1.7.1 Typical linear response increment from event k to next event

Here is the summary of a linear step in the event-to-event analysis process for linear elastic, perfectly-plastic element basic force-deformation response:

1) Set up the current stiffness of the structural model with basic force releases at all p_k locations where plastic hinges appeared in the events through k

$$\mathbf{K}^{(k)} = \mathbf{A}_f^T \mathbf{K}_s^{(k)} \mathbf{A}_f$$

2) Solve for the free dof displacements $\boldsymbol{U}'_f$ and the basic forces $\boldsymbol{Q}'$ under reference load

$$\begin{aligned} \boldsymbol{U}_f'^{(k)} &= \mathbf{K}^{(k)} \backslash \boldsymbol{P}_{ref} \\ \boldsymbol{Q}'^{(k)} &= \mathbf{K}_s^{(k)} \left[\mathbf{A}_f \boldsymbol{U}_f'^{(k)} \right] \end{aligned}$$

3) Determine the DC ratio under the reference load at locations m without plastic hinge

$$DC'_m = \frac{\boldsymbol{Q}_m'^{(k)}}{\operatorname{sgn} \boldsymbol{Q}_{pl,m}^{\text{sgn}} - \boldsymbol{Q}_m^{(k)}}$$

4) Determine the *load factor increment* $\mathit{\Delta}\lambda^{(k)}$ to next event

$$\mathit{\Delta}\lambda^{(k)} = \frac{1}{\max(DC'_m)}$$

5) Update the load factor, the free dof displacements and the basic forces to next event

$$\begin{aligned} \lambda^{(k+1)} &= \lambda^{(k)} + \mathit{\Delta}\lambda^{(k)} \\ \boldsymbol{U}_f^{(k+1)} &= \boldsymbol{U}_f^{(k)} + \mathit{\Delta}\lambda^{(k)} \boldsymbol{U}_f'^{(k)} \\ \boldsymbol{Q}^{(k+1)} &= \boldsymbol{Q}^{(k)} + \mathit{\Delta}\lambda^{(k)} \boldsymbol{Q}'^{(k)} \end{aligned}$$

6) Determine the plastic deformations at next event

$$\boldsymbol{V}_{hp}^{(k+1)} = \mathbf{A}_f \boldsymbol{U}_f^{(k+1)} - \mathbf{F}_s^{(0)} \boldsymbol{Q}^{(k+1)}$$

where $\mathbf{F}_s^{(0)}$ is the collection of *initial element flexibility matrices*.

4.3 Deformation State at Incipient Collapse

If the number of plastic hinges is such that a unique static solution for the basic forces $\boldsymbol{Q}_c$ results, then the structure is statically determinate at incipient collapse. In this case, the strain-dependent element deformations $\boldsymbol{V}_\varepsilon$ can be established in all elements. Just before the last hinge forms, the structure is stable and the number of unknown free dof displacements matches the number of available kinematic relations. It is, therefore, possible to determine the free dof displacements and plastic hinge deformations at incipient collapse from the basic forces $\boldsymbol{Q}_c$ and the corresponding element deformations $\boldsymbol{V}_\varepsilon$ just as for any statically determinate structure.

4.3.2 Solution Process

assumptions:

1) The applied loading consists only of the reference load $\boldsymbol{P}_{ref}$, which is increased monotonically by incrementing the load factor λ until reaching the collapse load factor λ_c.
2) Plastic hinges that are "open" under a load factor λ cannot "close" under a higher load factor: this means that plastic hinge deformations increase monotonically under the monotonically increasing reference load.

$$\boldsymbol{V}_\varepsilon = \mathbf{F}_s \boldsymbol{Q}_c + \boldsymbol{V}_0$$

The process for the determination of the last hinge to form consists of the following steps:

1) Select any hinge of the collapse mechanism as last to form and solve the kinematic relations for the corresponding free dof displacements $\boldsymbol{U}_f^{tr}$, where the superscript *tr* stands for *trial result.*
2) Determine the plastic hinge deformations $\boldsymbol{V}_{hp}^{tr}$ corresponding to $\boldsymbol{U}_f^{tr}$ with

$$\boldsymbol{V} = \boldsymbol{V}_\varepsilon + \boldsymbol{V}_{hp}^{tr} = \mathbf{A}_f \boldsymbol{U}_f^{tr}$$

3) If the sign of each plastic deformation matches the sign of the corresponding basic force $\boldsymbol{Q}_c$ from the equilibrium equations, the last hinge location is correct. The trial displacements and plastic deformations from steps (1) and (2) give the free dof displacements $\boldsymbol{U}_f$ and the plastic deformations $\boldsymbol{V}_{hp}$ at incipient collapse.
4) If the sign of one or more plastic deformations does not match the sign of the corresponding basic force, *correct the free dof displacements and plastic deformations* of Step (1) and (2) in a single step as described in the following.

If the sign of one or more plastic deformations does not match the sign of the corresponding basic force, the assumption about the last plastic hinge location in Step (1) is not correct. Consequently, the free dof displacements and plastic deformations of Steps (1) and (2) need to be corrected. The correction involves the addition of the deformation state of the collapse mechanism of the structure. The independent dof of the collapse mechanism is scaled so that *the total plastic deformations match everywhere the sign of the corresponding basic force, except for one location where the plastic deformation is zero, thus, identifying the location of the last plastic hinge to form.*

The determination of the scale factor for the collapse mechanism is described next. Consider the initial assumption about the deformation state of the structure at incipient collapse. The kinematic relations give

$$\boldsymbol{V}_{\varepsilon} + \boldsymbol{V}_{hp}^{tr} = \mathbf{A}_f \boldsymbol{U}_f^{tr} \tag{4.10}$$

From the comparison of the sign of the plastic deformations $\boldsymbol{V}_{hp}^{tr}$ with the sign of the corresponding basic forces $\boldsymbol{Q}_c$ we identify n locations of mismatch. We need to correct these plastic deformations by adding a deformation state *without changing the element deformations* $\boldsymbol{V}_{\varepsilon}$. Because $\Delta\boldsymbol{V}_{\varepsilon}$ is zero during the collapse mechanism displacement, we conclude that we need to superimpose to the deformation state in (4.10) the deformation state of the collapse mechanism with a single independent dof $\dot{U}_p$. For this deformation state it holds that

$$\Delta\boldsymbol{V}_{hp} = \mathbf{A}_{mp}\,\Delta U_p \tag{4.11}$$

where we have converted instantaneous rates of change to increments denoted with a Δ prefix for the corresponding variable. Consequently, ΔU_p is the displacement increment of the single independent dof of the collapse mechanism and $\Delta\boldsymbol{V}_{hp}$ are the increments of the corresponding plastic deformations.