APPLIED SIGNAL PROCESSING LABORATORY

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Detection of a signal

The goal of this assignment is to use correlation to detect the presence of an alarm signal generated by a sensor.

Correlation gives a measure of the similarity between two signals and is defined as:

$$c_{1,2} = \int s_1(t)s_2(t)$$

Furthermore, the correlation value is in between 0 and 1.
The correlation is 1 when the two signals have t he same shape and 0 if they are orthogonal.

Presence of a pulse inside a time window

Suppose to observe a time window which may or may not contain a pulse of a sensor. In both cases, thermal noise is present.

Two events have to be distinguish:

- H_0 the window contains only noise;
- H_1 the window contains a noisy version of the pulse.

Correlations needs to be compute between the observed signal and the original sensor pulse. If this correlation si very high then the window contains the pulse, otherwise only noise.

False Alarm and Missed Detection

So, we denote as Γ our correlation test.

A threshold is set and if $\Gamma \geq t$ the pulse is present (event H_1), otherwise the pulse is absent (event H_0).

Two scenarios are possible:

- False alarm: the pulse is absent but due to noise the test is above the threshold. The probability of false alarm is

$$P_{fa} = P_r(\Gamma \geq t | H_0)$$

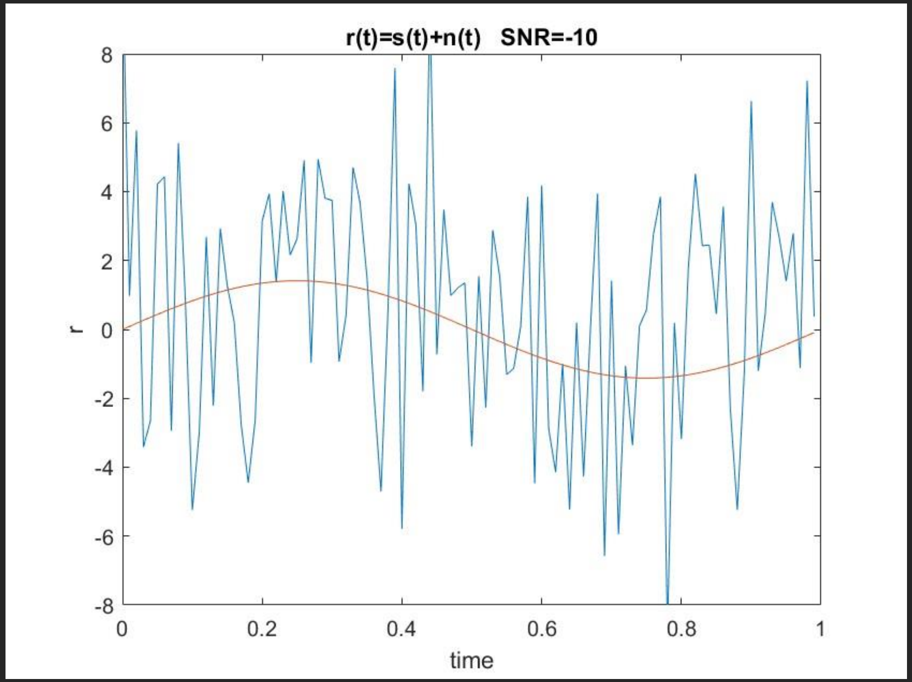
- Missed detection: the pulse is present () but due to noise the test is below the threshold. The probability of missed detection is

$$P_{md} = P_r(\Gamma < t | H_1)$$

Exercise

- After creating a signal and fixing the SNR value, generate and plot a noise signal realization;
- Generate num_{sim} noise realizations n(t), compute the normalized correlation test Γ for the observed signal under H_1 and under H_0 and plot the num_{sim} value of Γ under H_1 and under H_0 ;
- Plot the pdfs of Γ under H_1 and under H_0 using *histogram* function;
- Plot P_{fa} and P_{md} vs the threshold t;
- Plot the ROC curve;
- Repeat exercises using energy computed over time instead of the correlation.

Signal plus noise



The sensor outputs generates a sinusoidal signal with f_1=1 Hz and an amplitude of $A = \sqrt{2}$.

