Exercises

Answer Set Solving in Practice

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Exercise 1 (Introduction)

Quiz 1.1

	Answer	
1. Each positive logic program has some model.	**true**	**false**
2. Each positive logic program has some stable model.	**true**	**false**
3. If P is a positive logic program and $A(P)$ are the atoms occurring in P then $A(P)$ is a model of P.	**true**	**false**
4. If P is a positive logic program and $A(P)$ are the atoms occurring in P then $A(P)$ is a stable model of P.	**true**	**false**
5. If a positive rule is satisfied by a set of atoms, then each of its supersets satisfies the rule as well.	**true**	**false**
6. The stable model of a positive logic program is contained in each model of the program.	**true**	**false**

Quiz 1.2

		Answer	
1.	Each normal logic program has some model.	**true**	**false**
2.	Each normal logic program has some stable model.	**true**	**false**
3.	Given a normal logic program P, the reduct P^X is contained in P^Y for each subset X of a set Y of atoms.	**true**	**false**
4.	If a set X of atoms is a model of a normal logic program P, then X is a model of the reduct P^Y for each superset Y of X.	**true**	**false**
5.	Given a normal logic program P, each stable model of P is a $\subseteq$-minimal model of P.	**true**	**false**

Exercise 1.1 (Positive Logic Programs)

Determine the models of the following positive logic programs and decide which models are stable.

1.1–a $$P = \left\{ \begin{array}{r} rain \leftarrow \\ wet \leftarrow rain \\ wet \leftarrow sprinkler \end{array} \right\}$$

1.1–b $$P = \left\{ \begin{array}{r} coffee \leftarrow \\ lemon \leftarrow tea \\ sugar \leftarrow coffee \\ milk \leftarrow coffee, sugar \\ tea \leftarrow lemon \\ tea \leftarrow diet \end{array} \right\}$$

1.1–c $$P = \left\{ \begin{array}{r} shirt \leftarrow \\ sneakers \leftarrow \\ pants \leftarrow sneakers \\ skirt \leftarrow shirt, sandals \\ sandals \leftarrow dress \end{array} \right\}$$

1.1–d $$P = \left\{ \begin{array}{r} red \leftarrow \\ meat \leftarrow cabbage \\ meat \leftarrow red \\ fish \leftarrow asparagus \\ asparagus \leftarrow fish, white \\ white \leftarrow fish \end{array} \right\}$$

Exercise 1.2 (Normal Logic Programs)

Determine the stable models of the following normal logic programs.

1.2–a $$P = \left\{ \begin{array}{r@{\;}l} \mathit{sprinkler} & \leftarrow \neg \mathit{rain} \\ \mathit{rain} & \leftarrow \neg \mathit{sprinkler} \\ \mathit{wet} & \leftarrow \mathit{rain} \\ \mathit{wet} & \leftarrow \mathit{sprinkler} \end{array} \right\}$$

1.2–b $$P = \left\{ \begin{array}{r@{\;}l} \mathit{diet} & \leftarrow \neg \mathit{sugar} \\ \mathit{coffee} & \leftarrow \neg \mathit{tea} \\ \mathit{lemon} & \leftarrow \mathit{tea} \\ \mathit{sugar} & \leftarrow \mathit{coffee} \\ \mathit{milk} & \leftarrow \mathit{coffee}, \mathit{sugar} \\ \mathit{tea} & \leftarrow \mathit{lemon} \\ \mathit{tea} & \leftarrow \mathit{diet} \end{array} \right\}$$

1.2–c $$P = \left\{ \begin{array}{r@{\;}l} \mathit{dress} & \leftarrow \neg \mathit{shirt} \\ \mathit{shirt} & \leftarrow \neg \mathit{dress} \\ \mathit{sandals} & \leftarrow \neg \mathit{sneakers} \\ \mathit{sneakers} & \leftarrow \neg \mathit{sandals} \\ \mathit{pants} & \leftarrow \mathit{sneakers} \\ \mathit{skirt} & \leftarrow \mathit{shirt}, \mathit{sandals} \\ \mathit{sandals} & \leftarrow \mathit{dress} \end{array} \right\}$$

