CE-221 Midterm 1

1 Plastic Analysis

Upper Bound

$\dot{V}_{hp} = \mathbf{A}_f \dot{U}_f \quad \dot{U}_f = \mathbf{A}_{cp} \dot{U}_c$

$\dot{V}_{hp} = \mathbf{A}_f \mathbf{A}_{cp} \dot{U}_c = \mathbf{A}_{mp} \dot{U}_c$

$$\dot{\mathcal{W}}_e = \mathcal{D}_p$$
$$\dot{U}_f^T \left(\lambda P_{ref} + P_{cf}\right) = \left(\dot{V}_{hp}^+\right)^T Q_{pl}^+ + \left(\dot{V}_{hp}^-\right)^T Q_{pl}^-$$

$$\dot{\boldsymbol{V}}_{hp}^+ = \begin{cases} \dot{\boldsymbol{V}}_{hp} & \text{if } \dot{\boldsymbol{V}}_{hp} > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\dot{\boldsymbol{V}}_{hp}^- = \begin{cases} -\dot{\boldsymbol{V}}_{hp} & \text{if } \dot{\boldsymbol{V}}_{hp} < 0 \\ 0 & \text{otherwise} \end{cases}$$

$\dot{V}_{hp}^+ - \dot{V}_{hp}^- = \mathbf{A}_f \dot{U}_f$

$\lambda = \frac{\dot{V}_{hp}^T Q_{pl}}{\dot{U}_f^T P_{ref}} = \frac{\mathbf{A}_{mp}^T Q_{pl}}{\mathbf{A}_{cp}^T P_{ref}}$

Lower Bound

4 Advanced

4.2 P-M Interaction

4.3 Deformation at Collapse

assumptions:

1) The applied loading consists only of the reference load P_{ref}, which is increased monotonically by incrementing the load factor λ.
2) Plastic hinge deformations increase monotonically under the monotonically increasing reference load.

$$\boldsymbol{V}_{\boldsymbol{\varepsilon}} = \mathbf{F}, \boldsymbol{Q}_c + \boldsymbol{V}_0$$

3) Select any hinge of the collapse mechanism as last to form and solve the kinematic relations for the corresponding displacements U_f^t, where the superseript tr stands for trial result.
4) Determine the plastic hinge deformations V_{hp}^{tr} corresponding to U_f^{tr} with

$$\boldsymbol{V} = \boldsymbol{V}_\varepsilon + \boldsymbol{V}_{hp}^{tr} = \mathbf{A}_f \boldsymbol{U}_f^{tr}$$

5) If the sign of each plastic deformation matches the sign of the corresponding basic force Q_c from the equilibrium equations, the last hinge location is correct. The trial displacements and plastic deformations from steps (1) and (2) give the free dof displacements U_f and the plastic deformations $\boldsymbol{V}_{hp}$ at incipient collapse.
6) If the sign of one or more plastic deformations does not match the sign of the corresponding basic force, correct the free dof displacements and plastic deformations of Step (1) and (2) in a single step as described in the following.

If the sign of one or more plastic deformations does not match the sign of the corresponding basic

5 Event-to-Event

1) Set up the current stiffness of the structural model with basic force releases at all p_k locations where plastic hinges appeared in the events through k

$$\mathbf{K}^{(k)} = \mathbf{A}_f^T \mathbf{K}_s^{(k)} \mathbf{A}_f$$

2) Solve for the free dof displacements U_f' and the basic forces Q' under reference load

$$U_f^{(k)} = \mathbf{K}^{(k)} | P_{\text{ref}}$$
$$Q^{(k)} = \mathbf{K}_s^{(k)} \left[\mathbf{A}_f U_f^{(k)}\right]$$

3) Determine the DC ratio under the reference load at locations m without plastic hinge

$$DC_m' = \frac{Q_m^{(k)}}{\operatorname{sgn} Q_{p,i,m}^{\text{en}} - Q_m^{(k)}}$$

4) Determine the load factor increment $\Delta\lambda^{(k)}$ to next event

$$\Delta\lambda^{(k)} = \frac{1}{\max\left(DC_m'\right)}$$

5) Update the load factor, the free dof displacements and the basic forces to next event

$$\lambda^{(k+1)} = \lambda^{(k)} + \Delta\lambda^{(k)}$$
$$U_f^{(k+1)} = U_f^{(k)} + \Delta\lambda^{(k)} U_f^{(k)}$$
$$Q^{(k+1)} = Q^{(k)} + \Delta\lambda^{(k)} Q^{(k)}$$

6) Determine the plastic deformations at next event

$$\boldsymbol{V}_{hp}^{(k+1)} = \mathbf{A}_f \boldsymbol{U}_f^{(k+1)} - \mathbf{F}_s^{(0)} \boldsymbol{Q}^{(k+1)}$$

where $\mathbf{F}_a^{(0)}$ is the collection of initial element flexibility matrices.

