23. A. Takayama, *Mathematical Economics* (Hinsdale, IL: The Dryden Press, 1974), Theorem 4.D.3 on p. 393 and the discussion on p. 405. Also see Appendix B of Baumol, Bailey, and Willig, *op. cit.*, for a discussion of the relation between GS and other popular hypotheses in demand theory.
24. Baumol, Bailey, and Willig, *op. cit.*, p. 356.
25. Baumol, Bailey, and Willig, *op. cit.*, p. 357.
26. The qualification "against challenges $y \subseteq T$" must be added because DSD does not allow us to conclude: $D(y^m + y)\, y > C(y)$ implies $D(y)y > C(y)$. This is true in the one-dimensional case but complementarities in demand may falsify it in the multidimensional case.
27. See H. Sonnenschein, "Market Excess Demand Functions," *Econometrica*, April 1972, pp. 549–563; R. Mantel, "On the Characterization of Aggregate Excess Demand," *Journal of Economic Theory*, March 1974, pp. 348–353; G. Debreu, "Excess Demand Functions," *Journal of Mathematical Economics*, March 1974, pp. 15–21.
28. It is worth pointing out here that there is a relation between price and quantity sustainability and supportability of the cost function in the Sharkey–Telser sense. As they point out, supportability is necessary but not sufficient for price sustainability. Notice that supportability is not necessary for quantity sustainability because one-dimensional examples with U-shaped average cost may be found that are quantity sustainable at the Ramsey point but are not supportable there. All that needs to be done to create an example is to generate a one-dimensional, U-shaped average cost Panzar–Willig price nonsustainable example but make demand fall off rapidly enough beyond the Ramsey point so that the Ramsey point is quantity sustainable. See W. Sharkey and L. Telser, "Supportable Cost Functions for the Multiproduct Firm," *Journal of Economic Theory*, June 1978, pp. 23–37.
29. Panzar and Willig (1977), *op. cit.*, p. 7.
30. Baumol, Bailey, and Willig, *op. cit.*, pp. 357–358.
31. This is not so for the case of price nonsustainability. Suppose there is y such that $D(y^m)\, y > C(y)$ then the net gain is

$$B(y^m + y) - C(y^m) - C(y) - [B(y^m) - C(y^m)] \leq D(y^m) - C(y)$$

That is, although the upper bound to net gain $D(y^m)y - C(y)$ is positive it is easy to construct examples even in the one-dimensional case where net gain is negative. The point is that quantity nonsustainability implies a positive lower bound to net gain. Price nonsustainability only implies the existence of a positive upper bound.
32. Panzar, "Comment on Baseman," in G. Fromm, ed., *op. cit.*

Chapter 10

Multiproduct Cost Function Estimates and Natural Monopoly Tests for the Bell System*

David S. Evans and James J. Heckman

This chapter (*a*) reports multiproduct cost function estimates for the Bell System; (*b*) describes an empirically tractable test for subadditivity of the cost function; and (*c*) reports the results of this test for alternative Bell System cost function estimates. Subadditivity of the cost function is a necessary and sufficient condition for natural monopoly. AT&T has argued that the telephone industry is a natural monopoly:[1]

> The scientific principles applicable to telecommunications, the organization of the nationwide telecommunications network, and the engineering principles and practices by which telecommunications services are provided make a single interactive and interdependent network the most efficient means for providing all of the Nation's telecommunications services. . . . [S]uch a network can be planned, constructed, and managed most efficiently by an integrated enterprise that owns the major piece-parts of the facilities network. . . .

This assertion is true if and only if the cost function for the Bell System is subadditive.

Previous studies of the Bell System provide little information concerning whether the telephone industry is a natural monopoly.[2] These studies aggregate diverse telecommunications service outputs into a single measure of output, estimate single-product cost or production functions, and determine whether there are scale economies. There are two major problems with this approach. First, cost or production functions based on an aggregate measure of output are valid only under highly restrictive assumptions, which these studies do not test.[3] We

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have found that these assumptions do not hold for the Bell System. Second, the presence of scale economies for an aggregate measure of output does not imply the presence of scale economies for any of the components that comprise the aggregate measure of output. Christensen, Cummings, and Schoech claimed their finding of scale economies for an aggregate measure of output "is consistent with the view that the proliferation of suppliers of telecommunications would result in a large sacrifice of efficiency due to foregone scale economies."[3] Since aggregate scale economies do not imply that there are scale economies in intercity message toll or private line service—the two markets opened to competition by the FCC during the 1970s—their finding is consistent with the opposite view as well. As Baumol, Panzar, and Willig have observed, "We can see why analysts have attempted to use the analytically and statistically tractable concept of scale economies as a surrogate test of natural monopoly. Unfortunately, . . . such traditional tests simply can not do the job."[4]

This chapter is divided into three sections and three appendices. The first section reports multiproduct cost function estimates for the Bell System. The second section reports tests of whether these estimated cost functions as subadditive. The third section summarizes our results. Appendix A discusses alternative formulations of the cost function. Appendix B reports estimates from a cost function that was not restricted to satisfy the homogeneity and symmetry restrictions required by producer theory. Appendix C discusses the data that were used in this study.

Multiproduct Cost Function Estimates

The Bell System uses capital, labor, and materials to produce local and long-distance telephone service. Its cost function is[5,6]

$$C = f(L,T,r,m,w,t) \tag{10.1}$$

where L is local service output, T is long-distance service output, r is the capital rental rate, w is the wage rate, m is the price of materials, and t is an index of technological change. We have estimated a cost function rather than a production function in order to make our approach consistent with previous studies of the production characteristics of the telecommunications industry and because the theoretical literature on natural monopoly relies on the cost function rather than the production function. We have disaggregated output into local and long-distance service because these are the major services provided by the Bell System.[7] We decided upon this particular cost function because, as described in Appendix A, the data were not available to estimate cost functions that describe the structure of the Bell System more realistically.

Christensen, Jorgenson, and Lau claim that the translog cost function provides a useful second-order approximation to any twice differentiable cost function.[8] The translog cost function imposes fewer restrictions on the characteristics of the production structure than other commonly used cost function specifications

and is therefore more suitable for testing alternative hypotheses concerning the characteristics of the production structure. The translog approximation to (10.1) is

$$\begin{aligned}\ln(C) = \alpha_0 &+ \sum_i \alpha_i \ln(p_i) + \sum_k \beta_k \ln(q_k) + \mu \ln(t)\\ &+ \frac{1}{2}\sum_i\sum_j \gamma_{ij}\ln(p_i)\ln(p_j) + \frac{1}{2}\sum_k\sum_l \delta_{kl}\ln(q_k)\ln(q_l)\\ &+ \sum_i\sum_k \rho_{ik}\ln(p_i)\ln(q_k)\\ &+ \sum_i \lambda_i \ln(p_i)\ln(t) + \sum_k \theta_k \ln(q_k)\ln(t) + \tau[\ln(t)]^2\end{aligned} \tag{10.2}$$

where p denotes the vector of input prices (r,m,w) and q denotes the vector of outputs (L,t). We apply Shephard's Lemma

$$x_i = \frac{\partial C}{\partial p_i} \tag{10.3}$$

where x_i is the quantity demanded of the ith input, in order to obtain the input cost share equations

$$S_i = \frac{p_i x_i}{C} = \alpha_i + \sum_i \gamma_{ij}\ln p_j + \sum_k \rho_{ik}\ln q_k + \lambda_i \ln t \tag{10.4}$$

Assuming AT&T operates efficiently, the cost function and the associated input cost share equations are consistent with production theory if (1) the cost function is linear homogeneous in input prices; (2) the Hessian of the cost function with respect to input prices is symmetric; and (3) the α's, ρ's, γ's, and λ's are identical across equations.[9] Homogeneity requires

$$\sum_j \alpha_j = 1, \quad \sum_j \gamma_{ij} = 0, \quad \sum_j \rho_{jk} = 0, \quad \sum_j \lambda_j = 0 \tag{10.5}$$

Symmetry requires

$$\gamma_{ij} = \gamma_{ji} \tag{10.6}$$

This general specification of the cost function subsumes many special production structures. Table 10.1 lists alternative hypotheses concerning the structure of production and technological change and parameter restrictions these hypotheses imply.[10] Using standard statistical procedures, we can readily test between alternative hypotheses concerning the structure of production and technological change.

The separability and nonjointness hypotheses are of particular concern to this study. Separability implies that there exists an aggregate measure of output $Q = A(L,T)$ such that

$$C(L,T,r,w,m,t) = C[A(L,T),r,w,m,t] = C(Q,r,w,m,t) \tag{10.7}$$

Table 10.1 Alternative Hypotheses Concerning the Structure of Production and Technological Change

Hypotheses[a]	Coat function characteristics[b]	Translog parameter[c] restrictions	
Separability of inputs and outputs[d,e]	$C(q,p,t) = C[A(q),p,t]$	$\rho_{il}\beta_k = \rho_{ik}\beta_l$	$i = 1,2,3$ $k \neq 1$
Nonjointness[e]	$C(q,p,t) = \Sigma C_i(q_i,p,t)$	$\delta_{kl} = -\beta_k\beta_l$	$k \neq 1$
Homotheticity	$C(q,p,t) = A(q)g(p,t)$	$\rho_{il} = 0$	$i = 1,2,3$ $l = 1,2$
Homogeneity in outputs	$C(\kappa^r q,p,t) = \kappa^r C(q,p,t)$	$\rho_{il} = 0$ $\delta_{kl} = 0$	$i,l = 1,2,3$ $k,l = 1,2$
Unitary elasticities of substitution	$\frac{\partial^2 C}{\partial p_i \partial p_j} = 0$	$\gamma_{ij} = 0$	$i,j = 1,2,3$
Neutral technological change	$C(q,p,t) = h(t)C(q,p)$	$\theta_k = 0$ $\lambda_j = 0$	$i = 1,2$ $j = 1,2,3$
Nonfactor augmenting technological change	$C(q,p,t) = h(t)C[f(q,t),p]$	$\lambda_j = 0$	$j = 1,2,3$
Nonoutput augmenting technological change	$C(q,p,t) = h(t)C[q,g(p,t)]$	$\theta_k = 0$	$k = 1,2$

[a]The maintained hypothesis is that the homogeneity and symmetry restrictions implied by producer theory hold.
[b]p is the vector of input prices, q is the vector of outputs, t is technological change, and C is cost.
[c]See equation (10.2) in the text for a definition of the parameters.
[d]A sufficient condition for separability of inputs and outputs is the homotheticity of the cost function.
[e]Restriction holds at the point of expansion for the translog approximation to the cost function.

