  edge(1,3).  edge(1,4).  edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).  edge(4,1).  edge(4,2).  edge(5,3).
edge(5,4).  edge(5,6).  edge(6,2).  edge(6,3).  edge(6,5).

col(r).  col(b).  col(g).

1 {color(1,r), color(1,b), color(1,g)} 1.
1 {color(2,r), color(2,b), color(2,g)} 1.
1 {color(3,r), color(3,b), color(3,g)} 1.
1 {color(4,r), color(4,b), color(4,g)} 1.
1 {color(5,r), color(5,b), color(5,g)} 1.
1 {color(6,r), color(6,b), color(6,g)} 1.

 :- color(1,r), color(2,r).  :- color(2,g), color(5,g). ...  :- color(6,r), color(2,r).
 :- color(1,b), color(2,b).  :- color(2,r), color(6,r).      :- color(6,b), color(2,b).
 :- color(1,g), color(2,g).  :- color(2,b), color(6,b).      :- color(6,g), color(2,g).
 :- color(1,r), color(3,r).  :- color(2,g), color(6,g).      :- color(6,r), color(3,r).
 :- color(1,b), color(3,b).  :- color(3,r), color(1,r).      :- color(6,b), color(3,b).
 :- color(1,g), color(3,g).  :- color(3,b), color(1,b).      :- color(6,g), color(3,g).
 :- color(1,r), color(4,r).  :- color(3,g), color(1,g).      :- color(6,r), color(5,r).
 :- color(1,b), color(4,b).  :- color(3,r), color(4,r).      :- color(6,b), color(5,b).
 :- color(1,g), color(4,g).  :- color(3,b), color(4,b).      :- color(6,g), color(5,g).
 :- color(2,r), color(4,r).  :- color(3,g), color(4,g).
 :- color(2,b), color(4,b).  :- color(3,r), color(5,r).
 :- color(2,g), color(4,g).  :- color(3,b), color(5,b).
```

Graph Coloring: Solving

```
$ gringo color.lp | clasp 0
```

```
clasp version 1.2.1
Reading from stdin
Reading      : Done(0.000s)
Preprocessing: Done(0.000s)
Solving...
Answer: 1
color(1,b) color(2,r) color(3,r) color(4,g) color(5,b) color(6,g) node(1) ... edge(1,2) ... col(r) ...
Answer: 2
color(1,g) color(2,r) color(3,r) color(4,b) color(5,g) color(6,b) node(1) ... edge(1,2) ... col(r) ...
Answer: 3
color(1,b) color(2,g) color(3,g) color(4,r) color(5,b) color(6,r) node(1) ... edge(1,2) ... col(r) ...
Answer: 4
color(1,g) color(2,b) color(3,b) color(4,r) color(5,g) color(6,r) node(1) ... edge(1,2) ... col(r) ...
Answer: 5
color(1,r) color(2,b) color(3,b) color(4,g) color(5,r) color(6,g) node(1) ... edge(1,2) ... col(r) ...
Answer: 6
color(1,r) color(2,g) color(3,g) color(4,b) color(5,r) color(6,b) node(1) ... edge(1,2) ... col(r) ...

Models      : 6
Time        : 0.000  (Solving: 0.000)
```

Graph Coloring: Solving

```
$ gringo color.lp | clasp 0
```

```
clasp version 1.2.1
Reading from stdin
Reading      : Done(0.000s)
Preprocessing: Done(0.000s)
Solving...
Answer: 1
color(1,b) color(2,r) color(3,r) color(4,g) color(5,b) color(6,g) node(1) ... edge(1,2) ... col(r) ...
Answer: 2
color(1,g) color(2,r) color(3,r) color(4,b) color(5,g) color(6,b) node(1) ... edge(1,2) ... col(r) ...
Answer: 3
color(1,b) color(2,g) color(3,g) color(4,r) color(5,b) color(6,r) node(1) ... edge(1,2) ... col(r) ...
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color(1,r) color(2,g) color(3,g) color(4,b) color(5,r) color(6,b) node(1) ... edge(1,2) ... col(r) ...

Models      : 6
Time        : 0.000  (Solving: 0.000)
```

Basic Methodology

Generate and Test (or: Guess and Check) approach

Generator: Generate potential answer set candidates (typically through non-deterministic constructs)

Tester: Eliminate invalid candidates (typically through integrity constraints)

Nutshell

$$\text{Logic program} \quad = \quad \text{Data} + \text{Generator} + \text{Tester} \\ (+\ \text{Optimizer})$$

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Satisfiability

- Problem Instance: A propositional formula ϕ.
- Problem Class: Is there an assignment of propositional variables to true and false such that a given formula ϕ is true.

- Example: Consider formula $(a \vee \neg b) \wedge (\neg a \vee b)$.
- Logic Program:

Generator	Tester	Answer sets
$\{a,b\} \leftarrow$	$\leftarrow$ not a, b	$X_1 = \{a,b\}$
	$\leftarrow$ a, not b	$X_2 = \{\}$

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Generator	**Tester**	**Answer sets**
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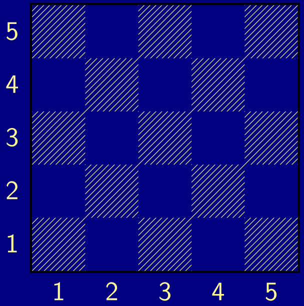
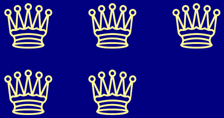
The n-Queens Problem



- Place n queens on an $n \times n$ chess board
- Queens must not attack one another



Defining the Field

queens.lp

```
row(1..n).
col(1..n).
```

- Create file `queens.lp`
- Define the field
  - n rows
  - n columns

Defining the Field

Running ...

```
$ clingo queens.lp -c n=5
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5)
SATISFIABLE

Models      : 1
Time        : 0.000
  Prepare   : 0.000
  Prepro.   : 0.000
  Solving   : 0.000
```

Placing some Queens

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
```

- Guess a solution candidate
- Place some queens on the board

Placing some Queens

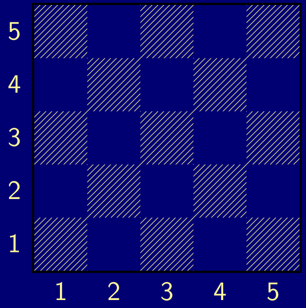
Running . . .

```
$ clingo queens.lp -c n=5 3
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5)
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) queen(1,1)
Answer: 3
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) queen(2,1)
SATISFIABLE

Models      : 3+
```

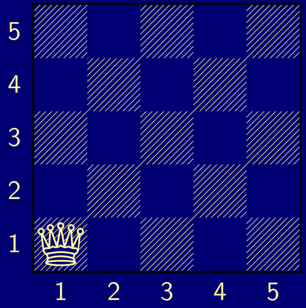
Placing some Queens: Answer 1

Answer 1



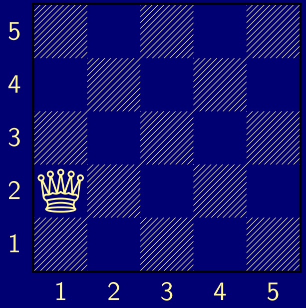
Placing some Queens: Answer 2

Answer 2



Placing some Queens: Answer 3

Answer 3



Placing n Queens

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not { queen(I,J) } == n.
```

- Place exactly *n* queens on the board

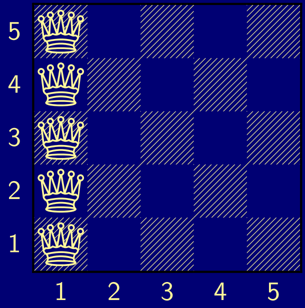
Placing n Queens

Running ...

```
$ clingo queens.lp -c n=5 2
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(5,1) queen(4,1) queen(3,1) \
queen(2,1) queen(1,1)
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(1,2) queen(4,1) queen(3,1) \
queen(2,1) queen(1,1)
...
```

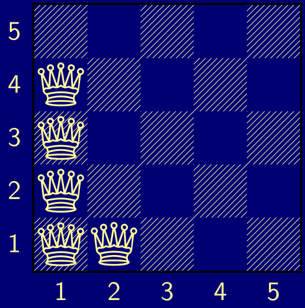
Placing n Queens: Answer 1

Answer 1



Placing n Queens: Answer 2

Answer 2



Horizontal and vertical Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not { queen(I,J) } == n.
:- queen(I,J), queen(I,JJ), J != JJ.
:- queen(I,J), queen(II,J), I != II.
```

- Forbid horizontal attacks
- Forbid vertical attacks

Horizontal and vertical Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not { queen(I,J) } == n.
:- queen(I,J), queen(I,JJ), J != JJ.
:- queen(I,J), queen(II,J), I != II.
```

- Forbid horizontal attacks
- Forbid vertical attacks

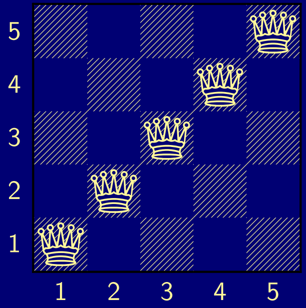
Horizontal and vertical Attack

Running ...

```
$ clingo queens.lp -c n=5
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(5,5) queen(4,4) queen(3,3) \
queen(2,2) queen(1,1)
...
```

Horizontal and vertical Attack: Answer 1

Answer 1



Diagonal Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not { queen(I,J) } == n.
:- queen(I,J), queen(I,JJ), J != JJ.
:- queen(I,J), queen(II,J), I != II.
:- queen(I,J), queen(II,JJ), (I,J) != (II,JJ),
   I-J == II-JJ.
:- queen(I,J), queen(II,JJ), (I,J) != (II,JJ),
   I+J == II+JJ.
```

- Forbid diagonal attacks

Diagonal Attack

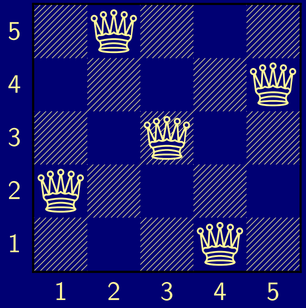
Running . . .

```
$ clingo queens.lp -c n=5
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(4,5) queen(1,4) queen(3,3) \
queen(5,2) queen(2,1)
SATISFIABLE

Models      : 1+
Time        : 0.000
  Prepare   : 0.000
  Prepro.   : 0.000
  Solving   : 0.000
```

Diagonal Attack: Answer 1

Answer 1



Optimizing

queens-opt.lp

```
{ queen(I,1..n) } == 1 :- I = 1..n.
{ queen(1..n,J) } == 1 :- J = 1..n.
:- { queen(D-J,J) } >= 2, D = 2..2*n.
:- { queen(D+J,J) } >= 2, D = 1-n..n-1.
```

- Encoding can be optimized
- Much faster to solve

- See Section *Tweaking N-Queens*

Reviewer Assignment
by Ilkka Niemelä

```
reviewer(r1). paper(p1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
reviewer(r2). paper(p2). classA(r1,p3). classB(r1,p4). coi(r1,p6).
...

3 { assigned(P,R) : reviewer(R) } 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- 9 { assigned(P,R) : paper(P) }  , reviewer(R).
 :-   { assigned(P,R) : paper(P) } 6, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 { assignedB(P,R) : paper(P) }, reviewer(R).

#minimize { assignedB(P,R) : paper(P) : reviewer(R) }.
```

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```

Simplistic STRIPS Planning

```
fluent(p).     fluent(q).    fluent(r).
action(a).     pre(a,p).     add(a,q).     del(a,p).
action(b).     pre(b,q).     add(b,r).     del(b,q).
init(p).       query(r).

time(1..k).    lasttime(T) :- time(T), not time(T+1).

holds(P,0) :- init(P).

1 { occ(A,T) : action(A) } 1 :- time(T).
 :- occ(A,T), pre(A,F), not holds(F,T-1).

ocdel(F,T) :- occ(A,T), del(A,F).
holds(F,T) :- occ(A,T), add(A,F).
holds(F,T) :- holds(F,T-1), not ocdel(F,T), time(T).

 :- query(F), not holds(F,T), lasttime(T).
```

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```

Simplistic STRIPS Planning with iASP

```
#base.

fluent(p).    fluent(q).    fluent(r).
action(a).    pre(a,p).     add(a,q).     del(a,p).
action(b).    pre(b,q).     add(b,r).     del(b,q).
init(p).      query(r).

holds(P,0) :- init(P).

#cumulative t.

1 { occ(A,t) : action(A) } 1.
 :- occ(A,t), pre(A,F), not holds(F,t-1).

ocdel(F,t) :- occ(A,t), del(A,F).
holds(F,t) :- occ(A,t), add(A,F).
holds(F,t) :- holds(F,t-1), not ocdel(F,t).

#volatile t.

 :- query(F), not holds(F,t).
```

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```
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holds(F,t) :- holds(F,t-1), not ocdel(F,t).

#volatile t.

 :- query(F), not holds(F,t).
```

Language Extensions
Overview

Language extensions

- The expressiveness of a language can be enhanced by introducing new constructs.
- To this end, we must address the following issues:
  - What is the syntax of the new language construct?
  - What is the semantics of the new language construct?
  - How to implement the new language construct?
- A way of providing semantics is to furnish a translation removing the new constructs, eg. classical negation.
- This translation might also be used for implementing the language extension. When is this feasible?

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Integrity Constraints

Purpose Integrity constraints eliminate unwanted solution candidates

Syntax An integrity constraints is of the form

$$\leftarrow A_1, \dots, A_m, \mathit{not}\ A_{m+1}, \dots, \mathit{not}\ A_n,$$

where $n \geq m \geq 1$, and each A_i $(1 \leq i \leq n)$ is a atom.

Example

```
:- edge(X,Y), color(X,C), color(Y,C).
```

Implementation For a new symbol x, map

$$\begin{array}{rrcl} & & \leftarrow & A_1, \dots, A_m, \mathit{not}\ A_{m+1}, \dots, \mathit{not}\ A_n \\ \mapsto & x & \leftarrow & A_1, \dots, A_m, \mathit{not}\ A_{m+1}, \dots, \mathit{not}\ A_n, \mathit{not}\ x \end{array}$$

Another example $\Pi = \{p \leftarrow \mathit{not}\ q,\ q \leftarrow \mathit{not}\ p\}$

versus $\Pi' = \Pi \cup \{\leftarrow p\}$ and $\Pi'' = \Pi \cup \{\leftarrow \mathit{not}\ p\}$

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Choice rules

Idea Choices over subsets.

Syntax

$$\{A_1,\dots,A_m\} \leftarrow A_{m+1},\dots,A_n, \mathit{not}\ A_{n+1},\dots,\mathit{not}\ A_o,$$

Informal meaning If the body is satisfied in an answer set, then any subset of $\{A_1,\dots,A_m\}$ can be included in the answer set.

Example `1 {color(X,C) : col(C)} 1 :- node(X).`

Another Example The program $\Pi = \{\ \{a\} \leftarrow b,\ \ b \leftarrow\}$ has two answer sets: $\{b\}$ and $\{a,b\}$.

Implementation lparse/gringo + smodels/cmodels/nomore/clasp

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Embedding in normal logic programs

- A choice rule of form

$$\{A_1,\dots,A_m\} \leftarrow A_{m+1},\dots,A_n, \mathit{not}\ A_{n+1},\dots,\mathit{not}\ A_o$$

can be translated into $2m+1$ rules

$$\begin{array}{rclcrcl}
A & \leftarrow & A_{m+1},\dots,A_n, \mathit{not}\ A_{n+1},\dots,\mathit{not}\ A_o & & & & \\
A_1 & \leftarrow & A, \mathit{not}\ \overline{A_1} & \dots & A_m & \leftarrow & A, \mathit{not}\ \overline{A_m} \\
\overline{A_1} & \leftarrow & \mathit{not}\ A_1 & \dots & \overline{A_m} & \leftarrow & \mathit{not}\ A_m
\end{array}$$

by introducing new atoms $A, \overline{A_1}, \dots, \overline{A_m}$.

Cardinality constraints

Syntax A (positive) cardinality constraint is of the form
$l\ \{A_1, \ldots, A_m\}\ u$

Informal meaning A cardinality constraint is satisfied in an answer set X, if the number of atoms from $\{A_1, \ldots, A_m\}$ satisfied in X is between l and u (inclusive).
More formally, if $l \leq |\{A_1, \ldots, A_m\} \cap X| \leq u$.

Conditions
$$l\ \{A_1 : B_1, \ldots, A_m : B_m\}\ u$$
where $B_1, \ldots, B_m$ are used for restricting instantiations of variables occurring in $A_1, \ldots, A_m$.

Example 2 {hd(a),...,hd(m)} 4

Implementation lparse/gringo + smodels/cmodels/nomore/clasp

Cardinality rules

Idea Control cardinality of subsets.

Syntax

$$A_0 \leftarrow l \; \{A_1, \dots, A_m, not \; A_{m+1}, \dots, not \; A_n\}$$

Informal meaning If at least l elements of the "body" are true in an answer set, then add A_0 to the answer set.

➥ l is a lower bound on the "body"

Example The program $\Pi = \{\; a \leftarrow 1\{b, c\}, \;\; b \leftarrow \}$ has one answer set: $\{a, b\}$.

Implementation lparse/gringo + smodels/cmodels/nomore/clasp

☞ gringo distinguishes sets and multi-sets!

Embedding in normal logic programs (ctd)

- Replace each cardinality rule

$$A_0 \leftarrow l\ \{A_1, \ldots, A_m\} \qquad \text{by} \qquad A_0 \leftarrow cc(A_1, l)$$

where atom $cc(A_i, j)$ represents the fact that at least j of the atoms in $\{A_i, \ldots, A_m\}$, that is, of the atoms that have an equal or greater index than i, are in a particular answer set.

- The definition of $cc(A_i, j)$ is given by the rules

$$\begin{array}{rcl} cc(A_i, j{+}1) & \leftarrow & cc(A_{i+1}, j), A_i \\ cc(A_i, j) & \leftarrow & cc(A_{i+1}, j) \\ cc(A_{m+1}, 0) & \leftarrow & \end{array}$$

- What about space complexity?

Embedding in normal logic programs (ctd)

- Replace each cardinality rule

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- What about space complexity?

. . . and vice versa

- A normal rule

$$A_0 \leftarrow A_1, \ldots, A_m, \mathit{not}\ A_{m+1}, \ldots, \mathit{not}\ A_n,$$

can be represented by the cardinality rule

$$A_0 \leftarrow n{+}m\ \{A_1, \ldots, A_m, \mathit{not}\ A_{m+1}, \ldots, \mathit{not}\ A_n\}.$$

Cardinality rules with upper bounds

- A rule of the form

$$A_0 \leftarrow l \ \{A_1, \dots, A_m, not\ A_{m+1}, \dots, not\ A_n\} \ u$$

stands for

$$\begin{array}{rcl} A_0 & \leftarrow & B, not\ C \\ B & \leftarrow & l \ \{A_1, \dots, A_m, not\ A_{m+1}, \dots, not\ A_n\} \\ C & \leftarrow & u{+}1 \ \{A_1, \dots, A_m, not\ A_{m+1}, \dots, not\ A_n\} \end{array}$$

Cardinality constraints as heads

- A rule of the form

$$l\ \{A_1,\dots,A_m\}\ u \leftarrow A_{m+1},\dots,A_n, not\ A_{n+1},\dots,not\ A_o,$$

stands for

$$\begin{array}{rcl} B & \leftarrow & A_{m+1},\dots,A_n, not\ A_{n+1},\dots,not\ A_o \\ \{A_1,\dots,A_m\} & \leftarrow & B \\ C & \leftarrow & l\ \{A_1,\dots,A_m\}\ u \\ & \leftarrow & B, not\ C \end{array}$$

Full-fledged cardinality rules

- A rule of the form

$$l_0 \ S_0 \ u_0 \leftarrow l_1 \ S_1 \ u_1, \dots, l_n \ S_n \ u_n$$

stands for $0 \leq i \leq n$

$$\begin{array}{rcl} B_i & \leftarrow & l_i \ S_i \\ C_i & \leftarrow & u_i{+}1 \ S_i \\ A & \leftarrow & B_1, \dots, B_n, \mathit{not} \ C_1, \dots, \mathit{not} \ C_n \\ & \leftarrow & A, \mathit{not} \ B_0 \\ & \leftarrow & A, C_0 \\ S_0 \cap \mathcal{A} & \leftarrow & A \end{array}$$

where $\mathcal{A}$ is the underlying alphabet.

Full-fledged cardinality rules

- A rule of the form

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Weight constraints

Syntax $l\ [A_1 = w_1, \dots, A_m = w_m,$
$\quad not\ A_{m+1} = w_{m+1}, \dots, not\ A_n = w_n]\ u$

Informal meaning A weight constraint is satisfied in an answer set X, if

$$l \leq \left(\sum_{1 \leq i \leq m, A_i \in X} w_i + \sum_{m < i \leq n, A_i \notin X} w_i \right) \leq u \ .$$

➥ Generalization of cardinality constraints.

Example 80 [hd(a)=50,...,hd(m)=100] 400

Implementation lparse/gringo + smodels/cmodels/nomore/clasp

☞ gringo distinguishes sets and multi-sets!

Optimization statements

Idea Compute optimal answer sets by minimizing or maximizing a weighted sum of given atoms, respectively.

Syntax

- $\#minimize\ [A_1 = w_1, \dots, A_m = w_m,$
  $not\ A_{m+1} = w_{m+1}, \dots, not\ A_n = w_n]$
- $\#maximize\ [A_1 = w_1, \dots, A_m = w_m,$
  $not\ A_{m+1} = w_{m+1}, \dots, not\ A_n = w_n]$

Several optimization statements are interpreted lexicographically.

Example

- #minimize [hd(a)=30,. . . ,hd(m)=50]
- #minimize [road(X,Y) : length(X,Y,L) = L]

Implementation lparse/gringo + smodels/cmodels/nomore/clasp

Weak integrity constraints

Syntax $:\sim\ A_1, \dots, A_m, not\ A_{m+1}, \dots, not\ A_n\ [w : l]$

Informal meaning

1. minimize the sum of weights of violated constraints in the highest level;
2. minimize the sum of weights of violated constraints in the next lower level;
3. etc

Implementation dlv

Conditional literals in lparse and gringo

- We often want to encode the contents of a (multi-)set rather than enumerating each of the elements.
- To support this, `lparse` and `gringo` allow for conditional literals.

Syntax

$$A_0 : A_1 : \dots : A_m : not\ A_{m+1} : \dots : not\ A_n$$

Informal meaning

List all ground instances of A_0 such that corresponding instances of $A_1, \dots, A_m, not\ A_{m+1}, \dots, not\ A_n$ are true.

Example

`gringo` instantiates the program:

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p(1). p(2). p(3).  q(2).
{r(X) : p(X) : not q(X)}.
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to:

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{r(1), r(3)}.
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Domain predicates in lparse and gringo

- The predicates of literals on the right-hand side of a colon (:) must be defined from facts without any negative recursion.
- Such domain predicates are fully evaluated by lparse and gringo.

Example

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p(1). p(2).
q(X) :- p(X), not p(X+1).
q(X) :- p(X), q(X+1).
r(X) :- p(X), not r(X+1).
```

- p/1 and q/1 are domain predicates because none of them negatively depends on itself.
- r/1 is not a domain predicate because it is defined in terms of not r(X+1).

See lparse and gringo documentations for further details.

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Normal form in `lparse` and `gringo`

- Consider a logic program consisting of
  - normal rules
  - choice rules
  - cardinality rules
- Such a format is obtained by `lparse` or `gringo`
  and directly implemented by `smodels` and `clasp`.

Classical Negation
Overview

Syntax

Status quo

- In logic programs *not* (or $\sim$) denotes default negation.

Generalization

- We allow classical negation for atoms (only!).
  - ➥ Logic programs in "negation normal form."
- Given an alphabet $\mathcal{A}$ of atoms, let $\overline{\mathcal{A}} = \{\neg A \mid A \in \mathcal{A}\}$.
  - ☞ We assume $\mathcal{A} \cap \overline{\mathcal{A}} = \emptyset$.
- The atoms A and $\neg A$ are complementary.
  - ➥ $\neg A$ is the classical negation of A, and vice versa.

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Semantics

- A set X of atoms is consistent, if $X \cap \{\neg A \mid A \in (\mathcal{A} \cap X)\} = \emptyset$, and inconsistent, otherwise.
- A set X of atoms is an answer set of a logic program Π over $\mathcal{A} \cup \overline{\mathcal{A}}$ if X is an answer set of $\Pi \cup \{B \leftarrow A, \neg A \mid A \in \mathcal{A}, B \in (\mathcal{A} \cup \overline{\mathcal{A}})\}$
  - ➥ The only inconsistent answer set (candidate) is $X = \mathcal{A} \cup \overline{\mathcal{A}}$.
- For a normal or disjunctive logic program Π over $\mathcal{A} \cup \overline{\mathcal{A}}$, exactly one of the following two cases applies:
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To cross or not to cross...?

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  - Answer set: $\{cross\}$
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  ☞ Π is a *nested* logic program.

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Disjunctive Logic Programs
Overview

Disjunctive logic programs

- A disjunctive rule, r, is an ordered pair of the form

$$A_1 \, ; \ldots ; A_m \leftarrow A_{m+1}, \ldots, A_n, not\ A_{n+1}, \ldots, not\ A_o,$$

  where $o \geq n \geq m \geq 0$, and each A_i $(0 \leq i \leq o)$ is an atom.
- A disjunctive logic program is a finite set of disjunctive rules.
- (Generalized) Notation

$$\begin{array}{rcl} head(r) & = & \{A_1, \ldots, A_m\} \\ body(r) & = & \{A_{m+1}, \ldots, A_n, not\ A_{n+1}, \ldots, not\ A_o\} \\ body^+(r) & = & \{A_{m+1}, \ldots, A_n\} \\ body^-(r) & = & \{A_{n+1}, \ldots, A_o\} \end{array}$$

- A program is called positive if $body^-(r) = \emptyset$ for all its rules.

Answer sets

- Positive programs:
  - A set X of atoms is closed under a positive program Π iff for any $r \in \Pi$, $head(r) \cap X \neq \emptyset$ whenever $body^+(r) \subseteq X$.
    - ➥ X corresponds to a model of Π (seen as a formula).
  - The set of all $\subseteq$-minimal sets of atoms being closed under a positive program Π is denoted by $\min_\subseteq(\Pi)$.
    - ➥ $\min_\subseteq(\Pi)$ corresponds to the $\subseteq$-minimal models of Π (ditto).
- Disjunctive programs:
  - The reduct, Π^X, of a disjunctive program Π relative to a set X of atoms is defined by

$$\Pi^X = \{head(r) \leftarrow body^+(r) \mid r \in \Pi \text{ and } body^-(r) \cap X = \emptyset\}.$$

  - A set X of atoms is an answer set of a disjunctive program Π if $X \in \min_\subseteq(\Pi^X)$.

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A "positive" example

$$\Pi = \left\{ \begin{array}{lcl} a & \leftarrow & \\ b\,;c & \leftarrow & a \end{array} \right\}$$

- The sets $\{a,b\}$, $\{a,c\}$, and $\{a,b,c\}$ are closed under Π.
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- $\Pi_2 = \{a\,;b\,;c \leftarrow\ ,\ \leftarrow a\}$ has answer sets $\{b\}$ and $\{c\}$.
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- $\Pi_1 = \{a\,;b\,;c \leftarrow\}$ has answer sets $\{a\}$, $\{b\}$, and $\{c\}$.
- $\Pi_2 = \{a\,;b\,;c \leftarrow\ ,\ \leftarrow a\}$ has answer sets $\{b\}$ and $\{c\}$.
- $\Pi_3 = \{a\,;b\,;c \leftarrow\ ,\ \leftarrow a\ ,\ b \leftarrow c\ ,\ c \leftarrow b\}$ has answer set $\{b, c\}$.
- $\Pi_4 = \{a\,;b \leftarrow c\ ,\ b \leftarrow \mathit{not}\ a, \mathit{not}\ c\ ,\ a\,;c \leftarrow \mathit{not}\ b\}$ has answer sets $\{a\}$ and $\{b\}$.

An example with variables

$$\Pi = \left\{ \begin{array}{lcl} a(1,2) & \leftarrow & \\ b(X)\,;c(Y) & \leftarrow & a(X,Y), \mathit{not}\ c(Y) \end{array} \right\}$$

$$\mathit{ground}(\Pi) = \left\{ \begin{array}{lcl} a(1,2) & \leftarrow & \\ b(1)\,;c(1) & \leftarrow & a(1,1), \mathit{not}\ c(1) \\ b(1)\,;c(2) & \leftarrow & a(1,2), \mathit{not}\ c(2) \\ b(2)\,;c(1) & \leftarrow & a(2,1), \mathit{not}\ c(1) \\ b(2)\,;c(2) & \leftarrow & a(2,2), \mathit{not}\ c(2) \end{array} \right\}$$

For every answer set X of Π, we have

- $a(1,2) \in X$ and
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Nested Logic Programs
Overview

Nested logic programs

- Formulas are formed from
  - propositional atoms and
  - $\top$ and $\bot$

  using
  - negation-as-failure (*not*),
  - conjunction (,), and
  - disjunction (;).
- A nested rule, r, is an ordered pair of the form $F \leftarrow G$ where F and G are formulas.
- A nested program is a finite set of rules.
- Notation: $head(r) = F$ and $body(r) = G$.

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Satisfaction relation

- The satisfaction relation $X \models F$ between a set of atoms and a formula F is defined recursively as follows:
  - $X \models F$ if $F \in X$ for an atom F,
  - $X \models \top$,
  - $X \not\models \bot$,
  - $X \models (F, G)$ if $X \models F$ and $X \models G$,
  - $X \models (F; G)$ if $X \models F$ or $X \models G$,
  - $X \models \mathit{not}\ F$ if $X \not\models F$.
- A set X of atoms satisfies a nested program Π, written $X \models \Pi$, iff for any $r \in \Pi$, $X \models \mathit{head}(r)$ whenever $X \models \mathit{body}(r)$.
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- The reduct, F^X, of a formula F relative to a set X of atoms is defined recursively as follows:
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$$\Pi^X = \{head(r)^X \leftarrow body(r)^X \mid r \in \Pi\}.$$

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Two examples

- $\Pi_1 = \{(p \,;\, \mathit{not}\ p) \leftarrow \top\}$

  For $X = \emptyset$, we get

  $\Pi_1^{\emptyset} = \{(p \,;\, \top) \leftarrow \top\}$
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- In general,

  $F \leftarrow G,\ \mathit{not}\ \mathit{not}\ H$ is equivalent to $F \,;\, \mathit{not}\ H \leftarrow G$
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➥ Intuitionistic Logics HT (Heyting, 1930) and G3 (Gödel, 1932)

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Some more examples

$$
\begin{array}{rcl}
\Pi_3 & = & \{p \leftarrow (q, r) \,;\, (not\ q, not\ s)\} \\
\Pi_4 & = & \{(p \,;\, not\ p), (q \,;\, not\ q), (r \,;\, not\ r) \leftarrow \top\} \\
\Pi_5 & = & \{(p \,;\, not\ p), (q \,;\, not\ q), (r \,;\, not\ r) \leftarrow \top,\ \bot \leftarrow p, q\}
\end{array}
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Propositional Theories
Overview

Propositional theories

- Formulas are formed from
  - propositional atoms and
  - $\bot$

  using
  - conjunction ($\wedge$),
  - disjunction ($\vee$), and
  - implication ($\rightarrow$).
- Notation

$$\begin{aligned} \top &= (\bot \rightarrow \bot) \\ \sim F &= (F \rightarrow \bot) \qquad \text{(or: } \textit{not } F\text{)} \end{aligned}$$

A propositional theory is a finite set of formulas.

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Reduct

- The satisfaction relation $X \models F$ between a set X of atoms and a (set of) formula(s) F is defined as in propositional logic.
- The reduct, F^X, of a formula F relative to a set X of atoms is defined recursively as follows:
  - $F^X = \bot$ if $X \not\models F$
  - $F^X = F$ if $F \in X$
  - $F^X = (G^X \circ H^X)$ if $X \models F$ and $F = (G \circ H)$ for $\circ \in \{\wedge, \vee, \rightarrow\}$
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- If X is an answer set of $\mathcal{F}$, then
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- $\mathcal{F}_1 = \{p \vee (p \rightarrow (q \wedge r))\}$
  - For $X = \{p, q, r\}$, we get
    $\mathcal{F}_1^{\{p,q,r\}} = \{p \vee (p \rightarrow (q \wedge r))\}$ and $\min_{\subseteq}(\mathcal{F}_1^{\{p,q,r\}}) = \{\emptyset\}$.
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Logic programs as propositional theories

- The normal logic program $\Pi = \{p \leftarrow \mathit{not}\ q,\ q \leftarrow \mathit{not}\ p\}$ corresponds to $\tau[\Pi] = \{\sim q \rightarrow p,\ \sim p \rightarrow q\}$.
  - ➥ Answer sets: $\{p\}$ and $\{q\}$

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Complexity

Let A be an atom and X be a set of atoms.

- For a positive normal logic program Π:
  - Deciding whether X is the answer set of Π is **P**-complete.
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Overview

Completion

Let Π be a normal logic program,
and recall that $atom(\Pi)$ denotes the set of atoms occurring in Π.
The completion of Π is defined as follows:

$$Comp(body(r)) = \bigwedge_{A \in body^+(r)} A \wedge \bigwedge_{A \in body^-(r)} \neg A$$

$$Comp(\Pi) = \{A \leftrightarrow \bigvee_{r \in \Pi, head(r)=A} Comp(body(r)) \mid A \in atom(\Pi)\}$$

- Every answer set of Π is a model of $Comp(\Pi)$, but not vice versa.
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A first example

$$\Pi = \left\{ \begin{array}{lcl} a & \leftarrow & \\ b & \leftarrow & a \\ c & \leftarrow & b \\ c & \leftarrow & d \\ d & \leftarrow & c, e \end{array} \right\} \qquad Comp(\Pi) = \left\{ \begin{array}{lcl} a & \leftrightarrow & \top \\ b & \leftrightarrow & a \\ c & \leftrightarrow & (b \vee d) \\ d & \leftrightarrow & (c \wedge e) \\ e & \leftrightarrow & \bot \end{array} \right\}$$

- The supported model of Π is $\{a, b, c\}$.
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$$\Pi = \left\{ \begin{array}{lcl} q & \leftarrow & \mathit{not}\ p \\ p & \leftarrow & \mathit{not}\ q, \mathit{not}\ x \end{array} \right\} \qquad \mathit{Comp}(\Pi) = \left\{ \begin{array}{lcl} q & \leftrightarrow & \neg p \\ p & \leftrightarrow & (\neg q \wedge \neg x) \\ x & \leftrightarrow & \bot \end{array} \right\}$$

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$\Pi = \{ \; p \leftarrow p \; \} \qquad Comp(\Pi) = \{ \; p \leftrightarrow p \; \}$

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Fitting operator: Basic idea

Idea Extend T_Π to normal logic programs.

Logical background Completion

- The head atom of a rule must be *true* if the rule's body is *true*.
- An atom must be *false* if the body of each rule having it as head is *false*.

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Fitting operator: Definition

Let Π be a normal logic program.

Define

$$\mathbf{\Phi}_\Pi\langle T,F\rangle = \langle \mathbf{T}_\Pi\langle T,F\rangle, \mathbf{F}_\Pi\langle T,F\rangle\rangle$$

where

$$\begin{aligned}
\mathbf{T}_\Pi\langle T,F\rangle &= \{\mathit{head}(r) \mid r \in \Pi, \mathit{body}^+(r) \subseteq T, \mathit{body}^-(r) \subseteq F\} \\
\mathbf{F}_\Pi\langle T,F\rangle &= \{A \in \mathit{atom}(\Pi) \mid \mathit{body}^+(r) \cap F \neq \emptyset \text{ or } \mathit{body}^-(r) \cap T \neq \emptyset \\
&\qquad\qquad \text{for each } r \in \Pi \text{ such that } \mathit{head}(r) = A\}
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$$\Pi_1 = \left\{ \begin{array}{lll} a \leftarrow & c \leftarrow a, not\ d & e \leftarrow b \\ b \leftarrow not\ a & d \leftarrow not\ c, not\ e & e \leftarrow e \end{array} \right\}$$

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Fitting semantics

Define the iterative variant of $\mathbf{\Phi}_\Pi$ analogously to T_Π:

$$\mathbf{\Phi}^0_\Pi\langle T,F\rangle = \langle T,F\rangle \qquad \mathbf{\Phi}^{i+1}_\Pi\langle T,F\rangle = \mathbf{\Phi}_\Pi\mathbf{\Phi}^i_\Pi\langle T,F\rangle$$

Define the Fitting semantics of a normal logic program Π as the partial interpretation:

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Fitting semantics: Properties

Let Π be a normal logic program.

- $\mathbf{\Phi}_\Pi\langle\emptyset,\emptyset\rangle$ is monotonic.
  That is, $\mathbf{\Phi}^i_\Pi\langle\emptyset,\emptyset\rangle \sqsubseteq \mathbf{\Phi}^{i+1}_\Pi\langle\emptyset,\emptyset\rangle$.
- The Fitting semantics of Π is
  - not conflicting,
  - and generally not total.

Fitting fixpoints

Let Π be a normal logic program,
and let $\langle T, F \rangle$ be a partial interpretation.
Define $\langle T, F \rangle$ as a Fitting fixpoint of Π if $\mathbf{\Phi}_{\Pi}\langle T, F \rangle = \langle T, F \rangle$.

- The Fitting semantics is the $\sqsubseteq$-least Fitting fixpoint of Π.
- Any other Fitting fixpoint extends the Fitting semantics.
- Total Fitting fixpoints correspond to supported models.

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Π_1 has three total Fitting fixpoints:

1. $\langle\{a, c\}, \{b, d, e\}\rangle$
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Π_1 has three supported models, two of them are answer sets.

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Properties of Fitting operator

Let Π be a normal logic program,
and let $\langle T, F\rangle$ be a partial interpretation.

- Let $\mathbf{\Phi}_\Pi\langle T, F\rangle = \langle T', F'\rangle$.
  If X is an answer set of Π such that $T \subseteq X$ and $X \cap F = \emptyset$,
  then $T' \subseteq X$ and $X \cap F' = \emptyset$.
- That is, $\mathbf{\Phi}_\Pi$ is answer set preserving.
  - $\mathbf{\Phi}_\Pi$ can be used for approximating answer sets and so for propagation in ASP-solvers.

