CE 220 - Structural Analysis

Homework Set 7 (due 10/23/2019)

1. Problem (4 points)

Fig. 1 shows a braced frame.

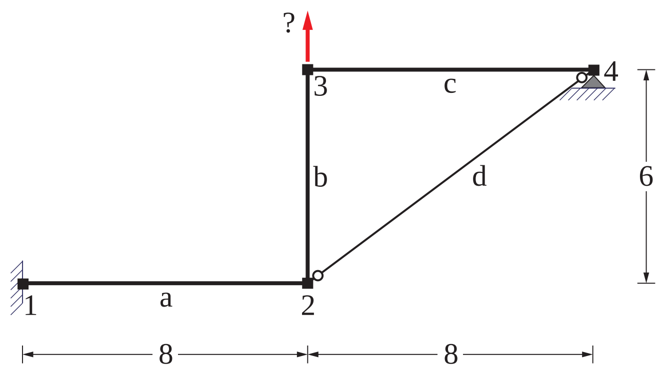

Figure 1: Braced frame under unknown vertical nodal force at node #3

The computer analysis of the frame under the assumption that the elements a through c are inextensible gives the following basic element forces $\boldsymbol{Q}$ of primary interest:

$$\boldsymbol{q}^{(a)} = \begin{pmatrix} 87.947 \\ 54.824 \end{pmatrix} \qquad \boldsymbol{q}^{(b)} = \begin{pmatrix} -54.824 \\ 22.844 \end{pmatrix} \qquad \boldsymbol{q}^{(c)} = -22.844 \qquad \boldsymbol{q}^{(d)} = 15.497$$

You are asked to answer the following questions noting that the value for the applied vertical nodal force at node #3 was not recorded by the analyst:

1. Draw the bending moment diagram.
2. Use the principle of virtual displacements to determine the horizontal support reaction at node #1 making use only of the given basic force values and making sure not to involve the unknown nodal force at node #3 in the external virtual work.
3. Use the principle of virtual displacements to determine the vertical support reaction at node #4 making use only of the given basic force values and making sure not to involve the unknown nodal force at node #3 in the external virtual work.
4. Determine the applied vertical nodal force value at node #3 with the principle of virtual displacements making sure not to involve the support reactions in the external virtual work.

2. Problem (8 points)

Fig. 2 shows a braced frame. The computer analysis of the frame under the assumption that the elements a through d are inextensible gives the following element deformations $\boldsymbol{V}$:

$$\boldsymbol{v}^{(a)} = \begin{pmatrix} -0.6937 \\ 1.3874 \end{pmatrix} 10^{-3} \qquad \boldsymbol{v}^{(b)} = \begin{pmatrix} 0.0765 \\ -2.9278 \end{pmatrix} 10^{-3} \qquad \boldsymbol{v}^{(c)} = \begin{pmatrix} -0.1310 \\ 3.1515 \end{pmatrix} 10^{-3}$$

$$\boldsymbol{v}^{(d)} = \begin{pmatrix} -3.5215 \\ -2.2149 \end{pmatrix} 10^{-3} \qquad \boldsymbol{v}^{(e)} = 6.2922 \cdot 10^{-3}$$

The corresponding basic element forces $\boldsymbol{Q}$ of primary interest are:

$$\boldsymbol{q}^{(a)} = 27.748 \quad \boldsymbol{q}^{(b)} = \begin{pmatrix} -27.748 \\ -57.791 \end{pmatrix} \quad \boldsymbol{q}^{(c)} = \begin{pmatrix} 57.791 \\ 123.440 \end{pmatrix} \quad \boldsymbol{q}^{(d)} = \begin{pmatrix} -123.440 \\ -106.019 \end{pmatrix} \quad \boldsymbol{q}^{(e)} = 12.584$$

You are asked to answer the following questions:

