V Variational Formulation

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10.1

Traction Equilibrium Condition: Weak Form

Virtual power balance,

$$\int_{\mathcal{B}} (\operatorname{sym} \nabla \mathbf{w}) : \mathbb{C}(\operatorname{sym} \nabla \mathbf{u}) dv - \int_{\mathcal{S}_2} \hat{\mathbf{t}} \cdot \mathbf{w} da - \int_{\mathcal{B}} \mathbf{b} \cdot \mathbf{w} dv = 0 \quad \forall \text{ admissible w.}$$

is satisfied if and only if $\operatorname{div} \sigma + \mathrm{b} = 0$ in $\mathcal{B}$ and $\sigma \mathbf{n} = \hat{\mathbf{t}}$ on $\mathcal{S}_2$

Where $\mathbf{w}$ is a virtual velocity field such that $\mathbf{w} = \mathbf{0}$ on $\mathcal{S}_1$.

10.2

Elastostatic Displacement Problem: Weak Form

Given $\mathbb{C}$, $\mathbf{b}$, and boundary data $\hat{\mathbf{u}}$ and $\hat{\mathbf{t}}$, find a displacement field $\mathbf{u}$ equal to $\hat{\mathbf{u}}$ on $\mathcal{S}_1$ such that:

$$\int_{\mathcal{B}} (\operatorname{sym} \nabla \mathbf{w}) : \mathbb{C}(\operatorname{sym} \nabla \mathbf{u}) dv - \int_{\mathcal{S}_2} \hat{\mathbf{t}} \cdot \mathbf{w} da - \int_{\mathcal{B}} \mathbf{b} \cdot \mathbf{w} dv = 0 \quad \forall \text{ admissible w.}$$