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MESTRADO EM ENGENHARIA INFORMÁTICA

# Exploiting IoT-enabled Inertial Sensors for Position Estimation

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3 de outubro de 2021

# Abstract

With the surge of inexpensive, widely accessible, and precise microelectromechanical systems (MEMS) in recent years, inertial systems tracking movement have become ubiquitous nowadays. Contrary to GPS-based positioning, Inertial Navigation Systems (INS) are intrinsically unaffected by signal jamming, blockage susceptibilities, and spoofing. Measurements from inertial sensors are also acquired at elevated sampling rates and may be numerically integrated to estimate position and orientation knowledge. These measurements are precise on a small-time scale but gradually accumulate errors over extended periods. Combining multiple inertial sensors in a method known as sensor fusion makes it possible to produce a more consistent and dependable understanding of the system, decreasing accumulative errors. Several sensor fusion algorithms occur in literature aimed at estimating the Attitude and Heading Reference System (AHRS) of a rigid body with respect to a reference frame. This work describes the development and implementation of a low-cost, multipurpose INS for position and orientation estimation. It presents an experimental comparison of the most popular sensor fusion solutions. Additionally, the study explores the prospect of integrating transmission Internet of Things (IoT) devices (based on an open radio frequency at 868 MHz) to broadcast in real-time and at a long distance the navigational data of a moving object and how this approach may be employed in a series of real-world scenarios.

**Keywords:** Sensor Fusion · Inertial Navigation System (INS) · Microelectromechanical system (MEMS) · Internet of Things (IoT) · Inertial Measurement Unit (IMU) · Aerial Assessments

# Resumo

**Keywords:** Ubiquitous Computing · Aerial Assessments · Wildlife Monitoring  
· Machine Learning · Application Development · Low-Altitude Balloons.

## Acknowledgements

Cláudio Rodrigues

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## 1 Introduction

Navigation systems have become a popular subject in the designing of unmanned and autonomous systems in recent years. Still, most navigation systems highly depend on the Global Navigation Satellite System (GNSS) receivers as the central resource of navigation data. Although the satellite system can deliver accurate and long-term positioning in open spaces, when relying on GNSS only localization, there are often circumstances when the satellite signal is obstructed or weakened, resulting in degradation or even loss of position estimate precision [14]. Such is especially threatening in high urbanized centers where satellite signals can suffer multipath propagation from tall glass-covered buildings [32]. Satellite positioning is especially impactful in battery autonomy, whereas GNSS receivers remain draining a substantial amount of electric current. Inevitably, there is a need to become less dependent on GNSS-based localization, particularly in autonomous settings. Substantial research has been conducted to enhance the localization precision of an object devoid of satellite signals [29] [6] [17] [5] [43]. Increased accuracy is a precursor to establishing autonomous agents for a diversity of functions. Inertial measurement units (IMU) have become standard in embedded inertial systems due to their low cost, lightweight, and low power consumption. They can provide short-term position and orientation changes. Furthermore, inertial systems have been employed in wearable applications with uses in unmanned aerial vehicles (UAV) [12] [24] [36], telemedicine [26], and robotics [42]. Despite these accomplishments, inertial systems suffer from rapid drift owing to the existence of disturbances and noise in the measurements. Practically all present commercial applications are restricted to minimal motion recognition. Position and orientation in real-world applications are rarely employed due to difficulties in precise integration. Recently, innovative fusion algorithms have emerged, which can diminish the impacts of noise and disturbances, broadening these devices' capabilities. There is an increasing need, which remains unmet, for inertial orientation and position applications since they are key to automation and the "Internet of Things" (IoT). We propose a low-cost, multipurpose inertial solution exploring the Internet of Things with modules for an Inertial Measurement Unit that may support maintaining high levels of orientation and location exactness even when satellite-based location is not possible.

## 1.1 Problem Statement

This dissertation explores three core challenges that remain with significant interest in literature.

### 1.1.0.1 A. GNSS only positioning is prone to signal jamming and impacts battery autonomy

Modern navigation relies greatly on the Global Navigation Satellite System (GNSS) constellations (such as GPS, Galileo, GLONASS, etc.); being straightforward to operate, accurate and trustworthy, it is widely employed in navigation systems. Nevertheless, GNSS signals still encounter numerous vulnerabilities and can often be compromised by natural and human sources [14]. Attacks against GNSS-based localization are becoming more frequent and of intensifying damage [33]. Satellite signals can also be affected by abnormal activity due to solar winds, creating temporary gaps in coverage [1]. Moreover, while GNSS offers seamless navigation with inexpensive receivers, they are also prone to signal jamming, where satellites are unable to detect the objects. Repeatedly, position accuracy is reduced or even lost, such as in tunnels or underground sections [34] [32]. Furthermore, GNSS cannot accurately determine altitude with the necessary exactness, which is essential to accurately depict a body in three-dimensional space. Lastly, localization technologies demand high processing capacity and communication costs. This is particularly impactful in autonomous settings, where battery autonomy is crucial, while GNSS receivers continue to drain a large amount of electric current [21]. Consequently, power optimization is critical, and there is a necessity to become less reliant on GNSS-based localization.

### 1.1.0.2 B. Dead reckoning is susceptible to cumulative errors and suffers from gimbal lock

When understanding the alternatives to GNSS for estimating the position, the dead reckoning technique is often employed to resolve the location of a moving object. Using sensor data (gyroscope, accelerometer, magnetometer, etc.), it is possible to assess current position even when GNSS positioning is not possible [32]. It has been recognized as a low-power alternative to GNSS localization that can deliver high-resolution position data [6]. It is possible to estimate the current position from an obtained distance (which may be estimated from velocity), the known starting point, and estimated drift. However, the precision of the dead-reckoning approach is continuously

worsening while measurement errors accumulate during the current position estimation [17]. The approach is also embedded in Kalman filtering and other fusion techniques (will be explored in Section II), which mathematically merges a series of navigation solutions to obtain the best estimate of the navigator's current position, velocity, attitude angles, etc [19].

Still, precise tracking of a moving and rotating body in three-dimensional space implies a superior degree of complexity (compared to two-dimensional tracking). It can be accomplished in a range of approaches. Most commonly, the orientation of bodies that move in three dimensions may be explained by a combination of their angle of rotation across each of their three axes (e.g., such as in trigonometry, where Euler angles can approximate yaw, roll, and pitch). Fundamentally, a specific movement could be defined by multiple rotations. In such case, to perform a rotation over a particular axis, rotational matrices, vector operations, and trigonometric functions are required [3]. This involves numerous complex mathematical operations and several clock cycles in a microprocessor that could negatively impact computational performance [16].

Nevertheless, the rotation axes are not always independent, and results are not necessarily distinctive or unique. This further implies that the plane of two gimbals (rotational axes) to align, which causes the recognized gimbal lock phenomenon, where two out of three gimbals are parallel or very nearly parallel (such as in figure 1.b), reducing the output to two degrees of freedom [11]. When gimbal lock happens, it is not possible to reorientate the axes without an external reference.

#### **1.1.0.3 C. Gravity acceleration greatly impacts sensor readings**

An accelerometer is generally utilized to estimate the velocity and position of a given body devoid of the usage of GNSS. These electromechanical devices can measure proper acceleration forces, which can be employed to determine a body's velocity and position relative to a starting point. In theory, this can be done by integrating the resultant of acceleration yielding velocity; double integrating will deliver the body's accumulative position [44]. In practice, these measurements are influenced by Earth's gravitational field and rotational components of acceleration, significantly magnifying numerical errors during the readings [30]. The gravity component will not be differentiated from the physical acceleration of the device and will eventually generate exceedingly elevated errors in the measured acceleration. Double integrating these

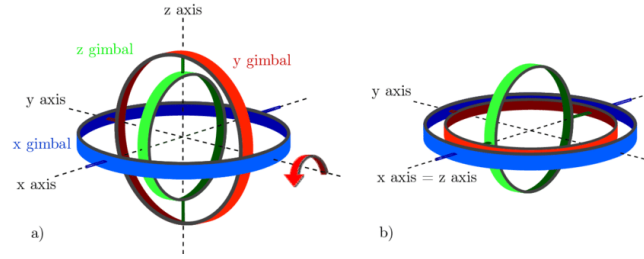


Fig. 1: Representation of the Gimbal Lock problematic [45] - The exterior blue gimbal characterizes the x-axis, the middle, red-colored gimbal the y axis, and the inner green gimbal the z-axis. In the initial arrangement a), every axis is perpendicular to one another. Following a rotation of  $90^\circ$  across the red arrow (y-axis), the blue and the green gimbals occupy the same rotation axis. This condition inhibits the clear determination of the rotation axes when subsequently rotating around the x or z-axis.

measurements will inherently amplify errors which will accumulate exponentially, translating into a yet greater offset in velocity and position estimates [39].

A potential solution to minimize this difficulty is to assume the body moves on a flat surface, thus greatly diminishing the influence of the gravity component on acceleration readings. Every inaccuracy that may arise from hiring such supposition can be regarded as noise and is smoothly filtered [30].

Understandably, such an assumption is not possible with a body moving through three-dimensional space since it is subject to many kinds of forces and movements. A numerical process is required for handling accelerometer measurements that utterly removes the effect of the gravity component and further undesirable acceleration vectors.

## 1.2 Research Questions

While research has been performed towards orientation estimation by the fusion of multiple sensors, few studies sought to assess position due to difficulties removing the measurement of normal forces that do not cause physical acceleration of the sensor. This investigation seeks to comprehend how different sensor fusion approaches perform in approximating the orientation of a moving system, along with how to overcome the positioning challenge through a technique that combines orientation estimation to filter non-physical acceleration. Additionally, we examine how the Internet of Things can be

applied to perform telemetry in real-time and at a long distance of a moving object.

- **[RQ1]. Estimate** - How to perform a low-cost orientation and position estimation of a moving object devoid of GNSS?

The study will emphasize on the possibility of estimating the orientation and position of a moving object in three-dimensional space short of GNSS-based positioning by combining multiple low-cost sensors to provide an object's navigation information.

- **[RQ2]. Comparing** - How distinct fusion techniques perform in orientation and position estimation?

An experimental comparison between various sensor fusion algorithms is used to evaluate how accurate each approach can approximate orientation and position.