However, $\mathbf{\Phi}_\Pi$ is still insufficient, because total fixpoints correspond to supported models, not necessarily answer sets.

☞ The problem is the same as with program completion.

The missing piece is non-circularity of derivations !

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$$\Pi = \left\{ \begin{array}{lcl} a & \leftarrow & b \\ b & \leftarrow & a \end{array} \right\}$$

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That is, Fitting semantics cannot assign *false* to a and b, although they can never become *true* !

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(Non-)cyclic derivations

- Cyclic derivations are causing the mismatch between supported models and answer sets.
- Atoms in an answer set can be "derived" from a program in a finite number of steps.
- Atoms in a cycle (not being "supported from outside the cycle") cannot be "derived" from a program in a finite number of steps.
  - ☞ But they do not contradict the completion of a program.

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- Atoms in an answer set can be "derived" from a program in a finite number of steps.
- Atoms in a cycle (not being "supported from outside the cycle") cannot be "derived" from a program in a finite number of steps.
  - ☞ But they do not contradict the completion of a program.

Non-cyclic derivations

Let X be an answer set of normal logic program Π.

- For every atom $A \in X$, there is a finite sequence of positive rules

  $$\langle r_1, \dots, r_n \rangle$$

  such that

  1. $head(r_1) = A$,
  2. $body^+(r_i) \subseteq \{head(r_j) \mid i < j \leq n\}$ for $1 \leq i \leq n$,
  3. $r_i \in \Pi^X$ for $1 \leq i \leq n$.
- That is, each atom of X has a non-cyclic derivation from Π^X.
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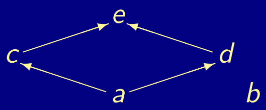
Positive atom dependency graph

Let Π be a normal logic program.
The positive atom dependency graph of Π is a directed graph $G(\Pi) = (V, E)$ such that

1. $V = atom(\Pi)$ and
2. $E = \{(p, q) \mid r \in \Pi, p \in body^+(r), head(r) = q\}$.

Examples

$$\Pi_2 = \left\{ \begin{array}{ll} a \leftarrow \mathit{not}\ b & b \leftarrow \mathit{not}\ a \\ c \leftarrow a, \mathit{not}\ d & d \leftarrow a, \mathit{not}\ c \\ e \leftarrow c, \mathit{not}\ a & e \leftarrow d, \mathit{not}\ b \end{array} \right\}$$



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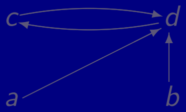

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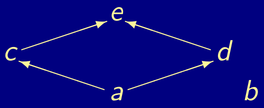

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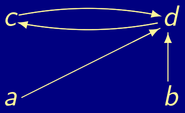

Tight programs

- A normal logic program Π is tight iff $G(\Pi)$ is acyclic.
- For example, Π_2 is tight, whereas Π_3 is not.
- If a normal logic program Π is tight, then

  X is an answer set of Π iff X is a model of $Comp(\Pi)$.

  That is, for tight programs, answer sets and supported models coincide.
- Also, for tight programs, $\mathbf{\Phi}_\Pi$ is sufficient for propagation.

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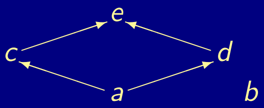

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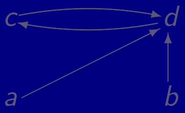

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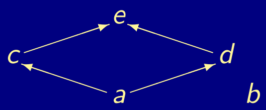

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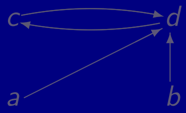

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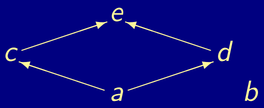

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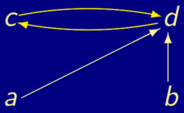

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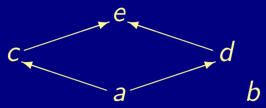

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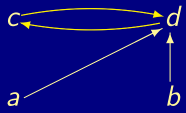

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Unfounded Sets
Overview

Unfounded sets

Let Π be a normal logic program,
and let $\langle T, F \rangle$ be a partial interpretation.

A set $U \subseteq atom(\Pi)$ is an unfounded set of Π with respect to $\langle T, F \rangle$ if, for each rule $r \in \Pi$, we have

1. $head(r) \notin U$,
2. $body^+(r) \cap F \neq \emptyset$ or $body^-(r) \cap T \neq \emptyset$, or
3. $body^+(r) \cap U \neq \emptyset$.

- Intuitively, $\langle T, F \rangle$ is what we already know about Π.
- Rules satisfying Condition 1 or 2 are not usable for further derivations.
- Condition 3 is the unfounded set condition treating cyclic derivations: All rules still being usable to derive an atom in U require an(other) atom in U to be true.

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Example

$$\Pi = \left\{ \begin{array}{ccc} a & \leftarrow & b \\ b & \leftarrow & a \end{array} \right\}$$

- $\emptyset$ is an unfounded set (by definition).
- $\{a\}$ is not an unfounded set of Π wrt $\langle \emptyset, \emptyset \rangle$.
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Greatest unfounded sets

Observation The union of two unfounded sets is an unfounded set.

Let Π be a normal logic program,
and let $\langle T, F\rangle$ be a partial interpretation.
The greatest unfounded set of Π with respect to $\langle T, F\rangle$, denoted by $\mathbf{U}_\Pi\langle T, F\rangle$, is the union of all unfounded sets of Π with respect to $\langle T, F\rangle$.

- Alternatively, we may define

$$\mathbf{U}_\Pi\langle T, F\rangle = \mathit{atom}(\Pi) \setminus \mathit{Cn}(\{r \in \Pi \mid \mathit{body}^+(r) \cap F = \emptyset\}^T).$$

- Observe that $\mathit{Cn}(\{r \in \Pi \mid \mathit{body}^+(r) \cap F = \emptyset\}^T)$ contains all non-circularly derivable atoms from Π wrt $\langle T, F\rangle$.

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Well-founded operator

Let Π be a normal logic program,
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Observation Condition 2 (in the definition of an unfounded set) corresponds to set $\mathbf{F}_\Pi\langle T, F \rangle$ of Fitting's $\mathbf{\Phi}_\Pi\langle T, F \rangle$.

Idea Extend (negative part of) Fitting's operator $\mathbf{\Phi}_\Pi$.
That is,

- keep definition of $\mathbf{T}_\Pi\langle T, F \rangle$ from $\mathbf{\Phi}_\Pi\langle T, F \rangle$ and
- replace $\mathbf{F}_\Pi\langle T, F \rangle$ from $\mathbf{\Phi}_\Pi\langle T, F \rangle$ by $\mathbf{U}_\Pi\langle T, F \rangle$.

In words, an atom must be *false*
if it belongs to the greatest unfounded set.

Definition $\mathbf{\Omega}_\Pi\langle T, F \rangle = \langle \mathbf{T}_\Pi\langle T, F \rangle, \mathbf{U}_\Pi\langle T, F \rangle \rangle$

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- keep definition of $\mathbf{T}_\Pi\langle T, F\rangle$ from $\mathbf{\Phi}_\Pi\langle T, F\rangle$ and
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In words, an atom must be *false*
if it belongs to the greatest unfounded set.

Definition $\mathbf{\Omega}_\Pi\langle T, F\rangle = \langle \mathbf{T}_\Pi\langle T, F\rangle, \mathbf{U}_\Pi\langle T, F\rangle\rangle$

Property $\mathbf{\Phi}_\Pi\langle T, F\rangle \sqsubseteq \mathbf{\Omega}_\Pi\langle T, F\rangle$

Well-founded operator

Let Π be a normal logic program,
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Observation Condition 2 (in the definition of an unfounded set) corresponds to set $\mathbf{F}_\Pi\langle T, F\rangle$ of Fitting's $\mathbf{\Phi}_\Pi\langle T, F\rangle$.

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$$\begin{array}{rcl} \mathbf{\Omega}_{\Pi_1}\langle \{c\}, \emptyset \rangle & = & \langle \{a\}, \{d\} \rangle \\ \mathbf{\Omega}_{\Pi_1}\langle \{a\}, \{d\} \rangle & = & \langle \{a, c\}, \{b, e\} \rangle \\ \mathbf{\Omega}_{\Pi_1}\langle \{a, c\}, \{b, e\} \rangle & = & \langle \{a\}, \{b, d, e\} \rangle \\ \mathbf{\Omega}_{\Pi_1}\langle \{a\}, \{b, d, e\} \rangle & = & \langle \{a, c\}, \{b, e\} \rangle \\ & \vdots & \end{array}$$

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Well-founded semantics

Define the iterative variant of $\mathbf{\Omega}_\Pi$ analogously to $\mathbf{\Phi}_\Pi$:

$$\mathbf{\Omega}^0_\Pi\langle T,F\rangle = \langle T,F\rangle \qquad \mathbf{\Omega}^{i+1}_\Pi\langle T,F\rangle = \mathbf{\Omega}_\Pi\mathbf{\Omega}^{i}_\Pi\langle T,F\rangle$$

Define the well-founded semantics of a normal logic program Π as the partial interpretation:

$$\bigsqcup_{i\geq 0}\mathbf{\Omega}^i_\Pi\langle\emptyset,\emptyset\rangle$$

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Well-founded semantics: Properties

Let Π be a normal logic program.

- $\mathbf{\Omega}_\Pi\langle\emptyset,\emptyset\rangle$ is monotonic.
  That is, $\mathbf{\Omega}^i_\Pi\langle\emptyset,\emptyset\rangle \sqsubseteq \mathbf{\Omega}^{i+1}_\Pi\langle\emptyset,\emptyset\rangle$.
- The well-founded semantics of Π is
  - not conflicting,
  - and generally not total.
- We have $\bigsqcup_{i\geq 0}\mathbf{\Phi}^i_\Pi\langle\emptyset,\emptyset\rangle \sqsubseteq \bigsqcup_{i\geq 0}\mathbf{\Omega}^i_\Pi\langle\emptyset,\emptyset\rangle$.

Well-founded fixpoints

Let Π be a normal logic program,
and let $\langle T, F\rangle$ be a partial interpretation.
Define $\langle T, F\rangle$ as a well-founded fixpoint of Π if $\mathbf{\Omega}_\Pi\langle T, F\rangle = \langle T, F\rangle$.

- The well-founded semantics is the $\sqsubseteq$-least well-founded fixpoint of Π.
- Any other well-founded fixpoint extends the well-founded semantics.
- Total well-founded fixpoints correspond to answer sets.

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Π_1 has two total well-founded fixpoints:

1. $\langle \{a, c\}, \{b, d, e\} \rangle$
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Both of them represent answer sets.

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Properties of well-founded operator

Let Π be a normal logic program,
and let $\langle T, F\rangle$ be a partial interpretation.

- Let $\mathbf{\Omega}_\Pi\langle T, F\rangle = \langle T', F'\rangle$.
  If X is an answer set of Π such that $T \subseteq X$ and $X \cap F = \emptyset$, then $T' \subseteq X$ and $X \cap F' = \emptyset$.
- That is, $\mathbf{\Omega}_\Pi$ is answer set preserving.
  - $\mathbf{\Omega}_\Pi$ can be used for approximating answer sets and so for propagation in ASP-solvers.

Unlike $\mathbf{\Phi}_\Pi$, operator $\mathbf{\Omega}_\Pi$ is sufficient for propagation because total fixpoints correspond to answer sets.

☞ In addition to $\mathbf{\Omega}_\Pi$, most ASP-solvers apply backward propagation, originating from program completion (although this is unnecessary from a formal point of view).

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Characterizing non-cyclic derivations

An alternative approach

Question Is there a propositional formula $F(\Pi)$ such that the models of $F(\Pi)$ correspond to the answer sets of Π ?

- ☞ If we consider the completion of a program, $Comp(\Pi)$, then the problem boils down to eliminating the circular support of atoms that are true in the supported models of Π.

Idea Add formulas to $Comp(\Pi)$ that prohibit circular support of sets of atoms.

- ☞ Circular support between atoms p and q is possible if p has a path to q and q has a path to p in a program's positive atom dependency graph.

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Loops

Let Π be a normal logic program, and
let $G(\Pi) = (atom(\Pi), E)$ be the positive atom dependency graph of Π.

- A set $\emptyset \subset L \subseteq atom(\Pi)$ is a loop of Π if it induces a non-trivial strongly connected subgraph of $G(\Pi)$.
- That is, each pair of atoms in L is connected by a path of non-zero length in $(L, E \cap (L \times L))$.
- We denote the set of all loops of Π by $Loop(\Pi)$.

Observation Program Π is tight iff $Loop(\Pi) = \emptyset$.

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Loop formulas

Let Π be a normal logic program.

- For $L \subseteq atom(\Pi)$, define the external supports of L for Π as

$$ES_\Pi(L) = \{ \, r \in \Pi \mid head(r) \in L, body^+(r) \cap L = \emptyset \, \}.$$

- The (disjunctive) loop formula of L for Π is

$$\begin{aligned} LF_\Pi(L) &= \left(\bigvee_{A \in L} A\right) \rightarrow \left(\bigvee_{r \in ES_\Pi(L)} Comp(body(r))\right) \\ &\equiv \left(\bigwedge_{r \in ES_\Pi(L)} \neg Comp(body(r))\right) \rightarrow \left(\bigwedge_{A \in L} \neg A\right). \end{aligned}$$

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Let Π be a normal logic program.

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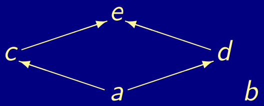
Lin-Zhao Theorem

Theorem

Let Π be a normal logic program and $X \subseteq atom(\Pi)$.
Then, X is an answer set of Π iff $X \models Comp(\Pi) \cup LF(\Pi)$.

Loops and loop formulas: Examples

$$\Pi_2 = \left\{ \begin{array}{ll} a \leftarrow not\ b & b \leftarrow not\ a \\ c \leftarrow a, not\ d & d \leftarrow a, not\ c \\ e \leftarrow c, not\ a & e \leftarrow d, not\ b \end{array} \right\}$$



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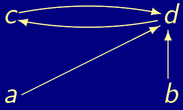

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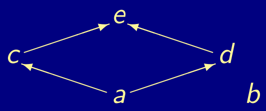

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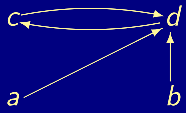

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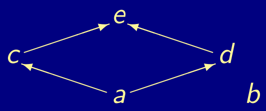

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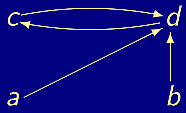

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Let X be a supported model of normal logic program Π.

Then, X is an answer set of Π iff

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If $\mathcal{P} \not\subseteq \mathcal{NC}^1/poly$,[1] then there is no translation $\mathcal{T}$ from logic programs to propositional formulas such that, for each normal logic program Π, both of the following conditions hold:

1. The propositional variables in $\mathcal{T}[\Pi]$ are a subset of $atom(\Pi)$.
2. The size of $\mathcal{T}[\Pi]$ is polynomial in the size of Π.

☞ Every vocabulary-preserving translation from normal logic programs to propositional formulas must be exponential (in the worst case).

Observations

- Translation $Comp(\Pi) \cup LF(\Pi)$ preserves the vocabulary of Π.
- The number of loops in $Loop(\Pi)$ may be exponential in $|atom(\Pi)|$.

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Tableau Calculi
Overview

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Goal Analyze computations in ASP-solvers

Wanted A declarative and fine-grained instrument for characterizing operations as well as strategies of ASP-solvers

Idea View answer set computations as derivations in an inference system

➥ Tableau-based proof system for analyzing ASP-solving

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Tableau calculi

- Traditionally, tableau calculi are used for
  - automated theorem proving and
  - proof theoretical analysis

  in classical as well as non-classical logics.
- **General idea:** Given an input, prove some property by decomposition. Decomposition is done by applying deduction rules.
- For details, see [14].

Tableau calculi: General definitions

- A tableau is a (mostly binary) tree.
- A branch in a tableau is a path from the root to a leaf.
- A branch containing $\gamma_1, \dots, \gamma_m$ can be extended by applying tableau rules of form:

$$\frac{\gamma_1, \dots, \gamma_m}{\begin{array}{c}\alpha_1\\ \vdots\\ \alpha_n\end{array}} \qquad\qquad \frac{\gamma_1, \dots, \gamma_m}{\beta_1 \mid \dots \mid \beta_n}$$

- Rules of the former format append entries $\alpha_1, \dots, \alpha_n$ to the branch.
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Tableau calculus: Example

A simple tableau calculus for proving unsatisfiability of propositional formulas, composed from $\neg$, $\wedge$, and $\vee$, consists of rules:

$$\frac{\neg\neg\alpha}{\alpha} \qquad \frac{\alpha_1 \wedge \alpha_2}{\begin{array}{c}\alpha_1\\ \alpha_2\end{array}} \qquad \frac{\beta_1 \vee \beta_2}{\beta_1 \mid \beta_2}$$

- All rules are semantically valid, interpreting entries in a branch as connected via "and" and distinct (sub-)branches as connected via "or".
- A propositional formula φ (composed from $\neg$, $\wedge$, and $\vee$) is unsatisfiable iff there is a tableau with φ as the root node such that
  1. all other entries can be produced by tableau rules and
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Tableau calculus: Example (ctd)

(1)	$a \wedge ((\neg b \wedge (\neg a \vee b)) \vee \neg\neg\neg a)$	$[\varphi]$
(2)	a	[1]
(3)	$(\neg b \wedge (\neg a \vee b)) \vee \neg\neg\neg a$	[1]

(4)	$\neg b \wedge (\neg a \vee b)$	[3]	**(9)**	$\neg\neg\neg a$	[3]
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Right branch:

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➥ $a \wedge ((\neg b \wedge (\neg a \vee b)) \vee \neg\neg\neg a)$ is unsatisfiable.

Tableaux and ASP: The idea

- A tableau rule captures an elementary inference scheme in an ASP-solver.
- A branch in a tableau corresponds to a successful or unsuccessful computation of an answer set.
- An entire tableau represents a traversal of the search space.

Tableaux and ASP: Specific definitions

- A (signed) tableau for a logic program Π is a binary tree such that
  - the root node of the tree consists of the rules in Π;
  - the other nodes in the tree are entries of the form $\mathbf{T}v$ or $\mathbf{F}v$, called signed literals, where v is a variable,
  - generated by extending a tableau using deduction rules (given below).
- An entry $\mathbf{T}v$ ($\mathbf{F}v$) reflects that variable v is *true* (*false*) in a corresponding variable assignment.
  - ➥ A set of signed literals constitutes a partial assignment.
- For a normal logic program Π,
  - atoms of Π in $atom(\Pi)$ and
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Tableau rules for ASP at a glance [42]

$$(FTB)\quad \frac{p \leftarrow l_1, \dots, l_n \quad \mathbf{t}l_1, \dots, \mathbf{t}l_n}{\mathbf{T}\{l_1, \dots, l_n\}} \qquad (BFB)\quad \frac{\mathbf{F}\{l_1, \dots, l_i, \dots, l_n\} \quad \mathbf{t}l_1, \dots, \mathbf{t}l_{i-1}, \mathbf{t}l_{i+1}, \dots, \mathbf{t}l_n}{\mathbf{f}l_i}$$

$$(FTA)\quad \frac{p \leftarrow l_1, \dots, l_n \quad \mathbf{T}\{l_1, \dots, l_n\}}{\mathbf{T}p} \qquad (BFA)\quad \frac{p \leftarrow l_1, \dots, l_n \quad \mathbf{F}p}{\mathbf{F}\{l_1, \dots, l_n\}}$$

$$(FFB)\quad \frac{p \leftarrow l_1, \dots, l_i, \dots, l_n \quad \mathbf{f}l_i}{\mathbf{F}\{l_1, \dots, l_i, \dots, l_n\}} \qquad (BTB)\quad \frac{\mathbf{T}\{l_1, \dots, l_i, \dots, l_n\}}{\mathbf{t}l_i}$$

$$(FFA)\quad \frac{\mathbf{F}B_1, \dots, \mathbf{F}B_m}{\mathbf{F}p}\ (\S) \qquad (BTA)\quad \frac{\mathbf{T}p \quad \mathbf{F}B_1, \dots, \mathbf{F}B_{i-1}, \mathbf{F}B_{i+1}, \dots, \mathbf{F}B_m}{\mathbf{T}B_i}$$

$$(WFN)\quad \frac{\mathbf{F}B_1, \dots, \mathbf{F}B_m}{\mathbf{F}p}\ (\dagger) \qquad (WFJ)\quad \frac{\mathbf{T}p \quad \mathbf{F}B_1, \dots, \mathbf{F}B_{i-1}, \mathbf{F}B_{i+1}, \dots, \mathbf{F}B_m}{\mathbf{T}B_i}$$

$$(FL)\quad \frac{\mathbf{F}B_1, \dots, \mathbf{F}B_m}{\mathbf{F}p}\ (\ddagger) \qquad (BL)\quad \frac{\mathbf{T}p \quad \mathbf{F}B_1, \dots, \mathbf{F}B_{i-1}, \mathbf{F}B_{i+1}, \dots, \mathbf{F}B_m}{\mathbf{T}B_i}$$

$$(Cut[X])\quad \frac{}{\mathbf{T}v \mid \mathbf{F}v}\quad (\sharp[X])$$

More concepts

- A **tableau calculus** is a set of tableau rules.
- A branch in a tableau is conflicting,
  if it contains both $\mathbf{T}v$ and $\mathbf{F}v$ for some variable v.
- A branch in a tableau is total for a program Π,
  if it contains either $\mathbf{T}v$ or $\mathbf{F}v$ for each $v \in atom(\Pi) \cup body(\Pi)$.
- A branch in a tableau of some calculus $\mathcal{T}$ is closed,
  if no rule in $\mathcal{T}$ other than *Cut* can produce any new entries.
- A branch in a tableau is complete,
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Example

Consider the program

$$\Pi = \left\{ \begin{array}{l} a \leftarrow \\ c \leftarrow \mathit{not}\ b, \mathit{not}\ d \\ d \leftarrow a, \mathit{not}\ c \end{array} \right\}$$

having two answer sets $\{a, c\}$ and $\{a, d\}$.

(Previewed) Example

$$a \leftarrow$$
$$c \leftarrow \mathit{not}\ b, \mathit{not}\ d$$
$$d \leftarrow a, \mathit{not}\ c$$

(FTB)		$\mathbf{T}\emptyset$		
(FTA)		$\mathbf{T}a$		
(FFA)		$\mathbf{F}b$		
(Cut[$atom(\Pi)$])		$\mathbf{T}c$		$\mathbf{F}c$
	(BTA)	$\mathbf{T}\{\mathit{not}\ b, \mathit{not}\ d\}$	(BFA)	$\mathbf{F}\{\mathit{not}\ b, \mathit{not}\ d\}$
	(BTB)	$\mathbf{F}d$	(BFB)	$\mathbf{T}d$
	(FFB)	$\mathbf{F}\{a, \mathit{not}\ c\}$	(FTB)	$\mathbf{T}\{a, \mathit{not}\ c\}$

Recall answer sets $\{a, c\}$ and $\{a, d\}$.

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\begin{array}{rccccc}
 & & & a \leftarrow & & \\
 & & & c \leftarrow not\ b, not\ d & & \\
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\text{(FTB)} & & & \mathbf{T}\emptyset & & \\
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$$

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Tableau rules: Auxiliary definitions

- The application of rules makes use of two conjugation functions, $\mathbf{t}$ and $\mathbf{f}$.
- For a literal l, define:

$$\mathbf{t}l = \begin{cases} \mathbf{T}l & \text{if } l \text{ is an atom} \\ \mathbf{F}p & \text{if } l = not\ p \text{ for an atom } p \end{cases}$$

$$\mathbf{f}l = \begin{cases} \mathbf{F}l & \text{if } l \text{ is an atom} \\ \mathbf{T}p & \text{if } l = not\ p \text{ for an atom } p \end{cases}$$

Examples

$\mathbf{t}p = \mathbf{T}p \qquad \mathbf{f}p = \mathbf{F}p \qquad \mathbf{t}not\ p = \mathbf{F}p \qquad \mathbf{f}not\ p = \mathbf{T}p$

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Tableau rules: Auxiliary definitions (ctd)

- Some tableau rules require conditions for their application.
  Such conditions are specified as provisos:

$$\frac{\mathit{prerequisites}}{\mathit{consequence}}\ (\mathit{proviso}) \qquad \mathit{proviso}\text{: some condition(s)}$$

☞ All tableau rules given in the sequel are answer set preserving.

Forward True Body (FTB)

Prerequisites All of a body's literals are *true*.

Consequence The body is *true*.

Tableau Rule FTB

$$\frac{\begin{array}{c} p \leftarrow l_1, \ldots, l_n \\ \mathbf{t}l_1, \ldots, \mathbf{t}l_n \end{array}}{\mathbf{T}\{l_1, \ldots, l_n\}}$$

Example

$$\frac{\begin{array}{c} a \leftarrow b, not\ c \\ \mathbf{T}b \\ \mathbf{F}c \end{array}}{\mathbf{T}\{b, not\ c\}}$$

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Backward False Body (BFB)

Prerequisites A body is *false*, and all its literals except for one are *true*.

Consequence The residual body literal is *false*.

Tableau Rule BFB

$$\frac{\mathbf{F}\{l_1,\dots,l_i,\dots,l_n\} \quad \mathbf{t}l_1,\dots,\mathbf{t}l_{i-1},\mathbf{t}l_{i+1},\dots,\mathbf{t}l_n}{\mathbf{f}l_i}$$

Examples

$$\frac{\mathbf{F}\{b, not\ c\} \quad \mathbf{T}b}{\mathbf{T}c} \qquad \frac{\mathbf{F}\{b, not\ c\} \quad \mathbf{F}c}{\mathbf{F}b}$$

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Examples

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Forward False Body (FFB)

Prerequisites Some literal of a body is *false*.

Consequence The body is *false*.

Tableau Rule FFB

$$\frac{\begin{array}{c} p \leftarrow l_1, \dots, l_i, \dots, l_n \\ \mathbf{f}\, l_i \end{array}}{\mathbf{F}\{l_1, \dots, l_i, \dots, l_n\}}$$

Examples

$$\frac{\begin{array}{c} a \leftarrow b, not\ c \\ \mathbf{F} b \end{array}}{\mathbf{F}\{b, not\ c\}} \qquad \frac{\begin{array}{c} a \leftarrow b, not\ c \\ \mathbf{T} c \end{array}}{\mathbf{F}\{b, not\ c\}}$$

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Examples

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Backward True Body (BTB)

Prerequisites A body is *true*.

Consequence The body's literals are *true*.

Tableau Rule BTB

$$\frac{\mathbf{T}\{l_1, \dots, l_i, \dots, l_n\}}{\mathbf{t}l_i}$$

Examples

$$\frac{\mathbf{T}\{b, not\ c\}}{\mathbf{T}b} \qquad \frac{\mathbf{T}\{b, not\ c\}}{\mathbf{F}c}$$

Backward True Body (BTB)

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Examples

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Reviewing tableau rules for bodies

Consider rule body $B = \{l_1, \dots, l_n\}$.

- Rules FTB and BFB amount to implication:

$$l_1 \wedge \cdots \wedge l_n \rightarrow B$$

- Rules FFB and BTB amount to implication:

$$B \rightarrow l_1 \wedge \cdots \wedge l_n$$

☞ Together they yield:

$$B \equiv l_1 \wedge \cdots \wedge l_n$$

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Forward True Atom (FTA)

Prerequisites Some of an atom's bodies is *true*.

Consequence The atom is *true*.

Tableau Rule FTA

$$\frac{\begin{array}{c} p \leftarrow l_1, \ldots, l_n \\ \mathbf{T}\{l_1, \ldots, l_n\} \end{array}}{\mathbf{T}p}$$

Examples

$$\frac{\begin{array}{c} a \leftarrow b, not\ c \\ \mathbf{T}\{b, not\ c\} \end{array}}{\mathbf{T}a} \qquad \frac{\begin{array}{c} a \leftarrow d, not\ e \\ \mathbf{T}\{d, not\ e\} \end{array}}{\mathbf{T}a}$$

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Backward False Atom (BFA)

Prerequisites An atom is *false*.

Consequence The bodies of all rules with the atom as head are *false*.

Tableau Rule BFA

$$\frac{\begin{array}{c} p \leftarrow l_1, \dots, l_n \\ \mathbf{F}p \end{array}}{\mathbf{F}\{l_1, \dots, l_n\}}$$

Examples

$$\frac{\begin{array}{c} a \leftarrow b, not\ c \\ \mathbf{F}a \end{array}}{\mathbf{F}\{b, not\ c\}} \qquad \frac{\begin{array}{c} a \leftarrow d, not\ e \\ \mathbf{F}a \end{array}}{\mathbf{F}\{d, not\ e\}}$$

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Examples

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Forward False Atom (FFA)

Prerequisites For some atom, the bodies of all rules with the atom as head are *false*.

Consequence The atom is *false*.

Tableau Rule FFA

$$\frac{\mathbf{F}B_1, \ldots, \mathbf{F}B_m}{\mathbf{F}p} \;(body(p) = \{B_1, \ldots, B_m\})$$

☞ For an atom p occurring in a logic program Π, we let $body(p) = \{body(r) \mid r \in \Pi, head(r) = p\}$.

Example

$$\frac{\begin{array}{c}\mathbf{F}\{b, not\ c\}\\ \mathbf{F}\{d, not\ e\}\end{array}}{\mathbf{F}a} \;(body(a) = \{\{b, not\ c\}, \{d, not\ e\}\})$$

Forward False Atom (FFA)

Prerequisites For some atom, the bodies of all rules with the atom as head are *false*.

Consequence The atom is *false*.

Tableau Rule FFA

$$\frac{\mathbf{F}B_1, \dots, \mathbf{F}B_m}{\mathbf{F}p} \;(body(p) = \{B_1, \dots, B_m\})$$

☞ For an atom p occurring in a logic program Π, we let $body(p) = \{body(r) \mid r \in \Pi, head(r) = p\}$.

Example

$$\frac{\begin{array}{c}\mathbf{F}\{b, not\ c\}\\ \mathbf{F}\{d, not\ e\}\end{array}}{\mathbf{F}a} \;(body(a) = \{\{b, not\ c\}, \{d, not\ e\}\})$$

Backward True Atom (BTA)

Prerequisites An atom is *true*, and the bodies of all rules with the atom as head except for one are *false*.

Consequence The residual body is *true*.

Tableau Rule BTA

$$\frac{\begin{array}{c}\mathbf{T}p\\ \mathbf{F}B_1,\dots,\mathbf{F}B_{i-1},\mathbf{F}B_{i+1},\dots,\mathbf{F}B_m\end{array}}{\mathbf{T}B_i}\ (body(p)=\{B_1,\dots,B_m\})$$

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Reviewing tableau rules for atoms

Consider an atom p such that $body(p) = \{B_1, \ldots, B_m\}$.

- Rules FTA and BFA amount to implication:

$$B_1 \vee \cdots \vee B_m \rightarrow p$$

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Relationship with Clark's completion

Let Π be a normal logic program.
The eight tableau rules introduced so far essentially provide:

- (straightforward) inferences from $Comp(\Pi)$ (cf. Page 302)
- inferences via *smodels*' **atleast**

Given the same partial assignment (of atoms),

- any literal derived by *smodels*' **atleast** is also derived by tableau rules, while the converse does not hold in general.

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Preliminaries for unfounded sets

Let Π be a normal logic program.

- For $\Pi' \subseteq \Pi$, define the greatest unfounded set, denoted by $GUS(\Pi')$, of Π with respect to Π' as:

$$GUS(\Pi') = atom(\Pi) \setminus Cn((\Pi')^{\emptyset})$$

- For a loop $L \in Loop(\Pi)$, define

$$EB(L) = \{body(r) \mid r \in \Pi, head(r) \in L, body^{+}(r) \cap L = \emptyset\}$$

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Well-Founded Negation (WFN)

Prerequisites An atom is in the greatest unfounded set with respect to rules whose bodies are *false*.

Consequence The atom is *false*.

Tableau Rule WFN

$$\frac{\mathbf{F}B_1, \dots, \mathbf{F}B_m}{\mathbf{F}p} \ (p \in GUS(\{r \in \Pi \mid body(r) \not\in \{B_1, \dots, B_m\}\}))$$

Examples

$$\frac{\begin{array}{c} a \leftarrow not\ b \\ \mathbf{F}\{not\ b\} \end{array}}{\mathbf{F}a} \ (*) \qquad \frac{\begin{array}{c} a \leftarrow a \\ a \leftarrow not\ b \\ \mathbf{F}\{not\ b\} \end{array}}{\mathbf{F}a} \ (*)$$

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Well-Founded Justification (WFJ)

Prerequisites A *true* atom is in the greatest unfounded set with respect to rules whose bodies are *false* if a particular body is made *false*.

Consequence The respective body is *true*.

Tableau Rule WFJ

$$\frac{\begin{array}{c}\mathbf{T}p\\ \mathbf{F}B_1,\dots,\mathbf{F}B_{i-1},\mathbf{F}B_{i+1},\dots,\mathbf{F}B_m\end{array}}{\mathbf{T}B_i}\quad (p \in GUS(\{r \in \Pi \mid body(r) \notin \{B_1,\dots,B_m\}\}))$$

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Reviewing well-founded tableau rules

Tableau rules WFN and WFJ ensure non-circular support for *true* atoms.
Note that

1. WFN subsumes falsifying atoms via FFA,
2. WFJ can be viewed as "backward propagation" for unfounded sets,
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Relationship with well-founded operator

Let Π be a normal logic program, $\langle T, F\rangle$ a partial interpretation, and $\Pi' = \{r \in \Pi \mid body^+(r) \cap F = \emptyset, body^-(r) \cap T = \emptyset\}$.
Then the following conditions are equivalent:

1. $p \in \mathbf{U}_\Pi\langle T, F\rangle$; (cf. Page 380)
2. $p \in GUS(\Pi')$.
   - ➥ Well-founded operator, *smodels*' **atmost**, and WFN coincide.

- ☞ In contrast to the former, WFN does not necessarily require a rule body to contain a *false* literal for the rule being inapplicable.

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Forward Loop (FL)

Prerequisites The external bodies of a loop are *false*.

Consequence The atoms in the loop are *false*.

Tableau Rule FL

$$\frac{\mathbf{F}B_1,\dots,\mathbf{F}B_m}{\mathbf{F}p}\ (p \in L, L \in Loop(\Pi), EB(L) = \{B_1,\dots,B_m\})$$

Example

$$\begin{array}{c} a \leftarrow a \\ a \leftarrow not\ b \\ \hline \end{array}$$

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For a loop L such that $EB(L) = \{B_1, \dots, B_m\}$,
they amount to implication:

$$\bigvee_{p \in L} p \rightarrow B_1 \vee \cdots \vee B_m$$

Comparison to well-founded tableau rules yields:

- FL (plus FFA and FFB) is equivalent to WFN (plus FFB),
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Relationship with loop formulas

Tableau rules FL and BL essentially provide:

- (straightforward) inferences from loop formulas (cf. Page 422)
  - ☞ But impractical to precompute exponentially many loop formulas !
- an application of the Lin-Zhao Theorem (cf. Page 426)

In practice, ASP-solvers such as `smodels`:

- exploit strongly connected components of positive atom dependency graphs
  - ➥ Can be viewed as an interpolation of FL.
- do not directly implement BL (and neither WFJ)
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Case analysis by *Cut*

Up to now, all tableau rules are deterministic.
That is, rules extend a single branch but cannot create sub-branches.

☞ In general, closing a branch leads to a partial assignment.

Case analysis is done by $Cut[\mathcal{C}]$ where $\mathcal{C} \subseteq atom(\Pi) \cup body(\Pi)$.

Tableau Rule $Cut[\mathcal{C}]$

$$\frac{}{\mathbf{T}v \mid \mathbf{F}v}\ (v \in \mathcal{C})$$

Examples $Cut[\mathcal{C}]$

$$\frac{\begin{array}{c} a \leftarrow not\ b \\ b \leftarrow not\ a \end{array}}{\mathbf{T}a \mid \mathbf{F}a}\ (\mathcal{C} = atom(\Pi)) \qquad \frac{\begin{array}{c} a \leftarrow not\ b \\ b \leftarrow not\ a \end{array}}{\mathbf{T}\{not\ b\} \mid \mathbf{F}\{not\ b\}}\ (\mathcal{C} = body(\Pi))$$

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Well-known tableau calculi

Fitting's operator $\mathbf{\Phi}$ applies forward propagation without sophisticated unfounded set checks. We have:

$$\mathcal{T}_{\mathbf{\Phi}} = \{FTB, FTA, FFB, FFA\}$$

Well-founded operator $\mathbf{\Omega}$ replaces negation of single atoms with negation of unfounded sets. We have:

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"Local" propagation via a program's completion can be determined by elementary inferences on atoms and rule bodies. We have:

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Tableau calculi characterizing ASP-solvers

ASP-solvers combine propagation with case analysis.
We obtain the following tableau calculi characterizing [2, 59, 48, 74, 55, 52, 1]:

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- SAT-based ASP-solvers, `assat` and `cmodels`, incrementally add loop formulas to a program's completion.
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Proof complexity

The notion of proof complexity is used for describing the relative efficiency of different proof systems.
It compares proof systems based on minimal refutations.

- ➥ Proof complexity does not depend on heuristics.

A proof system $\mathcal{T}$ polynomially simulates a proof system $\mathcal{T}'$ if every refutation of $\mathcal{T}'$ can be polynomially mapped to a refutation of $\mathcal{T}$.
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For showing that proof system $\mathcal{T}$ does not polynomially simulate $\mathcal{T}'$, we have to provide an infinite witnessing family of programs such that minimal refutations of $\mathcal{T}$ asymptotically are exponentially larger than minimal refutations of $\mathcal{T}'$.
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For showing that proof system $\mathcal{T}$ does not polynomially simulate $\mathcal{T}'$, we have to provide an infinite witnessing family of programs such that minimal refutations of $\mathcal{T}$ asymptotically are exponentially larger than minimal refutations of $\mathcal{T}'$.
The size of tableaux is simply the number of their entries.

☞ We do not need to know the precise number of entries:
Counting required *Cut* applications is sufficient !

Proof complexity

The notion of proof complexity is used for describing the relative efficiency of different proof systems.
It compares proof systems based on minimal refutations.

➥ Proof complexity does not depend on heuristics.

