$$tg\,30^\circ + \sec 30^\circ = \frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

15. Demostrar:

a. $\dfrac{1}{1-\cos\alpha} + \dfrac{1}{1+\cos\alpha} = 2\,cosec^2\,\alpha$

<u>Demostración:</u> Si observamos la identidad es preferible iniciar la demostración desde la expresión izquierda del signo =. Por lo tanto:

$$\frac{1}{1-\cos\alpha} + \frac{1}{1+\cos\alpha} = \frac{1+\cos\alpha+1-\cos\alpha}{(1-\cos\alpha)(1+\cos\alpha)} = \frac{2}{1-\cos^2\alpha} = \frac{2}{sen^2\alpha} = 2\,cosec^2\,\alpha$$

Luego, aplicando identidades trigonométricas básicas, hemos demostrado que

$$\frac{1}{1-\cos\alpha} + \frac{1}{1+\cos\alpha} = 2\,cosec^2\,\alpha$$

b. $cosec\,\alpha\,\sec\alpha - \dfrac{\cos\alpha}{sen\,\alpha} = tg\,\alpha$

<u>Demostración:</u> Análogamente a la demostración anterior, tenemos:

$$cosec\,\alpha \cdot \sec\alpha - \frac{\cos\alpha}{sen\,\alpha} = \frac{1}{sen\,\alpha}\cdot\frac{1}{\cos\alpha} - \frac{\cos\alpha}{sen\,\alpha}$$

$$= \frac{1-\cos^2\alpha}{sen\,\alpha\,\cos\alpha}$$

$$= \frac{sen^2\alpha}{sen\,\alpha\cdot\cos\alpha} = \frac{sen\,\alpha}{\cos\alpha} = tg\,\alpha$$

$$\therefore cosec\,\alpha \cdot \sec\alpha - \frac{\cos\alpha}{sen\,\alpha} = tg\,\alpha$$

c. $\dfrac{2\,sen\theta\,\cos\theta - \cos\theta}{1 - sen\theta + sen^2\theta - \cos^2\theta} = \cot g\,\theta$

Demostración: tenemos:

$$\frac{2\,sen\theta\cdot\cos\theta-\cos\theta}{1-sen\theta+sen^2\theta-\cos^2\theta}=\frac{2\,sen\theta\,\cos\theta-\cos\theta}{\left(1-\cos^2\theta\right)-sen\theta+sen^2\theta}=$$

$$=\frac{2\,sen\theta\,\cos\theta-\cos\theta}{sen^2\theta-sen\theta+sen^2\theta}=\frac{\cos\theta\,(2\,sen\theta-1)}{sen\,\theta(2\,sen\theta-1)}$$

$$=\frac{\cos\theta}{sen\theta}=\cot g\,\theta.$$

d. $sen^2\alpha\cdot\cos^2\beta-\cos^2\alpha\cdot sen^2\beta=sen^2\alpha-sen^2\beta$

Demostración: tenemos:

$$sen^2\alpha\cdot\cos^2\beta-\cos^2\alpha\cdot sen^2\beta=sen^2\alpha\left(1-sen^2\beta\right)-\left(1-sen^2\alpha\right)sen^2\beta=$$

$$sen^2\alpha-sen^2\alpha\cdot sen^2\beta-sen^2\beta+sen^2\alpha\cdot sen^2\beta=sen^2\alpha-sen^2\beta$$

$$\therefore\ sen^2\alpha\cdot\cos^2\beta-\cos^2\alpha\cdot sen^2\beta=sen^2\alpha-sen^2\beta$$

e. $\dfrac{sen^6\alpha-\cos^6\alpha}{2\,sen^2\alpha-1}=1-sen^2\alpha\cdot\cos^2\alpha$