1. Draw the bending moment diagram.
2. Use the free dof equilibrium equations to determine the applied nodal forces that the analyst forgot to record.
3. Determine the vertical support reaction at node #1 by selecting a suitable virtual displacement field with the principle of virtual work.
4. Determine the horizontal and vertical translation at nodes #2, #3 and #4 from the kinematic relations $\boldsymbol{V} = \mathbf{A}_f \boldsymbol{U}_f$ *without determining the node rotations*.
5. Confirm the vertical and horizontal translation at node #3 by the principle of virtual forces.
6. Use the translations to draw the element chords, and then use the element deformations $\boldsymbol{V}$ to carefully draw the deformed shape of the structural model *without determining the node rotations*.

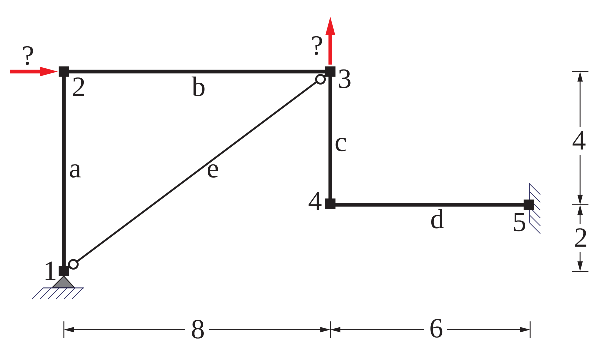

Figure 2: Braced frame under unknown nodal forces

3. Problem (3 points)

The computer analysis of the structural model in Fig. 3 gives the following deformation values under the assumption that the elements a through e are inextensible:

$$\boldsymbol{v}_\varepsilon^{(a)} = \begin{pmatrix} 1.1861 \\ -0.1302 \end{pmatrix} 10^{-3} \qquad \boldsymbol{v}_\varepsilon^{(b)} = \begin{pmatrix} -2.5024 \\ 1.2512 \end{pmatrix} 10^{-3} \qquad \boldsymbol{v}_\varepsilon^{(c)} = \begin{pmatrix} 0.5445 \\ 2.6814 \end{pmatrix} 10^{-3}$$

$$\boldsymbol{v}_\varepsilon^{(d)} = \begin{pmatrix} -0.3655 \\ 0.1828 \end{pmatrix} 10^{-3} \qquad \boldsymbol{v}_\varepsilon^{(e)} = \begin{pmatrix} -2.1568 \\ 1.0784 \end{pmatrix} 10^{-3} \qquad \boldsymbol{v}_\varepsilon^{(f)} = -14.6252 \cdot 10^{-3}$$

1. What is the degree of static indeterminacy of the model?
2. Use the principle of virtual forces to determine the horizontal translation at node 4.
3. Give a geometric interpretation of the results under (2).
4. Repeat (2) for the vertical translation at node 4.
5. Use the information about the translation of node 4 to draw the element chords in the deformed configuration and proceed to draw the deformed shape of the structural model for the given element deformations.

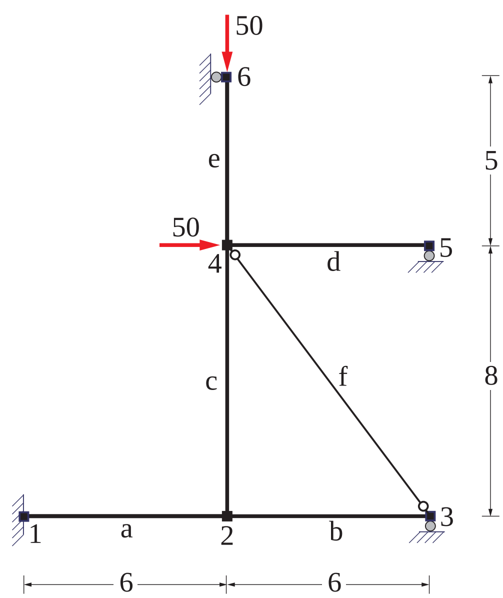

Figure 3