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$\mathcal{T}_{smodels}$ versus $\mathcal{T}_{noMoRe}$

Recall that $\mathcal{T}_{smodels}$ restricts *Cut* to $atom(\Pi)$ and $\mathcal{T}_{noMoRe}$ to $body(\Pi)$.
Are both approaches similar or is one of them superior to the other?
Let $\{\Pi_a^n\}$, $\{\Pi_b^n\}$, and $\{\Pi_c^n\}$ be infinite families of programs as follows:

$$\Pi_a^n = \left\{\begin{array}{l} x \leftarrow not\ x \\ x \leftarrow a_1, b_1 \\ \vdots \\ x \leftarrow a_n, b_n \end{array}\right\} \quad \Pi_b^n = \left\{\begin{array}{ll} \multicolumn{2}{l}{x \leftarrow c_1, \dots, c_n, not\ x} \\ c_1 \leftarrow a_1 & c_1 \leftarrow b_1 \\ \vdots & \vdots \\ c_n \leftarrow a_n & c_n \leftarrow b_n \end{array}\right\} \quad \Pi_c^n = \left\{\begin{array}{l} a_1 \leftarrow not\ b_1 \\ b_1 \leftarrow not\ a_1 \\ \vdots \\ a_n \leftarrow not\ b_n \\ b_n \leftarrow not\ a_n \end{array}\right\}$$

In minimal refutations for $\Pi_a^n \cup \Pi_c^n$, the number of applications of $Cut[body(\Pi_a^n \cup \Pi_c^n)]$ with $\mathcal{T}_{noMoRe}$ is linear in n, whereas $\mathcal{T}_{smodels}$ requires exponentially many applications of $Cut[atom(\Pi_a^n \cup \Pi_c^n)]$.
Vice versa, minimal refutations for $\Pi_b^n \cup \Pi_c^n$ require linearly many applications of $Cut[atom(\Pi_b^n \cup \Pi_c^n)]$ with $\mathcal{T}_{smodels}$ and exponentially many applications of $Cut[body(\Pi_b^n \cup \Pi_c^n)]$ with $\mathcal{T}_{noMoRe}$.

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Relative efficiency

As witnessed by $\{\Pi_a^n \cup \Pi_c^n\}$ and $\{\Pi_b^n \cup \Pi_c^n\}$, respectively, $\mathcal{T}_{smodels}$ and $\mathcal{T}_{noMoRe}$ do not polynomially simulate one another.
Any refutation of $\mathcal{T}_{smodels}$ or $\mathcal{T}_{noMoRe}$ is a refutation of $\mathcal{T}_{nomore^{++}}$ (but not vice versa).

It follows that

- both $\mathcal{T}_{smodels}$ and $\mathcal{T}_{noMoRe}$ are polynomially simulated by $\mathcal{T}_{nomore^{++}}$ and
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$\mathcal{T}_{smodels}$: Example tableau

(r_1)	$a \leftarrow not\ b$	(r_2)	$b \leftarrow d, not\ a$	(r_3)	$c \leftarrow b, d$
(r_4)	$c \leftarrow g$	(r_5)	$d \leftarrow c$	(r_6)	$d \leftarrow g$
(r_7)	$e \leftarrow f, not\ c$	(r_8)	$f \leftarrow not\ g$	(r_9)	$g \leftarrow not\ a, not\ f$

(1)	$\mathbf{T}a$	$[Cut]$
(2)	$\mathbf{T}\{not\ b\}$	$[BTA: r_1, 1]$
(3)	$\mathbf{F}b$	$[BTB: 2]$
(4)	$\mathbf{F}\{d, not\ a\}$	$[BFA: r_2, 3]$
(5)	$\mathbf{F}\{not\ a, not\ f\}$	$[FFB: r_9, 1]$
(6)	$\mathbf{F}g$	$[FFA: r_9, 5]$
(7)	$\mathbf{T}\{not\ g\}$	$[FTB: r_8, 6]$
(8)	$\mathbf{T}f$	$[FTA: r_8, 7]$
(9)	$\mathbf{F}\{b, d\}$	$[FFB: r_3, 3]$
(10)	$\mathbf{F}\{g\}$	$[FFB: r_4, r_6, 6]$
(11)	$\mathbf{F}c$	$[FFA: r_3, r_4, 9, 10]$
(12)	$\mathbf{F}\{c\}$	$[FFB: r_5, 11]$
(13)	$\mathbf{F}d$	$[FFA: r_5, r_6, 10, 12]$
(14)	$\mathbf{T}\{f, not\ c\}$	$[FTB: r_7, 8, 11]$
(15)	$\mathbf{T}e$	$[FTA: r_7, 14]$

(16)	$\mathbf{F}a$	$[Cut]$
(17)	$\mathbf{F}\{not\ b\}$	$[BFA: r_1, 16]$
(18)	$\mathbf{T}b$	$[BFB: 17]$
(19)	$\mathbf{T}\{d, not\ a\}$	$[BTA: r_2, 18]$
(20)	$\mathbf{T}d$	$[BTB: 19]$
(21)	$\mathbf{T}\{b, d\}$	$[FTB: r_3, 18, 20]$
(22)	$\mathbf{T}c$	$[FTA: r_3, 21]$
(23)	$\mathbf{F}\{f, not\ c\}$	$[FFB: r_7, 22]$
(24)	$\mathbf{F}e$	$[FFA: r_7, 23]$
(25)	$\mathbf{T}\{c\}$	$[FTB: r_5, 22]$

(26)	$\mathbf{T}f$	$[Cut]$
(27)	$\mathbf{F}\{not\ a, not\ f\}$	$[FFB: r_9, 26]$
(28)	$\mathbf{F}c$	$[WFN: 27]$

(29)	$\mathbf{F}f$	$[Cut]$
(30)	$\mathbf{T}\{not\ a, not\ f\}$	$[FTB: r_9, 16, 29]$
(31)	$\mathbf{T}g$	$[FTA: r_9, 30]$
(32)	$\mathbf{T}\{g\}$	$[FTB: r_4, r_6, 31]$
(33)	$\mathbf{F}\{not\ g\}$	$[FFB: r_8, 31]$

$\mathcal{T}_{noMoRe}$: Example tableau

(r_1)	$a \leftarrow not\ b$	(r_2)	$b \leftarrow d, not\ a$	(r_3)	$c \leftarrow b, d$
(r_4)	$c \leftarrow g$	(r_5)	$d \leftarrow c$	(r_6)	$d \leftarrow g$
(r_7)	$e \leftarrow f, not\ c$	(r_8)	$f \leftarrow not\ g$	(r_9)	$g \leftarrow not\ a, not\ f$

(1)	$\mathbf{T}\{not\ b\}$	$[Cut]$
(2)	$\mathbf{T}a$	$[FTA: r_1, 1]$
(3)	$\mathbf{F}b$	$[BTB: 1]$
(4)	$\mathbf{F}\{d, not\ a\}$	$[BFA: r_2, 3]$
(5)	$\mathbf{F}\{not\ a, not\ f\}$	$[FFB: r_9, 2]$
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(11)	$\mathbf{F}c$	$[FFA: r_3, r_4, 9, 10]$
(12)	$\mathbf{F}\{c\}$	$[FFB: r_5, 11]$
(13)	$\mathbf{F}d$	$[FFA: r_5, r_6, 10, 12]$
(14)	$\mathbf{T}\{f, not\ c\}$	$[FTB: r_7, 8, 11]$
(15)	$\mathbf{T}e$	$[FTA: r_7, 14]$

(16)	$\mathbf{F}\{not\ b\}$	$[Cut]$
(17)	$\mathbf{F}a$	$[FFA: r_1, 16]$
(18)	$\mathbf{T}b$	$[BFB: 16]$
(19)	$\mathbf{T}\{d, not\ a\}$	$[BTA: r_2, 18]$
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(32)	$\mathbf{T}\{g\}$	$[FTB: r_4, r_6, 31]$
(33)	$\mathbf{F}f$	$[FFA: r_8, 30]$
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(1)	$\mathbf{T}a$	$[Cut]$
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Conflict-Driven Answer Set Solving
Overview

Motivation

Goal New approach to computing answer sets of logic programs, based on concepts from

- Constraint Processing (CSP) and
- Satisfiability Checking (SAT)

Idea View inferences in Answer Set Programming (ASP) as unit propagation on nogoods.

Benefits

- A uniform constraint-based framework for different kinds of inferences in ASP
- Advanced techniques from the areas of CSP and SAT
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Assignments

- An assignment A over $dom(A) = atom(\Pi) \cup body(\Pi)$ is a sequence

  $$(\sigma_1, \dots, \sigma_n)$$

  of signed literals σ_i of form $\mathbf{T}p$ or $\mathbf{F}p$ for $p \in dom(A)$ and $1 \leq i \leq n$.
  - ☞ $\mathbf{T}p$ expresses that p is *true* and $\mathbf{F}p$ that it is *false*.
- The complement, $\overline{\sigma}$, of a literal σ is defined as $\overline{\mathbf{T}p} = \mathbf{F}p$ and $\overline{\mathbf{F}p} = \mathbf{T}p$.
- $A \circ B$ denotes the concatenation of assignments A and B.
- Given $A = (\sigma_1, \dots, \sigma_{k-1}, \sigma_k, \dots, \sigma_n)$, we let $A[\sigma_k] = (\sigma_1, \dots, \sigma_{k-1})$.
- We sometimes identify an assignment with the set of its literals. Given this, we access *true* and *false* propositions in A via

  $$A^{\mathbf{T}} = \{p \in dom(A) \mid \mathbf{T}p \in A\} \text{ and } A^{\mathbf{F}} = \{p \in dom(A) \mid \mathbf{F}p \in A\}\ .$$

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  of signed literals σ_i of form $\mathbf{T}p$ or $\mathbf{F}p$ for $p \in dom(A)$ and $1 \leq i \leq n$.
  - ☞ $\mathbf{T}p$ expresses that p is *true* and $\mathbf{F}p$ that it is *false*.
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- $A \circ B$ denotes the concatenation of assignments A and B.
- Given $A = (\sigma_1, \dots, \sigma_{k-1}, \sigma_k, \dots, \sigma_n)$, we let $A[\sigma_k] = (\sigma_1, \dots, \sigma_{k-1})$.
- We sometimes identify an assignment with the set of its literals. Given this, we access *true* and *false* propositions in A via

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Nogoods, Solutions, and Unit Propagation

- A nogood is a set $\{\sigma_1, \dots, \sigma_n\}$ of signed literals, expressing a constraint violated by any assignment containing $\sigma_1, \dots, \sigma_n$.
- An assignment A such that $A^{\mathbf{T}} \cup A^{\mathbf{F}} = dom(A)$ and $A^{\mathbf{T}} \cap A^{\mathbf{F}} = \emptyset$ is a solution for a set Δ of nogoods, if $\delta \not\subseteq A$ for all $\delta \in \Delta$.
- For a nogood δ, a literal $\sigma \in \delta$, and an assignment A, we say that $\overline{\sigma}$ is unit-resulting for δ wrt A, if
  1. $\delta \setminus A = \{\sigma\}$ and
  2. $\overline{\sigma} \notin A$.
- For a set Δ of nogoods and an assignment A, unit propagation is the iterated process of extending A with unit-resulting literals until no further literal is unit-resulting for any nogood in Δ.

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Nogoods from logic programs

via Clark's completion

The completion of a logic program Π can be defined as follows:

$$\begin{aligned} & \{p_\beta \leftrightarrow p_1 \wedge \dots \wedge p_m \wedge \neg p_{m+1} \wedge \dots \wedge \neg p_n \mid \\ & \qquad \beta \in \mathit{body}(\Pi), \beta = \{p_1, \dots, p_m, \mathit{not}\ p_{m+1}, \dots, \mathit{not}\ p_n\}\} \\ \cup\ & \{p \leftrightarrow p_{\beta_1} \vee \dots \vee p_{\beta_k} \mid \\ & \qquad p \in \mathit{atom}(\Pi), \mathit{body}(p) = \{\beta_1, \dots, \beta_k\}\}\ , \end{aligned}$$

where $\mathit{body}(p) = \{\mathit{body}(r) \mid r \in \Pi, \mathit{head}(r) = p\}$.

Nogoods from logic programs (ctd)

via Clark's completion

Let $\beta = \{p_1, \dots, p_m, \mathit{not}\ p_{m+1}, \dots, \mathit{not}\ p_n\}$ be a body.
The equivalence

$$p_\beta \leftrightarrow p_1 \wedge \dots \wedge p_m \wedge \neg p_{m+1} \wedge \dots \wedge \neg p_n$$

can be decomposed into two implications.

1. We get

$$p_\beta \rightarrow p_1 \wedge \dots \wedge p_m \wedge \neg p_{m+1} \wedge \dots \wedge \neg p_n \ ,$$

which is equivalent to the conjunction of

$$\neg p_\beta \vee p_1, \ \dots, \ \neg p_\beta \vee p_m, \ \neg p_\beta \vee \neg p_{m+1}, \ \dots, \ \neg p_\beta \vee \neg p_n \ .$$

This set of clauses expresses the following set of nogoods:

$$\Delta(\beta) = \{ \{\mathbf{T}\beta, \mathbf{F}p_1\}, \dots, \{\mathbf{T}\beta, \mathbf{F}p_m\}, \{\mathbf{T}\beta, \mathbf{T}p_{m+1}\}, \dots, \{\mathbf{T}\beta, \mathbf{T}p_n\} \} \ .$$

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2. The converse of the previous implication, viz.

$$p_1 \wedge \cdots \wedge p_m \wedge \neg p_{m+1} \wedge \cdots \wedge \neg p_n \rightarrow p_\beta \ ,$$

gives rise to the nogood

$$\delta(\beta) = \{\mathbf{F}\beta, \mathbf{T}p_1, \dots, \mathbf{T}p_m, \mathbf{F}p_{m+1}, \dots, \mathbf{F}p_n\} \ .$$

Intuitively, $\delta(\beta)$ is a constraint enforcing the truth of body β, or the falsity of a contained literal.

Nogoods from logic programs (ctd)

via Clark's completion

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Nogoods from logic programs (ctd)

via Clark's completion

Proceeding analogously with the atom-based equivalences, viz.

$$p \leftrightarrow p_{\beta_1} \vee \cdots \vee p_{\beta_k}$$

we obtain for an atom $p \in atom(\Pi)$ along with its bodies $body(p) = \{\beta_1, \ldots, \beta_k\}$ the nogoods

$$\Delta(p) = \{\, \{\mathbf{F}p, \mathbf{T}\beta_1\}, \ldots, \{\mathbf{F}p, \mathbf{T}\beta_k\} \,\} \text{ and } \delta(p) = \{\mathbf{T}p, \mathbf{F}\beta_1, \ldots, \mathbf{F}\beta_k\} \,.$$

Nogoods from logic programs

atom-oriented nogoods

For an atom p where $body(p) = \{\beta_1, \ldots, \beta_k\}$, recall that

$$\begin{array}{rcl} \delta(p) & = & \{\mathbf{T}p, \mathbf{F}\beta_1, \ldots, \mathbf{F}\beta_k\} \\ \Delta(p) & = & \{\,\{\mathbf{F}p, \mathbf{T}\beta_1\}, \ldots, \{\mathbf{F}p, \mathbf{T}\beta_k\}\,\}\ . \end{array}$$

For example, for atom x with $body(x) = \{\{y\}, \{not\ z\}\}$, we obtain

$$\boxed{\begin{array}{lcl} x & \leftarrow & y \\ x & \leftarrow & not\ z \end{array}}$$

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For nogood $\delta(x) = \{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{not\ z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{not\ z\})$ and
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Nogoods from logic programs

atom-oriented nogoods

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Nogoods from logic programs

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For nogood $\delta(x) = \{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{not\ z\}\}$, the signed literal

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Nogoods from logic programs

body-oriented nogoods

For a body $\beta = \{p_1, \dots, p_m, not\ p_{m+1}, \dots, not\ p_n\}$, recall that

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$$\boxed{\begin{array}{l} \dots \leftarrow x, not\ y \\ \qquad\vdots \\ \dots \leftarrow x, not\ y \end{array}}$$

$$\begin{aligned}\delta(\{x, not\ y\}) &= \{\mathbf{F}\{x, not\ y\}, \mathbf{T}x, \mathbf{F}y\}\\ \Delta(\{x, not\ y\}) &= \{\,\{\mathbf{T}\{x, not\ y\}, \mathbf{F}x\}, \{\mathbf{T}\{x, not\ y\}, \mathbf{T}y\}\,\}\end{aligned}$$

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$$\begin{array}{r}\delta(\{x, \mathit{not}\ y\}) = \{\mathbf{F}\{x, \mathit{not}\ y\}, \mathbf{T}x, \mathbf{F}y\} \\ \Delta(\{x, \mathit{not}\ y\}) = \{\,\{\mathbf{T}\{x, \mathit{not}\ y\}, \mathbf{F}x\}, \{\mathbf{T}\{x, \mathit{not}\ y\}, \mathbf{T}y\}\,\}\end{array}$$

For nogood $\delta(\{x, \mathit{not}\ y\}) = \{\mathbf{F}\{x, \mathit{not}\ y\}, \mathbf{T}x, \mathbf{F}y\}$, the signed literal

- $\mathbf{T}\{x, \mathit{not}\ y\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}y)$ and
- $\mathbf{T}y$ is unit-resulting wrt assignment $(\mathbf{F}\{x, \mathit{not}\ y\}, \mathbf{T}x)$.

Characterization of answer sets

for tight logic programs

Let Π be a logic program and

$$\begin{array}{rcl}\Delta_\Pi & = & \{\delta(p) \mid p \in atom(\Pi)\} \cup \{\delta \in \Delta(p) \mid p \in atom(\Pi)\} \\ & \cup & \{\delta(\beta) \mid \beta \in body(\Pi)\} \cup \{\delta \in \Delta(\beta) \mid \beta \in body(\Pi)\}\ .\end{array}$$

Theorem

Let Π be a tight logic program. Then,
$X \subseteq atom(\Pi)$ is an answer set of Π iff
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☞ The set Δ_Π of nogoods captures inferences from (program Π and) Clark's completion.

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Atom-oriented nogoods and tableau rules

- Tableau rules FTA, BFA, FFA, and BTA are atom-oriented.
- For an atom p such that $body(p) = \{\beta_1, \dots, \beta_k\}$, consider the equivalence: $\quad p \leftrightarrow p_{\beta_1} \vee \cdots \vee p_{\beta_k}$
- Inferences from nogoods $\Delta(p) = \{\, \{\mathbf{F}p, \mathbf{T}\beta_1\}, \dots, \{\mathbf{F}p, \mathbf{T}\beta_k\} \,\}$ correspond to those from tableau rules FTA and BFA:

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$$\frac{p \leftarrow l_1, \dots, l_n \quad \mathbf{t}l_1, \dots, \mathbf{t}l_n}{\mathbf{T}\{l_1, \dots, l_n\}} \qquad \frac{\mathbf{F}\{l_1, \dots, l_n\} \quad \mathbf{t}l_1, \dots, \mathbf{t}l_{i-1}, \mathbf{t}l_{i+1}, \dots, \mathbf{t}l_n}{\mathbf{f}l_i}$$

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Nogoods from logic programs

via loop formulas (cf. Page 422)

Let Π be a normal logic program and recall that:

- For $L \subseteq atom(\Pi)$, the external supports of L for Π are

$$ES_\Pi(L) = \{r \in \Pi \mid head(r) \in L, body^+(r) \cap L = \emptyset\}.$$

- The (disjunctive) loop formula of L for Π is

$$\begin{aligned} LF_\Pi(L) &= \left(\bigvee_{A \in L} A\right) \rightarrow \left(\bigvee_{r \in ES_\Pi(L)} Comp(body(r))\right) \\ &\equiv \left(\bigwedge_{r \in ES_\Pi(L)} \neg Comp(body(r))\right) \rightarrow \left(\bigwedge_{A \in L} \neg A\right). \end{aligned}$$

  ☞ The loop formula of L enforces all atoms in L to be *false* whenever L is not externally supported.

- The external bodies of L for Π are

$$\begin{aligned} EB(L) &= \{body(r) \mid r \in \Pi, head(r) \in L, body^+(r) \cap L = \emptyset\} \\ &= \{body(r) \mid r \in ES_\Pi(L)\}. \end{aligned}$$

Nogoods from logic programs

loop nogoods

For a logic program Π and some $\emptyset \subset U \subseteq atom(\Pi)$,
define the loop nogood of an atom $p \in U$ as

$$\lambda(p, U) \quad = \quad \{\mathbf{T}p, \mathbf{F}\beta_1, \ldots, \mathbf{F}\beta_k\}$$

where $EB(U) = \{\beta_1, \ldots, \beta_k\}$.

In all, we get the following set of loop nogoods for Π:

$$\Lambda_\Pi \quad = \quad \bigcup_{\emptyset \subset U \subseteq atom(\Pi)} \{\lambda(p, U) \mid p \in U\}$$

☞ The set Λ_Π of loop nogoods denies cyclic support among *true* atoms.

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Example

Consider

$$\Pi = \left\{ \begin{array}{ll} x \leftarrow not\ y & u \leftarrow x \\ & u \leftarrow v \\ y \leftarrow not\ x & v \leftarrow u, y \end{array} \right\}$$

For u in the set $\{u, v\}$, we obtain the loop nogood:

$$\lambda(u, \{u, v\}) \quad = \quad \{\mathbf{T}u, \mathbf{F}\{x\}\}$$

Similarly for v in $\{u, v\}$, we get:

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Characterization of answer sets

For a logic program Π,
let Δ_Π and Λ_Π as defined on Page 582 and Page 594, respectively.

Theorem

Let Π be a logic program. Then,
$X \subseteq atom(\Pi)$ is an answer set of Π iff
$X = A^{\mathbf{T}} \cap atom(\Pi)$ for a (unique) solution A for $\Delta_\Pi \cup \Lambda_\Pi$.

Some remarks

- Nogoods in Λ_Π augment Δ_Π with conditions checking for unfounded sets, in particular, those being loops. While $|\Delta_\Pi|$ is linear in the size of Π, Λ_Π may contain exponentially many (non-redundant) loop nogoods !

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Conflict-driven search

Boolean constraint solving algorithms pioneered for SAT led to:

Traditional approach

- (Unit) propagation
- Exhaustive (chronological) backtracking

☞ DPLL [17, 16]

State of the art

- (Unit) propagation
- Conflict analysis (via resolution)
- Learning + Backjumping + Assertion

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Idea

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Outline of CDNL-ASP algorithm

- Keep track of deterministic consequences by unit propagation on:
  - Clark's completion [Δ_Π]
  - Loop nogoods, determined and recorded on demand [Λ_Π]
    - ☞ Dedicated unfounded set detection !
  - Dynamic nogoods, derived from conflicts and unfounded sets [∇]
- When a nogood in $\Delta_\Pi \cup \nabla$ becomes violated:
  - Analyze the conflict by resolution until reaching the First Unique Implication Point (First-UIP) [63]
  - Learn the derived conflict nogood δ
  - Backjump to the earliest (heuristic) choice such that the complement of the First-UIP is unit-resulting for δ
  - Assert the complement of the First-UIP and proceed (by unit propagation)
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  - Deriving a conflict independently of (heuristic) choices

Algorithm 1: CDNL-ASP

Input : A logic program Π.
Output : An answer set of Π or "no answer set".

$A \leftarrow \emptyset$ // *assignment over* $atom(\Pi) \cup body(\Pi)$
$\nabla \leftarrow \emptyset$ // *set of (dynamic) nogoods*
$dl \leftarrow 0$ // *decision level*
loop
    $(A, \nabla) \leftarrow \textsc{NogoodPropagation}(\Pi, \nabla, A)$
    if $\varepsilon \subseteq A$ for some $\varepsilon \in \Delta_\Pi \cup \nabla$ **then**
        if $dl = 0$ **then return** no answer set
        $(\delta, k) \leftarrow \textsc{ConflictAnalysis}(\varepsilon, \Pi, \nabla, A)$
        $\nabla \leftarrow \nabla \cup \{\delta\}$ // *learning*
        $A \leftarrow (A \setminus \{\sigma \in A \mid k < dl(\sigma)\})$ // *backjumping*
        $dl \leftarrow k$
    else if $A^{\mathbf{T}} \cup A^{\mathbf{F}} = atom(\Pi) \cup body(\Pi)$ **then**
        return $A^{\mathbf{T}} \cap atom(\Pi)$ // *answer set*
    else
        $\sigma_d \leftarrow \textsc{Select}(\Pi, \nabla, A)$ // *heuristic choice of* $\sigma_d \notin A$
        $dl \leftarrow dl + 1$
        $A \leftarrow A \circ (\sigma_d)$ // $dl(\sigma_d) = dl$

Observations

- Decision level dl, initially set to 0, is used to count the number of heuristically chosen literals in assignment A.
- For a heuristically chosen literal $\sigma_d = \mathbf{T}p$ or $\sigma_d = \mathbf{F}p$, respectively, we require $p \in (atom(\Pi) \cup body(\Pi)) \setminus (A^{\mathbf{T}} \cup A^{\mathbf{F}})$.
- For any literal $\sigma \in A$, $dl(\sigma)$ denotes the decision level of σ, viz. the value dl had when σ was assigned.
- A conflict is detected from violation of a nogood $\varepsilon \subseteq \Delta_\Pi \cup \nabla$.
- A conflict at decision level 0 (where A contains no heuristically chosen literals) indicates non-existence of answer sets.
- A nogood δ derived by conflict analysis is asserting, that is, some literal is unit-resulting for δ at a decision level $k < dl$.
  - ➥ After learning δ and backjumping to decision level k, at least one literal is newly derivable by unit propagation.
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$$\Pi = \left\{ \begin{array}{llll} x \leftarrow not\ y & u \leftarrow x, y & v \leftarrow x & w \leftarrow not\ x, not\ y \\ y \leftarrow not\ x & u \leftarrow v & v \leftarrow u, y & \end{array} \right\}$$

dl	σ_d	$\overline{\sigma}$	δ
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Outline of NogoodPropagation

- Derive deterministic consequences via:
  - Unit propagation on Δ_Π and ∇;
  - Unfounded sets $U \subseteq atom(\Pi)$.
- Note that U is unfounded if $EB(U) \subseteq A^{\mathbf{F}}$.
  - ☞ For any $p \in U$, we have $(\lambda(p, U) \setminus \{\mathbf{T}p\}) \subseteq A$.
- An "interesting" unfounded set U satisfies:

$$\emptyset \subset U \subseteq (atom(\Pi) \setminus A^{\mathbf{F}}) .$$

- Wrt a fixpoint of unit propagation,
  such an unfounded set contains some loop of Π.
  - ➥ Tight programs do not yield "interesting" unfounded sets !
- Given an unfounded set U and some $p \in U$, adding $\lambda(p, U)$ to ∇ triggers a conflict or further derivations by unit propagation.
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Algorithm 2: NogoodPropagation

Input : A logic program Π, a set ∇ of nogoods, and an assignment A.
Output : An extended assignment and set of nogoods.

$U \leftarrow \emptyset$ // *set of unfounded atoms*
loop
  repeat
    if $\delta \subseteq A$ for some $\delta \in \Delta_\Pi \cup \nabla$ **then return** (A, ∇) // *conflict*
    $\Sigma \leftarrow \{\delta \in \Delta_\Pi \cup \nabla \mid (\delta \setminus A) = \{\sigma\}, \overline{\sigma} \notin A\}$ // *unit-resulting nogoods*
    if $\Sigma \neq \emptyset$ **then**
      let $\sigma \in (\delta \setminus A)$ for some $\delta \in \Sigma$ **in**
        $A \leftarrow A \circ (\overline{\sigma})$ // $dl(\overline{\sigma}) = max(\{dl(\rho) \mid \rho \in (\delta \setminus \{\sigma\})\} \cup \{0\})$
  until $\Sigma = \emptyset$
  if Π is tight **then return** (A, ∇) // *no unfounded set* $\emptyset \subset U \subseteq (atom(\Pi) \setminus A^{\mathbf{F}})$
  else
    $U \leftarrow (U \setminus A^{\mathbf{F}})$
    if $U = \emptyset$ **then** $U \leftarrow$ UnfoundedSet(Π, A)
    if $U = \emptyset$ **then return** (A, ∇) // *no unfounded set* $\emptyset \subset U \subseteq (atom(\Pi) \setminus A^{\mathbf{F}})$
    let $p \in U$ **in**
      $\nabla \leftarrow \nabla \cup \{\lambda(p, U)\}$ // *record unit-resulting or violated loop nogood*

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- Implementations of UNFOUNDEDSET must guarantee the following for a result U:
  1. $U \subseteq (atom(\Pi) \setminus A^{\mathbf{F}})$;
  2. $EB(U) \subseteq A^{\mathbf{F}}$;
  3. $U = \emptyset$ iff there is no nonempty unfounded subset of $(atom(\Pi) \setminus A^{\mathbf{F}})$.
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    Usually, the latter option is implemented in ASP solvers !

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Outline of ConflictAnalysis

- Conflict analysis is triggered whenever some nogood $\delta \in \Delta_\Pi \cup \nabla$ becomes violated, viz. $\delta \subseteq A$, at a decision level $dl > 0$.

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- Iterated resolution stops as soon as it generates a nogood δ containing exactly one literal σ assigned at decision level dl.

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  ➥ If $\sigma \in \delta$ has been unit-resulting for ε, we obtain a new violated nogood by resolving δ and ε as follows:

$$(\delta \setminus \{\sigma\}) \cup (\varepsilon \setminus \{\overline{\sigma}\}) \ .$$

- Resolution is directed by resolving first over the literal $\sigma \in \delta$ derived last, viz. $(\delta \setminus A[\sigma]) = \{\sigma\}$.

  ☞ Iterated resolution progresses in inverse order of assignment.

- Iterated resolution stops as soon as it generates a nogood δ containing exactly one literal σ assigned at decision level dl.

  - This literal σ is called First Unique Implication Point (First-UIP).

  ☞ All literals in $(\delta \setminus \{\sigma\})$ are assigned at decision levels smaller than dl.

Algorithm 3: ConflictAnalysis

Input : A violated nogood δ, a logic program Π, a set ∇ of nogoods, and an assignment A.

Output : A derived nogood and a decision level.

```
loop
    let σ ∈ δ such that (δ \ A[σ]) = {σ} in
        k ← max({dl(ρ) | ρ ∈ δ \ {σ}} ∪ {0})
        if k = dl(σ) then
            let ε ∈ Δ_Π ∪ ∇ such that (ε \ A[σ]) = {σ̄} in
                δ ← (δ \ {σ}) ∪ (ε \ {σ̄})                    // resolution
        else return (δ, k)
```

Example: ConflictAnalysis

Consider

$$\Pi = \left\{ \begin{array}{llll} x \leftarrow \mathit{not}\ y & u \leftarrow x, y & v \leftarrow x & w \leftarrow \mathit{not}\ x, \mathit{not}\ y \\ y \leftarrow \mathit{not}\ x & u \leftarrow v & v \leftarrow u, y & \end{array} \right\}$$

dl	σ_d	$\overline{\sigma}$	δ	
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		$\mathbf{F}w$	$\{\mathbf{T}w, \mathbf{F}\{\mathit{not}\ x, \mathit{not}\ y\}\} = \delta(w)$	
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Remarks

- There always is a First-UIP at which conflict analysis terminates.

☞ In the worst, resolution stops at the heuristically chosen literal assigned at decision level dl.

- The nogood δ containing First-UIP σ is violated by A, viz. $\delta \subseteq A$.
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The `clasp` system

- Native ASP solver combining conflict-driven search with sophisticated reasoning techniques:
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  - Lookback-based decision heuristics
  - Restart policies
  - Nogood deletion
  - Progress saving
  - Dedicated data structures for binary and ternary nogoods
  - Lazy data structures (watched literals) for long nogoods
  - Dedicated data structures for cardinality and weight constraints
  - Lazy unfounded set checking based on “source pointers”
  - Tight integration of unit propagation and unfounded set checking
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☞ Many of these techniques are configurable !

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Beyond deciding answer set existence, `clasp` allows for:

- Optimization
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- Brave and Cautious Reasoning determining the
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Front-ends also admit `clasp` to solve:

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  of all answer sets by computing only linearly many of them

☞ Reasoning applicable wrt answer sets as well as supported models

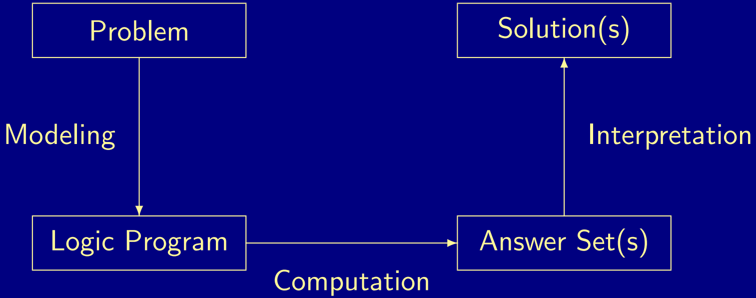
Front-ends also admit `clasp` to solve:

- Propositional CNF formulas
- Pseudo-Boolean formulas

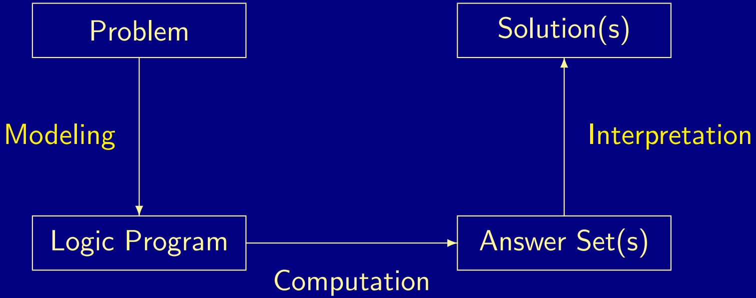
Find `clasp` at: `http://potassco.sourceforge.net`

Effective Modeling
Overview

Modeling and Interpreting



Modeling and Interpreting



Problem $\longmapsto$ Logic Program

For solving a problem class P for a problem instance I,
encode

1. the problem instance I as a set C(I) of facts and
2. the problem class P as a set C(P) of rules

such that the solutions to P for I can be (polynomially) extracted from the answer sets of $C(I) \cup C(P)$.

☞ A uniform encoding C(P) is a first-order logic program, encoding the solutions to P for any set C(I) of facts.

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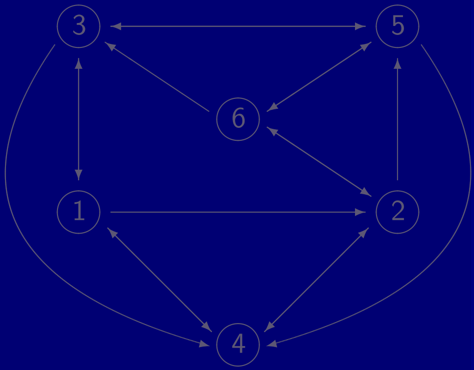
☞ A uniform encoding C(P) is a first-order logic program, encoding the solutions to P for any set C(I) of facts.

N-Colorability

Problem Instance as Facts

Given: a (directed) graph G

$$G = \begin{pmatrix} V = \{1,2,3,4,5,6\}, & \\ E = \{(1,2),(1,3),(1,4), & \\ \quad (2,4),(2,5),(2,6), & \\ \quad (3,1),(3,4),(3,5), & \\ \quad (4,1),(4,2), & \\ \quad (5,3),(5,4),(5,6), & \\ \quad (6,2),(6,3),(6,5)\} & \end{pmatrix}$$



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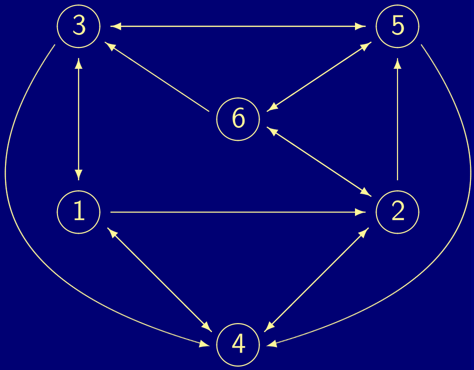


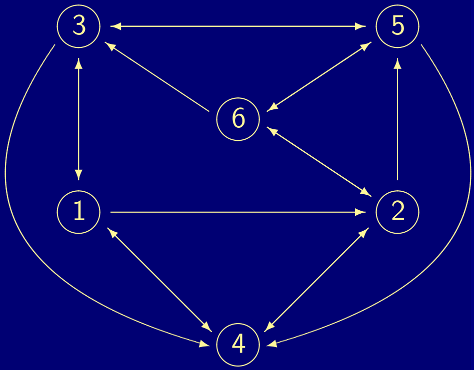
N-Colorability

Problem Instance as Facts

Given: a (directed) graph G

```
node(1).  node(2).  node(3).
node(4).  node(5).  node(6).

edge(1,2). edge(1,3). edge(1,4).
edge(2,4). edge(2,5). edge(2,6).
edge(3,1). edge(3,4). edge(3,5).
edge(4,1). edge(4,2). edge(5,3).
edge(5,4). edge(5,6). edge(6,2).
edge(6,3). edge(6,5).
```



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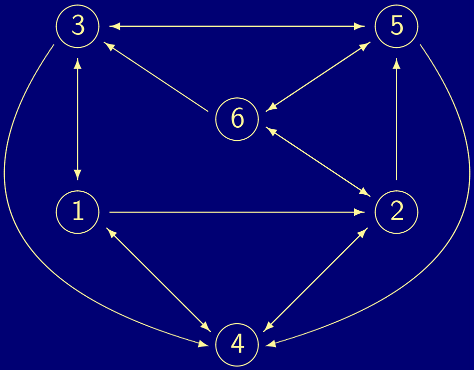

N-Colorability

(Extended) Problem Encoding

Natural Language

1. Each node has a unique color.
2. Any two connected nodes must not have the same color.
3. Let there be three colors.
4. A solution is a coloring.

