Introduction to Feedback

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1 Preliminaries

In electronics the term **feedback** refers to the situation where a signal **sensed** (or derived or sampled or measured) from the output port of an amplifier is **returned** to the input port, where it is **combined** with the externally applied input signal to create a new signal to be processed by the amplifier itself.

- when the returned signal is added to the external signal, we have positive feedback (a.k.a. regenerative feddback)
- when the returned signal is subtracted we have negative feedback (a.k.a. degenerative feedback)

Negative feedback was conceived in 1928 by Harold Black.

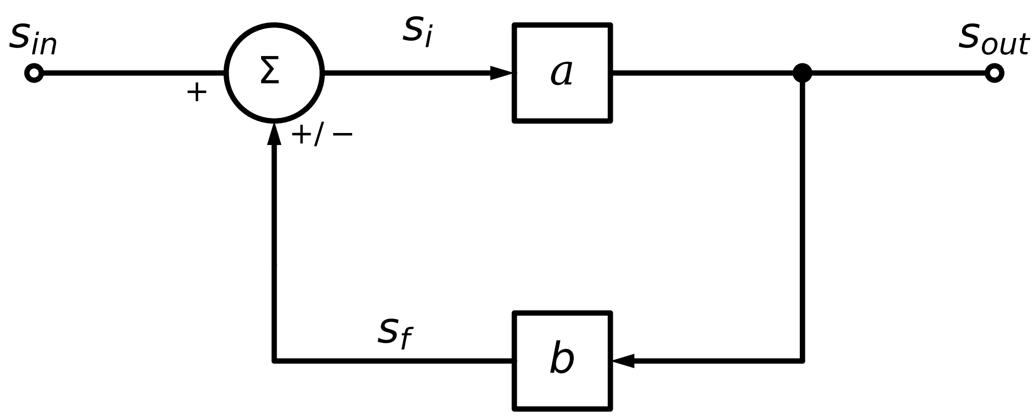


Figure 1: Block diagram of a feedback system.

2 Systematic Feedback Analysis Frameworks

- Two-port Analysis
  - Proposed by Harold Black
  - Helpful for gaining basic intuition about feedback

- Map the feedback network onto one of four topologies
    - voltage-voltage (serie-shunt), current-voltage (shunt-shunt), voltage-current (series-series), current-current (shunt-series), depending on the desired input/output quantities
- In practice, the feedback network "b" (and possibly the forward amplifier "a") are not unilateral, so mapping arbitrary circuits into a generic "ab" block diagram that uses two-ports is not obvious
- The feedback network causes loading at the input and output of the basic amplifier
    - model the feedback network as an ideal two-port and absorb the loading effects into the forward amplifier. This procedure is often quite tedious.
- Some feedback circuits cannot be modeled using two-ports
    - e.g. bias circuits with feedback loops tend to have only one port

- Return Ratio Analysis
    - Proposed by Hendrik Bode
    - "Asymptotic" method. It does no attempt to break the circuit into pieces
    - The closed-loop properties of the feedback circuit are described in terms of the *return ratio* of a dependent source in an active device
    - The block diagram used to analyze the closed loop properties of the feedback circuit is extended to handle the feedforward through the feedback network
    - Often easier than two-port analysis

3 Negative Feedback

- Basic Idea
    - trade off gain for other desirable properties
- Desirable properties
    - desentisize gain
    - extend the bandwidth
    - control input and output impedance
    - reduce NL distorsion
- Drawbacks (undesired properties)
    - reduced gain
    - under certain circumstances the negative feedback in an amplifier can become positive and of such a magnitude as to cause oscillations (instability)

NOTE: it should not be implied that positive feedback always lead to instability

4 General negative feedback structure

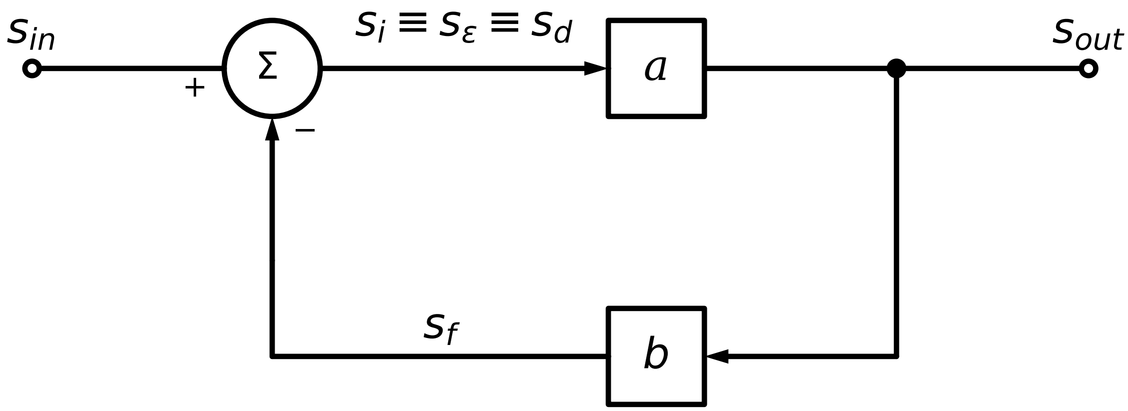


Figure 2: Block diagram of a negative feedback system.

Basic Terminology:

s_{in} = input signal applied to the feedback system

s_{out} = output signal from the feedback system

$a = \frac{s_{out}}{s_i}$ = open-loop gain = gain of "basic" amplifier = gain of feedforward amplifier

b = feedback factor

$a \cdot b = L$ = loop gain (it is always unit-less)

$1 + ab = 1 + L$ = amount of feedback

$s_i \equiv s_\epsilon \equiv s_d = s_{in} - s_f$ = input to "basic" amplifier = error = difference

$A = \frac{s_{out}}{s_{in}}$ = closed-loop gain = gain of feedback system

Ideal feedback assumptions

- the feedback network does not load the basic amplifier output
- the feedback network does not load the basic amplifier input
- the feedback network is unilateral
- the basic amplifier is unilateral

Closed-loop gain expression

The feedback network measures (or samples or senses) the output signal s_{out} and provides (returns) a feedback signal s_f (that is related to s_{out} by the **feedback factor** b)

$$
\begin{aligned}
s_{out} &= a \cdot s_i \\
s_f &= b \cdot s_{out} \\
s_\epsilon \equiv s_d \equiv s_i &= s_{in} - s_f
\end{aligned}
$$

$$
\begin{aligned}
A = \frac{s_{out}}{s_{in}} = \frac{s_{out}}{s_i + s_f} = \frac{1}{\dfrac{s_i}{s_{out}} + \dfrac{s_f}{s_{out}}} = \frac{1}{\dfrac{1}{a} + b} = \frac{a}{1+ab} = \\
= \frac{1}{b} \cdot \frac{1}{1 + 1/(ab)} = \frac{1}{b} \cdot \frac{1}{1 + \dfrac{1}{L}} = \frac{1}{b} \cdot \frac{L}{1+L} \approx \frac{1}{b}
\end{aligned}
$$

The most important benefit coming from the use of negative feedback is under the condition $L \gg 1$:

- for $L \gg 1$ the actual gain $A (\approx 1/b)$ is virtually independent of the open-loop gain a. The open-loop gain a is typically an extremely inaccurate parameter, and varies significantly subject to drift with temperature, supply voltage, fabrication process, and DC biasing conditions.

Although physically unattainable the limit $L \to \infty$ represent the ideal condition:

$$A_{ideal} = \lim_{L \to \infty} A = \frac{1}{b}$$

Key result: when the loop gain L is large, the closed loop gain A approaches the ideal closed loop gain A_{ideal}, which is equal to $1/b$

To achieve gain we need $b \leq 1$.

$b \leq 1$ is easy to implement. We can for example use a wire ($b = 1$), or a resistive divider (ratiometric) or a capacitive divider (ratiometric).

Example 1 - voltage buffer

$v_f = v_{out} \Rightarrow b = v_f / v_{out} = 1$

$v_{out} = a \cdot (v_{in} - v_{out})$

$v_{out}(1 + a) = a \cdot v_{in}$

$\dfrac{v_{out}}{v_{in}} = \dfrac{a}{1+a} \approx 1$

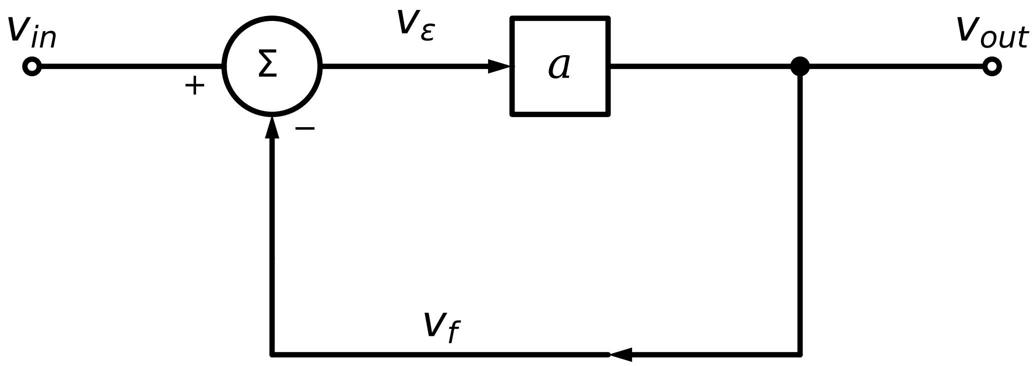


Figure 3: Block diagram of a voltage buffer.

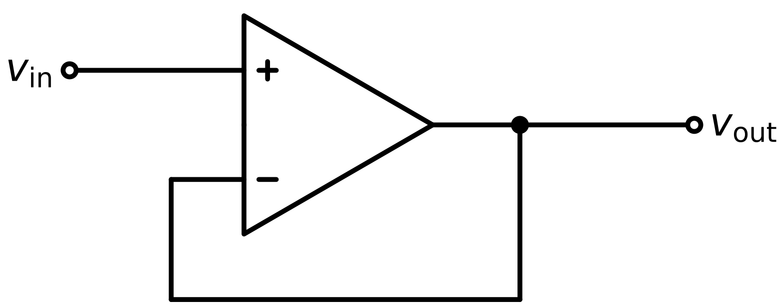


Figure 4: op amp based voltage buffer

Example 2 - non-inverting voltage amplifier

$b = \frac{v_f}{v_{out}} = \frac{R_1}{R_1 + R_2}$

$\frac{v_{out}}{v_{in}} \approx \frac{1}{b} = 1 + \frac{R_2}{R_1}$

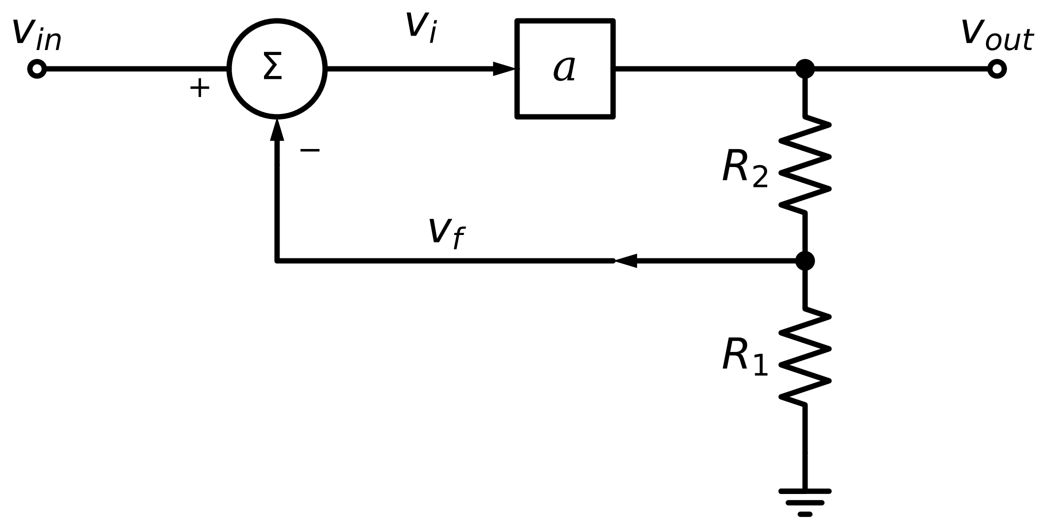


Figure 5: Block diagram of a non inverting voltage amplifier (series-shunt feedback)

If we regard $1/L$ as an error term, then L gives a measure of how close the actual gain A is to the ideal gain $A_{ideal} = 1/b$.

$$A = \frac{1}{b} \cdot \frac{1}{1 + \frac{1}{L}} \approx \frac{1}{b} \cdot \left(1 - \frac{1}{L}\right)$$

The % gain error between actual gain A and ideal gain A_{ideal} is usually defined as follows:

$$gainerror\% = 100 \times \frac{A - A_{ideal}}{A_{ideal}} = 100 \times \frac{A_{ideal} \cdot \left(\frac{1}{1 + 1/L}\right) - A_{ideal}}{A_{ideal}} =$$
$$= 100 \times \frac{-1}{1 + L} \approx 100 \times \frac{-1}{L}$$

Example 3 - op amp based non inverting voltage amplifier

KLC at node v_{out}:

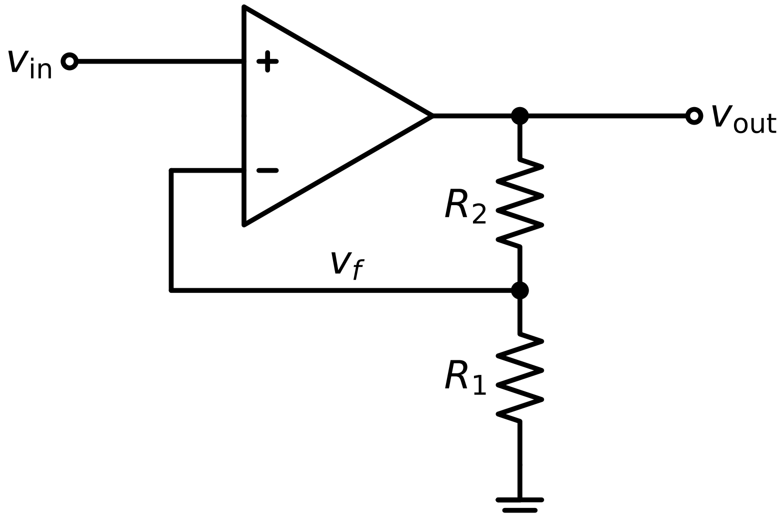


Figure 6: op amp based non inverting voltage amplifier

$$\frac{v_N - v_{out}}{R_2} = \frac{v_{out} - a_D \cdot (v_{in} - v_N)}{r_o}$$

KCL at node v_N:

$$\frac{v_N - v_{in}}{r_i} + \frac{v_N}{R_1} + \frac{v_N - v_{out}}{R_2} = 0$$

For $a_D = 10^6$, $r_i = 10M\Omega$, $r_o = 10\Omega$, $R_1 = 20k\Omega$ and $R_2 = 80k\Omega$:

$$A_{ideal} = \frac{R_1 + R_2}{R_1}$$

$$A = \frac{v_{out}}{v_{in}} = \frac{a_D \cdot r_i(R_1 + R_2) + R_1 r_o}{a_D \cdot R_1 r_i + r_o(R_1 + r_i) + R_1 R_2 + r_i(R_1 + R_2)} = 4.999975$$

$$gainerror = \frac{4.999975 - 5}{5} = -5 \cdot 10^{-6} = -5ppm$$

The exact analysis is very tedious. A better way to obtain the same insight is to use the simpler approach shown in example 4.