- **[RQ3]. Communicate** - How to transmit navigational data in real-time at a long-distance?

Lastly, this dissertation will explore the prospect of amalgamating transmission IoT devices (based on open radio frequency at 868 MHz) to broadcast in real-time and at a long distance the navigational data of a moving object and how this approach may be employed in a series of real-world scenarios.

## 2 Background

The subsequent section presents background knowledge regarding different concepts and notions that will be adopted throughout the dissertation. First, a definition of orientation frames and coordinate systems will be presented, followed by an introduction to Euler angles and quaternions, building a foundation of understanding wherein the mathematics and arithmetic behind attitude representation. An introduction to inertial sensors and how they can be employed to estimate orientation is then exhibited. The chapter concludes with an analysis and summary of different sensor fusion algorithms utilized to determine orientation.



## 2.1 Frames of coordinates

This section will focus on defining and distinguishing the different concepts of frame coordinate systems. An emphasis will be given to East North Up (ENU), Earth Centered, Earth Fixed (ECEF), and the World Geodetic System (WGS84). The notion of body frame will also be defined. The ECEF and WGS84 can be considered supplementary frame systems applied to describe the ENU frame, which can be understood as the world frame.

### 2.1.1 ECEF and ENU frame

ECEF coordinate system describes a referential axis where the origin of the coordinates is at the center of mass of the Earth, also known as barycenter. Mathematically, this translates to the integral of the position vector times the density over the Earth being zero (equation 1).

$$\int \vec{x} \rho \, dx^3 = 0 \quad (1)$$

The X-axis is described by the intersection of the zero-latitude line (Equator) plan and the zero-longitude line (prime meridian) plan. The orientation of the X-axis is deemed to be positive from the center towards the point defined by zero latitude and longitude. Z-axis is expressed by the line interconnecting North and South Poles, staying positive in the Earth's barycenter to the North Pole. Y-axis lies in the equatorial plane and is perpendicular to the plane described by the X and Z-axis, and it is in a positive direction. The right-hand rule explains its orientation.

The East North Up (ENU) system is a geographic coordinate structure where the origin is placed at an empirical point in the ECEF coordinate system. In the ENU coordinate system, the X-axis points towards the East, and the Y-axis aims over the North Pole. The plane described by the X and Y-axis is tangential to the WGS84 frame on the origin of ENU. Z-axis designates the elevation from a defined geographical plane (figure 2). The ENU frame is considered in this work as the reference frame.

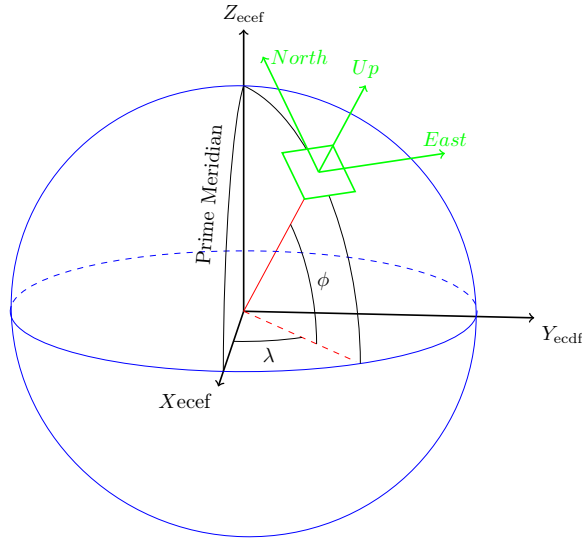


Fig. 2: An illustrative diagram for the WGS84, ECEF, and ENU coordinate systems for the Earth and their transformation correlation (PM line is the Prime Meridian;  $\phi$  and  $\lambda$  are latitude and longitude in WGS84; X, Y, Z for ECEF; and E, N, U for ENU).

Considering a point of reference in the ECEF frame, it is required to discover the equivalent latitude and longitude of the reference point ( $X_r$ ,  $Y_r$ ,  $Z_r$ ). The parameters of WGS84 required to perform such transformation are presented in table 1.

Parameter	Notation	Value
Semi-major axis	$a$	6 378 137.0 m
Reciprocal of flattening	$1/f$	298.257 223 563
Semi-minor axis	$b$	6 356 752.3142 m
First eccentricity squared	$e^2$	6.694 379 990 14x10-3
Second eccentricity squared	$e'^2$	6.739 496 742 28x10-3

Table 1: WGS 84 needed parameters to convert ECEF coordinates into ENU.

Applying the set of equations (2) to approximate latitude ( $\psi_r$ ) and longitude ( $\phi_r$ ) for the reference coordinate point. The last transformation outcome in applying equation (3) to ECEF physical quantities.

$$\begin{aligned}
p &= \sqrt{(X_r)^2 + (Y_r)^2} \\
\theta &= \arctan(Z_r \frac{a}{pb}) \\
\lambda_r &= \arctan(Y_r, X_r) \\
\varphi_r &= \arctan(\frac{Z_r + e^2 b \sin^3(\theta)}{p - e^2 a \cos^3(\theta)})
\end{aligned} \tag{2}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{ENU} = \begin{bmatrix} -\sin(\lambda_r) & \cos(\phi_r) & 0 \\ -\sin(\phi_r) \cos(\lambda_r) & -\sin(\phi_r) \sin(\lambda_r) & \cos(\phi_r) \\ \cos(\phi_r) \cos(\lambda_r) & \cos(\phi_r) \sin(\lambda_r) & \sin(\phi_r) \end{bmatrix} \begin{bmatrix} X_p - X_r \\ Y_p - Y_r \\ Z_p - Z_r \end{bmatrix}_{ECEF} \tag{3}$$

### 2.1.2 Body frame

Each sensor present in the razor board is aligned to match the sensor axes printed in the board as seen in figure 3.1. For simplicity, the razor sensors are placed as possible near the middle point of the bisector segment between the two wheel axes in such a way that YY axis as marked in the figure 3.1 is pointing towards the front of vehicle, XX axis is pointing to the right side of the car and ZZ axis is pointing to the top. This way, if the Euler angles describing the orientation of body frame related to world frame are all equal to zero, it means the axes in each frame are coincident apart from an offset in origin. Rotation angles are considered positive following the right hand rule in each axis. The origin of body frame is equal to intersection of rear wheel axis with the bisector defined above.

## 2.2 Orientation

The attitude orientation of a UAV is a critical aspect in autonomous flight. In a low-cost AHRS, accuracy and low complexity are important in calculating the attitude of the UAV. There are various ways to represent attitude including: rotational matrices, Euler angles and quaternions.

### 2.2.1 Rotation Matrix

The rotation matrix is a concept employed to express the transformation of coordinates from one frame to another. In addition, it can also convey

orientation of one frame relative to another. Any orientation can be attained by composing three elemental rotations, beginning from a known standard orientation. Analogously, any rotation matrix  $R$  can be decomposed as a product of three elemental rotation matrices (equation 4).

$$R = X(\alpha)Y(\beta)Z(\gamma) \quad (4)$$

In equation 4,  $R$  is a rotation matrix that can be applied to represent a composition of intrinsic rotations about axes  $X$ ,  $Y$ ,  $Z$ , (in that order), or a composition of extrinsic rotations about axes  $Z$ ,  $Y$ ,  $X$  (in that order). The convention used in this work is represented by equation 5 and maps quantities described in frame  $b$  to frame  $a$ . Comparing the structure of (2.3) with figure 3 it is seen that columns of  ${}^a_b R$  represent each unity vector defining all axes of frame  $b$ .

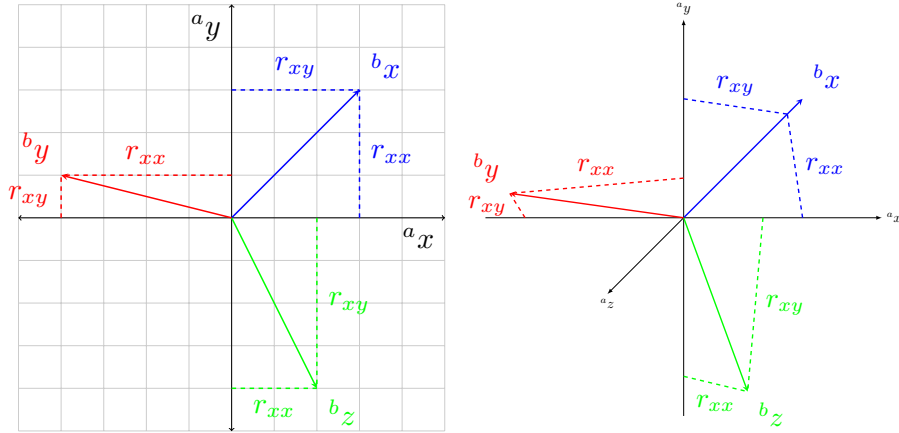


Fig. 3: Sensor data on each axis (blue is X-axis, red is Y-axis, green is Z-axis) obtained by the accelerometer, gyroscope, and magnetometer at 100 Hz sampling rate. Accelerometer provided the system's proper acceleration; gyroscope supplied the body's angular rate, and the magnetometer presented the detected magnetic flux.

$${}^a_b R = \begin{bmatrix} {}^a r_{xx} & {}^a r_{yx} & {}^a r_{zx} \\ {}^a r_{xy} & {}^a r_{yy} & {}^a r_{zy} \\ {}^a r_{xz} & {}^a r_{yz} & {}^a r_{zz} \end{bmatrix} \quad (5)$$

Rotation matrices belong to the orthonormal group and one important property is that  ${}^bR_a {}^bR^T = I$  meaning  ${}^aR^T = {}^bR^{-1}$ . Also a  ${}^bR^T$  is equal to  ${}^bR^T$  [36][37].