Logical Language

1.
```
color(X,C) :- iscol(C),
node(X), not other(X,C).

other(X,C) :- iscol(C),
color(X,D), D != C.
```
2.
```
:- color(X,C), color(Y,C),
   edge(X,Y).
```
3.
```
#const n=3.
iscol(1..n).
```
4.
```
#hide.
#show color/2.
```

N-Colorability

(Extended) Problem Encoding

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N-Colorability

(Extended) Problem Encoding

Natural Language	Logical Language
1 Each node has a unique color.	1 `color(X,C) :- iscol(C), node(X), not other(X,C).` `other(X,C) :- iscol(C), color(X,D), D != C.`
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N-Colorability

(Extended) Problem Encoding

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4 A solution is a coloring.	4 `#hide.` `#show color/2.`

N-Colorability

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2 Any two connected nodes must not have the same color.	2 `:- color(X,C), color(Y,C), edge(X,Y).`
3 Let there be three colors.	3 `#const n=3.` `iscol(1..n).`
4 A solution is a coloring.	4 `#hide.` `#show color/2.`

N-Colorability

(Extended) Problem Encoding

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1 Each node has a unique color.	1 `color(X,C) :- iscol(C), node(X), not other(X,C).` `other(X,C) :- iscol(C), color(X,D), D != C.`
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3 Let there be three colors.	3 `#const n=3.` `iscol(1..n).`
4 A solution is a coloring.	4 `#hide.` `#show color/2.`

N-Colorability

(Extended) Problem Encoding

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```
:- color(X,C), color(Y,C),
   edge(X,Y).
```
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```
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iscol(1..n).
```
4.
```
#hide.
#show color/2.
```

N-Colorability

(Extended) Problem Encoding

Natural Language

1. Each node has a unique color.
2. Any two connected nodes must not have the same color.
3. Let there be three colors.
4. A solution is a coloring.

Logical Language

1.
```
1 #count{ color(X,C) :
                  iscol(C) } 1
              :- node(X).
```
2.
```
:- color(X,C), color(Y,C),
   edge(X,Y).
```
3.
```
#const n=3.
iscol(1..n).
```
4.
```
#hide.
#show color/2.
```

N-Colorability

Recapitulation I

Instance as Facts (in `graph.lp`)

```
node(1).  node(2).  node(3).  node(4).  node(5).  node(6).

edge(1,2). edge(1,3). edge(1,4).
edge(2,4). edge(2,5). edge(2,6).
edge(3,1). edge(3,4). edge(3,5).
edge(4,1). edge(4,2).
edge(5,3). edge(5,4). edge(5,6).
edge(6,2). edge(6,3). edge(6,5).
```

N-Colorability

Recapitulation II

Uniform Encoding (in `color.lp`)

```
% DOMAIN
#const n=3. iscol(1..n).

% GENERATE
1 #count{ color(X,C) : iscol(C) } 1 :- node(X).
% color(X,C) :- iscol(C), node(X), not other(X,C).
% other(X,C) :- iscol(C), color(X,D), D != C.

% TEST
:- color(X,C), color(Y,C), edge(X,Y).

% DISPLAY
#hide. #show color/2.
```

N-Colorability

Let's Run it!

```
gringo graph.lp color.lp | clasp 0
```

```
clasp version 2.0.2
Reading from stdin
Solving...
Answer: 1
color(6,2) color(5,3) color(4,2) color(3,1) color(2,1) color(1,3)
Answer: 2
color(6,1) color(5,3) color(4,1) color(3,2) color(2,2) color(1,3)
Answer: 3
color(6,3) color(5,2) color(4,3) color(3,1) color(2,1) color(1,2)
Answer: 4
color(6,1) color(5,2) color(4,1) color(3,3) color(2,3) color(1,2)
Answer: 5
color(6,3) color(5,1) color(4,3) color(3,2) color(2,2) color(1,1)
Answer: 6
color(6,2) color(5,1) color(4,2) color(3,3) color(2,3) color(1,1)
```

N-Colorability

Let's Run it!

```
gringo graph.lp color.lp | clasp 0
```

```
clasp version 2.0.2
Reading from stdin
Solving...
Answer: 1
color(6,2) color(5,3) color(4,2) color(3,1) color(2,1) color(1,3)
Answer: 2
color(6,1) color(5,3) color(4,1) color(3,2) color(2,2) color(1,3)
Answer: 3
color(6,3) color(5,2) color(4,3) color(3,1) color(2,1) color(1,2)
Answer: 4
color(6,1) color(5,2) color(4,1) color(3,3) color(2,3) color(1,2)
Answer: 5
color(6,3) color(5,1) color(4,3) color(3,2) color(2,2) color(1,1)
Answer: 6
color(6,2) color(5,1) color(4,2) color(3,3) color(2,3) color(1,1)
```

N-Colorability

Let's Interpret it!

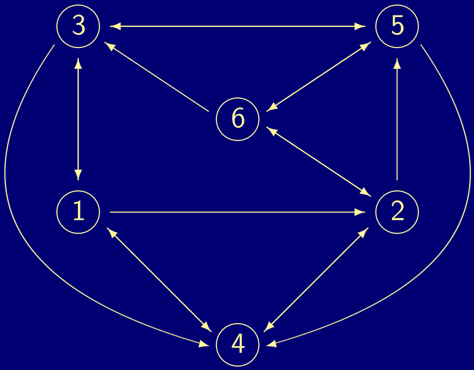
Found: 3-coloring(s)

```
Answer: 1

color(1,3) color(5,3)

color(2,1) color(3,1)

color(4,2) color(6,2)
```



N-Colorability

Let's Interpret it!

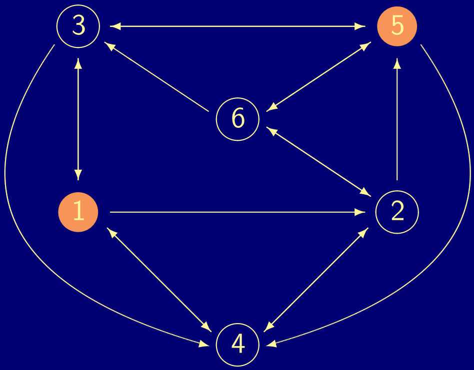
Found: 3-coloring(s)

```
Answer: 1

color(1,3) color(5,3)

color(2,1) color(3,1)

color(4,2) color(6,2)
```



N-Colorability

Let's Interpret it!

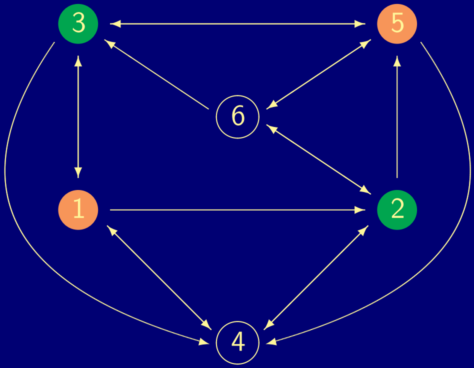
Found: 3-coloring(s)

```
Answer: 1

color(1,3) color(5,3)

color(2,1) color(3,1)

color(4,2) color(6,2)
```



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Let's Interpret it!

Found: 3-coloring(s)

```
Answer: 1

color(1,3) color(5,3)

color(2,1) color(3,1)

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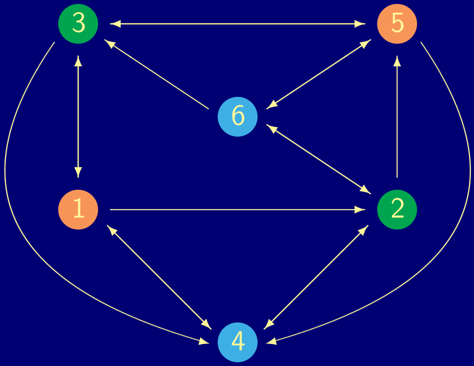
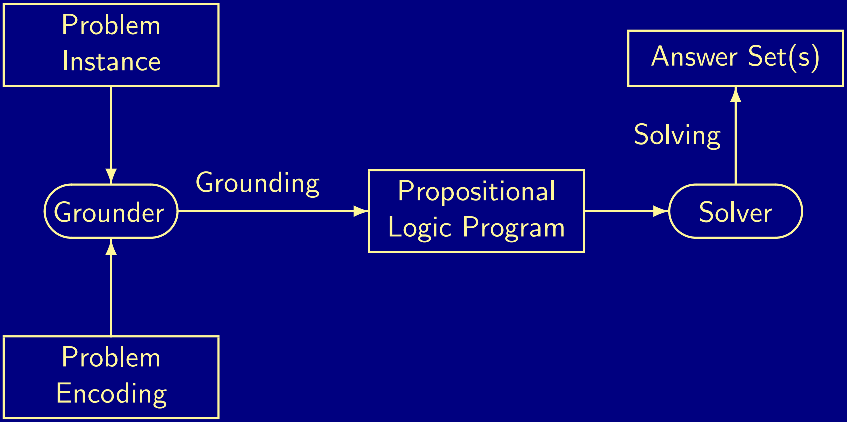
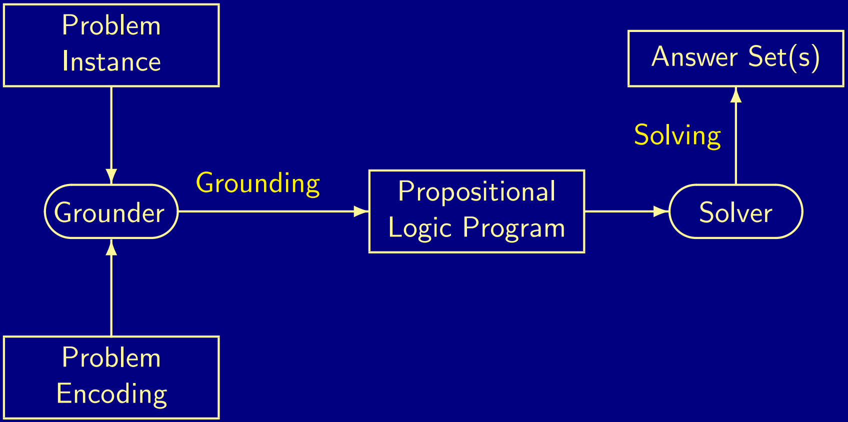
```



Interlude: Answer Set(s) Computation



Interlude: Answer Set(s) Computation



N-Colorability

Grounding

```
gringo -t graph.lp color.lp
```

```
node(1).   node(2).   node(3).   node(4).   node(5).   node(6).
edge(1,2). edge(1,3). edge(1,4). edge(2,4). edge(2,5). ...

iscol(1).  iscol(2).  iscol(3).

1 #count{ color(1,1), color(1,2), color(1,3) } 1.
1 #count{ color(2,1), color(2,2), color(2,3) } 1.
1 #count{ color(3,1), color(3,2), color(3,3) } 1.
1 #count{ color(4,1), color(4,2), color(4,3) } 1.
1 #count{ color(5,1), color(5,2), color(5,3) } 1.
1 #count{ color(6,1), color(6,2), color(6,3) } 1.

:- color(1,1), color(2,1).
:- color(1,2), color(2,2).
:- color(1,3), color(2,3). ...
```

N-Colorability

Grounding

```
gringo -t graph.lp color.lp
```

```
node(1).   node(2).   node(3).   node(4).   node(5).   node(6).
edge(1,2). edge(1,3). edge(1,4). edge(2,4). edge(2,5). ...

iscol(1).  iscol(2).  iscol(3).

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1 #count{ color(6,1), color(6,2), color(6,3) } 1.

:- color(1,1), color(2,1).
:- color(1,2), color(2,2).
:- color(1,3), color(2,3). ...
```

N-Colorability

Solving

```
gringo graph.lp color.lp | clasp --stats 0
```

```
...
Models       : 6
Time         : 0.001s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time     : 0.000s
Choices      : 5
Conflicts    : 0
Restarts     : 0

Atoms        : 63
Rules        : 113    (1: 95 2: 12 3: 6)
Bodies       : 64
Equivalences: 106    (Atom=Atom: 31 Body=Body: 6 Other: 69)
Tight        : Yes

Variables    : 63     (Eliminated: 0 Frozen:   30)
Constraints  : 45     (Binary: 73.3% Ternary:  0.0% Other: 26.7%)
Lemmas       : 0      (Binary:  0.0% Ternary:  0.0% Other:  0.0%)
```

N-Colorability

Solving

```
gringo graph.lp color.lp | clasp --stats 0
```

```
...
Models       : 6
Time         : 0.001s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time     : 0.000s
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N-Colorability

Solving

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```
...
Models       : 6
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```

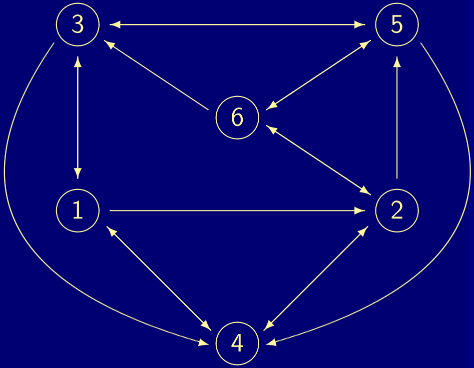
Hamiltonian Cycle

Problem Instance as Facts

Recall: a directed graph G

```
node(1).  node(2).  node(3).
node(4).  node(5).  node(6).

edge(1,2). edge(1,3). edge(1,4).
edge(2,4). edge(2,5). edge(2,6).
edge(3,1). edge(3,4). edge(3,5).
edge(4,1). edge(4,2). edge(5,3).
edge(5,4). edge(5,6). edge(6,2).
edge(6,3). edge(6,5).
```



Hamiltonian Cycle

Engineering an Encoding

Problem Specification

A (directed) graph $G = (V, E)$ is Hamiltonian if it contains a cycle C that visits every node of V exactly once.

- ☞ C traverses exactly one incoming and one outgoing edge per node.
- ☞ C traverses every node of V (starting from an arbitrary node in V).

Problem Encoding

```
1 #count{ cycle(X,Y) : edge(X,Y) } 1 :- node(Y).
1 #count{ cycle(X,Y) : edge(X,Y) } 1 :- node(X).
```

Hamiltonian Cycle

Engineering an Encoding

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- ☞ C traverses exactly one incoming and one outgoing edge per node.
- ☞ C traverses every node of V (starting from an arbitrary node in V).

Problem Encoding

```
reach(X) :- first(X).
reach(Y) :- reach(X), cycle(X,Y).
```

Hamiltonian Cycle

Engineering an Encoding

Problem Specification

A (directed) graph $G = (V, E)$ is Hamiltonian if it contains a cycle C that visits every node of V exactly once.

- ☞ C traverses exactly one incoming and one outgoing edge per node.
- ☞ C traverses every node of V (starting from an arbitrary node in V).

Problem Encoding

```
reach(X) :- first(X).
reach(Y) :- reach(X), cycle(X,Y).
```

- ☞ The definition of `reach` is recursive!

Hamiltonian Cycle

Engineering an Encoding

Problem Specification

A (directed) graph $G = (V, E)$ is Hamiltonian if it contains a cycle C that visits every node of V exactly once.

- ☞ C traverses exactly one incoming and one outgoing edge per node.
- ☞ C traverses every node of V (starting from an arbitrary node in V).

Problem Encoding

```
reach(X) :- first(X).
reach(Y) :- reach(X), cycle(X,Y).

first(X) :- X = #min[ node(Y) = Y ].
```

Hamiltonian Cycle

Engineering an Encoding

Problem Specification

A (directed) graph $G = (V, E)$ is Hamiltonian if it contains a cycle C that visits every node of V exactly once.

- ☞ C traverses exactly one incoming and one outgoing edge per node.
- ☞ C traverses every node of V (starting from an arbitrary node in V).

Problem Encoding

```
reach(X) :- first(X).
reach(Y) :- reach(X), cycle(X,Y).

:- node(Y), not reach(Y).
```

Hamiltonian Cycle

The Complete Picture

Uniform Encoding (in `cycle.lp`)

```
% DOMAIN
first(X) :- X = #min[ node(Y) = Y ].

% GENERATE
1 #count{ cycle(X,Y) : edge(X,Y) } 1 :- node(X).
1 #count{ cycle(X,Y) : edge(X,Y) } 1 :- node(Y).

% DEFINE
reach(X) :- first(X).
reach(Y) :- reach(X), cycle(X,Y).

% TEST
:- node(Y), not reach(Y).

% DISPLAY
#hide. #show cycle/2.
```

Hamiltonian Cycle

Let's Run it!

```
gringo graph.lp cycle.lp | clasp --stats
```

```
Answer: 1
cycle(6,5) cycle(5,3) cycle(4,2) cycle(3,1) cycle(2,6) cycle(1,4)
SATISFIABLE

Models       : 1+
Time         : 0.001s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time     : 0.000s
Choices      : 3
Conflicts    : 0
Restarts     : 0

Atoms        : 84
Rules        : 117    (1: 84 2: 21 3: 12)
Bodies       : 81
Equivalences: 174    (Atom=Atom: 36 Body=Body: 12 Other: 126)
Tight        : No     (SCCs: 1 Nodes: 20)
```

Hamiltonian Cycle

Let's Run it!

```
gringo graph.lp cycle.lp | clasp --stats
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Answer: 1
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Tight        : No     (SCCs: 1 Nodes: 20)
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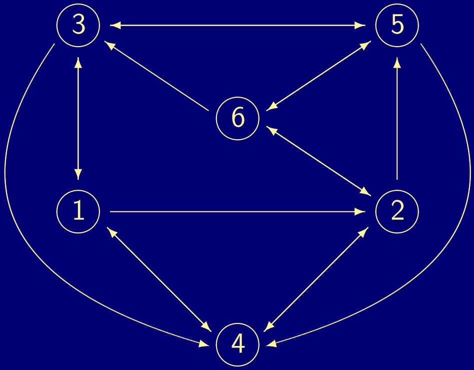
Hamiltonian Cycle

Let's Interpret it!

Found: Hamiltonian cycle

```
Answer: 1

cycle(1,4)
cycle(4,2)
cycle(2,6)
cycle(6,5)
cycle(5,3)
cycle(3,1)
```



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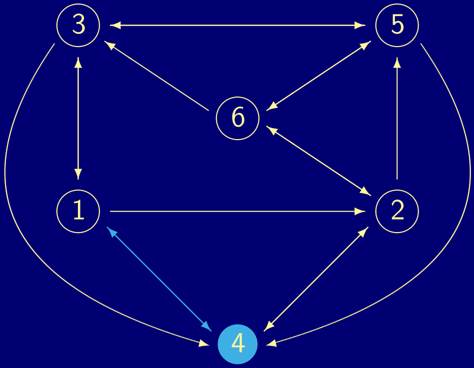

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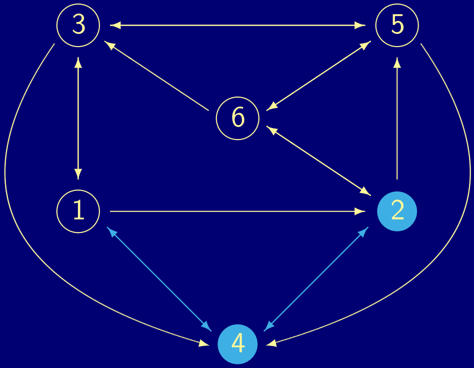

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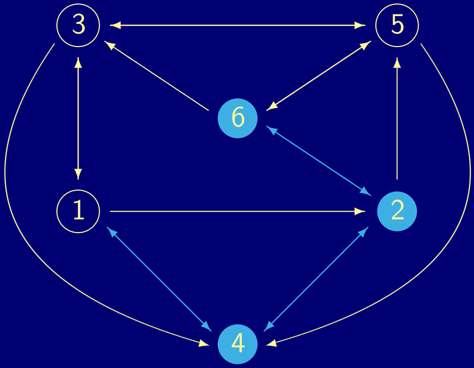

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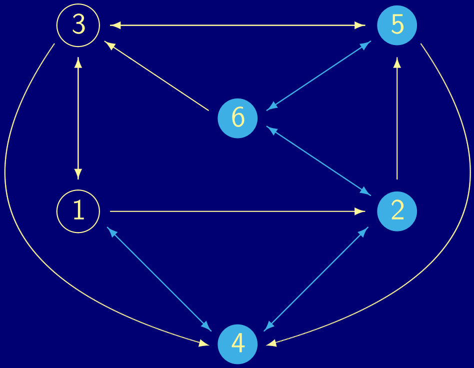

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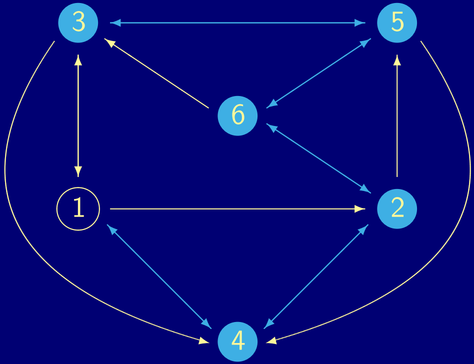

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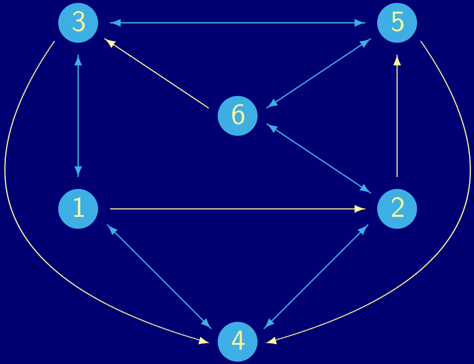


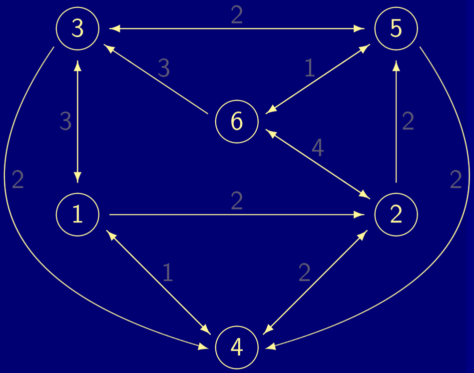
Mr Hamilton as Traveling Salesperson

Problem Instance as Facts

Given: a directed graph *G* plus edge costs

```
node(1).  node(2).  node(3).
node(4).  node(5).  node(6).

edge(1,2). edge(1,3). edge(1,4).
edge(2,4). edge(2,5). edge(2,6).
edge(3,1). edge(3,4). edge(3,5).
edge(4,1). edge(4,2). edge(5,3).
edge(5,4). edge(5,6). edge(6,2).
edge(6,3). edge(6,5).
```



Mr Hamilton as Traveling Salesperson

Problem Instance as Facts

Given: a directed graph G plus edge costs

```
cost(1,2,2).
cost(1,3,3). cost(3,1,3).
cost(1,4,1). cost(4,1,1).
cost(2,4,2). cost(4,2,2).
cost(2,5,2).
cost(2,6,4). cost(6,2,4).
cost(3,4,2).
cost(3,5,2). cost(5,3,2).
cost(5,4,2).
cost(5,6,1). cost(6,5,1).
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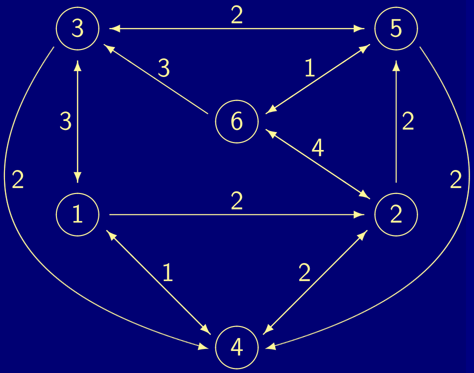

Mr Hamilton as Traveling Salesperson

Solution Optimization

Optimization Objective

A Hamiltonian cycle is optimal if its accumulated edge costs are minimal.

☞ Use `#minimize` (and/or `#maximize`) to associate each answer set with objective value(s).

Optimization Encoding

```
% OPTIMIZE
#minimize[ cycle(X,Y) : cost(X,Y,C) = C@1 ].
```

Target: minimal sum of costs `C` (at priority level `1`) associated with instances of `cycle` in an answer set

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Mr Hamilton as Traveling Salesperson

Let's Run it!

```
gringo graph.lp costs.lp cycle.lp price.lp | clasp --stats 0
```

```
Answer: 1
cycle(6,5) cycle(5,3) cycle(4,2) cycle(3,1) cycle(2,6) cycle(1,4)
Optimization: 13
Answer: 2
cycle(6,5) cycle(5,3) cycle(4,1) cycle(3,4) cycle(2,6) cycle(1,2)
Optimization: 12
Answer: 3
cycle(6,3) cycle(5,6) cycle(4,1) cycle(3,4) cycle(2,5) cycle(1,2)
Optimization: 11
OPTIMUM FOUND

Models      : 1
  Enumerated: 3
  Optimum   : yes
Optimization: 11
Time        : 0.004s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.000s
```

Mr Hamilton as Traveling Salesperson

Let's Run it!

```
gringo graph.lp costs.lp cycle.lp price.lp | clasp --stats 0
```

```
Answer: 1
cycle(6,5) cycle(5,3) cycle(4,2) cycle(3,1) cycle(2,6) cycle(1,4)
Optimization: 13
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cycle(6,5) cycle(5,3) cycle(4,1) cycle(3,4) cycle(2,6) cycle(1,2)
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Time        : 0.004s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
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```

Mr Hamilton as Traveling Salesperson

Let's Interpret it!

Found: optimal Hamiltonian cycle

```
Answer: 1

cycle(1,4)
cycle(4,2)
cycle(2,6)
cycle(6,5)
cycle(5,3)
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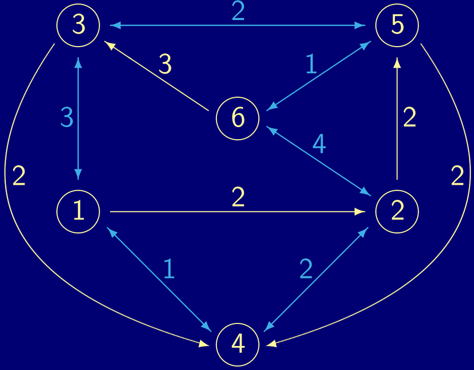
```



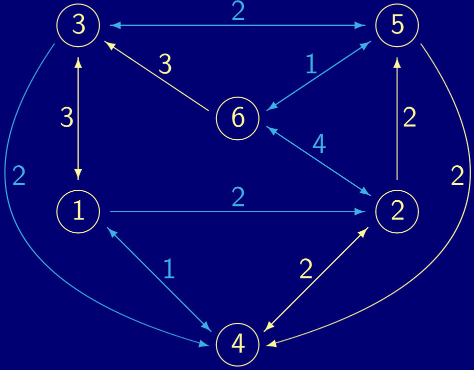
Mr Hamilton as Traveling Salesperson

Let's Interpret it!

Found: optimal Hamiltonian cycle

```
Answer: 2

cycle(1,2)
cycle(2,6)
cycle(6,5)
cycle(5,3)
cycle(3,4)
cycle(4,1)
```



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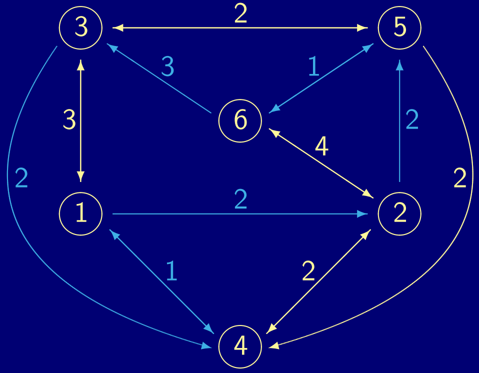
Let's Interpret it!

Found: optimal Hamiltonian cycle

```
Answer: 3

cycle(1,2)
cycle(2,5)
cycle(5,6)
cycle(6,3)
cycle(3,4)
cycle(4,1)
```



Take-Home Messages

For solving a problem (class) in ASP, provide

1. facts describing an instance and
2. a (uniform) encoding of solutions.

Encodings are often structured by the following logical parts:

1. Domain information (by deduction from facts)
2. Generator providing solution candidates (choice rules)
3. Define rules analyzing properties of candidates (normal rules)
4. Tester eliminating invalid candidates (integrity constraints)
5. Display statements projecting answer sets (onto characteristic atoms)
6. Optimizer evaluating answer sets (`#minimize`/`#maximize`)

In a Nutshell

Logic Program $\subseteq$ (Data + Deduction) + (Generation + Analysis) + Selection + Projection [+ Optimization]

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The Camel through the Eye of a Needle

ASP offers

1. rich yet easy modeling languages
2. efficient instantiation procedures
3. powerful search engines

Question: Anything left to worry about?

Answer: Yes! (unfortunately)

☞ Even in declarative programming, the problem encoding matters.

Consider sorting $[4, 7, 2, 5, 1, 8, 6, 3]$

- divide-and-conquer (Quicksort) $\sim 8(\log_2 8) = 16$ "operations"
- permutation guessing $\sim 8!/2 = 20,160$ "operations"

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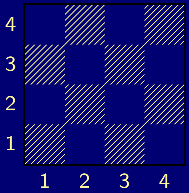
N-Queens Problem

Problem Specification

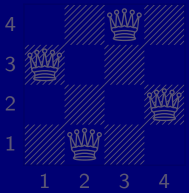
Given an $N \times N$ chessboard,
place N queens such that they do not attack each other
(neither horizontally, vertically, nor diagonally).

$N = 4$

Chessboard



Placement



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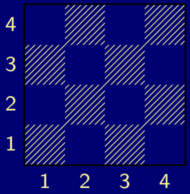
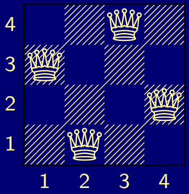


Placement



A First Encoding

1. Each square may host a queen.
2. No row, column, or diagonal hosts two queens.
3. A placement is given by instances of `queen` in an answer set.

queens_0.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- queen(X1,Y1), queen(X1,Y2), Y1 < Y2.

% DISPLAY
#hide. #show queen/2.
```

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0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- queen(X1,Y1), queen(X1,Y2), Y1 < Y2.

% DISPLAY
#hide. #show queen/2.
```

A First Encoding

1. Each square may host a queen.
2. No row, column, or diagonal hosts two queens.
3. A placement is given by instances of `queen` in an answer set.

queens_0.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- queen(X1,Y1), queen(X2,Y1), X1 < X2.

% DISPLAY
#hide. #show queen/2.
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% GENERATE
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% TEST
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.

% DISPLAY
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A First Encoding

1. Each square may host a queen.
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queens_0.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
[...]

% DISPLAY
#hide. #show queen/2.
```

A First Encoding

1. Each square may host a queen.
2. No row, column, or diagonal hosts two queens.
3. A placement is given by instances of `queen` in an answer set.

queens_0.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
[...]

% DISPLAY
#hide. #show queen/2.
```

Anything missing?

A First Encoding

1. Each square may host a queen.
2. No row, column, or diagonal hosts two queens.
3. A placement is given by instances of `queen` in an answer set.
4. We have to place (at least) N queens.

queens_0.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
[...]
:- not n #count{ queen(X,Y) }.

% DISPLAY
#hide. #show queen/2.
```

A First Encoding

Let's Place 8 Queens!

```
gringo -c n=8 queens_0.lp | clasp --stats
```

```
Answer: 1
queen(1,6) queen(2,3) queen(3,1) queen(4,7)
queen(5,5) queen(6,8) queen(7,2) queen(8,4)
SATISFIABLE

Models      : 1+
Time        : 0.006s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.000s
Choices     : 18
Conflicts   : 13
Restarts    : 0

Variables   : 793
Constraints : 729
```

A First Encoding

Let's Place 8 Queens!

```
gringo -c n=8 queens_0.lp | clasp --stats
```

```
Answer: 1
queen(1,6) queen(2,3) queen(3,1) queen(4,7)
queen(5,5) queen(6,8) queen(7,2) queen(8,4)
SATISFIABLE

Models      : 1+
Time        : 0.006s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
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Choices     : 18
Conflicts   : 13
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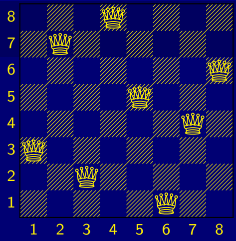
Variables   : 793
Constraints : 729
```

A First Encoding

Let's Place 8 Queens!

```
gringo -c n=8 queens_0.lp | clasp --stats
```




```
Answer: 1
queen(1,6) queen(2,3) queen(3,1) queen(4,7)
queen(5,5) queen(6,8) queen(7,2) queen(8,4)
SATISFIABLE

Models      : 1+
Time        : 0.006s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
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Let's Place 8 Queens!

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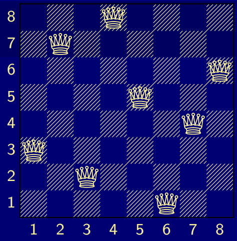
Models      : 1+
Time        : 0.006s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.000s
Choices     : 18
Conflicts   : 13
Restarts    : 0

Variables   : 793
Constraints : 729
```



A First Encoding

Let's Place 22 Queens!

```
gringo -c n=22 queens_0.lp | clasp --stats
```

```
Answer: 1
queen(1,10) queen(2,6) queen(3,16) queen(4,14) queen(5,8) ...
SATISFIABLE

Models      : 1+
Time        : 150.531s (Solving: 150.37s 1st Model: 150.34s Unsat: 0.00s)
CPU Time    : 147.480s
Choices     : 594960
Conflicts   : 574565
Restarts    : 19

Variables   : 17271
Constraints : 16787
```

A First Encoding

Let's Place 22 Queens!

```
gringo -c n=22 queens_0.lp | clasp --stats
```

```
Answer: 1
queen(1,10) queen(2,6) queen(3,16) queen(4,14) queen(5,8) ...
SATISFIABLE

Models      : 1+
Time        : 150.531s (Solving: 150.37s 1st Model: 150.34s Unsat: 0.00s)
CPU Time    : 147.480s
Choices     : 594960
Conflicts   : 574565
Restarts    : 19

Variables   : 17271
Constraints : 16787
```

A First Refinement

At least *N* queens? Exactly one queen per row and column!

queens_0.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- queen(X1,Y1), queen(X1,Y2), Y1 < Y2.
:- queen(X1,Y1), queen(X2,Y1), X1 < X2.
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.
:- not n #count{ queen(X,Y) }.

% DISPLAY
#hide. #show queen/2.
```

A First Refinement

At least *N* queens? Exactly one queen per row and column!

queens_0.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

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0 #count{ queen(X,Y) } 1 :- square(X,Y).

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:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- queen(X1,Y1), queen(X2,Y1), X1 < X2.
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.
:- not n #count{ queen(X,Y) }.

% DISPLAY
#hide. #show queen/2.
```

A First Refinement

At least *N* queens? Exactly one queen per row and column!

queens_0.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.
:- not n #count{ queen(X,Y) }.

% DISPLAY
#hide. #show queen/2.
```

A First Refinement

At least N queens? Exactly one queen per row and column!

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.


% DISPLAY
#hide. #show queen/2.
```

A First Refinement

Let's Place 22 Queens!

```
gringo -c n=22 queens_1.lp | clasp --stats
```

```
Answer: 1
queen(1,18) queen(2,10) queen(3,21) queen(4,3) queen(5,5) ...
SATISFIABLE

Models      : 1+
Time        : 0.113s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.020s
Choices     : 132
Conflicts   : 105
Restarts    : 1

Variables   : 7238
Constraints : 6710
```

A First Refinement

Let's Place 22 Queens!

```
gringo -c n=22 queens_1.lp | clasp --stats
```

```
Answer: 1
queen(1,18) queen(2,10) queen(3,21) queen(4,3) queen(5,5) ...
SATISFIABLE

Models      : 1+
Time        : 0.113s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.020s
Choices     : 132
Conflicts   : 105
Restarts    : 1

Variables   : 7238
Constraints : 6710
```

A First Refinement

Let's Place 122 Queens!

```
gringo -c n=122 queens_1.lp | clasp --stats
```

```
Answer: 1
queen(1,24) queen(2,52) queen(3,37) queen(4,60) queen(5,76) ...
SATISFIABLE

Models      : 1+
Time        : 79.475s (Solving: 1.06s 1st Model: 1.06s Unsat: 0.00s)
CPU Time    : 6.930s
Choices     : 1373
Conflicts   : 845
Restarts    : 4

Variables   : 1211338
Constraints : 1196210
```

A First Refinement

Let's Place 122 Queens!

```
gringo -c n=122 queens_1.lp | clasp --stats
```

```
Answer: 1
queen(1,24) queen(2,52) queen(3,37) queen(4,60) queen(5,76) ...
SATISFIABLE

Models      : 1+
Time        : 79.475s (Solving: 1.06s 1st Model: 1.06s Unsat: 0.00s)
CPU Time    : 6.930s
Choices     : 1373
Conflicts   : 845
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A First Refinement

Let's Place 122 Queens!

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gringo -c n=122 queens_1.lp | clasp --stats
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Answer: 1
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SATISFIABLE

Models      : 1+
Time        : 79.475s (Solving: 1.06s 1st Model: 1.06s Unsat: 0.00s)
CPU Time    : 6.930s
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Conflicts   : 845
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Variables   : 1211338
Constraints : 1196210
```

A First Refinement

Where Time Has Gone

```
time(gringo -c n=122 queens_1.lp | clasp --stats

1241358 7402724 24950848

real 1m15.468s
user 1m15.980s
sys  0m0.090s
```

☞ Grounding makes the problem!

A First Refinement

Where Time Has Gone

```
time(gringo -c n=122 queens_1.lp | wc)

1241358 7402724 24950848

real 1m15.468s
user 1m15.980s
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```

☞ Grounding makes the problem!

A First Refinement

Where Time Has Gone

```
time(gringo -c n=122 queens_1.lp | wc)

1241358 7402724 24950848

real 1m15.468s
user 1m15.980s
sys  0m0.090s
```

Just kidding :-)

☞ Grounding makes the problem!

A First Refinement

Where Time Has Gone

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time(gringo -c n=122 queens_1.lp | wc)
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Where Time Has Gone

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time(gringo -c n=122 queens_1.lp | wc)
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Where Time Has Gone

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time(gringo -c n=122 queens_1.lp | wc)
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☞ Grounding makes the problem!

