Politecnico di Torino

Computational linear algebra for large scale problems
HW3_SVD for image classification

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Indice

1 Introduction

The objective of this optional homework is to apply the knowledge gained this semester on information compression using techniques like SVD (Singular Value Decomposition) and PCA (Principal Component Analysis).
The focus of this analysis is the application of these techniques to images. Specifically, a famous dataset from Kaggle known as ORL Faces has been chosen. This dataset contains 40 subfolders, each corresponding to a different person. Each subfolder contains 10 images of that person, featuring different facial expressions.
The objective of this project is to apply the methods mentioned above for image recognition.

2 SVD

The `Singular Value Decomposition` (SVD) is a matrix decomposition technique.
Given a matrix of any size $A \in \mathbb{R}^{m \times n}$, the SVD decomposes the matrix as the product of three matrices:

$$A = U \Sigma V^T \tag{1}$$

where:

- $U \in \mathbb{R}^{m \times m}$ is an orthogonal matrix containing the left singular vectors of A,
- $\Sigma \in \mathbb{R}^{m \times n}$ is a diagonal matrix containing the singular values of A arranged along the diagonal. The singular values are non-negative numbers and are ordered in decreasing order,
- $V \in \mathbb{R}^{n \times n}$ is an orthogonal matrix containing the right singular vectors of A.

Dimensionality reduction via SVD can be achieved by selecting the first k singular values, where k is the desired number of principal components. In this case, the matrix A is approximated as follows:

$$A_k \approx U_k \Sigma_k V_k^T \tag{2}$$

where U_k contains the first k left singular vectors, Σ_k is the diagonal matrix containing the first k singular values, and V_k contains the first k right singular vectors. The approximated matrix A_k thus has a reduced rank and retains most of the information present in A, with a significantly smaller number of dimensions.

SVD is particularly powerful in data compression applications, such as in the case of images, where the use of SVD allows reducing the amount of data while preserving the main structure of the image.

3 PCA

`Principal Component Analysis` (PCA) is a statistical technique used for dimensionality reduction while preserving as much variance in the data as possible. The goal of PCA is to transform a dataset into a new coordinate system, such that the greatest variance of the data lies along the first coordinate (principal component), the second greatest variance along the second coordinate, and so on.

4 Code

4.1 Singular value importance

The first operation performed in the code was to test SVD on various images. In particular, what was done was to plot the importance of each component in the decomposition.
Below, I present the result of applying the code to two images.

As seen on the left, there is the image of the person in question.

The central plot shows how "important" each component of the decomposition is. The first singular value is much larger than the others. The subsequent values rapidly decay towards zero. This