Denny and Pinto have shown that, at the point of expansion for the translog approximation to the cost function, the cost function is separable if[11]

$$\rho_{il}\beta_k = \rho_{ik}\beta_l, \quad i = 1,2,3,\ k \neq 1 \tag{10.8}$$

If these restrictions are accepted, it may be possible to form an aggregate output measure, estimate a single-product cost function, and use scale-economy estimates to test whether there is a natural monopoly.[12] Nonjointness implies that the cost of producing several outputs equals the sum of the costs of producing the outputs separately

$$C(L,T,r,w,m,t) = C_L(L,r,w,m,t) + C_T(T,r,w,m,t) \tag{10.9}$$

Denny and Pinto have shown that, at the point of expansion of the translog approximation to the cost function, the cost function is nonjoint if[13]

$$\delta_{kl} = -\beta_k\beta_l \tag{10.10}$$

If this restriction is accepted, the cost function exhibits neither economies nor diseconomies of scope.[14]

Several researchers have criticized the translog cost function specification. Burgess found that different results were obtained depending upon whether a translog cost function or a translog production function was estimated.[15] Gallant argued that the translog specification yields potentially biased parameter estimates.[16] Guilkey and Lovell found that "there is a pervasive, but not pronounced, upward bias to translog estimates of . . . returns to scale."[17] Fuss and Waverman found that a slight modification to the translog cost function reduced the aggregate scale elasticity estimate from around 1.4 to 1.0.[18]

In order to test the sensitivity of our results to the translog cost function specification, we estimated the modified translog cost function and the Box–Tidwell cost function.[19] The modified translog cost function performs a Box–Cox transformation on the output variables. The Box–Tidwell cost function performs a Box–Cox transformation on the right-hand side variables. The Box–Cox transformation of a variable y is given by

$$y^* = \frac{y^\eta - 1}{\eta} \tag{10.11}$$

The ordinary translog cost function is, following (10.2),

$$\ln(C_{OTL}) = g[\ln(L),\ln(T),\ln(r),\ln(w),\ln(m),\ln(t)] \tag{10.12}$$

The modified translog cost function is

$$\ln(C_{MTL}) = g[L^*,T^*,\ln(r),\ln(w),\ln(m),\ln(t)] \tag{10.13}$$

The Box–Tidwell cost function is

$$\ln(C_{BT}) = g(L^*,T^*,r^*,w^*,m^*,t^*) \tag{10.14}$$

With substitution of the transformed variables for the logarithmic variables in (10.2), the cost share equations (10.4) and cross equation restrictions (10.5) and (10.6) apply to the modified translog cost function. For convenience in estimation, we impose linear homogeneity in prices on the Box–Tidwell cost function by normalizing cost and input prices by the price of materials. Linear homogeneity in prices requires

$$C(\xi w,\xi r,\xi m,L,T,t) = \xi C(w,r,m,L,T,t) \tag{10.15}$$

Letting $\xi = \frac{1}{m}$ we have

$$C\left(\frac{w}{m},\frac{r}{m},1,L,T,t\right) = \frac{1}{m}C\left(\frac{w}{m},\frac{r}{m}, L,T,t\right) \tag{10.16}$$

Define

$$W = \frac{w}{m}, \quad R = \frac{r}{m}, \quad P = (W,R) \tag{10.17}$$

Then

$$\ln\left(\frac{C}{m}\right) = \alpha_0 + \sum_i \beta_i P_i^* + \sum_k \beta_k q_k^* + \frac{1}{2}\sum_i\sum_j \gamma_{ij} P_i^* P_j^* + \frac{1}{2}\sum_k\sum_l \delta q_k^* q_l^* + \sum\sum \rho_{ik} p_i^* q_k^* + \sum_i \lambda P_i^* t^* + \sum_i \theta_i q_i^* t^* + \tau (t^*)^2 \quad (10.18)$$

Applying Shephard's Lemma (10.3)

$$S_i = P_i^\eta [\alpha_i + \sum_j \gamma_{ij} P_j^* + \sum_k \rho_{ik} q_k^* + \lambda_i t_i^*] \quad (10.19)$$

We specify additive disturbance terms $\varepsilon_C, \varepsilon_K, \varepsilon_L$, and ε_M for the cost, capital share, labor share, and materials share equations. We assume that these disturbances are temporally uncorrelated, contemporaneously correlated, and multinormally distributed with

$$\begin{aligned} &E\,\varepsilon_{it} = 0 && i = C,K,L,M \\ &E\,\varepsilon_{it}\varepsilon_{jt} = \sigma_{ij} && i,j = C,K,L,M \\ &E\,\varepsilon_{it}\varepsilon_{jt'} = 0 && i,j = C,K,L,M \\ & && t \neq t' \end{aligned} \quad (10.20)$$

where t denotes time. Second, the disturbances are generated by the following first-order autoregressive process

$$\varepsilon_{it} = u_{it} + \nu_i \varepsilon_{i,t-1}, \quad i = C,K,L,M \quad (10.21)$$

where the u_{it} are multinormally distributed with

$$\begin{aligned} &Eu_{it} = 0 && i = C,K,L,M \\ &Eu_{it}u_{jt} = \sigma_{ij} && i,j = C,K,L,M \\ &Eu_{it}u_{jt'} = 0 && i,j = C,K,L,M \\ & && t \neq t' \end{aligned} \quad (10.22)$$

Under (10.21), the disturbances are contemporaneously correlated across equations, temporally correlated within equations, and temporally uncorrelated across equations.[20]

For both error processes, the equations were estimated by iterating Zellner's two-step procedure for estimating seemingly unrelated regressions.[21] This iterated method is computationally equivalent to maximum likelihood estimation.[22] Because the input cost shares sum to one, the covariance matrix is singular. In order to provide a nonsingular covariance matrix, we deleted the materials share equation. Barten has shown that maximum likelihood estimates are invariant to the equation deleted when the error process is given by (10.20).[23] Berndt and Savin have shown that maximum likelihood estimates are invariant to the equation deleted if $\nu_K = \nu_L = \nu_M$ in the error process given by (10.21).[24] The hypothesis $\nu_K = \nu_L = \nu_M$ was, unfortunately, always rejected at the 99% level or better. Nonetheless, rather than report the results for estimations based on the deletion of different share equations, we report estimates based on $\nu_K = \nu_L = \nu_M$ and note where results would differ when this restriction is not imposed.[25]

The equations were estimated with yearly data on the Bell System from 1947–1977. These data were obtained from Christensen who calculated Tornqvist indices of cost, outputs, and prices from detailed Bell System data. In order to represent the level of technology, this study used an index based on a distributed lag of research and development expenditures proposed by Vinod.[26] Several alternative measures of technological change were tested but yielded substantially less satisfactory results.[27] The data, their construction, and their limitations are described more fully in Appendix C.

Table 10.2 presents estimates for three cost function specifications under the

Table 10.2 Parameter Estimates for Alternative Cost Function Specifications with No Serial Correlation

Parameter[a]	Translog[b]		Modified[c] translog		Box–Tidwell[c]	
Constant	9.057	(.196)	9.054	(.004)	9.054	(.004)
Capital	.536	(.004)	.537	(.004)	.537	(.004)
Labor	.354	(.004)	.354	(.004)	.353	(.003)
Local	.294	(.261)	.260	(.350)	.542	(.204)
Toll	.420	(.197)	.462	(.299)	.110	(.140)
Technology	−.161	(.070)	−.193	(.108)	−.008	(.073)
Capital^2	.197	(.024)	.190	(.027)	−.145	(.085)
Labor^2	.176	(.025)	.171	(.027)	−.028	(.037)
Capital · Labor	−.163	(.021)	−.158	(.023)	−.246	(.027)
Toll^2	−5.276	(1.700)	−6.531	(4.905)	−2.999	(1.432)
Local^2	−2.640	(1.132)	−3.951	(4.118)	.491	(.567)
Local · Toll	7.764	(2.700)	10.233	(8.828)	−.287	(1.185)
Technology^2	.412	(.799)	−.126	(1.547)	−.260	(.424)
Capital · Toll	.354	(.097)	.399	(.131)	.264	(.045)
Capital · Local	−.352	(.089)	−.390	(.114)	−.374	(.034)
Labor · Toll	−.221	(.087)	−.263	(.116)	−.038	(.028)
Labor · Local	.209	(.080)	.244	(.103)	.104	(.016)
Capital · Tech.	.106	(.037)	.119	(.044)	−.006	(.008)
Labor · Tech.	−.108	(.034)	−.120	(.039)	.020	(.074)
Tech. · Toll	−.967	(1.204)	−1.924	(2.990)	2.440	(1.062)
Tech. · Local	.358	(1.202)	1.513	(3.130)	−.678	(.498)
ν_C	—	—	—	—	—	—
$\nu_L = \nu_K$	—	—	—	—	—	—
η	—	—	−.031	(.114)	.725	(.110)

[a]Standard errors in parentheses.
[b]Maximum likelihood estimates obtained by an iterative Zellner method.
[c]Nonlinear seemingly unrelated regression estimates.

assumption that the disturbance terms are serially uncorrelated. The Durbin–Watson test for positive autocorrelation was inconclusive for the cost equation but indicated positive autocorrelation for the capital and labor share equations. Table 10.3 presents estimates for the three cost function specifications under the assumption that the disturbance terms are generated by a first-order autoregressive process, as given in (10.21). Table 10.4 reports summary statistics for each of the six estimated equations.

