Sound Synthesis and Spatial Processing

Homework 4

Wave Digital Filter modeling of a three way crossover network

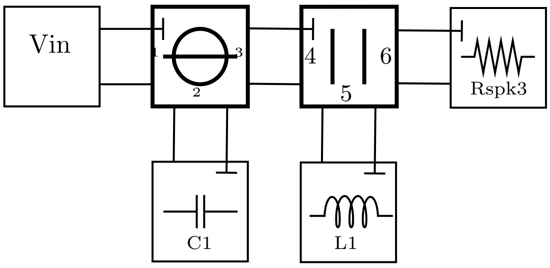
Student Name	**Student ID**
Mattia Massimi	102635

1 Problem Formulation

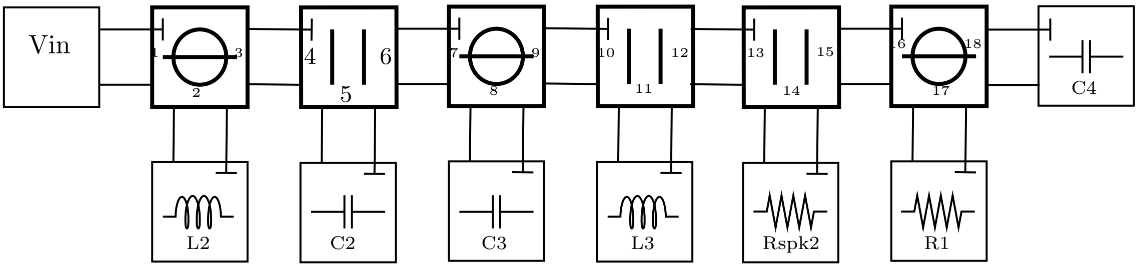
The aim of this project is to model a three way crossover network in the wave digital domain, starting from a reference analog circuit.

2 WDF schemes

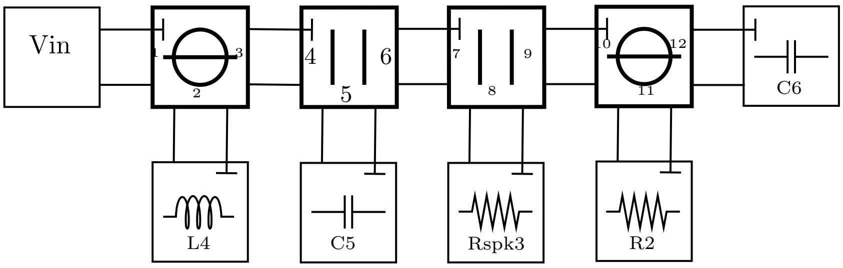
Here are represented the schemes used to implement the analog circuit with all the free parameters set in order to adapt every linear element. The circuits are implemented separately so the variables are always identified by the letter L, M or H (Low, Mid, High).



Highpass scheme



Lowpass scheme



Lowpass scheme

Highpass free parameters:
$\mathbf{z2H} = \frac{T_S}{2C1}$, $\mathbf{z5H} = \frac{2L_1}{T_s}$, $\mathbf{z6H} = RspkHigh$, $\mathbf{z4H} = \frac{z6H \cdot z5H}{z6H + z5H}/$, $\mathbf{z3H} = z4H$, $\mathbf{z1H} = z3H + z2H$.

Bandpass free parameters:
$\mathbf{z18M} = \frac{T_s}{2C4}$, $\mathbf{z17M} = R1$, $\mathbf{z16M} = z18M + z17M$, $\mathbf{z15M} = z16M$, $\mathbf{z14M} = RspkMid$, $\mathbf{z13M} = \frac{z14M \cdot z15M}{z14M + z15M}$, $\mathbf{z12} = z13M$, $\mathbf{z11M} = \frac{2L3}{T_s}$, $\mathbf{z10M} = \frac{z11M \cdot z12M}{z11M + z12M}$, $\mathbf{z9M} = z10M$, $\mathbf{z8M} = \frac{T_s}{2C3}$, $\mathbf{z7M} = z8M + z9M$, $\mathbf{z6M} = z7M$, $\mathbf{z5M} = \frac{T_s}{2C2}$, $\mathbf{z4M} = \frac{z5M \cdot z6M}{z5M + z6M}$, $\mathbf{z3M} = z4M$, $\mathbf{z2M} = \frac{2L2}{T_s}$, $\mathbf{z1M} = z2M + z3M$.

