ÁLGEBRA LINEAR

TÓPICO 5 – MATRIZES E DETERMINANTES

ATIVIDADE 11

Nome: Claudio Ferreira da Silva dos Santos RA:2040482323060

1) Determine a matriz A = $(aij)_{3x3}$ tal que aij = 5i – 3j.

$\begin{pmatrix} 2 & -1 & -4 \\ 7 & 4 & 1 \\ 12 & 9 & 6 \end{pmatrix}$

| | | |
|---|---|---|
| 5x1 – 3x1 = 2 | 5x2 – 3x1 = 7 | 5x3 – 3x1 = 12 |
| 5x1 – 3x2 = -1 | 5x2 – 3x2 = 4 | 5x3 – 3x2 = 9 |
| 5x1 – 3x3 = -4 | 5x2 – 3x3 = 1 | 5x3 – 3x3 = 6 |

2) Construa as seguintes matrizes:

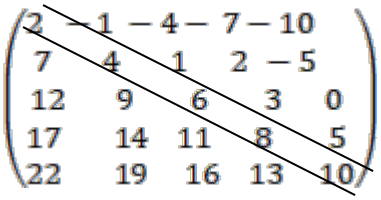
A = $(aij)_{3x3}$ tal que aij = $\begin{cases} 1, se\ i = j \\ 0, se\ i \neq j \end{cases}$ $\begin{pmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

B = $(bij)_{3x3}$ tal que bij = $\begin{cases} i + 2j, se\ i \neq j \\ i - 3j, se\ i = j \end{cases}$ $\begin{pmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{pmatrix} = \begin{pmatrix} -2 & 5 & 7 \\ 4 & -4 & 8 \\ 5 & 7 & -6 \end{pmatrix}$,

| | | |
|---|---|---|
| 1 – 3 x 1 = -2 | 2 + 2 x 1 = 4 | 3 + 2 x 1 = 5 |
| 1 + 2 x 2 = 5 | 2 – 3 x 2 = -4 | 3 + 2 x 2 = 7 |
| 1 + 2 x 3 = 7 | 2 + 2 x 3 = 8 | 3 - 3 x 3 = -6 |

3) Seja a matriz A = $(aij)_{5x5}$ tal que aij = 5i – 3j. Determine a soma dos elementos da diagonal principal dessa matriz.

| | | | | |
|---|---|---|---|---|
| 5 x 1 – 3 x 1 = 2 | 5 x 2 – 3 x 1 = 7 | 5 x 3 – 3 x 1 = 12 | 5 x 4 – 3 x 1 = 17 | 5 x 5 – 3 x 1 = 22 |
| 5 x 1 - 3 x 2 = -1 | 5 x 2 - 3 x 2 = 4 | 5 x 3 - 3 x 2 = 9 | 5 x 4 - 3 x 2 = 14 | 5 x 5 - 3 x 2 = 19 |
| 5 x 1 - 3 x 3 = -4 | 5 x 2 - 3 x 3 = 1 | 5 x 3 - 3 x 3 = 6 | 5 x 4 - 3 x 3 = 11 | 5 x 5 - 3 x 3 = 16 |
| 5 x 1 - 3 x 4 = -7 | 5 x 2 - 3 x 4 = -2 | 5 x 3 - 3 x 4 = 3 | 5 x 4 - 3 x 4 = 8 | 5 x 5 - 3 x 4 = 13 |
| 5 x 1 - 3 x 5 = -10 | 5 x 2 - 3 x 5 = -5 | 5 x 3 - 3 x 5 = 0 | 5 x 4 - 3 x 5 = 5 | 5 x 5 - 3 x 5 = 10 |




2+4+6+8+10 = 30
A soma total dos elementos da diagonal é 30

4) Sejam A = $\begin{pmatrix} 2 & 3 \\ 4 & -1 \\ 0 & 2 \end{pmatrix}$ e B = $\begin{pmatrix} -2 & 0 \\ 7 & -1 \\ 8 & 5 \end{pmatrix}$, determine $(A + B)^t$.

$\begin{pmatrix} 2 & 3 \\ 4 & -1 \\ 0 & 2 \end{pmatrix} + \begin{pmatrix} -2 & 0 \\ 7 & -1 \\ 8 & 5 \end{pmatrix}$ A+B = $\begin{pmatrix} 0 & 3 \\ 11 & -2 \\ 8 & 7 \end{pmatrix}$ $(A + B)^t = \begin{pmatrix} 0 & 11 & 8 \\ 3 & -2 & 7 \end{pmatrix}$

5) Dada a matriz A = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & -2 \end{bmatrix}$, obtenha a matriz x tal que x = A + $A^t$.