A First Refinement

Grounding Time ∼ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                    O(n×n)

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).                          O(n×n)

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.                        O(n×n)
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.                        O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.         O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

A First Refinement

Grounding Time ∼ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                        O(n×n)

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0 #count{ queen(X,Y) } 1 :- square(X,Y).                              O(n×n)

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:- X = 1..n, not 1 #count{ queen(X,Y) } 1.                            O(n×n)
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% DISPLAY
#hide. #show queen/2.
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A First Refinement

Grounding Time ∼ Space

queens_1.lp

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#const n=4. square(1..n,1..n).                                   O(n×n)

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).                         O(n×n)

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:- X = 1..n, not 1 #count{ queen(X,Y) } 1.                       O(n×n)
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.                       O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.        O(n^2×n^2)

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A First Refinement

Grounding Time ∼ Space

queens_1.lp

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% DOMAIN
#const n=4. square(1..n,1..n).                                          O(n×n)

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).                                O(n×n)

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A First Refinement

Grounding Time ∼ Space

queens_1.lp

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#const n=4. square(1..n,1..n).                                        O(n×n)

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0 #count{ queen(X,Y) } 1 :- square(X,Y).                              O(n×n)

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.                            O(n×n)
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:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.          O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

A First Refinement

Grounding Time ∼ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                              O(n×n)

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).                                     O(n×n)

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.                                   O(n×n)
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.                                   O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.                    O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

A First Refinement

Grounding Time ∼ Space

queens_1.lp

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% DOMAIN
#const n=4. square(1..n,1..n).                                      O(n×n)

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% DISPLAY
#hide. #show queen/2.
```

A First Refinement

Grounding Time ∼ Space

queens_1.lp

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% DOMAIN
#const n=4. square(1..n,1..n).                                   O(n×n)

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:- X = 1..n, not 1 #count{ queen(X,Y) } 1.                       O(n×n)
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% DISPLAY
#hide. #show queen/2.
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A First Refinement

Grounding Time ∼ Space

queens_1.lp

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% DOMAIN
#const n=4. square(1..n,1..n).                                      O(n×n)

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).                            O(n×n)

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.                          O(n×n)
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.                          O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.          O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

A First Refinement

Grounding Time ∼ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                    O(n×n)

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).                         O(n×n)

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.                        O(n×n)
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.                        O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.     O(n^2×n^2)

% DISPLAY
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A First Refinement

Grounding Time ∼ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                        O(n×n)

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).                              O(n×n)

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.                            O(n×n)
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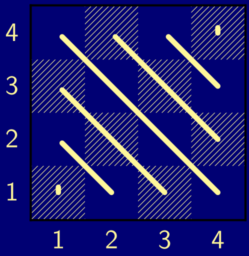
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.             O(n^2×n^2)

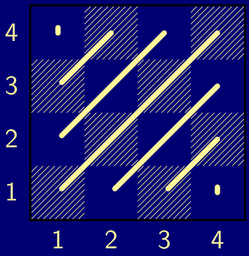
% DISPLAY
#hide. #show queen/2.
```

Diagonals make trouble!

A Nomenclature for Diagonals

$N = 4$





$\#\text{diagonal}_1 =$
$(\#\text{row} + \#\text{column}) - 1$

$\#\text{diagonal}_2 =$
$(\#\text{row} - \#\text{column}) + N$

☞ $\#\text{diagonal}_{1/2}$ can be determined in this way for arbitrary N.

A Nomenclature for Diagonals

$N = 4$

4	**4**	**5**	**6**	**7**
3	**3**	**4**	**5**	**6**
2	**2**	**3**	**4**	**5**
1	**1**	**2**	**3**	**4**
	1	2	3	4

$\#\text{diagonal}_1 =$
$(\#\text{row} + \#\text{column}) - 1$

4	**7**	**6**	**5**	**4**
3	**6**	**5**	**4**	**3**
2	**5**	**4**	**3**	**2**
1	**4**	**3**	**2**	**1**
	1	2	3	4

$\#\text{diagonal}_2 =$
$(\#\text{row} - \#\text{column}) + N$

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A Nomenclature for Diagonals

$N = 4$

4	**4**	**5**	**6**	**7**
3	**3**	**4**	**5**	**6**
2	**2**	**3**	**4**	**5**
1	**1**	**2**	**3**	**4**
	1	2	3	4

$\#\text{diagonal}_1 =$
$(\#\text{row} + \#\text{column}) - 1$

4	**7**	**6**	**5**	**4**
3	**6**	**5**	**4**	**3**
2	**5**	**4**	**3**	**2**
1	**4**	**3**	**2**	**1**
	1	2	3	4

$\#\text{diagonal}_2 =$
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A Nomenclature for Diagonals

$N = 4$

4	**4**	**5**	**6**	**7**
3	**3**	**4**	**5**	**6**
2	**2**	**3**	**4**	**5**
1	**1**	**2**	**3**	**4**
	1	2	3	4

$\#\text{diagonal}_1 =$
$(\#\text{row} + \#\text{column}) - 1$

4	**7**	**6**	**5**	**4**
3	**6**	**5**	**4**	**3**
2	**5**	**4**	**3**	**2**
1	**4**	**3**	**2**	**1**
	1	2	3	4

$\#\text{diagonal}_2 =$
$(\#\text{row} - \#\text{column}) + N$

☞ $\#\text{diagonal}_{1/2}$ can be determined in this way for arbitrary N.

A Second Refinement

Let's go for Diagonals!

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.


% DISPLAY
#hide. #show queen/2.
```

A Second Refinement

Let's go for Diagonals!

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X+Y)-1 }. % Diagonal 1


% DISPLAY
#hide. #show queen/2.
```

A Second Refinement

Let's go for Diagonals!

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X+Y)-1 }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X-Y)+n }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Second Refinement

Let's go for Diagonals!

queens_2.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X+Y)-1 }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X-Y)+n }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Second Refinement

Let's Place 122 Queens!

```
gringo -c n=122 queens_2.lp | clasp --stats
```

```
Answer: 1
queen(1,98) queen(2,54) queen(3,89) queen(4,83) queen(5,59) ...
SATISFIABLE

Models      : 1+
Time        : 2.211s (Solving: 0.13s 1st Model: 0.13s Unsat: 0.00s)
CPU Time    : 0.210s
Choices     : 11036
Conflicts   : 499
Restarts    : 3

Variables   : 16098
Constraints : 970
```

A Second Refinement

Let's Place 122 Queens!

```
gringo -c n=122 queens_2.lp | clasp --stats
```

```
Answer: 1
queen(1,98) queen(2,54) queen(3,89) queen(4,83) queen(5,59) ...
SATISFIABLE

Models      : 1+
Time        : 2.211s (Solving: 0.13s 1st Model: 0.13s Unsat: 0.00s)
CPU Time    : 0.210s
Choices     : 11036
Conflicts   : 499
Restarts    : 3

Variables   : 16098
Constraints : 970
```

A Second Refinement

Let's Place 122 Queens!

```
gringo -c n=122 queens_2.lp | clasp --stats
```

```
Answer: 1
queen(1,98) queen(2,54) queen(3,89) queen(4,83) queen(5,59) ...
SATISFIABLE

Models      : 1+
Time        : 2.211s (Solving: 0.13s 1st Model: 0.13s Unsat: 0.00s)
CPU Time    : 0.210s
Choices     : 11036
Conflicts   : 499
Restarts    : 3

Variables   : 16098
Constraints : 970
```

A Second Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_2.lp | clasp --stats
```

```
Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models      : 1+
Time        : 35.450s (Solving: 6.69s 1st Model: 6.68s Unsat: 0.00s)
CPU Time    : 7.250s
Choices     : 141445
Conflicts   : 7488
Restarts    : 9

Variables   : 92994
Constraints : 2394
```

A Second Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_2.lp | clasp --stats
```

```
Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models      : 1+
Time        : 35.450s (Solving: 6.69s 1st Model: 6.68s Unsat: 0.00s)
CPU Time    : 7.250s
Choices     : 141445
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Restarts    : 9

Variables   : 92994
Constraints : 2394
```

A Second Refinement

Let's Place 300 Queens!

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gringo -c n=300 queens_2.lp | clasp --stats
```

```
Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models      : 1+
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CPU Time    : 7.250s
Choices     : 141445
Conflicts   : 7488
Restarts    : 9

Variables   : 92994
Constraints : 2394
```

A Third Refinement

Let's Precompute Diagonals!

queens_2.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).
diag1(X,Y,(X+Y)-1) :- square(X,Y). diag2(X,Y,(X-Y)+n) :- square(X,Y).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X+Y)-1 }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X-Y)+n }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Third Refinement

Let's Precompute Diagonals!

queens_2.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).
diag1(X,Y,(X+Y)-1) :- square(X,Y). diag2(X,Y,(X-Y)+n) :- square(X,Y).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X+Y)-1 }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X-Y)+n }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Third Refinement

Let's Precompute Diagonals!

queens_2.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).
diag1(X,Y,(X+Y)-1) :- square(X,Y). diag2(X,Y,(X-Y)+n) :- square(X,Y).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : diag1(X,Y,D) }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : diag2(X,Y,D) }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Third Refinement

Let's Precompute Diagonals!

queens_3.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).
diag1(X,Y,(X+Y)-1) :- square(X,Y). diag2(X,Y,(X-Y)+n) :- square(X,Y).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : diag1(X,Y,D) }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : diag2(X,Y,D) }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Third Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_3.lp | clasp --stats
```

```
Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models      : 1+
Time        : 8.889s (Solving: 6.61s 1st Model: 6.60s Unsat: 0.00s)
CPU Time    : 7.320s
Choices     : 141445
Conflicts   : 7488
Restarts    : 9

Variables   : 92994
Constraints : 2394
```

A Third Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_3.lp | clasp --stats
```

```
Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models      : 1+
Time        : 8.889s (Solving: 6.61s 1st Model: 6.60s Unsat: 0.00s)
CPU Time    : 7.320s
Choices     : 141445
Conflicts   : 7488
Restarts    : 9

Variables   : 92994
Constraints : 2394
```

A Third Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_3.lp | clasp --stats
```

```
Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models      : 1+
Time        : 8.889s (Solving: 6.61s 1st Model: 6.60s Unsat: 0.00s)
CPU Time    : 7.320s
Choices     : 141445
Conflicts   : 7488
Restarts    : 9

Variables   : 92994
Constraints : 2394
```

A Third Refinement

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
```

```
Answer: 1
queen(1,477) queen(2,365) queen(3,455) queen(4,470) queen(5,237) ...
SATISFIABLE

Models      : 1+
Time        : 76.798s (Solving: 65.81s 1st Model: 65.75s Unsat: 0.00s)
CPU Time    : 68.620s
Choices     : 869379
Conflicts   : 25746
Restarts    : 12

Variables   : 365994
Constraints : 4794
```

A Third Refinement

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
```

```
Answer: 1
queen(1,477) queen(2,365) queen(3,455) queen(4,470) queen(5,237) ...
SATISFIABLE

Models      : 1+
Time        : 76.798s (Solving: 65.81s 1st Model: 65.75s Unsat: 0.00s)
CPU Time    : 68.620s
Choices     : 869379
Conflicts   : 25746
Restarts    : 12

Variables   : 365994
Constraints : 4794
```

A Case for Oracles

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
--heuristic=vsids --trans-ext=dynamic
```

```
Answer: 1
queen(1,477) queen(2,365) queen(3,455) queen(4,470) queen(5,237) ...
SATISFIABLE

Models      : 1+
Time        : 76.798s (Solving: 65.81s 1st Model: 65.75s Unsat: 0.00s)
CPU Time    : 68.620s
Choices     : 869379
Conflicts   : 25746
Restarts    : 12

Variables   : 365994
Constraints : 4794
```

A Case for Oracles

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
--heuristic=vsids --trans-ext=dynamic
```

```
Answer: 1
queen(1,477) queen(2,365) queen(3,455) queen(4,470) queen(5,237) ...
SATISFIABLE

Models      : 1+
Time        : 76.798s (Solving: 65.81s 1st Model: 65.75s Unsat: 0.00s)
CPU Time    : 68.620s
Choices     : 869379
Conflicts   : 25746
Restarts    : 12

Variables   : 365994
Constraints : 4794
```

A Case for Oracles

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
--heuristic=vsids --trans-ext=dynamic
```

```
Answer: 1
queen(1,422) queen(2,458) queen(3,224) queen(4,408) queen(5,405) ...
SATISFIABLE

Models      : 1+
Time        : 37.454s (Solving: 26.38s 1st Model: 26.26s Unsat: 0.00s)
CPU Time    : 29.580s
Choices     : 961315
Conflicts   : 3222
Restarts    : 7

Variables   : 365994
Constraints : 4794
```

A Case for Oracles

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
--heuristic=vsids --trans-ext=dynamic
```

```
Answer: 1
queen(1,422) queen(2,458) queen(3,224) queen(4,408) queen(5,405) ...
SATISFIABLE

Models      : 1+
Time        : 37.454s (Solving: 26.38s 1st Model: 26.26s Unsat: 0.00s)
CPU Time    : 29.580s
Choices     : 961315
Conflicts   : 3222
Restarts    : 7

Variables   : 365994
Constraints : 4794
```

A Case for Oracles

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
--heuristic=vsids --trans-ext=dynamic
```

```
Answer: 1
queen(1,90) queen(2,452) queen(3,494) queen(4,145) queen(5,84) ...
SATISFIABLE

Models      : 1+
Time        : 22.654s (Solving: 10.53s 1st Model: 10.47s Unsat: 0.00s)
CPU Time    : 15.750s
Choices     : 1058729
Conflicts   : 2128
Restarts    : 6

Variables   : 403123
Constraints : 49636
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. check all properties explicitly ... obsolete if properties change
2. use variable-sized conjunction (via ':') ... adapts to changing facts
3. use negation of complement ... adapts to changing facts

Example: vegetables to buy

```
veg(asparagus).          veg(cucumber).
pro(asparagus,cheap).    pro(cucumber,cheap).
pro(asparagus,fresh).    pro(cucumber,fresh).
pro(asparagus,tasty).    pro(cucumber,tasty).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. check all properties explicitly . . . obsolete if properties change
2. use variable-sized conjunction (via ':') . . . adapts to changing facts
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Example: vegetables to buy

```
veg(asparagus).        veg(cucumber).
pro(asparagus,cheap).  pro(cucumber,cheap).
pro(asparagus,fresh).  pro(cucumber,fresh).
pro(asparagus,tasty).  pro(cucumber,tasty).


buy(X) :- veg(X), pro(X,cheap), pro(X,fresh), pro(X,tasty).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. check all properties explicitly ... obsolete if properties change ✗
2. use variable-sized conjunction (via ':') ... adapts to changing facts
3. use negation of complement ... adapts to changing facts

Example: vegetables to buy

```
veg(asparagus).         veg(cucumber).
pro(asparagus,cheap).   pro(cucumber,cheap).
pro(asparagus,fresh).   pro(cucumber,fresh).
pro(asparagus,tasty).   pro(cucumber,tasty).
pro(asparagus,clean).

buy(X) :- veg(X), pro(X,cheap), pro(X,fresh), pro(X,tasty), pro(X,clean).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✗
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✔
3. use negation of complement ... adapts to changing facts ✔

Example: vegetables to buy

```
veg(asparagus).         veg(cucumber).
pro(asparagus,cheap).   pro(cucumber,cheap).
pro(asparagus,fresh).   pro(cucumber,fresh).
pro(asparagus,tasty).   pro(cucumber,tasty).
pro(asparagus,clean).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✗
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✓
3. use negation of complement ... adapts to changing facts

Example: vegetables to buy

```
veg(asparagus).         veg(cucumber).
pro(asparagus,cheap).   pro(cucumber,cheap).   pre(cheap).
pro(asparagus,fresh).   pro(cucumber,fresh).   pre(fresh).
pro(asparagus,tasty).   pro(cucumber,tasty).   pre(tasty).
pro(asparagus,clean).

buy(X) :- veg(X), pro(X,P) : pre(P).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✗
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✓
3. use negation of complement ... adapts to changing facts ✓

Example: vegetables to buy

```
veg(asparagus).        veg(cucumber).
pro(asparagus,cheap).  pro(cucumber,cheap).  pre(cheap).
pro(asparagus,fresh).  pro(cucumber,fresh).  pre(fresh).
pro(asparagus,tasty).  pro(cucumber,tasty).  pre(tasty).
pro(asparagus,clean).                        pre(clean).

buy(X) :- veg(X), pro(X,P) : pre(P).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✗
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✔
3. use negation of complement ... adapts to changing facts ✔

Example: vegetables to buy

```
veg(asparagus).         veg(cucumber).
pro(asparagus,cheap).   pro(cucumber,cheap).   pre(cheap).
pro(asparagus,fresh).   pro(cucumber,fresh).   pre(fresh).
pro(asparagus,tasty).   pro(cucumber,tasty).   pre(tasty).
pro(asparagus,clean).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✗
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✔
3. use negation of complement ... adapts to changing facts ✔

Example: vegetables to buy

```
veg(asparagus).        veg(cucumber).
pro(asparagus,cheap). pro(cucumber,cheap). pre(cheap).
pro(asparagus,fresh). pro(cucumber,fresh). pre(fresh).
pro(asparagus,tasty). pro(cucumber,tasty). pre(tasty).
pro(asparagus,clean).

buy(X) :- veg(X), not bye(X).     bye(X) :- veg(X), pre(P), not pro(X,P).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✗
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✔
3. use negation of complement ... adapts to changing facts ✔

Example: vegetables to buy

```
veg(asparagus).         veg(cucumber).
pro(asparagus,cheap).   pro(cucumber,cheap).   pre(cheap).
pro(asparagus,fresh).   pro(cucumber,fresh).   pre(fresh).
pro(asparagus,tasty).   pro(cucumber,tasty).   pre(tasty).
pro(asparagus,clean).                          pre(clean).

buy(X) :- veg(X), not bye(X).     bye(X) :- veg(X), pre(P), not pro(X,P).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✘
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✔
3. use negation of complement ... adapts to changing facts ✔

Example: vegetables to buy

```
veg(asparagus).          veg(cucumber).
pro(asparagus,cheap).    pro(cucumber,cheap).    pre(cheap).
pro(asparagus,fresh).    pro(cucumber,fresh).    pre(fresh).
pro(asparagus,tasty).    pro(cucumber,tasty).    pre(tasty).
pro(asparagus,clean).                            pre(clean).

buy(X) :- veg(X), not bye(X).     bye(X) :- veg(X), pre(P), not pro(X,P).
```

Projecting Irrelevant Details Out

A Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).



% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- square(X1,Y1), N = 1..n, not num(X1,Y2,N) : square(X1,Y2).
:- square(X1,Y1), N = 1..n, not num(X2,Y1,N) : square(X2,Y1).
```

☞ unreused "singleton variables"

`gringo latin_0.lp | wc`	`gringo latin_1.lp | wc`
105480 2558984 14005258	42056 273672 1690522

Projecting Irrelevant Details Out

A Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).



% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- square(X1,Y1), N = 1..n, not num(X1,Y2,N) : square(X1,Y2).
:- square(X1,Y1), N = 1..n, not num(X2,Y1,N) : square(X2,Y1).
```

☞ unreused "singleton variables"

```
gringo latin_0.lp | wc
```

105480 2558984 14005258

```
gringo latin_1.lp | wc
```

42056 273672 1690522

Projecting Irrelevant Details Out

A Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).



% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- square(X1,Y1), N = 1..n, not num(X1,Y2,N) : square(X1,Y2).
:- square(X1,Y1), N = 1..n, not num(X2,Y1,N) : square(X2,Y1).
```

☞ unreused "singleton variables"

```
gringo latin_0.lp | wc
```

105480 2558984 14005258

```
gringo latin_1.lp | wc
```

42056 273672 1690522

Projecting Irrelevant Details Out

A Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
squareX(X) :- square(X,Y).      squareY(Y) :- square(X,Y).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- squareX(X1)  , N = 1..n, not num(X1,Y2,N) : square(X1,Y2).
:- squareY(Y1)  , N = 1..n, not num(X2,Y1,N) : square(X2,Y1).
```

☞ unreused "singleton variables"

```
gringo latin_0.lp | wc
```

105480 2558984 14005258

```
gringo latin_1.lp | wc
```

42056 273672 1690522

Projecting Irrelevant Details Out

A Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
squareX(X) :- square(X,Y).      squareY(Y) :- square(X,Y).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- squareX(X1)  , N = 1..n, not num(X1,Y2,N) : square(X1,Y2).
:- squareY(Y1)  , N = 1..n, not num(X2,Y1,N) : square(X2,Y1).
```

☞ unreused "singleton variables"

```
gringo latin_0.lp | wc
```

105480 2558984 14005258

```
gringo latin_1.lp | wc
```

42056 273672 1690522

Unraveling Symmetric Inequalities

Another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 != Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 != X2.
```

☞ duplicate ground rules (swapping Y1/Y2 and X1/X2 gives the "same")

```
gringo latin_2.lp | wc
```

2071560 12389384 40906946

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

Unraveling Symmetric Inequalities

Another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 != Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 != X2.
```

☞ duplicate ground rules (swapping `Y1`/`Y2` and `X1`/`X2` gives the "same")

```
gringo latin_2.lp | wc
```

2071560 12389384 40906946

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gringo latin_3.lp | wc
```

1055752 6294536 21099558

Unraveling Symmetric Inequalities

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% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 != Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 != X2.
```

☞ duplicate ground rules (swapping `Y1`/`Y2` and `X1`/`X2` gives the "same")

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2071560 12389384 40906946

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gringo latin_3.lp | wc
```

1055752 6294536 21099558

Unraveling Symmetric Inequalities

Another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 <  Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 <  X2.
```

☞ duplicate ground rules (swapping Y1/Y2 and X1/X2 gives the "same")

```
gringo latin_2.lp | wc
```

2071560 12389384 40906946

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

Unraveling Symmetric Inequalities

Another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 <  Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 <  X2.
```

☞ duplicate ground rules (swapping `Y1`/`Y2` and `X1`/`X2` gives the "same")

```
gringo latin_2.lp | wc
```

2071560 12389384 40906946

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 < Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 < X2.
```

☞ uniqueness of `N` in a row/column checked by ENUMERATING PAIRS!

`gringo latin_3.lp | wc`

1055752 6294536 21099558

`gringo latin_4.lp | wc`

228360 1205256 4780744

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 < Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 < X2.
```

☞ uniqueness of `N` in a row/column checked by ENUMERATING PAIRS!

gringo latin_3.lp | wc	gringo latin_4.lp | wc
1055752 6294536 21099558	228360 1205256 4780744

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 < Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 < X2.
```

☞ uniqueness of `N` in a row/column checked by ENUMERATING PAIRS!

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

```
gringo latin_4.lp | wc
```

228360 1205256 4780744

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
gtX(X-1,Y,N) :- num(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- num(X,Y,N), 1 < Y.
gtX(X-1,Y,N) :- gtX(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- gtY(X,Y,N), 1 < Y.
 :- num(X,Y,N), gtX(X,Y,N).               :- num(X,Y,N), gtY(X,Y,N).
```

☞ uniqueness of N in a row/column checked by ENUMERATING PAIRS!

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

```
gringo latin_4.lp | wc
```

228360 1205256 4780744

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
gtX(X-1,Y,N) :- num(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- num(X,Y,N), 1 < Y.
gtX(X-1,Y,N) :- gtX(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- gtY(X,Y,N), 1 < Y.
 :- num(X,Y,N), gtX(X,Y,N).              :- num(X,Y,N), gtY(X,Y,N).
```

☞ uniqueness of N in a row/column checked by ENUMERATING PAIRS!

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

```
gringo latin_4.lp | wc
```

228360 1205256 4780744

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
gtX(X-1,Y,N) :- num(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- num(X,Y,N), 1 < Y.
gtX(X-1,Y,N) :- gtX(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- gtY(X,Y,N), 1 < Y.
 :- num(X,Y,N), gtX(X,Y,N).              :- num(X,Y,N), gtY(X,Y,N).
```

☞ uniqueness of `N` in a row/column checked by ENUMERATING PAIRS!

```
gringo latin_3.lp | wc
```

```
1055752 6294536 21099558
```

```
gringo latin_4.lp | wc
```

```
228360 1205256 4780744
```

Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
sigma(S) :- S = #sum[ square(X,n) = X ].

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
occY(Y,N,C) :- Y = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
:- occX(X,N,C), C != 1.  :- occY(Y,N,C), C != 1.

% DISPLAY
#hide. #show num/3. #show sigma/1.
```

```
gringo latin_5.lp | wc          gringo latin_6.lp | wc
```

Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
sigma(S) :- S = #sum[ square(X,n) = X ].

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
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:- occX(X,N,C), C != 1.  :- occY(Y,N,C), C != 1.

% DISPLAY
#hide. #show num/3. #show sigma/1.
```

```
gringo latin_5.lp | wc          gringo latin_6.lp | wc
```

Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
sigma(S) :- S = #sum[ square(X,n) = X ].                          ✔

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
occY(Y,N,C) :- Y = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
:- occX(X,N,C), C != 1.  :- occY(Y,N,C), C != 1.

% DISPLAY
#hide. #show num/3. #show sigma/1.
```

```
gringo latin_5.lp | wc                gringo latin_6.lp | wc
```

Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
sigma(S) :- S = #sum[ square(X,n) = X ].

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
occY(Y,N,C) :- Y = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
:- occX(X,N,C), C != 1.  :- occY(Y,N,C), C != 1.

% DISPLAY
#hide. #show num/3. #show sigma/1.
```

```
gringo latin_5.lp | wc          gringo latin_6.lp | wc
```

Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
sigma(S) :- S = #sum[ square(X,n) = X ].

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C #count{ num(X,Y,N) } C, C = 0..n.
occY(Y,N,C) :- Y = 1..n, N = 1..n, C #count{ num(X,Y,N) } C, C = 0..n.
:- occX(X,N,C), C != 1.  :- occY(Y,N,C), C != 1.

% DISPLAY
#hide. #show num/3. #show sigma/1.
```

☞ internal transformation by gringo

Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
sigma(S) :- S = #sum[ square(X,n) = X ].

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.           ✗
occY(Y,N,C) :- Y = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.           ✗
:- occX(X,N,C), C != 1.  :- occY(Y,N,C), C != 1.

% DISPLAY
#hide. #show num/3. #show sigma/1.
```

```
gringo latin_5.lp | wc
```

```
gringo latin_6.lp | wc
```

Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
occY(Y,N,C) :- Y = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
:- occX(X,N,C), C != 1.  :- occY(Y,N,C), C != 1.

% DISPLAY
#hide. #show num/3.
```

```
gringo latin_5.lp | wc
```

```
gringo latin_6.lp | wc
```

48136 373768 2185042

Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

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occY(Y,N,C) :- Y = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
:- occX(X,N,C), C != 1.  :- occY(Y,N,C), C != 1.

% DISPLAY
#hide. #show num/3.
```

```
gringo latin_5.lp | wc
```

304136 5778440 30252505

```
gringo latin_6.lp | wc
```

48136 373768 2185042

Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.



% DISPLAY
#hide. #show num/3.
```

```
gringo latin_5.lp | wc
```

```
304136 5778440 30252505
```

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gringo latin_6.lp | wc
```

Assigning Aggregate Values

Yet another Latin square encoding

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#const n=32. square(1..n,1..n).

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:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.


% DISPLAY
#hide. #show num/3.
```

```
gringo latin_5.lp | wc
```

304136 5778440 30252505

```
gringo latin_6.lp | wc
```

48136 373768 2185042

Breaking Symmetries

The ultimate Latin square encoding?

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

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:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.


% DISPLAY
#hide. #show num/3.
```

Breaking Symmetries

The ultimate Latin square encoding?

```
% DOMAIN
#const n=32. square(1..n,1..n).

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1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.


% DISPLAY
#hide. #show num/3.
```

☞ many symmetric solutions (mirroring, rotation, value permutation)

Breaking Symmetries

The ultimate Latin square encoding?

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.


% DISPLAY
#hide. #show num/3.
```

☞ easy and safe to fix a full row/column!

Breaking Symmetries

The ultimate Latin square encoding?

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- square(1,Y), not num(1,Y,Y). % Symmetry Breaking

% DISPLAY
#hide. #show num/3.
```

☞ easy and safe to fix a full row/column!

Breaking Symmetries

The ultimate Latin square encoding?

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- square(1,Y), not num(1,Y,Y). % Symmetry Breaking

% DISPLAY
#hide. #show num/3.
```

☞ Let's compare enumeration speed!

Breaking Symmetries

The ultimate Latin square encoding?

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.


% DISPLAY
#hide. #show num/3.
```

```
gringo -c n=5 latin_6.lp | clasp -q 0
```

Breaking Symmetries

The ultimate Latin square encoding?

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.


% DISPLAY
#hide. #show num/3.
```

```
gringo -c n=5 latin_6.lp | clasp -q 0
```

```
Models : 161280     Time : 2.078s
```

Breaking Symmetries

The ultimate Latin square encoding?

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- square(1,Y), not num(1,Y,Y). % Symmetry Breaking

% DISPLAY
#hide. #show num/3.
```

```
gringo -c n=5 latin_7.lp | clasp -q 0
```

```
Models : 161280    Time : 2.078s
```

Breaking Symmetries

The ultimate Latin square encoding?

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- square(1,Y), not num(1,Y,Y). % Symmetry Breaking

% DISPLAY
#hide. #show num/3.
```

```
gringo -c n=5 latin_7.lp | clasp -q 0
```

```
Models : 1344      Time : 0.024s
```

Encode With Care!

1. Create a working encoding
   - Q1: Do you need to modify the encoding if the facts change?
   - Q2: Are all variables significant (or statically functionally dependent)?
   - Q3: Can there be (many) identic ground rules?
   - Q4: Do you enumerate pairs of values (to test uniqueness)?
   - Q5: Do you assign dynamic aggregate values (to check a fixed bound)?
   - Q6: Do you admit (obvious) symmetric solutions?
   - Q7: Do you have additional domain knowledge simplifying the problem?
   - Q8: Are you aware of anything else that, if encoded, would reduce grounding and/or solving efforts?
2. Revise until no "Yes" answer!
   - ☞ If the format of facts makes encoding painful (for instance, abusing grounding for "scientific calculations"), revise the fact format as well.

Encode With Care!

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   - Q5: Do you assign dynamic aggregate values (to check a fixed bound)?
   - Q6: Do you admit (obvious) symmetric solutions?
   - Q7: Do you have additional domain knowledge simplifying the problem?
   - Q8: Are you aware of anything else that, if encoded, would reduce grounding and/or solving efforts?
2. Revise until no “Yes” answer!
   - ☞ If the format of facts makes encoding painful (for instance, abusing grounding for “scientific calculations”), revise the fact format as well.

Some Hints on (Preventing) Debugging

Kinds of errors

- syntactic ... follow error messages by the grounder
- semantic ... (most likely) encoding/intention mismatch

Ways to identify semantic errors (early)

- develop and test incrementally
  - prepare toy instances with "interesting features"
  - build the encoding bottom-up and verify additions (eg. new predicates)
- compare the encoded to the intended meaning
  - check whether the grounding fits (use `gringo -t`)
  - if answer sets are unintended, investigate conditions that fail to hold
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Overcoming Performance Bottlenecks

Grounding

- monitor time spent by and output size of `gringo`
  1. system tools (eg. `time(gringo [...] | wc)`)
  2. profiling info (eg. `gringo --gstats --verbose=3 [...] > /dev/null`)

☞ once identified, reformulate "critical" logic program parts

Solving

- check solving statistics (use `clasp --stats`)

  if great search efforts (`Conflicts`/`Choices`/`Restarts`), then

  - try auto-configuration (offered by `claspfolio`)
  - try manual fine-tuning (requires expert knowledge!)
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Hitori

A Japanese Grid Puzzle (Beyond Sudoku)

The Puzzle

Given: an $N \times N$ board of numbered squares

4	8	1	6	3	2	5	7
3	6	7	2	1	6	5	4
2	3	4	8	2	8	6	1
4	1	6	5	7	7	3	5
7	2	3	1	8	5	1	2
3	5	6	7	3	1	8	4
6	4	2	3	5	4	7	8
8	7	1	4	2	3	5	6

Wanted: a set of black squares such that

1. no black squares are horizontally or vertically adjacent
2. numbers of white squares are unique for each row and column
3. every pair of white squares is connected via a path (not passing black squares)

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Fact and Solution Format

Facts provide instances of `state(X,Y,N)` to express that the square in column `X` and row `Y` contains number `N`.

Example Instance

```
state(1,1,4). state(2,1,8). ... state(8,1,7).
state(1,2,3). state(2,2,6). ... state(8,2,4).
state(1,3,2). state(2,3,3). ... state(8,3,1).
state(1,4,4). state(2,4,1). ... state(8,4,5).
state(1,5,7). state(2,5,2). ... state(8,5,2).
state(1,6,3). state(2,6,5). ... state(8,6,4).
state(1,7,6). state(2,7,4). ... state(8,7,8).
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Example Solution

Black squares given by instances of `blackOut(X,Y)`:

```
blackOut(1,1)   blackOut(2,5)   ...
blackOut(1,3)   ...   blackOut(8,4)
blackOut(1,6)   ...   blackOut(8,6)
```

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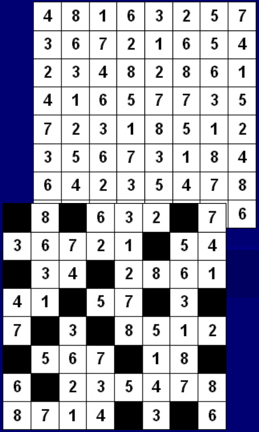

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A Working Encoding I

Found on the WWW (and Adapted to gringo Syntax)

hitori_0.lp (under GNU GPL: COPYING)

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% (A) Adjacent grid locations %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Domain predicate (evaluated upon grounding)
adjacent(X,Y,X+1,Y) :- state(X,Y,_), state(X+1,Y,_).
adjacent(X,Y,X,Y+1) :- state(X,Y,_), state(X,Y+1,_).
adjacent(X2,Y2,X1,Y1) :- adjacent(X1,Y1,X2,Y2).

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% (B) Generate solution candidate %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Every square is blacked out or normal
1 { blackOut(X,Y), -blackOut(X,Y) } 1 :- state(X,Y,_).
```

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A Working Encoding II

Found on the WWW (and Adapted to gringo Syntax)

hitori_0.lp

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% (C.1) Test eliminating adjacent blanks %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Can't have adjacent black squares
 :- adjacent(X1,Y1,X2,Y2), blackOut(X1,Y1), blackOut(X2,Y2).

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% (C.2) Tests eliminating number recurrences %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Can't have the same number twice in the same row
 :- state(X1,Y,N), state(X2,Y,N), -blackOut(X1,Y), -blackOut(X2,Y), X1 != X2.

% Can't have the same number twice in the same column
 :- state(X,Y1,N), state(X,Y2,N), -blackOut(X,Y1), -blackOut(X,Y2), Y1 != Y2.
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 :- state(X,Y1,N), state(X,Y2,N), -blackOut(X,Y1), -blackOut(X,Y2), Y1 != Y2.
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 :- state(X,Y1,N), state(X,Y2,N), -blackOut(X,Y1), -blackOut(X,Y2), Y1 != Y2.
```

Already spot something?

A Working Encoding III

Found on the WWW (and Adapted to gringo Syntax)

hitori_0.lp

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% (C.3) Test eliminating disconnected numbers %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Define mutual reachability
reachable(X1,Y1,X2,Y2) :- adjacent(X1,Y1,X2,Y2), -blackOut(X1,Y1),
     -blackOut(X2,Y2).
reachable(X1,Y1,X3,Y3) :- reachable(X1,Y1,X2,Y2), reachable(X2,Y2,X3,Y3).

% Can't have mutually unreachable non-black squares
 :- -blackOut(X1,Y1), -blackOut(X2,Y2), not reachable(X1,Y1,X2,Y2),
     (X1,Y1) != (X2,Y2).
```

☞ Answer sets (of `hitori_0.lp` plus instance) match Hitori solutions. ✔

A Working Encoding III

Found on the WWW (and Adapted to gringo Syntax)

hitori_0.lp

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
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A Working Encoding

Let's Run it!

```
gringo hitori_0.lp instance.lp | clasp --stats
```

```
Answer: 1
blackOut(1,1) blackOut(1,3) blackOut(1,6) blackOut(2,5)
blackOut(2,7) ...           blackOut(8,4) blackOut(8,6)
SATISFIABLE

Models      : 1+
Time        : 13.485s (Solving: 11.77s 1st Model: 11.77s Unsat: 0.00s)
CPU Time    : 13.290s
Choices     : 458
Conflicts   : 323
Restarts    : 2

Variables   : 260625
Constraints : 1018953
```

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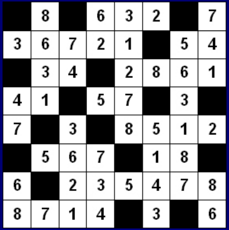

Why Classical Negation?

hitori_0.lp

```
% Every square is blacked out or normal
1 { blackOut(X,Y), -blackOut(X,Y) } 1 :- state(X,Y,_).

:- blackOut(X,Y), -blackOut(X,Y).
```

☞ no internal transformation by gringo

```
gringo hitori_0.lp instance.lp | wc
```

267534 1608172 5535208

```
gringo hitori_1.lp instance.lp | wc
```

267470 1607788 5534184

☞ no noticeable effect on grounding/solving performance

Why Classical Negation?

hitori_0.lp

```
% Every square is blacked out or normal
1 { blackOut(X,Y), -blackOut(X,Y) } 1 :- state(X,Y,_).

:- blackOut(X,Y), -blackOut(X,Y).
```

☞ internal transformation by gringo

```
gringo hitori_0.lp instance.lp | wc
```

267534 1608172 5535208

```
gringo hitori_1.lp instance.lp | wc
```

267470 1607788 5534184

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```

☞ `blackOut(X,Y)` and `-blackOut(X,Y)` exclusive in view of upper bound!

```
gringo hitori_0.lp instance.lp | wc
```

267534 1608172 5535208

```
gringo hitori_1.lp instance.lp | wc
```

267470 1607788 5534184

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```

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```
gringo hitori_0.lp instance.lp | wc
```

267534 1608172 5535208

```
gringo hitori_1.lp instance.lp | wc
```

267470 1607788 5534184

☞ no noticeable effect on grounding/solving performance

Why Classical Negation?

hitori_1.lp

```
% Every square is blacked out or normal
1 { blackOut(X,Y), negBlackOut(X,Y) } 1 :- state(X,Y,_).