1.2–d $$P = \left\{ \begin{array}{r@{\;}l} \mathit{asparagus} & \leftarrow \neg \mathit{cabbage} \\ \mathit{cabbage} & \leftarrow \neg \mathit{asparagus} \\ \mathit{red} & \leftarrow \neg \mathit{white} \\ \mathit{meat} & \leftarrow \mathit{cabbage} \\ \mathit{meat} & \leftarrow \mathit{red} \\ \mathit{fish} & \leftarrow \mathit{asparagus} \\ \mathit{asparagus} & \leftarrow \mathit{fish}, \mathit{white} \\ \mathit{white} & \leftarrow \mathit{fish} \end{array} \right\}$$

Exercise 1.3 (Positive Logic Programs with Variables)

Determine the Herbrand universe $\mathcal{T}$, the Herbrand base $\mathcal{A}$, the ground instantiation, and the stable models of the following positive logic programs with variables.

1.3–a $$P = \left\{ \begin{array}{r} \mathit{fish}(\mathit{blinky}) \leftarrow \\ \mathit{bird}(\mathit{tweety}) \leftarrow \\ \mathit{flies}(X) \leftarrow \mathit{bird}(X) \end{array} \right\}$$

1.3–b $$P = \left\{ \begin{array}{r} \mathit{next}(0,1) \leftarrow \\ \mathit{next}(1,0) \leftarrow \\ \mathit{even}(0) \leftarrow \\ \mathit{even}(Y) \leftarrow \mathit{next}(X,Y), \mathit{odd}(X) \\ \mathit{odd}(Y) \leftarrow \mathit{next}(X,Y), \mathit{even}(X) \end{array} \right\}$$

1.3–c $$P = \left\{ \begin{array}{r} \mathit{friend}(\mathit{alice}, \mathit{bob}) \leftarrow \\ \mathit{friend}(\mathit{bob}, \mathit{alice}) \leftarrow \\ \mathit{friend}(\mathit{eve}, \mathit{alice}) \leftarrow \\ \mathit{invite}(\mathit{alice}) \leftarrow \\ \mathit{invite}(Y) \leftarrow \mathit{invite}(X), \mathit{friend}(X,Y) \end{array} \right\}$$

1.3–d $$P = \left\{ \begin{array}{r} \mathit{next}(0,1) \leftarrow \\ \mathit{next}(1,2) \leftarrow \\ \mathit{before}(X) \leftarrow \mathit{next}(X,Y) \\ \mathit{between}(Y) \leftarrow \mathit{next}(X,Y), \mathit{before}(Y) \end{array} \right\}$$

Exercise 1.4 (Normal Logic Programs with Variables)

Determine the ground instantiation and the stable models of the following normal logic programs with variables.

1.4–a $$P = \left\{ \begin{array}{r@{\;}l} \mathit{sparrow}(\mathit{jack}) & \leftarrow \\ \mathit{penguin}(\mathit{tweety}) & \leftarrow \\ \mathit{bird}(X) & \leftarrow \mathit{sparrow}(X) \\ \mathit{bird}(X) & \leftarrow \mathit{penguin}(X) \\ \mathit{flies}(X) & \leftarrow \mathit{bird}(X), \neg \mathit{penguin}(X) \end{array} \right\}$$

1.4–b $$P = \left\{ \begin{array}{r@{\;}l} \mathit{next}(0,1) & \leftarrow \\ \mathit{next}(1,0) & \leftarrow \\ \mathit{even}(X) & \leftarrow \neg \mathit{odd}(X) \\ \mathit{odd}(Y) & \leftarrow \mathit{next}(X,Y), \mathit{even}(X) \end{array} \right\}$$

1.4–c $$P = \left\{ \begin{array}{r@{\;}l} \mathit{next}(0,1) & \leftarrow \\ \mathit{next}(1,2) & \leftarrow \\ \mathit{next}(2,0) & \leftarrow \\ \mathit{select}(X) & \leftarrow \mathit{next}(X,Y), \neg \mathit{select}(Y) \end{array} \right\}$$