### 2.2.2 Euler angles

Euler angles are a well-known form to represent attitude with respect to a fixed coordinate system. Mathematically, Euler angles represent three composed sequential rotations that stir a reference frame to a given referred frame. It expresses a rotation in three angles, often referred to as pitch, roll and yaw, denoted by  $\phi$ ,  $\theta$  and  $\psi$  (The roll will have the range of  $\pm 180^\circ$ , this allows the pitch to have the range of  $\pm 90^\circ$ . The yaw is represented using the range of  $\pm 180^\circ$ ). These angles depict three successive rotations about the axes of the coordinate frame, for instance  $x - y - z$ : First rotate  $\phi$  radians about the x-axis, then rotate  $\theta$  radians about the y-axis and finally rotate  $\psi$  radians about the z-axis. As stated by the differential equations of Euler angles given at (equation 6),  $\phi$ ,  $\theta$ , and  $\psi$  can be found based on angular rate measurements. Merely three differential equations require solving prior to the attitude gets distinctly approximated.

$$\begin{bmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \frac{1}{\cos(\theta)} \begin{bmatrix} \sin(\phi) & 0 & -\cos(\phi) \\ \cos(\theta) \cos(\phi) & 0 & \cos(\theta) \sin(\phi) \\ \sin(\theta) \sin(\phi) & 1 & \sin(\theta) \cos(\phi) \end{bmatrix} \begin{bmatrix} \omega_{nbx}^b \\ \omega_{nby}^b \\ \omega_{nbz}^b \end{bmatrix} \quad (6)$$

$${}^b_a R = {}^b_2 R_x(\phi) {}^2_1 R_y(\theta) {}^1_a R_z(\psi) = \begin{bmatrix} \sin(\phi) & 0 & -\cos(\phi) \\ \cos(\theta) \cos(\phi) & 0 & \cos(\theta) \sin(\phi) \\ \sin(\theta) \sin(\phi) & 1 & \sin(\theta) \cos(\phi) \end{bmatrix} \quad (7)$$

Nevertheless, there is a significant flaw with this approach. As  $\cos(\theta)$  approaches zero, the differential equations degrade rapidly and the output solution becomes vastly imprecise, which indicates that these equations cannot provide effective attitude results at unique points in space. This is also known as Gimbal lock. Euler angles can be represented as a gimbaled system, where the three axes can be thought of as three distinct gimbals attached together. Gimbal lock happens when two axes line up, such as when the pitch and the yaw axis are aligned, and as the roll gimbal is rotated, the pitch and the yaw angle are both affected simultaneously, consequently losing orientation (figure

4). Euler angles commonly involve a large number of complex mathematical operations, such as matrix manipulations, which typically take up several clock cycles in a CPU, such can negatively impact computational performance. In this dissertation, a different kind of orientation representation will be used: quaternions.

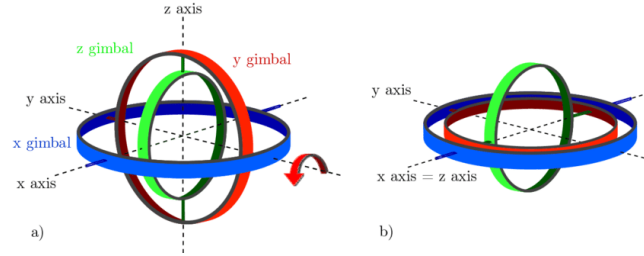


Fig. 4: Representation of the Gimbal Lock problematic [45] - The exterior blue gimbal characterizes the x-axis, the middle, red-colored gimbal the y axis, and the inner green gimbal the z-axis. In the initial arrangement a), every axis is perpendicular to one another. Following a rotation of  $90^\circ$  across the red arrow (y-axis), the blue and the green gimbals occupy the same rotation axis. This condition inhibits the clear determination of the rotation axes when subsequently rotating around the x or z-axis.

### 2.2.3 Quaternions

Another commonly used attitude representation is the quaternion. To comprehend quaternions, one must first grasp the complex number relationship. A complex number can depict a rotation in a 2-dimensional coordinate frame with a real x-axis and an imaginary y-axis (or vice versa) (figure 5). A quaternion builds upon this concept, but rather than one imaginary axis, it makes use of three imaginary axes: similar to merging three complex numbers into one. Four components are required to progress from a two-dimensional definition to a three-dimensional plane: one real component  $q_0$  and three imaginary  $q_1$ ,  $q_2$  and  $q_3$ . Quaternions are mathematically denoted as equation 8, and are commonly represented as a vector (equation 9), where  $q_0$  is the norm,  $q_1$ ,  $q_2$ , and  $q_3$  are complex coordinates with  $i$ ,  $j$ ,  $k$  being the axis versors. Equation (2.1) shows this relation with the complex number  $x$  having a real part  $a$  and imaginary part  $b$ ; and the quaternion  $q$  containing a real scalar part  $s$  and an imaginary vector component  $v$ .

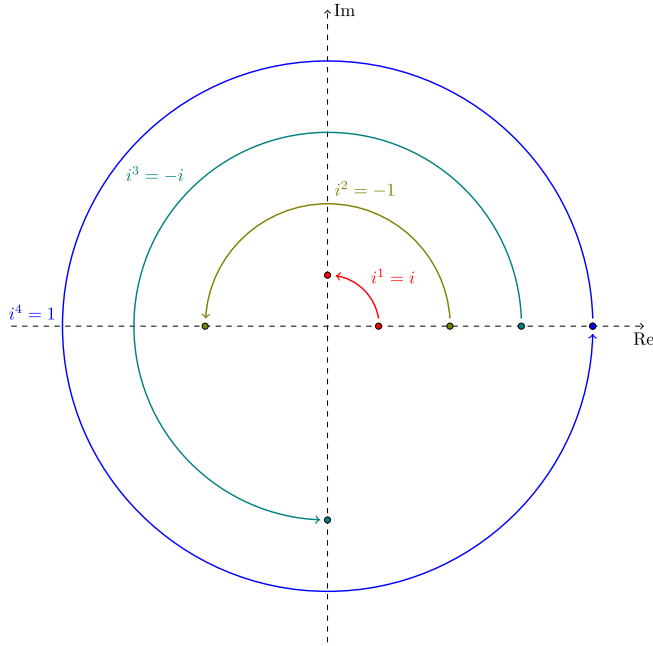


Fig. 5: Representation of the Gimbal Lock problematic [45] - The exterior blue gimbal characterizes the x-axis, the middle, red-colored gimbal the y axis, and the inner green gimbal the z-axis. In the initial arrangement a), every axis is perpendicular to one another. Following a rotation of  $90^\circ$  across the red arrow (y-axis), the blue and the green gimbals occupy the same rotation axis. This condition inhibits the clear determination of the rotation axes when subsequently rotating around the x or z-axis.

$$q = q_0 + q_1 i + q_2 j + q_3 k \quad (8)$$

$$q = \begin{bmatrix} q_0 & q_1 & q_2 & q_3 \end{bmatrix} \quad (9)$$

Specifically, when used for attitude description the quaternions should have a norm equal to 1:

$$\|q\| = \sqrt{q_0^2 + q_1^2 + q_2^2 + q_3^2} = 1 \quad (10)$$

Normalizing a quaternion is identical to normalizing a vector. The quaternion  $q$  is divided by the norm of the quaternion  $\|q\|$ . When a quaternion is normalized, it is denoted with a circumflex accent ( $\hat{q}$ ):

$$\hat{q} = \frac{q}{\|q\|} \quad (11)$$

A quaternion's conjugate ( $q^*$ ) is expressed by:

$$q^* = [q_0 \ -q_1 \ -q_2 \ -q_3] \quad (12)$$

The inverse of a quaternion ( $q^{-1}$ ) is therefore given by:

$$q^{-1} = \frac{q^*}{\|q\|} \quad (13)$$

Applying the notation as declared previously,  ${}^w_b q$  constitutes the orientation of body frame with respect to world frame. From equation 8, the subsequent list of properties/operations is derived:

Every quaternion operation ought to follow the succeeding list of properties:

$$\begin{aligned} i^2 &= j^2 = k^2 = ijk = -1 \\ ij &= -ji = k \\ jk &= -kj = i \\ ki &= -ik = j \end{aligned} \quad (14)$$

The quaternions multiplication, denoted by  $\cdot$ , is defined by the Hamilton product as in equation 2.8

	1	i	j	k
1	1	i	j	k
i	i	-1	k	-j
j	j	-k	-1	i
k	k	j	-i	-1

Table 2: Quaternion multiplication table.



$$a \otimes b = [a_1 a_2 a_3 a_4] \otimes [b_1 b_2 b_3 b_4] = \begin{bmatrix} a_1 b_1 - a_2 b_1 - a_3 b_3 - a_4 b_4 \\ a_1 b_2 + a_2 b_2 + a_3 b_4 - a_4 b_3 \\ a_1 b_3 - a_2 b_3 + a_3 b_1 + a_4 b_4 \\ a_1 b_4 + a_2 b_4 - a_3 b_2 + a_4 b_1 \end{bmatrix} \quad (15)$$

The quaternion conjugate describes the inverse rotation and is defined as equation 2.9.

$${}^w_b q^* = {}^b_w q = [q_1 - q_2 - q_3 - q_4] \quad (16)$$

Let's define the following quaternion representation of the same vector but in each referential by using is pure quaternion as  $wv = [0 \ wx \ wy \ wz]$  and  $bv = [0 \ bx \ by \ bz]$ . The rotation of vector  $v$  from one frame to other, using a quaternion, is performed by equation 2.10.

$${}^b_v = {}^w_b \hat{q} \otimes {}^w_v \otimes {}^w_b \hat{q}^* \quad (17)$$

The composition of rotations can be described in quaternions as the product between quaternions. For example the sequence  $a \ b \ q^\wedge \rightarrow b \ c \ q^\wedge$  is equal to  $a \ c \ q^\wedge$  and is defined as equation 2.11.

$${}^a_c \hat{q} = {}^b_c \hat{q} \otimes {}^a_b \hat{q} \quad (18)$$

As stated previously, to eliminate the singularities at  $\pm 90^\circ$  on the pitch and roll axes, and to keep the computational efficiency optimal, quaternions are used for state attitude representation. To understand quaternions, one must first understand the complex number relationship. A complex number can represent a rotation in 2 dimensional coordinate frame with a real x-axis and imaginary axis y-axis or vise versa. A quaternion is the same concept, except instead of one imaginary axis, there are three imaginary axes; in a sense its like combining three complex numbers into one. To go from a 2D interpretation to a 3D, four components are required; one real component  $q_s$  and three imaginary  $q_x \ q_y \ q_z$  which can also be represented as a vector  $\sim v$ . Equation (2.1) shows this relation with the complex number  $x$  having a real part  $a$  and imaginary part  $b$ ; and the quaternion  $q$  containing a real scalar part  $s$  and an

imaginary vector component  $v$ . To visualize a quaternion operation, think of Euler angles in the sense that there are three orthogonal angles each on a separate mutually exclusive axis,  $x, y$  and  $z$  as shown in figure 2.2. The rotation is not simultaneous, each rotation must happen sequentially, one axis at a time. In navigation terms, the plane must roll, then pitch, then yaw. In quaternion terms, the sequence can be thought of the same way as a rotating a vector laying on the  $x$ -axis, then rotating a vector on the  $y$ -axis, and finally rotating a vector on the  $z$ -axis.