Demostración: tenemos:

$$\frac{sen^6\alpha-\cos^6\alpha}{2\,sen^2\alpha-1}=\frac{\left(sen^3\alpha\right)^2-\left(\cos^3\alpha\right)^2}{2\,sen^2\alpha-1}=\frac{\left(sen^3\alpha-\cos^3\alpha\right)\left(sen^3\alpha+\cos^3\alpha\right)}{2\,sen^2\alpha-1}=$$

factorizamos la diferencia y suma de cubos

$$=\frac{(sen\alpha-\cos\alpha)\left(sen^2\alpha+sen\alpha\,\cos\alpha+\cos^2\alpha\right)(sen\alpha+\cos\alpha)\left(sen^2\alpha-sen\alpha\cos\alpha+\cos^2\alpha\right)}{2\,sen^2\alpha-1}$$

$$= \frac{(sen\alpha - \cos\alpha)\ (1 + sen\alpha\cos\alpha)\ (sen\alpha + \cos\alpha)(1 - sen\alpha\cos\alpha)}{2\ sen^2\alpha - 1}$$

$$= \frac{[\ (sen\alpha - \cos\alpha)\ (sen\alpha + \cos\alpha)\ ][\ (1 + sen\alpha\cos\alpha)\ (1 - sen\alpha\ \cos\alpha)\ ]}{2 sen^2\alpha - 1}$$

$$= \frac{(\ sen^2\alpha - \cos^2\alpha)(1 - sen^2\alpha\cos^2\alpha)}{2 sen^2\alpha - 1} = \frac{[\ sen^2\alpha - (1 - sen^2\alpha)](1 - sen^2\alpha \cdot \cos^2\alpha)}{2 sen^2\alpha - 1}$$

$$= \frac{(2 sen^2\alpha - 1)(1 - sen^2\alpha\ \cos^2\alpha)}{2\ sen^2\alpha - 1}$$

$$= 1 - sen^2\alpha \cdot \cos^2\alpha$$

$$\therefore \frac{sen^6\alpha - \cos^6\alpha}{2 sen^2\alpha - 1} = 1 - sen^2\alpha \cdot \cos^2\alpha.$$

f. $\dfrac{tg\,\beta + \sec^3\beta - \sec\beta}{\sec\beta} = tg^2\beta + sen\,\beta.$

<u>Demostración:</u> tenemos:

$$\frac{tg\,\beta + \sec^3\beta - \sec\beta}{\sec\beta} = \frac{tg\beta}{\sec\beta} + \frac{\sec^3\beta}{\sec\beta} - \frac{\sec\beta}{\sec\beta}$$

$$= sen\,\beta + (\sec^2\beta - 1) = sen\,\beta + tg^2\beta$$

En general, una identidad trigonométrica puede demostrarse utilizando distintas formas; como ejemplo, en el ejercicio anterior hacemos:

$$\frac{tg\beta + \sec^3\beta - \sec\beta}{\sec\beta} = \frac{\dfrac{sen\beta}{\cos\beta} + \dfrac{1}{\cos^3\beta} - \dfrac{1}{\cos\beta}}{\dfrac{1}{\cos\beta}} =$$

$$= \frac{\dfrac{sen\beta\cos^2\beta + (1 - \cos^2\beta)}{\cos^3\beta}}{\dfrac{1}{\cos\beta}} = \frac{sen\,\beta\ \cos^2\beta + sen^2\beta}{\cos^2\beta}$$

$$= \frac{sen\,\beta\,\cos^2\beta}{\cos^2\beta} + \frac{sen^2\beta}{\cos^2\beta} =$$

$$= sen\,\beta + tg^2\beta.$$

g. $(1 - sen\,\alpha + \cos\alpha)^2 = 2(1 - sen\,\alpha)(1 + \cos\alpha).$

Demostración: La iniciaremos elevando al cuadrado el trinomio:

$$(1 - sen\,\alpha + \cos\alpha)^2 = 1 + (sen^2\alpha + \cos^2\alpha) - 2\,sen\,\alpha + 2\cos\alpha - 2\,sen\,\alpha\,\cos\alpha$$

$$= (2 - 2\,sen\,\alpha) + (2\cos\alpha - 2\,sen\,\alpha\,\cos\alpha)$$

$$= 2\,(1 - sen\,\alpha) + 2\cos\alpha\,(1 - sen\,\alpha)$$

$$= (1 - sen\,\alpha)(2 + 2\cos\alpha)$$

$$= 2(1 - sen\,\alpha)(1 + \cos\alpha)$$

$$\therefore\ (1 - sen\,\alpha + \cos\alpha)^2 = 2(1 - sen\,\alpha)(1 + \cos\alpha).$$