Dept. of Civil and Environmental Engineering Instructor: Filip C. Filippou

CE 220 - Structural Analysis Homework Set 4 (due 10/2/2019)

1. Problem (4 points)

The structural model in Fig. 1 is to be analyzed. You are asked to proceed as follows:

- 1. Number the global degrees of freedom (dofs) without including trivial equations and without including those involving axial basic forces in elements a through d. Only flexural basic forces should appear in these equations (Hint: there should be only two equations and two flexural basic forces).
- 2. Set up the equilibrium equations in terms of the basic element forces Q and the equivalent nodal forces P_{wf} due to the uniformly distributed element loading w of 10 units of force/unit of length.
- 3. Solve the equilibrium equations for the flexural basic forces Q.
- 4. Draw the bending moment diagram.
- 5. Determine the dependent element forces and use node equilibrium to determine the axial basic forces in elements a through d.
- 6. With the axial and flexural basic forces determine the support reactions and check global equilibrium.

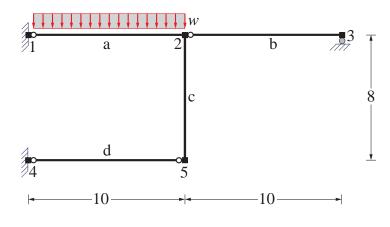


Figure 1

2. Problem (2 points)

For the structural models in Fig. 2 determine the degree of static indeterminacy NOS. To show that NOS is a property of the structural model answer the following questions:

- 1. Number all available equations of equilibrium that do not involve support reactions and the corresponding basic forces (including trivial equations). The difference between number of basic forces and available free dof equilibrium equations is the degree of static indeterminacy NOS.
- 2. Remove the trivial equations and the corresponding basic forces. Is *NOS* the same as under question (1)?
- 3. Number all available equilibrium equations that do not involve the axial basic forces in frame elements a through d in the two models. Number the corresponding basic forces of primary interest. Is NOS still the same?

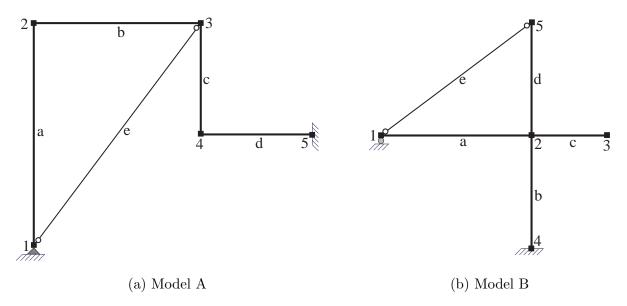


Figure 2: Two structural models

3. Problem (4 points)

The degree of static indeterminacy NOS of the structural model in Fig. 3 is 2.

- 1. Number the equilibrium equations without including trivial equations. How many basic forces and equations are there?
- 2. Set up the equilibrium equations in terms of the basic forces of primary interest Q that do not involve the axial basic forces in frame elements a, b, and c.
- 3. Select the end moment at the right end of element b and the end moment at the lower end of element c as redundant basic forces.
- 4. Determine the particular solution under the given loading.
- 5. Determine the homogeneous solution for a unit value of each redundant basic force.
- 6. Explain how you can arrive at the same results by studying the force flow in the model and draw the resulting bending moment diagram for the particular solution and each homogeneous solution separately.

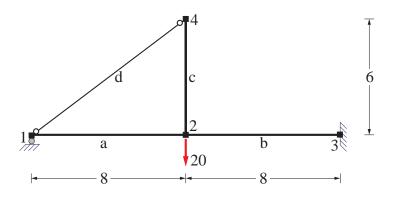


Figure 3

4. Problem (3 points)

The structural model in Fig. 4 is subjected to a concentrated force of 20 units at node 3.

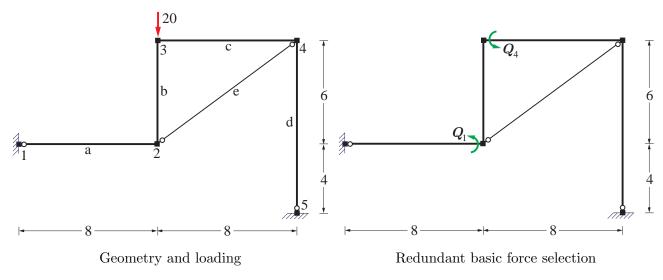


Figure 4

Answer the following questions under the assumption that the axial forces in elements a through d are of secondary importance and should not be included in the reduced set of free dof equilibrium equations:

- 1. Confirm that the degree of static indeterminacy NOS of the model is 2 by confirming that the number of basic forces of primary interest is 7 and the number of the corresponding free dof equilibrium equations is 5.
- 2. Set up the equilibrium equations at the free dofs of the structural model.
- 3. Select Q_1 and Q_4 as redundant basic forces and use the FEDEASLab function BbariBbarx_matrix to get the particular and homogeneous solutions of the equilibrium equations following the example in the Box 2.7 of the Section 2.5.4 in the FEDEASLab Guide. Use your equilibrium equations from step (2) to confirm the FEDEASLab results.
- 4. Determine the support reactions for each homogeneous solution separately. Comment on why the support reactions are zero for one homogeneous solution but not the other.

5. Problem (2 points)

The third problem of the last homework studied the response of the Howe truss with rigid connections in Fig. 5.

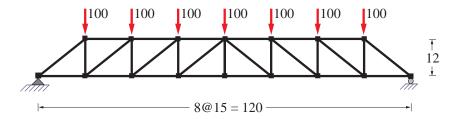


Figure 5: Howe truss with rigid connections

The present problem addresses the response of the structure that results from the removal of the diagonals from a truss with rigid connections in Fig. 6. This type of structure is known as a *Vierendeel truss*, even though its behavior is dependent on the rigid connections between the posts and the upper and lower chord elements.

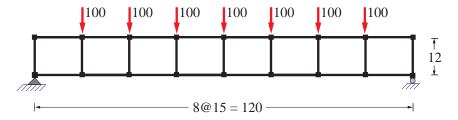


Figure 6: Vierendeel truss

The Matlab® script file Hw4P5.m uses the displacement method for the analysis of the Vierendeel truss in Fig. 6. All connections are rigid by selecting the element type Lin2dFrm, i.e. the linear 2d frame element in the FEDEASLab library. The element properties of the Vierendeel truss elements are: $EA = 5 \cdot 10^5$ and $EI = 4 \cdot 10^5$.

Compare the response of the Howe truss with rigid connections from the last homework with the response of the Vierendeel truss having the same element properties (recall that one of the questions of the last homework required doubling the original flexural stiffness of $EI = 2 \cdot 10^5$). Focus in particular on the deformed shape of the two structures and the resulting midspan deflection and comment.