#### **2.2.4 Accelerometer**

#### **2.2.5 Gyroscope**

#### **2.2.6 Magnetometer**

### **2.3 Sensor Fusion**

#### **2.3.1 Sensor Fusion Algorithms**

##### **2.3.1.1 Kalman Filter**

The Kalman filter algorithm is a set of mathematical equations that provides a computationally efficient approach to estimate some unknown variables by the detected measurements [41]. Kalman filters operate recursive functions to predict the present state of a linear problem by monitoring the current input data, the previous input data, and the previous state prediction. Two generally assigned methods for Kalman filter-based sensor fusion are state-vector fusion and measurement fusion. The state-vector fusion method (figure 8a) applies a group of Kalman filters to acquire individual sensor-based state estimates, which are subsequently fused to obtain an enhanced combined state estimate. Measurement fusion (figure 8b) approach directly combines the sensor data to achieve a joint measurement and later uses a single Kalman filter to get hold of the final state estimate centered on the fused measurement [28]. When applied appropriately, Kalman filters offer highly precise orientation, even with the existence of substantial noise. Nevertheless, Kalman filters are computationally expensive rising hardware cost and latency. They are also of complex implementation, which, shared with computational overhead, can make the algorithm unfeasible for computationally restricted applications. They are regularly useful in a wide-ranging variety of applications and have become a standard method in sensor fusion. Several studies examine the possibility of using Kalman filters to predict a body's orientation and position by combining

multiple sensors. The Kalman filter is founded on recursive Bayesian filtering. Consequently, the system's noise is assumed to be Gaussian. Therefore, the Kalman filter is generally suggested for linear systems. For this reason, an extension of the classic Kalman Filter designed for non-linear systems has emerged, recognized as Extended Kalman filter [42].

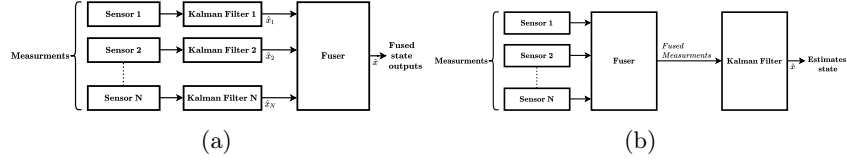


Fig. 6: Kalman-filter-based multi-sensor data fusion. (a) State-vector fusion. (b) Measurement fusion. [28]

### 2.3.1.2 Complementary Filter

The complementary filter is considered a simpler approach relatively to the Kalman filter since it is a computationally lightweight solution and straightforward to implement [13]. This filter takes as input two noisy sensor measurements and assumes one input is mainly formed by high-frequency signals whereas the other is mostly by low-frequency signals. Through a low pass filter, the high-frequency noise of the first input is filtered out. An identical procedure occurs with the second signal, but this time with a high pass filter to remove low-frequency noises, as illustrated in figure 9. Yet, the complementary filter is not especially robust to noisy or biased data since it simply uses currently available information, therefore, has no direct method of compensating for sensor noise [42]. A conventional application of the complementary filter is to bring together measurements of vertical acceleration and barometric readings to attain an approximation of vertical velocity. Similar to the Kalman filter, new versions built upon the principles of the classic complementary filter have emerged in recent times, such as the Extended Complementary Filter (ECF). They promise a high level of accuracy and enhanced robustness to noise while preserving computational efficiency.

### 2.3.1.3 Optimization Filters

Up until recently, there remained mainly two distinct AHRS fusion approaches. One category including the complementary filters, and the other is related to

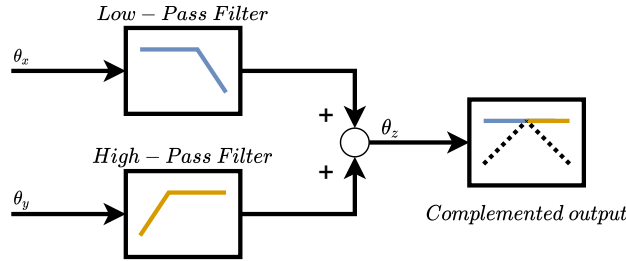


Fig. 7: Basic complementary filter [13] - Two different measurement sources for estimating one variable. The noise properties of the two measurements are such that one source gives good information only in low frequency region while the other is good only in high frequency region.

Kalman filtering. Some recent AHRS algorithms have emerged in the literature over the past years. Two of the most prominent are the Mahony and Madgwick algorithms, which have been categorized as optimization filters. Optimization filters obtain orientation by assessing a vector representative of the sensor output at the present orientation and lessening the disparity concerning predicted and observed outputs. Optimization filters are well established for linking accuracy with computational expense and simplicity of implementation [26]. Both methods make use of a quaternion representation, which is a four-dimensional complex number representing of an object orientation. Quaternions involve fewer computation time because of their minimal quantity of calculation parameters [22]. Additionally, vector rotations are easily executed by quaternion multiplications. Madgwick et al. [25] pioneered a gradient descent fusion algorithm, frequently recognized as ‘Madgwick Algorithm.’ This gradient descent fusion algorithm first obtains a quaternion estimation of the gyroscope output integration and later corrects it with a quaternion from the accelerometer and magnetometer data. Madgwick’s approach guarantees decent attitude estimation at a low computational cost. Further, it tackles the difficulty of the local magnetic disturbances that can influence all the orientation components. By reducing the constraint of the magnetic field vector rotation, it can limit the effect of the magnetic disturbances to only affect the yaw component of the orientation.

#### **2.3.1.4 Other Filters**

### **2.4 Low-Cost Inertial Measurement Units**

#### **2.4.1 MPU-9150 Evaluation Board 9DOF**

## **3 Related Work**

The literature review is divided into subsequent sections: 1) Position estimation using inertial sensors systems. 2) Sensor fusion in position and orientation estimation.

### **3.1 Position estimation using inertial sensor systems**

Inertial sensor systems have been thoroughly researched with the purpose of delivering position estimation of a moving body. Inertial systems are autonomous and independent and do not rely on external information, such as radio signals or electromagnetic waves. Their navigation data have short-term high accuracy, great constancy, and elevated data update rate. They may be applied in an array of distinct positioning methods.

#### **3.1.1 Pedestrian Dead Reckoning**

Pedestrian dead reckoning (PDR) is among the most explored. PDR combines step detection, step length estimation, and orientation approximations to calculate the absolute position and heading of a walking user. PDR can operate with a single accelerometer, although superior precision and robustness are obtained with more sensors. An Inertial Measurement Unit (IMU) containing several accelerometers, gyroscopes, magnetometers, and even pressure sensors are commonly employed to recognize steps and orientation. Sensors might be body-mounted or shoe-mounted. Pedestrian navigation systems can aid the blind and visually impaired, locating and rescuing firefighters and other emergency workers, hiking, sports, and others. Pedestrian dead reckoning is commonly reviewed in the literature, being subject to studies in various settings.

Ladetto et al. [20] applied PDR in urban and indoor areas seeking to assist blind people reaching unfamiliar locations along with aiming to facilitate emergency coordinators to track rescue workers. The study integrated a GPS receiver with a body-mounted IMU applying pattern recognition to accelerometer signals, determining a user's step signature.

Stirling et al. [38] illustrate an experiment exploiting a shoe-mounted sensor prototype that calculates stride length with accelerometers and magnetometers. Their system measures angular acceleration by manipulating pairs of accelerometers as an alternative to gyroscopes.

Several other studies investigate the prospect of using inertial sensor systems to estimate the absolute position and heading of a walking user for multiple purposes [37] [40] [18] [4]. The main drawback of PDR is its dependence on step prediction algorithms that must distinguish step direction and step lengths as the user changes pace.

### 3.1.2 Strapdown Inertial Integration

Strapdown inertial integration or strapdown inertial navigation system (SNIS) is another prevalent position method. With SNIS, sensors are usually tightly strapped or attached to the axes of the moving body's structure, lowering costs and enhancing the system's reliability. This technique integrates accelerometer and gyroscope measurements to distinguish the variation of position and heading. The strapdown system demands a high-level measurement rate, on average, beyond 2000 Hz. Typically, higher measurement rates translate into more accurate integration readings of position and attitude. Strapdown systems are currently employed in commercial and military applications (airplanes, vessels, ROVs, projectiles) and are a topic of study among scholars.

Jameian et al. [15] introduced strapdown inertial navigation system to nautical environments, proposing a compensation method against disturbing forces affecting vessel motion caused by rough sea conditions. They aim to resolve attitude determination offset through self-alignment of SNIS by establishing vector observations. The implementation makes use of a quaternion estimator for attitude determination, significantly diminishing computational complexity.

An indoor strapdown inertial navigation with small foot-mounted and self-contained sensor systems was described by Bird et al. [2]. Similar to pedestrian dead reckoning, SNIS also has applications in pedestrian navigation systems, although operating in an utterly distinct fashion. Unlike PDR, the strapdown navigation algorithm traces the entire movement of the foot in between steps. Any movement like walking, running, climbing up or down, moving backward or sideways, sliding, and even jumping can be tracked. This

is possible because of a zero-velocity update algorithm (ZVU) which exploits the brief periods of zero velocity when the feet are stationary on the ground.

SNIS and PDR may also be used together, sharing the same inertial sensors. In this case, inertial navigation is incorporated within the multi-sensor integration architecture as the reference system and PDR as an aiding sensor.