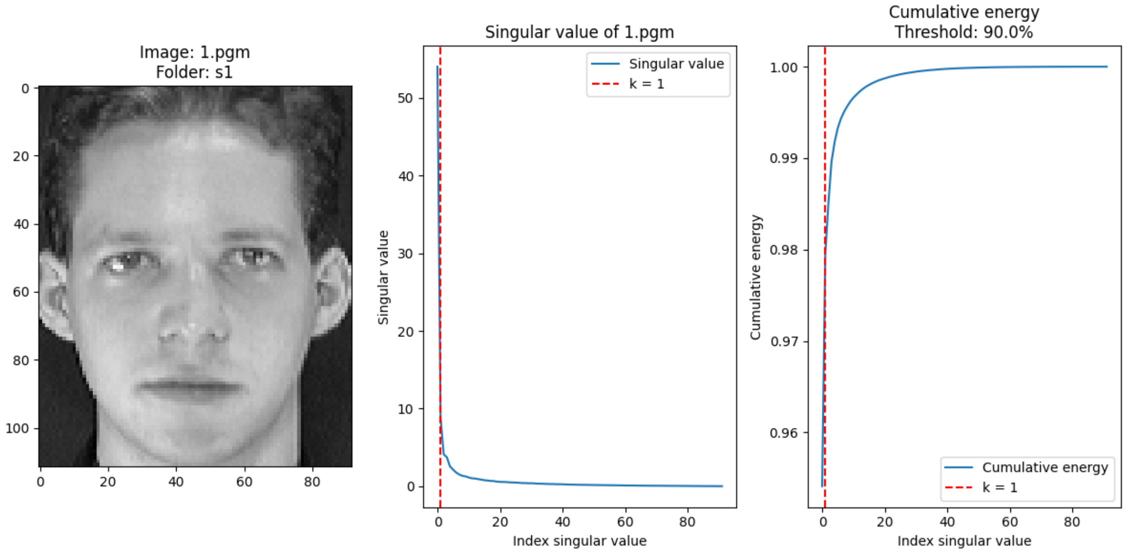


Figura 1: SVD

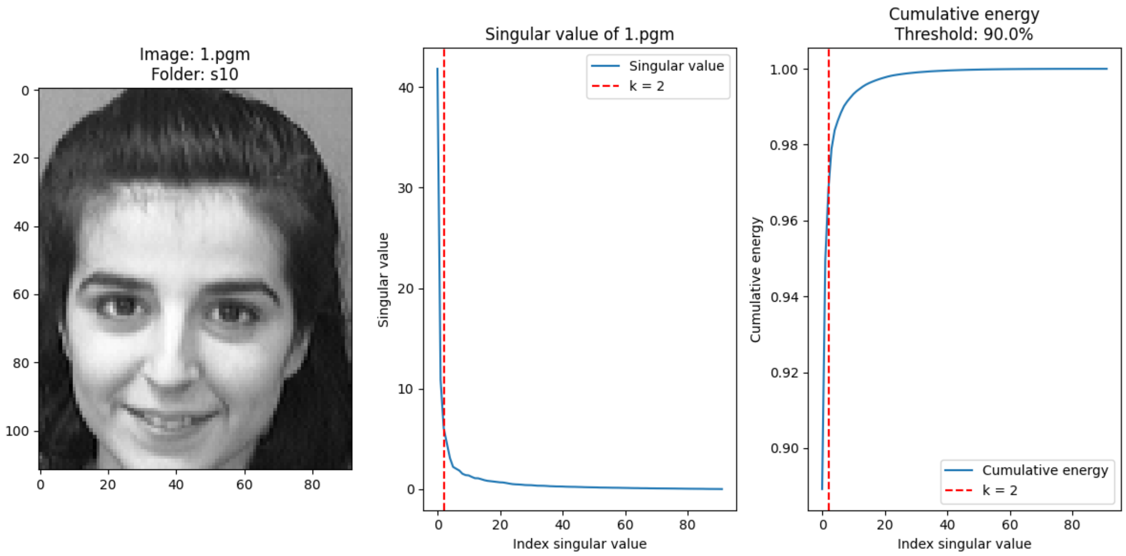


Figura 2: SVD

means that the amount of the image information contained in the following singular values is lower.

On the right plot is shown how much of the original information is captured by using the first k components. It is clear that with just a few components (around 2), 90% of the energy is already captured. This means the image can be well approximated using only the first few components. The energy of a matrix is the sum of the squares of its singular values, representing the total "power" or magnitude of the matrix. This implies that the quality of the approximation of a matrix A can be evaluated through the singular value σ_{k+1}, allowing us to quantify how well the matrix A_k approximates A.

$$A \approx \sum_{i=1}^{k} \sigma_1 A_1 \tag{3}$$

By this simple plots, we can understand the importance of this application: instead of storing all the pixels, we could use only the first k principal components. We would obtain a good approximation of the image with much less memory.

4.2 Classification

Our dataset is now divided into train (70%) and test (30%). For each image, instead of saving the entire matrix, the first k principal components are saved, both for SVD and PCA. A model is then trained, in this case, we used the `KNeighborsClassifier`, and it was trained to recognize people from the images giving them a label. Below are the values of the model based on the `accuracy_score` metric.

