

L^AT_EX 2_ε Cheat Sheet

Document classes

book	Default is two-sided.
report	No \part divisions.
article	No \part or \chapter divisions.
letter	Letter (?).
slides	Large sans-serif font.

Used at the very beginning of a document:
`\documentclass{class}`. Use `\begin{document}` to start
contents and `\end{document}` to end the document.

II Kinematics

Displacement:

$$\mathbf{u}(\mathbf{X}, t) = \chi(\mathbf{X}, t) - \mathbf{X}, \quad u_i(X_1, X_2, X_3) = \chi_i(X_1, X_2, X_3) - X_i$$

Velocity/Acceleration

$$\dot{\mathbf{u}}(\mathbf{X}, t) = \frac{\partial \chi(\mathbf{X}, t)}{\partial t} \quad (3.8.3)$$

$$\ddot{\mathbf{u}}(\mathbf{X}, t) = \frac{\partial^2 \chi(\mathbf{X}, t)}{\partial t^2} \quad (3.8.4)$$

3.2 Deformation/Displacement Gradient

$$\mathbf{F}(\mathbf{X}, t) = \frac{\partial}{\partial \mathbf{X}} \chi(\mathbf{X}, t), \quad F_{ij} = \frac{\partial}{\partial X_j} \chi_i(X_1, X_2, X_3, t), \quad \det \mathbf{F}(\mathbf{X}, t) > 0$$

$$\mathbf{H}(\mathbf{X}, t) = \frac{\partial}{\partial \mathbf{X}} \mathbf{u}(\mathbf{X}, t), \quad H_{ij} = \frac{\partial}{\partial X_j} u_i(X_1, X_2, X_3, t)$$

$$\mathbf{H}(\mathbf{X}, t) = \mathbf{F}(\mathbf{X}, t) - \mathbf{1}, \quad H_{ij} = F_{ij} - \delta_{ij}$$

$$J \equiv \det \left(\frac{\partial \chi}{\partial \mathbf{X}} \right) = \det \mathbf{F} = \frac{dv}{dv_R} \neq 0$$

3.3 Stretch & Rotation

Polar Decomposition: $\mathbf{F} = \mathbf{R}\mathbf{U} = \mathbf{V}\mathbf{R}$

$$\mathbf{C} = \mathbf{U}^2 = \mathbf{F}^T \mathbf{F}, \quad C_{ij} = F_{ki} F_{kj} = \frac{\partial \chi_k}{\partial X_i} \frac{\partial \chi_k}{\partial X_j}$$
$$\mathbf{B} = \mathbf{V}^2 = \mathbf{F} \mathbf{F}^T, \quad B_{ij} = F_{ik} F_{jk} = \frac{\partial \chi_i}{\partial X_k} \frac{\partial \chi_j}{\partial X_k}$$

$$\lambda \stackrel{\text{def}}{=} \frac{ds}{dS} = |\mathbf{U}\mathbf{e}| = \sqrt{\mathbf{e} \cdot \mathbf{C}(\mathbf{X})\mathbf{e}}$$

where $dS = |d\mathbf{X}|$, $ds = |d\mathbf{x}|$, $\mathbf{e} = \frac{d\mathbf{X}}{|d\mathbf{X}|}$

$$\textbf{Engineering shear: } \gamma = \sin^{-1} \left[\frac{\mathbf{e}^{(1)} \cdot \mathbf{C} \mathbf{e}^{(2)}}{\lambda(\mathbf{e}^{(1)}) \lambda(\mathbf{e}^{(2)})} \right]$$

3.4 Strain

$$\textbf{Green strain: } \mathbf{E} \stackrel{\text{def}}{=} \frac{1}{2} (\mathbf{F}^T \mathbf{F} - \mathbf{1}) = \frac{1}{2} (\mathbf{H} + \mathbf{H}^T + \mathbf{H}^T \mathbf{H}).$$

$$\textbf{Hencky's Log strain: } \ln \mathbf{U} \stackrel{\text{def}}{=} \sum_{i=1}^3 (\ln \lambda_i) \mathbf{r}_i \otimes \mathbf{r}_i$$

3.5.2 Infinitesimal Strain

ϵ' : distortion $\epsilon_M \delta_{ij}$: dilation

$$\epsilon = \frac{1}{2} [\mathbf{H} + \mathbf{H}^T], \quad \epsilon = \epsilon^\top, \quad |\mathbf{H}| \ll 1$$

$$\epsilon_{ij} = \frac{1}{2} \left[\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i} \right], \quad \epsilon_{ji} = \epsilon_{ij}, \quad \left| \frac{\partial u_i}{\partial X_j} \right| \ll 1$$

3.A Linearization

$$\lim_{\alpha \rightarrow 0} Y_o f(\mathbf{Y}) = f(\mathbf{Y}_o) + \frac{d}{d\alpha} f(\mathbf{Y}_o + \alpha(\mathbf{Y} - \mathbf{Y}_o)) \Big|_{\alpha=0}$$

$$\lim_{\alpha \rightarrow 0} f(\mathbf{H}) = f(0) + \frac{d}{d\alpha} f(\alpha \mathbf{H}) \Big|_{\alpha=0}$$

3.B Compatibility

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<http://wch.github.io/latexsheet/>