6 Nonlinear Geometry

$$\delta v = \frac{\partial v}{\partial u} \delta u = \mathrm{a}_g(u) \delta u$$

$$\mathbf{k}_m = \mathbf{a}_g^T(\boldsymbol{u}) \frac{\partial \boldsymbol{q}}{\partial v} \mathbf{a}_g(\boldsymbol{u})$$

$$\begin{aligned}\mathbf{k}_e = \frac{\partial \boldsymbol{p}}{\partial \boldsymbol{u}} = \frac{\partial\left[\mathbf{a}_g^T(\boldsymbol{u}) q\right]}{\partial \boldsymbol{u}} &= \mathbf{a}_g^T(\boldsymbol{u}) \frac{\partial q}{\partial \boldsymbol{u}} + \frac{\partial\left[\mathbf{a}_g^T(\boldsymbol{u})\right]}{\partial \boldsymbol{u}} q \\ &= \mathbf{a}_g^T(\boldsymbol{u}) \frac{\partial q}{\partial v} \frac{\partial v}{\partial \boldsymbol{u}} + \frac{\partial\left[\mathbf{a}_g^T(\boldsymbol{u})\right]}{\partial \boldsymbol{u}} q \\ &= \mathbf{a}_g^T(\boldsymbol{u}) \frac{\partial q}{\partial v} \mathbf{a}_g(\boldsymbol{u}) + \frac{\partial\left[\mathbf{a}_g^T(\boldsymbol{u})\right]}{\partial \boldsymbol{u}} q \\ &= \mathbf{k}_m \quad + \mathbf{k}_g\end{aligned}$$

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$$e = \mathbf{B}_0 \mathbf{u} + \frac{1}{2} \mathbf{u}^T \mathbf{H} \mathbf{u}$$

$$\mathbf{K} = A_0 L_0 \mathbf{B}^T \frac{\partial s}{\partial \mathbf{u}} + A_0 L_0 s \frac{\partial \mathbf{B}^T}{\partial \mathbf{u}} = \mathbf{K}_M + \mathbf{K}_G$$

$$\mathbf{K} = \frac{\partial \mathbf{p}}{\partial \mathbf{u}} = \frac{\partial}{\partial \mathbf{u}}\left(V_0 s \frac{\partial e}{\partial \mathbf{u}}\right) = V_0 E \frac{\partial e}{\partial \mathbf{u}} \otimes \frac{\partial e}{\partial \mathbf{u}} + V_0 s \frac{\partial^2 e}{\partial \mathbf{u} \partial \mathbf{u}} = \mathbf{K}_M + \mathbf{K}_G$$

6.2 Truss Kinematics

Green-Lagrange Strain

$$v_{GL} = \frac{\Delta X}{L} \Delta \boldsymbol{u}_x + \frac{\Delta Y}{L} \Delta \boldsymbol{u}_y + \frac{\left(\Delta \boldsymbol{u}_x\right)^2}{2L} + \frac{\left(\Delta \boldsymbol{u}_y\right)^2}{2L}$$

7 Solution Strategies

1. Load Incrementation: $P_f^{(k)} = P_f^{(k-1)} + \Delta\lambda P_{\text{ref}}$
2. Use the solution at $k-1$ as initial guess $U_0^{(k)} = U^{(k-1)}$
3. Structure State Determination: $P_r^{(k)} = P_r\left(U_0^{(k)}\right)$ and $K_t^{(k)} = K_t\left(U_0^{(k)}\right)$
4. Determine $P_u^{(k)} = P_f^{(k)} - P_r^{(k)}$
5. Determine $\Delta U_0^{(k)} = K_t^{(k)} \backslash P_u^{(k)}$
6. Update the solution $U_1^{(k)} = U_0^{(k)} + \Delta U_0^{(k)}$
7. Equilibrium Iterations For $i = 1 \ldots n$ and constant k without superscript (k) except for P_f.
   - State Determination: $P_r\left(U_i\right)$ and $K_t\left(U_i\right)$
   - Nodal force unbalance $P_u\left(U_i\right) = P_f^{(k)} - P_r\left(U_i\right)$
   - Solution correction $\Delta U_i = K_t\left(U_i\right) \backslash P_u\left(U_i\right)$
   - Update estimate $U_{i+1} = U_i + \Delta U_i$

   Repeat steps (a)-(d) until the error norm satisfies the specified tolerance.
8. On convergence of the equilibrium iterations determine the resisting forces for the final $U^{(k)}$

http://claudioperez.github.io/