:- blackOut(X,Y), -blackOut(X,Y).
```

☞ no internal transformation by gringo

```
gringo hitori_0.lp instance.lp | wc
```

267534 1608172 5535208

```
gringo hitori_1.lp instance.lp | wc
```

267470 1607788 5534184

☞ no noticeable effect on grounding/solving performance

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hitori_1.lp

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:- blackOut(X,Y), -blackOut(X,Y).
```

☞ no internal transformation by gringo

```
gringo hitori_0.lp instance.lp | wc
```

267534 1608172 5535208

```
gringo hitori_1.lp instance.lp | wc
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267470 1607788 5534184

☞ no noticeable effect on grounding/solving performance

Why Classical Negation?

hitori_1.lp

```
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:- blackOut(X,Y), -blackOut(X,Y).
```

☞ no internal transformation by gringo

```
gringo hitori_0.lp instance.lp | wc
```

267534 1608172 5535208

```
gringo hitori_1.lp instance.lp | wc
```

267470 1607788 5534184

☞ no noticeable effect on grounding/solving performance

Why Not Default Negation?

hitori_1.lp

```
% Every square is blacked out or normal
1 { blackOut(X,Y), negBlackOut(X,Y) } 1 :- state(X,Y,_).

% Can't have the same number twice in the same row
 :- state(X1,Y,N), state(X2,Y,N), negBlackOut(X1,Y), negBlackOut(X2,Y), X1 != X2.
 ...
```

☞ `blackOut(X,Y)` and `negBlackOut(X,Y)` are two sides of the same coin

Why Not Default Negation?

hitori_1.lp

```
% Every square is blacked out or normal
1 { blackOut(X,Y), negBlackOut(X,Y) } 1 :- state(X,Y,_).

% Can't have the same number twice in the same row
 :- state(X1,Y,N), state(X2,Y,N), negBlackOut(X1,Y), negBlackOut(X2,Y), X1 != X2.
 ...
```

☞ blackOut(X,Y) and negBlackOut(X,Y) are two sides of the same coin

Why Not Default Negation?

hitori_2.lp

```
% Every square is blacked out or normal
 { blackOut(X,Y) } :- state(X,Y,_).

% Can't have the same number twice in the same row
 :- state(X1,Y,N), state(X2,Y,N), not blackOut(X1,Y), not blackOut(X2,Y), X1 != X2.
 ...
```

☞ replace `negBlackOut(X,Y)` by "`not blackOut(X,Y)`"

A First Improvement

```
gringo hitori_1.lp instance.lp | clasp --stats
```

```
Answer: 1
blackOut(1,1) blackOut(1,3) blackOut(1,6) blackOut(2,5)
blackOut(2,7) ...           blackOut(8,4) blackOut(8,6)
SATISFIABLE

Models      : 1+
Time        : 13.485s (Solving: 11.77s 1st Model: 11.77s Unsat: 0.00s)
CPU Time    : 13.290s
Choices     : 458
Conflicts   : 323
Restarts    : 2

Variables   : 260625
Constraints : 1018953
```

A First Improvement

```
gringo hitori_2.lp instance.lp | clasp --stats
```

```
Answer: 1
blackOut(1,1) blackOut(1,3) blackOut(1,6) blackOut(2,5)
blackOut(2,7) ...           blackOut(8,4) blackOut(8,6)
SATISFIABLE

Models      : 1+
Time        : 13.485s (Solving: 11.77s 1st Model: 11.77s Unsat: 0.00s)
CPU Time    : 13.290s
Choices     : 458
Conflicts   : 323
Restarts    : 2

Variables   : 260625
Constraints : 1018953
```

A First Improvement

```
gringo hitori_2.lp instance.lp | clasp --stats
```

```
Answer: 1
blackOut(1,1) blackOut(1,3) blackOut(1,6) blackOut(2,5)
blackOut(2,7) ...           blackOut(8,4) blackOut(8,6)
SATISFIABLE

Models      : 1+
Time        : 10.177s (Solving: 8.42s 1st Model: 8.41s Unsat: 0.00s)
CPU Time    : 9.990s
Choices     : 344
Conflicts   : 264
Restarts    : 2

Variables   : 260433
Constraints : 1018825
```

Remember Symmetric Inequalities

hitori_2.lp

```
% Can't have the same number twice in the same row
 :- state(X1,Y,N), state(X2,Y,N), not blackOut(X1,Y), not blackOut(X2,Y), X1 != X2.

% Can't have the same number twice in the same column
 :- state(X,Y1,N), state(X,Y2,N), not blackOut(X,Y1), not blackOut(X,Y2), Y1 != Y2.
```

☞ no noticeable effect on grounding/solving performance

Remember Symmetric Inequalities

hitori_3.lp

```
% Can't have the same number twice in the same row
 :- state(X1,Y,N), state(X2,Y,N), not blackOut(X1,Y), not blackOut(X2,Y), X1 < X2.

% Can't have the same number twice in the same column
 :- state(X,Y1,N), state(X,Y2,N), not blackOut(X,Y1), not blackOut(X,Y2), Y1 < Y2.
```

☞ no noticeable effect on grounding/solving performance

Remember Symmetric Inequalities

hitori_3.lp

```
% Can't have the same number twice in the same row
 :- state(X1,Y,N), state(X2,Y,N), not blackOut(X1,Y), not blackOut(X2,Y), X1 < X2.

% Can't have the same number twice in the same column
 :- state(X,Y1,N), state(X,Y2,N), not blackOut(X,Y1), not blackOut(X,Y2), Y1 < Y2.
```

☞ no noticeable effect on grounding/solving performance

Let's Use Counting

hitori_3.lp

```
% Can't have the same number twice in the same row
 :- state(X1,Y,N), state(X2,Y,N), not blackOut(X1,Y), not blackOut(X2,Y), X1 < X2.

% Can't have the same number twice in the same column
 :- state(X,Y1,N), state(X,Y2,N), not blackOut(X,Y1), not blackOut(X,Y2), Y1 < Y2.
```

Let's Use Counting

hitori_4.lp

```
% Can't have the same number twice in the same row or column
 :- state(X1,Y1,N), 2 { not blackOut(X1,Y2) : state(X1,Y2,N) }.
 :- state(X1,Y1,N), 2 { not blackOut(X2,Y1) : state(X2,Y1,N) }.
```

A Second Improvement?

```
gringo hitori_3.lp instance.lp | clasp --stats
```

```
Answer: 1
blackOut(1,1) blackOut(1,3) blackOut(1,6) blackOut(2,5)
blackOut(2,7) ...           blackOut(8,4) blackOut(8,6)
SATISFIABLE

Models      : 1+
Time        : 10.182s (Solving: 8.47s 1st Model: 8.47s Unsat: 0.00s)
CPU Time    : 10.010s
Choices     : 344
Conflicts   : 264
Restarts    : 2

Variables   : 260433
Constraints : 1018825
```

A Second Improvement?

```
gringo hitori_4.lp instance.lp | clasp --stats
```

```
Answer: 1
blackOut(1,1) blackOut(1,3) blackOut(1,6) blackOut(2,5)
blackOut(2,7) ...           blackOut(8,4) blackOut(8,6)
SATISFIABLE

Models      : 1+
Time        : 10.182s (Solving: 8.47s 1st Model: 8.47s Unsat: 0.00s)
CPU Time    : 10.010s
Choices     : 344
Conflicts   : 264
Restarts    : 2

Variables   : 260433
Constraints : 1018825
```

A Second Improvement?

```
gringo hitori_4.lp instance.lp | clasp --stats
```

```
Answer: 1
blackOut(1,1) blackOut(1,3) blackOut(1,6) blackOut(2,5)
blackOut(2,7) ...           blackOut(8,4) blackOut(8,6)
SATISFIABLE

Models      : 1+
Time        : 9.781s (Solving: 7.99s 1st Model: 7.99s Unsat: 0.00s)
CPU Time    : 9.610s
Choices     : 278
Conflicts   : 227
Restarts    : 1

Variables   : 260432
Constraints : 1018828
```

Why Double-Check Reachability?

hitori_4.lp

```
% Define mutual reachability
reachable(X1,Y1,X2,Y2) :- adjacent(X1,Y1,X2,Y2),
                          not blackOut(X1,Y1), not blackOut(X2,Y2).
reachable(X1,Y1,X3,Y3) :- reachable(X1,Y1,X2,Y2), reachable(X2,Y2,X3,Y3).




% Can't have mutually unreachable non-black squares
 :- not blackOut(X1,Y1), not blackOut(X2,Y2), not reachable(X1,Y1,X2,Y2),
    (X1,Y1) != (X2,Y2),  state(X1,Y1,_), state(X2,Y2,_).
```

☞ reachable(X1,Y1,X2,Y2) and reachable(X2,Y2,X1,Y1) hold jointly

Why Double-Check Reachability?

hitori_4.lp

```
% Define mutual reachability
reachable(X1,Y1,X2,Y2) :- adjacent(X1,Y1,X2,Y2),
                          not blackOut(X1,Y1), not blackOut(X2,Y2).
reachable(X1,Y1,X3,Y3) :- reachable(X1,Y1,X2,Y2), reachable(X2,Y2,X3,Y3).




% Can't have mutually unreachable non-black squares
 :- not blackOut(X1,Y1), not blackOut(X2,Y2), not reachable(X1,Y1,X2,Y2),
    (X1,Y1) != (X2,Y2),  state(X1,Y1,_), state(X2,Y2,_).
```

☞ `reachable(X1,Y1,X2,Y2)` and `reachable(X2,Y2,X1,Y1)` hold jointly

Why Double-Check Reachability?

hitori_4.lp

```
% Define mutual reachability
reachable(X1,Y1,X2,Y2) :- adjacent(X1,Y1,X2,Y2), (X1,Y1) < (X2,Y2),
                          not blackOut(X1,Y1), not blackOut(X2,Y2).
reachable(X1,Y1,X3,Y3) :- reachable(X1,Y1,X2,Y2), reachable(X2,Y2,X3,Y3).
reachable(X1,Y1,X3,Y3) :- reachable(X1,Y1,X2,Y2), reachable(X3,Y3,X2,Y2),
                          (X1,Y1) < (X3,Y3).
reachable(X2,Y2,X3,Y3) :- reachable(X1,Y1,X2,Y2), reachable(X1,Y1,X3,Y3),
                          (X2,Y2) < (X3,Y3).

% Can't have mutually unreachable non-black squares
 :- not blackOut(X1,Y1), not blackOut(X2,Y2), not reachable(X1,Y1,X2,Y2),
    (X1,Y1) != (X2,Y2),  state(X1,Y1,_), state(X2,Y2,_).
```

☞ enforce `(X1,Y1) < (X2,Y2)` for instances of `reachable(X1,Y1,X2,Y2)`

Why Double-Check Reachability?

hitori_4.lp

```
% Define mutual reachability
reachable(X1,Y1,X2,Y2) :- adjacent(X1,Y1,X2,Y2), (X1,Y1) < (X2,Y2),
                          not blackOut(X1,Y1), not blackOut(X2,Y2).
reachable(X1,Y1,X3,Y3) :- reachable(X1,Y1,X2,Y2), reachable(X2,Y2,X3,Y3).
reachable(X1,Y1,X3,Y3) :- reachable(X1,Y1,X2,Y2), reachable(X3,Y3,X2,Y2),
                          (X1,Y1) < (X3,Y3).
reachable(X2,Y2,X3,Y3) :- reachable(X1,Y1,X2,Y2), reachable(X1,Y1,X3,Y3),
                          (X2,Y2) < (X3,Y3).

% Can't have mutually unreachable non-black squares
 :- not blackOut(X1,Y1), not blackOut(X2,Y2), not reachable(X1,Y1,X2,Y2),
    (X1,Y1) != (X2,Y2),  state(X1,Y1,_), state(X2,Y2,_).
```

☞ enforce `(X1,Y1) < (X2,Y2)` for instances of `reachable(X1,Y1,X2,Y2)`

Why Double-Check Reachability?

hitori_5.lp

```
% Define mutual reachability
reachable(X1,Y1,X2,Y2) :- adjacent(X1,Y1,X2,Y2), (X1,Y1) < (X2,Y2),
                          not blackOut(X1,Y1), not blackOut(X2,Y2).
reachable(X1,Y1,X3,Y3) :- reachable(X1,Y1,X2,Y2), reachable(X2,Y2,X3,Y3).
reachable(X1,Y1,X3,Y3) :- reachable(X1,Y1,X2,Y2), reachable(X3,Y3,X2,Y2),
                          (X1,Y1) < (X3,Y3).
reachable(X2,Y2,X3,Y3) :- reachable(X1,Y1,X2,Y2), reachable(X1,Y1,X3,Y3),
                          (X2,Y2) < (X3,Y3).

% Can't have mutually unreachable non-black squares
 :- not blackOut(X1,Y1), not blackOut(X2,Y2), not reachable(X1,Y1,X2,Y2),
    (X1,Y1) <  (X2,Y2),  state(X1,Y1,_), state(X2,Y2,_).
```

☞ enforce `(X1,Y1) < (X2,Y2)` for instances of `reachable(X1,Y1,X2,Y2)`

A Real Breakthrough?

```
gringo hitori_4.lp instance.lp | clasp --stats
```

```
Answer: 1
blackOut(1,1) blackOut(1,3) blackOut(1,6) blackOut(2,5)
blackOut(2,7) ...           blackOut(8,4) blackOut(8,6)
SATISFIABLE

Models      : 1+
Time        : 9.781s (Solving: 7.99s 1st Model: 7.99s Unsat: 0.00s)
CPU Time    : 9.610s
Choices     : 278
Conflicts   : 227
Restarts    : 1

Variables   : 260432
Constraints : 1018828
```

A Real Breakthrough?

```
gringo hitori_5.lp instance.lp | clasp --stats
```

```
Answer: 1
blackOut(1,1) blackOut(1,3) blackOut(1,6) blackOut(2,5)
blackOut(2,7) ...           blackOut(8,4) blackOut(8,6)
SATISFIABLE

Models      : 1+
Time        : 9.781s (Solving: 7.99s 1st Model: 7.99s Unsat: 0.00s)
CPU Time    : 9.610s
Choices     : 278
Conflicts   : 227
Restarts    : 1

Variables   : 260432
Constraints : 1018828
```

A Real Breakthrough?

```
gringo hitori_5.lp instance.lp | clasp --stats
```

```
Answer: 1
blackOut(1,1) blackOut(1,3) blackOut(1,6) blackOut(2,5)
blackOut(2,7) ...           blackOut(8,4) blackOut(8,6)
SATISFIABLE

Models      : 1+
Time        : 4.054s (Solving: 3.07s 1st Model: 3.07s Unsat: 0.00s)
CPU Time    : 3.810s
Choices     : 438
Conflicts   : 318
Restarts    : 2

Variables   : 129328
Constraints : 504573
```

Two Orders of Magnitude!

hitori_5.lp

```
% Define mutual reachability
reachable(X1,Y1,X2,Y2) :- adjacent(X1,Y1,X2,Y2), (X1,Y1) < (X2,Y2),
                          not blackOut(X1,Y1), not blackOut(X2,Y2).
reachable(X1,Y1,X3,Y3) :- reachable(X1,Y1,X2,Y2), reachable(X2,Y2,X3,Y3).
reachable(X1,Y1,X3,Y3) :- reachable(X1,Y1,X2,Y2), reachable(X3,Y3,X2,Y2),
                          (X1,Y1) < (X3,Y3).
reachable(X2,Y2,X3,Y3) :- reachable(X1,Y1,X2,Y2), reachable(X1,Y1,X3,Y3),
                          (X2,Y2) < (X3,Y3).
```

☞ grounding size: $O(8^6)$

Two Orders of Magnitude!

hitori_5.lp

```
% Define mutual reachability
reachable(X1,Y1,X2,Y2) :- adjacent(X1,Y1,X2,Y2), (X1,Y1) < (X2,Y2),
                          not blackOut(X1,Y1), not blackOut(X2,Y2).
reachable(X1,Y1,X3,Y3) :- reachable(X1,Y1,X2,Y2), reachable(X2,Y2,X3,Y3).
reachable(X1,Y1,X3,Y3) :- reachable(X1,Y1,X2,Y2), reachable(X3,Y3,X2,Y2),
                          (X1,Y1) < (X3,Y3).
reachable(X2,Y2,X3,Y3) :- reachable(X1,Y1,X2,Y2), reachable(X1,Y1,X3,Y3),
                          (X2,Y2) < (X3,Y3).
```

☞ grounding size: $O(8^6)$

Two Orders of Magnitude!

hitori_6.lp

```
% Define mutual reachability
reachable(X1,Y1,X2,Y2) :- adjacent(X1,Y1,X2,Y2), (X1,Y1) < (X2,Y2),
                          not blackOut(X1,Y1), not blackOut(X2,Y2).

reachable(X1,Y1,X3,Y3) :- reachable(X1,Y1,X2,Y2), adjacent(X2,Y2,X3,Y3),
                          (X1,Y1) < (X3,Y3), not blackOut(X3,Y3).
reachable(X2,Y2,X3,Y3) :- reachable(X1,Y1,X2,Y2), adjacent(X1,Y1,X3,Y3),
                          (X2,Y2) < (X3,Y3), not blackOut(X3,Y3).
```

☞ grounding size: $O(8^6)$

Two Orders of Magnitude!

hitori_6.lp

```
% Define mutual reachability
reachable(X1,Y1,X2,Y2) :- adjacent(X1,Y1,X2,Y2), (X1,Y1) < (X2,Y2),
                          not blackOut(X1,Y1), not blackOut(X2,Y2).

reachable(X1,Y1,X3,Y3) :- reachable(X1,Y1,X2,Y2), adjacent(X2,Y2,X3,Y3),
                          (X1,Y1) < (X3,Y3), not blackOut(X3,Y3).
reachable(X2,Y2,X3,Y3) :- reachable(X1,Y1,X2,Y2), adjacent(X1,Y1,X3,Y3),
                          (X2,Y2) < (X3,Y3), not blackOut(X3,Y3).
```

☞ grounding size: $O(8^4)$

A First Breakthrough

```
gringo hitori_5.lp instance.lp | clasp --stats
```

```
Answer: 1
blackOut(1,1) blackOut(1,3) blackOut(1,6) blackOut(2,5)
blackOut(2,7) ...           blackOut(8,4) blackOut(8,6)
SATISFIABLE

Models      : 1+
Time        : 4.054s (Solving: 3.07s 1st Model: 3.07s Unsat: 0.00s)
CPU Time    : 3.810s
Choices     : 438
Conflicts   : 318
Restarts    : 2

Variables   : 129328
Constraints : 504573
```

A First Breakthrough

```
gringo hitori_6.lp instance.lp | clasp --stats
```

```
Answer: 1
blackOut(1,1) blackOut(1,3) blackOut(1,6) blackOut(2,5)
blackOut(2,7) ...           blackOut(8,4) blackOut(8,6)
SATISFIABLE

Models      : 1+
Time        : 4.054s (Solving: 3.07s 1st Model: 3.07s Unsat: 0.00s)
CPU Time    : 3.810s
Choices     : 438
Conflicts   : 318
Restarts    : 2

Variables   : 129328
Constraints : 504573
```

A First Breakthrough

```
gringo hitori_6.lp instance.lp | clasp --stats
```

```
Answer: 1
blackOut(1,1) blackOut(1,3) blackOut(1,6) blackOut(2,5)
blackOut(2,7) ...           blackOut(8,4) blackOut(8,6)
SATISFIABLE

Models      : 1+
Time        : 0.093s (Solving: 0.01s 1st Model: 0.01s Unsat: 0.00s)
CPU Time    : 0.040s
Choices     : 64
Conflicts   : 23
Restarts    : 0

Variables   : 11231
Constraints : 32234
```

Let's Think a Bit More

hitori_6.lp

```
reachable(X1,Y1,X2,Y2) :- adjacent(X1,Y1,X2,Y2), (X1,Y1) < (X2,Y2),
                          not blackOut(X1,Y1), not blackOut(X2,Y2).
reachable(X1,Y1,X3,Y3) :- reachable(X1,Y1,X2,Y2), adjacent(X2,Y2,X3,Y3),
                          (X1,Y1) < (X3,Y3), not blackOut(X3,Y3).
reachable(X2,Y2,X3,Y3) :- reachable(X1,Y1,X2,Y2), adjacent(X1,Y1,X3,Y3),
                          (X2,Y2) < (X3,Y3), not blackOut(X3,Y3).

% Can't have unreachable non-black square
 :- not blackOut(X1,Y1), not blackOut(X2,Y2), not reachable(X1,Y1,X2,Y2),
    (X1,Y1) < (X2,Y2),   state(X1,Y1,_), state(X2,Y2,_).
```

Q: How many squares adjacent to `(1,1)` can possibly be black?

A:

	8		6	3	2		7
3	6	7	2	1		5	4
	3	4		2	8	6	1
4	1		5	7		3	
7		3		8	5	1	2
	5	6	7		1	8	
6		2	3	5	4	7	8
8	7	1	4		3		6

Let's Think a Bit More

hitori_6.lp

```
reachable(X1,Y1,X2,Y2) :- adjacent(X1,Y1,X2,Y2), (X1,Y1) < (X2,Y2),
                          not blackOut(X1,Y1), not blackOut(X2,Y2).
reachable(X1,Y1,X3,Y3) :- reachable(X1,Y1,X2,Y2), adjacent(X2,Y2,X3,Y3),
                          (X1,Y1) < (X3,Y3), not blackOut(X3,Y3).
reachable(X2,Y2,X3,Y3) :- reachable(X1,Y1,X2,Y2), adjacent(X1,Y1,X3,Y3),
                          (X2,Y2) < (X3,Y3), not blackOut(X3,Y3).

% Can't have unreachable non-black square
 :- not blackOut(X1,Y1), not blackOut(X2,Y2), not reachable(X1,Y1,X2,Y2),
    (X1,Y1) < (X2,Y2),   state(X1,Y1,_), state(X2,Y2,_).
```

Q: How many squares adjacent to (1,1) can possibly be black?

A: At most one!

	8		6	3	2		7
3	6	7	2	1		5	4
	3	4		2	8	6	1
4	1		5	7		3	
7		3		8	5	1	2
	5	6	7		1	8	
6		2	3	5	4	7	8
8	7	1	4		3		6

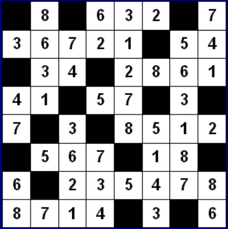
Let's Think a Bit More

hitori_6.lp

```
reachable(1,1).
reachable(X2,Y2) :- reachable(X1,Y1), adjacent(X1,Y1,X2,Y2), not blackOut(X2,Y2).




% Can't have unreachable non-black square
 :- not blackOut(X1,Y1), not blackOut(X2,Y2), not reachable(X1,Y1,X2,Y2),
    (X1,Y1) < (X2,Y2),   state(X1,Y1,_), state(X2,Y2,_).
```



Q: How many squares adjacent to (1,1) can possibly be black?

A: At most one!

Let's Think a Bit More

hitori_7.lp

```
reachable(1,1).
reachable(X2,Y2) :- reachable(X1,Y1), adjacent(X1,Y1,X2,Y2), not blackOut(X2,Y2).




% Can't have unreachable non-black square
 :- state(X,Y,_), not blackOut(X,Y), not reachable(X,Y).
```

Q: How many squares adjacent to `(1,1)` can possibly be black?

A: At most one!

	8		6	3	2		7
3	6	7	2	1		5	4
	3	4		2	8	6	1
4	1		5	7		3	
7		3		8	5	1	2
	5	6	7		1	8	
6		2	3	5	4	7	8
8	7	1	4		3		6

Not That Much Left to Save

```
gringo hitori_6.lp instance.lp | clasp --stats
```

```
Answer: 1
blackOut(1,1) blackOut(1,3) blackOut(1,6) blackOut(2,5)
blackOut(2,7) ...           blackOut(8,4) blackOut(8,6)
SATISFIABLE

Models      : 1+
Time        : 0.093s (Solving: 0.01s 1st Model: 0.01s Unsat: 0.00s)
CPU Time    : 0.040s
Choices     : 64
Conflicts   : 23
Restarts    : 0

Variables   : 11231
Constraints : 32234
```

Not That Much Left to Save

```
gringo hitori_7.lp instance.lp | clasp --stats
```

```
Answer: 1
blackOut(1,1) blackOut(1,3) blackOut(1,6) blackOut(2,5)
blackOut(2,7) ...           blackOut(8,4) blackOut(8,6)
SATISFIABLE

Models      : 1+
Time        : 0.093s (Solving: 0.01s 1st Model: 0.01s Unsat: 0.00s)
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Choices     : 64
Conflicts   : 23
Restarts    : 0

Variables   : 11231
Constraints : 32234
```

Not That Much Left to Save

```
gringo hitori_7.lp instance.lp | clasp --stats
```

```
Answer: 1
blackOut(1,1) blackOut(1,3) blackOut(1,6) blackOut(2,5)
blackOut(2,7) ...           blackOut(8,4) blackOut(8,6)
SATISFIABLE

Models      : 1+
Time        : 0.009s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.000s
Choices     : 77
Conflicts   : 25
Restarts    : 0

Variables   : 539
Constraints : 1137
```

Let's Reach All Squares (Anyway)

hitori_7.lp

```
% Define reachability
reachable(1,1).
reachable(X2,Y2) :- reachable(X1,Y1), adjacent(X1,Y1,X2,Y2), not blackOut(X2,Y2).

% Can't have unreachable non-black square
 :- state(X,Y,_), not blackOut(X,Y), not reachable(X,Y).
```

☞ require all white squares to be reached

Let's Reach All Squares (Anyway)

hitori_7.lp

```
% Define reachability
reachable(1,1). reachable(1,2).
reachable(X2,Y2) :- reachable(X1,Y1), adjacent(X1,Y1,X2,Y2), not blackOut(X1,Y1).

% Can't have unreachable non-black square
 :- state(X,Y,_), not blackOut(X,Y), not reachable(X,Y).
```

☞ require all white squares to be reached

Let's Reach All Squares (Anyway)

hitori_8.lp

```
% Define reachability
reachable(1,1). reachable(1,2).
reachable(X2,Y2) :- reachable(X1,Y1), adjacent(X1,Y1,X2,Y2), not blackOut(X1,Y1).

% Can't have unreachable square
 :- state(X,Y,_), not reachable(X,Y).
```

☞ require all white squares to be reached

The Final Result

```
gringo hitori_7.lp instance.lp | clasp --stats
```

```
Answer: 1
blackOut(1,1) blackOut(1,3) blackOut(1,6) blackOut(2,5)
blackOut(2,7) ...           blackOut(8,4) blackOut(8,6)
SATISFIABLE

Models      : 1+
Time        : 0.009s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.000s
Choices     : 77
Conflicts   : 25
Restarts    : 0

Variables   : 539
Constraints : 1137
```

The Final Result

```
gringo hitori_8.lp instance.lp | clasp --stats
```

```
Answer: 1
blackOut(1,1) blackOut(1,3) blackOut(1,6) blackOut(2,5)
blackOut(2,7) ...           blackOut(8,4) blackOut(8,6)
SATISFIABLE

Models      : 1+
Time        : 0.009s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.000s
Choices     : 77
Conflicts   : 25
Restarts    : 0

Variables   : 539
Constraints : 1137
```

The Final Result

```
gringo hitori_8.lp instance.lp | clasp --stats

Answer: 1
blackOut(1,1) blackOut(1,3) blackOut(1,6) blackOut(2,5)
blackOut(2,7) ...           blackOut(8,4) blackOut(8,6)
SATISFIABLE

Models      : 1+
Time        : 0.006s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.000s
Choices     : 16
Conflicts   : 5
Restarts    : 0

Variables   : 317
Constraints : 315
```

The Final Encoding (Pretty-Printed) I

hitori_9.lp

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% (A) Adjacent grid locations %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Domain predicate (evaluated upon grounding)
adjacent(X,Y,X+1,Y) :- state(X,Y,_;;X+1,Y,_).
adjacent(X,Y,X,Y+1) :- state(X,Y,_;;X,Y+1,_).
adjacent(X2,Y2,X1,Y1) :- adjacent(X1,Y1,X2,Y2).

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% (B) Generate solution candidate %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Every square is blacked out or normal
{ blackOut(X,Y) } :- state(X,Y,_).
```

The Final Encoding (Pretty-Printed) II

hitori_9.lp

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% (C.1) Test eliminating adjacent blanks %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Can't have adjacent black squares
 :- adjacent(X1,Y1,X2,Y2), blackOut(X1,Y1;;X2,Y2).

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% (C.2) Tests eliminating number recurrences %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Can't have the same number twice in the same row or column
 :- state(X1,Y1,N), 2 { not blackOut(X1,Y2) : state(X1,Y2,N) }.
 :- state(X1,Y1,N), 2 { not blackOut(X2,Y1) : state(X2,Y1,N) }.
```

The Final Encoding (Pretty-Printed) III

hitori_9.lp

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% (C.3) Test eliminating disconnected numbers %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Define reachability
reachable(1,1).
reachable(1,2).
reachable(X2,Y2) :- reachable(X1,Y1), adjacent(X1,Y1,X2,Y2),
                    not blackOut(X1,Y1).

% Can't have unreachable square
 :- state(X,Y,_), not reachable(X,Y).
```

Recall Where We Started

```
gringo hitori_0.lp instance.lp | clasp --stats

Answer: 1
blackOut(1,1) blackOut(1,3) blackOut(1,6) blackOut(2,5)
blackOut(2,7) ...           blackOut(8,4) blackOut(8,6)
SATISFIABLE

Models      : 1+
Time        : 13.485s (Solving: 11.77s 1st Model: 11.77s Unsat: 0.00s)
CPU Time    : 13.290s
Choices     : 458
Conflicts   : 323
Restarts    : 2

Variables   : 260625
Constraints : 1018953
```

And Where We Came

```
gringo hitori_9.lp instance.lp | clasp --stats
```

```
Answer: 1
blackOut(1,1) blackOut(1,3) blackOut(1,6) blackOut(2,5)
blackOut(2,7) ...           blackOut(8,4) blackOut(8,6)
SATISFIABLE

Models      : 1+
Time        : 0.006s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.000s
Choices     : 16
Conflicts   : 5
Restarts    : 0

Variables   : 317
Constraints : 315
```

And Where We Came

```
gringo hitori_9.lp instance.lp | clasp --stats
```

```
Answer: 1
blackOut(1,1) blackOut(1,3) blackOut(1,6) blackOut(2,5)
blackOut(2,7) ...           blackOut(8,4) blackOut(8,6)
SATISFIABLE

Models      : 1+
Time        : 0.006s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.000s
Choices     : 16
Conflicts   : 5
Restarts    : 0

Variables   : 317
Constraints : 315
```

The encoding matters!

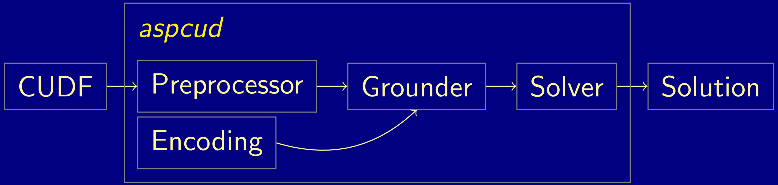
Some Real-World Applications
Overview

Motivation

- difficulties in maintaining packages of modern Linux distributions
    - complex dependencies
    - large package repositories
    - ever changing in view of software development
- challenges for package configuration tools
    - large problem size
    - soft (and hard) constraints
    - multiple optimization criteria
- advantages of ASP
    - uniform modeling by encoding plus instance(s)
    - solving techniques for multi-criteria optimization

Overview

aspcud tool for solving package configuration problems



Preprocessor converts CUDF input to ASP instance

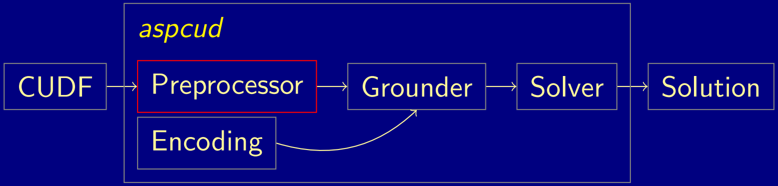
Encoding first-order problem specification

Grounder instantiates first-order variables

Solver searches for (optimal) answer sets

Overview

aspcud tool for solving package configuration problems



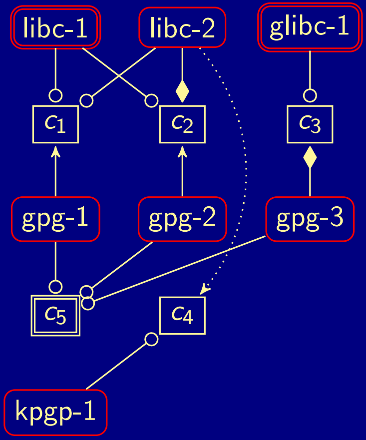
Preprocessor converts CUDF input to ASP instance

Encoding first-order problem specification

Grounder instantiates first-order variables

Solver searches for (optimal) answer sets

Instance Format



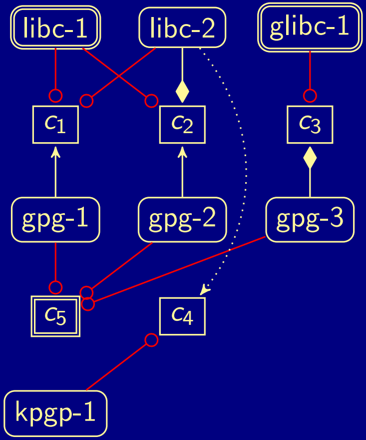
Installable Packages:

```
package(libc,1).
package(libc,2).

package(glibc,1).

package(gpg,1).
package(gpg,2).
package(gpg,3).

package(kpgp,1).
```



Instance Format

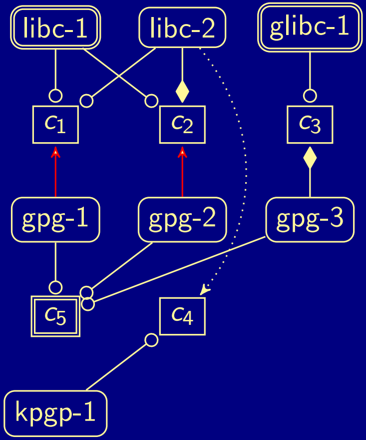
Package Clauses:

```
satisfies(libc,1,c1).
satisfies(libc,1,c2).
satisfies(libc,2,c1).

satisfies(glibc,1,c3).

satisfies(gpg,1,c5).
satisfies(gpg,2,c5).
satisfies(gpg,3,c5).

satisfies(kpgp,1,c4).
```

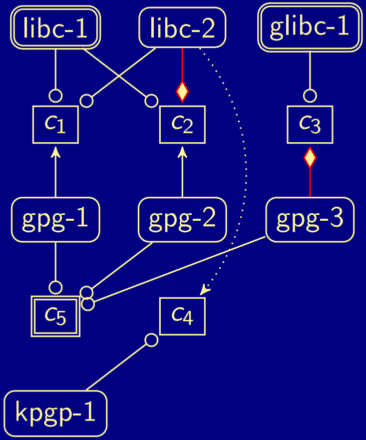


Instance Format

Package Dependencies:

```
depends(gpg,1,c1).
depends(gpg,2,c2).
```

Instance Format

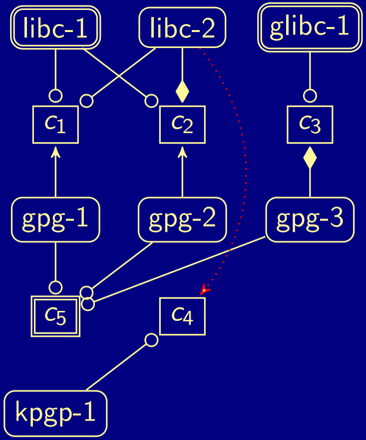


Package Conflicts:

```
conflicts(libc,2,c2).

conflicts(gpg,3,c3).
```

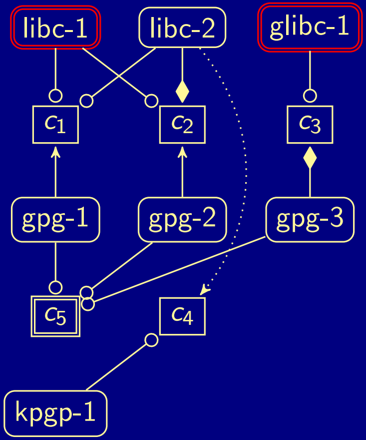
Instance Format



Package Recommendations:

```
recommends(libc,2,c4).
```

Instance Format

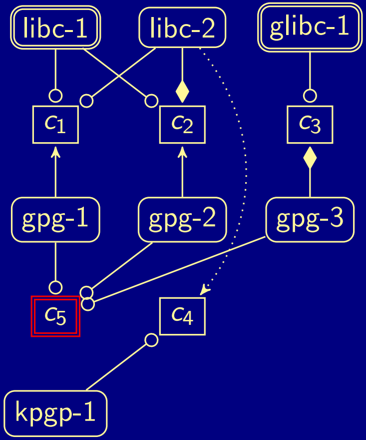


Installed Packages:

```
installed(libc,1).

installed(glibc,1).
```

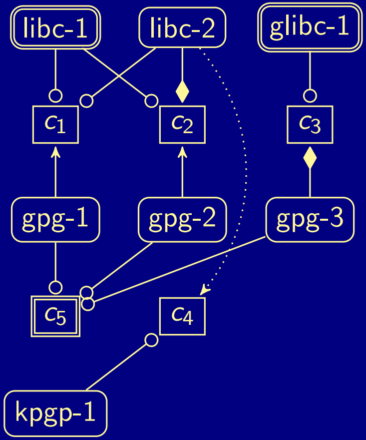
Instance Format



Requests:

```
requested(c5).
```

Instance Format

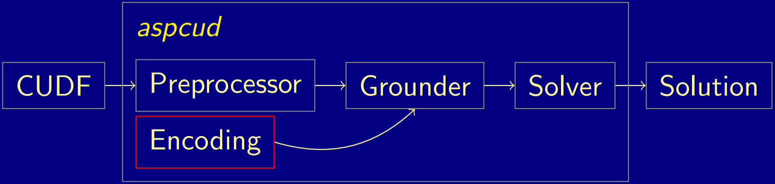


Optimization Criteria:

```
utility(delete,-1).
utility(change,-2).
```

Overview

aspcud tool for solving package configuration problems



Preprocessor converts CUDF input to ASP instance

Encoding first-order problem specification

Grounder instantiates first-order variables

Solver searches for (optimal) answer sets

Hard Constraints

```
% choose packages to install
{ install(N,V) } :- package(N,V).

% derive required clauses
exclude(C) :- install(N,V), conflicts(N,V,C).
include(C) :- install(N,V), depends(N,V,C).
% derive satisfied clauses
satisfy(C) :- install(N,V), satisfies(N,V,C).

% assert required clauses to be (un)satisfied
 :- exclude(C),     satisfy(C).
 :- include(C), not satisfy(C).
 :- request(C), not satisfy(C).
```

