

Identidades trigonométricas

$$(1) \operatorname{sen}^2(\alpha) + \cos^2(\alpha) = 1$$

$$(2) \operatorname{sen}(\alpha + \beta) = \operatorname{sen}\alpha \cos \beta + \operatorname{sen}\beta \cos \alpha$$

$$(3) \operatorname{sen}(\alpha - \beta) = \operatorname{sen}\alpha \cos \beta - \operatorname{sen}\beta \cos \alpha$$

$$(4) \cos(\alpha + \beta) = \cos \alpha \cos \beta - \operatorname{sen}\alpha \operatorname{sen}\beta$$

$$(5) \cos(\alpha - \beta) = \cos \alpha \cos \beta + \operatorname{sen}\alpha \operatorname{sen}\beta$$

$$\text{en } (2) \alpha = \beta$$

$$\operatorname{sen}(\alpha + \alpha) = \operatorname{sen}\alpha \cos \alpha + \operatorname{sen}\alpha \cos \alpha$$

$$(6) \operatorname{sen}(2\alpha) = 2\operatorname{sen}\alpha \cos \alpha$$

$$\text{en } (4) \alpha = \beta$$

$$\cos(\alpha + \alpha) = \cos \alpha \cos \alpha - \operatorname{sen}\alpha \operatorname{sen}\alpha$$

$$(7) \cos(2\alpha) = \cos^2(\alpha) - \operatorname{sen}^2(\alpha)$$

$$(8) \cos(2\alpha) = 1 - 2\operatorname{sen}^2(\alpha)$$

$$(9) \cos(2\alpha) = 2\cos^2(\alpha) - 1$$

$$\text{desde } (8)$$

$$\cos(2\alpha) = 1 - 2\operatorname{sen}^2(\alpha)$$

$$\operatorname{sen}^2(\alpha) = \frac{1 - \cos(2\alpha)}{2}$$

$$(10) |\operatorname{sen}(\alpha)| = \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\text{desde } (9)$$

$$(11) |\cos(\alpha)| = \sqrt{\frac{1 + \cos \alpha}{2}}$$

$$(2) \operatorname{sen}(\alpha + \beta) = \operatorname{sen} \alpha \cos \beta + \operatorname{sen} \beta \cos \alpha$$

$$(3) \operatorname{sen}(\alpha - \beta) = \operatorname{sen} \alpha \cos \beta - \operatorname{sen} \beta \cos \alpha$$

$$(2) + (3)$$

$$\operatorname{sen}(\alpha + \beta) + \operatorname{sen}(\alpha - \beta) = 2 \operatorname{sen} \alpha \cos \beta$$

$$\left. \begin{array}{l} u = \alpha + \beta \\ w = \alpha - \beta \end{array} \right\}$$

$$\frac{u + w}{2} = \alpha$$

$$u - \frac{u + w}{2} = \beta$$

$$\frac{u - w}{2} = \beta$$

$$\operatorname{sen}(\alpha + \beta) + \operatorname{sen}(\alpha - \beta) = 2 \operatorname{sen} \alpha \cos \beta$$

$$(12) \operatorname{sen}(u) + \operatorname{sen}(w) = 2 \operatorname{sen} \left(\frac{u + w}{2} \right) \cos \left(\frac{u - w}{2} \right)$$

$$(2) \operatorname{sen}(\alpha + \beta) = \operatorname{sen} \alpha \cos \beta + \operatorname{sen} \beta \cos \alpha$$

$$(3) \operatorname{sen}(\alpha - \beta) = \operatorname{sen} \alpha \cos \beta - \operatorname{sen} \beta \cos \alpha$$

$$(2) - (3)$$

$$\operatorname{sen}(\alpha + \beta) - \operatorname{sen}(\alpha - \beta) = 2 \operatorname{sen} \beta \cos \alpha$$

$$(13) \operatorname{sen}(u) - \operatorname{sen}(w) = 2 \operatorname{sen} \left(\frac{u - w}{2} \right) \cos \left(\frac{u + w}{2} \right)$$

$$(4) \cos(\alpha + \beta) = \cos \alpha \cos \beta - \operatorname{sen} \alpha \operatorname{sen} \beta$$

$$(5) \cos(\alpha - \beta) = \cos \alpha \cos \beta + \operatorname{sen} \alpha \operatorname{sen} \beta$$

$$(4) + (5) \cos(\alpha + \beta) + \cos(\alpha - \beta) = 2 \cos \alpha \cos \beta$$

$$(14) \cos(u) + \cos(w) = 2 \cos \left(\frac{u + w}{2} \right) \cos \left(\frac{u - w}{2} \right)$$

$$(4) \cos(\alpha + \beta) = \cos \alpha \cos \beta - \operatorname{sen} \alpha \operatorname{sen} \beta$$

$$(5) \cos(\alpha - \beta) = \cos \alpha \cos \beta + \operatorname{sen} \alpha \operatorname{sen} \beta$$