With a focus on low-cost inertial motion sensors, Coyte et al. [5] applied PDR to sporting training and rehabilitation. They propose solutions to acceleration noise accumulation and gyroscope angle error problems. To improve the accuracy of displacement estimation with a low-grade IMU, they developed a zero-velocity update algorithm.

### **3.2 Sensor fusion in position and orientation estimation**

Sensor fusion defines the blending of sensory information from two or more sources in a way that generates a more consistent and dependable understanding of the system. One that would otherwise not be possible when these sources were used individually [10]. Fusing multiple inertial systems has raised significant interest and consideration in location and attitude performance improvement. Numerous methods arose in recent times that merge information from various systems such as inertial sensors, GNSS, radar, radio telescopes, signal of opportunity systems like Angle of Arrival (AOA), Time of Arrival (TOA), Received Signal Strength (RSS), and Signal to Noise Ratio (SNR). The combination of multiple sources can help reduce noise with two different sensor types. These separate systems are integrated by fusion filter algorithms which process each input and generate a more precise and reliable output [8]. A substantial sum of distinct solutions designed to assess the orientation of a rigid body regarding a reference frame exist in literature. Two main approaches aimed at sensor fusion exist, Kalman and complementary related filters. A comprehensive analysis of the literature will be conducted seeking to better understand the distinction between algorithms and how do they compare.

#### **3.2.1 Kalman filter**

The Kalman filter algorithm is a set of mathematical equations that provides a computationally efficient approach to estimate some unknown variables by the detected measurements [41]. Kalman filters operate recursive functions to

predict the present state of a linear problem by monitoring the current input data, the previous input data, and the previous state prediction. Two generally assigned methods for Kalman filter-based sensor fusion are state-vector fusion and measurement fusion. The state-vector fusion method (figure 8a) applies a group of Kalman filters to acquire individual sensor-based state estimates, which are subsequently fused to obtain an enhanced combined state estimate. Measurement fusion (figure 8b) approach directly combines the sensor data to achieve a joint measurement and later uses a single Kalman filter to get hold of the final state estimate centered on the fused measurement [28]. When applied appropriately, Kalman filters offer highly precise orientation, even with the existence of substantial noise. Nevertheless, Kalman filters are computationally expensive rising hardware cost and latency. They are also of complex implementation, which, shared with computational overhead, can make the algorithm unfeasible for computationally restricted applications. They are regularly useful in a wide-ranging variety of applications and have become a standard method in sensor fusion. Several studies examine the possibility of using Kalman filters to predict a body's orientation and position by combining multiple sensors. The Kalman filter is founded on recursive Bayesian filtering. Consequently, the system's noise is assumed to be Gaussian. Therefore, the Kalman filter is generally suggested for linear systems. For this reason, an extension of the classic Kalman Filter designed for non-linear systems has emerged, recognized as Extended Kalman filter [42].

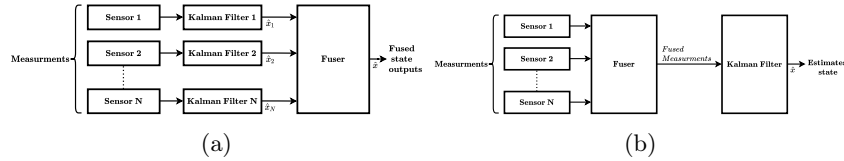


Fig. 8: Kalman-filter-based multi-sensor data fusion. (a) State-vector fusion. (b) Measurement fusion. [28]

Several research works have been conducted on Inertial Navigation Systems and Global Navigation Satellite System integration through data fusion, particularly using the Kalman filter. To overcome the shortcomings linked to the detached functioning of GNSS and INS, Wong et al. [43] Qi et al. [35], and Nassar et al. [29] combined both systems so that their disadvantages were



lessened or eradicated, complementing one another. While GNSS was comparatively more stable and consistent for long periods, INS had a more reliable and comprehensive short-term signal. Updating INS position and velocity with GNSS data corrected error expansion at the same time it delivered more precise estimates. The Kalman Filter attempted to adjust INS information based on the system error model whenever GNSS signals were interrupted or limited. These studies have demonstrated success in satisfying the accuracy requirements of low-precision applications. However, they could not deliver the high-precision positioning some applications required. Hence, other studies attempted to achieve better performance of integrated INS/GNSS systems through the exploration of extended and adaptive Kalman filtering techniques. Mohamed and Schwarz [27] performed an analysis on INS/GNSS alternative integration through an adaptive Kalman filtering technique. Findings reveal that their adaptive Kalman filter outperformed by almost 50% the conventional filter.

The use of data fusion in autonomous flying units such as Unmanned Aerial Vehicles (UAV) has recently gained particular concern due to the dissemination of consumer-grade quadcopters. In autonomous aerial settings, an accurate altitude reading is crucial to control the position of the flying system. However, such measurements are repeatedly corrupted with signal noise produced by the vehicle's motors. Hetényi et al. [12] applied a Kalman Filter to fuse the sonar and accelerometer signals, obtaining a considerably improved altitude estimate with minimal error. Similarly, Luo et al. [24] combined the UAV sensor system and received signal strength (RSS) in a Kalman filter solution. The study sought to increase position and altitude estimation as well as collision avoidance precision by approximating the distance between the receiver and the transmitter via the use of radiofrequency signals reducing the noise component.

Sharma et al. [36] present an experiment of Kalman filter-based sensor fusion for extrapolation of a robot's orientation and depth to obstacle by fusing the inputs from three infrared sensors and an inertial sensor system. The combination of multiple sensor inputs allowed the robot to operate in fault-tolerant applications and enhanced its obstacle avoidance decision making, localization, and orientation estimations.

### 3.2.2 Complementary filter

The complementary filter is considered a simpler approach relatively to the Kalman filter since it is a computationally lightweight solution and straightforward to implement [13]. This filter takes as input two noisy sensor measurements and assumes one input is mainly formed by high-frequency signals whereas the other is mostly by low-frequency signals. Through a low pass filter, the high-frequency noise of the first input is filtered out. An identical procedure occurs with the second signal, but this time with a high pass filter to remove low-frequency noises, as illustrated in figure 9. Yet, the complementary filter is not especially robust to noisy or biased data since it simply uses currently available information, therefore, has no direct method of compensating for sensor noise [42]. A conventional application of the complementary filter is to bring together measurements of vertical acceleration and barometric readings to attain an approximation of vertical velocity. Similar to the Kalman filter, new versions built upon the principles of the classic complementary filter have emerged in recent times, such as the Extended Complementary Filter (ECF). They promise a high level of accuracy and enhanced robustness to noise while preserving computational efficiency.

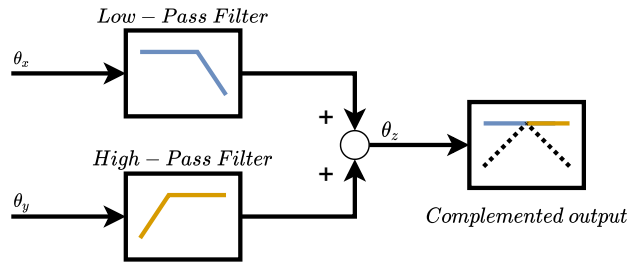


Fig. 9: Basic complementary filter [13] - Two different measurement sources for estimating one variable. The noise properties of the two measurements are such that one source gives good information only in low frequency region while the other is good only in high frequency region.

Madgwick et al. [26] outlined the formulation of an extended complementary filter algorithm and exhibited its applicability as a human motion monitoring wearable. Their design fused magnetic, angular rate, and gravity sensor data to remotely estimate limb orientation in stroke patients performing rehabilitation exercises. They analyzed performance under a range of circumstances and

benchmarked alongside other frequently utilized sensor fusion algorithms. They claim an improved computational efficiency of over 30% when compared with standard alternative algorithms.

A complementary filter designed for sensor fusion in quadrotor UAV employing a low-cost inertial measurement system was proposed by Noordin et al. [31]. The complementary filter filtered high-frequency signals associated with the gyroscope and low-frequency signals linked to the accelerometer. Findings demonstrate that the complementary filter technique overcame the over drift conundrum related to gyroscopes and was capable of computing attitude angles efficiently. Euston et al. [9] conducted an analogous study with a non-linear complementary filter for attitude estimation in a UAV utilizing a low-cost IMU. They broadened the experiment to incorporate a model of the longitudinal angle-of-attack corresponding to the UAV’s airframe acceleration using airspeed data. As a result, they could estimate the acceleration of the UAV during continuous turns based on gyroscope and airspeed data. They accomplished attitude filtering performance of similar quality as an extended Kalman filter that fused GPS/INS at a far less computational cost.

### 3.2.3 Optimization filters

Up until recently, there remained mainly two distinct AHRS fusion approaches. One category including the complementary filters, and the other is related to Kalman filtering. Some recent AHRS algorithms have emerged in the literature over the past years. Two of the most prominent are the Mahony and Madgwick algorithms, which have been categorized as optimization filters. Optimization filters obtain orientation by assessing a vector representative of the sensor output at the present orientation and lessening the disparity concerning predicted and observed outputs. Optimization filters are well established for linking accuracy with computational expense and simplicity of implementation [26]. Both methods make use of a quaternion representation, which is a four-dimensional complex number representing of an object orientation. Quaternions involve fewer computation time because of their minimal quantity of calculation parameters [22]. Additionally, vector rotations are easily executed by quaternion multiplications. Madgwick et al. [25] pioneered a gradient descent fusion algorithm, frequently recognized as ‘Madgwick Algorithm.’ This gradient descent fusion algorithm first obtains a quaternion estimation of the gyroscope output integration and later corrects it

with a quaternion from the accelerometer and magnetometer data. Madgwick's approach guarantees decent attitude estimation at a low computational cost. Further, it tackles the difficulty of the local magnetic disturbances that can influence all the orientation components. By reducing the constraint of the magnetic field vector rotation, it can limit the effect of the magnetic disturbances to only affect the yaw component of the orientation.