The translog cost function is a special case of the modified translog cost function and the Box–Tidwell cost function. To see this, observe that

$$\lim_{\eta \to 0} \frac{y^\eta - 1}{\eta} = \ln y$$

and substitute ln (y) for y^* in equations (10.13) and (10.14). Because of the numerical procedures we used, our likelihood estimates never converge to $\eta =$

Table 10.3 Parameter Estimates for Alternative Cost Function Specifications with First Order Serial Correlation

Parameter[a]	Translog[b]		Modified[c] translog		Box–Tidwell[c]	
Constant	9.054	(.005)	9.053	(.005)	9.053	(.006)
Capital	.535	(.008)	.535	(.009)	.538	(.009)
Labor	.355	(.007)	.354	(.007)	.352	(.007)
Local	.260	(.309)	.282	(.394)	.209	(.358)
Toll	.462	(.226)	.401	(.326)	.426	(.280)
Technology	−.193	(.086)	−.146	(.121)	−.121	(.108)
Capital²	.219	(.024)	.216	(.027)	.211	(.062)
Labor²	.174	(.027)	.162	(.030)	.154	(.067)
Capital · Labor	−.180	(.019)	−.179	(.019)	−.185	(.020)
Toll²	−8.018	(2.170)	−6.837	(4.892)	−14.545	(7.853)
Local²	−4.241	(1.314)	−3.249	(3.788)	−6.848	(5.983)
Local · Toll	11.663	(3.144)	9.411	(8.233)	9.969	(6.469)
Technology²	−.176	(1.033)	−.007	(.057)	−.006	(.827)
Capital · Toll	.337	(.138)	.335	(.141)	.337	(.158)
Capital · Local	−.359	(.122)	−.355	(.118)	−.388	(.126)
Labor · Toll	−.179	(.083)	−.170	(.087)	−.132	(.091)
Labor · Local	.164	(.071)	.161	(.068)	.133	(.069)
Capital · Tech.	.083	(.053)	.074	(.057)	.002	(.015)
Labor · Tech.	−.057	(.047)	−.054	(.048)	.061	(.124)
Toll · Tech.	−1.404	(1.497)	−.640	(2.464)	−.693	(2.158)
Local · Tech.	1.207	(1.431)	.591	(2.821)	.599	(2.443)
υ_C	.187	(.105)	.219	(.056)	.212	(.112)
$\upsilon_L = \upsilon_K$	.712	(.094)	.713	(.050)	.724	(.074)
η	—	—	.038	(.175)	.032	(.134)

[a]Standard errors in parentheses.
[b]Maximum likelihood estimates obtained by an iterative Zellner method.
[c]Nonlinear seemingly unrelated regression estimates.

Table 10.4 Diagnostic Statistics on Alternative Cost Function Specifications

	R^2	Durbin–Watson[a] (Durbin-H)	Degrees of freedom Durbin–Watson[a] (Durbin-H)	Generalized variance[b] $\times 10^{-5}$
Translog − AR(0)				22.338
Cost function	.9998	1.17	15	
Capital share	.9463	.51	27	
Labor share	.9570	.49	27	
Translog − AR(1)				10.568
Cost function	.9999	(.65)	14	
Capital share	.9756	(1.50)	26	
Labor share	.9835	(1.37)	26	
Modified translog − AR(0)				23.962
Cost function	.9999	1.05	14	
Capital share	.9447	.53	26	
Labor share	.9612	.47	26	
Modified translog − AR(1)				10.790
Cost function	.9999	(1.24)	13	
Capital share	.9755	(1.77)	25	
Labor share	.9839	(1.77)	25	
Box–Tidwell − AR(0)				36.647
Cost function	.9997	1.78	14	
Capital share	.9443	.58	26	
Labor share	.9618	.73	26	
Box–Tidwell − AR(1)				11.560
Cost function	.9997	(1.09)	13	
Capital share	.9729	(1.34)	25	
Labor share	.9823	(1.52)	25	

[a]The upper and lower bounds for the Durbin–Watson statistic are .90 and 1.60 for the share equations and .45 and 3.38 for the cost equation. We accept the hypothesis of no positive autocorrelation when the Durbin–Watson statistic exceeds the upper bound. The Durbin–Watson statistic is biased for the equations estimated under the assumption that the error term follows an AR(1) process. For these equations, we calculated the Durbin-H statistic, which is asymptotically distributed as a standard normal deviate. We reject the hypothesis of no first-order autocorrelation when this statistic exceeds 1.96.
[b]Maximum likelihood estimation which was used for the translog specification minimizes the generalized variance. Seemingly unrelated nonlinear regression methods which were used for the modified translog and Box–Tidwell specifications provide an asymptotically unbiased estimate of the generalized variance.

0 even when this value provides a global maximum of the likelihood function. Direct comparison of the generalized variances for the specifications listed in Table 10.2 indicates that the translog specification provides the lowest generalized variance.[28] In order to check that $\eta = 0$ provides a global maximum for the likelihood function, we maximized the likelihood function conditional on several alternative values of η. Table 10.5 reports the generalized variances from

Table 10.5 Generalized Variance for Alternative Functional Forms

	−.03	−.01	0.0	.01	.03	.50	.75	1.00
Modified translog AR(0)	24.547	24.974	22.338	31.775	28.868	44.391	30.916	28.059
Modified translog[a] AR(1)	28.311	23.069	10.568	28.233	27.730	54.422	37.180	[b]
Box–Tidwell AR(0)	39.885	38.318	22.338	37.215	36.207	32.402	36.764	79.043
Box–Tidwell[a] AR(1)	393.279	385.807	10.568	378.942	1016.051	155.617	43.922	306.456

[a]Maximum likelihood estimates conditional on $\nu_C = .187 \quad \nu_L = \nu_K = .712$
[b]Singular covariance matrix.

Table 10.6 Tests of Alternative Hypotheses Concerning the Structure of Production and Technological Change

Hypothesis[a]	Likelihood ratio statistic for translog-AR(0)	translog-AR(1)[c]	Number of restrictions	Critical values of χ^2 .05	.01
Separability	11.73	8.95	2	5.99	9.21
Nonjointness	12.00	19.68	1	3.84	6.63
Homotheticity	33.96	11.95	4	9.49	13.30
Homogeneity in outputs	54.93	30.20	7	14.10	18.50
Unitary elasticities of substitution	51.98	50.20	3	7.81	11.30
Neutral technological change	14.61	4.70[b]	4	9.49	13.30
Nonoutput augmenting technological change	2.66[b]	3.36[b]	2	5.99	9.21
Nonfactor augmenting technological change	11.59	1.63[b]	2	5.99	9.21
Homogeneity and symmetry	210.97	188.71	21	32.70	38.90

[a]Except for the test of homogeneity and symmetry, the maintained hypothesis is that the general model with homogeneity and symmetry imposed is the true model.
[b]Indicates hypothesis acceptable at 5% level or better.
[c]Entries are equal to the likelihood ratio statistic $-2 \ln [|\Sigma_r|/|\Sigma_u|]^{-T/2} = T \ln[|\Sigma_r|/|\Sigma_u|]$ where $|\Sigma_r|$ denotes the generalized variance of the restricted system, $|\Sigma_u|$ denotes the generalized variance of the unrestricted system, and T denotes the number of observations which is always equal to 31. This statistic is distributed asymptotically as χ_r^2 where r is the number of restrictions.

this estimation. These statistics confirm that $\eta = 0$ minimizes the generalized variance and thereby maximizes the likelihood function.

For the translog specification, Table 10.6 reports tests of several hypotheses concerning the structure of production and technological change in the Bell System. We resoundingly reject the homogeneity and symmetry restrictions implied by producer theory.[29] We also reject these restrictions for the single-output cost function estimated by Christensen, Cummings, and Schoech. Rejection of homogeneity and symmetry may indicate that the translog cost function is a poor approximation to the true cost function; that the cost function is misspecified in some other basic way, or that firms do not behave as assumed by producer theory. We cannot resolve these issues in this chapter. Like other researchers, we restrict our cost function estimates to satisfy homogeneity and symmetry.

Given homogeneity and symmetry, we reject separability at the 1% level when the disturbances are assumed to be temporally uncorrelated and at the 5% level

Table 10.7 Parameter Estimates for Cost Functions with Alternative Technological Change Specifications

Parameter[a]	Translog-AR(0)[b] nonoutput augmenting technological change		Translog-AR(1)[b] neutral technological change	
Constant	9.057	(.005)	9.053	(.005)
Capital	.537	(.004)	.538	(.008)
Labor	.354	(.004)	.353	(.007)
Toll	.345	(.181)	.422	(.205)
Local	.368	(.247)	.237	(.282)
Technology	−.120	(.052)	−.120	(.055)
Capital²	−.193	(.023)	.220	(.024)
Labor²	.173	(.025)	.174	(.026)
Capital · Labor	−.161	(.021)	−.181	(.019)
Toll²	−.231	(.084)	−.142	(.080)
Local²	.216	(.078)	.139	(.067)
Local · Toll	.367	(.094)	.390	(.139)
Technology²	−.361	(.086)	−.397	(.117)
Capital · Toll	−1.932	(.590)	−3.243	(.773)
Capital · Local	−4.178	(1.886)	−7.543	(1.947)
Labor · Toll	5.756	(1.843)	9.984	(2.483)
Labor · Local	.177	(.129)	.218	(.141)
Capital · Tech.	.107	(.037)	—	—
Labor · Tech.	−.109	(.033)	—	—
Toll · Tech.	—	—	—	—
Local · Tech.	—	—	—	—
ν_C	—	—	.191	(.081)
$\nu_L = \nu_K$	—	—	.712	(.070)

[a]Standard errors in parentheses.
[b]Maximum likelihood estimates obtained by an iterative Zellner method.

when the disturbances are assumed to be temporally correlated.[30] These results indicate that it is not possible to form a valid aggregate $A(L,T)$ of local and long-distance telephone service outputs and that scale-economy estimates based on such an aggregate provide no reliable evidence concerning whether the cost function exhibits subadditivity. We also reject nonjointness, homotheticity, and homogeneity in outputs for both the autoregressive and nonautoregressive specifications. The nonautoregressive specification exhibits no output-augmenting technological change while the autoregressive specification exhibits neither factor-augmenting nor output-augmenting technological change. Table 10.7 reports parameter estimates for the nonautoregressive specification with the output-technology variables excluded and for the autoregressive specification with the output-technology and price-technology variables excluded. Excluding these variables has only a modest impact on the parameter estimates for the remaining variables. Consequently, we focus our attention on the general translog cost function estimates reported in Table 10.2 and 10.3.