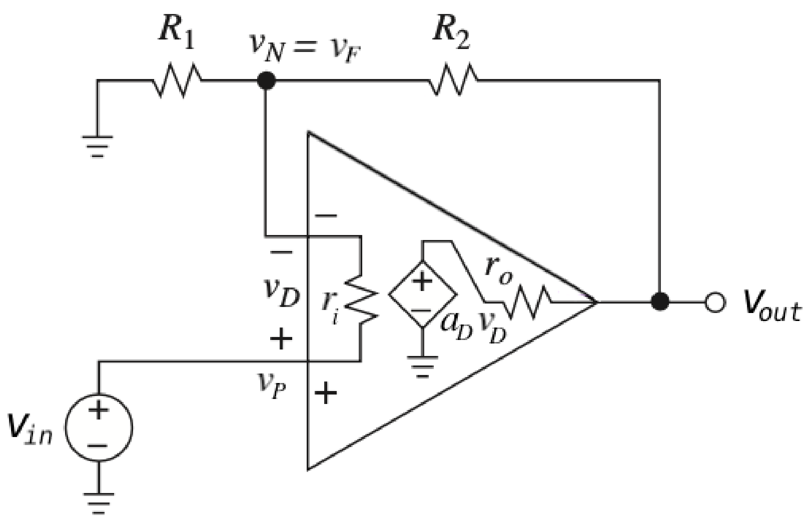


Figure 7: Non inverting voltage amplifier with non ideal op amp (Franco 2015)

5 The error signal s_ϵ and the feedback signal s_f

From Figure 2

$$\begin{aligned} s_\epsilon &= \frac{s_{out}}{a} = \frac{A \cdot s_{in}}{a} = \frac{\cancel{a}}{1+L} \cdot \frac{s_{in}}{\cancel{a}} \\ &= \frac{s_{in}}{1+L} \end{aligned}$$

also

$$\begin{aligned} s_f &= b \cdot s_{out} = b \cdot A \cdot s_{in} = \cancel{b} \cdot \frac{1}{\cancel{b}} \frac{1}{1+1/L} \cdot s_{in} \\ &= \frac{s_{in}}{1+1/L} \end{aligned}$$

These results show that for a large loop gain (ideally for $L \to \infty$), the error signal becomes very small (ideally $s_\epsilon \to 0$), causing the feedback signal s_f to closely follow the input signal s_{in} ($s_f \to s_{in}$).

If the feedback signal closely tracks the input signal, then the output signal is also a close "replica" of the input signal, and therefore feedback is also effective in reducing distorsion due to small signals. Low frequency distorsion is caused by changes in the slope of the basic-amplifier transfer characteristic (i.e. changes in the small signal gain of the basic-amplifier).

6 Benefits of negative feedback

6.1 Gain Desensitivity

To investigate the effect that a variation on the open-loop gain a causes on the closed-loop gain A we differentiate A w.r.t. a

$$A = \frac{a}{1+ab}$$

$$\frac{dA}{da} = \frac{1+ab-ab}{(1+ab)^2} = \frac{1}{(1+ab)^2}$$

$$\Delta A = \frac{\Delta a}{(1+ab)^2}$$

$$\frac{\Delta A}{A} = \frac{\Delta a}{A} \cdot \frac{1}{(1+ab)^2} = \frac{\Delta a}{\frac{a}{1+ab}} \cdot \frac{1}{(1+ab)^2} = \frac{\Delta a/a}{(1+ab)}$$

Thanks to feedback, a fractional change in the gain of the basic amplifier is reduced by a factor $1 + L = 1 + ab$ in the closed-loop. Conceptually, the loop gain L can be be made arbitrarily large. The loop gain is a key parameter and as we will see later it plays an important role also in bandwidth and impedance calculations.

6.2 Effect of feedback on non-linearity

Amplifiers are made up of transistors. Since, transistors are non-linear devices, the transfer charateristic of any practical amplifier is non-linear. Assume a basic amplifier that besides the desired linear relationship, also exhibits a quadratic and cubic relationship between its input and output.

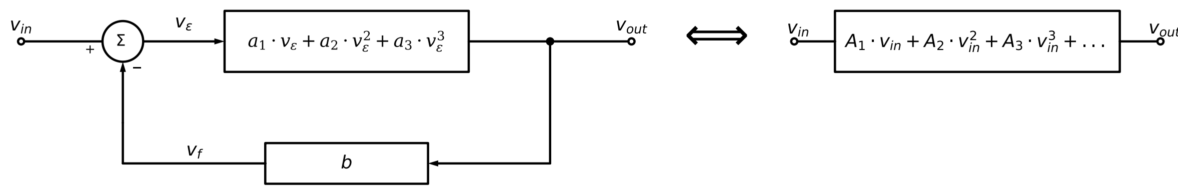


Figure 8: Effect of negative feedback on non-linearity

$$v_{out} = a_1(v_{in} - bv_{out}) + a_2(v_{in} - bv_{out})^2 + a_3(v_{in} - bv_{out})^3 \tag{1}$$

$$v_{out} = A_1 v_{in} + A_2 v_{in}^2 + A_3 v_{in}^3 + ... \tag{2}$$

Substituting Equation 2 into Equation 1 we get:

$$\begin{aligned} v_{out} = & a_1(v_{in} - bA_1 v_{in} - bA_2 v_{in}^2 - bA_3 v_{in}^3 - ...) + \\ & a_2(v_{in} - bA_1 v_{in} - bA_2 v_{in}^2 - bA_3 v_{in}^3 - ...)^2 + \\ & a_3(v_{in} - bA_1 v_{in} - bA_2 v_{in}^2 - bA_3 v_{in}^3 - ...)^3 \end{aligned}$$

and equating back into Equation 2:

for the linear term

$$\begin{aligned} & A_1 v_{in} = a_1 v_{in} - a_1 b A_1 v_{in} \Leftrightarrow A_1(1 + a_1 b) = a_1 \\ & A_1 = \frac{a_1}{1 + a_1 b} \end{aligned} \tag{3}$$

or:

$$\begin{aligned} & A_1 = a_1 - a_1 b A_1 \Leftrightarrow A_1 = a_1(1 - bA_1) \Leftrightarrow \\ & 1 - bA_1 = \frac{A_1}{a_1} = \frac{1}{1 + a_1 b} \end{aligned} \tag{4}$$

for the quadratic term

$$\begin{aligned} & A_2 v_{in}^2 = -a_1 b A_2 v_{in}^2 + a_2 \left(v_{in}^2 + b^2 A_1^2 v_{in}^2 - 2bA_1 v_{in}^2\right) \Leftrightarrow \\ & A_2(1 + a_1 b) = a_2(1 - bA_1)^2 \Leftrightarrow A_2 = a_2 \frac{(1 - bA_1)^2}{1 + a_1 b} \Leftrightarrow \\ & A_2 = \frac{a_2}{(1 + a_1 b)^3} \end{aligned} \tag{5}$$

for the cubic term

$$\begin{aligned} & A_3 \cdot v_{in}^3 = -a_1 b A_3 v_{in}^3 + a_3(1 - bA_1)^3 v_{in}^3 - 2a_2 b A_2 (1 - bA_1) v_{in}^3 \Leftrightarrow \\ & A_3(1 + a_1 b) = a_3(1 - bA_1)^3 - 2a_2 b A_2 (1 - bA_1) \Leftrightarrow \\ & A_3 = \frac{a_3(1 - bA_1)^3 - 2a_2 b A_2 (1 - bA_1)}{(1 + a_1 b)} \Leftrightarrow \\ & A_3 = \frac{a_3}{(1 + a_1 b)^4} - \frac{2a_2^2 b}{(1 + a_1 b)^5} \end{aligned} \tag{6}$$

The linear term a_1 (as expected) is reduced by $1 + L$, where $L \equiv a_1 b$

The quadratic term a_2 is reduced by $(1 + L)^3$

The cubic term a_3 is reduced by $(1+L)^4$, but ther is also an extra term due to the interaction with the second-order term a_2

If at different signal levels, the basic amplifier's small-signal gain varies, since feedback reduces the overall gain variations w.r.t. the relative variations of the basic amplifier gain, then feedback also reduces distorsion.

Note:

- feedback reduces distorsion without reducing the output voltage range. The overall gain is reduced, but additional gain can be provided with a preamplifier that operates with smaller signal swings, and therefore less distorsion.
- feedback does not help with saturation. In the case an amplifier's output shows hard saturation (i.e., the output become independent of the input), the incremental gain of the basic amplifier $\to 0$, so the amount of feedback also $\to 0$, and as a result negative feedback cannot improve the situation.

6.3 Effect of feedback on bandwidth

As an example, assume the transfer function of the basic amplifier is the following:

$$a(j\omega) = \frac{a_0}{1 + \dfrac{j\omega}{\omega_p}}$$

Then, the closed-loop transfer function is

$$A(j\omega) = \frac{a(j\omega)}{1 + a(j\omega) \cdot b} = \frac{a_0}{1 + a_0 b} \cdot \frac{1}{1 + \dfrac{j\omega}{\omega_p} \cdot \dfrac{1}{(1 + a_0 b)}}$$

The gain is reduced by $1 + L_0$, but the bandwidth is increased by $1 + L_0$, where $L_0 = a_0 b$.

The product of gain and bandwidth remains constant.

6.4 Effect of feedback on input/output resistances

In actual applications the input and output resistances (a.k.a. terminal resistances) of the amplifier play a key role. When an amplifier is driven by a non ideal source and drives an output load, the input resistance forms a divider with the source's resistance, and the output resistance forms a divider with the load, therefore reducing the overall gain from source to load. This reduction is usually referred as loading effect. Negative feedback modifies the terminal resistances in ways that tend to reduce the impact of loading.

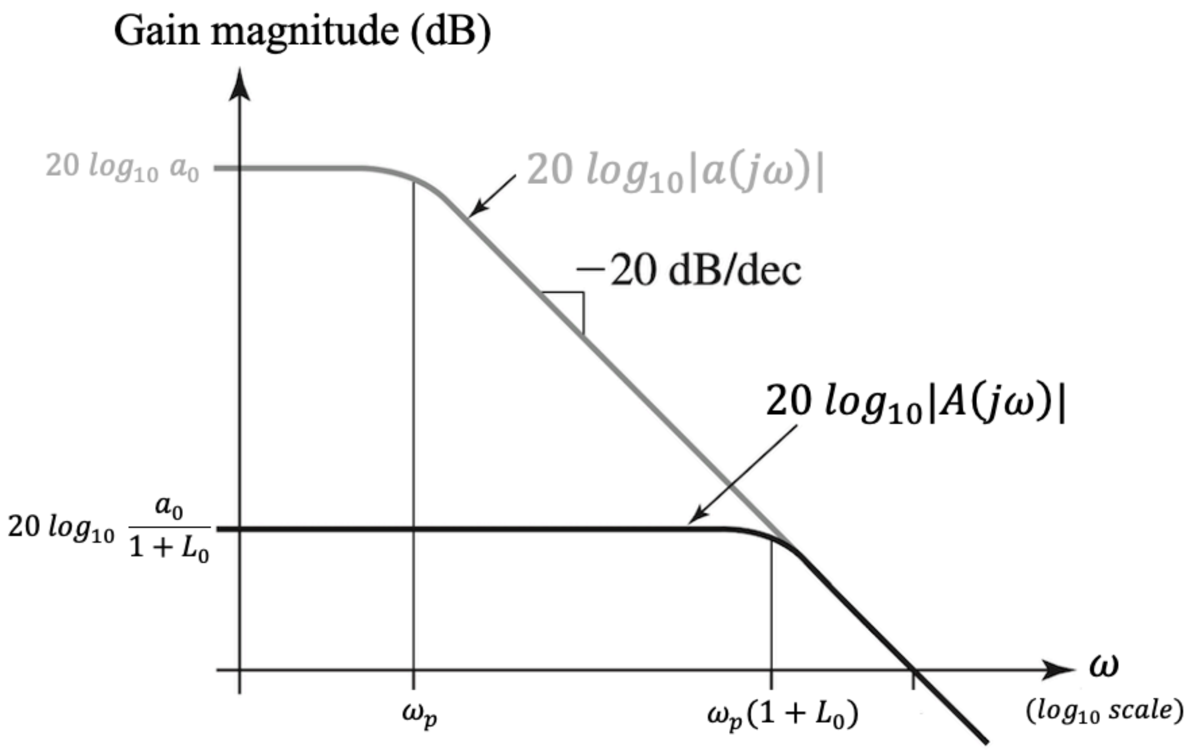


Figure 9: Gain magnitude vs. frequency for the basic amplifier and the feedback amplifier

6.4.1 Series-shunt (voltage-voltage) feedback configuration - Voltage Amplifier

Let's consider the application of negative feedback around a voltage amplifier. In this case feedback is performed sampling the output voltage and then feeding back a voltage that is a scaled version of the output voltage "into" the external input voltage source driving the amplifier. Note that the operation of voltage sensing at the output is performed in parallel, or shunt (when we measure a voltage we place the voltmeter in parallel, never in series), while to combine the external input voltage source and the voltage returned by the feedback network we connect them in series (to connect two voltage sources we place them in series, never in parallel).

To focus on the effect of negative feedback on r_i and r_o of the basic amplifier, we assume there is no loading effect on both ports of the basic amplifier. To neglect loading at the input's of the basic amplifier, we assume that the sources v_{in} and bv_{out} have *zero* series resistances. To neglect loading at the output port of the basic amplifier, we assume the output port of the basic amplifier is left open and the input port of the feedback network has *infinite* resistance.

Note: for the series-shunt feeback configuration, b is unitless.

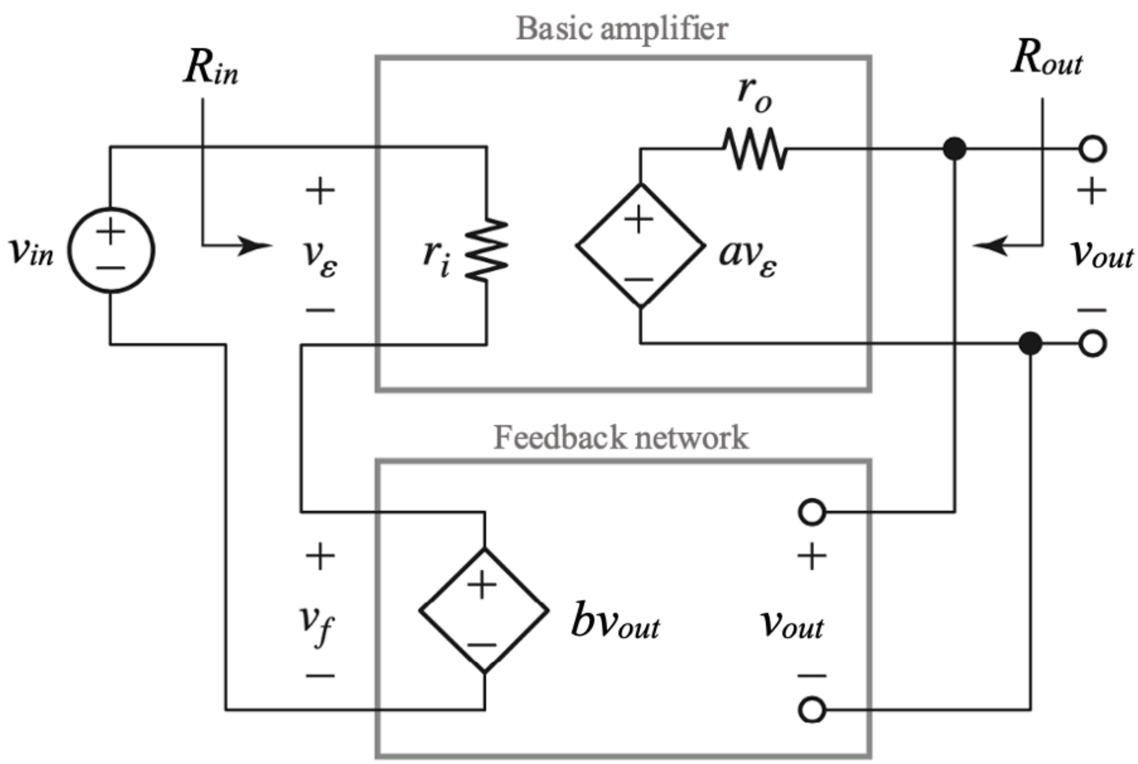


Figure 10: The ideal series-shunt configuration or voltage amplifier (Franco 2015)

It can be shown:

$$R_{in} = \frac{v_x}{i_x} = r_i(1+L)$$

$$R_{out} = \frac{v_t}{i_t} = \frac{r_o}{1+L}$$

For $L \to \infty$ the series-shunt configuration gives $R_{in} \to \infty$ and $R_{out} \to 0$

6.4.2 Shunt-shunt (current-voltage) feedback configuration - Transimpedance Amplifier

Let's consider the application of negative feedback around a transimpedance amplifier. In this case feedback is performed sampling the output voltage and then feeding back a current that is a scaled version of the output voltage "into" the external input current source driving the amplifier. Note that to combine the external input current source and the current returned by the feedback network we connect them in parallel or shunt (to connect two current sources we place them in parallel, never in series).