The signal is:

$$s(t) = Asin(2\pi f_1 t)$$

The noise is generated using:

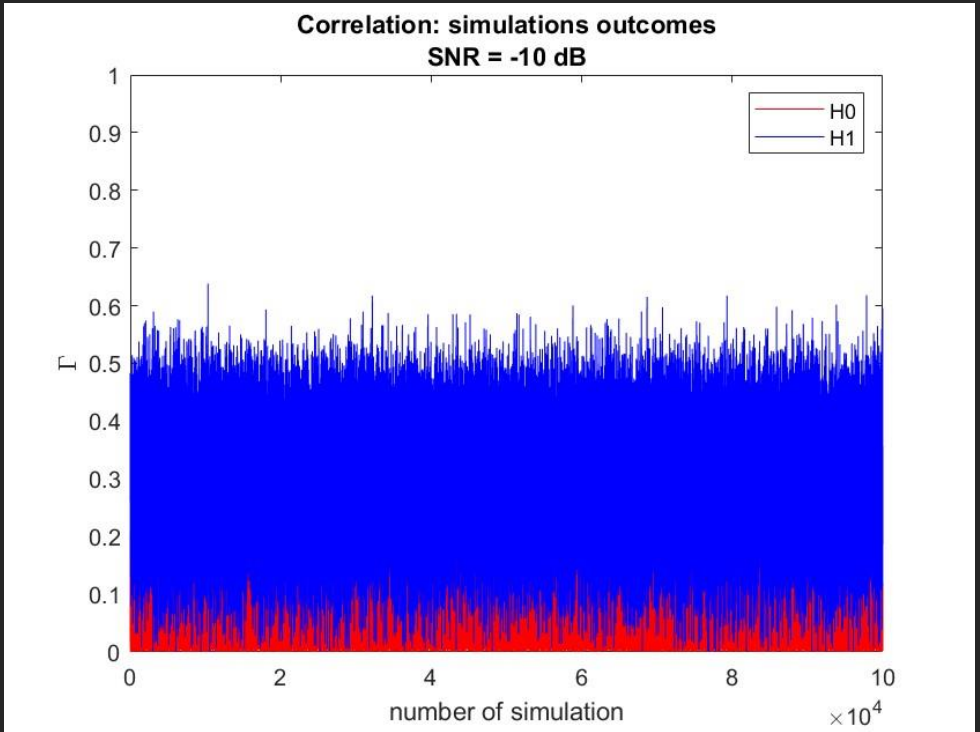
$$n(t) = \sqrt{\frac{P_s}{SNR}} randn(1, N)$$

We call:

$$r(t) = s(t) + n(t)$$

the noisy signal.

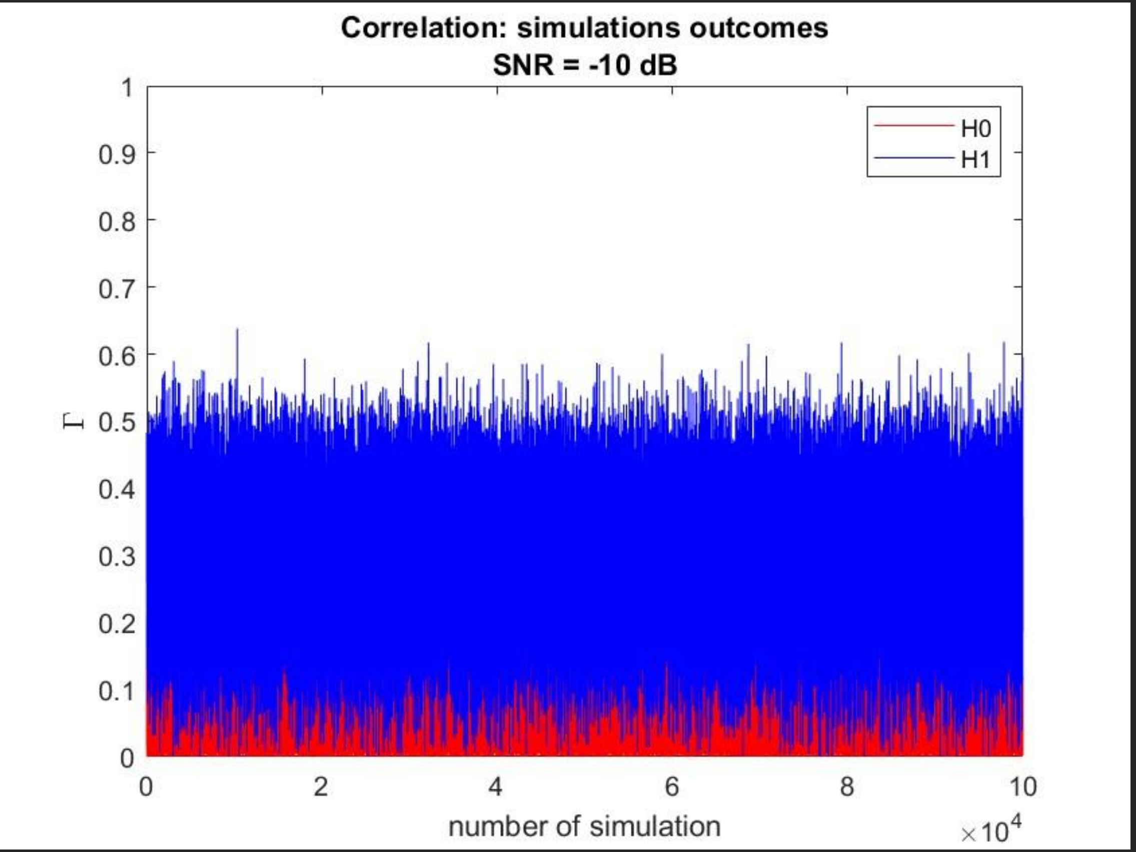
Correlation




```matlab
%% correlation
vector= 1:num_sim;
for i=vector
    n= sqrt(p_n)*randn(1,N); %noise
    r=s+n; %create total signal
    %compute the energy in order to normalize
    E_n= sum(n.^2)*Ts; %energy of the noise
    E_r= sum(r.^2)*Ts; %energy of the total signal
    norm_n= n/sqrt(E_n); %normalized to have unitary energy
    norm_r= r/sqrt(E_r);%normalized to have unitary energy
    Gamma_H0(i)= abs(sum(norm_n.*s)*Ts); %compute gammma_H0
    Gamma_H1(i)= abs(sum(norm_r.*s)*Ts); %compute gamma
end
```