### 3.2.4 Sensor fusion algorithms comparison

Some studies have conducted comparison experiments between the sensor fusion algorithms in distinct settings to assess their performance in that unique condition. Ludwig et al. [22] compared Madgwick and Mahony in a foot-mounted experiment. Their findings revealed that Madgwick achieved better heading orientation than Mahony when compared to the ground truth. Nonetheless, the performance of Mahony was superior to Madgwick. The same authors tested on [23] quadcopters the Extended Kalman Filter, Madgwick, and Mahony filters. Results showed that Mahony delivered a more precise orientation estimation and faster execution time than Madgwick and EKF. Diaz et al. [7] present a comparison among Madgwick and Mahony, a basic AHRS estimation algorithm, and the recent algorithm proposed by the authors. The study centered around comparing the performance of Madgwick, Mahony, Extended Kalman Filter, and their own sensor fusion algorithm, emphasizing the behavior under magnetic perturbations. Various examples of movement were analyzed, from carrying the sensor at separate places such as pocket, shoe, and hand. They concluded that their algorithm was slightly less influenced by magnetic perturbations than the others, but overall, the algorithms performed similarly.

### 3.3 Thesis Contribution

With this study, we aim to design and build a low-cost, multipurpose Inertial Navigation System, intending to estimate the orientation and position of a moving object in three-dimensional space. This research additionally proposes introducing an experimental comparison among several established AHRS sensor fusion algorithms such as the Extended Kalman Filter, Madgwick, and Mahony algorithms. Furthermore, a quaternion-based gravity compensation filter will be presented, diminishing the influence of the gravity component on acceleration readings. Our key contribution is to merge IoT with our Inertial

System through Long Range (LoRa) communication to perform real-time and long-distance telemetry.

## 4 Methodology

This section provides a hardware and software solution outline, describing the development and implementation of a low-cost, multipurpose Inertial Navigation System with experimental tests. Resultant findings are analyzed and validated to assess the implemented system's positioning and orientation (AHRS) effectiveness and precision. Several fusion algorithms will be benchmarked by performance and accuracy. This development was achieved mainly in three phases, system design, hardware application, and software implementation.

### 4.1 Evaluation Methodology

#### 4.1.1 Obtaining AHRS from sensor fusion

The first experiment involved obtaining orientation (AHRS) from the fusion of the three sensor outputs. The accelerometer provided the system's proper acceleration; the gyroscope supplied the body's angular rate, and the magnetometer presented the detected magnetic flux. In this first experiment, we first utilized the Madgwick algorithm to sensor fusion for its well-recognized proficiency to merge accuracy with computational cost and simplicity of implementation. A simple test was performed using a box containing the inertial navigation system and rotating around its three inertial axes, pitch, roll, and yaw, with a sampling frequency of 20 Hz. The raw output measurements from the sensors are visualized in figure 15 and are subsequently taken as input by the sensor fusion algorithm.

The fusion output is shown in figure ???. About sample #140 there was a shift in pitch to  $-40^\circ$  and later jumping to  $40^\circ$ . Around sample #350 there is a change in roll to  $60^\circ$  and then  $-40^\circ$ , and around sample #540 there is a variation in yaw value. This output is analogous to the movement made on the board.

#### 4.1.2 Estimating position

Accelerometer readings can measure proper acceleration forces, which can be employed to determine a body's velocity and position relative to a starting

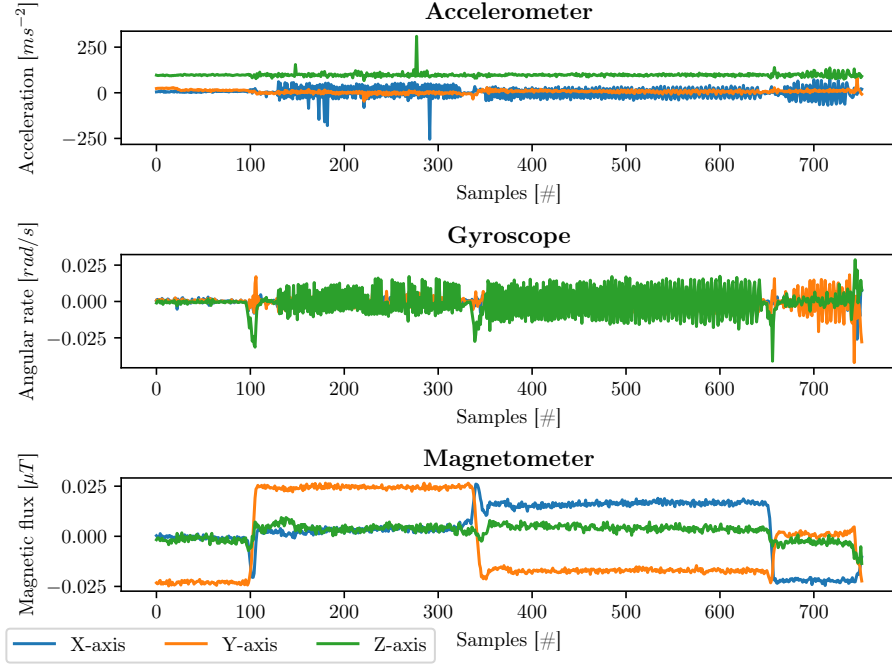


Fig. 10: Sensor data on each axis (blue is X-axis, red is Y-axis, green is Z-axis) obtained by the accelerometer, gyroscope, and magnetometer at 100 Hz sampling rate. Accelerometer provided the system's proper acceleration; gyroscope supplied the body's angular rate, and the magnetometer presented the detected magnetic flux.

point. Integrating the resultant of acceleration will yield velocity, and double integrating will provide the body's accumulative position.

Once the measured inertial-frame acceleration is attained, it can be integrated to obtain inertial frame velocity  $v_i$  and position  $x_i$  can be calculated:

$$v_i = v_0 + \int_0^t a_i dt \quad (19)$$

$$x_i = x_0 + \int \int_0^t a_i dt \quad (20)$$

In practice, data is acquired at discrete time periods ( $\Delta t$ ) so the approximate velocity and position are estimated by:

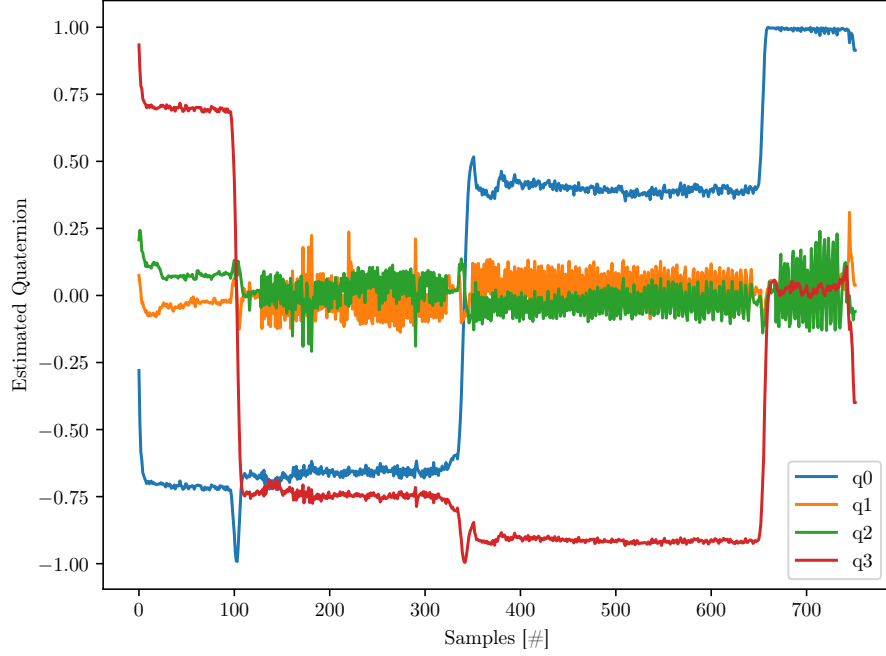


Fig. 11: Sensor data on each axis (blue is X-axis, red is Y-axis, green is Z-axis) obtained by the accelerometer, gyroscope, and magnetometer at 100 Hz sampling rate. Accelerometer provided the system's proper acceleration; gyroscope supplied the body's angular rate, and the magnetometer presented the detected magnetic flux.

$$v_i[k+1] = v_i[k] + a_i[k]\Delta t \quad (21)$$

$$x_i[k+1] = x_i[k] + v_i[k]\Delta t \quad (22)$$

However, these measurements are affected by the Earth's gravitational field (as seen in figure 15 in the acceleration readings) and rotational components of acceleration, considerably amplifying numerical errors. The gravity component will not be differentiated from the physical acceleration of the device and will eventually generate exceedingly elevated errors in the measured acceleration. To overcome this challenge, a gravity compensation algorithm is crucial for subtracting the impact of the gravity component on acceleration readings.

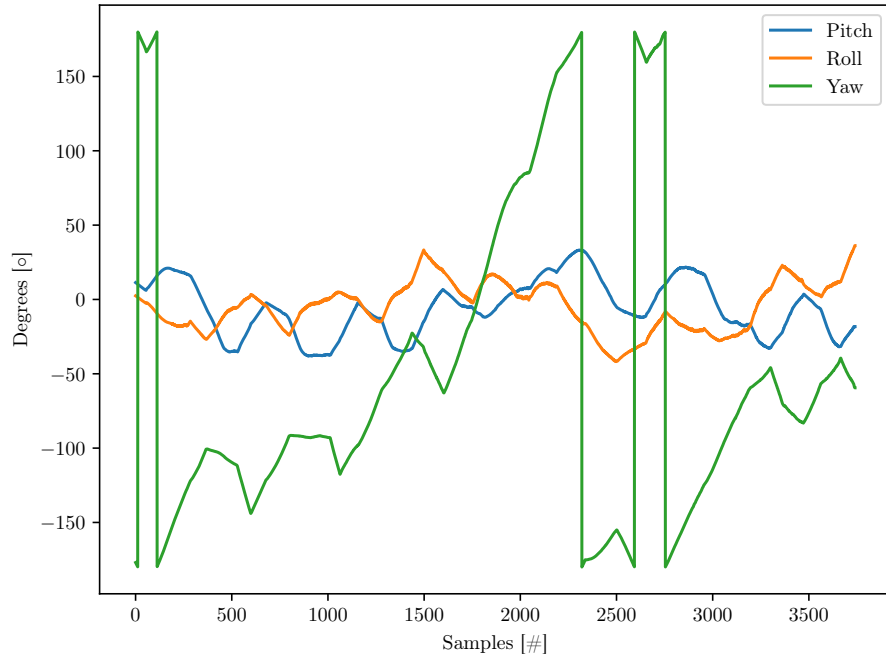


Fig. 12: Sensor data on each axis (blue is X-axis, red is Y-axis, green is Z-axis) obtained by the accelerometer, gyroscope, and magnetometer at 100 Hz sampling rate. Accelerometer provided the system's proper acceleration; gyroscope supplied the body's angular rate, and the magnetometer presented the detected magnetic flux.