The own-price elasticities of demand for capital, labor, and materials were negative in all years as required by producer theory. It is useful to contrast these results with Christensen, Cummings, and Schoech's most general single-product specification.[31] They found that the own-price elasticities of demand for capital and labor were positive, contrary to producer theory. By relaxing their assumption that there exists a consistent translog aggregate of local and long-distance service output, we find that the translog cost function specification provides more reasonable estimates.[32] Table 10.8 reports factor demand elasticities calculated from our estimated general translog cost function specifications for the middle year of our sample.

Table 10.8 Factor Demand Elasticities[a] Evaluated at 1961 Observation[b]

With respect to the price of	Elasticity of the demand for: Capital	Labor	Materials
Capital	−.097 / −.056	.076 / .28	.227 / .181
Labor	0.50 / .019	−.149 / −.151	.236 / .410
Materials	.047 / .183	.073 / .127	−.463 / −590

[a]Top triangle is for the translog specification with no serial correlation (first column of Table 10.2); bottom triangle is for the translog specification with serial correlation (first column of Table 10.3).
[b]Own-price factor demand elasticities were negative for every year between 1947 and 1977

Natural Monopoly Tests

When all firms have access to the same technology, certain properties of the cost function reveal whether one firm can produce given levels of output more cheaply than several firms can. The cost function $C(Q_1,Q_2)$ is locally subadditive if and only if

$$\sum_{ij} C(a_iQ_1, b_jQ_2) > C(Q_1,Q_2), \quad i,j, = 1, \ldots, n$$
$$\sum_i a_i = 1, \quad \sum_j b_j = 1 \qquad (10.23)$$

at least two a_i or b_j not equal to zero for given levels of Q_1 and Q_2. It is locally superadditive if the inequality is reversed (<) and locally additive if the inequality is replaced by equality (=). A firm with a locally additive cost function is not a natural monopoly. The cost function is globally subadditive if and only if the cost function is locally subadditive for all feasible levels of Q_1 and Q_2.[33] Local subadditivity is a necessary and sufficient condition for local natural monopoly. Global subadditivity is a necessary and sufficient condition for global natural monopoly.

Assuming that a firm acts efficiently, it will never produce an output configuration at which the cost function is superadditive, unless there are firm-specific fixed factors. Accordingly, the interesting statistical question centers on testing between local additivity and local subadditivity.

In order to determine whether the necessary and sufficient conditions for local natural monopoly are satisfied, we require global information concerning the cost function. The single-output average cost schedule drawn in Figure 10.1 illustrates this proposition. At point A, the cost function exhibits diseconomies of scale. Yet, as is readily verified, a single firm can produce Q^* more cheaply than two or more firms can. In order to determine whether the necessary and sufficient conditions for global natural monopoly are satisfied, we require global information concerning both the cost function and the demand function. If the demand curve in Figure 10.1 shifted rightward, perhaps because of increasing social wealth, this industry would be able to support several firms. At point B, two firms, each producing Q_m, could together produce $2Q_m$ more cheaply than a single firm could. Unfortunately, global information about cost and demand is seldom available.

Baumol, Panzar, and Willig have derived necessary conditions for subadditivity and sufficient conditions for subadditivity which require somewhat less information. Economies of scope is a necessary condition for subadditivity because, without economies of scope, two single-product monopolies could produce more cheaply than a multiproduct monopoly.[34] Economies of scope and product-specific scale economies are sufficient conditions for subadditivity; splitting production between several firms would reduce synergies from joint production and scale economies from volume production.[35] Baumol, Panzar, and Willig suggest testing the necessary and sufficient conditions separately. If the necessary con-

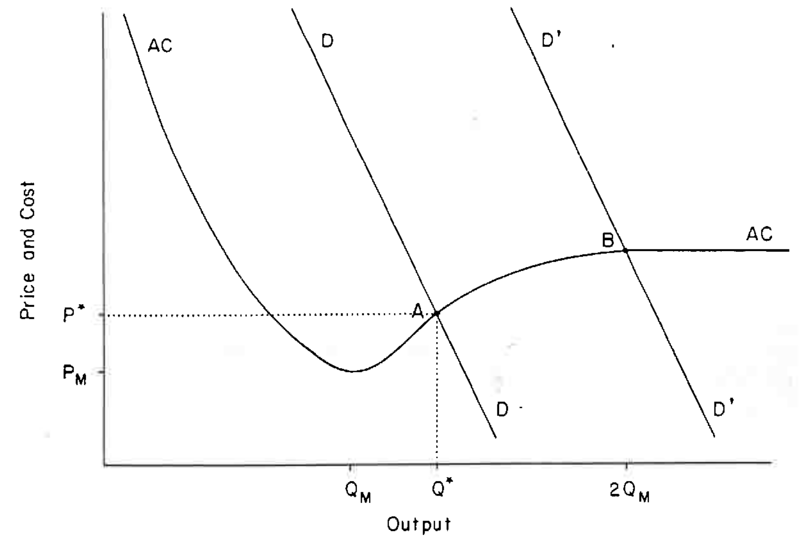

Figure 10.1 Local global subadditivity.

dition is rejected, subadditivity is decisively rejected. If the sufficient conditions are accepted, subadditivity is decisively accepted. If the necessary condition is accepted and the sufficient conditions are rejected, the test for subadditivity is inconclusive.

The Baumol, Panzar, and Willig test of natural monopoly suffers from two problems. First, estimates of economies of scope and product-specific scale economies require information about the costs of separate production $C(0,Q_2)$ and $C(Q_1,0)$. Reliable data on separate production are seldom available for cases of interest.[36] Second, when data are available we may frequently find that the test is inconclusive.

We have developed a more direct and empirically less demanding test of *within sample subadditivity*. The test is based on the definition of subadditivity given in equation (10.23). Within the range of admissible sample variation defined below, we evaluate equation (10.23) using the estimated cost function. Rejection of (10.23) within a region leads to a rejection of global subadditivity. Acceptance of (10.23) within a region obviously does not prove the existence of global subadditivity. Our test is thus less demanding of the data than the one proposed by Baumol, Panzar, and Willig and, for that reason, is more likely to provide useful information.

We describe the test for the simple case of two-firm production versus one-firm production of two outputs. It is straightforward to extend the test to the multifirm and multiproduct case. Let $Q_t^* = (Q_{1t}^*, Q_{2t}^*)$ denote the output vector realized in year t. Let $Q_M = (Q_{1M}, Q_{2M}) = (\min_t Q_{1t}^*, \min_t Q_{2t}^*)$ be the smallest output vector observed in the sample. Firm A produces

$$Q_{At} = (\phi Q_{1t} + Q_{1M},\ \omega Q_{2t} + Q_{2M}) \tag{10.24}$$

Firm B produces

$$Q_{BT} = [(1-\phi)Q_{1t} + Q_{1M},\ (1-\omega)Q_{2t} + Q_{2M}] \tag{10.25}$$

The parameters (ϕ,ω) satisfy $0 \leqslant \phi \leqslant 1$ and $0 \leqslant \omega \leqslant 1$. In order to avoid extrapolating the cost function to unobserved output configurations, we require both firms A and B to produce Q_1 and Q_2 in a ratio within the range of the data. Thus, we require

$$\begin{aligned} R_L &\leqslant \frac{\phi Q_{1t} + Q_{1M}}{\omega Q_{2t} + Q_{2M}} \leqslant R_U \\ R_L &\leqslant \frac{(1-\phi)Q_{1t} + Q_{1M}}{(1-\omega)Q_{2t} + Q_{2M}} \leqslant R_U \end{aligned} \tag{10.26}$$

where $R_L = \min(Q_{1t}/Q_{2t})$ and $R_U = \max(Q_{1t}/Q_{2t})$ where the min and max are taken over all t. Together, firm A and firm B produce

$$\begin{aligned} Q_{1t} + 2Q_{1M} &= Q_{1t}^* \\ Q_{2t} + 2Q_{2M} &= Q_{2t}^* \end{aligned} \tag{10.27}$$

so that

$$\begin{aligned} Q_{1t} &= Q_{1t}^* - 2Q_{1M} \\ Q_{2t} &= Q_{2t} - 2Q_{2M} \end{aligned} \tag{10.28}$$

This allocation is possible only for $Q_i^* > 2Q_{iM}$. We restrict the test to output levels which satisfy this constraint.

Let

$$\begin{aligned} C_{At}(\phi,\omega) &= C(Q_{At}) \\ C_{Bt}(\phi,\omega) &= C(Q_{Bt}) \\ C_t^* &= C(Q_{At} + Q_{Bt}) = C(Q_t^*) \end{aligned} \tag{10.29}$$

We measure the degree of subadditivity by

$$\mathrm{Sub}_t(\phi,\omega) = \frac{C_t^* - C_{At}(\phi,\omega) - C_{Bt}(\phi,\omega)}{C_t^*} \tag{10.30}$$

If $\mathrm{Sub}_t(\phi,\omega)$ is less than zero, the industry configuration given by (ϕ,ω) is less efficient than the monopoly configuration.

Our proposed statistical test is as follows. Calculate $\mathrm{Sub}_t(\phi,\omega)$. If $\mathrm{MaxSub}_t(\phi,\omega)$ is negative and statistically significantly different from zero, then we can accept the hypothesis that the cost function is locally subadditive. If MaxSub_t is not statistically significantly different from zero, we reject the hypothesis that the cost function is locally subadditive but we do not reject the hypothesis that the cost function locally additive.