To neglect loading at the input of the basic amplifier, we assume that the sources i_{in} and bv_{out} have *infinite* parallel resistances. To neglect loading at the output port of the basic amplifier, we assume the output port of the basic amplifier is left open and the input port of the feedback network has *infinite* resistance.

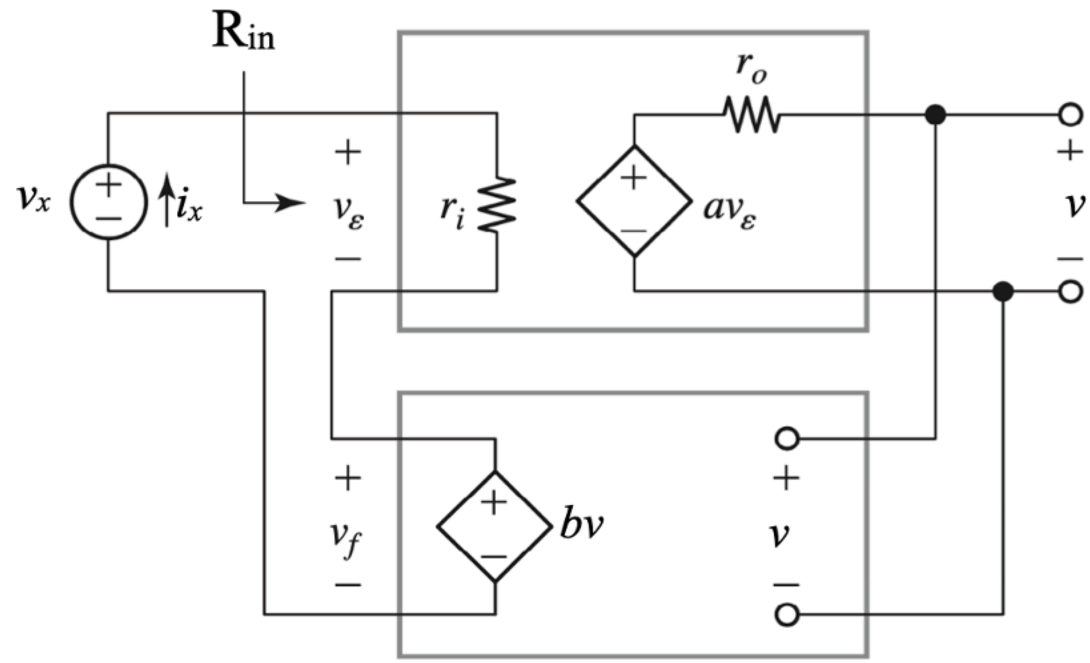
Note: for the shunt-shunt feeback configuration, b has dimensions of $1/\Omega$.

It can be shown:

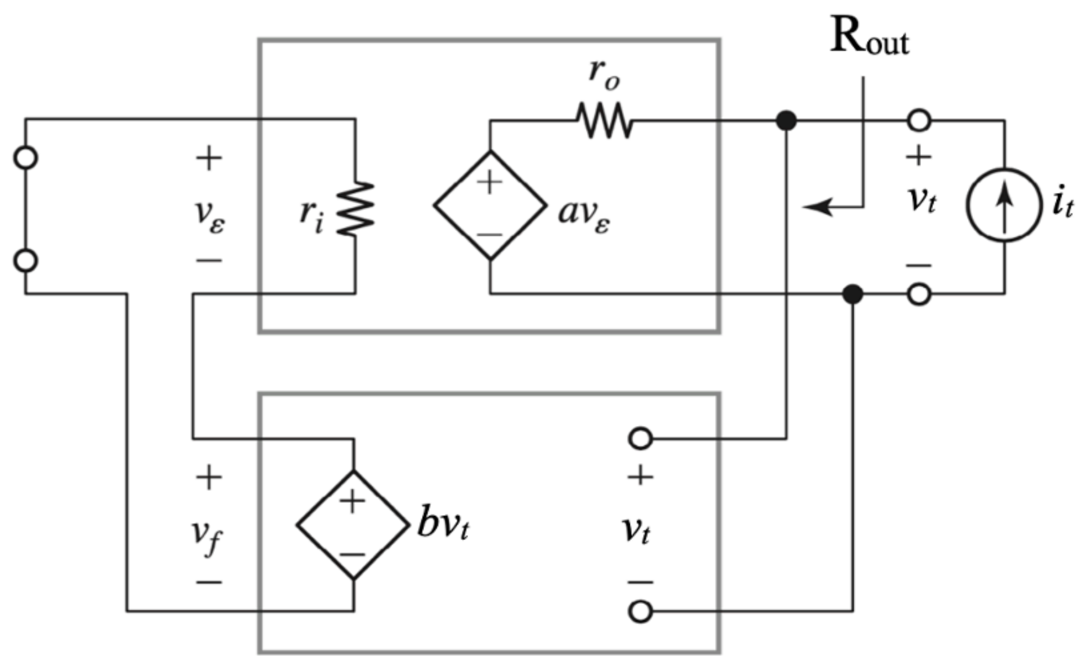
$$R_{in} = \frac{r_i}{1+L}$$

$$R_{out} = \frac{r_o}{1+L}$$

For $L \to \infty$ the shunt-shunt configuration gives $R_{in} \to 0$ and $R_{out} \to 0$



(a)



(b)

Figure 11: (a) test circuit for the calculation of the input impedance and (b) test circuit for the calculation of the output impedance

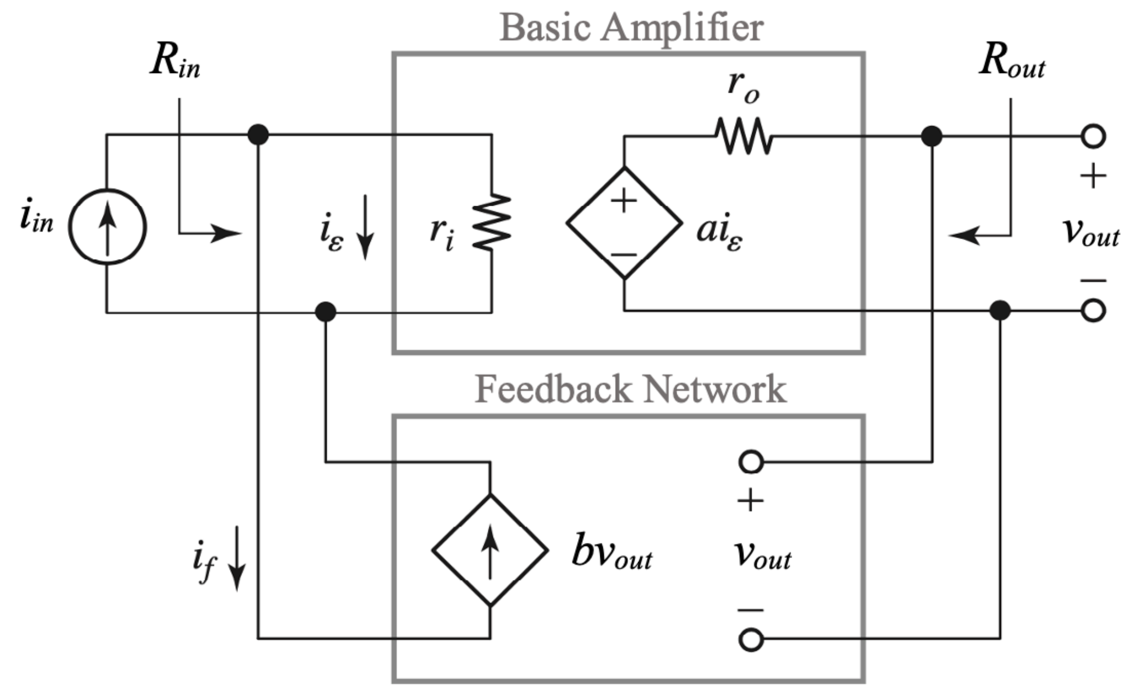


Figure 12: The ideal shunt-shunt configuration or transimpedance amplifier (Franco 2015)

6.4.3 Series-series (voltage-current) feedback configuration - Transconductance Amplifier

Let's consider the application of negative feedback around a transconductance amplifier. In this case feedback is performed sampling the output current and then feeding back a voltage that is a scaled version of the output current "into" the external input voltage source driving the amplifier. Note that the operation of current sensing at the output is performed in series, or shunt (when we measure a current we place the ampmeter in series, never in parallel).

To neglect loading at the input of the basic amplifier, we assume that the sources v_{in} and bi_{out} have *zero* series resistances. To neglect loading at the output port of the basic amplifier, we assume the output port of the basic amplifier is shorted and the input port of the feedback network has *zero* resistance.

Note: for the series-series feedback configuration, b has dimensions of Ω.

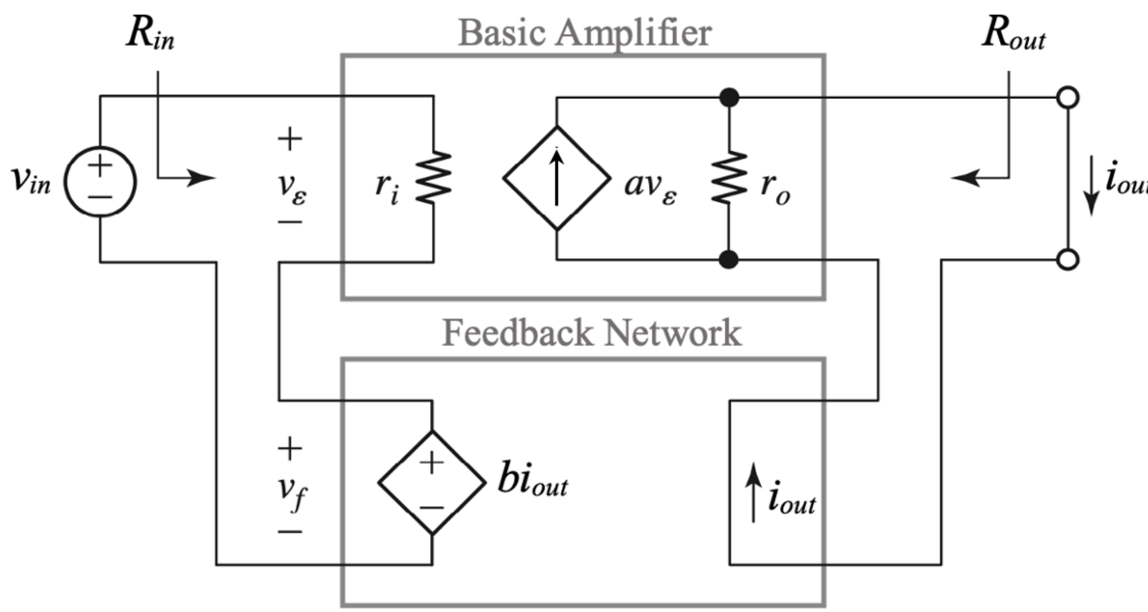


Figure 13: The ideal series-series configuration or transconductance amplifier (Franco 2015)

It can be shown:

$$R_{in} = r_i(1 + L)$$

$$R_{out} = r_o(1 + L)$$

For $L \to \infty$ the series-series configuration gives $R_{in} \to \infty$ and $R_{out} \to \infty$

6.4.4 Shunt-series (current-current) feedback configuration - Current Amplifier

Let's consider the application of negative feedback around a current amplifier. In this case feedback is performed sampling the output current and then feeding back a current that is a scaled version of the output current "into" the external input current source driving the amplifier.

To neglect loading at the input of the basic amplifier, we assume that the sources i_{in} and bi_{out} have *infinite* parallel resistances. To neglect loading at the output port of the basic amplifier, we assume the output port of the basic amplifier is shorted and the input port of the feedback network has *zero* resistance.

Note: for the shunt-series feedback configuration, b is unitless.

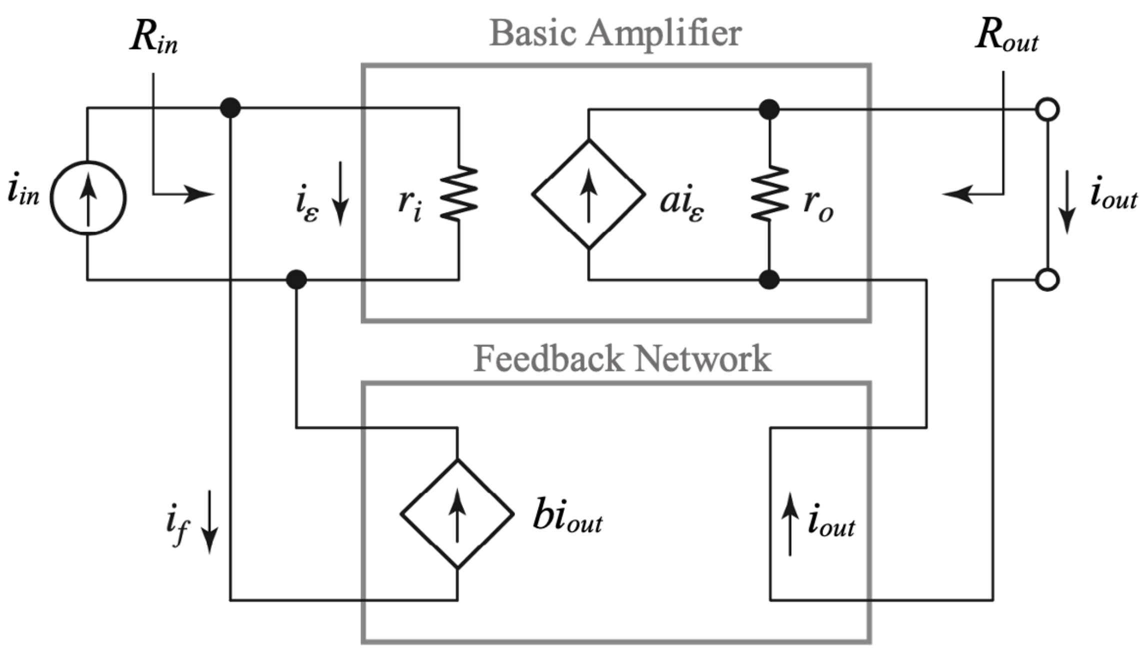


Figure 14: The shunt-series configuration or current amplifier (Franco 2015)

It can be shown:

$$R_{in} = \frac{r_i}{1+L}$$

$$R_{out} = r_o(1 + L)$$

For $L \to \infty$ the shunt-series configuration gives $R_{in} \to 0$ and $R_{out} \to \infty$

7 Feedback analysis of op amp circuits

In typical op amp circuits r_i and r_o are negligible compared to the resistances used in the feedback network.