This portion of the code generates a num_{sim} noise realizations n(t) and then, for both n(t) and r(t), the normalized correlations test Γ is computed.

Correlation: plot



Let's analyze the plot in details:

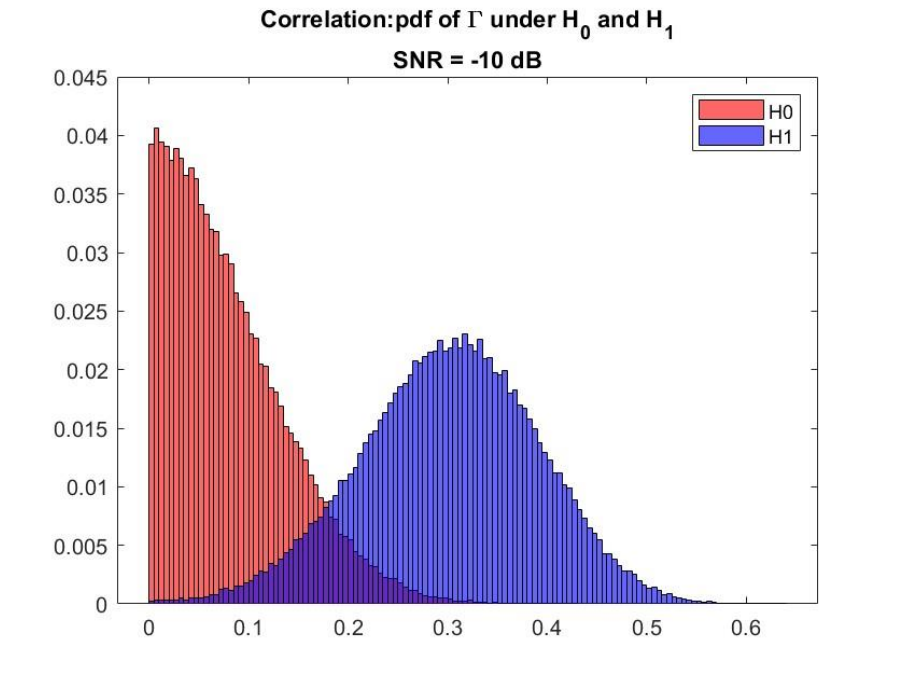
- H_1: is the event where the time window contains a noisy version of the pulse. Because in this case the signal sensor is present and we are computing correlation with the original signal, the H_1 values are higher;

- H_0: is the event where the time window contains only noise. We are still computing the correlation with the original signal and due to the absence of the original signal, H_0 values are around zero.

Correlation: histogram

The plot represents the histogram of the distribution of the correlation Γ under H_1 and under H_0.