1.4–d $$P = \left\{ \begin{array}{r@{\;}l} \mathit{friend}(\mathit{alice}, \mathit{bob}) & \leftarrow \\ \mathit{friend}(\mathit{bob}, \mathit{alice}) & \leftarrow \\ \mathit{friend}(\mathit{eve}, \mathit{alice}) & \leftarrow \\ \mathit{refuse}(X) & \leftarrow \neg \mathit{invite}(X) \\ \mathit{invite}(X) & \leftarrow \neg \mathit{refuse}(X) \\ \mathit{invite}(Y) & \leftarrow \mathit{invite}(X), \mathit{friend}(X,Y) \end{array} \right\}$$

Exercise 1.5 (Safety)

Determine which rules of the following programs are not safe.

1.5–a
$$P = \left\{ \begin{array}{r@{\ }l} \mathit{sparrow}(\mathit{jack}) \leftarrow & \\ \mathit{penguin}(\mathit{tweety}) \leftarrow & \\ \mathit{bird}(X) \leftarrow & \mathit{sparrow}(X) \\ \mathit{bird}(X) \leftarrow & \mathit{penguin}(X) \\ \mathit{animal}(X) \leftarrow & \\ \mathit{flies}(X) \leftarrow & \neg \mathit{penguin}(X) \end{array} \right\}$$

1.5–b
$$P = \left\{ \begin{array}{r@{\ }l} \mathit{next}(0,1) \leftarrow & \\ \mathit{next}(1,0) \leftarrow & \\ \mathit{greater\_equal}(1,X) \leftarrow & \\ \mathit{even}(X) \leftarrow & \neg \mathit{odd}(X) \\ \mathit{odd}(Y) \leftarrow & \mathit{next}(X,Y), \mathit{even}(X), \neg \mathit{odd}(X) \end{array} \right\}$$

1.5–c
$$P = \left\{ \begin{array}{l} \mathit{next}(0,1) \leftarrow \\ \mathit{next}(1,2) \leftarrow \\ \mathit{next}(2,0) \leftarrow \\ \mathit{select}(X) \leftarrow \neg \mathit{next}(X,Y), \neg \mathit{select}(Y) \end{array} \right\}$$

1.5–d
$$P = \left\{ \begin{array}{r@{\ }l} \mathit{friend}(\mathit{alice},\mathit{bob}) \leftarrow & \\ \mathit{friend}(\mathit{bob},\mathit{alice}) \leftarrow & \\ \mathit{friend}(\mathit{eve},X) \leftarrow & \\ \mathit{refuse}(X) \leftarrow & \neg \mathit{invite}(X) \\ \mathit{invite}(X) \leftarrow & \neg \mathit{refuse}(X) \\ \mathit{invite}(Y) \leftarrow & \mathit{invite}(X), \neg \mathit{friend}(X,Y) \end{array} \right\}$$

Solution

Quiz 1.1

		Answer	
1.	Each positive logic program has some model.	**true**	
2.	Each positive logic program has some stable model.	**true**	
3.	If P is a positive logic program and $A(P)$ are the atoms occurring in P then $A(P)$ is a model of P.	**true**	
4.	If P is a positive logic program and $A(P)$ are the atoms occurring in P then $A(P)$ is a stable model of P.		**false**
5.	If a positive rule is satisfied by a set of atoms, then each of its supersets satisfies the rule as well.		**false**
6.	The stable model of a positive logic program is contained in each model of the program.	**true**	

Solution

Quiz 1.2

		Answer	
1.	Each normal logic program has some model.	**true**	
2.	Each normal logic program has some stable model.		**false**
3.	Given a normal logic program P, the reduct P^X is contained in P^Y for each subset X of a set Y of atoms.		**false**
4.	If a set X of atoms is a model of a normal logic program P, then X is a model of the reduct P^Y for each superset Y of X.	**true**	
5.	Given a normal logic program P, each stable model of P is a $\subseteq$-minimal model of P.	**true**	