We have applied this test to the Bell System. Table 10.9 reports cost and output data on the Bell System between 1947 and 1977. Between these years, costs increased more than fourteenfold, toll service increased almost fourteenfold, and local service increased more than fivefold. These data provide information on a reasonably large portion of the cost function. Both local and toll output doubled by 1958 making this year the first feasible year for our test. The ratio

Table 10.9 Local and Toll Output and Costs for the Bell System (1947–1977)

Year	Local	Toll	Cost
1947	.410	.346	2550.7
1948	.458	.372	2994.9
1949	.487	.383	3291.1
1950	.520	.416	3563.2
1951	.556	.466	4047.1
1952	.591	.501	4616.2
1953	.625	.522	4935.1
1954	.656	.550	5258.8
1955	.702	.619	5770.5
1956	.756	.683	6305.4
1957	.803	.740	6351.2
1958	.842	.776	6788.4
1959	.896	.862	7334.7
1960	.953	.935	7912.5
1961	1.000	1.000	8516.5
1962	1.054	1.082	9018.7
1963	1.110	1.174	9508.1
1964	1.159	1.317	10524.0
1965	1.228	1.474	11207.0
1966	1.306	1.684	11954.2
1967	1.383	1.842	12710.9
1968	1.465	2.055	13814.1
1969	1.558	2.334	14940.4
1970	1.638	2.536	16485.8
1971	1.709	2.697	17951.8
1972	1.804	2.969	20161.2
1973	1.912	3.316	21221.7
1974	2.007	3.605	23168.4
1975	2.075	3.864	27378.7
1976	2.173	4.244	31304.5
1977	2.291	4.684	36078.0

of local to toll has been between 0.5 and 1.3. For each year $t = 1958, \ldots, 1977$ we calculated $\mathrm{Sub}_t(\phi,\omega)$ for the unique combinations of (ϕ,ω) corresponding to[37]

$$\begin{aligned} \phi &= 0, .1, .2, \ldots, .9, 1.0 \\ \omega &= 0, .1, .2, \ldots, .9, 1.0 \end{aligned} \tag{10.31}$$

from three estimated cost functions: (1) the general translog cost function with nonautoregressive errors; (2) the general translog cost function with first order autoregressive errors; and (3) the Box–Tidwell cost function with nonautoregressive errors. We calculated Sub_t for the third cost function because, although this function performed substantially more poorly than the other two cost function, it was the only cost function we estimated that exhibited cost complementarities between local and toll and is therefore the cost function most likely to exhibit subadditivity.

Table 10.10 Percentage of Gain or Loss from Multifirm Versus Single-Firm Production for Alternative Industry Configurations[a,b] Translog—AR(1) 1961

ω = \ ϕ =	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
0.0	8 (21)											0.0
0.1	8 (19)	8 (20)										0.1
0.2	9 (18)	8 (18)	8 (19)									0.2
0.3	12 (16)	10 (17)	9 (18)	9 (18)								0.3
0.4	15 (15)	13 (13)	10 (17)	9 (18)	9 (18)							0.4
0.5	20 (14)	16 (15)	13 (16)	11 (17)	9 (18)	9 (18)						0.5
0.6	25 (14)	21 (15)	17 (16)	14 (17)	11 (17)	10 (18)	9 (18)					0.6
0.7			23 (16)	18 (16)	15 (17)	12 (18)	10 (18)	9 (18)				0.7
0.8					20 (17)	16 (18)	12 (18)	10 (19)	8 (19)			0.8
0.9							17 (19)	13 (19)	10 (20)	8 (20)		0.9
1.0										10 (21)	8 (21)	1.0
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	

[a]Entries equal Sub_{1961} so that a positive number indicates that multifirm production is much more efficient than single-firm production.
[b]Standard errors are reported in parentheses.

Table 10.11 Percentage of Gain or Loss from Multifirm Versus Single-Firm Production for Alternative Industry Configurations[a,b]
Translog—AR(0) 1961

ω = \ φ =	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
0.0	−17 (16)											0.0
0.1	−16 (14)	−16 (14)										0.1
0.2	−13 (12)	−14 (13)	−15 (13)									0.2
0.3	− 9 (11)	−12 (12)	−13 (12)	−14 (12)								0.3
0.4	− 5 (11)	− 9 (11)	−11 (12)	−13 (12)	−13 (12)							0.4
0.5	0 (11)	− 5 (11)	− 8 (11)	−11 (12)	−13 (12)	−13 (12)						0.5
0.6	5 (11)	0 (11)	− 4 (11)	− 8 (12)	−11 (12)	−13 (12)	−13 (13)					0.6
0.7			1 (13)	− 4 (12)	− 8 (12)	−11 (12)	−13 (12)	−14 (12)				0.7
0.8					− 4 (14)	− 8 (13)	−12 (13)	−14 (13)	−15 (13)			0.8
0.9							− 8 (15)	−12 (15)	14 (14)	−16 (14)		0.9
1.0										−16 (16)	−17 (16)	1.0
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	

[a]Entries equal Sub_{1961} so that a positive number indicates that multifirm production is more efficient than single-firm production.
[b]Standard errors are reported in parentheses.

We find that $MaxSub_t$ is greater than zero for all three cost functions at the output configurations observed between 1958 and 1977. $MaxSub_t$, however, is never statistically significantly different from zero at conventional significance levels. Thus, we reject the hypothesis that the Bell System cost function is locally subadditive and that the Bell System is a natural monopoly at the output levels observed between 1958 and 1977. We do not reject the hypothesis that the Bell System cost function is locally additive. Tables 10.10 and 10.11, and 10.12 report the values of Sub_{1961} for the three cost functions we examined. Standard errors are reported in parentheses. The fact that Sub_{1961} is never significantly different from zero is consistent with the hypothesis that the cost function is locally additive at the level of demand observed in 1961.[38]

Table 10.12 Percentage of Gain or Loss from Multifirm Versus Single-Firm Production for Alternative Industry Configurations[a,b]
Box–Tidwell AR(0) 1961

ω = \ φ =	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	10 (14)										
0.1	10 (14)	11 (14)									
0.2	11 (14)	11 (14)	11 (14)								
0.3	11 (14)	11 (14)	11 (14)	11 (14)							
0.4	11 (14)	11 (14)	11 (14)	11 (14)	11 (14)						
0.5	11 (14)	11 (14)	11 (14)	11 (14)	11 (14)	11 (14)					
0.6	10 (14)	11 (14)	11 (14)	11 (14)	11 (14)	11 (14)	11 (14)				
0.7			11 (14)	11 (14)	11 (14)	11 (14)	11 (14)	11 (14)			
0.8					11 (14)	11 (14)	11 (14)	11 (14)	11 (14)		
0.9							11 (14)	11 (14)	11 (14)	11 (14)	
1.0										10 (14)	10 (14)
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0

[a]Entries equal Sub_{1961} so that positive number indicates that multifirm production is more efficient than single firm production.
[b]Standard errors are reported in parentheses.

Summary

We estimated several alternative multiproduct cost functions for the Bell System. We rejected the modified translog cost function and the Box–Tidwell cost function in favor of the translog cost function. We tested and rejected the hypothesis that there exists an aggregate measure of local and long-distance telecommunications service. Therefore (*a*) the single-output cost functions estimated by previous researchers were rejected by the data and (*b*) scale economy estimates from these single-output cost functions provide little information about the optimal structure of the telecommunications industry.

We developed a test for natural monopoly that is more direct and more empirically tractable than the previous tests reported in the literature. For observed output configurations, our test compares the single-firm cost of production with the multifirm cost of production. We applied this test to the Bell System. We found that the Bell System did not have a natural monopoly over any of the output configurations which were realized between 1958 and 1977. Two firms were always able to produce these output configurations more cheaply than a single firm.

Appendix A
Alternative Formulations of the Cost Function

The Bell System provides numerous telecommunications services to consumers and businesses. Its operating companies provide local service through their exchange facilities; public telephone service; directory advertising; and interstate and intrastate long-distance services including various private line services. Its Long Lines Department provides some interstate long-distance and private line services and most wide-area toll services. These entities own different portions of the telecommunications network.

It is useful to view the telecommunications network as consisting of numerous nodes, which represent the local exchange facilities, interconnected by lines, which represent the long-distance facilities. This idealized network provides local service—including local telephone service, public telephone service, and local private line services—through the exchange facilities and long-distance service—including toll service, private line service, and wide-area toll service—through the long-distance facilities. AT&T and the Bell Operating Companies operate and maintain different chunks of this integrated network.

AT&T owns 100% of the stock of 17 of its operating companies, more than 85% of the stock of four others, and a minority interest in one. Its Long Lines Division and General Departments provide overall coordination and direction for the operating companies. It therefore ultimately owns and coordinates the numerous local exchanges as well as the long-distance facilities shared by the local exchanges. In examining whether there is a telecommunications natural monopoly, these considerations suggest the following cost function

$$C = C(l_1, \ldots, l_n, T) \tag{10.32}$$

where l_i represents the local service produced the ith local exchange and T is the long-distance (or toll) service produced by the shared long-distance facilities. If synergies arise from the joint production of local exchange services with each other and with the long-distance service, a natural monopoly may exist. Separation of the local exchange facilities from each other and from the shared long-distance service would decrease efficiency. With data on a cross section of telephone conglomerates having different numbers of local exchanges and not all having a shared long-distance facility, it is possible to estimate (10.32) and test whether there are economies of scope between local exchanges and long-distance facilities. Such data are not presently available. With time-series data on a conglomerate such as AT&T, it is possible to estimate (10.32) and test whether there are cost complementarities between local exchange facilities and long-distance facilities. For reasons discussed below, available data are not even sufficient for this purpose.