- H_1: due to the fact that the signal is present, the correlation is more similar to a normal gaussian;
- H_0: due to the absence of the signal, the correlation has a peak on the zero value. This means a lower correlation, as expected.



False Alarm and Missed Detection

```
%% probability
tmin=min(Gamma_H0);
tmax=max(Gamma_H1);
N_bins=100;
t=linspace(tmax,tmin,N_bins);

for i= 1:N_bins
    if(length(find(Gamma_H0>=t(i)))<30)
        P_fa(i) = 0;
    else
        P_fa(i) = length(find(Gamma_H0 >= t(i)))/num_sim;
    end

%missed detection probability : Pmd = Pr (Γ < t| H1)
    if (length(find(Gamma_H1<t(i)))<30)
        P_md(i) = 0;
    else
        P_md(i)  = length(find(Gamma_H1 < t(i)))/num_sim;
    end
end
```

To consider the probability of false alarm P_{fa} and the probability of missed detection P_{md} , we consider 100 threshold between the minimum and the maximum of Γ.

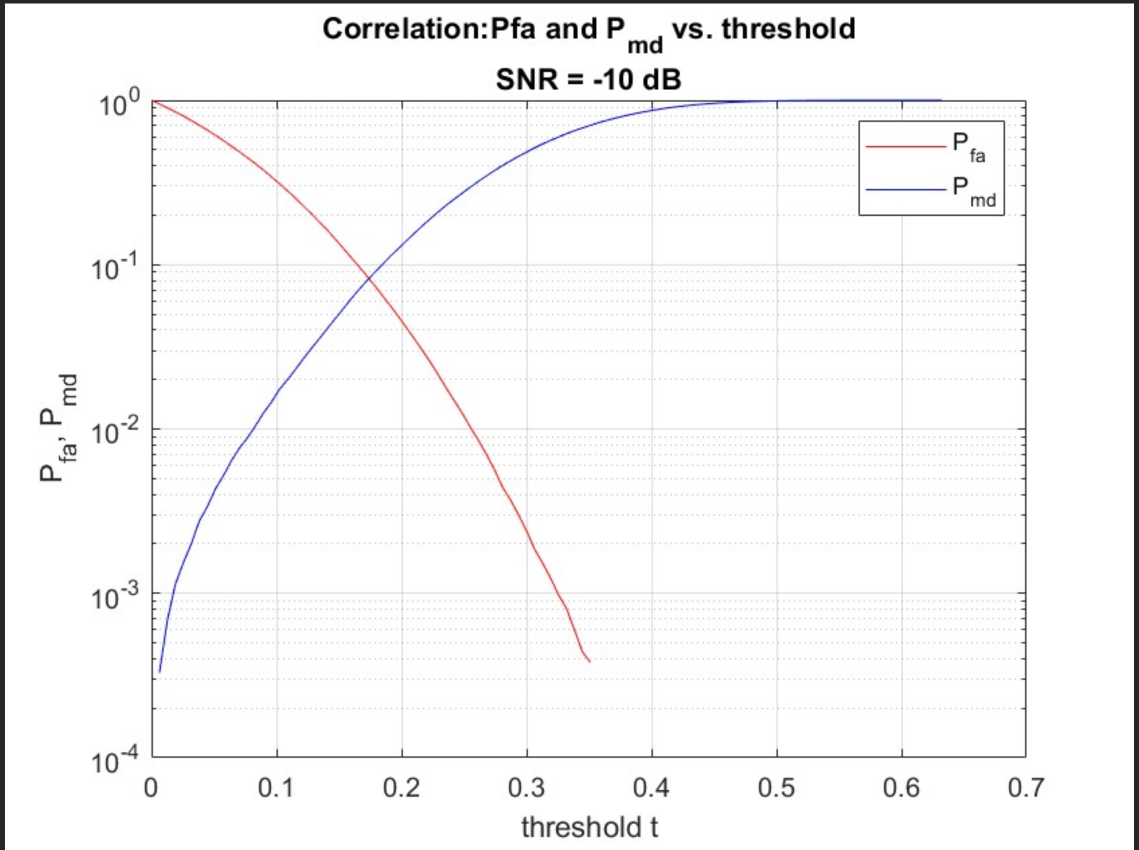
Then we compute

$$P_{fa}(t) = lenght(find(Gamma(H0) \geq t))/num_{sim}$$

$$P_{md}(t) = lenght(find(Gamma(H1) < t))/num_{sim}$$

In order to have a good reliability of the measured probability, an **if** statement is implemented.