Solution 1.1

1.1–a Models:

- $\{rain, wet\}$
- $\{rain, wet, sprinkler\}$

Stable Model: $\{rain, wet\}$

1.1–b Models:

- $\{coffee, sugar, milk\}$
- $\{coffee, sugar, milk, tea, lemon\}$
- $\{coffee, sugar, milk, tea, lemon, diet\}$

Stable Model: $\{coffee, sugar, milk\}$

1.1–c Models:

- $\{shirt, sneakers, pants\}$
- $\{shirt, sneakers, pants, skirt\}$

- $\{shirt, sneakers, pants, skirt, sandals\}$
- $\{shirt, sneakers, pants, skirt, sandals, dress\}$

Stable Model: $\{shirt, sneakers, pants\}$

1.1–d Models:

- $\{red, meat\}$
- $\{red, meat, cabbage\}$
- $\{red, meat, white\}$
- $\{red, meat, white, cabbage\}$
- $\{red, meat, white, fish, asparagus\}$
- $\{red, meat, white, fish, asparagus, cabbage\}$

Stable Model: $\{red, meat\}$

Solution 1.2

1.2–a Stable Models:

- $\{rain, wet\}$
- $\{sprinkler, wet\}$

1.2–b Stable Models:

- $\{coffee, sugar, milk\}$
- $\{tea, lemon, diet\}$

1.2–c Stable Models:

- $\{shirt, sneakers, pants\}$
- $\{shirt, sandals, skirt\}$
- $\{dress, sandals\}$

1.2–d Stable Models:

- $\{red, meat, cabbage\}$
- $\{white, fish, asparagus\}$

Solution 1.3

1.3–a Herbrand Universe $\mathcal{T} = \{blinky, tweety\}$
Herbrand Base:

$$\mathcal{A} = \left\{ \begin{array}{l} fish(blinky), fish(tweety), \\ bird(blinky), bird(tweety), \\ flies(blinky), flies(tweety) \end{array} \right\}$$

Ground instantiation:

$$ground(P) = \left\{ \begin{array}{rcl} fish(blinky) & \leftarrow & \\ bird(tweety) & \leftarrow & \\ flies(blinky) & \leftarrow & bird(blinky) \\ flies(tweety) & \leftarrow & bird(tweety) \end{array} \right\}$$

Stable Models:

- $\{fish(blinky), bird(tweety), flies(tweety)\}$

1.3–b Herbrand Universe $\mathcal{T} = \{0, 1\}$
Herbrand Base:

$$\mathcal{A} = \left\{ \begin{array}{l} next(0,0), next(0,1), next(1,0), next(1,1) \\ even(0), even(1), odd(0), odd(1) \end{array} \right\}$$

Ground instantiation:

$$
ground(P) = \left\{ \begin{array}{r@{}l}
next(0,1) \leftarrow & \\
next(1,0) \leftarrow & \\
even(0) \leftarrow & \\
even(0) \leftarrow & \; next(0,0), odd(0) \\
even(1) \leftarrow & \; next(0,1), odd(0) \\
even(0) \leftarrow & \; next(1,0), odd(1) \\
even(1) \leftarrow & \; next(1,1), odd(1) \\
odd(0) \leftarrow & \; next(0,0), even(0) \\
odd(1) \leftarrow & \; next(0,1), even(0) \\
odd(0) \leftarrow & \; next(1,0), even(1) \\
odd(1) \leftarrow & \; next(1,1), even(1)
\end{array} \right\}
$$

Stable Models:

- $\{next(0,1), next(1,0), even(0), odd(1)\}$

1.3–c Herbrand Universe $\mathcal{T} = \{alive, bob, eve\}$
Herbrand Base:

$$
\mathcal{A} = \left\{ \begin{array}{l}
friend(alice, alice), friend(alice, bob), friend(alice, eve), \\
friend(bob, alice), friend(bob, bob), friend(bob, eve), \\
friend(eve, alice), friend(eve, bob), friend(eve, eve), \\
invite(alice), invite(bob), invite(eve)
\end{array} \right\}
$$