The Bell operating companies are distinct financial units although they are commonly owned by AT&T. They provide both local and long-distance services within their respective territories. In 1979, they earned about 21 billion dollars in local service revenue and 21 billion dollars in long-distance service revenue. AT&T Long Lines supplements the long-distance service provided by the operating companies. In 1979, it earned slightly more than three billion dollars in long-distance service revenues. These considerations suggest the following formulation of the cost function

$$C_i = C_i(l_i, t_i, L_R, T_R), \quad i = 1, \ldots, n \tag{10.33}$$

where l_i is the local service output of the ith operating company, t_i is the long-distance service output of the ith operating company, L_R is the local service output of the other operating companies, T_R is the long-distance service output of the other operating companies and the Long Lines Division and C_i is the cost incurred by the ith operating company. This system of equations could be estimated with time series cross section data on the Bell operating companies and AT&T Long Lines. If increases in L_R and T_R decrease the costs of producing given levels of l_i and t_i and if there are product-specific scale economies in producing l_i and t_i, there may be a natural monopoly over the telecommunications system.

Unfortunately, reliable data are not available for estimating (10.33). The costs incurred and the revenues received by the various entities that provide telecommunications service are determined by the "separations and settlement" process. This process involves periodic negotiations between the Bell operating companies, AT&T, the independent telephone companies, the FCC, and the state regulatory commissions. These negotiations assign the costs of operating the network to the state rate bases examined by the state commissions and to the interstate rate base examined by the FCC. These negotiations also assign the revenues earned by the network to the assets residing in the various regulatory jurisdictions. The costs and revenues that appear on the financial record of AT&T Long Lines and the Bell operating companies are determined by a political rather than an economic process. The assignment formulas have changed over time as the political powers of the participants in the negotiations have ebbed and flowed. Therefore, the correspondence between reported costs and revenues and economic costs and revenues for the individual entities providing telecommunications services is tenuous and changeable.

These data limitations dictate the following formulation of the cost function

$$C = C(L,T) \tag{10.34}$$

where L is the aggregate local service provided by the Bell operating companies,

T is the aggregate long-distance service provided by the Bell operating companies and the Long Lines Department, and C is the aggregate cost incurred by the Bell System. This formulation assumes that costs are independent of the allocation of a given level of output across the Bell operating companies and Long Lines. Because the levels of local and long-distance services have probably increased in tandem across the Bell operating companies, the aggregation of l_i and t_i into L and T and the estimation of an aggregate cost function is probably not too unreasonable.

Appendix B
Unrestricted Cost Function Estimates

When the homogeneity and symmetry restrictions implied by producer theory were relaxed, the parameter estimates changed dramatically. Table 10.13 reports the unrestricted parameter estimates for the nonautoregressive translog cost function.

Table 10.13 Parameter Estimates for the Translog Cost Function Unrestricted for Homogeneity and Symmetry with No Serial Correlation

Parameter*	Cost		Capital share		Labor share	
Constant	9.051	(.002)				
Capital	.197	(.050)	.539	(.004)		
Labor	.163	(.155)			.352	(.003)
Toll	−1.168	(.183)				
Local	.760	(.159)				
Technology	1.226	(.083)				
Capital²	2.484	(.394)	.197	(.025)		
Labor²	4.722	(3.487)			.203	(.062)
Materials²	10.811	(4.057)				
Capital · Labor	−2.836	(.373)	−.218	(.063)	−.166	(.025)
Capital · Materials	−3.892	(1.042)	.208	(.115)		
Labor · Materials	−13.377	(3.542)			−.165	(.113)
Capital · Toll	12.723	(1.264)	−.499	(.103)		
Labor · Toll	4.210	(3.622)			.406	(.102)
Materials · Toll	−11.562	(6.477)				
Capital · Local	−11.814	(1.352)	.568	(.100)		
Labor · Local	−4.033	(3.448)			−.500	(.098)
Materials · Local	8.764	(5.783)				
Toll²	36.462	(3.455)				
Local²	45.660	(4.374)				
Toll · Local	−77.513	(7.387)				
Technology²	16.041	(1.126)				
Capital · Tech.	−3.334	(.290)	.223	(.043)		
Labor · Tech.	.795	(.975)			−.192	(0.42)
Materials · Tech	14.942	(2.810)				
Toll · Tech	−29.646	(2.435)				
Local · Tech.	20.436	(2.086)				

tion. The estimates are markedly different from the corresponding estimates in the first column of Table 10.2. The homogeneity and symmetry restrictions are rejected by the data at a high level of confidence. These restrictions require parameters to be identical across equations. As we see by comparing the columns of Table 10.13 parameter estimates are wildly different between equations.

Appendix C
Bell System Data

Our data on costs, input prices, and output quantities were obtained from Christensen.[39] Christensen calculated Tornqvist indices of output quantities and input quantities for the Bell system based on detailed yearly data for the period 1947–1977. The Tornqvist index can be written as

$$\ln\left(\frac{X_t}{X_{t-1}}\right) = \sum \overline{w}_{it} \ln\left(\frac{x_{it}}{x_{it-1}}\right)$$

where

$$\overline{w}_{it} = \frac{1}{2}\left(\frac{p_{it}x_{it}}{\Sigma p_{it}x_{it}} + \frac{p_{it-1}x_{it-1}}{\Sigma p_{it-1}x_{it-1}}\right)$$

and where X_t is an aggregate quantity index for period t, x_{it} is one of individual quantity indices, and p_{it} is the price of quantity i in period t. The quantity index X_t can be obtained from this formula by normalizing one of the X_t to one and calculating the X_t recursively.

Christensen collected operating revenue data for five categories: local, interstate toll, intrastate toll, directory advertising, and miscellaneous. In order to obtain quantity data from these revenue data he divided by price indices supplied by AT&T. He formed an aggregate measure of output from these quantity data and the price indices using the Tornqvist procedure described above. For our multiproduct study, we used the Tornqvist procedure to calculate aggregate output indices for local and long-distance service. Our local service output is based on local revenue, directory advertising revenue, and miscellaneous revenue.

Christensen collected data on hours worked by Bell System employees broken down by occupation and years of service. Using Bell System wage rates, he then calculated an index of labor output. He collected data on twenty different types of owned tangible assets. He says,

> For each of the twenty categories we obtained a time series of investment expenditures, which we then deflated by specific price indexes. The resulting real investment figures were used in conjunction with capital stock benchmarks and rates of replacement to obtain capital stock series via the perpetual inventory method. The benchmarks and replacement rates were based on surveys of Bell

System Capital Stock for 1958, 1965, and 1970. These capital stocks, their asset prices, and rates of replacement were used along with the Bell System's cost of capital and tax information to compute capital service price weights. These weights were constructed following the methodology originally proposed by Christensen and Jorgenson and modified for regulated firms by Caves, Christensen, and Swanson. We computed capital input for the Bell System as a Tornqvist index of the twenty types of owned capital, and one category of rented capital, using service price weights.[40]

He included data on seven categories of materials: electricity, accounting, machines, advertising, stationery and postage, services from Bell Labs, and mis-

Table 10.14 Bell System Data Used for Multiproduct Cost Function Estimates[a]

Year	Cost	Local output	Toll output	Capital price
1947	2550.68	.41014	.36642	.49948
1948	2994.94	.45783	.34642	.55879
1949	3291.06	.48703	.38296	.57440
1950	3563.20	.52004	.41592	.61810
1951	4047.07	.55560	.46552	.70031
1952	4616.23	.59149	.50116	.79500
1953	4935.13	.62452	.52271	.80853
1954	5258.76	.65669	.55000	.81269
1955	5770.47	.70289	.61941	.86056
1956	6305.44	.75645	.68394	.88033
1957	6351.19	.80355	.74006	.81997
1958	6788.40	.84224	.77663	.87304
1959	7334.71	.89657	.86274	.91051
1960	7912.48	.95314	.93512	.95733
1961	8516.46	1.00000	1.00000	1.00000
1962	9018.66	1.05411	1.08231	1.01457
1963	9508.12	1.11068	1.17451	1.00832
1964	10524.00	1.15909	1.31715	1.07804
1965	11207.00	1.22822	1.47436	1.06139
1966	11954.20	1.30609	1.68434	1.04475
1967	12710.90	1.38312	1.84266	1.04058
1968	13814.10	1.46568	2.05511	1.08325
1969	14940.40	1.55869	2.33437	1.04579
1970	16485.80	1.63899	2.53682	1.04891
1971	17951.80	1.70956	2.69772	1.04058
1972	20161.20	1.80454	2.96927	1.09157
1973	21221.70	1.91210	3.31628	1.00312
1974	23168.40	2.00785	3.60503	1.00104
1975	27376.70	2.07532	3.86421	1.18939
1976	31304.50	2.17307	4.24442	1.32778
1977	36078.00	2.29155	4.68449	1.53590

Source: See Appendix C.

[a]Arithmetic values of Tornqvist indices normalized to equal one in 1961.
[b]Tornqvist index of local and toll used for estimating single-product cost function.

cellaneous. Using Bell System price indices, he formed an index of materials inputs. He obtained price indices for capital, labor, and materials by dividing total expenditures on these items by the associated price index. We normalized input prices to equal one in 1961.