False Alarm and Missed Detection: output



The plot shows how these two probabilities change as the threshold varies. In particular, P_{fa} decreases as the threshold increases, while P_{md} increases.

This means that raising the threshold makes the detection more prone to missing real signals.

Furthermore, it reflects the fact that for lower values of t, pure noise may be detected and this condition increases false alarm.

ROC curve

Defining the probability of correctly detecting the signal as:

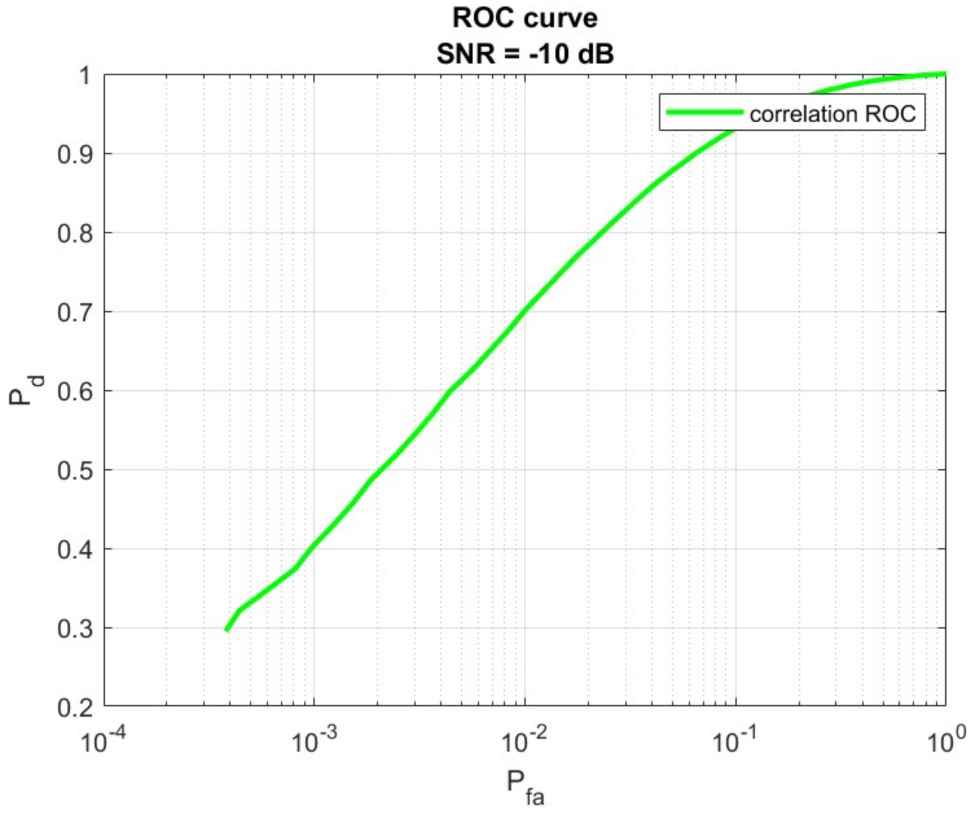
$$P_d = (1 - P_{md})$$

we plot the ROC curve.

The ROC curve illustrates the diagnostic ability of a binary classifier system as its discrimination threshold is varied.

The closer this curve is to the top left corner, the better is the performance of the binary classifier.

Due to SNR=-10, we are far from ideality.