7.1 Non Inverting configuration

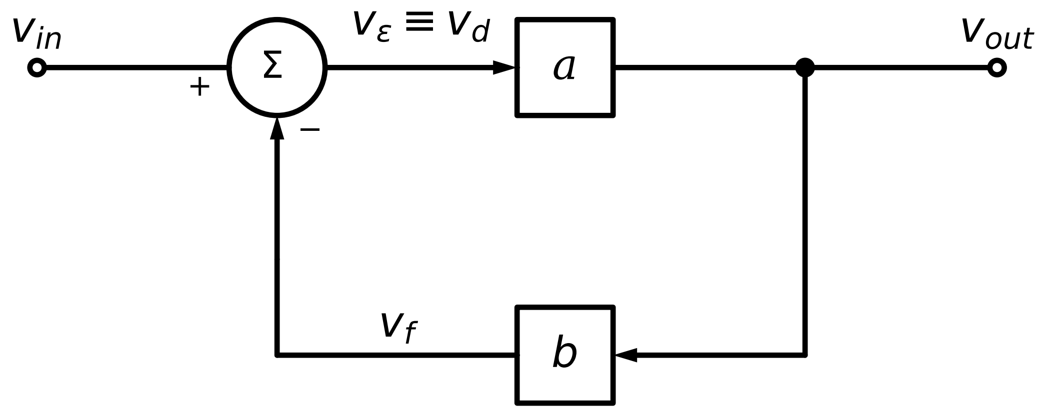


Figure 15: Block diagram of negative feedback system.

For identifying a and b directly from the circuit, we can set $v_{in} = 0$ and note that with $v_{in} = 0$

$i_{out} = \dfrac{v_{out}}{R_2 + R_1||r_i} = \dfrac{a_D v_D}{r_o + R_2 + R_1||r_i}$

$v_f = -v_D = i_{out} \cdot (R_1||r_i)$

$v_{out} = i_{out} \cdot \big(R_2 + R_1||r_i\big)$

So,

$b = \dfrac{v_f}{v_{out}} = \dfrac{R_1||\cancel{r_i}}{R_1||\cancel{r_i} + R_2} \approx \dfrac{R_1}{R_1 + R_2}$

$a = \dfrac{v_{out}}{v_D} = a_D \cdot \dfrac{R_2 + R_1||\cancel{r_i}}{R_2 + R_1||\cancel{r_i} + \cancel{r_o}} \approx a_D$

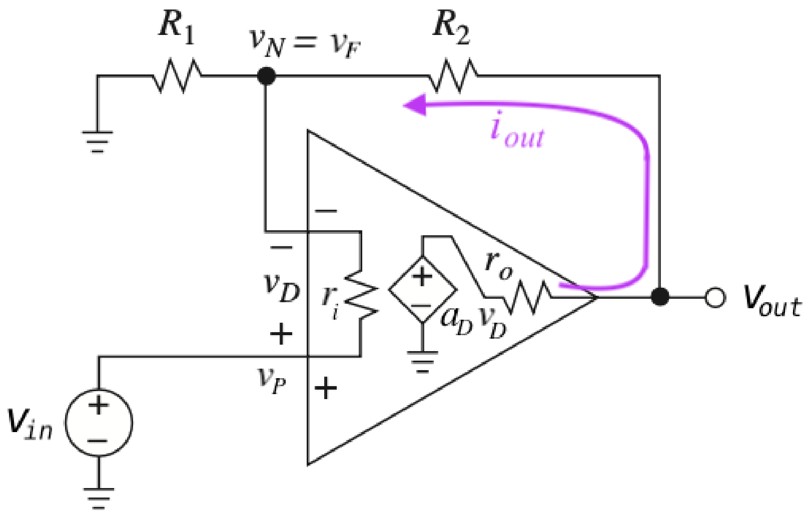


Figure 16: Non inverting op amp amplifier (Franco 2015).

In practice, for identifying a and b, it is better to think directly in terms of loop gain L

$$L = ab = -\frac{v_r}{v_t}$$

The minus sign is due to the $-$ at the summing node

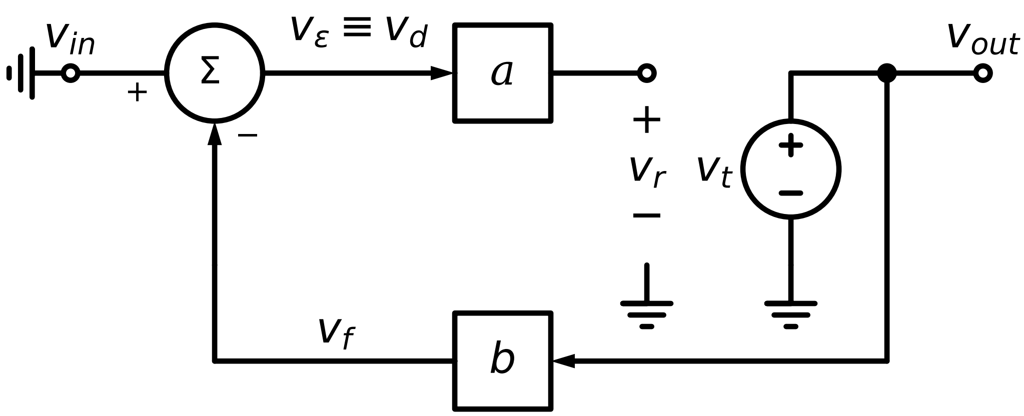


Figure 17: Breaking the Loop.

Note: To find the loop gain, it is best to break the loop at the opamp's voltage controlled voltage source. This approach preserves all of the node impedances in the circuit.

$$L = -\frac{v_r}{v_t} = a_D \cdot \frac{R_1 || \cancel{r_i}}{R_1 || \cancel{r_i} + R_2 + \cancel{r_o}}$$

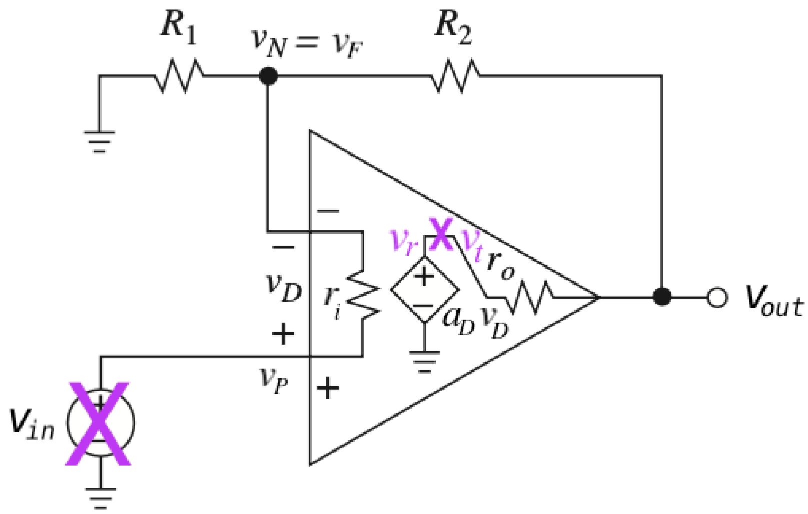


Figure 18: Non inverting op amp amplifier with v_{in} nulled and the loop broken at the opamp's vcvs.

$A = A_{ideal}\frac{L}{1+L} \approx A_{ideal}\Big(1 - \frac{1}{L}\Big)$

$gainerror \approx \frac{A_{ideal}\Big(1-\frac{1}{L}\Big) - A_{ideal}}{A_{ideal}} = -\frac{1}{L}$

The value of A_{ideal} is already known from the ideal op amp analysis

Infinite opamp gain ($a_D \to \infty$) implies infinite open-loop gain ($a \approx a_D$) and therefore infinite loop gain $L = ab$

$$A_{ideal} = \frac{1}{b} \approx 1 + \frac{R_2}{R_1}$$

Example 4 - non inverting op amp based voltage amplifier

For $a_D = 10^6$, $r_i = 10M\Omega$, $r_o = 10\Omega$, $R_1 = 20k\Omega$ and $R_2 = 80k\Omega$:

$L = -\frac{v_r}{v_t} = a_D \cdot \frac{R_1||\cancel{r_i}}{R_1||\cancel{r_i} + R_2 + \cancel{r_o}} \approx 10^6\frac{20k\Omega}{20k\Omega + 80k\Omega}$

The exact value of L is 199600, so the above approximation has 0.2% error.

$A = 5 \cdot \frac{200000}{1+200000} = 4.999975$

$gainerror \approx -\frac{1}{200000} = -5ppm$

This is the same result as before, except we did not have to go through a tedious nodal analysis and plug in numbers in a "high entropy" espression.

What if the op amp gain changes ?

If the gain is cut in half :

$$A = 5 \cdot \frac{100000}{1 + 100000} = 4.999950$$

If the gain doubles:

$$A = 5 \cdot \frac{400000}{1 + 400000} = 4.999988$$

- The closed loop gain (A) is immune to large variations in the op amp gain
- The voltage gain of the overall circuit (= closed loop gain) is primarily defined by the divider ratio of the resistive feedback
  - A quantity that we can control very precisely

7.2 Inverting configuration

In this configuration the resistors affect both the input and the feedback path.

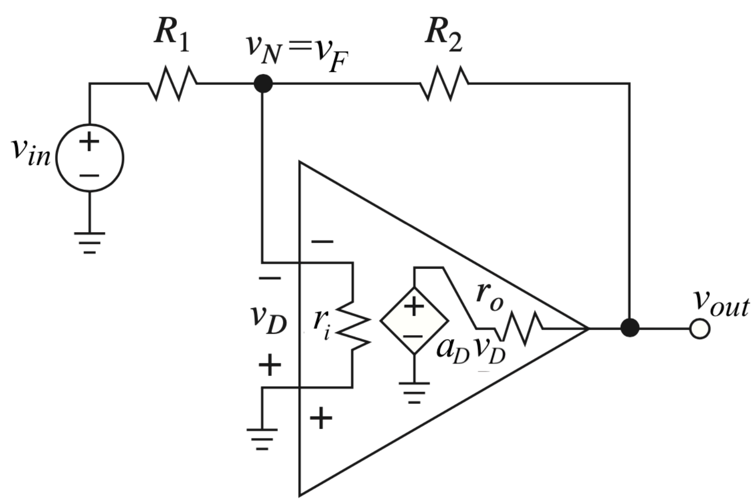


Figure 19: Inverting op amp amplifier. (Franco 2015})

It is not clear how to map the circuit into the block diagram representation. The flow of the signal through the circuit elements is not unidirectional.

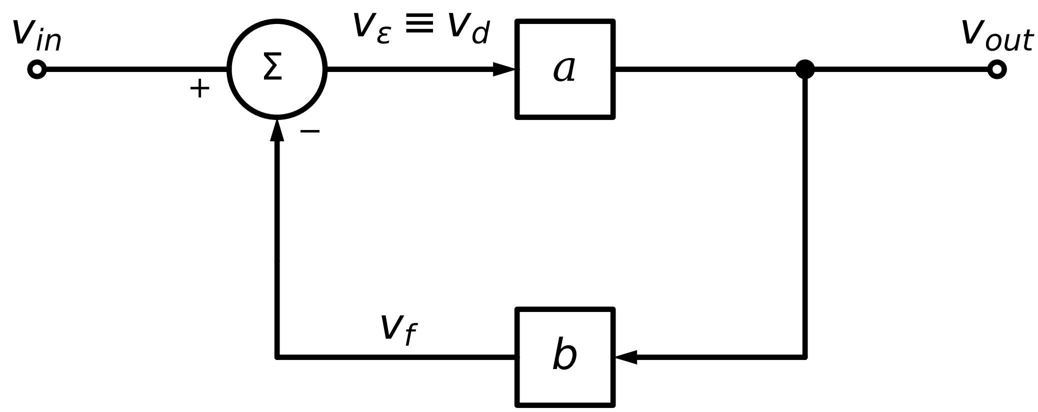


Figure 20: Block diagram of negative feedback system.

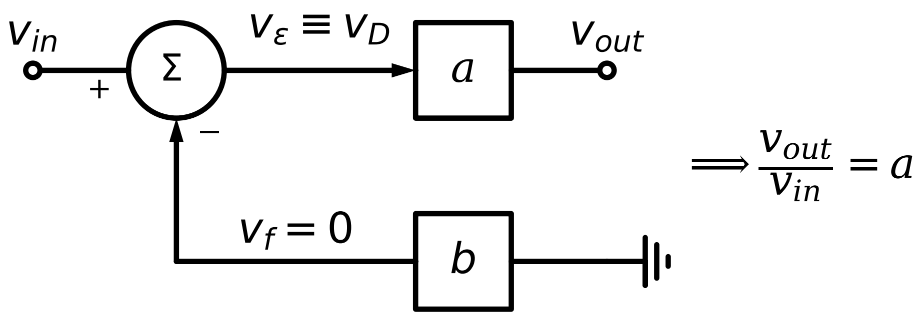


Figure 21: Breaking the loop ($v_f = 0$)

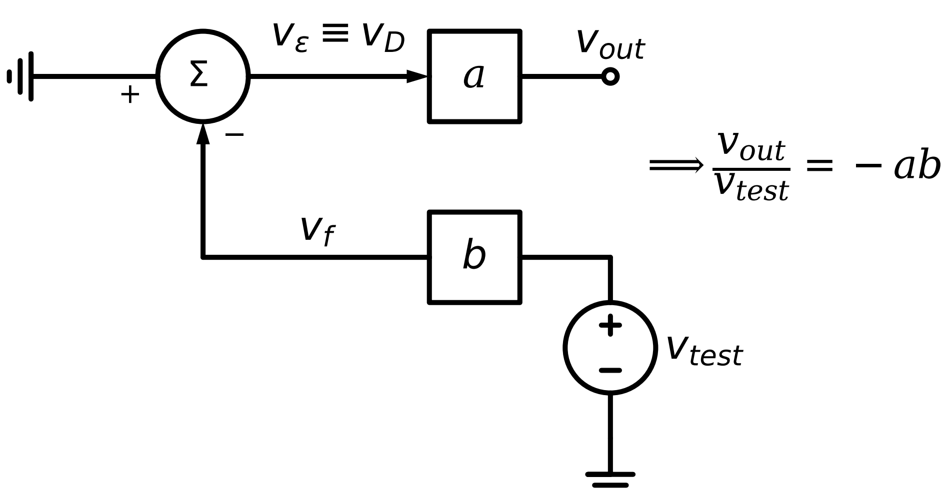


Figure 22: Breaking the loop ($v_{in} = 0$)

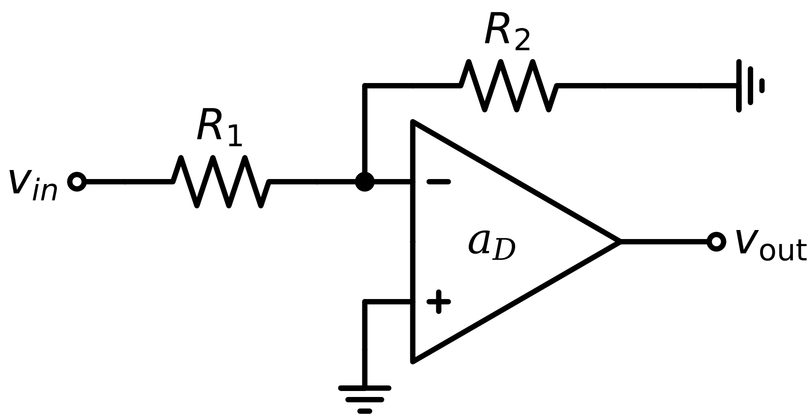


Figure 23: op amp based inverting amplifier with break in the loop ($v_f = 0$)

We can still try to make things work using superposition.