The research and development index was calculated

$$A_t = \left[\sum_{k=0}^{N_t} \left(\frac{\lambda^{k-1}}{\Gamma(k)} \right) \frac{Rand_{t-k}}{CPI_{t-k}} \right] \Big/ \sum_{k=0}^{N_t} \left(\frac{\lambda^{k-1}}{\Gamma(k)} \right)$$

Table 10.14 *(continued)*

Labor price	Materials price	R&D index	Capital share	Labor share	Aggregate[b] output
.53566	.66952	.57955	.39552	.49635	.37200
.58236	.75117	.55445	.40430	.48286	.41109
.60959	.74530	.55261	.41936	.47113	.43262
.63164	.76525	.56980	.44096	.45352	.46579
.66926	.81572	.59576	.45338	.44230	.50814
.70946	.82863	.62057	.46670	.43159	.54461
.73411	.84389	.63873	.46436	.43614	.57320
.76134	.85563	.65059	.46596	.42866	.60369
.80674	.87558	.66162	.47840	.41414	.66093
.81063	.90493	.68018	.47642	.41045	.72066
.84824	.93896	.71436	.47138	.41365	.77258
.85084	.95305	.76830	.50754	.38849	.81106
.91958	.97417	.83934	.52030	.37321	.88010
.95979	.99061	.91902	.53120	.36083	.94477
1.00000	1.00000	1.00000	.54381	.34605	1.00000
1.03632	1.01995	1.08533	.55077	.33966	1.06707
1.07393	1.03404	1.18984	.55139	.33353	1.13972
1.12970	1.08451	1.32815	.56240	.32693	1.22792
1.17121	1.10681	1.49998	.55286	.32925	1.33575
1.22827	1.14085	1.16877	.54302	.33698	1.47162
1.29702	1.17371	1.86844	.54079	.34058	1.58499
1.36057	1.21948	2.02744	.54614	.33406	1.72439
1.49416	1.28286	2.16342	.51402	.35802	1.89881
1.62387	1.35211	2.28416	.49799	.37133	2.03198
1.80415	1.42019	2.40026	.48313	.38304	2.14140
2.06226	1.47653	2.52124	.47953	.39061	2.31167
2.26329	1.56221	2.65447	.44558	.41442	2.52082
2.51621	1.74061	2.80468	.434068	.42485	2.69770
2.85473	1.91315	2.97195	.46178	.40606	2.84174
3.21920	2.01408	3.15081	.469773	.39508	3.05380
3.40726	2.12911	3.33422	.48680	.37808	3.30393

where $\Gamma(k)$ is the gamma function evaluated at k, $Rand_t$ is the research and development expenditure by Bell Labs charged to AT&T, CPI_t is the consumer price index, $\lambda = 6$ and

$$N_t = \begin{cases} 22 - (1958\text{-}t) & t = 1947\text{–}1957 \\ 22 & t = 1958\text{–}1977 \end{cases}$$

A_t is based on fewer than 22 lagged values prior to 1958 because we had data on $Rand_t$ only from 1936. We tried deflating by an R & D-specific deflator rather than the CPI but we obtained poor statistical results. We also tried different values of the lag parameter but found that our results were insensitive to this parameter. This index was initially proposed by Vinod.[41] Our long-distance service output is based on interstate and intrastate toll service. Table 10.14 reports the data we used.

NOTES

1. AT&T, *Defendants' Third Statement of Contentions and Proof*, in *US* v. *AT&T*, p. 35.
2. The major studies are Laurits Christensen, Diane Cummings, and Philip Schoech, "Econometric Estimation of Scale Economies in Telecommunications," SSRI Working Paper No. 8013 (Madison, WI: Social Systems Research Institute, University of Wisconsin at Madison August 1981) and M. Ishaq Nadiri and Mark A. Shankerman, "The Structure of Production, Technological Change, and the Rate of Growth of Total Factor Productivity in the US Bell System," in T. Cowing and R. Stevenson, eds., *Productivity Measurement in Regulated Industries* (New York: Academic Press, 1981). H.D. Vinod has published two studies that rely on ridge regression: "Applications of New Ridge Regression Methods to a Study of Bell System Scale Economies," *Journal of the American Statistical Association*, December 1976, pp. 835–841 and "Bell System Scale Economies and the Economics of Joint Production," FCC Docket 20003, Bell Exhibit No. 59. The latter paper is the only major study which estimates the parameters of a multiproduct technology for the Bell System. Unfortunately, this study is seriously flawed because it relies on canonical ridge regression which has no known optimality property, as Vinod admits in "Canonical Ridge and the Econometrics of Joint Production," *Journal of Econometrics*, August 1976, pp. 147–166. Several studies have estimated multiproduct cost functions for Bell Canada. See Melvyn Fuss and Leonard Waverman, *The Regulation of Telecommunications in Canada*, Technical Report No. 7, Economic Council of Canada, March 1981 and the references cited therein. See Chapter 6 of this volume for a critique of the existing studies on the cost characteristics of the telecommunications industry.
3. Christensen, Cummings, and Schoech, *ibid*., p. 4.
4. W. Baumol, J. Panzar, and R. Willig, *Contestable Markets and the Theory of Industry Structure* (San Diego: Harcourt Brace Jovanovich, 1982).
5. This function is estimated under the assumption that AT&T has exogenously given output levels and exogenously given factor prices. The assumption that input prices are exogenously given is questionable. The Communications Workers of America represent most of the Bell System's nonmanagerial employees. The wage rates faced by the Bell System are therefore determined by negotiations between a powerful corporation and a powerful trade union rather than by competitive market forces. The Bell System purchases most of its capital, materials, and research and development from Western Electric and Bell Labs, both of which are owned by AT&T and both of which are large purchasers in their respective factor markets. The Bell System may have some control over the prices it faces for capital, materials, and research and development. The assumption that output levels are exogenously given is also questionable. The justification usually given for this assumption is that the prices charged for telephones service are regulated and that the Bell System is obligated to meet all demand. But the Bell System can file for tariff increases which may be approved and instituted quite quickly. Given the institutional evidence against the hypothesis of output level and input price endogeneity, it is important to test this hypothesis. Using a Wu test, we reject the hypothesis that output and input prices are endogenous.

6. We have not modeled the regulatory process explicity because a sensible, empirically tractable model of regulation is presently lacking and we did not have the resources to develop one. Previous studies of regulated industries have used the Averch–Johnson theory of regulation to model the impact of regulation on profit maximization and input demand. These studies have had rather mixed success. See Fuss and Waverman, *op. cit.*; R. Spann, "Rate of Return Regulation and Efficiency in Production: An Empirical Test of the Averch–Johnson Thesis," *The Bell Journal of Economics and Management Science*, Spring 1974, pp. 38–52; and T. Cowing, "The Effectiveness of Rate-of-Return Regulation: An Empirical Test Using Profit Functions," in M. Fuss and D. McFadden, *Production Economics: A Dual Approach to Theory and Applications* (Amsterdam: North Holland, 1978). The Averch–Johnson model misconstrues several aspects of the regulatory process. In telecommunications, for example, regulators control prices directly through the tariff-setting mechanism and the rate of return only indirectly through these tariffs. See Paul Joskow, "Inflation and Environmental Concern: Structural Change in the Process of Public Utility Regulation," *Journal of Law and Economics*, October 1974, pp. 291–328 and Paul Joskow and Roger Noll, "Theory and Practice in Public Regulation: A Current Overview," in G. Fromm, ed., *Studies in Public Regulation* (Cambridge: MIT Press, 1981) for further discussion.
7. We combined local telephone service, directory advertising, and miscellaneous service into an aggregate called *local service* and interstate and intrastate long distance into an aggregate called *long distance*.
8. L.R. Christensen, D.W. Jorgenson, and L.J. Lau, "Transcendental Logarithmic Production Functions," *Review of Economics and Statistics*, August 1973, pp. 28–49.
9. These restrictions are termed the homogeneity and symmetry restrictions in what follows although they include several other restrictions as well. Symmetry refers to the independence of the second-order derivatives of cost with respect to price to the order of differentiation, that is,

$$\frac{\partial C}{\partial p_i\, \partial p_j} = \frac{\partial C}{\partial p_j\, \partial p_i} \text{ which implies } \gamma_{ij} = \gamma_{ji}.$$

Since only $(\gamma_{ij} + \gamma_{ji})$ is identified in the cost and share equations, symmetry is usually tested by determining whether the γ_{ij}'s are identical across equations.
10. See Ronald W. Shephard, *Theory of Cost and Production Functions* (Princeton: Princeton University Press, 1970), for definitions.
11. Michael Denny and Cheryl Pinto, "An Aggregate Model with Multiproduct Technologies," in M. Fuss and D. McFadden, *op. cit.*, p. 256. Note that the indices in their derivation are in an improper order.
12. The separability test tells us whether an aggregator function exists but not the form of the aggregator function. It is possible to accept separability but reject particular aggregator functions such as the translog aggregator used by Christensen, Cummings, and Schoech.
13. Denny and Pinto, *op cit*., p. 258
14. Economies of scope are a necessary condition for natural monopoly. Economies of scope exist for the cost function $C(Q_1,Q_2)$ if $C(Q,0) + C(0,Q_2)$ exceeds $C(Q_1,Q_2)$ so that joint production is cheaper than separate production.
15. David F. Burgess, "Duality Theory and Pitfalls in the Specifications of Technologies," *Journal of Econometrics*, May 1975, pp. 105–123.
16. A.R. Gallant, "On the Bias in Flexible Functional Forms and an Essentially Unbiased Form," *Journal of Econometrics*, February 1981, pp. 211–246.
17. David K. Guilkey and C.A. Knox Lovell, "On the Flexibility of the Translog Approximation," *International Economic Review*, February 1980, pp. 137–147.
18. Melvyn Fuss and Leonard Waverman, *op. cit.*.
19. Douglas W. Caves, Laurits R. Christensen, and Michael Trethway, "Flexible Cost Functions for Multiproduct Firms," *Review of Economics and Statistics*, August 1980, pp. 477–481.

20. Because the disturbances for the share equations must lie in the unit interval, the assumption that the ε_i in (10.16) and the u_i in (10.17) are multinormally distributed is clearly inappropriate. Woodland, however, found that parameter estimates obtained under the assumption that the error terms of the share equations are multinormally distributed were close to parameter estimates obtained under the assumption that the error terms are Dirichlet distributed. The Dirichlet distribution lies within the unit sphere. See A.D. Woodland, "Stochastic Specification of the Estimation of Share Equations." *Journal of Econometrics*, August 1979, pp. 361–384. We note, however, that the Dirichlet distribution is rather restrictive since it imposes negative covariances between the disturbances. Also, using the method proposed by H. White it is possible to correct for nonnormality of the error distribution terms in forming estimates of the standard error. H. White, "A Heteroskedasticity Consistent Covariance Matrix Estimator and a Direct Test of Heteroskedasticity," *Econometrica*, May 1980, 817–830.