Lowpass free parameters:
$\mathbf{z12L} = \frac{T_s}{2C6}$, $\mathbf{z11L} = R2$, $\mathbf{z10L} = z11L + z12L$, $\mathbf{z9L} = z10L$, $\mathbf{z8L} = RspkLow$, $\mathbf{z7L} = \frac{z8L \cdot z9L}{z8L + z9L}$, $\mathbf{z6L} = z7L$, $\mathbf{z5L} = \frac{T_s}{2C5}$, $\mathbf{z4L} = \frac{z5L \cdot z6L}{z5L + z6L}$, $\mathbf{z3L} = z4L$, $\mathbf{z2L} = \frac{2L4}{T_s}$, $\mathbf{z1L} = z2L + z3L$.

3 Answers

3.1 Error in each frequency range

The low frequency error has a peak around the half of the total length and the others both have roughly the same shape with the maximum at the end of the signal, corresponding to the high frequencies. We can notice also that the low frequencies have the error spread throughout the whole length (and so the bandwidth) even when there's a very low signal (figure 1, 2, 3). As expected the least accurate is the high frequency one, that has the error in the order of 0.02, a thousand times bigger then the mid one. This is mainly due to the time derivative approximation method used (i.e. trapezoidal rule). This mapping is defined by:

$$\omega = \frac{2}{T_s} \tan\left(\tilde{\omega}\frac{T_s}{2}\right) \tag{1}$$

So ω is really close to $\tilde{\omega}$ at low frequencies and will differ more at high frequencies, because of the tangent. This explains the big error in the high frequencies plot and the fact that error is still present even when there's a very low signal in the low frequency plot.

3.2 Sampling frequency

Increasing the sampling frequency gives us better results in terms of error in the 3 bands because as we can see in (1) if we increase the sampling frequency $F_s = 1/T_s$, T_s become smaller and the range in which ω and $\tilde{\omega}$ are more similar becomes larger. This is confirmed by the plots shown in figures 1,2 and 3.

3.3 Root elements

If we add a diode in parallel to the speaker resistor we will have a WDF with 2 elements that cannot be adapted: the ideal generator and the diode so we would have 2 roots. In this case we can only use iterative methods to solve the problem (since the two elements cannot be grouped in the same root). In the second case, if we use a resistive generator that can be adapted we can solve the circuit using the diode (the only element that can't be adapted) as the root instead of the generator.

3.4 Backward Euler method

Here are reported all the mathematical steps to needed to derive the WD model of an inductor using the backward Euler method:

$$s = j\omega \leftarrow \frac{1 - z^{-1}}{T_s} \tag{2}$$

- Continuous time domain constitutive equation:

$$V(s) = sLI(s) \tag{3}$$

- Backward Euler mapping and approximation obtained by applying (2) in (3)

$$V[k] = \frac{L}{T_s}I[k] - \frac{L}{T_s}I[k-1] \tag{4}$$

- Referring to the generic one port element constitutive equation we have:

$$R_e = \frac{L}{T_s},\ V_e[k] = -\frac{L}{T_s}I[k-1] \tag{5}$$

- Non adapted scattering relation

$$b[k] = \frac{L/T_s - Z[k]}{L/T_s + Z[k]}a[k] + \frac{b[k-1] - a[k-1]}{L/T_s + Z(k)}\frac{L}{T_s} \tag{6}$$

- Adapted scattering relation with $Z[k] = \frac{L}{T_s}$

$$b[k] = \frac{b[k-1] - a[k-1]}{2} \tag{7}$$

Constitutive Eq.	Wave mapping in case of adaptation	Adaptation condition
$v(t) = L\frac{di(t)}{dt}$	$b[k] = \frac{b[k-1]-a[k-1]}{2}$	$Z[k] = \frac{L}{T_s}$