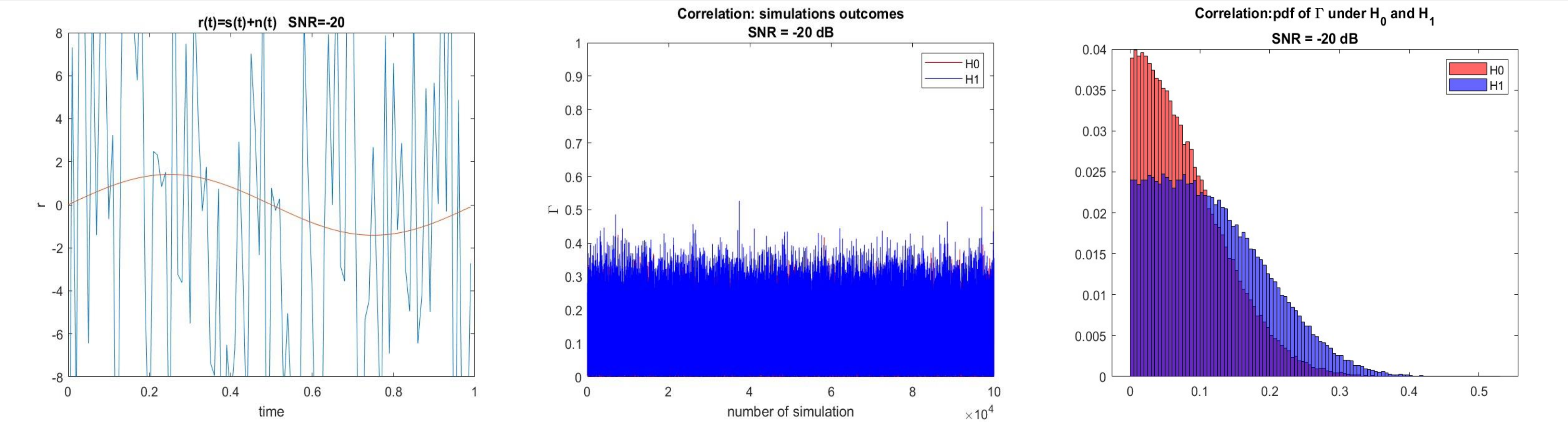


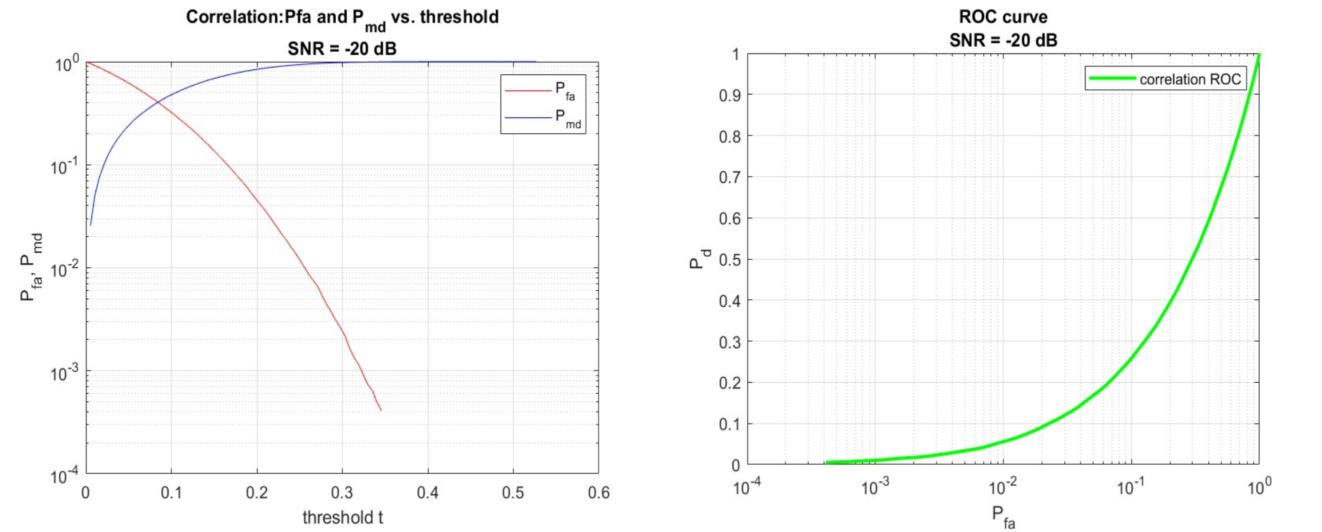
Example: SNR=-20dB

If we decrease the SNR value, the noise has an higher contribution on the signal, as consequence:

- The resulting signal is very noisy;
- Correlation is more overlapped, this increase the difficulty to detect the signal correctly;
- ROC curve less closer to the top left corner and more close to zero. Then, there is a more reliable detection only for high false alarm.

