X Viscoelasticity

$\sigma(t) = \eta\dot{\epsilon}(t)$

29 Linear Viscoelasticity

29.2 Stress relaxation & Creep

Stress (Relaxation test), given $\epsilon(t) = \epsilon_0 h(t), \quad \dot{\epsilon}(t) = \epsilon_0 \delta(t)$

$\sigma(t) = \sum_{i=1}^{N} h\left(t - t_i\right) \Delta\sigma_i$

$\sigma(t) = \int_{0^-}^{t} E_r(t-\tau)\frac{d\epsilon(\tau)}{d\tau}d\tau = \left(E_r * \dot{\epsilon}\right)(t)$

Strain (Creep test), given $\sigma(t) = \sigma_0 h(t)$

$\epsilon(t) = \sum_{i=1}^{N} J_c\left(t - t_i\right) \Delta\sigma_i \quad (p497)$

$\epsilon(t) = \int_{0-}^{t} J_c(t-\tau)\frac{d\sigma(\tau)}{d\tau}d\tau \quad (29.2.6)$

$J_c(t) \stackrel{\text{def}}{=} \frac{\epsilon(t)}{\sigma_0}$

29.5 Correspondence Principle (1D)

$$\bar{\sigma}(s) = \bar{E}_r^*(s)\bar{\epsilon}(s)$$

where

$$\bar{E}_r^*(s) = s\bar{E}_r(s)$$

$$\bar{J}_c(s)\bar{E}_r(s) = \frac{1}{s^2}$$

29.7 Oscillatory Response

$J^* E^* = 1$

Applied stress, $\sigma(t) = \sigma_0 \cos(\omega t)$

$$\begin{aligned}\epsilon(t) &= \epsilon_0 \cos(\omega t - \delta) \\ &= \epsilon_0 \cos(\delta)\cos(\omega t) - \epsilon_0 \sin(\delta)\sin(\omega t) \\ &= \sigma_0 \left(J' \cos(\omega t) + J'' \sin(\omega t)\right)\end{aligned}$$

Storage compliance, $J' \stackrel{\text{def}}{=} \frac{\epsilon_0}{\sigma_0}\cos(\delta)$.

Loss compliance, $J'' \stackrel{\text{def}}{=} \frac{\epsilon_0}{\sigma_0}\sin(\delta)$.

Applied strain, $\epsilon(t) = \epsilon_0 \cos(\omega t)$

$$\begin{aligned}\sigma(t) &= \sigma_0 \cos(\omega t + \delta) \\ &= \sigma_0 \cos(\delta)\cos(\omega t) - \sigma_0 \sin(\delta)\sin(\omega t) \\ &= \epsilon_0 \left(E' \cos(\omega t) - E'' \sin(\omega t)\right)\end{aligned}$$

Storage modulus, $E' \stackrel{\text{def}}{=} \frac{\sigma_0}{\epsilon_0}\cos(\delta)$.

Loss modulus, $E'' \stackrel{\text{def}}{=} \frac{\sigma_0}{\epsilon_0}\sin(\delta)$.

29.8 Complex formulation of oscillatory response

Applied stress, $\sigma(t) = \sigma_0 e^{i\omega t}$

$$\begin{aligned}\epsilon(t) &= \epsilon_0 e^{i(\omega t - \delta)} \\ &= \sigma_0 \left[J' - iJ''\right] e^{i\omega t} \\ &= J^* \sigma(t)\end{aligned}$$

Complex compliance, $J^*(\omega) \stackrel{\text{def}}{=} \frac{\epsilon(t)}{\sigma_0 e^{i\omega t}} = J' - iJ''$

Applied strain, $\epsilon(t) = \epsilon_0 e^{i\omega t}$

$$\begin{aligned}\sigma(t) &= \sigma_0 e^{i(\omega t+\delta)} \\ &= \epsilon_0 \left[E' + iE''\right] e^{i\omega t} \\ &= E^* \epsilon(t)\end{aligned}$$

Complex modulus, $E^*(\omega) \stackrel{\text{def}}{=} \frac{\sigma(t)}{\epsilon_0 e^{i\omega t}} = E' + iE''$.

29.8.1 Energy dissipation under oscillatory conditions

29.9 More on complex variable representation

	Kelvin-Voight	Maxwell	Std	Gen
$E'(\omega)$	E	$\frac{\tau^2\omega^2}{\tau^2\omega^2+1}E$	$\frac{E_{re}+E_{rg}\left(\tau_R^{(1)}\omega\right)^2}{1+\left(\tau_R^{(1)}\omega\right)^2}$	$E^{(0)} + \sum_\alpha \frac{E^{(\alpha)}\left(\omega\tau_R^{(\alpha)}\right)^2}{1+\left(\omega\tau_R^{(\alpha)}\right)^2}$
$E''(\omega)$	$\eta\omega = E\tau_R\omega$	$\frac{\tau\omega}{\tau^2\omega^2+1}E$	$\frac{(E_{rg}-E_{re})\left(\tau_R^{(1)}\omega\right)}{1+\left(\tau_R^{(1)}\omega\right)^2}$	$\sum_\alpha \frac{E^{(\alpha)}\left(\omega\tau_R^{(\alpha)}\right)}{1+\left(\omega\tau_R^{(\alpha)}\right)^2}$
$\tan\delta(\omega)$			$\frac{(E_{rg}-E_{re})\left(\tau_R^{(1)}\omega\right)}{E_{re}+E_{rg}\left(\tau_R^{(1)}\omega\right)^2}$	$\frac{E''(\omega)}{E'(\omega)}$

29.10 Time-integration

29.11 3D Constitutive equation

29.11.1 BVP for isotropic linear viscoelasticity

29.11.2 Correspondence principle in three dimensions