Hard Constraints

```
% choose packages to install
{ install(N,V) } :- package(N,V).

% derive required clauses
exclude(C) :- install(N,V), conflicts(N,V,C).
include(C) :- install(N,V), depends(N,V,C).
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 :- exclude(C),     satisfy(C).
 :- include(C), not satisfy(C).
 :- request(C), not satisfy(C).
```

Hard Constraints

```
% choose packages to install
{ install(N,V) } :- package(N,V).

% derive required clauses
exclude(C) :- install(N,V), conflicts(N,V,C).
include(C) :- install(N,V), depends(N,V,C).
% derive satisfied clauses
satisfy(C) :- install(N,V), satisfies(N,V,C).

% assert required clauses to be (un)satisfied
 :- exclude(C),     satisfy(C).
 :- include(C), not satisfy(C).
 :- request(C), not satisfy(C).
```

"Redundant" Hard Constraints

```
% lift package interdependencies (applying to all version)
pconflicts(N,C) :-  conflicts(N,V,C).
 conflicts(N,C) :- pconflicts(N,  C), conflicts(N,V,C) : package(N,V).

pdepends(N,C)   :-  depends(N,V,C).
 depends(N,C)   :- pdepends(N,  C),     depends(N,V,C) : package(N,V).

psatisfies(N,C) :-  satisfies(N,V,C).
 satisfies(N,C) :- psatisfies(N,  C), satisfies(N,V,C) : package(N,V).

% lifted derivations of required and satisfied clauses
install(N) :- install(N,V).

exclude(C) :- install(N), conflicts(N,C).
include(C) :- install(N), depends(N,C).

satisfy(C) :- install(N), satisfies(N,C).
```

"Redundant" Hard Constraints

```
% lift package interdependencies (applying to all version)
pconflicts(N,C) :-  conflicts(N,V,C).
 conflicts(N,C) :- pconflicts(N,  C), conflicts(N,V,C) : package(N,V).

pdepends(N,C)   :-  depends(N,V,C).
 depends(N,C)   :- pdepends(N,  C),     depends(N,V,C) : package(N,V).

psatisfies(N,C) :-  satisfies(N,V,C).
 satisfies(N,C) :- psatisfies(N,  C), satisfies(N,V,C) : package(N,V).

% lifted derivations of required and satisfied clauses
install(N) :- install(N,V).

exclude(C) :- install(N), conflicts(N,C).
include(C) :- install(N), depends(N,C).

satisfy(C) :- install(N), satisfies(N,C).
```

Soft Constraints

```
% auxiliary definition
installed(N) :- installed(N,V).

% derive optimization criteria violations
violate(newpkg,N) :-
    utility(newpkg,L), install(N), not installed(N).
violate(delete,N) :-
    utility(delete,L), installed(N), not install(N).
% similar for other criteria
...

% impose soft constraints
#minimize[ violate(U,T) = 1 @ -L : utility(U,L) : L < 0 ].
#maximize[ violate(U,T) = 1 @  L : utility(U,L) : L > 0 ].
```

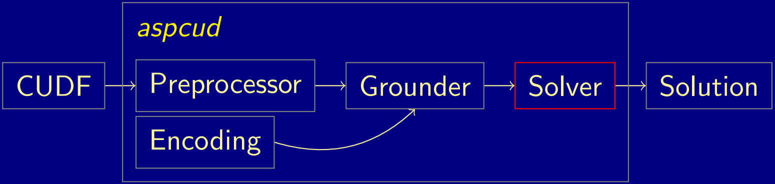
Soft Constraints

```
% auxiliary definition
installed(N) :- installed(N,V).

% derive optimization criteria violations
violate(newpkg,N) :-
    utility(newpkg,L), install(N), not installed(N).
violate(delete,N) :-
    utility(delete,L), installed(N), not install(N).
% similar for other criteria
...

% impose soft constraints
#minimize[ violate(U,T) = 1 @ -L : utility(U,L) : L < 0 ].
#maximize[ violate(U,T) = 1 @  L : utility(U,L) : L > 0 ].
```

Optimization Algorithm



- package configuration problems often under-constrained
- lexicographical optimization algorithm enumerates too much

Alternative Approach

- optimize criteria in the order of significance
- decrease upper bounds (costs) w.r.t. witnesses
- proceed to next criterion upon unsatisfiability

Design Goals

- incorporate into conflict-driven solving
- keep as much learned information as possible
- build upon standard features like assumptions

Experimental Results

- optimization of (Debian) Linux installations wrt. multiple criteria
- approaches of participants include
  1. Maximum Satisfiability: *cudf2msu*
  2. Pseudo-Boolean Optimization: *cudf2pbo*, *p2cudf*
  3. Answer Set Programming: *aspcud*
     - configurable optimization strategies and heuristics
- benchmarks and scoring from the 3rd MISC-live run (5 tracks)
  - MISC(-live) regularly organized by mancoosi consortium

	paranoid		trendy		user1		user2		user3	
Solver	S	T/O	S	T/O	S	T/O	S	T/O	S	T/O
$clasp_0^0$-r	431	2,287/6	1730	23,829/ 80	935	14,349/35	525	5,097/12	1031	14,184/37
$clasp_0^0$	416	2,294/6	2375	29,781/105	1727	21,897/73	1224	14,697/45	671	11,178/21
$clasp_0^1$-r	**410**	2,210/6	1560	22,660/ 73	898	13,466/30	502	4,654/ 9	980	13,682/35
$clasp_0^1$	**410**	2,326/6	2079	26,471/ 92	1723	21,525/72	922	10,767/31	658	10,675/23
$clasp_0^2$-r	427	2,135/6	712	16,867/ 51	527	5,891/11	426	2,981/ 5	587	7,628/20
$clasp_0^3$-r	429	**2,134**/6	740	17,079/ 52	507	5,863/12	425	3,044/ 6	576	7,769/21
$clasp_1^0$-r	425	2,428/6	579	16,713/ 50	550	5,819/14	434	3,000/ 6	710	8,958/25
$clasp_1^0$	417	2,418/6	549	16,544/ 50	475	5,318/12	411	2,538/ 5	502	6,279/16
$clasp_1^1$-r	429	2,405/6	622	17,304/ 50	518	5,908/13	438	2,976/ 6	676	8,938/23
$clasp_1^1$	427	2,372/6	613	16,946/ 49	490	5,478/12	416	2,562/ 5	496	6,144/16
$clasp_1^2$-r	427	2,352/6	571	16,646/ 50	518	5,358/13	418	2,582/ 5	471	6,356/16
$clasp_1^3$-r	429	2,346/6	547	16,386/ 50	499	5,306/12	413	2,498/ 5	497	6,255/16
$clasp_2^0$-r	425	2,392/6	806	16,598/ 50	523	5,583/13	421	2,677/ 6	479	5,548/12
$clasp_2^0$	417	2,364/7	748	17,132/ 50	487	5,823/14	422	2,583/ 5	482	5,592/15
$clasp_2^1$-r	416	2,378/6	752	17,269/ 52	492	5,663/12	414	2,409/ 5	451	5,349/11
$clasp_2^1$	425	2,365/6	864	17,128/ 51	517	6,151/15	412	2,681/ 5	463	5,972/14
$clasp_2^2$-r	445	2,402/6	706	16,551/ 50	528	5,788/13	419	2,700/ 5	436	5,519/13
$clasp_2^3$-r	434	2,345/6	748	16,982/ 51	518	5,850/14	415	2,559/ 5	457	5,360/13
cudf2msu	610	3,051/8	669	5,318/ 8	1270	8,709/18	548	3,238/ 7	504	4,750/ 9
cudf2pbo	465	**2,727**/7	1082	21,302/ 68	520	6,168/13	462	3,575/ 7	537	3,487/ 8
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	paranoid		trendy		user1		user2		user3	
Solver	*S*	*T/O*	*S*	*T/O*	*S*	*T/O*	*S*	*T/O*	*S*	*T/O*
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Regulatory Networks vs Experimental Profiles

Molecular Biology

- Repositories of biochemical reactions and genetic regulations
  - *Often established experimentally*
- High-throughput methods for collecting experimental profiles
  - *Often incompatible with biological knowledge*
- Incompatibilities due to unreliable data or missing reactions
  - *It is still a common practice to shift the task of making biological sense out of experimental profiles on human experts!*

Qualitative Approach

- Represent regulatory networks by influence graphs
- Represent experimental profiles by observed variations
- An experimental profile is consistent with a regulatory network **iff** each observed variation can be explained by some influence
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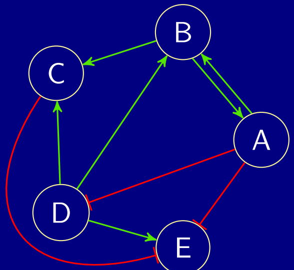
Influence Graphs

Vertices: genes, metabolites, proteins

Edges: regulations

- activation
- inhibition

Example:

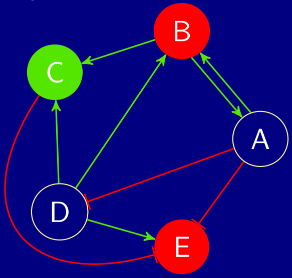


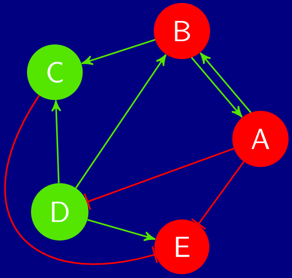
Observations

Labels: variations found in genetic profiles

- increase
- decrease

Examples:



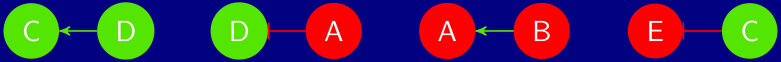


Note: Observations and regulation labelings can be partial

Sign Consistency Constraints (SCCs)

Local Consistency:

- A variation is consistent **iff** it is explained by some influence



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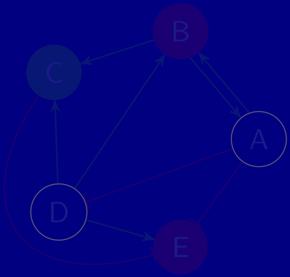

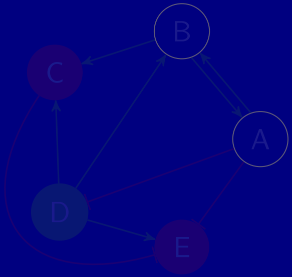

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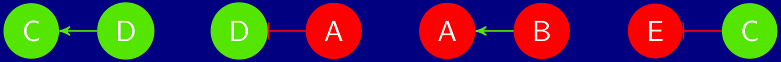

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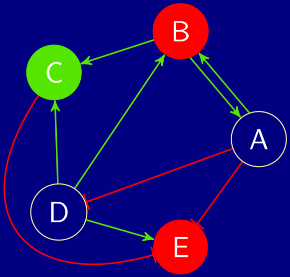

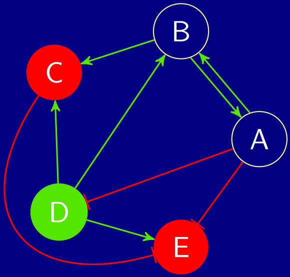

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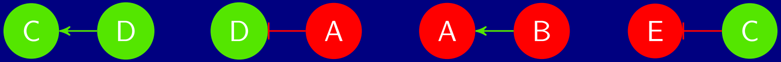

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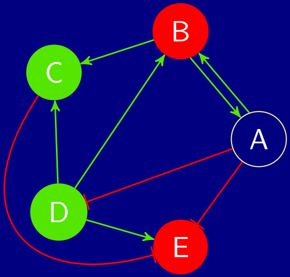

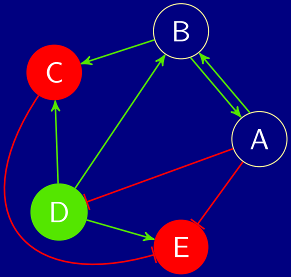

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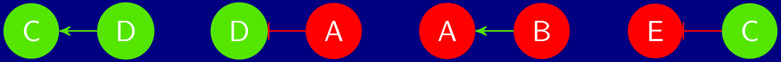

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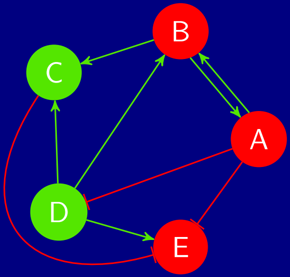

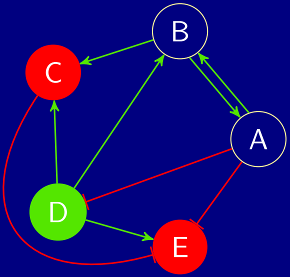

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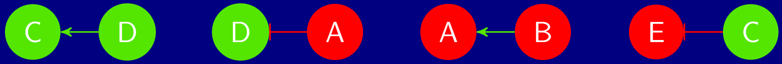

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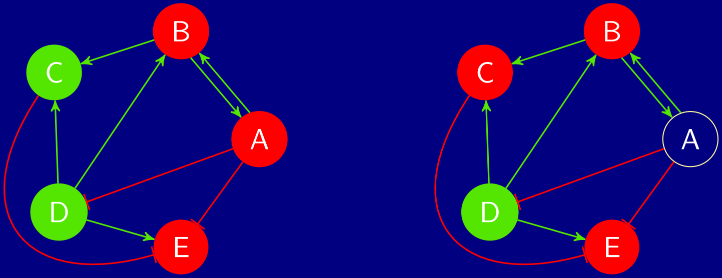

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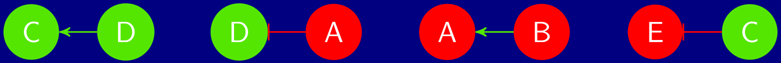

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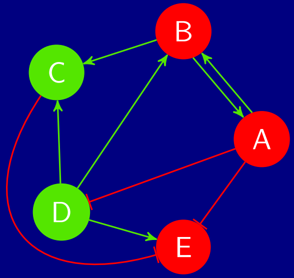
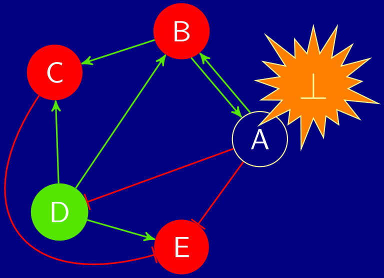
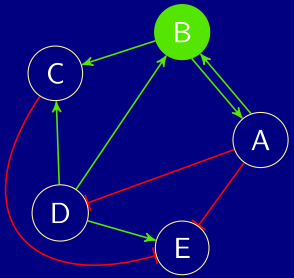




Predicting Variations
under Consistency

A partially labeled influence graph may admit several solutions.

Example:



Predicted Variations:

Predicting Variations
under Consistency

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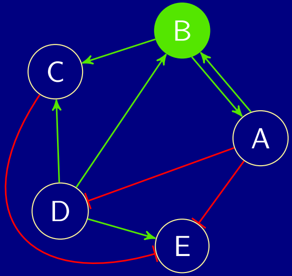

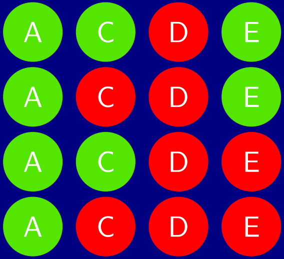

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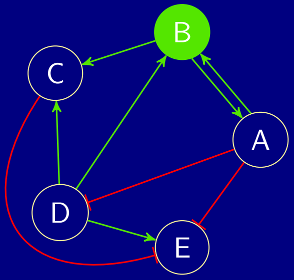

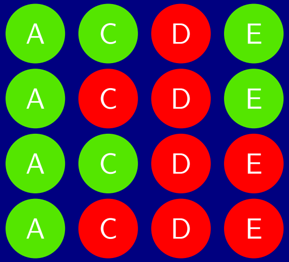

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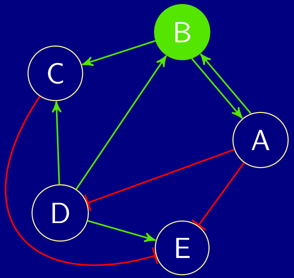

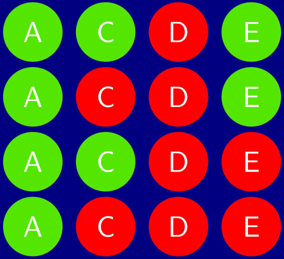

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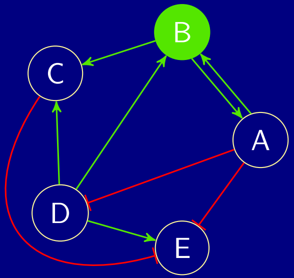

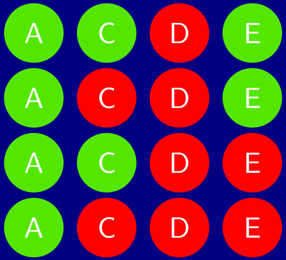

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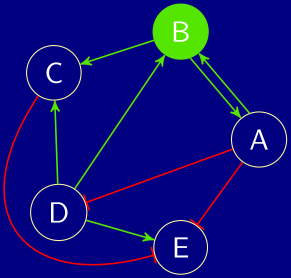

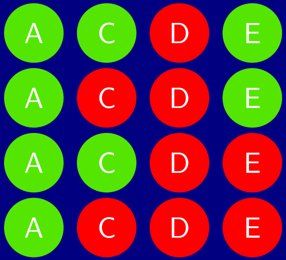

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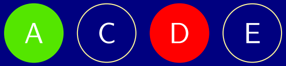

Influence Graphs and Variations

Vertices: *vertex*(i).

Edges: *edge*(j, i).

- — *observedE*(j, i, +1).
- — *observedE*(j, i, −1).

Variations:

- ● *observedV*(i, +1).
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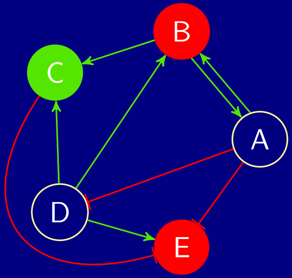

Example:

vertex(A). ... *vertex*(E).

edge(A, B). *edge*(A, D). ... *edge*(D, C). *edge*(D, E).

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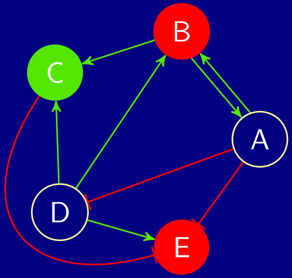

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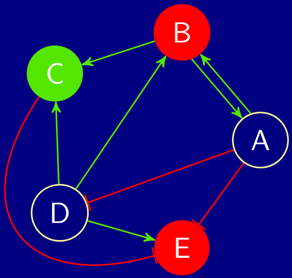

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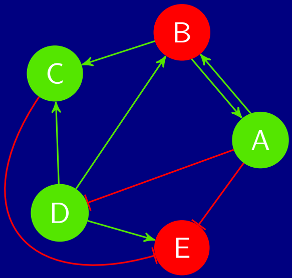

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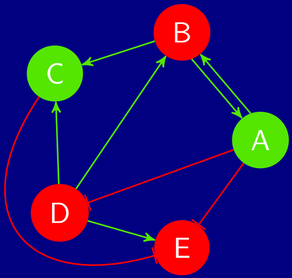

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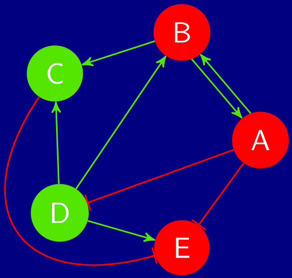

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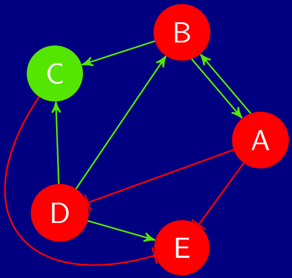

Testing Total Labelings

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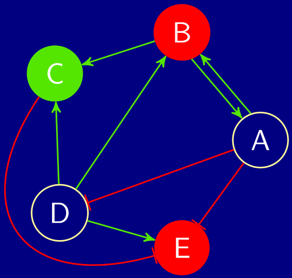

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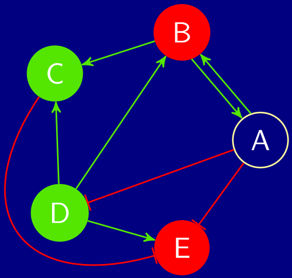

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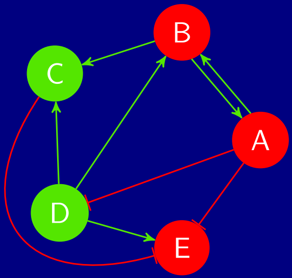

Motivation

Observation: Regulatory networks and experimental profiles are often inconsistent with each other!

Question: How to predict unobserved variations in this case?

Idea:

1. Repair inconsistencies
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Repairing Networks and/or Profiles

Network Repair:

- Adding edges completes an incomplete network (w.r.t. profiles)
- Flipping edge labels curates an improper network
- Making vertices input indicates incompleteness or oscillations

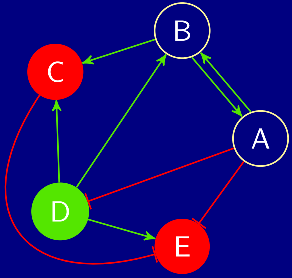
Profile Repair:

- Flipping vertex labels indicates aberrant experimental data

Repair Operations

Adding Edges

$$rep(add\_e(U, V)) \leftarrow vertex(U), vertex(V), U \neq V, not\ edge(U, V).$$



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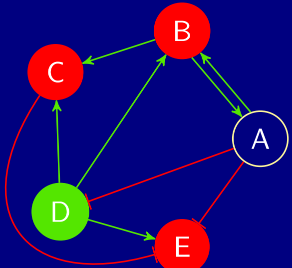

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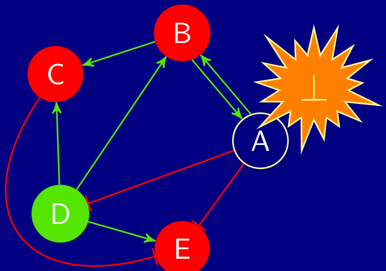

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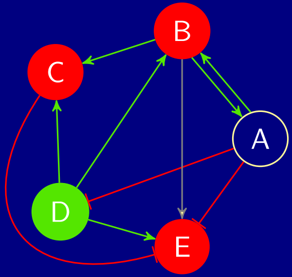

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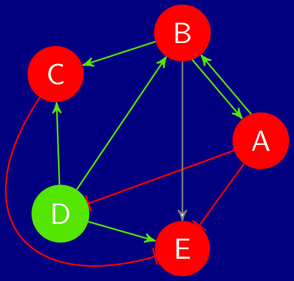
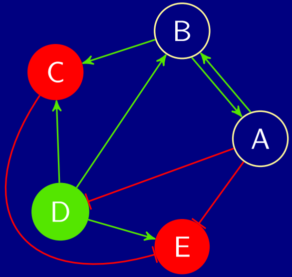


Repair Operations
Flipping Edge Labels

$$rep(flip\_e(U,V,S)) \leftarrow observedE(U,V,S).$$



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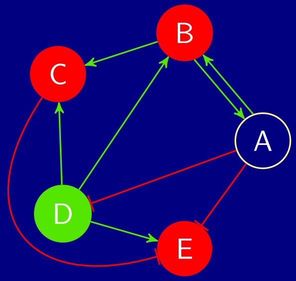

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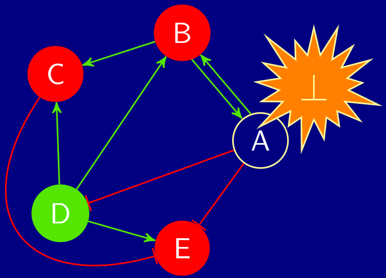

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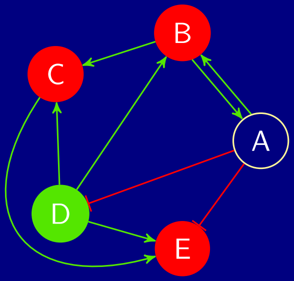

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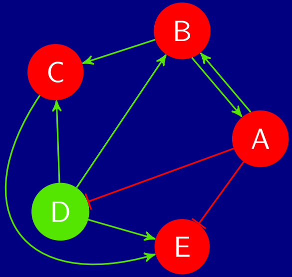

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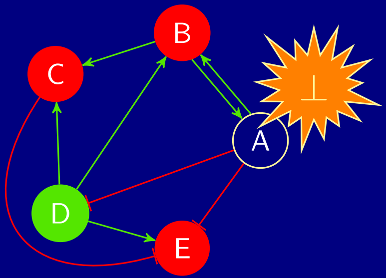

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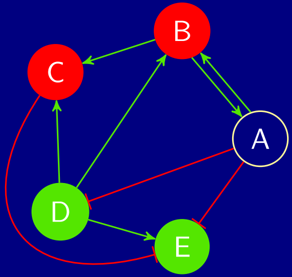

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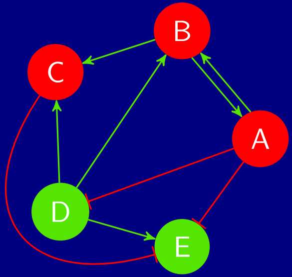
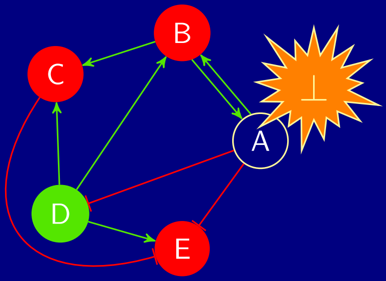


Repair Operations
Making Vertices Input

$$rep(inp\_v(V)) \leftarrow vertex(V), not\ input(V).$$



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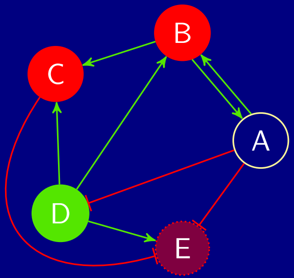

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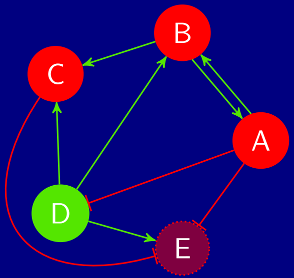

Generating Total Labelings under Repair

Applying Repair Operations:

$0\{app(R)\}1 \leftarrow rep(R).$

Generating Edge Labelings:

$1\{labelE(U,V,+1), labelE(U,V,-1)\}1 \leftarrow edge(U,V).$
$1\{labelE(U,V,+1), labelE(U,V,-1)\}1 \leftarrow app(add\_e(U,V)).$
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Testing Total Labelings under Repair

Enforcing Sign Consistency Constraints:

$$\begin{aligned} receive(I, S*T) &\leftarrow labelE(J, I, S), labelV(J, T). \\ &\leftarrow labelV(I, S), not\ receive(I, S), \\ &\quad\ not\ input(V), not\ app(inp\_v(V)). \end{aligned}$$

Minimal Repair

Goal:

Minimal change of networks/profiles
(re)establishing consistency

Implementation (cardinality minimality):

$$\#minimize\{app(R) : rep(R)\}.$$

- See KR'10 paper for disjunctive subset minimality encoding
- **NEW**(@ICLP'11): subset minimality via meta-programming

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Predicting under Repair

Two Phase Approach:

1. Compute minimal number of required repair operations
2. Intersect consistent labelings under minimal repair
   - Cautious reasoning (supported by answer set solver `clasp`)

Predicting Variations

under Inconsistency

- Transcriptional network of *Escherichia coli*, obtained from RegulonDB by Gama-Castro *et al.* [2008], consisting of
  - 5150 interactions between 1914 genes
- Two datasets
  - Exponential-Stationary growth shift by Bradley *et al.* [2007]
  - Heatshock by Allen *et al.* [2003]
- The data of both experiments is highly noisy and inconsistent with the (well-curated) RegulonDB model
- For enabling prediction rate and accuracy assessment, we randomly select samples of significantly expressed genes (3%,6%,9%,12%,15% of the whole data, 200 samples each) and use them for testing both our repair modes and prediction

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Repair and Prediction Times

	Repair			Exponential-Stationary 3%	6%	9%	12%	15%	Heatshock 3%	6%	9%	12%	15%
Repair	e			6.58	8.44	11.60	14.88	26.20	25.54	42.76	50.46	69.23	84.77
		i		2.18	2.15	2.21	2.23	2.21	2.10	2.13	2.13	2.05	2.08
			v	1.41	1.40	1.40	1.41	1.37	1.41	1.47	1.42	1.37	1.39
	e	i		73.16	202.66	392.97	518.50	574.85	120.91	374.69	553.00	593.20	595.99
	e		v	28.53	85.17	189.27	327.98	470.48	67.92	236.05	465.92	579.88	596.17
		i	v	2.09	2.14	2.45	3.08	6.06	2.27	4.94	60.63	257.68	418.93
	e	i	v	133.84	391.60	538.93	593.33	600.00	232.29	542.48	593.88	600.00	600.00
Prediction	e			13.27	12.19	14.76	15.34	25.90	25.77	37.18	29.09	36.23	41.88
		i		6.18	5.26	4.77	4.60	4.42	6.57	5.93	5.17	4.86	4.54
			v	4.64	4.45	4.39	4.40	4.30	4.86	5.06	5.34	5.42	5.52
	e	i		35.25	97.66	293.80	456.55	550.33	85.47	293.28	524.19	591.81	594.74
	e		v	14.35	26.17	90.17	200.25	363.36	23.32	111.99	338.95	545.56	591.23
		i	v	6.43	5.75	6.27	6.69	8.61	6.91	6.63	30.33	176.14	371.95
	e	i	v	42.51	248.30	468.71	579.58	—	101.82	466.91	585.64	—	—

'e': flipping edge labels 'i': making vertices input 'v': flipping vertex labels

Repair and Prediction Times

	Repair	Exponential-Stationary 3%	6%	9%	12%	15%	Heatshock 3%	6%	9%	12%	15%
Repair	e	6.58	8.44	11.60	14.88	26.20	25.54	42.76	50.46	69.23	84.77
	i	2.18	2.15	2.21	2.23	2.21	2.10	2.13	2.13	2.05	2.08
	v	1.41	1.40	1.40	1.41	1.37	1.41	1.47	1.42	1.37	1.39
	e i	73.16	202.66	392.97	518.50	574.85	120.91	374.69	553.00	593.20	595.99
	e v	28.53	85.17	189.27	327.98	470.48	67.92	236.05	465.92	579.88	596.17
	i v	2.09	2.14	2.45	3.08	6.06	2.27	4.94	60.63	257.68	418.93
	e i v	133.84	391.60	538.93	593.33	600.00	232.29	542.48	593.88	600.00	600.00
Prediction	e	13.27	12.19	14.76	15.34	25.90	25.77	37.18	29.09	36.23	41.88
	i	6.18	5.26	4.77	4.60	4.42	6.57	5.93	5.17	4.86	4.54
	v	4.64	4.45	4.39	4.40	4.30	4.86	5.06	5.34	5.42	5.52
	e i	35.25	97.66	293.80	456.55	550.33	85.47	293.28	524.19	591.81	594.74
	e v	14.35	26.17	90.17	200.25	363.36	23.32	111.99	338.95	545.56	591.23
	i v	6.43	5.75	6.27	6.69	8.61	6.91	6.63	30.33	176.14	371.95
	e i v	42.51	248.30	468.71	579.58	—	101.82	466.91	585.64	—	—

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Repair and Prediction Times

	Repair			Exponential-Stationary 3%	Exponential-Stationary 6%	Exponential-Stationary 9%	Exponential-Stationary 12%	Exponential-Stationary 15%	Heatshock 3%	Heatshock 6%	Heatshock 9%	Heatshock 12%	Heatshock 15%
Repair	e			6.58	8.44	11.60	14.88	26.20	25.54	42.76	50.46	69.23	84.77
Repair		i		2.18	2.15	2.21	2.23	2.21	2.10	2.13	2.13	2.05	2.08
Repair			v	1.41	1.40	1.40	1.41	1.37	1.41	1.47	1.42	1.37	1.39
Repair	e	i		73.16	202.66	392.97	518.50	574.85	120.91	374.69	553.00	593.20	595.99
Repair	e		v	28.53	85.17	189.27	327.98	470.48	67.92	236.05	465.92	579.88	596.17
Repair		i	v	2.09	2.14	2.45	3.08	6.06	2.27	4.94	60.63	257.68	418.93
Repair	e	i	v	133.84	391.60	538.93	593.33	600.00	232.29	542.48	593.88	600.00	600.00
Prediction	e			13.27	12.19	14.76	15.34	25.90	25.77	37.18	29.09	36.23	41.88
Prediction		i		6.18	5.26	4.77	4.60	4.42	6.57	5.93	5.17	4.86	4.54
Prediction			v	4.64	4.45	4.39	4.40	4.30	4.86	5.06	5.34	5.42	5.52
Prediction	e	i		35.25	97.66	293.80	456.55	550.33	85.47	293.28	524.19	591.81	594.74
Prediction	e		v	14.35	26.17	90.17	200.25	363.36	23.32	111.99	338.95	545.56	591.23
Prediction		i	v	6.43	5.75	6.27	6.69	8.61	6.91	6.63	30.33	176.14	371.95
Prediction	e	i	v	42.51	248.30	468.71	579.58	—	101.82	466.91	585.64	—	—

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Repair and Prediction Times

It's feasible!

				Exponential-Stationary					Heatshock				
Repair				3%	6%	9%	12%	15%	3%	6%	9%	12%	15%
Repair	e			6.58	8.44	11.60	14.88	26.20	25.54	42.76	50.46	69.23	84.77
		i		2.18	2.15	2.21	2.23	2.21	2.10	2.13	2.13	2.05	2.08
			v	1.41	1.40	1.40	1.41	1.37	1.41	1.47	1.42	1.37	1.39
	e	i		73.16	202.66	392.97	518.50	574.85	120.91	374.69	553.00	593.20	595.99
	e		v	28.53	85.17	189.27	327.98	470.48	67.92	236.05	465.92	579.88	596.17
		i	v	2.09	2.14	2.45	3.08	6.06	2.27	4.94	60.63	257.68	418.93
	e	i	v	133.84	391.60	538.93	593.33	600.00	232.29	542.48	593.88	600.00	600.00
Prediction	e			13.27	12.19	14.76	15.34	25.90	25.77	37.18	29.09	36.23	41.88
		i		6.18	5.26	4.77	4.60	4.42	6.57	5.93	5.17	4.86	4.54
			v	4.64	4.45	4.39	4.40	4.30	4.86	5.06	5.34	5.42	5.52
	e	i		35.25	97.66	293.80	456.55	550.33	85.47	293.28	524.19	591.81	594.74
	e		v	14.35	26.17	90.17	200.25	363.36	23.32	111.99	338.95	545.56	591.23
		i	v	6.43	5.75	6.27	6.69	8.61	6.91	6.63	30.33	176.14	371.95
	e	i	v	42.51	248.30	468.71	579.58	—	101.82	466.91	585.64	—	—

'e': flipping edge labels 'i': making vertices input 'v': flipping vertex labels

Prediction Rate and Accuracy in Percent

	Repair			Exponential-Stationary					Heatshock				
				3%	6%	9%	12%	15%	3%	6%	9%	12%	15%
Rate	e			15.00	18.51	20.93	22.79	23.94	15.47	19.54	21.87	23.17	24.78
Rate		i		15.00	18.51	20.93	22.79	23.93	15.48	19.62	21.89	23.20	24.80
Rate			v	14.90	18.37	20.86	22.73	23.77	15.32	19.59	21.37	22.13	23.79
Rate	e	i		14.92	18.61	20.55	21.96	22.80	15.37	19.62	22.83	23.44	24.05
Rate	e		v	14.89	18.33	21.07	22.52	23.74	15.33	19.21	21.00	22.65	24.90
Rate		i	v	14.89	18.33	20.79	22.59	23.66	15.41	19.47	21.36	21.81	23.55
Rate	e	i	v	14.58	19.00	20.29	21.13	—	15.01	19.11	22.52	—	—
Accuracy	e			90.93	91.98	92.42	92.70	92.81	91.87	92.93	92.92	92.83	92.71
Accuracy		i		90.93	91.98	92.42	92.70	92.81	91.93	92.90	92.94	92.87	92.76
Accuracy			v	90.99	92.05	92.44	92.73	92.89	92.29	93.27	93.88	94.27	94.36
Accuracy	e	i		91.09	91.90	92.57	93.03	93.19	91.99	92.49	91.16	93.62	94.44
Accuracy	e		v	90.99	92.03	92.50	92.82	92.94	92.30	93.37	93.66	94.36	94.35
Accuracy		i	v	90.99	92.03	92.42	92.71	92.87	92.24	93.34	93.90	94.26	94.38
Accuracy	e	i	v	91.35	92.29	92.52	93.04	—	92.26	93.04	91.78	—	—

'e': flipping edge labels 'i': making vertices input 'v': flipping vertex labels

Prediction Rate and Accuracy in Percent

	Repair	Exponential-Stationary 3%	6%	9%	12%	15%	Heatshock 3%	6%	9%	12%	15%
Rate	e	15.00	18.51	20.93	22.79	23.94	15.47	19.54	21.87	23.17	24.78
	i	15.00	18.51	20.93	22.79	23.93	15.48	19.62	21.89	23.20	24.80
	v	14.90	18.37	20.86	22.73	23.77	15.32	19.59	21.37	22.13	23.79
	e i	14.92	18.61	20.55	21.96	22.80	15.37	19.62	22.83	23.44	24.05
	e v	14.89	18.33	21.07	22.52	23.74	15.33	19.21	21.00	22.65	24.90
	i v	14.89	18.33	20.79	22.59	23.66	15.41	19.47	21.36	21.81	23.55
	e i v	14.58	19.00	20.29	21.13	—	15.01	19.11	22.52	—	—
Accuracy	e	90.93	91.98	92.42	92.70	92.81	91.87	92.93	92.92	92.83	92.71
	i	90.93	91.98	92.42	92.70	92.81	91.93	92.90	92.94	92.87	92.76
	v	90.99	92.05	92.44	92.73	92.89	92.29	93.27	93.88	94.27	94.36
	e i	91.09	91.90	92.57	93.03	93.19	91.99	92.49	91.16	93.62	94.44
	e v	90.99	92.03	92.50	92.82	92.94	92.30	93.37	93.66	94.36	94.35
	i v	90.99	92.03	92.42	92.71	92.87	92.24	93.34	93.90	94.26	94.38
	e i v	91.35	92.29	92.52	93.04	—	92.26	93.04	91.78	—	—

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Prediction Rate and Accuracy in Percent

	Repair			Exponential-Stationary					Heatshock				
				3%	6%	9%	12%	15%	3%	6%	9%	12%	15%
Rate	e			15.00	18.51	20.93	22.79	23.94	15.47	19.54	21.87	23.17	24.78
Rate		i		15.00	18.51	20.93	22.79	23.93	15.48	19.62	21.89	23.20	24.80
Rate			v	14.90	18.37	20.86	22.73	23.77	15.32	19.59	21.37	22.13	23.79
Rate	e	i		14.92	18.61	20.55	21.96	22.80	15.37	19.62	22.83	23.44	24.05
Rate	e		v	14.89	18.33	21.07	22.52	23.74	15.33	19.21	21.00	22.65	24.90
Rate		i	v	14.89	18.33	20.79	22.59	23.66	15.41	19.47	21.36	21.81	23.55
Rate	e	i	v	14.58	19.00	20.29	21.13	—	15.01	19.11	22.52	—	—
Accuracy	e			90.93	91.98	92.42	92.70	92.81	91.87	92.93	92.92	92.83	92.71
Accuracy		i		90.93	91.98	92.42	92.70	92.81	91.93	92.90	92.94	92.87	92.76
Accuracy			v	90.99	92.05	92.44	92.73	92.89	92.29	93.27	93.88	94.27	94.36
Accuracy	e	i		91.09	91.90	92.57	93.03	93.19	91.99	92.49	91.16	93.62	94.44
Accuracy	e		v	90.99	92.03	92.50	92.82	92.94	92.30	93.37	93.66	94.36	94.35
Accuracy		i	v	90.99	92.03	92.42	92.71	92.87	92.24	93.34	93.90	94.26	94.38
Accuracy	e	i	v	91.35	92.29	92.52	93.04	—	92.26	93.04	91.78	—	—

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Prediction Rate and Accuracy in Percent

	Repair			Exponential-Stationary 3%	6%	9%	12%	15%	Heatshock 3%	6%	9%	12%	15%
Rate	e			15.00	18.51	20.93	22.79	23.94	15.47	19.54	21.87	23.17	24.78
Rate		i		15.00	18.51	20.93	22.79	23.93	15.48	19.62	21.89	23.20	24.80
Rate			v	14.90	18.37	20.86	22.73	23.77	15.32	19.59	21.37	22.13	23.79
Rate	e	i		14.92	18.61	20.55	21.96	22.80	15.37	19.62	22.83	23.44	24.05
Rate	e		v	14.89	18.33	21.07	22.52	23.74	15.33	19.21	21.00	22.65	24.90
Rate		i	v	14.89	18.33	20.79	22.59	23.66	15.41	19.47	21.36	21.81	23.55
Rate	e	i	v	14.58	19.00	20.29	21.13	—	15.01	19.11	22.52	—	—
Accuracy	e			90.93	91.98	92.42	92.70	92.81	91.87	92.93	92.92	92.83	92.71
Accuracy		i		90.93	91.98	92.42	92.70	92.81	91.93	92.90	92.94	92.87	92.76
Accuracy			v	90.99	92.05	92.44	92.73	92.89	92.29	93.27	93.88	94.2[illegible]	94.36
Accuracy	e	i		91.09	91.90	92.57	93.03	93.19	91.99	92.49	91.16	93.[illegible]2	94.44
Accuracy	e		v	90.99	92.03	92.50	92.82	92.94	92.30	93.37	93.66	94.36	94.35
Accuracy		i	v	90.99	92.03	92.42	92.71	92.87	92.24	93.34	93.90	94.2[illegible]	94.38
Accuracy	e	i	v	91.35	92.29	92.52	93.04	—	92.26	93.0[illegible]	91.78	—	—

Accuracy over 90%!