SNR=-20dB: outputs



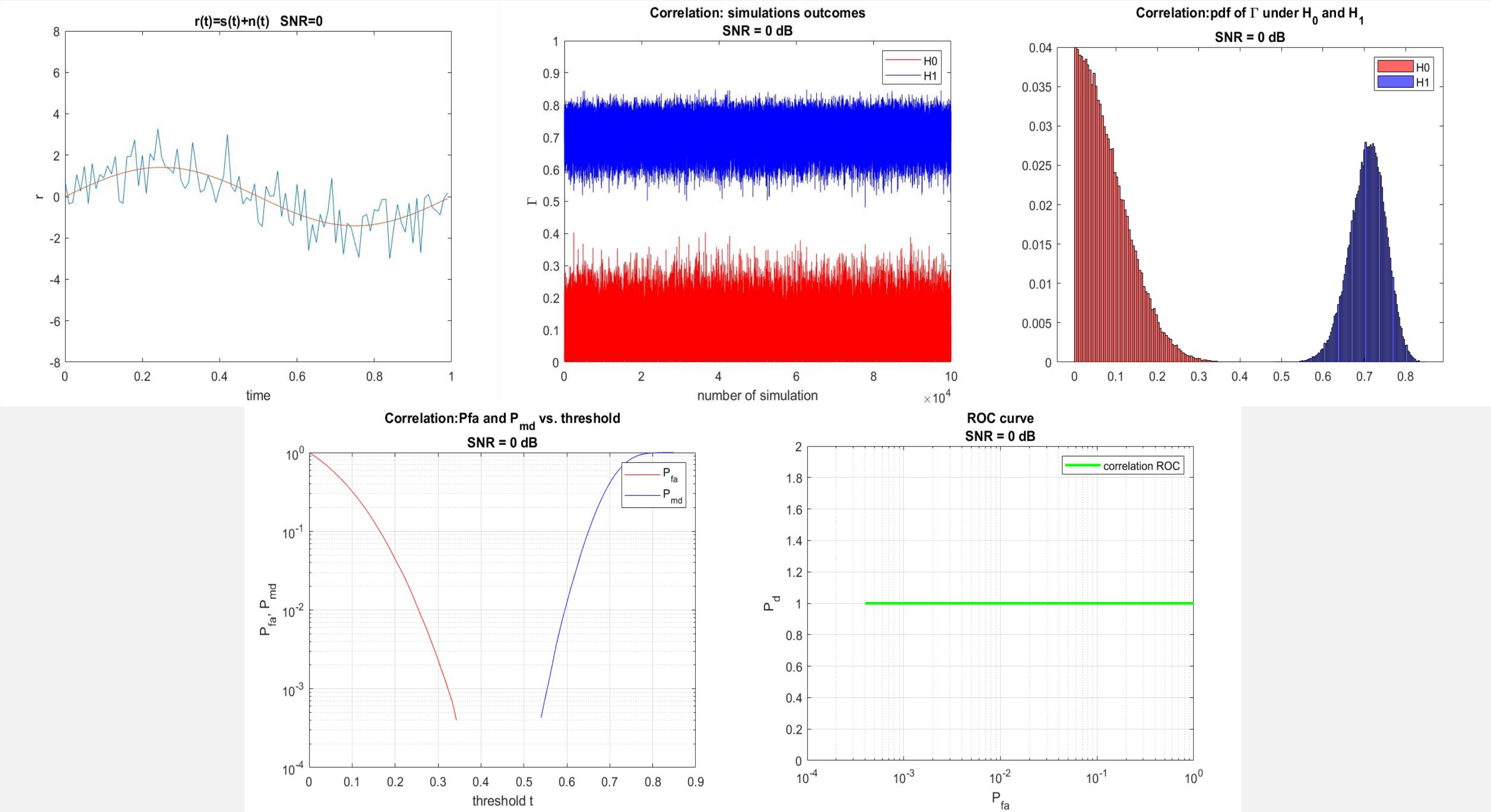


SNR= 0dB

If we increase the SNR value, the noise has a lower contribute on the signal, as consequence:

- Resulting signal is less conditioned by the noise. This produce a more easy detection;
- False alarm and missed detection are no more overlapped. So, if we set the threshold between 0.4 and 0.5 it is possible to obtain a zero false alarm and missed detection probability;
- ROC curve is equal to 1 and this means that the correct detection is always performed.