$$a = \frac{v_{out}}{v_{in}} = -\frac{R_2}{R_1 + R_2} \cdot a_D$$

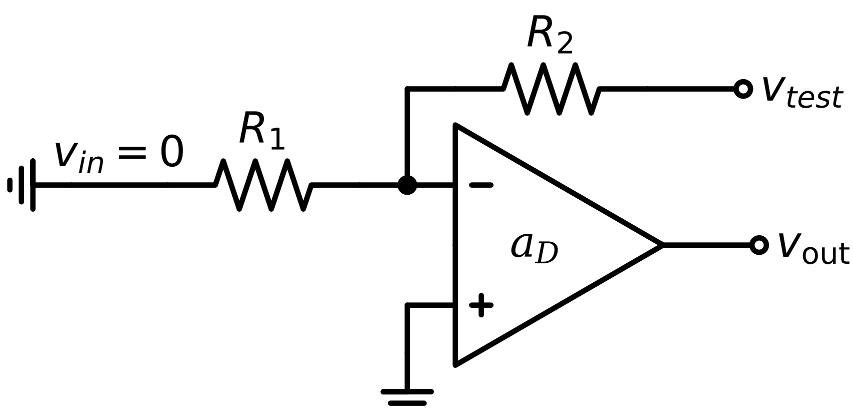


Figure 24: op amp based inverting amplifier with break in the loop ($v_{in} = 0$)

$$-ab = \frac{v_{out}}{v_{test}} = -\frac{R_1}{R_1 + R_2} \cdot a_D$$

Note: the loop gain is the same as the one of the non-inverting configuration.

$$A_{ideal} = \frac{1}{b} = \frac{a}{ab} = -\frac{R_2}{R_1}$$

Beyond A_{ideal} the only other value we need to know is the loop gain $L = ab$ so we can compute the deviation from ideality

7.3 Comparison between non-inverting and inverting topology

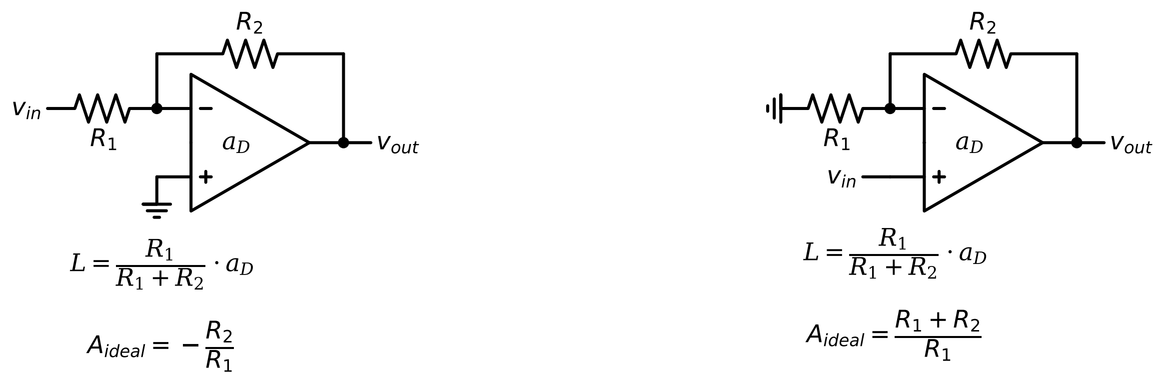


Figure 25: comparison between inverting and non-inverting topology

The following model is valid for both topologies:

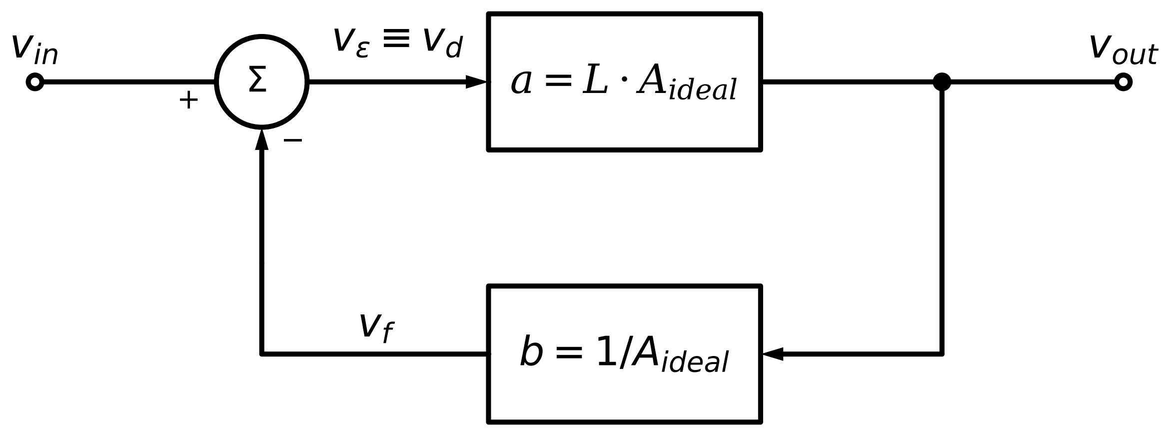


Figure 26: Block diagram model valid for both topologies.

In summary op amp circuits can be analyzed as follows:

- Find A_{ideal} using nodal analysis, assuming infinite op amp gain
- Find the loop gain to compute the deviation from the ideal case
  - This is usually straight forward, especially when there are ideal breakpoints that do not alter the impedance loading around the loop
  - The best breakpoint for a voltage amplifier is right at the controlled voltage source

$$A = \frac{v_{out}}{v_{in}} = A_{ideal} \cdot \frac{L}{L+1} = A_{ideal} \cdot \frac{1}{1 + \frac{1}{L}}$$

7.4 The four feedback configurations using op amps

Sometimes the operations of output sensing and input returning are not obvious. Even though strictly speaking the op amp is a voltage amplifier, it can be used in any of the four feedback configurations.

7.4.1 Series-Shunt (voltage-voltage) configuration (Voltage Amplifier)

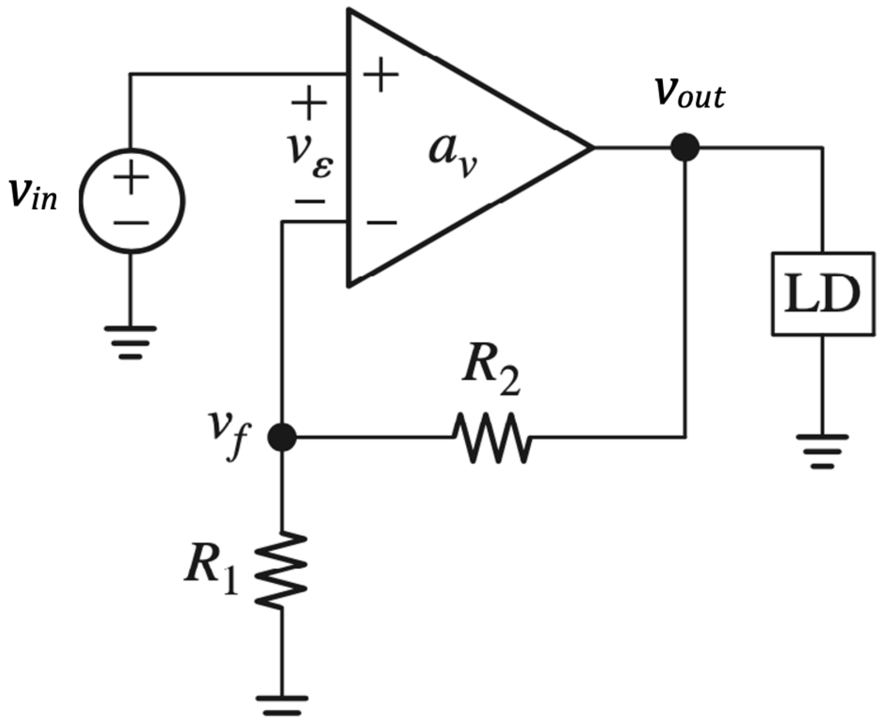


Figure 27: Series-shunt configuration: $b = \frac{v_f}{v_{out}} = \frac{R_1}{R_1 + R_2}$

For $a_v \to \infty$ this circuit gives:

$A_v = \frac{v_{out}}{v_{in}} = \frac{1}{b} = 1 + \frac{R_2}{R_1}$

$R_{in} \to \infty$

$R_{out} \to 0$

This circuit is the popular non inverting op amp based voltage amplifier.

7.4.2 Shunt-Shunt (current-voltage) configuration (Transimpedance Amplifier)

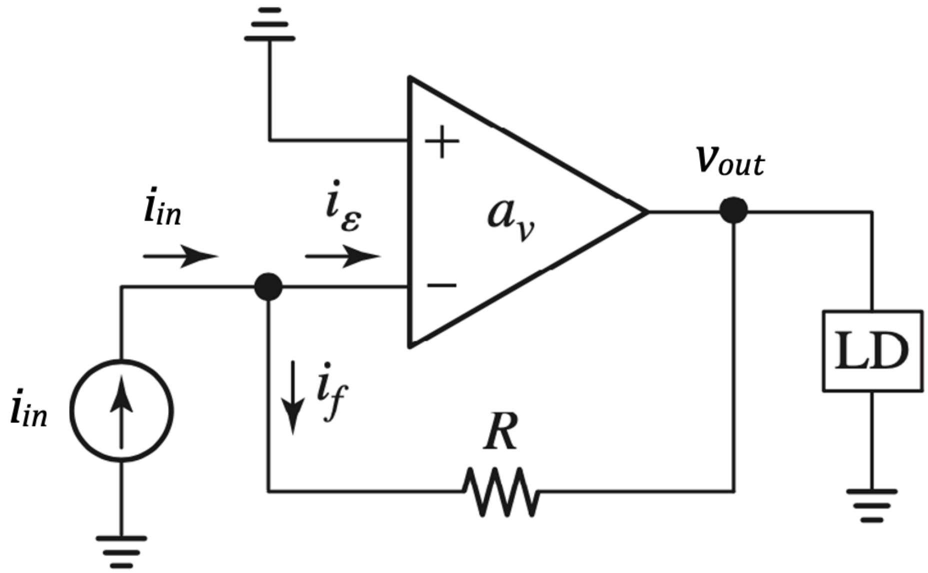


Figure 28: Shunt-shunt configuration: $b = \dfrac{i_f}{v_{out}} = -\dfrac{1}{R}$

For $a_v \to \infty$ this circuit (= TIA) gives:

$R_m = \dfrac{v_{out}}{i_{in}} = \dfrac{1}{b} = -R$

$R_{in} \to 0$

$R_{out} \to 0$

Even though the shunt-shunt configuration is a TIA, it forms the basis of the popular inverting op amp based voltage amplifier.

This becomes more evident if we perform a source transformation to convert the voltage source v_{in} in Figure 29 into a current source $i_{in} = \dfrac{v_{in}}{R_1}$ as shown in Figure 30.

For the inverting voltage amplifier as $a_v \to \infty$

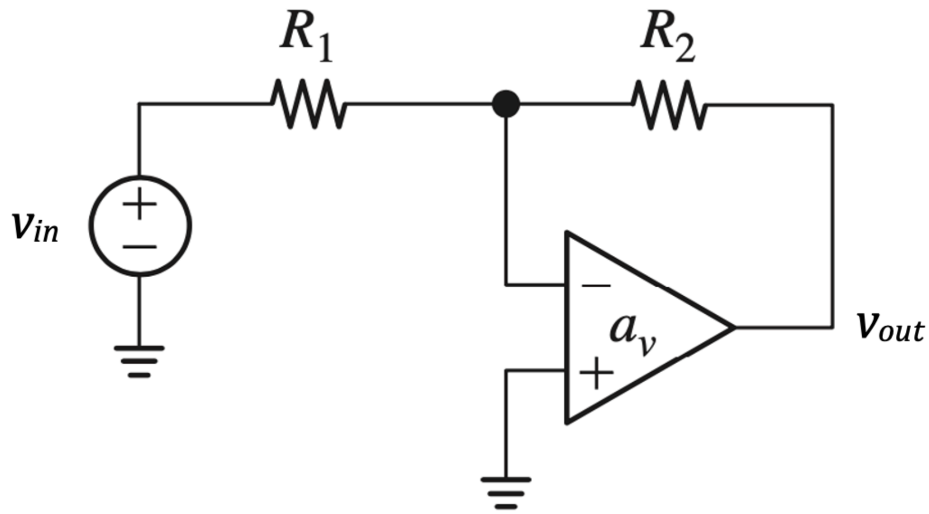


Figure 29: inverting op amp based voltage amplifier

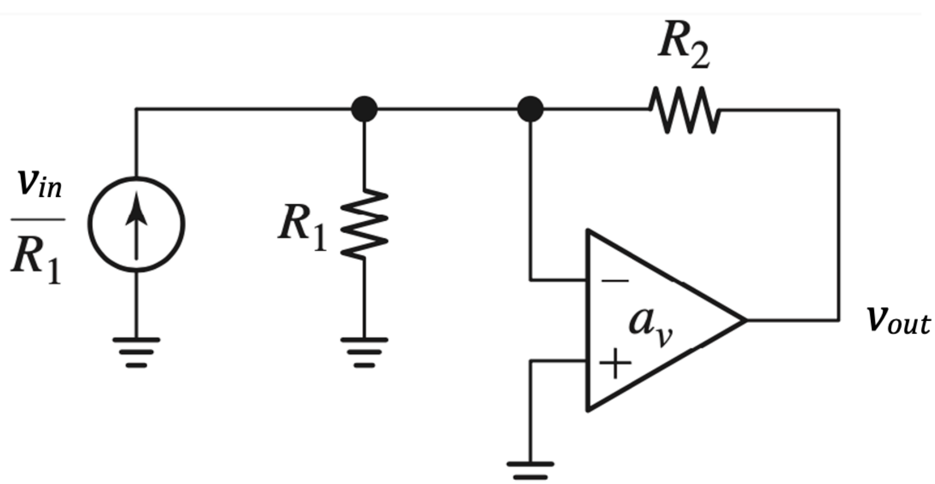


Figure 30: inverting op amp based voltage amplifier with the input voltage source transformed into a current source

$A_v = \frac{v_{out}}{v_{in}} = \frac{v_{out}}{i_{in}} \times \frac{i_{in}}{v_{in}} = -\frac{R_2}{R_1}$