Ground instantiation:

$$
ground(P) = \left\{ \begin{array}{r@{}l}
friend(alice, bob) \leftarrow & \\
friend(bob, alice) \leftarrow & \\
friend(eve, alice) \leftarrow & \\
invite(alice) \leftarrow & \\
invite(alice) \leftarrow & \; invite(alice), friend(alice, alice) \\
invite(bob) \leftarrow & \; invite(alice), friend(alice, bob) \\
invite(eve) \leftarrow & \; invite(alice), friend(alice, eve) \\
invite(alice) \leftarrow & \; invite(bob), friend(bob, alice) \\
invite(bob) \leftarrow & \; invite(bob), friend(bob, bob) \\
invite(eve) \leftarrow & \; invite(bob), friend(bob, eve) \\
invite(alice) \leftarrow & \; invite(eve), friend(eve, alice) \\
invite(bob) \leftarrow & \; invite(eve), friend(eve, bob) \\
invite(eve) \leftarrow & \; invite(eve), friend(eve, eve)
\end{array} \right\}
$$

Stable Models:

- $\left\{ \begin{array}{l} friend(alice, bob), friend(bob, alice), friend(eve, alice), \\ invite(alice), invite(bob) \end{array} \right\}$

1.3–d Herbrand Universe $\mathcal{T} = \{0, 1, 2\}$
Herbrand Base:

$$
\mathcal{A} = \left\{ \begin{array}{l}
next(0,0), next(0,1), next(0,2), \\
next(1,0), next(1,1), next(1,2), \\
next(2,0), next(2,1), next(2,2), \\
before(0), before(1), before(2), \\
between(0), between(1), between(2)
\end{array} \right\}
$$

Ground instantiation:

$$
ground(P) = \left\{ \begin{array}{r@{\ }l}
next(0,1) \leftarrow & \\
next(1,2) \leftarrow & \\
before(0) \leftarrow & next(0,0) \\
before(0) \leftarrow & next(0,1) \\
before(0) \leftarrow & next(0,2) \\
before(1) \leftarrow & next(1,0) \\
before(1) \leftarrow & next(1,1) \\
before(1) \leftarrow & next(1,2) \\
before(2) \leftarrow & next(2,0) \\
before(2) \leftarrow & next(2,1) \\
before(2) \leftarrow & next(2,2) \\
between(0) \leftarrow & next(0,0), before(0) \\
between(1) \leftarrow & next(0,1), before(1) \\
between(2) \leftarrow & next(0,2), before(2) \\
between(0) \leftarrow & next(1,0), before(0) \\
between(1) \leftarrow & next(1,1), before(1) \\
between(2) \leftarrow & next(1,2), before(2) \\
between(0) \leftarrow & next(2,0), before(0) \\
between(1) \leftarrow & next(2,1), before(1) \\
between(2) \leftarrow & next(2,2), before(2)
\end{array} \right\}
$$

Stable Models:

- $\{next(0,1), next(1,2), before(0), before(1), between(1)\}$

Solution 1.4

1.4–a

$$
ground(P) = \left\{ \begin{array}{r@{\ }l}
sparrow(jack) \leftarrow & \\
penguin(tweety) \leftarrow & \\
bird(jack) \leftarrow & sparrow(jack) \\
bird(tweety) \leftarrow & sparrow(tweety) \\
bird(jack) \leftarrow & penguin(jack) \\
bird(tweety) \leftarrow & penguin(tweety) \\
flies(jack) \leftarrow & bird(jack), \neg penguin(jack) \\
flies(tweety) \leftarrow & bird(tweety), \neg penguin(tweety)
\end{array} \right\}
$$

Stable Models:

- $\{sparrow(jack), penguin(tweety), bird(jack), bird(tweety), flies(jack)\}$

1.4–b

$$
ground(P) = \left\{ \begin{array}{r@{\ }l}
next(0,1) \leftarrow & \\
next(1,0) \leftarrow & \\
even(0) \leftarrow & \neg odd(0) \\
even(1) \leftarrow & \neg odd(1) \\
odd(0) \leftarrow & next(0,0), even(0) \\
odd(1) \leftarrow & next(0,1), even(0) \\
odd(0) \leftarrow & next(1,0), even(1) \\
odd(1) \leftarrow & next(1,1), even(1)
\end{array} \right\}
$$