Table 1: WD inductor model based on Backward Euler Method

4 Plots

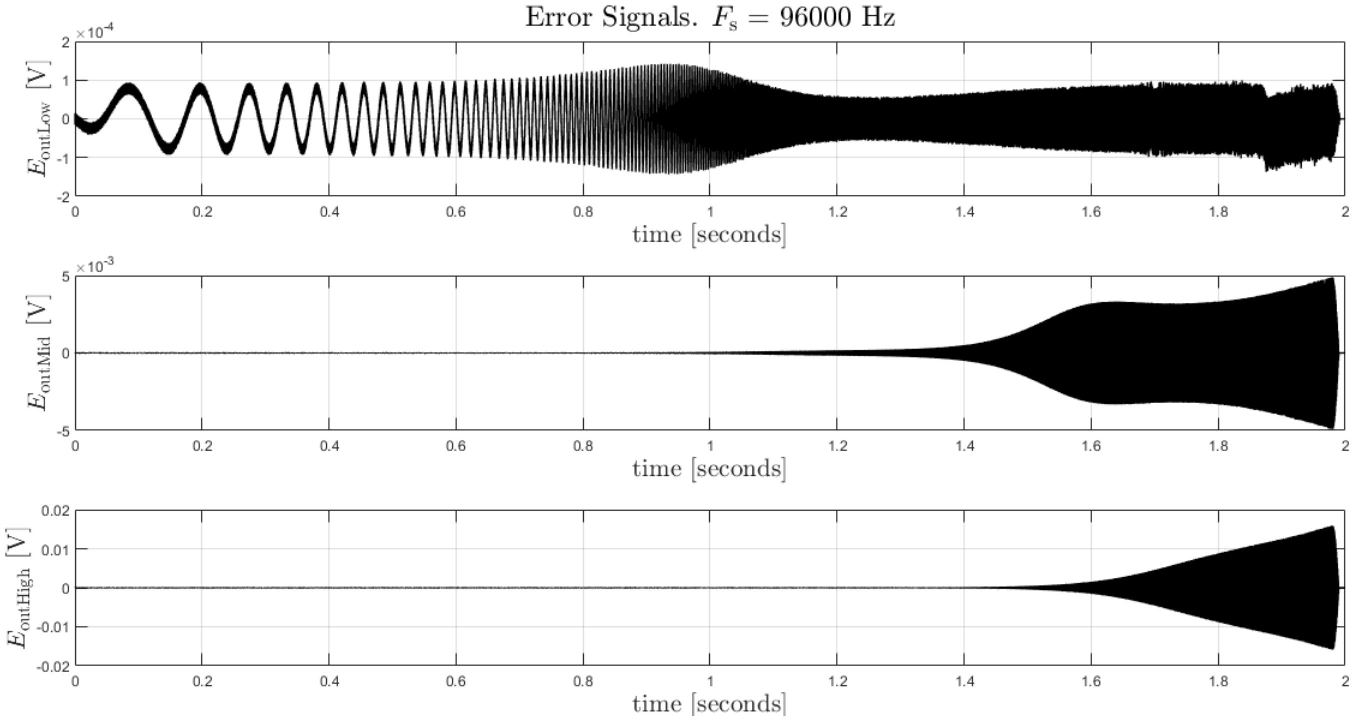


Figure 1: Error signals plots for $F_s = F_{LTspice}/2$