$R_{in} = R_1$

$R_{out} \to 0$

7.4.3 Series-Series (voltage-current) configuration (Transconductance Amplifier)

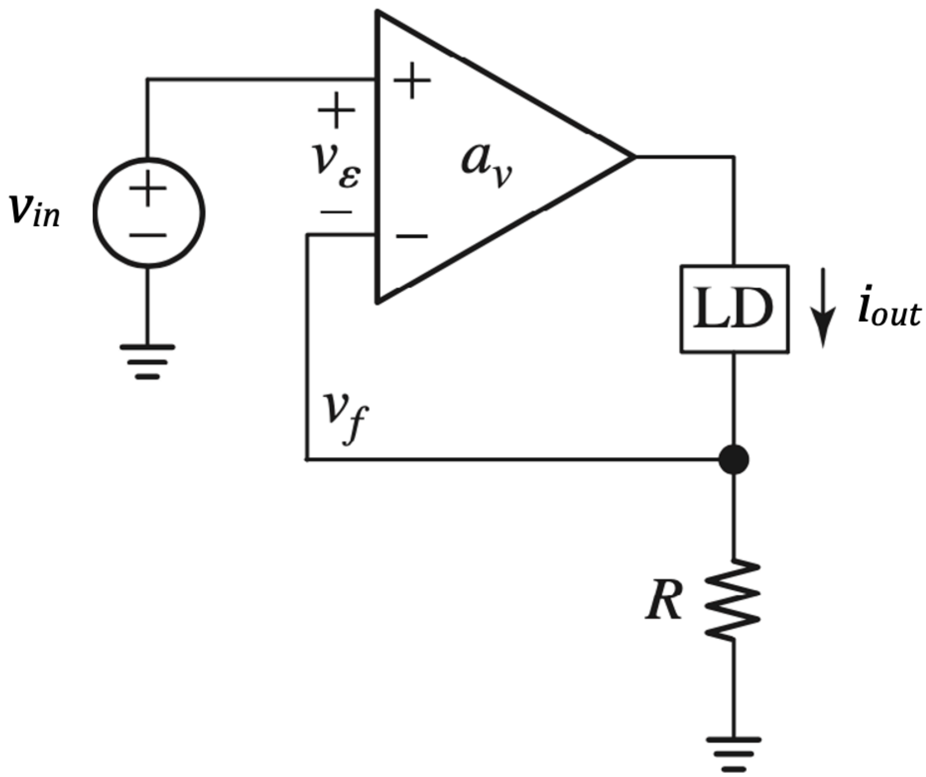


Figure 31: series-series configuration: $b = \frac{v_f}{i_{out}} = R$

For $a_v \to \infty$

$G_m = \frac{i_{out}}{v_{in}} = \frac{1}{b} = \frac{1}{R}$

$R_{in} \to \infty$

$R_{out} \to \infty$

7.4.4 Shunt-Series (current-current) configuration (Current Amplifier)

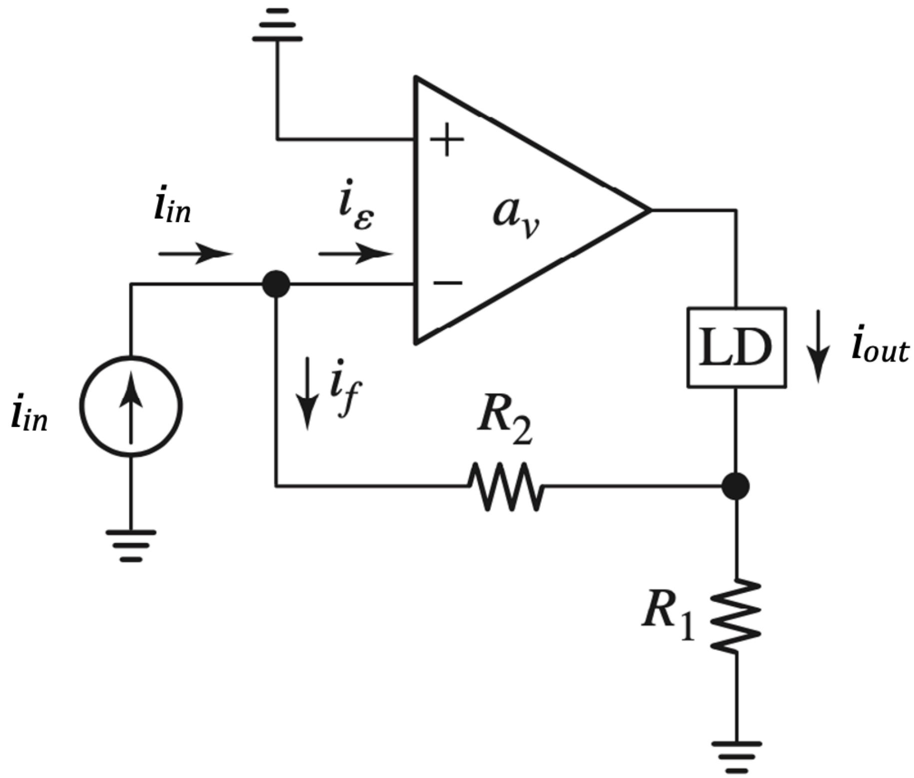


Figure 32: shunt-series configuration: $b = \dfrac{i_f}{i_{out}} = -\dfrac{R_1}{R_1 + R_2} = -\dfrac{1}{1 + R_2/R_1}$

For $a_v \to \infty$

$A_i = \dfrac{i_{out}}{i_{in}} = \dfrac{1}{b} = -(1 + \dfrac{R_2}{R_1})$

$R_{in} \to 0$

$R_{out} \to \infty$

8 Return Ratio Analysis

8.1 Closed-loop gain using return ratio

Given a feedback amplifier, with a controlled source of value k (see Figure 33), the closed loop gain can be derived as shown in (Gray et al. 2009):

$$A = \frac{s_{out}}{s_{in}} = A_{\infty} \cdot \frac{T}{1+T} + \frac{d}{1+T}$$

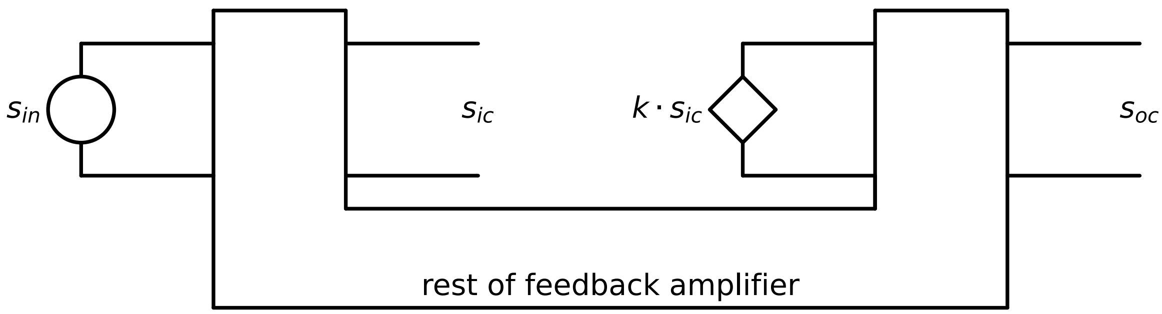


Figure 33: Feedback amplifier: return ratio framework to derive the closed-loop gain

Visually, the expression for the closed loop gain can be summarized through the block diagram in Figure 34:

$$\left(s_{in} - \frac{s_{out}}{A_{\infty}}\right) \cdot TA_{\infty} + d \cdot s_{in} = s_{out}$$

$$A = \frac{s_{out}}{s_{in}} = A_{\infty} \cdot \frac{T}{1+T} + \frac{d}{1+T}$$

The three terms needed to find the closed loop gain (A_{∞}, T, d) are all directly computable and measurable (SPICE), and do not rely on any idealization of the feedback network.

In practical circuits the feedback network loads both the input and output of the amplifier, so idealizing its effect is not realistic. With two-port analysis incorporating the effect of loading without the use of suitable approximations/idealizations of the feedback network becomes extremely tedious.

The return ratio analysis does not try to identify the transfer function of the basic amplifier and the feedback network separately. It aims to identify directly the gain around the feedback loop. From the loop gain of a circuit, we can determine:

- stability
- closed-loop gain
- nodal impedances

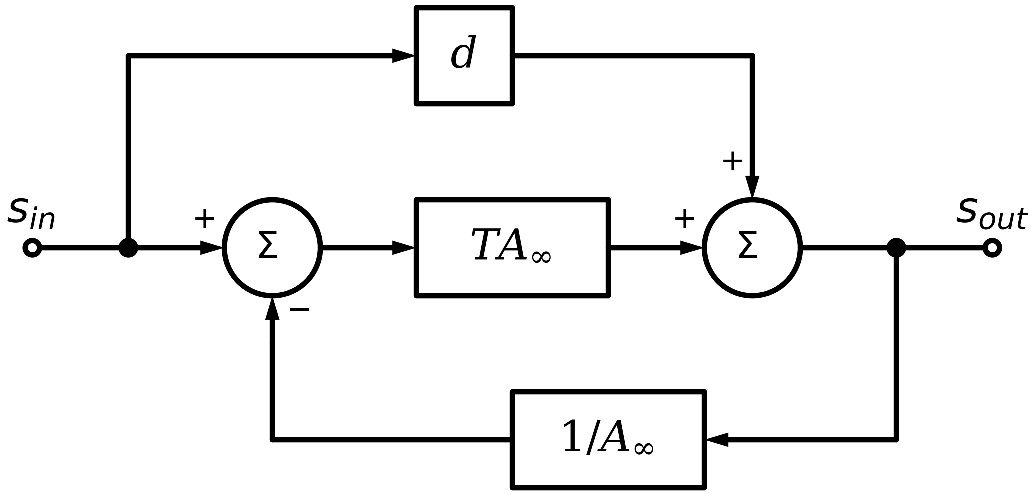


Figure 34: Block diagram for closed-loop gain using return ratio framework

The return ratio analysis can be applied to any arbitrary feedback circuit, independent of topology and port structure.

The return ratio for a dependent source in a feedback loop is found as follows:

1. Set all independent sources to zero
2. Identify a dependent source in the feedback loop that you want to analyze and break the loop by disconnecting the dependent source from the rest of the circuit. Leave the dependent source open-circuited if it is of the voltage type, or short-circuited if it is of the current-type
3. On the side of the break that is not connected to the dependent source, inject an independent test source s_t of the same sign and type as the dependent source
4. Find the return signal s_r, generated at the controlled source that was disconnected
5. The return ratio T for the dependent source is $T = -s_r/s_t$

    - Provided that we have chosen a controlled source that breaks the loop globally, the return ratio of the dependent source is equal to the loop gain of the circuit.
    - For the stabilty analysis of a feedback circuit we must have the loop gain not just any return ratio. Unless, we have a single loop feedback circuit, the return ratios computed for different dependent sources are not necessarily equal (Hurst 1991), so in general, the return ratio is not a global property of the loop (Hajimiri 2023).

The direct feedthrough d is given by:

$$d = \frac{s_{out}}{s_{in}}|_{k=0}$$

which is the transfer function from input to output evaluated for k = 0. In other words, the direct feedthrough d is the signal transfer from input to output through the passive elements, it represent a signal path from input to output that goes around rather than through the controlled source k.

The value of A_∞ is the closed loop gain when the feedback circuit is ideal (that is when $T \to \infty$). If $T \to \infty$ then $A = A_\infty$, because $\frac{T}{1+T} \to 1$ and $\frac{d}{1+T} \to 0$

Since letting $k \to \infty$ causes $T \to \infty$ (Gray et al. 2009) the value of A_∞ can be easily found by evaluating

$$\frac{s_{out}}{s_{in}}|_{k\to\infty}$$

When $k \to \infty$, the controlling signal s_{ic} for the dependent source must be zero for the output of the controlled source to be finite. The controlled source output will be finite if the feedback is negative.

The key difference between the return ratio analysis and the two-port analysis can be seen comparing Figure 34 and Figure 35 In the two-port analysis all forward signal transmission is lumped in the block a. In the return ratio analysis, there are two forward signal paths: one path (d) for the feedback network and another path (TA_∞) for the forward gain.

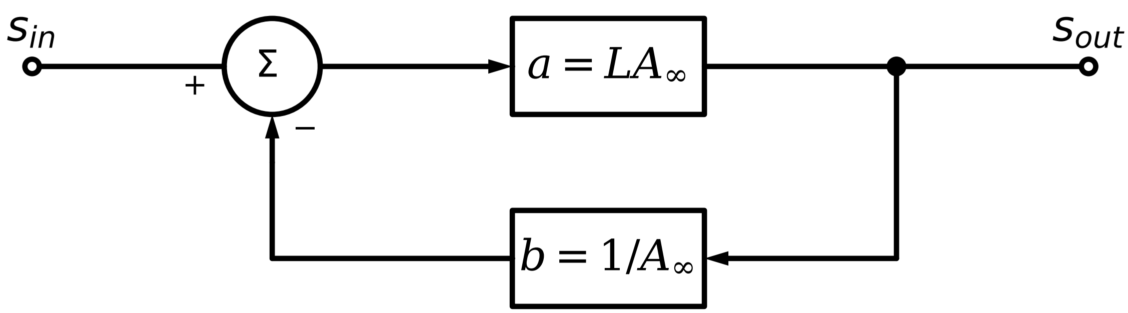


Figure 35: Block diagram model for closed-loop gain using the two-port framework

Example 5 - common source stage with current source biasing

Consider the circuit in Figure 36 and its AC equivalent model in Figure 37

Current source biasing in this circuit doesn't work without feedback setting the drain voltage.

Given the finite gain and the nature of the impedances of the MOS, in this circuit is hard to decouple a(s) and b(s), so using the two-port analysis is not a good option.

$$A(s) = \frac{a(s)}{1 + a(s)b(s)}$$

In this case, since the circuit is not very complex we can use exact nodal analysis.

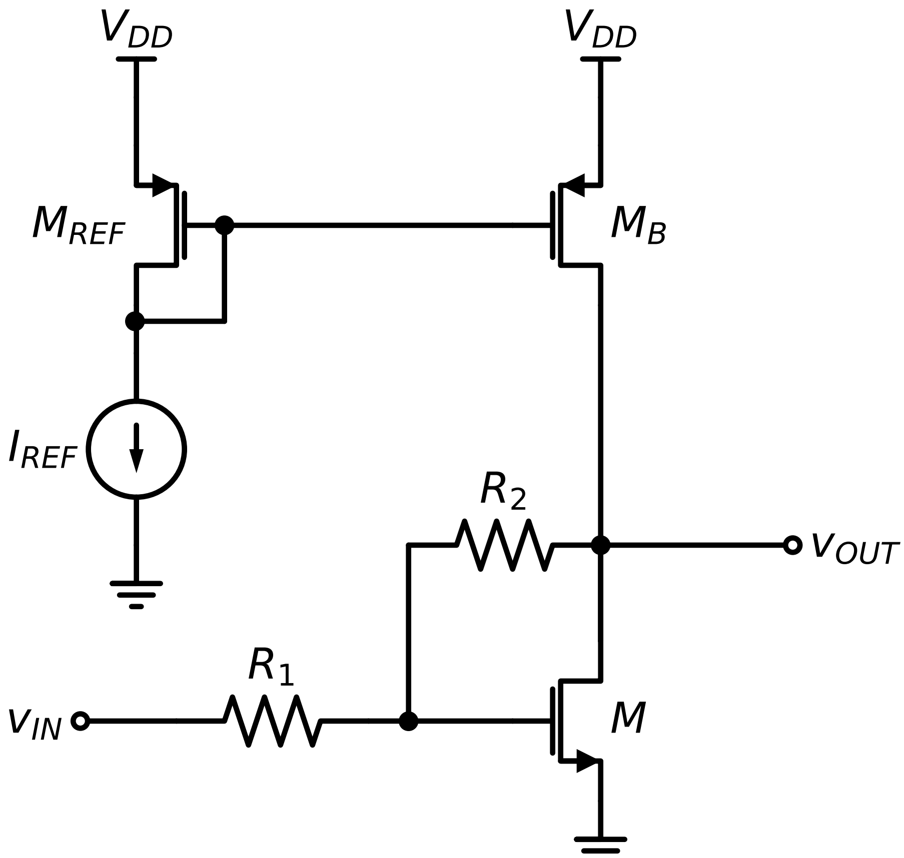


Figure 36: Common source with current-source biasing

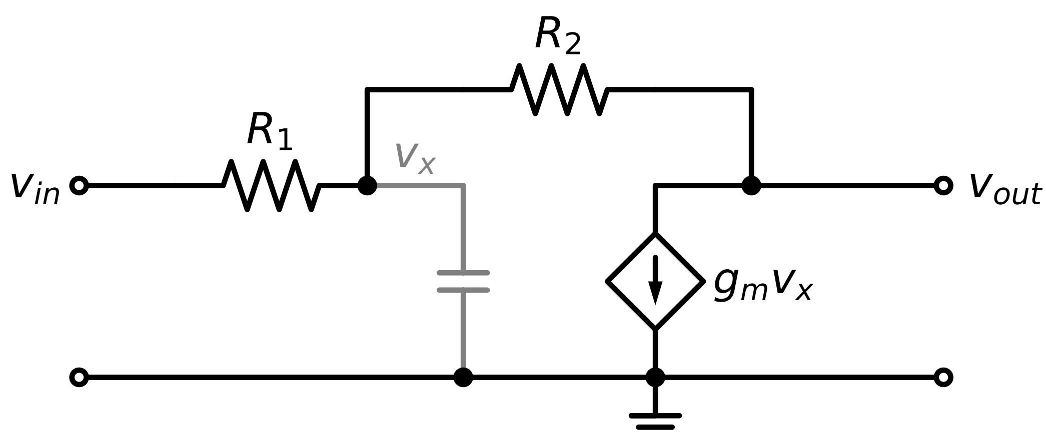


Figure 37: AC equivalent model of the feedback circuit in Figure 36

$$\frac{v_{in} - v_x}{R_1} + \frac{v_{out} - v_x}{R_2} = 0 \Leftrightarrow \frac{v_{in}}{R_1} - \left(\frac{1}{R_1} + \frac{1}{R_2}\right) \cdot v_x + \frac{v_{out}}{R_2} = 0 \tag{7}$$

$$\frac{v_x - v_{out}}{R_2} = g_m v_x \Leftrightarrow v_x = \frac{v_{out}}{1 - g_m R_2} \tag{8}$$

Substituting Equation 7 in Equation 8:

$$\frac{v_{out}}{v_{in}} = \frac{1 - g_m R_2}{1 + g_m R_1} = -\frac{R_2}{R_1} \left(\frac{1 - \frac{1}{g_m R_2}}{1 + \frac{1}{g_m R_1}} \right)$$

If the $g_m R$ terms are much greater than 1, the result is the same as the op amp solution.