Through orientation estimation determined previously, it is possible to find the orientation of the Earth frame with respect to the sensor frame. Therefore, compute the expected direction of gravity and then subtract that from the accelerometer readings (figure ??). Resulting in a linear acceleration that corresponds to the physical acceleration of the device. This linear acceleration can be numerically integrated returning velocity and double integrating delivering the position of the device (figure 14).

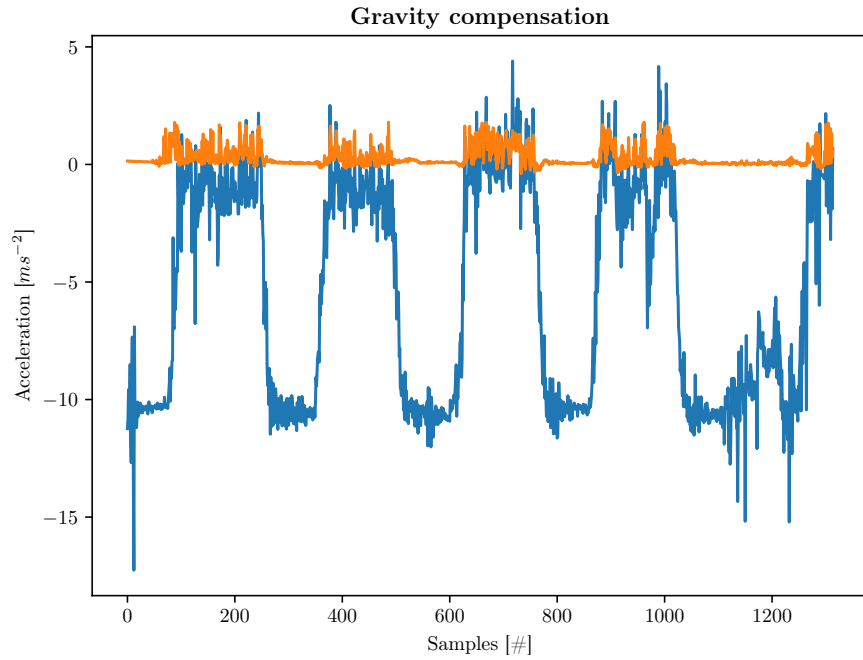


Fig. 13: Sensor data on each axis (blue is X-axis, red is Y-axis, green is Z-axis) obtained by the accelerometer, gyroscope, and magnetometer at 100 Hz sampling rate. Accelerometer provided the system's proper acceleration; gyroscope supplied the body's angular rate, and the magnetometer presented the detected magnetic flux.

The next step consisted of experimentally testing the inertial system seeking to achieve position estimation merely by integrating the accelerometer measurements with the gravity compensation filter. Numerous experiments were performed in several settings. The tests involved walking on a straight line carrying a box containing the inertial system (with the inertial X-axis parallel

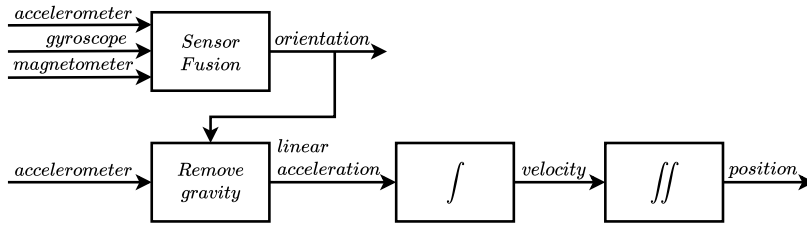


Fig. 14: Overview of the position estimation method.

to the path) for around 30 meters, evaluating the computed acceleration, velocity, and accumulative position estimation. Specific tests were executed at walking pace, others at running pace, and some with a combination of both.

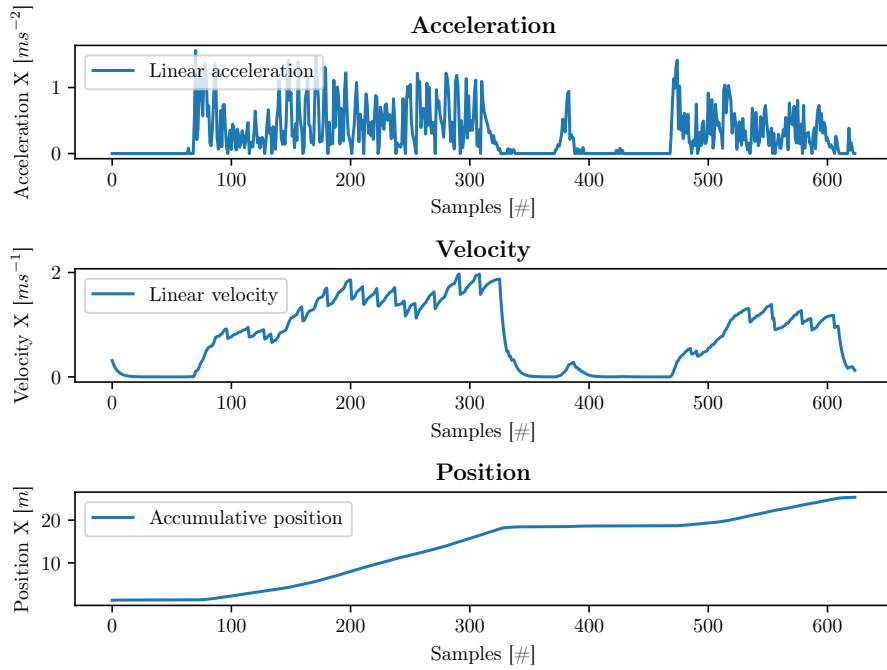


Fig. 15: Sensor data on each axis (blue is X-axis, red is Y-axis, green is Z-axis) obtained by the accelerometer, gyroscope, and magnetometer at 100 Hz sampling rate. Accelerometer provided the system's proper acceleration; gyroscope supplied the body's angular rate, and the magnetometer presented the detected magnetic flux.

First, one-dimensional experiments were conducted, pursuing to estimate accumulative position when moving on a straight line for 30 meters. These tests revealed an average error margin from 10% to 15% of the actual distance. Different movements and walking paces were tested, such as stopping at different time intervals. With the accomplishment of this first experiment, we decided to expand the testing to two dimensions. An experiment result is observed in figure ??, where the final observed accumulative position was 25 meters.

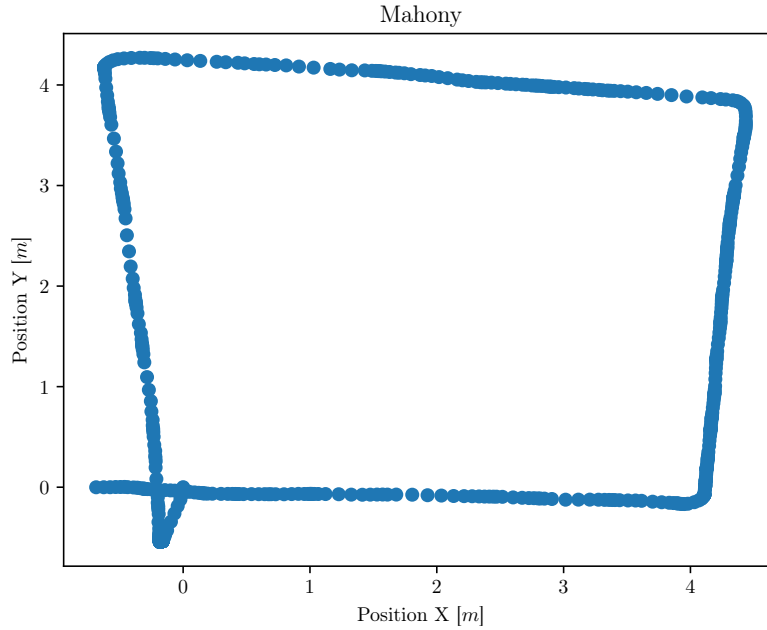


Fig. 16: Sensor data on each axis (blue is X-axis, red is Y-axis, green is Z-axis) obtained by the accelerometer, gyroscope, and magnetometer at 100 Hz sampling rate. Accelerometer provided the system's proper acceleration; gyroscope supplied the body's angular rate, and the magnetometer presented the detected magnetic flux.

In two-dimensional tests, the output from X-axis and Y-axis accelerometer measurements were integrated with orientation to acquire accumulative position on both axes. These tests implicated moving on a linear pattern, generally 10 meters on each axis. Integrated axis position was then plotted into

a scatter plot that combines the accumulative position on the two axes. Figure ?? shows an experiment performed with two-dimensional positioning.



Fig. 17: 5 meter side ground square used for baseline of accuracy of inertial system.

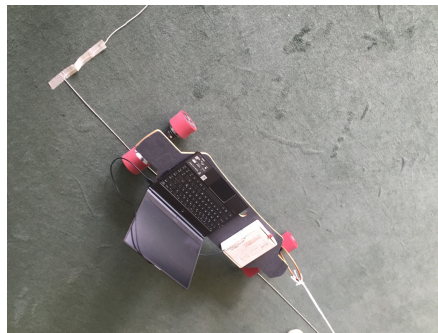


Fig. 18: Experiment made with skateboard to assess localization accuracy.

Finally, only the three-dimensional experiment was left combining the output readings from the Z-axis accelerometer. The same previous procedure was employed in this experimentation; the output from X-axis, Y-axis, and Z-axis accelerometer measurements were combined with orientation and integrated to obtain an accumulative position on all axes. A similar movement to the two-dimensional test was performed, but this time wobbling the inertial unit vertically to better visualize the position variation on the Z-axis. An example of an experimental test conducted in three dimensions in visible is figure 24.



Fig. 19: Manually pulling the skateboard on each side of the ground square.

