1 The code

Using MATLAB, I produced my own implementation of PCG, GMRES and Preconditioned GMRES methods. I created three functions, and then I used them in three different files. For PCG, I used Cholesky preconditioner, while for the Preconditioned GMRES method I used the ILU preconditioner.

2 PCG

The Conjugate Gradient (CG) method is the iterative method of choice for solving large and sparse symmetric positive denite linear systems. It is also one of the best known iterative methods. Described in one sentence, it is the realization of an orthogonal projection technique onto the Krylov subspace generated by the initial residual:

$$x_k \in x_0 + K_k(A, r0)$$

subject to

$$r_k \perp K_k(A, r0)$$

When a preconditioner is used, the method is called Preconditioned Conjugate Gradient (PCG) method. PCG is expected to converge only for SPD matrices.

3 GMRES

In order to solve non-symmetric matrices, it is possible to use GMRES: a methods that provide orthogonality + minimization by using a long-term recurrence.

The GMRES (Generalized Minimal RESidual) method finds the solution of the linear system

$$Ax = b$$

by minimizing the norm of the residual

$$r_k = b - Ax_k$$

over all the vectors x_k written as

$$x_k = x_0 + y$$

$$y \in K_k(v_1)$$

where x_0 is an arbitrary initial vector and K_k is the Krylov subspace generated by the normalized initial residual ($v_1 = \frac{r_0}{||r_0||}$).