Let's now confirm that return ratio analysis produces the same result.

Return ratio calculation

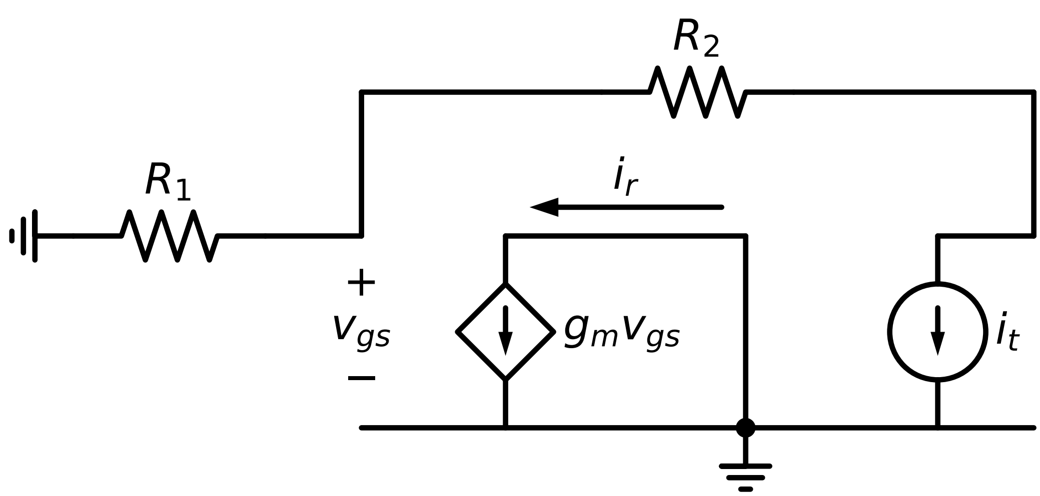


Figure 38: return ratio calculation

$v_{gs} = -R_1 i_t$

$i_r = g_m v_{gs} = -g_m R_1 i_t$

$\dfrac{i_r}{i_t} = -g_m R_1$

$T \equiv -\dfrac{i_r}{i_t} = g_m R_1$

Ideal closed loop gain calculation

A_∞ is the transfer function when the gain element of the controlled source becomes ∞. In our case this corresponds to $g_m \to \infty$. If the gain element becomes ∞ the controlling signal must be zero. In our case this correspond to the input node $v_{gs} = 0$, therefore:

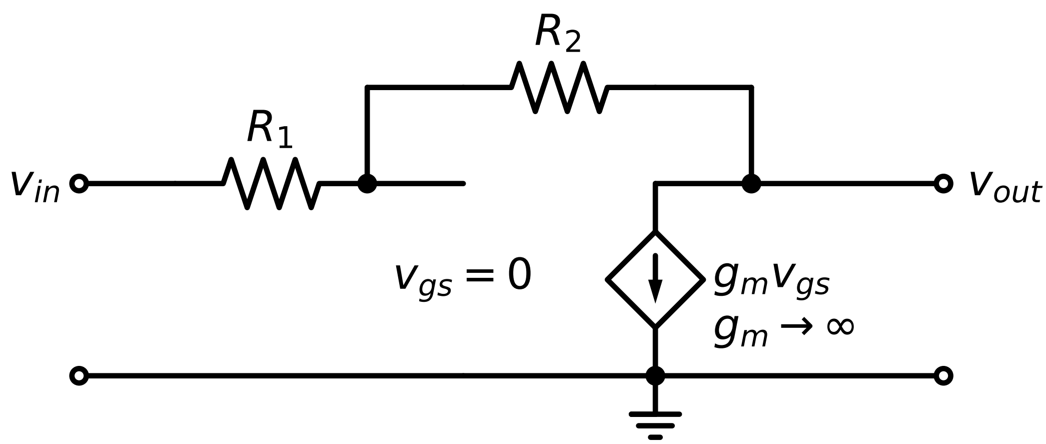


Figure 39: Ideal closed loop gain calculation

$\frac{v_{in}}{R_1} = -\frac{v_{out}}{R_2} \Leftrightarrow A_\infty = \frac{v_{out}}{v_{in}} = -\frac{R_2}{R_1}$

Direct feedtrough (d) calculation

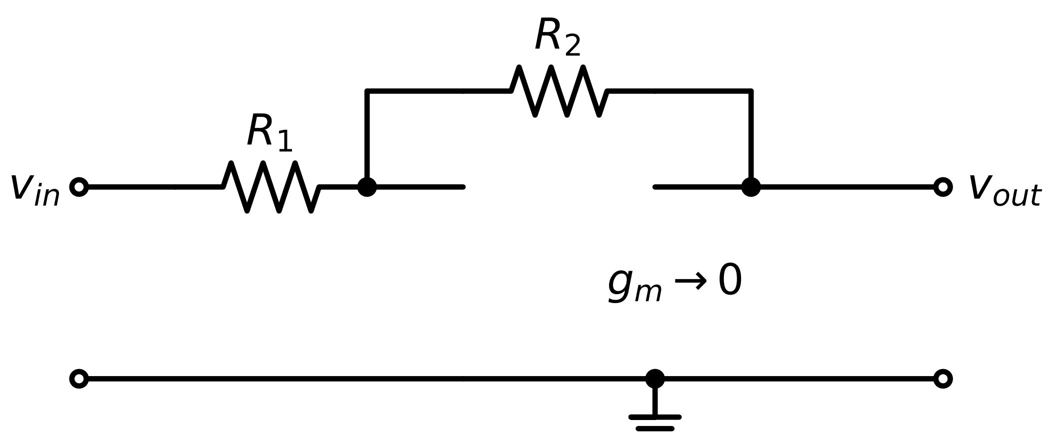


Figure 40: Direct feedthrough calculation

d is defined as the transfer function when the gain element becomes zero. In our case for $g_m = 0$

$v_{out} = v_{in} \Leftrightarrow d = \frac{v_{out}}{v_{in}} = 1$

In summary:

- $T = g_m R_1$
- $A_\infty = -\dfrac{R_2}{R_1}$
- $d = 1$

Therefore, as expected the closed loop gain computed using the return ratio framework matches with the result obtained using exact nodal analysis:

$$
\begin{aligned}
A &= A_\infty \cdot \frac{T}{1+T} + \frac{d}{1+T} \\
&= -\frac{R_2}{R_1} \cdot \frac{g_m R_1}{1+g_m R_1} + \frac{1}{1+g_m R_1} \\
&= \frac{1-g_m R_2}{1+g_m R_1}
\end{aligned}
$$

Example 6 - constant-g_m reference

In this example, we will use the return ratio framework to evaluate the stability of the bias circuit in Figure 41. The bias circuit in Figure 41 is know as constant-g_m reference or ΔV_{GS} reference.

Note: this circuit has only one-port so it cannot be analyzed using the two-ports framework.

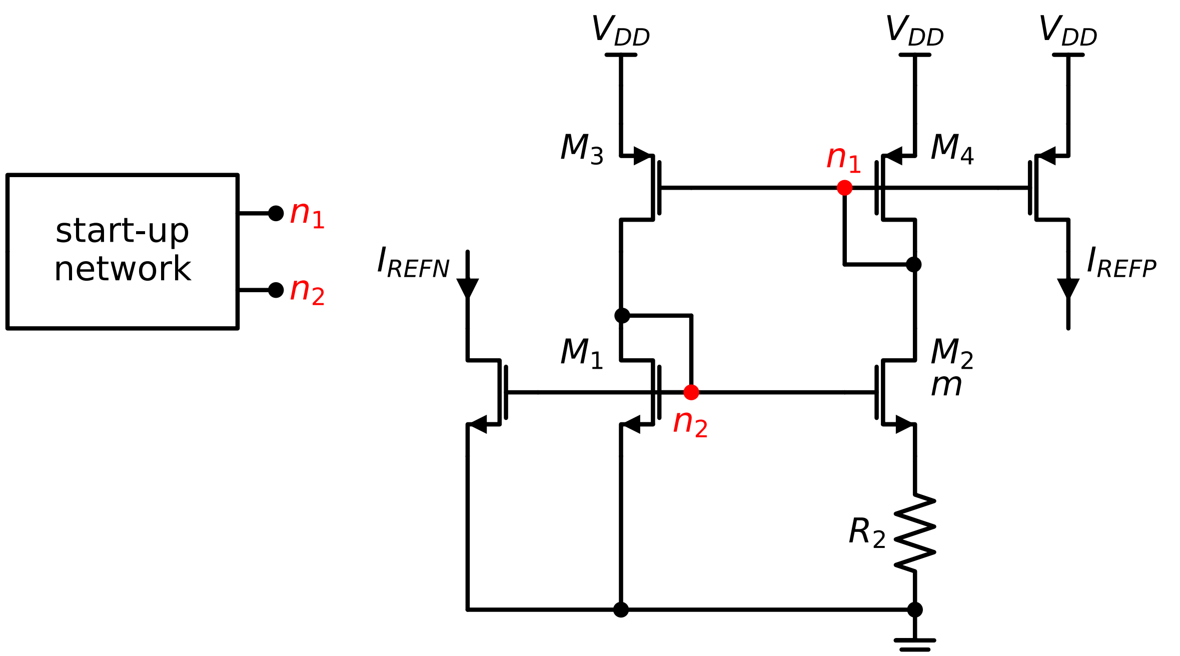


Figure 41: Constant-g_m reference

$$
\begin{aligned}
I_{REF} \cdot R_2 &= V_{GS1} - V_{GS2} = V_{OV1} - V_{OV2} \\
&\approx V_{OV1}\left(1 - \frac{1}{\sqrt{m}}\right)
\end{aligned}
$$

$$
I_{REF} \approx \frac{V_{OV1}\left(1 - \frac{1}{\sqrt{m}}\right)}{R_2}
$$

- This circuit has positive feedback.
- A positive feedback system is stable only if its loop gain is less than 1 ($T < 1$)

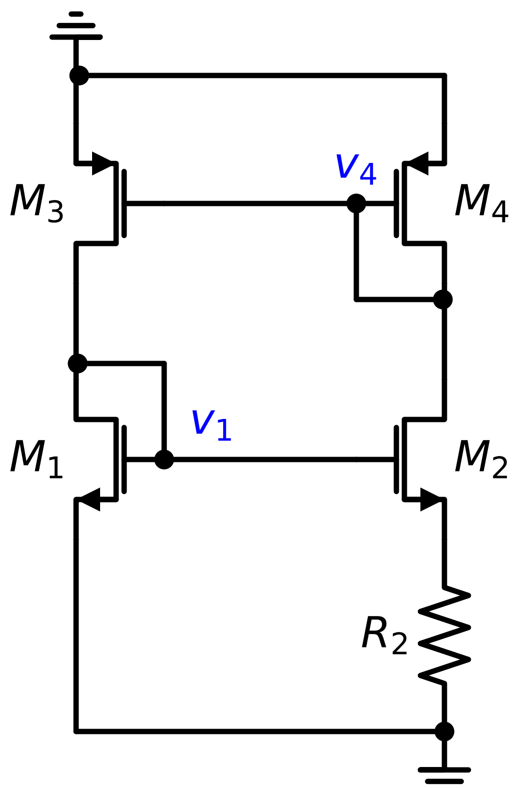


Figure 42: AC equivalent model of the constant-g_m bias circuit

To identify which dependent source to use for computing the loop gain T of the circuit, the feedback loop we break must be global.

- M_2 and R_2 degeneration, form a local feedback loop, so M_2 cannot be used to break the main loop

- M_1 and M_4 are diode-connected; their transcondactance elements are equivalent to a resistance $1/g_m$, so cannot be used to break the main loop
- M_3 is the only *"normal"* CS gain stage in the circuit, so it is the only element that can be used to break the main loop

In general, **the return ratio of CG, CD, or degenerated CS cannot be used** to find the "global" loop gain of a circuit.

To compute the loop gain T of the circuit, it is convenient to first unroll and linearize the AC equivalent model in Figure 42:

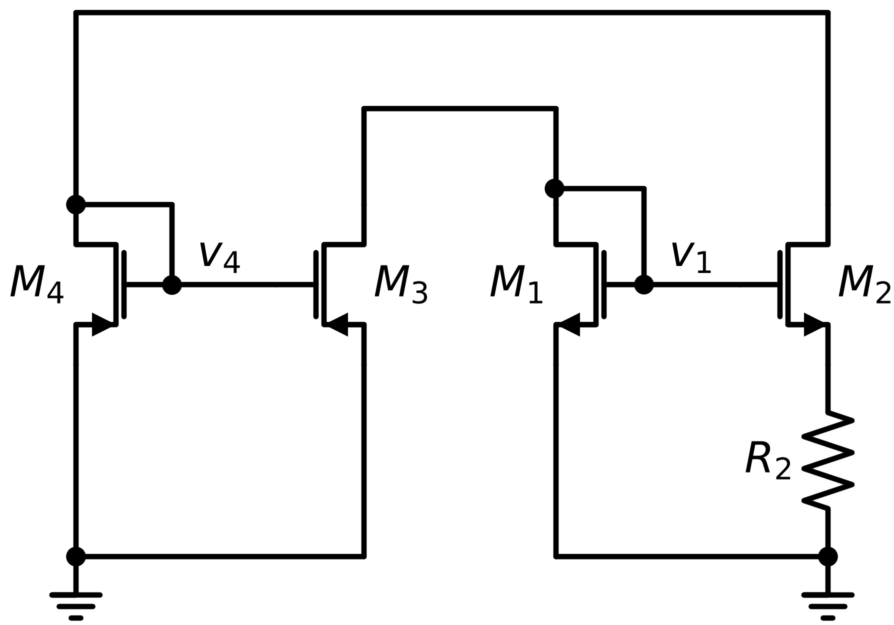


Figure 43: Unrolled AC equivalent model for the constant-g_m bias circuit

and then insert the test source:

$i_r = g_{m3}v_4$

$i_t = -g_{m1}v_1$

$v_4 = -i_2\left(1/g_{m4}\right)$

$i_2 = g_{m2}v_2$

$v_1 = v_2 + g_{m2}v_2R_2 \Leftrightarrow v_2 = \dfrac{v_1}{(1+g_{m2}R_2)}$

Putting the pieces together:

$$\frac{i_r}{i_t} = \left(-\frac{g_{m3}}{g_{m1}}\right)\left(-\frac{g_{m2}}{g_{m4}}\right)\left(\frac{1}{1+g_{m2}R_2}\right)$$