21. See L. Christensen and W. Greene, "Economies of Scale in U.S. Electric Power Generation," *Journal of Political Economy*, October 1976, pp 655–676, for a discussion of the estimation of the translog cost function using the iterated Zellner method. The Zellner method for estimating a seemingly unrelated regression system involves two steps: (1) form a consistent estimate S_1 of the cross-equation covariance matrix Σ, and (2) using S_1 in place of Σ calculate the joint generalized least squares estimate of the parameter vector β. The iterated method forms a new estimate S_2 of Σ from the residuals of the second step and then forms a new estimate of the parameter vector β from S_2. This iteration continues until estimates of β and Σ converge. The Zellner method and the iterated Zellner method were developed for systems which are linear in their parameters but can be readily extended to systems which are nonlinear in their parameters. See A. Ronald Gallant, "Seemingly Unrelated Nonlinear Regressions," *Journal of Econometrics*, April 1975, pp. 35–50.

22. The likelihood function for the modified translog and Box–Tidwell cost functions exhibited considerable nonlinearities and plateaus. The likelihood optimization routines usually either failed to converge or converged to local but not global maxima. Seemingly unrelated nonlinear regression techniques were more successful for these specifications. Consequently, the estimates reported for the modified translog and Box–Tidwell cost functions were obtained from seemingly unrelated nonlinear regressions rather than maximum likelihood esimation. Asymptotically, both methods yield identical results.

23. A.P. Barten, "Maximum Likelihood Estimation of Complete Systems of Demand Equations," *European Economic Review*, May 1967, pp. 7–73.

24. E.R. Berndt and N.E. Savin, "Estimation and Hypothesis Testing in Singular Equation Systems with Autoregressive Disturbances," *Econometrica*, September–November 1975, pp. 937–957.

25. The specification $\nu_L = \nu_K$ successfully purged the autocorrelation from the residuals whereas the specification $\nu_L \neq \nu_K$ continued to exhibit autocorrelation.

26. H.D. Vinod, 1976, *op. cit.*

27. As alternative measures of technological changes, we used time, the percent of telephones with access to long-distance dialing facilities, and the percent of telephones connected to modern switching facilities, singly and in combination. Generally, these specifications gave less stable and less plausible estimates (e.g., negative marginal cost estimates and positive own-price factor demand elasticities) and higher generalized variances than the specifications reported below. Fuss and Waverman, *op. cit.*, assumed that technological change was output augmenting, with increases in direct distance dialing increasing the value of toll calls and increases in modern switching facilities increasing the value of local calls. This specification performed poorly with U.S. data. Christensen, Cummings, and Schoech's single-output study, *op. cit.*, estimated, *inter alia*, a factor augmenting technological change specification where increases in R & D lowered the effective prices for capital, labor, and materials, In the two-output case with autoregressive disturbances, this specification exhibited constant returns to scale. The generalized variance was, however, considerably higher than in a specification which included general technological change. This study also experimented with different lag parameters for the R & D index but found that estimates were relatively insensitive to the value of the lag parameter.

28. These comparisons are not entirely reliable since the translog specifications were estimated with maximum likelihood while the other specifications were estimated with seemingly unrelated nonlinear regression techniques. In order to compare generalized variances obtained from iden-

tical estimation procedures, we also estimated the translog specification with seemingly unrelated nonlinear regression. We found that the modified translog cost function with $\eta = .03$ gave the lowest generalized variance under the assumption that the disturbances are not temporally correlated. The modified translog had a generalized variance of 23.962 whereas the translog had a generalized variance of 24.378. Obviously, it is not possible to reject the hypothesis that $\eta = 0$. We found that the translog cost function continued to give the lowest generalized variance under the assumption that the disturbances are temporally correlated. Also, the translog cost function performed better than the Box–Tidwell cost function for both error specifications.

29. We usually rejected homogeneity and symmetry separately; rejected homogeneity with symmetry imposed; and rejected symmetry with homogeneity imposed. Table 10.13 in Appendix B reports estimates for the translog cost function with nonautoregressive disturbances when homogeneity and symmetry are not imposed.

30. When $\nu_L \neq \nu_K$ it was possible to accept separability. The generalized variance under the hypothesis of homogeneity and symmetry was 8.4177×10^{-15} and under the hypothesis of homogeneity, symmetry, and separability was 9.8276×10^{-15}. The likelihood ratio test statistic for separability was 4.79 compared with a critical value at the five percent level with two restrictions of 5.99. The acceptance of separability suggests that a consistent aggregate $A(L,T)$ exists but does not tell us whether the translog aggregate used by Christensen, Cummings, and Schoech and by Nadiri and Schankerman is appropriate. We tested the validity of the translog aggregation in the following fashion. Let $A(L,T) = \ln Q = W_L \ln L + W_T \ln T$ where $\ln Q$ is the aggregate formed by Christensen, Cummings, and Schoech. We chose W_L and W_T to satisfy this relationship. Under our general specification we have

$$\beta \ln Q = \beta_1 W_L^z \ln L + \beta_2 W_T^z \ln T \text{ and } \delta(\ln Q)^2 = \delta_{11}(W_L^z \ln L)^2 + \delta_{22}(W_T^z \ln T)^2 + 2\delta_{12} W_T^z W_L^z \ln L \ln T$$

with $z = 0$. Under the single-output specification, we have $\beta_1 = \beta_2 = \beta, \delta_{11} = \delta_{22} = 2\delta_{12}$, and $z = 1$. The generalized variance under the general multioutput specification is 9.827. The generalized variance under the single-output specification is 22.725. Separability, homogeneity, and symmetry were imposed on both specifications. Letting $z, \beta_1, \beta_2, \delta_{11}, \delta_{22}, \delta_{12}$, be unconstrained we obtained a generalized variance of 8.5281. The point estimate of z was -5.43 with a standard error of 1.92 leading to rejection of $z = 1$ at the .0001 level of significance.

31. Christensen, Cummings, and Schoech *op. cit.*, Table 1, column 12. At the sample mean and using their notation, $\varepsilon_{KK} = (\gamma_{KK} + \beta_K^2 - \beta_K)/\beta_K = .1494$, the elasticity of demand for labor is $\varepsilon_{LL} = (\gamma_{LL} + \beta_L^2 - \beta_L)/\beta_L = .0269$, and the elasticity of demand for materials is $\varepsilon_{MM} = (\gamma_{MM} + \beta_M^2 - \beta_M)/\beta_M = -.0979$. They claim that the cost function is better behaved when technological change is allowed to be factor augmenting in the following fashion; $p_i = p_i^{\lambda_i A}$, i $= L,K,M$ and where A denotes the technological change proxy. Their reported estimates (see column 10), however, show that the elasticity of demand for capital is positive: $\varepsilon_{KK} = (\gamma_{KK} + \beta_K^2 - \beta_K)/\beta_K = [.266 + (.518)^2 - (.518)]/.518 = .0315$ at the sample mean.

32. The estimated general translog cost functions were monotonic with respect to input prices in all years. They were also quasi-concave in all years. Quasi-concavity requires that the following conditions hold where

$$C_{ij} = \frac{\partial C}{\partial p_i\, \partial p_j}, \quad i,j = 1,2,3, = K,L,M \quad \text{for capital, labor, and materials, respectively}$$

$$(1)\ C_{KK} \leq 0 \qquad (2)\ C_{LL} \leq 0 \qquad (3)\ C_{MM} \leq 0$$

$$(4)\begin{vmatrix} C_{KK} & C_{KL} \\ C_{KL} & C_{LL} \end{vmatrix} \geq 0 \qquad (5)\begin{vmatrix} C_{KK} & C_{KM} \\ C_{KM} & C_{MM} \end{vmatrix} \geq 0 \qquad (6)\begin{vmatrix} C_{LL} & C_{LM} \\ C_{LM} & C_{MM} \end{vmatrix} \geq 0$$

$$(7)\begin{vmatrix} C_{KK} & C_{KL} & C_{KM} \\ C_{KL} & C_{LL} & C_{LM} \\ C_{KM} & C_{LM} & C_{MM} \end{vmatrix} \leq 0$$

where we have made use of the symmetry of the C_{ij}'s. The first three quantities were strictly less than zero in every year. The second three quantities were strictly greater than zero in every year. The last quantity was virtually zero in every year. The value of the determinant (7) was of the order $\pm\ 10^{-8}$ although typical elements were of the order 10^3. In the case of the nonautoregressive specification, the determinant (7) was exactly zero in the middle year of the sample. In the nonautoregressive specification the determinant (7) was slightly negative in 14 years, zero in two years, and positive in 15 years. In the autoregressive specification, the determinant (7) was slightly negative in 19 years, and slightly positive in 14 years. Thus, estimated cost functions are both quasi-concave. Monotonicity and quasi-concavity of the cost function are sufficient conditions for cost minimization.

33. Q_1 and Q_2 is feasible if there exist prices p_1 and p_2 such that $Q_1 = D(p_1)$, $Q_2 = D(p_2)$ and $R(p_1, p_2) = p_1Q_1 + p_2Q_2 - C[Q_1(p_1), Q_2(p_2)] \geqslant 0$ where $R(p_1, p_2)$ is the firm's profit function.
34. See Note 14 for a definition of economies of scope.
35. There are product-specific scale economies in product two if

$$\frac{C(Q_1,Q_2) - C(Q_1,0)}{Q_2\frac{\partial C}{\partial Q_2}} > 1$$

and similarly for product one.

36. See Fuss and Waverman, *op. cit.*, for a tentative test of economies of scope.
37. Excluding the complements of (ϕ,ω) and the values which do not satisfy the inequalities given by (10.26).
38. Of course, it is possible that the cost function is subadditive at output vectors other than those realized between 1958 and 1977. If introducing competition into the telecommunications industry lead to radical changes in prices, it is possible that an output vector could be realized at which the cost function is subadditive. If the introduction of competition leads to only minor changes in prices and if the demand is fairly stable over time, our results suggest that several firms could meet demand more cheaply than a single firm could. Our results are too imprecise to evaluate the relative costs of *particular* industry configurations although they show that *some* multifirm configurations would be more efficient than a single firm.
39. These data were submitted as an appendix to Christensen's written testimony in *US* v. *AT&T*.
40. Christensen, Cummings, and Schoech, *op cit.*, p. 9. In calculating the user cost of capital, he "used the embedded cost of capital used for capital budget planning in the Bell System." See Appendix 2 to Christensen's testimony in *US* v. AT&T.
41. Vinod (1976), *op. cit.*

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