1 Indicial Notation

- Free Indices

  1. A free index appears once and only once within each additive term in an expression.

  2. A free index implies three distinct equations, such that the index assumes the values of 1,2, and 3.

  3. The same letter must be used for the free index in every additive term. You can rename this index (i.e. going from i to j) if you rename it in every single term.

  4. Terms in an expression may have more than one free index, which will indicate the dimension/rank of the term.

  5. For example, if we consider a second order tensor $\mathbf{A}$, then there will be two free indexes. The first index will correspond to the row, and the second index corresponds to the column.

  $$\mathbf{A} \rightarrow A_{ij} \rightarrow \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

  where the full expression for the tensor given appropriate basis vectors is

  $$\mathbf{A} = A_{ij}\mathbf{e}_i \otimes \mathbf{e}_j$$

  6. Similarly, for a vector $\mathbf{v}$ there will be just one free index corresponding to the row of the entry.

  $$\mathbf{v} \rightarrow v_i \rightarrow \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

- Dummy Indices

  1. A dummy index appears twice within an additive term of an expression. A dummy index can only appear twice, so an individual term with 3 dummy indices is not allowed.

  2. A dummy index implies a summation over the range from 1 to 3. For example,

  $$a_{ii} = a_{11} + a_{22} + a_{33}$$

  3. A dummy index may be renamed to any letter not currently being used as a free index (or already in use as another dummy index pair in that term). The dummy index is local to an individual additive term, such that one can rename the dummy index in one term and it would not need to be renamed in other terms.