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & -2 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 0 \\ -1 & 3 & 1 \\ 0 & 4 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 6 & 5 \\ 0 & 5 & -4 \end{bmatrix}$$

6) Dadas as matrizes A = $\begin{pmatrix} 0 & 4 & -2 \\ 6 & 2 & 8 \end{pmatrix}$, B = $\begin{pmatrix} -3 & 6 & 9 \\ 12 & -6 & 0 \end{pmatrix}$ e C = $\begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 2 \end{pmatrix}$, calcule o resultado da equação matricial

2A + 2X = – B + 3C

2 x 0 + 2X = – (-3) + 3 x 0 x = 3/2
2 x 4 + 2X = –6 + 3 x -1 x = -17/2
2 x 2 + 2X = – 9 + 3 x 0 x = 13/2
2 x 6 + 2X = –12 + 3 x 1 x = -21/2
2 x 2 + 2X = – (-6) + 3 x -1 x = -1/2
2 x 8 + 2X = –0 + 3 x 2 x = -10

7) Efetue as seguintes multiplicações:

a) $\begin{pmatrix} 5 & -3 \\ -1 & 4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ $\frac{5 \times 3 + (-3) \times 2}{-1 \times 3 + 4 \times -2} = \frac{21}{-11}$

b) $\begin{pmatrix} 5 & 2 \\ -1 & 4 \end{pmatrix} \cdot \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$ $\frac{5 \times 2 + 2 \times 0 \mid 5 \times -1 + 2 \times 3}{-1 \times 2 + 4 \times 0 \mid -1 \times -1 + 4 \times 3} = \frac{10 \mid 1}{-2 \mid 13}$

c) $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 2 \end{pmatrix}$ $\frac{1 \times 2 + 0 \times 1 + 0 \times 2 \mid 1 \times 2 + 0 \times 2 + 0 \times 1 \mid 1 \times 1 + 0 \times 2 + 0 \times 2}{1 \times 2 + 1 \times 1 + 0 \times 2 \mid 1 \times 2 + 1 \times 2 + 0 \times 1 \mid 1 \times 1 + 1 \times 2 + 0 \times 2} = \frac{2 \mid 2 \mid 1}{3 \mid 4 \mid 3}$
$\frac{}{0 \times 2 + 1 \times 1 + 1 \times 2 \mid 0 \times 2 + 1 \times 2 + 1 \times 1 \mid 0 \times 1 + 1 \times 2 + 1 \times 2} \quad \frac{}{3 \mid 3 \mid 4}$

8) Determine a e b para que a igualdade $\begin{pmatrix} a+4 & b^3 \\ 10 & 7 \end{pmatrix} = \begin{pmatrix} 2a & b \\ 10 & 7 \end{pmatrix}$ seja verdadeira.

a + 4 = 2a 4 +4 = 8 $b^3 = b$
2a – a = 4 2 x 4 = 8 $0^3 = 0$
a = 4

9) Dadas as matrizes A = $\begin{pmatrix} 3 & 1 \\ 4 & -2 \end{pmatrix}$ e B = $\begin{pmatrix} x+y & x-y \\ 1 & -2 \end{pmatrix}$, determine x e y para que A = $B^t$.

$\begin{pmatrix} 3 & 1 \\ 4 & -2 \end{pmatrix} = \begin{pmatrix} x+y & 1 \\ X-y & -2 \end{pmatrix}$
A $B^t$

3 = x +y
4 = x - y

x = 4 + y
3 = x +y
3 = 4 + y +y
2y = 3 – 4
y = -1/2

x = 4 + y
x = 4 – ½
x = $\frac{8-1}{2}$
X = 7/2

x + y
$\frac{7}{2} \frac{-1}{2} = \frac{6}{2} = 3$

x - y
$\frac{7}{2} \frac{(-)-1}{2} = \frac{8}{2} = 4$

10) Determine os valores de x e y na equação matricial: $\begin{pmatrix} 2 & x \\ y & 3 \end{pmatrix} + \begin{pmatrix} -4 & -4 \\ -7 & 5 \end{pmatrix} = 2.\begin{pmatrix} -1 & 2 \\ -3 & 4 \end{pmatrix}$.

$\begin{pmatrix} 2 & x \\ y & 3 \end{pmatrix} + \begin{pmatrix} -4 & -4 \\ -7 & 5 \end{pmatrix} = 2.\begin{pmatrix} -1 & 2 \\ -3 & 4 \end{pmatrix}$

$\begin{pmatrix} -2 & x-4 \\ y-7 & 8 \end{pmatrix} = \begin{pmatrix} -2 & 4 \\ -6 & 8 \end{pmatrix}$.

x - 4 = 4 y - 7 = -6
x = 8 y = 1