SNR=0dB: outputs

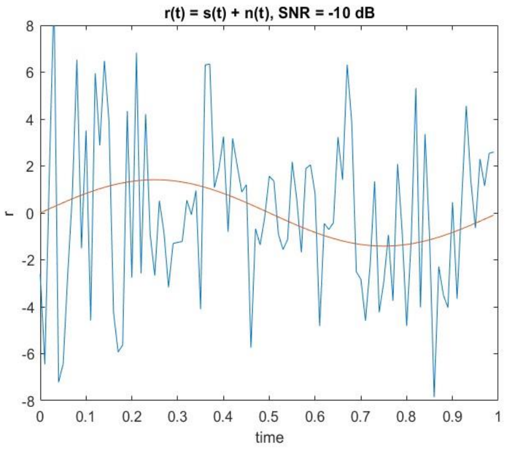


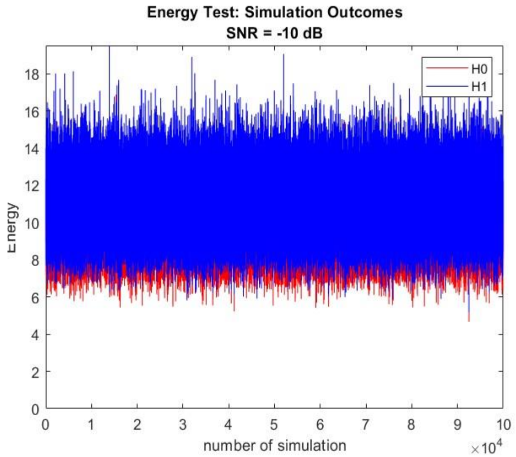
Energy instead of correlation

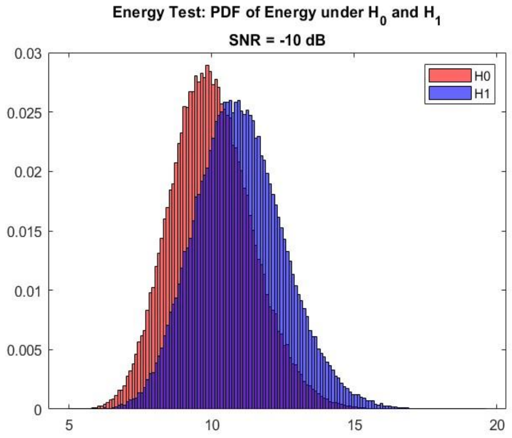
We repeated the entire exercise by using as statistical test Γ the energy computed over the time window instead of the correlation.

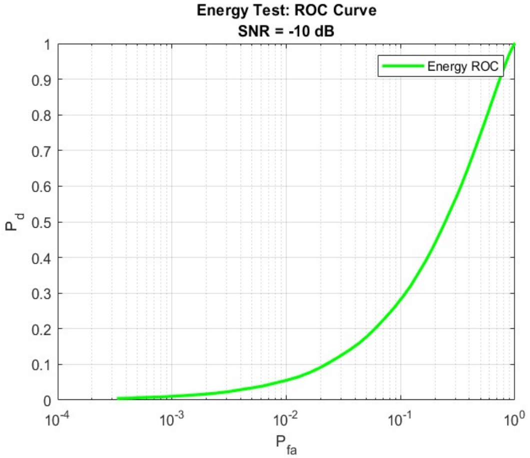
In order to do that, we did not normalize but instead used the energy of the total signal and the noise.

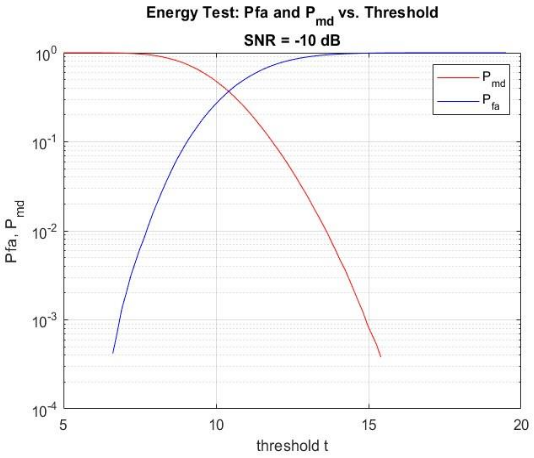
Energy test


r(t) = s(t) + n(t), SNR = -10 dB
r
time


Energy Test: Simulation Outcomes
SNR = -10 dB
H0
H1
number of simulation


Energy Test: PDF of Energy under H0 and H1
SNR = -10 dB
H0
H1


Energy Test: ROC Curve
SNR = -10 dB
Energy ROC
Pd
Pfa


Energy Test: Pfa and Pmd vs. Threshold
SNR = -10 dB
Pmd
Pfa
Pfa, Pmd
threshold t

ROC COMPARISON

The graphs look different because we're using two different ways to check for the alarm signal. One way looks at how similar the signals are over time, while the other focuses more on the total power of the signal.

Considering our scenario, the correlation method might be preferable if we anticipate clear patterns in the signals.