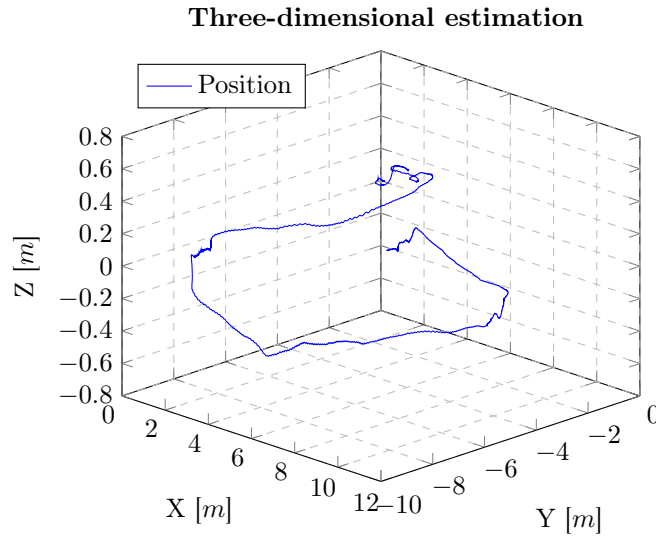


Fig. 20: Same experiment conducted in figure ?? with Z-axis altitude estimation.

## 4.2 System Architecture

As our projected solution involved a low-cost navigation system, the hardware selection criteria were primarily founded on availability and cost. The cost reduction normally concedes in accurateness and reliability, although with the recent surge of inexpensive, widely accessible, and precise microelectromechanical systems (MEMS), that is no longer the case. Still, we aim to employ commercially available tools at the lowest possible cost without compromising the design of a robust and accurate inertial navigation system. A

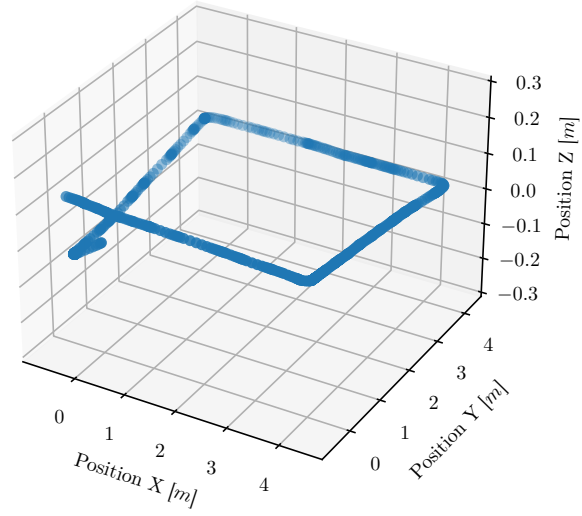


Fig. 21: Sensor data on each axis (blue is X-axis, red is Y-axis, green is Z-axis) obtained by the accelerometer, gyroscope, and magnetometer at 100 Hz sampling rate. Accelerometer provided the system's proper acceleration; gyroscope supplied the body's angular rate, and the magnetometer presented the detected magnetic flux.

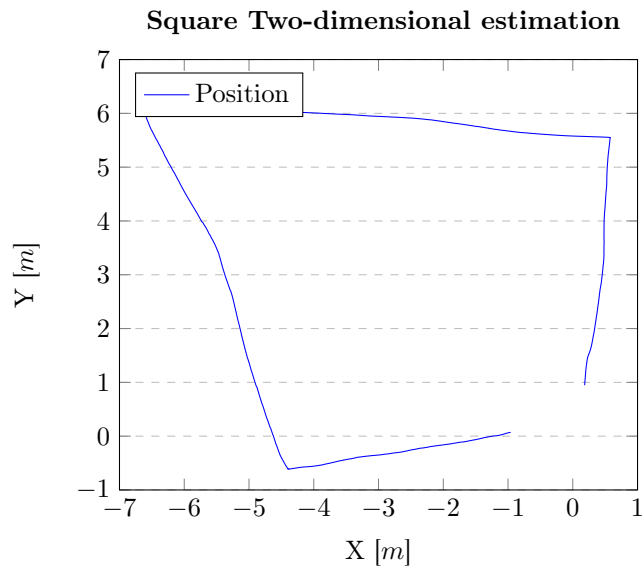


Fig. 22: Same experiment conducted in figure ?? with Z-axis altitude estimation.

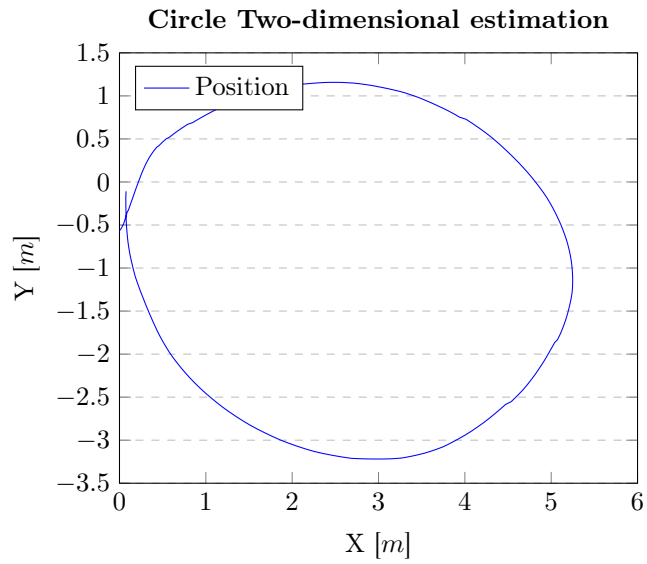


Fig. 23: Same experiment conducted in figure ?? with Z-axis altitude estimation.

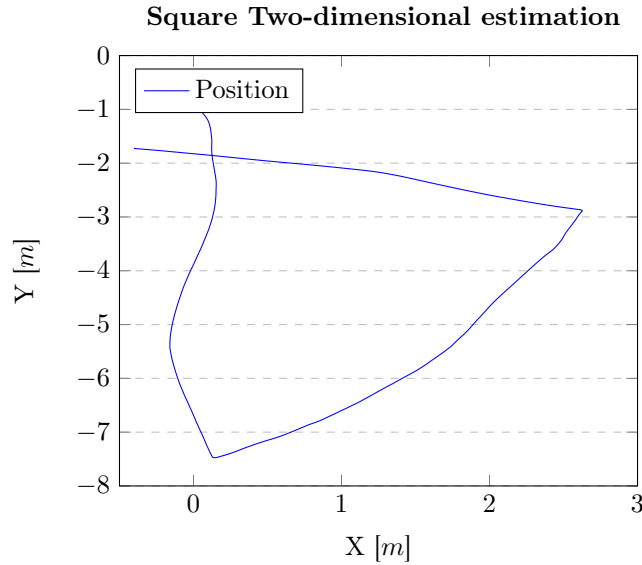


Fig. 24: Same experiment conducted in figure ?? with Z-axis altitude estimation.

LoPy microcontroller was selected as the navigational computing device of the inertial system, meeting the envisioned requirements for low power with flexible and diverse computational capabilities. The LoPy development board interfaces with the external physical inertial sensor through connection pins (figure 25.a) and communicates remotely by LoRa protocol. A PySense expansion board connects with the LoPy module providing a programable interface for the microcontroller. The inertial navigation system encompasses a small MPU-9250 (figure 25.b) IMU (3x3x1mm) with nine degrees of freedom comprising three accelerometers, three orthogonal gyros, three magnetometers as well as temperature and pressure sensors. This chip is extensively employed in wearable sensors for health, fitness, and sports, motion-based game controllers, and portable gaming. A correct calibration of such sensors is essential for the compensation of their systematic errors, bias, and scale factor. Each time prior to an experiment, the inertial sensor is calibrated while the system is stationary and stabilized to compensate for static error that might corrupt the measurements.

The microcontroller operates MicroPython, a barebones and efficient implementation of Python 3, which incorporates a small subset of the Python



standard library. It is optimized to run on microcontrollers and in constrained environments. The inertial module's raw measurements are interpreted by the microcontroller through Inter-Integrated Circuit (I2C) MicroPython driver serial allowing to read the peripherals memory addresses synchronously. The readings of each sensor are later averaged and linearized to better detect and reduce the presence of outlier readings. A fusion algorithm takes as input the averaged data of the accelerometer, gyroscope, and magnetometer. It returns the estimated inertial angles (pitch, roll, and yaw) as well as the projected linear acceleration with a gravity compensation numerical method that utterly removes the effect of the gravity component. Numerically integrating the resultant linear acceleration yields velocity, and double integrating will deliver the body's accumulative position. Merging the AHRS with an accumulative position allows tracking a moving body in three dimensions over time. Moreover, the navigation system is equipped with a LoRa antenna enabling the system to transmit at 868MHz/915MHz LoRa bands in real-time at a long distance the position and orientation information to an external gateway (figure 26). A visualization of the entire hardware solution is provided at image 27.

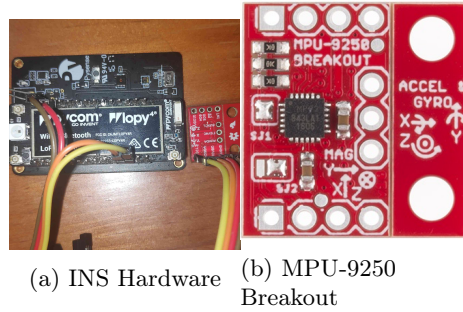


Fig. 25: Pin connection between the microcontroller (left) and the IMU (right). Modules are linked through SCL, CLK, VDD and GND pins.

## 5 Discussion

TBD

## 6 Results

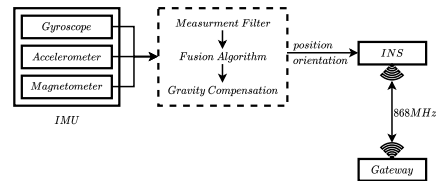


Fig. 26: System overview - Raw measurements from IMU are fused together and numerically integrated to obtain position and orientation. This INS data is transmitted in real-time to a remote node gateway.



Fig. 27: Complete hardware solution - Box containing the full inertial navigation system, at the left the antenna suitable for use in the 868MHz/915MHz LoRa bands.

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