Stable Models:

- $\{next(0,1), next(1,0), even(0), odd(1)\}$
- $\{next(0,1), next(1,0), even(1), odd(0)\}$

1.4–c
$$ground(P) = \left\{ \begin{array}{r} next(0,1) \leftarrow \\ next(1,2) \leftarrow \\ next(2,0) \leftarrow \\ select(0) \leftarrow next(0,0), \neg select(0) \\ select(0) \leftarrow next(0,1), \neg select(1) \\ select(0) \leftarrow next(0,2), \neg select(2) \\ select(1) \leftarrow next(1,0), \neg select(0) \\ select(1) \leftarrow next(1,1), \neg select(1) \\ select(1) \leftarrow next(1,2), \neg select(2) \\ select(2) \leftarrow next(2,0), \neg select(0) \\ select(2) \leftarrow next(2,1), \neg select(1) \\ select(2) \leftarrow next(2,2), \neg select(2) \end{array} \right\}$$

Stable Models: None

1.4–d

$$ground(P) = \left\{\begin{array}{r@{}l} friend(alice, bob) &\leftarrow \\ friend(bob, alice) &\leftarrow \\ friend(eve, alice) &\leftarrow \\ refuse(alice) &\leftarrow \neg invite(alice) \\ refuse(bob) &\leftarrow \neg invite(bob) \\ refuse(eve) &\leftarrow \neg invite(eve) \\ invite(alice) &\leftarrow \neg refuse(alice) \\ invite(bob) &\leftarrow \neg refuse(bob) \\ invite(eve) &\leftarrow \neg refuse(eve) \\ invite(alice) &\leftarrow invite(alice), friend(alice, alice) \\ invite(bob) &\leftarrow invite(alice), friend(alice, bob) \\ invite(eve) &\leftarrow invite(alice), friend(alice, eve) \\ invite(alice) &\leftarrow invite(bob), friend(bob, alice) \\ invite(bob) &\leftarrow invite(bob), friend(bob, bob) \\ invite(eve) &\leftarrow invite(bob), friend(bob, eve) \\ invite(alice) &\leftarrow invite(eve), friend(eve, alice) \\ invite(bob) &\leftarrow invite(eve), friend(eve, bob) \\ invite(eve) &\leftarrow invite(eve), friend(eve, eve) \end{array}\right\}$$

Stable Models:

- $\left\{\begin{array}{l} friend(alice, bob), friend(bob, alice), friend(eve, alice), \\ refuse(alice), refuse(bob), refuse(eve) \end{array}\right\}$
- $\left\{\begin{array}{l} friend(alice, bob), friend(bob, alice), friend(eve, alice), \\ invite(alice), invite(bob), refuse(eve) \end{array}\right\}$
- $\left\{\begin{array}{l} friend(alice, bob), friend(bob, alice), friend(eve, alice), \\ invite(alice), invite(bob), invite(eve) \end{array}\right\}$

Solution 1.5

1.5–a The following rules are not safe:

$$\begin{array}{r@{}l} animal(X) &\leftarrow \\ flies(X) &\leftarrow \neg penguin(X) \end{array}$$

1.5–b The following rules are not safe:

$$\begin{array}{r@{}l} greater\_equal(1, X) &\leftarrow \\ even(X) &\leftarrow \neg odd(X) \end{array}$$

1.5–c The following rule is not safe:

$$select(X) \leftarrow \neg next(X, Y), \neg select(Y)$$

1.5–d The following rules are not safe:

$$\begin{array}{r@{}l} friend(eve, X) &\leftarrow \\ refuse(X) &\leftarrow \neg invite(X) \\ invite(X) &\leftarrow \neg refuse(X) \\ invite(Y) &\leftarrow invite(X), \neg friend(X, Y) \end{array}$$