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Subset-Minimal Repairs

Direct Encoding versus Meta-Programming

- 100 samples per repair mode
- 4,000 seconds time(out) per run

Repair	direct Σ time	direct Σ out	meta Σ time	meta Σ out
e	365,227	78	366,798	79
i	45,736	0	42,203	2
v	315,801	72	4,823	0

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	direct		**meta**	
Repair	Σ time	Σ out	Σ time	Σ out
e	365,227	78	366,798	79
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v	**315,801**	**72**	**4,823**	**0**

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Incremental Grounding and Solving
Overview

Motivation

Many real-world applications, having exponential state spaces, like

- bio-informatics,
- planning,
- model checking,
- etc.

have associated PSPACE-decision problems.

☞ For instance, the plan existence problem of deterministic planning is PSPACE-complete.

But the problem of whether there is a plan having a length bounded by a given polynomial is in NP.

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Motivation

State of the Art In ASP such problems are dealt with by iterative deepening search.
That is, considering one problem instance after another by gradually increasing the bound on the solution size.

Problem This approach
- is prone to redundancies in grounding and solving, and
- cannot harness modern look-back techniques regarding conflict-driven learning and heuristics.

Goal Avoiding redundancy by gradually processing the extensions to a problem rather than repeatedly re-processing the entire extended problem.

Proposal An incremental approach to both grounding and solving in ASP.

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Problem Specification

- A (parameterized) domain description is a triple (B, P, Q) of logic programs, among which P and Q contain a (single) parameter k ranging over the natural numbers.
  We sometimes denote P and Q by $P[k]$ and $Q[k]$.
- The base program B is meant to describe static knowledge, independent of parameter k.
  The role of P is to capture knowledge accumulating with increasing k, whereas Q is specific for each value of k.
- One goal is then to decide, for instance, whether the program

$$R[k/i] = B \cup \bigcup_{1 \leq j \leq i} P[k/j] \cup Q[k/i]$$

  has an answer set for some (minimum) integer $i \geq 1$.
  We write $R[i]$ rather than $R[k/i]$ whenever clear from the context.

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Grounding and Solving, classically

Input A domain description $R[k] = (B, P[k], Q[k])$.

Output A non-empty set of answer sets of $R[k/i]$, for instance.

1. Ground $B \cup P[1] \cup Q[1]$ and Solve $B \cup P[1] \cup Q[1]$
2. Ground $B \cup P[1] \cup P[2] \cup Q[2]$ and Solve $B \cup P[1] \cup P[2] \cup Q[2]$
3. Ground $B \cup P[1] \cup P[2] \cup P[3] \cup Q[3]$ and Solve $B \cup P[1] \cup P[2] \cup P[3] \cup Q[3]$

i. etc. until an answer set is obtained.

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3. Ground $B \cup P[1] \cup P[2] \cup P[3] \cup Q[3]$ and Solve $B \cup P[1] \cup P[2] \cup P[3] \cup Q[3]$

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Grounding and Solving, classically

Input A domain description $R[k] = (B, P[k], Q[k])$.

Output A non-empty set of answer sets of $R[k/i]$, for instance.

1. Ground $B \cup P[1] \cup Q[1]$ and Solve $B \cup P[1] \cup Q[1]$
2. Ground $B \cup P[1] \cup P[2] \cup Q[2]$ and Solve $B \cup P[1] \cup P[2] \cup Q[2]$
3. Ground $B \cup P[1] \cup P[2] \cup P[3] \cup Q[3]$ and
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i. etc. until an answer set is obtained.

Grounding and Solving, incrementally

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Output A non-empty set of answer sets of $R[k/i]$, for instance.

1. Ground B and Keep B
2. Ground $P[1] \cup Q[1]$, Solve $\underline{B} \cup P[1] \cup Q[1]$,
   Keep $B \cup P[1]$, and Discard $Q[1]$
3. Ground $P[2] \cup Q[2]$, Solve $\underline{B \cup P[1]} \cup P[2] \cup Q[2]$,
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An Example

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\left.\begin{array}{l}
a \textbf{ causes } p \\
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\end{array}\right\} \mapsto
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Module

A module $\mathbb{P}$ is a triple (P, I, O) consisting of

- a (ground) program P over $grd(\mathcal{A})$ and
- sets $I, O \subseteq grd(\mathcal{A})$ such that
  - $I \cap O = \emptyset$,
  - $atom(P) \subseteq I \cup O$, and
  - $head(P) \subseteq O$.

The elements of I and O are called input and output atoms,

- also denoted by $I(\mathbb{P})$ and $O(\mathbb{P})$, respectively;
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Recall: Ground Instantiation

- The ground instantiation of a program P is defined as

$$grd(P) = \{r\theta \mid r \in P, \theta : var(r) \to \mathcal{U}\} \text{ , where}$$
$$\mathcal{U} = \{t \in \mathcal{T} \mid var(t) = \emptyset\} \text{ .}$$

- Analogously, $grd(\mathcal{A}) = \{a \in \mathcal{A} \mid var(a) = \emptyset\}$.

- Note that the set $\mathcal{T}$ of terms includes the natural numbers !

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Formal Setting

- For a program P over $grd(\mathcal{A})$ and a set $X \subseteq grd(\mathcal{A})$,

$$P|_X = \{head(r) \leftarrow body^+(r) \cup L \mid \\ r \in P, body^+(r) \subseteq X, L = \{not\ c \mid c \in body^-(r) \cap X\}\} .$$

☞ $P|_X$ projects the bodies of rules in P to the atoms of X.

- For a program P over $\mathcal{A}$ and $I \subseteq grd(\mathcal{A})$, define $\mathbb{P}(I)$ as the module

$$(\, grd(P)|_Y, I, head(grd(P)|_X)\,) \ ,$$

where $X = I \cup head(grd(P))$ and $Y = I \cup head(grd(P)|_X)$.

- Let $\mathbb{P}(I) = (P', I, O)$. Then, we have

$$O \subseteq grd(\mathcal{A}) \quad \text{and} \quad atom(P') \subseteq I \cup O \ .$$

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A Simple Example

Consider

$$P[k] = \{\ p(k) \leftarrow p(Y), \mathit{not}\ p(2) \qquad p(k) \leftarrow p(2)\ \}$$

and note that $grd(P[1])$ is infinite !

For $P[1]$ and $I = \{\ p(0)\ \}$, we get the module

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Modular Domain Description

- Define the join of two modules $\mathbb{P}$ and $\mathbb{Q}$, $\mathbb{P} \sqcup \mathbb{Q}$, as the module

$$(P(\mathbb{P}) \cup P(\mathbb{Q}) , \; I(\mathbb{P}) \cup (I(\mathbb{Q}) \setminus O(\mathbb{P})) , \; O(\mathbb{P}) \cup O(\mathbb{Q})) ,$$

provided that $(I(\mathbb{P}) \cup O(\mathbb{P})) \cap O(\mathbb{Q}) = \emptyset$.
  - ☞ Recursion between two modules to be joined is disallowed.
  - ☞ Recursion is allowed within each module.
- A domain description $(B, P[k], Q[k])$ is modular, if the modules

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are defined for $i \geq 1$, where $\mathbb{P}_0 = \mathbb{B}(\emptyset)$.

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A Pragmatic Approach

A domain description $(B, P[k], Q[k])$ is modular, if

- atoms defined in B comprise dedicated predicates or 0 as argument,
- atoms defined in $P[k]$ comprise k as argument, and
- atoms defined in $Q[k]$ comprise dedicated predicates and k as argument.

The above conditions can be formalized as follows:

- $atom(grd(B)) \cap \left(\bigcup_{1 \leq i} head(grd(P[i] \cup Q[i]))\right) = \emptyset$,
- $\left(\bigcup_{1 \leq i} atom(grd(P[i]))\right) \cap \left(\bigcup_{1 \leq j} head(grd(Q[j]))\right) = \emptyset$,
- $atom(grd(P[i])) \cap \left(\bigcup_{i < j} head(grd(P[j]))\right) = \emptyset$ for all $1 \leq i$, and
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Incremental ASP Solving (made very easy)

See [28] for formal details!

Grounding For a program P over $\mathcal{A}$ and $I \subseteq grd(\mathcal{A})$, an incremental grounder is a partial function

$$\texttt{ground} : (P, I) \mapsto (P', O) \ ,$$

where P' is a program over $grd(\mathcal{A})$ and $O \subseteq grd(\mathcal{A})$.

Solving For programs R, R' over $grd(\mathcal{A})$ and a set L of literals over $grd(\mathcal{A})$, an incremental solver is a pair of total functions

$$\texttt{add} : R \mapsto R' \quad \text{and} \quad \texttt{solve} : L \mapsto \chi,$$

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Algorithm 4: `isolve`

Input : A domain description $(B, P[k], Q[k])$.
Output : A nonempty set of answer sets.
Internal : A grounder $\textsc{Grounder}$.
Internal : A solver $\textsc{Solver}$.

$i \leftarrow 0$
$(P_0, O) \leftarrow \textsc{Grounder}.\mathtt{ground}(B, \emptyset)$
$\textsc{Solver}.\mathtt{add}(P_0)$
loop
  $i \leftarrow i + 1$
  $(P_i, O_i) \leftarrow \textsc{Grounder}.\mathtt{ground}(P[i], O)$
  $\textsc{Solver}.\mathtt{add}(P_i)$
  $O \leftarrow O \cup O_i$
  $(Q_i, O'_i) \leftarrow \textsc{Grounder}.\mathtt{ground}(Q[i], O)$
  $\textsc{Solver}.\mathtt{add}(Q_i(\alpha_i) \cup \{\{\alpha_i\} \leftarrow\} \cup \{\leftarrow \alpha_{i-1}\})$
  $\mathcal{X} \leftarrow \textsc{Solver}.\mathtt{solve}(\{\alpha_i\})$
  if $\mathcal{X} \neq \emptyset$ **then return** $\{X \setminus \{\alpha_i\} \mid X \in \mathcal{X}\}$

Example Reloaded

$$
\left.\begin{array}{l}
a \textbf{ causes } p \\
\textbf{exogenous } a \\
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\end{array}\right\}
\mapsto
\left\{\begin{array}{rcl}
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Example Reloaded

i	Rules				L
0	B	$p(0)$	$\leftarrow$	$not\ \neg p(0)$	
		$\neg p(0)$	$\leftarrow$	$not\ p(0)$	
			$\leftarrow$	$p(0), \neg p(0)$	
1	$P[1]$	$a(1)$	$\leftarrow$	$not\ \neg a(1)$	
		$\neg a(1)$	$\leftarrow$	$not\ a(1)$	
		$p(1)$	$\leftarrow$	$a(1)$	
		$p(1)$	$\leftarrow$	$p(0), not\ \neg p(1)$	
		$\neg p(1)$	$\leftarrow$	$\neg p(0), not\ p(1)$	
			$\leftarrow$	$p(1), \neg p(1)$	
			$\leftarrow$	$a(1), \neg a(1)$	
	$Q[1](\alpha_1)$		$\leftarrow$	$not\ \neg p(0), \alpha_1$	α_1
			$\leftarrow$	$not\ p(1), \alpha_1$	
			$\leftarrow$	$not\ \neg a(1), \alpha_1$	
		$\{\alpha_1\}$	$\leftarrow$		
			$\leftarrow$	α_0	

Example Reloaded

i	Rules				L
0	B	$p(0)$	$\leftarrow$	$not\ \neg p(0)$	
		$\neg p(0)$	$\leftarrow$	$not\ p(0)$	
			$\leftarrow$	$p(0), \neg p(0)$	
1			$\vdots$		
2	$P[2]$	$a(2)$	$\leftarrow$	$not\ \neg a(2)$	
		$\neg a(2)$	$\leftarrow$	$not\ a(2)$	
		$p(2)$	$\leftarrow$	$a(2)$	
		$p(2)$	$\leftarrow$	$p(1), not\ \neg p(2)$	
		$\neg p(2)$	$\leftarrow$	$\neg p(1), not\ p(2)$	
			$\leftarrow$	$p(2), \neg p(2)$	
			$\leftarrow$	$a(2), \neg a(2)$	
	$Q[2](\alpha_2)$		$\leftarrow$	$not\ \neg p(0), \alpha_2$	α_2
			$\leftarrow$	$not\ p(2), \alpha_2$	
			$\leftarrow$	$not\ \neg a(2), \alpha_2$	
		$\{\alpha_2\}$	$\leftarrow$		
			$\leftarrow$	α_1	

Example with iclingo

incremental.lp

```
#base.
p(0)  :- not -p(0).
-p(0) :- not p(0).
:- p(0), -p(0).

#cumulative k.
a(k)  :- not -a(k).
-a(k) :- not a(k).
p(k)  :- a(k).
p(k)  :- p(k-1), not -p(k).
-p(k) :- -p(k-1), not p(k).
:- p(k), -p(k).
:- a(k), -a(k).

#volatile k.
:- not -p(0).
:- not p(k).
:- not -a(k).
```

```
$ iclingo -V[erbose] incremental.lp

iclingo version 2.0.2 (clasp 1.1.1)
Reading from incremental.lp...
=============== step 1 ===============
Grounding...
Preprocessing...
Solving...
=============== step 2 ===============
Grounding...
Preprocessing...
Solving...
Answer: 1
-p(0) a(1) p(1) -a(2) p(2)
=============== Summary ===============

Models      : 1
Total Steps : 2
Time        : 0.000
```

Experiments

We consider iclingo in four settings, keeping over successive solving steps

1. learned constraints,
2. learned constraints and heuristic values,
3. heuristic values only, and
4. neither.

We compare these variants with iterative deepening search using

- clingo, the direct combination of gringo and clasp via an internal interface, as well as
- gringo and clasp via a textual interface (using the output language of lparse).

Experiments

Name	n	*iclingo* (1)	*iclingo* (2)	*iclingo* (3)	*iclingo* (4)	*clingo*	*gringo*|*clasp*
Blocks-world	20	2.61	2.61	2.62	2.62	37.09	42.41
	25	6.78	6.84	6.80	6.80	124.35	138.68
	30	15.68	15.80	15.71	15.81	330.15	362.39
	35	32.43	32.36	32.29	32.31	753.90	821.96
	40	60.99	60.75	60.71	61.04	-	-
	Σ	118.49	118.36	118.13	118.58	2445.49	2565.44
Queens	80	19.46	65.83	39.98	47.79	144.28	153.61
	90	36.72	135.19	70.81	81.70	249.13	264.21
	100	49.25	227.69	111.99	128.62	409.69	431.23
	110	64.05	424.03	176.16	201.67	636.91	669.75
	120	99.54	612.76	274.29	354.00	958.34	1003.67
	Σ	269.02	1465.50	673.23	813.78	2398.35	2522.47

Experiments

Name	n	*iclingo (1)*	*iclingo (2)*	*iclingo (3)*	*iclingo (4)*	*clingo*	*gringo | clasp*
Sokoban	16	243.22	287.46	320.07	334.08	376.74	384.41
	12	26.50	37.55	50.61	28.19	27.83	28.43
	16	124.26	124.44	320.97	341.94	189.48	194.12
	16	135.72	164.70	128.66	183.74	120.60	123.57
	18	140.80	145.07	233.71	275.12	236.60	242.19
	16	26.86	40.60	29.41	27.88	45.94	47.04
	17	1165.67	906.00	734.44	730.09	887.26	904.75
	14	119.95	140.11	106.40	213.22	96.26	98.10
	14	35.42	42.74	58.79	46.81	70.16	71.81
	21	286.46	200.43	600.19	777.68	278.97	285.09
	17	120.33	140.44	139.19	156.85	171.01	174.90
	14	39.09	36.21	36.00	47.48	66.12	67.43
	Σ	2464.28	2265.75	2758.44	3163.08	2566.97	2621.84

Experiments

Name	n	*iclingo (1)*	*iclingo (2)*	*iclingo (3)*	*iclingo (4)*	*clingo*	*gringo* | *clasp*
Sokoban	16	-	-	-	-	-	-
back	12	51.23	44.62	98.09	57.42	72.59	74.30
	16	264.81	201.48	265.21	359.38	296.45	302.46
	16	148.19	121.19	150.06	145.40	148.25	151.43
	18	723.07	-	-	-	1059.02	1081.34
	16	243.81	185.00	340.97	190.32	402.27	410.72
	17	599.74	714.40	1051.60	825.61	-	-
	14	149.37	126.04	164.98	191.33	170.36	173.74
	14	29.73	69.46	73.03	28.04	43.06	43.89
	21	346.56	428.43	400.81	295.69	402.78	411.70
	17	181.00	143.20	172.83	317.82	234.21	239.56
	14	15.06	58.45	39.27	17.50	59.63	60.78
	Σ	3952.57	4492.27	5156.85	4828.51	5288.62	5349.92

Experiments

Name	n	*iclingo (1)*	*iclingo (2)*	*iclingo (3)*	*iclingo (4)*	*clingo*	*gringo*|*clasp*
Towers	33	38.00	42.96	48.46	27.15	31.98	32.76
	34	61.40	36.78	47.09	45.95	61.77	63.39
	36	81.26	60.77	88.52	131.29	86.56	88.46
	39	223.46	155.76	184.63	204.13	216.89	222.74
	41	429.82	327.74	392.47	342.11	459.97	471.22
	Σ	833.94	624.01	761.17	750.63	857.17	878.57
Towers back	33	4.62	6.42	5.68	5.80	12.59	12.79
	34	55.79	33.42	56.27	42.39	52.80	54.00
	36	16.66	16.46	14.69	17.11	24.81	25.38
	39	27.88	25.43	28.60	32.83	46.01	46.85
	41	48.20	36.38	62.75	40.62	83.78	85.60
	Σ	153.15	118.11	167.99	138.75	219.99	224.62
	$\Sigma\Sigma$	7791.45	9084.00	9635.81	9813.33	13776.59	14162.86

Conclusion

- Tackling bounded problems in ASP, paving the way for more ambitious real-world applications.
- Module theory provides us with
  - a natural semantics for non-ground, parameterized program slices and
  - makes precise their composition by appeal to input/output interfaces.
- First experimental results indicate the computational impact of our incremental approach, *but* more needs to be done !

☞ Incremental problems differ from traditional ones !

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☞ Incremental problems differ from traditional ones !

Constraint Answer Set Programming
Overview

Motivation

Observation

Certain applications are more naturally modeled by mixing Boolean with non-Boolean constructs, eg., accounting for

- resources,
- fine timings, or
- functions over finite domains.

Introduction

Groundbreaking Work in ASP [5, 64, 65]

- semantics for multi-sorted, first-order language
- algorithms using DPLL-style backtracking

SAT Modulo Theories [69]

- no modelling language
- algorithms using CDCL-style backjumping and learning

Our ASP approach [40]

- propositional semantics
- algorithms using CDCL-style backjumping and learning
- use off-the-shelf CP solvers

SAT Modulo Theories

SAT Modulo Theories (SMT)

- logical formulas with respect to combinations of background theories
- real numbers, integers, lists, arrays, bit vectors ...

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SAT

$$(x \vee y \vee z) \wedge (\neg x \vee \neg y \vee \neg z)$$

SAT Modulo Theories

SAT Modulo Theories (SMT)

- logical formulas with respect to combinations of background theories
- real numbers, integers, lists, arrays, bit vectors ...

SMT

$$(\sin(x)^3 = \cos(\log(y) \cdot x) \qquad \vee\ b \qquad \vee\ -x^2 \geq 2.3y)$$

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SMT

$$(\quad a \quad \lor b \quad \lor -x^2 \geq 2.3y)$$

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- logical formulas with respect to combinations of background theories
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SMT

$$(\quad a \quad \vee b \quad \vee \quad c \quad)$$

Outline

Constraint Satisfaction Problem

Definition

A *Constraint Satisfaction Problem* (CSP) consists of

- a set V of variables,
- a set D of domains, and
- a set C of constraints

such that

- each *variable* $v \in V$ has an associated *domain* $dom(v) \in D$;
- a *constraint* c is a pair (S, R) consisting of a k-ary *relation* R on a vector $S \subseteq V^k$ of variables, called the *scope* of R.
  - ☞ For $S = (v_1, \ldots, v_k)$, we have $R \subseteq dom(v_1) \times \cdots \times dom(v_k)$.

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Example

$$
\begin{array}{ccccc}
 & s & e & n & d \\
+ & m & o & r & e \\
\hline
m & o & n & e & y
\end{array}
$$

Each letter corresponds exactly to one digit and all variables have to be pairwisely distinct.

$V = \{s, e, n, d, m, o, r, y\}$ $\qquad$ $dom(v) = 0..9$ for all $v \in V$

$$
\begin{aligned}
C = \{ & (V, \quad allDistinct(V)), \\
& (V, \quad s \times 1000 + e \times 100 + n \times 10 + d + \\
& \qquad m \times 1000 + o \times 100 + r \times 10 + e == \\
& \qquad m \times 10000 + o \times 1000 + n \times 100 + e \times 10 + y), \\
& ((m), m == 1)\}
\end{aligned}
$$

Example

$$\begin{array}{ccccc} & s & e & n & d \\ + & m & o & r & e \\ \hline m & o & n & e & y \end{array}$$

Each letter corresponds exactly to one digit and all variables have to be pairwisely distinct.
The example has exactly one solution.

$$\{ s \mapsto 9, e \mapsto 5, n \mapsto 6, d \mapsto 7, m \mapsto 1, o \mapsto 0, r \mapsto 8, y \mapsto 2 \}$$

$$\begin{array}{ccccc} & 9 & 5 & 6 & 7 \\ + & 1 & 0 & 8 & 5 \\ \hline 1 & 0 & 6 & 5 & 2 \end{array}$$

Constraint Satisfaction Problem

Notation

We use $S(c) = S$ and $R(c) = R$ to access the scope and the relation of a constraint $c = (S, R)$.

Definition

For an assignment $A : V \to \bigcup_{v \in V} dom(v)$ and a constraint (S, R) with scope $S = (v_1, \ldots, v_k)$, define

$$sat_C(A) = \{c \in C \mid A(S(c)) \in R(c)\}$$

where $A(S) = (A(v_1), \ldots, A(v_k))$.

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Constraint Answer Set Programming

Definition

A *Constraint Logic Program* P is a logic program over an extended alphabet $\mathcal{A} \cup \mathcal{C}$ where

- $\mathcal{A}$ is a set of *regular atoms* and
- $\mathcal{C}$ is a set of *constraint atoms*,

such that $head(r) \in \mathcal{A}$ for each $r \in P$.

Auxiliary Definition

Given a set of literals B and some set $\mathcal{B}$ of atoms, we define
$B|_{\mathcal{B}} = (B^+ \cap \mathcal{B}) \cup \{not\ a \mid a \in B^- \cap \mathcal{B}\}$.

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Definition

We identify constraint atoms with constraints via a function

$$\gamma : \mathcal{C} \rightarrow C$$

furthermore, $\gamma(Y) = \{\gamma(c) \mid c \in Y\}$ for any $Y \subseteq \mathcal{C}$.

Note

- Unlike regular atoms $\mathcal{A}$, constraint atoms $\mathcal{C}$ are not subject to the unique names assumption, eg.

  $$\gamma(x < y) \;=\; \gamma(((-y - 1) \leq -(x + 1)) \wedge (x \neq y))$$

- A constraint logic program P is associated with a CSP as follows
  - $C[P] = \gamma(atom(P) \cap \mathcal{C})$,
  - $V[P]$ is obtained from the constraint scopes in $C[P]$,
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Definition

Let P be a constraint logic program over $\mathcal{A} \cup \mathcal{C}$ and let $A : V[P] \rightarrow D[P]$ be an assignment.
We define the *constraint reduct* of as P wrt A as follows.

$$\begin{aligned} P^A \quad = \quad \{ \; & head(r) \leftarrow body(r)|_{\mathcal{A}} \mid r \in P, \\ & \gamma(body(r)|_{\mathcal{C}}{}^{+}) \subseteq sat_{C[P]}(A), \\ & \gamma(body(r)|_{\mathcal{C}}{}^{-}) \cap sat_{C[P]}(A) = \emptyset \; \} \end{aligned}$$

Definition

A set $X \subseteq \mathcal{A}$ of (regular) atoms is a *constraint answer set* of P wrt A, if X is an answer set of P^A.

☞ That is, if X is the $\subseteq$-smallest model of $(P^A)^X$.

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Outline

Modeling Language

Note

Although our semantics is propositional, the atoms in $\mathcal{A}$ and $\mathcal{C}$ are constructible from a multi-sorted, first-order signature given by:

- a set $\mathcal{P}_\mathcal{A} \cup \mathcal{P}_\mathcal{C}$ of *predicate symbols* such that $\mathcal{P}_\mathcal{A} \cap \mathcal{P}_\mathcal{C} = \emptyset$,
- a set $\mathcal{F}_\mathcal{A} \cup \mathcal{F}_\mathcal{C}$ of *function symbols* (including constant symbols),
- a set $\mathcal{V}_\mathcal{A}$ of *regular variable symbols*, and
- a set $\mathcal{V}_\mathcal{C} \subseteq \mathcal{T}(\mathcal{F}_\mathcal{A})$ of *constraint variable symbols*, where $\mathcal{T}(\mathcal{F}_\mathcal{A})$ denotes the set of all ground terms over $\mathcal{F}_\mathcal{A}$.

As common in ASP, the atoms in $\mathcal{A} \cup \mathcal{C}$ are obtained by a grounding process.

An Example

$$
\begin{array}{rcl}
time(0..t_{max}) & & \\
bucket(a) \quad bucket(b) & & \\
1\ \{pour(B,T) : bucket(B)\}\ 1 & \leftarrow & time(T), T < t_{max} \\
1 \leq^{\$} amt(B,T) & \leftarrow & pour(B,T), T < t_{max} \\
amt(B,T) \leq^{\$} 3 & \leftarrow & pour(B,T), T < t_{max} \\
amt(B,T) =^{\$} 0 & \leftarrow & not\ pour(B,T), T < t_{max} \\
vol(B,T{+}1) =^{\$} vol(B,T) + amt(B,T) & \leftarrow & time(T) < t_{max} \\
down(B,T) & \leftarrow & vol(C,T) <^{\$} vol(B,T) \\
up(B,T) & \leftarrow & not\ down(B,T) \\
vol(a,0) =^{\$} 0 \quad vol(b,0) =^{\$} 1 & & \\
 & \leftarrow & up(a, t_{max})
\end{array}
$$

An Example

- Consider the signature of our exemplary program:

$$
\begin{aligned}
\{B, C, T\} &\subseteq \mathcal{V}_{\mathcal{A}} \\
\{0, \dots, t_{max}, +, a, b, amt, vol\} &\subseteq \mathcal{F}_{\mathcal{A}} \\
\{<, time, bucket, pour, up, down\} &\subseteq \mathcal{P}_{\mathcal{A}}
\end{aligned}
$$

$$
\begin{aligned}
\{0, 1, 3, +\} &\subseteq \mathcal{F}_{\mathcal{C}} \\
\{=^{\$}, <^{\$}, \leq^{\$}\} &\subseteq \mathcal{P}_{\mathcal{C}}
\end{aligned}
$$

- With substitution $\{B \mapsto b, T \mapsto 1, t_{max} \mapsto 2\}$, we get:

$$
\begin{aligned}
amt(b,1) =^{\$} 0 &\leftarrow not\ pour(b,1) \\
vol(b,2) =^{\$} vol(b,1) + amt(b,1) &\leftarrow
\end{aligned}
$$

- and, among others, our signature hence contains

$$
\begin{aligned}
\{amt(b,1), vol(b,1), vol(b,2)\} &\subseteq \mathcal{V}_{\mathcal{C}} \\
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\end{array}$$

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- and, among others, our signature hence contains

$$\begin{array}{rcl} \{amt(b,1), vol(b,1), vol(b,2)\} & \subseteq & \mathcal{V}_{\mathcal{C}} \\ \{pour(b,1)\} & \subseteq & \mathcal{A} \\ \{amt(b,1) =^{\$} 0,\ vol(b,2) =^{\$} vol(b,1) + amt(b,1)\} & \subseteq & \mathcal{C} \end{array}$$

An Example

- Consider the signature of our exemplary program:

$$
\begin{array}{rcl}
\{B, C, T\} & \subseteq & \mathcal{V}_{\mathcal{A}} \\
\{0, \dots, t_{max}, +, a, b, amt, vol\} & \subseteq & \mathcal{F}_{\mathcal{A}} \\
\{<, time, bucket, pour, up, down\} & \subseteq & \mathcal{P}_{\mathcal{A}} \\
\{0, 1, 3, +\} & \subseteq & \mathcal{F}_{\mathcal{C}} \\
\{=^{\$}, <^{\$}, \leq^{\$}\} & \subseteq & \mathcal{P}_{\mathcal{C}}
\end{array}
$$

- With substitution $\{B \mapsto b, T \mapsto 1, t_{max} \mapsto 2\}$, we get:

$$
\begin{array}{rcl}
amt(b,1) =^{\$} 0 & \leftarrow & not\ pour(b,1) \\
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An Example

For $t_{max} = 2$, our program has eleven constraint answer sets, summarized as follows

$up(a,0)$	$pour(a,0)$	$amt(a,0)$	$up(a,1)$	$pour(a,1)$	$amt(a,1)$	$up(a,2)$
T	**T**	1	**T**	**T**	$1,2,3$	**F**
T	**T**	$2,3$	**F**	**T**	$1,2,3$	**F**
T	**T**	3	**F**	**F**	0	**F**
T	**F**	0	**T**	**T**	3	**F**

An Example

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T	**T**	1	**T**	**T**	$1,2,3$	**F**
T	**T**	$2,3$	**F**	**T**	$1,2,3$	**F**
T	**T**	3	**F**	**F**	0	**F**
T	**F**	0	**T**	**T**	3	**F**

Outline

CDNL

Main Algorithm

loop

PROPAGATE

if no conflict **then**

if partial assignment **then** DECIDE
else return solution

else if some decisions made **then**

ANALYZE CONFLICT
RECORD REASON
BACKJUMP

else exit

CDNL

Main Algorithm

loop

PROPAGATE

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ASP Propagation

Propagation Algorithm

loop

  UNIT-PROPAGATION

  if conflict **then return**

  else if UNFOUNDED-SET **then**

    RECORD LOOP-NOGOOD

    if conflict **then return**

  else return

Constraint ASP Propagation

Propagation Algorithm

loop
- UNIT-PROPAGATION
- **if** conflict **then return**
- **else if** UNFOUNDED-SET **then**
  - RECORD LOOP-NOGOOD
  - **if** conflict **then return**
- **else if** CONSTRAINT-PROPAGATION **then**
  - **if** conflict **then return**
- **else return**

CDNL

Main Algorithm

loop

    PROPAGATE
    if no conflict **then**
        if partial assignment **then** DECIDE
        else return solution
    else if some decisions made **then**
        ANALYZE CONFLICT
        RECORD REASON
        BACKJUMP
    else exit

Constraint CDNL

Main Algorithm

loop

PROPAGATE

if no conflict **then**

if partial assignment **then** DECIDE
else if CSP-SOLVE **then return** solution
else

ANALYZE CONFLICT
RECORD REASON
BACKJUMP

else if some decisions made **then**

ANALYZE CONFLICT
RECORD REASON
BACKJUMP

else exit

Outline

Example on Grounding

John goes to work either by car (30-40 minutes), or by bus (at least 60 minutes).
Fred goes to work either by car (20-30 minutes), or in a car pool (40-50 minutes).
Today John left home between 7:10 and 7:20, and Fred arrived between 8:00 and 8:10.
We also know that John arrived at work about 10-20 minutes after Fred left home.

We wish to answer queries such as:

- Is the information in the story consistent?
- Is it possible that John took the bus, and Fred used the carpool?
- What are the possible times at which Fred left home?

Example on Grounding

- Number of Lines Output

maxtime	gringo	clingcon
10	626	19
100	47132	19
500	1137332	19
1000	4525082	19

NASA Advisor: 40 seconds realtime

	clingo	*adsolver*				*clingcon*				
Benchmark	5	5	7	11	13	5	7	11	13	20
3-0/025	162.84	14.74	51.42	460.57	365.37	1.19	1.97	4.21	5.99	17.84
3-0/050	173.28	31.39	108.21	471.41	—	1.26	2.32	6.80	11.85	27.36
3-0/100	175.94	448.90	188.33	—	—	1.32	2.35	10.11	12.04	38.78
3-0/125	165.64	19.78	60.07	224.60	—	1.18	1.94	4.05	10.00	133.99
5-0/025	174.12	28.78	107.41	—	—	1.28	2.90	5.87	14.27	66.55
5-0/050	163.25	13.57	42.00	204.34	497.64	1.18	1.97	4.71	10.04	241.59
5-0/100	168.16	21.50	66.10	282.36	514.08	1.20	1.98	4.13	6.45	25.32
5-0/125	174.38	32.02	104.32	429.72	—	1.34	2.95	6.39	9.70	81.17
8-0/025	177.82	41.57	140.93	—	—	1.30	2.73	11.00	12.69	222.49
8-0/050	167.72	18.83	54.76	215.43	—	1.18	1.93	4.02	7.76	457.86
8-0/100	165.55	13.72	41.03	208.74	—	1.21	2.00	5.05	6.10	26.17
8-0/125	162.29	16.81	53.40	246.64	519.59	1.20	1.99	4.15	6.69	17.82
∅	169.25	58.47	84.83	378.65	558.06	1.24	2.25	5.87	9.47	113.08

clingo standard ASP grounder and ASP solver
adsolver ASPmCSP solver based on smodels [Mellarkod and Gelfond, '08]
clingcon ASPmCSP solver based on clingo and gecode

http://potassco.sourceforge.net

Potassco, the Potsdam Answer Set Solving Collection,
bundles tools for ASP developed at the University of Potsdam,
for instance:

- *Grounder*: Gringo, pyngo
- *Solver*: clasp, claspD, claspar
- *Grounder+Solver*: Clingo, iClingo, oClingo, Clingcon
- *Further Tools*: claspre, claspfolio, coala, inca, plasp, sbass, xorro

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Summary

- ASP is emerging as a viable tool for Knowledge Representation and Reasoning
- ASP offers efficient and versatile off-the-shelf solving technology
  - `http://potassco.sourceforge.net`
  - ASP'09, PB'09, and SAT'09
- ASP offers an expanding functionality and ease of use
  - Rapid application development tool
- ASP has a growing range of applications

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$$\text{ASP} = \text{KR} + \text{DB} + \text{Search}$$

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