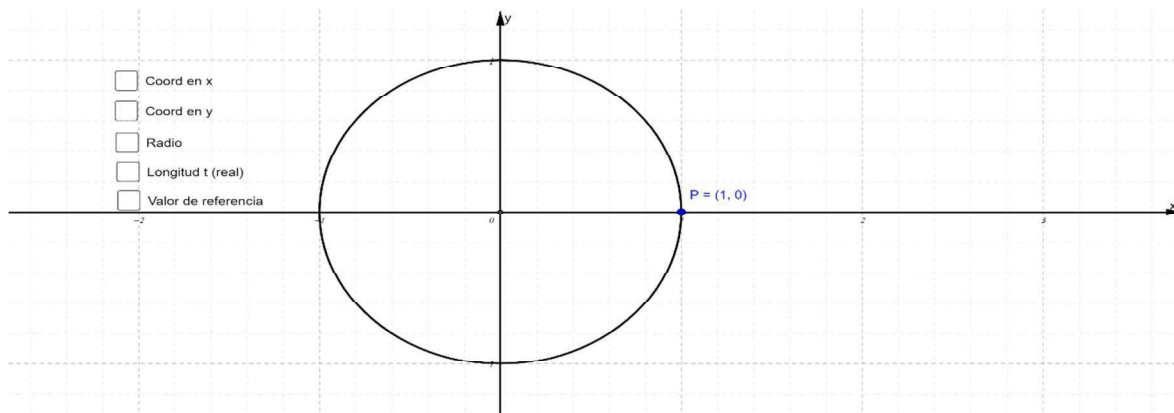
$$(4) - (5)$$

$$\cos(\alpha + \beta) - \cos(\alpha - \beta) = -2 \operatorname{sen} \alpha \operatorname{sen} \beta$$

$$(15) \cos(u) - \cos(w) = -2 \operatorname{sen} \left(\frac{u+w}{2} \right) \operatorname{sen} \left(\frac{u-w}{2} \right)$$

Valores fundamentales

	0°	30°	45°	60°	90°	180°	270°	360°	
sen	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0	
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1	
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞	0	$-\infty$	0	



Ecuaciones elementales

$$\operatorname{sen} x = 0$$

$$x = k\pi; k \in \mathbb{Z}$$

$$\operatorname{sen} x = 1$$

$$x = \frac{\pi}{2} + k \cdot 2\pi; k \in \mathbb{Z}$$

$$\operatorname{sen} x = -1$$

$$x = \frac{3\pi}{2} + k \cdot 2\pi$$

$$\cos(\alpha) = 0$$

$$\alpha = \frac{\pi}{2} + k \cdot \pi$$

$$\cos \alpha = 1$$

$$\alpha = 0 + k \cdot 2\pi$$

$$\cos \alpha = -1$$

$$\alpha = \pi + k \cdot 2\pi$$

Resolver:

$$\operatorname{Sen}\left(2x + \frac{\pi}{3}\right) = 0$$

$$\operatorname{Sen}\left(2x + \frac{\pi}{3}\right) = 0$$

$$2x + \frac{\pi}{3} = k\pi; \quad k \in \mathbb{Z}$$

$$2x = k\pi - \frac{\pi}{3}$$

$$x = \frac{k\pi}{2} - \frac{\pi}{6}$$

II	I
Sen+	Sen+
Cos--	Cos+
Tan--	Tan+
III	IV
Sen—	Sen—
Cos—	Cos+
Tan+	Tan--

II	I
$180^\circ - S_I$ $(\pi - S_I)$	S_I
III	IV
$180^\circ + S_I$ $(\pi + S_I)$	$360^\circ - S_I$ $(2\pi - S_I)$

$$\operatorname{sen} x = \frac{1}{2}$$

$$x \in \begin{cases} I \\ II \end{cases}$$

$$S_I = \operatorname{sen}^{-1}\left(\frac{1}{2}\right) = 30^\circ = \frac{\pi}{6}$$

$$x = \begin{cases} \frac{\pi}{6} + k \cdot 2\pi \\ \pi - \frac{\pi}{6} + k \cdot 2\pi \end{cases}$$

$$x = \begin{cases} \frac{\pi}{6} + k \cdot 2\pi \\ \frac{5\pi}{6} + k \cdot 2\pi \end{cases}$$

$$k = 0$$

$$x = \frac{\pi}{6}, x = \frac{5\pi}{6}$$

$$k = 1$$

$$x = \frac{\pi}{6} + 2\pi = \frac{13\pi}{6}; x = \frac{5\pi}{6} + 2\pi = \frac{17\pi}{6}$$

etc

$$\operatorname{sen} 2x = \frac{-\sqrt{2}}{2}$$

$$2x \in \begin{cases} III \\ IV \end{cases}$$

$$S_I = \operatorname{sen}^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45^\circ = \frac{\pi}{4}$$

$$2x = \begin{cases} \pi + \frac{\pi}{4} + k \cdot 2\pi \\ 2\pi - \frac{\pi}{4} + k \cdot 2\pi \end{cases}$$

$$2x = \begin{cases} \frac{5\pi}{4} + k \cdot 2\pi \\ \frac{7\pi}{4} + k \cdot 2\pi \end{cases}$$

$$x = \begin{cases} \frac{5\pi}{8} + k \cdot \pi \\ \frac{7\pi}{8} + k \cdot \pi \end{cases}$$

Resuelva:

$$2 \cos^2 x + \cos x - 1 = 0$$

$$u = \cos x$$

$$2u^2 + u - 1 = 0$$

$$u = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4} =$$

$$u_1 = -1$$

$$u_2 = \frac{1}{2}$$

$$\cos x = -1 \vee \cos x = \frac{1}{2}$$