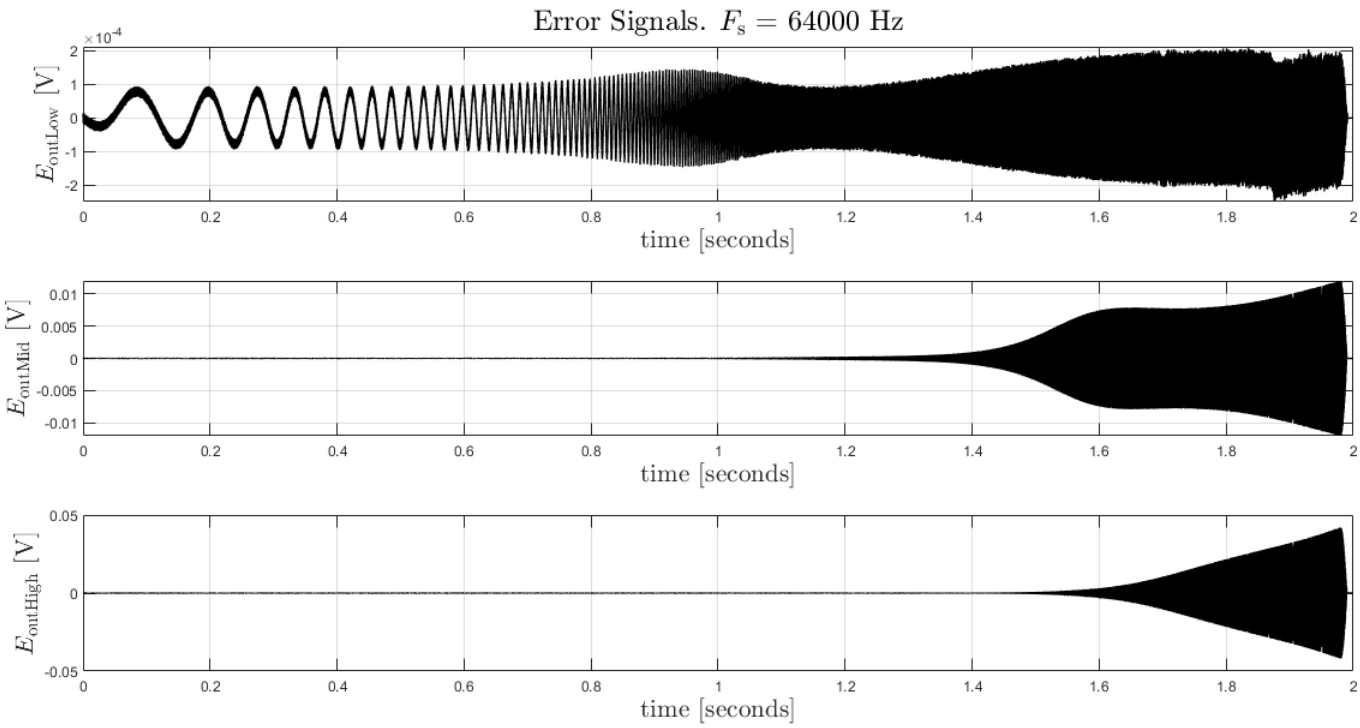


Figure 2: Error signals plots for $F_s = F_{LTspice}/3$

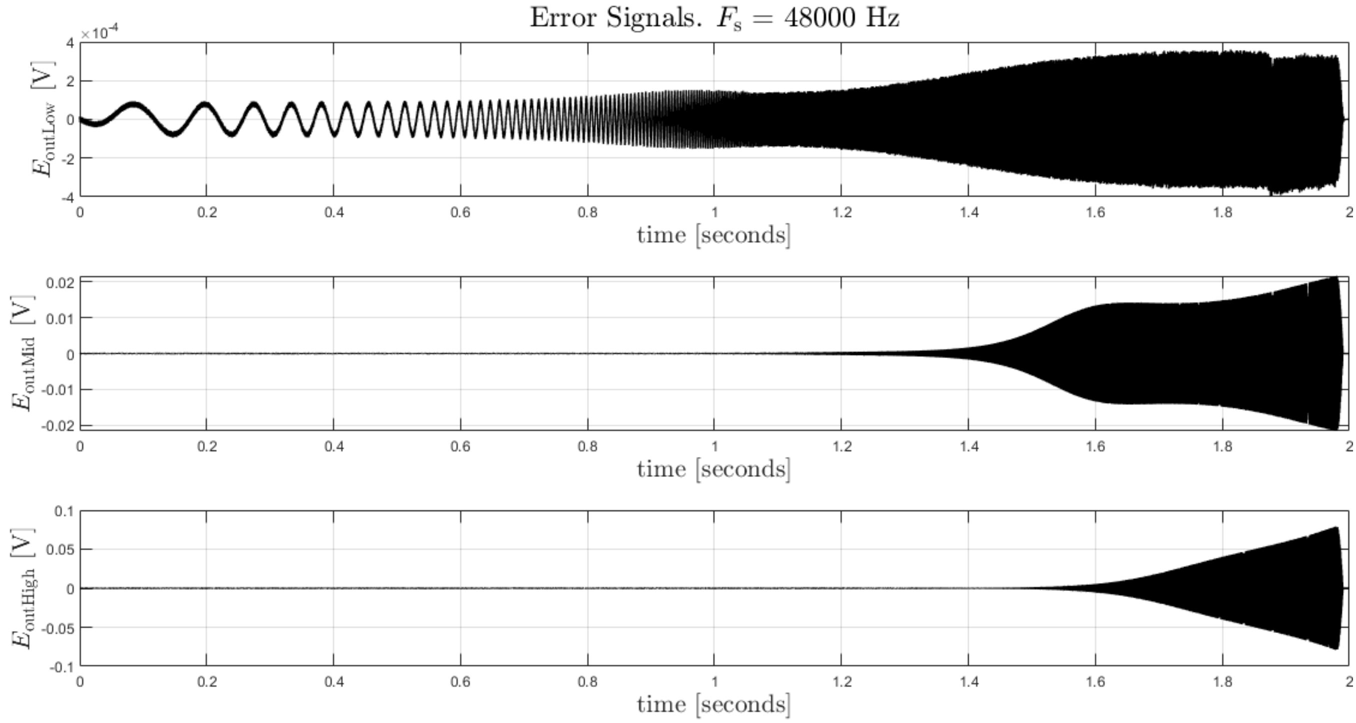


Figure 3: Error signals plots for $F_s = F_{LTspice}/4$