Metric	**Value**
Accuracy	91.67%
Macro avg (Precision)	0.94
Macro avg (Recall)	0.92
Macro avg (F1-Score)	0.91
Weighted avg (Precision)	0.94
Weighted avg (Recall)	0.92
Weighted avg (F1-Score)	0.91

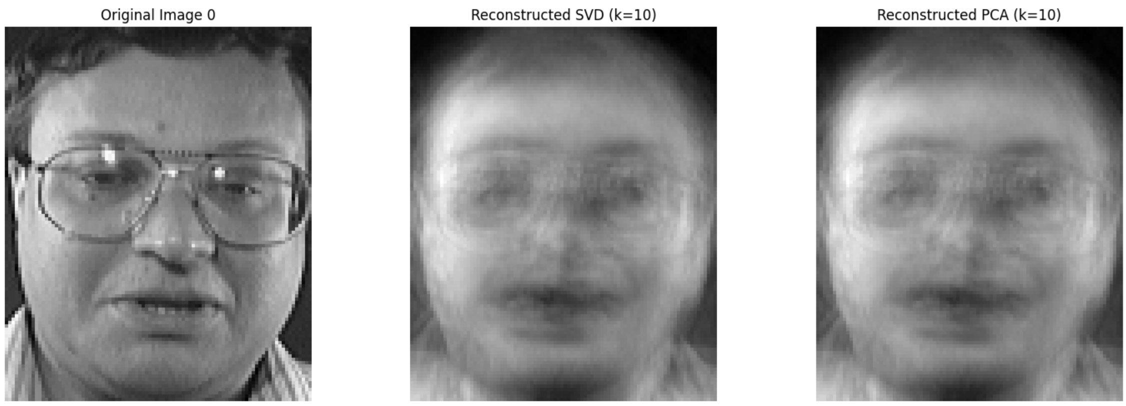
Tabella 1: Performance of the model using SVD $k = 10$

Metric	**Value**
Accuracy	90.83%
Macro avg (Precision)	0.94
Macro avg (Recall)	0.91
Macro avg (F1-Score)	0.90
Weighted avg (Precision)	0.94
Weighted avg (Recall)	0.91
Weighted avg (F1-Score)	0.90

Tabella 2: Performance of the model using PCA $k = 10$

As we can observe, both models are able to correctly classify most of the people. The model using SVD performs better than the one using PCA, demonstrating its importance in capturing the most significant information.

We found it interesting to also visualize what the models have learned. Therefore, for each image, we plotted the corresponding model using SVD and PCA. This allows us to see how each method represents the data and captures the essential features that the classifier uses for recognition. By comparing the visualizations, we can gain insight into how each dimensionality reduction technique affects the learning process and the resulting feature representations.



However, there does not seem to be a noticeable difference between what the model has learned using SVD and what it has learned using PCA. This is because we chose an extremely high value of k for both methods. For instance, with SVD, we achieved 90% of the image information using just 2 singular values.

For demonstration purposes, we now choose a much higher $k = 90$ for SVD and a much lower $k = 2$ for PCA to better highlight the differences between the two methods in terms of how they capture the essential features of the images.

The difference between using many components (as in the case of SVD with $k = 90$) or just a few (as in PCA with $k = 2$) is dramatically visible in the quality of the reconstruction.

In the case of SVD with $k = 90$, the model retains almost all the key details of the image. Since $k = 90$ includes the majority of the significant singular values, the reconstruction process captures most of the variance in the data, allowing for a detailed representation of the original image. The higher number of components ensures that fine details such as textures, subtle lighting variations, and the features of the person's face are preserved. This provides a clear and accurate representation of the original image, making it almost indistinguishable from the original.

On the other hand, when using PCA with $k = 2$, only two principal components are kept. This drastic reduction in dimensionality means that much of the detail and information is lost. The two components capture only the most general features of the image, such as the broad shape or the overall structure. As a result, the reconstructed image appears blurry, lacking the finer details, and may not clearly represent the person's unique features. The image will be highly abstracted, with only the most significant variance captured, and may not preserve the individual characteristics that differentiate one person from another. .