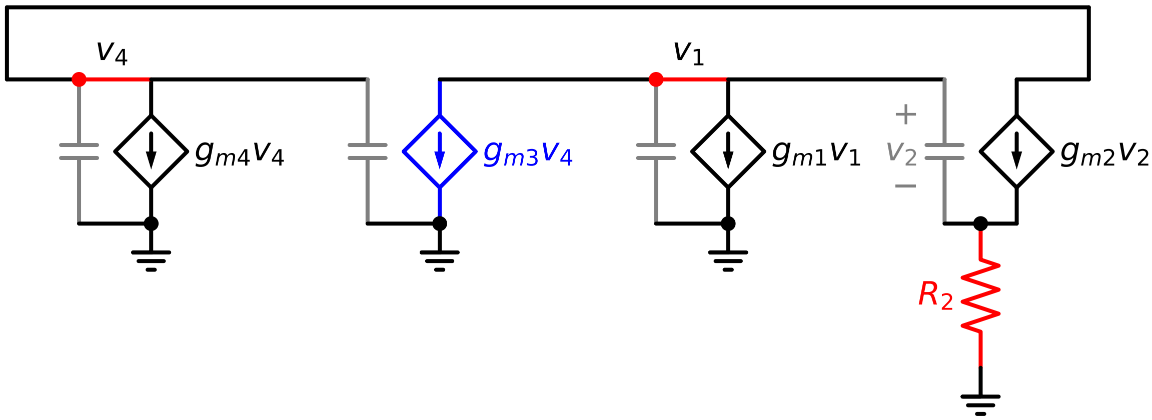


Figure 44: Small signal equivalent model for the constant-g_m bias circuit, (a) red: circuit features that make s_t insertion not possible, (b) blue: best place for s_t insertion

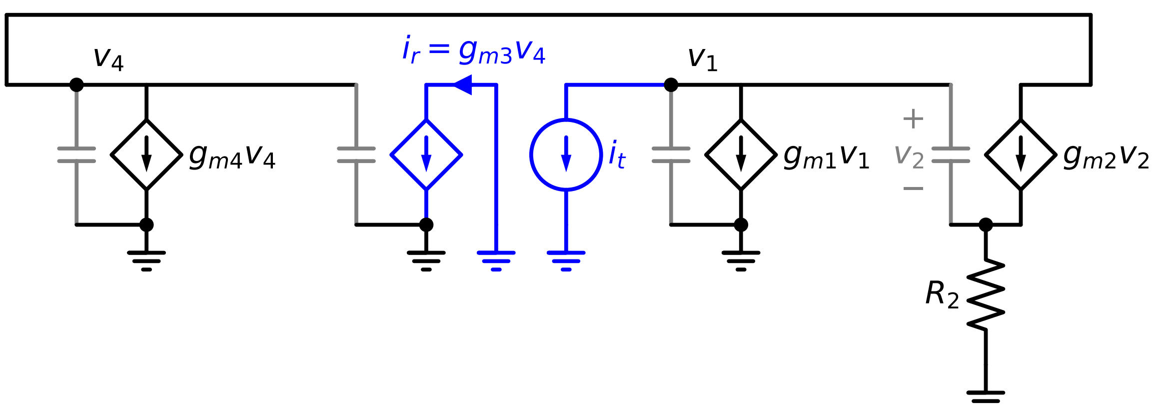


Figure 45: Computing the return ratio for the constant-g_m bias circuit

Therefore: $T = -\frac{i_r}{i_t} = -\left(\frac{g_{m3}}{g_{m1}}\right)\left(\frac{g_{m2}}{g_{m4}}\right)\left(\frac{1}{1+g_{m2}R_2}\right)$

The resulting return ratio is negative (i.e. the feedback is definitely not negative, even at zero frequency). The challange is to keep the magnitude of the return ratio less than unity ($|T| < 1$).

The loop has two inverting gain blocks, each loaded with a "diode connected" MOS device. The gain block formed by M_3 is loaded by the diode connected MOS M_1 and the gain block formed by M_2 and R_2 is loaded by the diode connected MOS M_4.

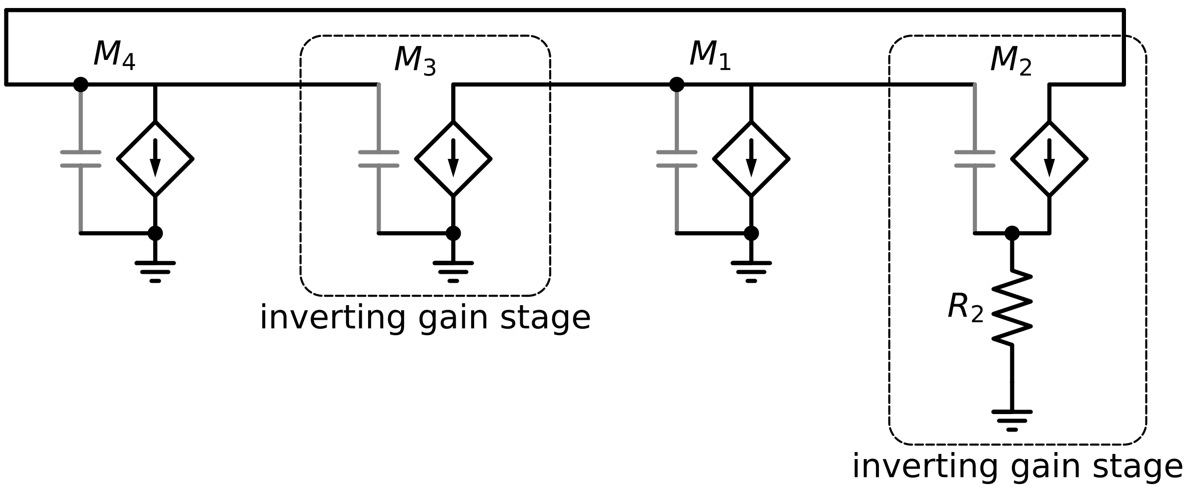


Figure 46: Small signal equivalent model for the constant-g_m bias circuit

Example 7 - potential stability issue of the constant-g_m reference

Consider the design of the constant-g_m reference circuit in Figure 47:

Assume: $I(M_3) = I(M_4) = I_{REFN} = 100\mu A$

$V_{OV1} = 200mV$

$m = 2$

For the given circuit:

$g_{m4} = g_{m3} = g_{m1} = \frac{2 \cdot 100\mu A}{200mV} = 1mS$

Recalling that (assuming square law):

$V_{OV2} = \frac{V_{OV1}}{\sqrt{m}}$

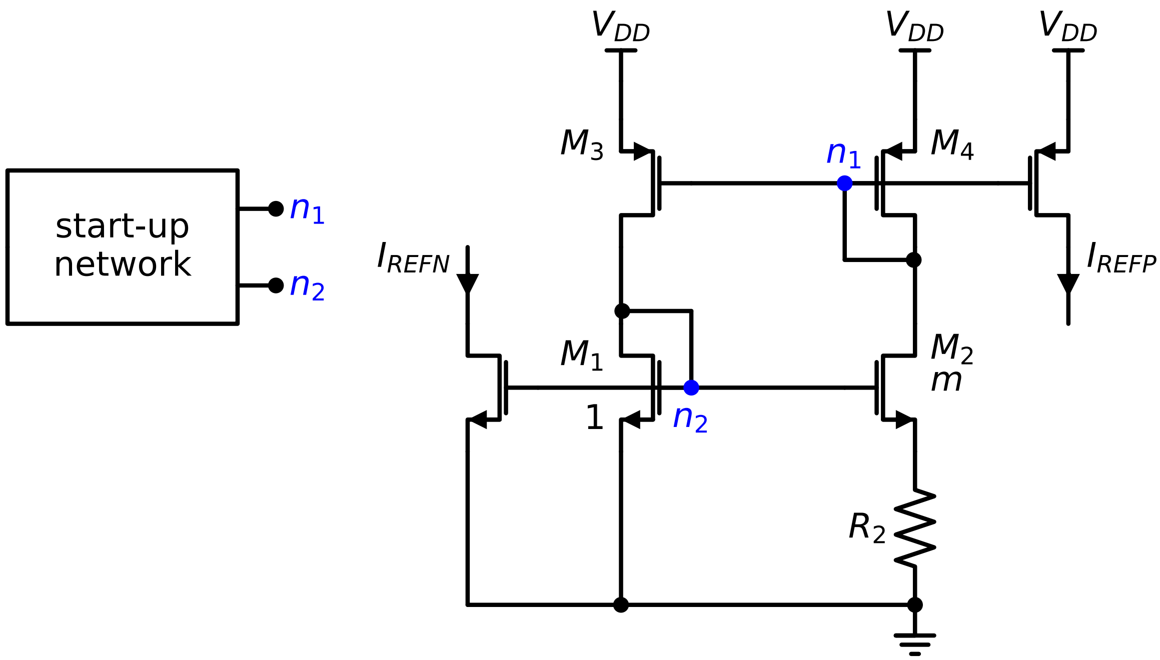


Figure 47: Constant-g_m reference: design example

$$I_{REF} = \frac{V_{OV1}\left(1 - \frac{1}{\sqrt{m}}\right)}{R_2}$$

for m=2:

$V_{OV2} = 141mV$

$R_2 = 586\Omega$

$$g_{m2} = \frac{2 \cdot 100\mu A}{141mV} = 1.41mS$$

$$\frac{g_{m2}}{1 + g_{m2}R_2} = 772\mu S$$

The magnitude of the return ratio is less than 1:

$$|T| = \frac{i_r}{i_t} = \left(\frac{g_{m2}}{1 + g_{m2}R_2}\right)\left(\frac{1}{g_{m4}}\right)\left(\frac{\cancel{g_{m3}}}{\cancel{g_{m1}}}\right) = 0.772(< 1)$$

Therefore, the circuit designed is stable, **but is possible to have problems**. Consider, Figure 48, if the parasitic capacitance C shorts R_2, the return ratio becomes larger than 1 and the circuit becomes unstable.

$$\frac{i_r}{i_t} = \cancelto{1}{\left(\frac{1}{1+g_{m2}R_2}\right)} \left(\frac{g_{m2}}{g_{m4}}\right) \left(\frac{\cancel{g_{m3}}}{\cancel{g_{m1}}}\right) \to 1.41$$

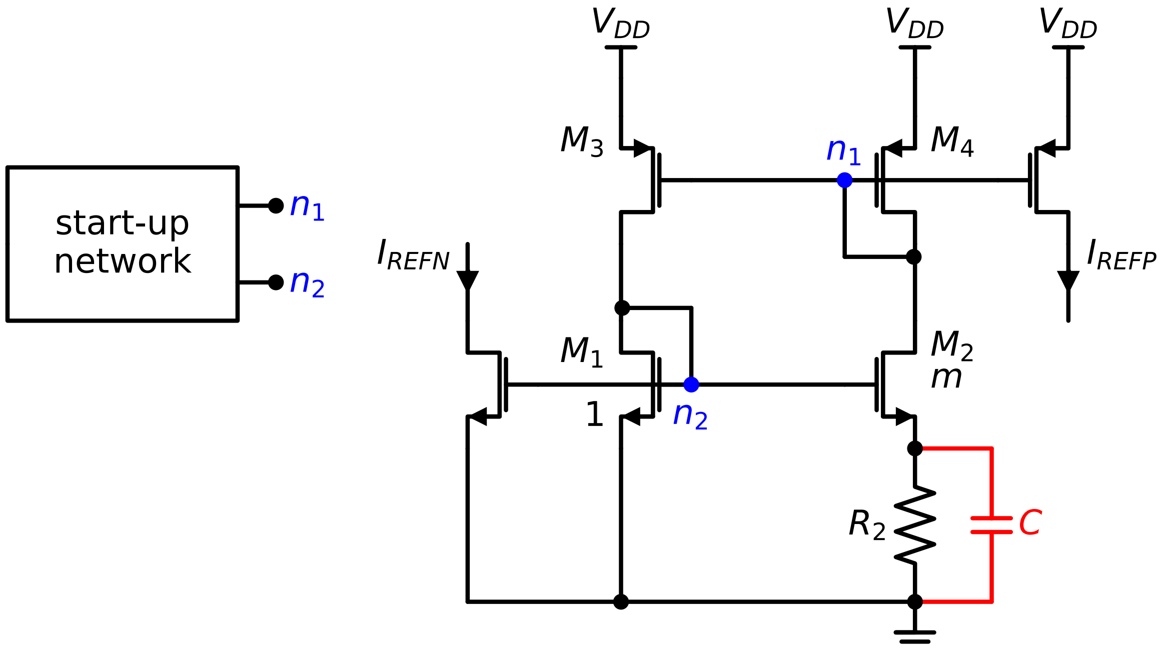


Figure 48: Constant-g_m reference: possible problem

8.2 Closed-loop impedances using return ratio

Feedback can be used to modify the port impedances of a circuit.

Given a feedback circuit with a controlled source of value k (see Figure 49), the impedance at any port, including the input and output ports, can be computed (Gray et al. 2009) using the following expression (a.k.a. **Blackman's formula**):

$$Z_{port} = Z_{port}(k=0) \cdot \left[\frac{1 + T(portshorted)}{1 + T(portopen)}\right]$$

- $Z_{port}(k=0)$ is the port impedance with the return ratio's gain element k set to 0 (either set $g_m = 0$ or $a_v = 0$)
- $T(portshorted)$ is the return ratio with the port under consideration *shorted*
- $T(portopen)$ is the return ratio with the port under consideration *open*

Often, one of the two return ratios in the formula is zero, in these cases feedback increases or decreases the impedance by a factor $(1 + T)$

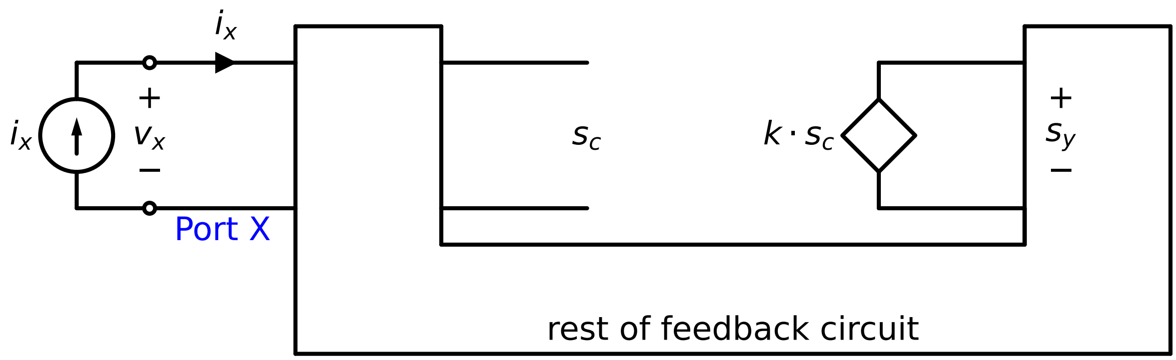


Figure 49: Feedback circuit to derive Blackman's formula with respect to port x

In summary, Blackman's impedance formula:

- it applies to any feedback circuit, regardless of the type of feedback
- it is extremely useful and easy to use
- it is based on return ratio calculations

Example 8 - Input resistance of non-inverting op amp configuration

$$R_{in} = R_{in0} \cdot \left[\frac{1 + T(portshorted)}{1 + T(portopen)} \right]$$

- To find R_{in0} set the op amp gain a_v to zero:

$$R_{in0} = r_i + R_1 || (R_2 + r_o || R_L) \approx r_i$$

- To compute $T(portshorted)$ short the node v_p to ground and find the return ratio:

$$T_{sc} = -\frac{v_r}{v_t}|_{sc} = a_v \cdot \frac{R_1}{r_o + R_L||(R_2 + r_i||R_1)} \approx a_v \cdot \frac{R_1}{R_1 + R_2}$$

- To compute $T(portopen)$ leave the node v_p floating and find the return ratio:

$$T_{open} = -\frac{v_r}{v_t}|_{open} = 0$$

Feedback increases the input impedance of the circuit significantly.

$$R_{in} \approx r_i \cdot \frac{1 + a_v \cdot \frac{R_1}{R_1 + R_2}}{1 + 0} = r_i \cdot \left(1 + a_v \cdot \frac{R_1}{R_1 + R_2} \right)$$

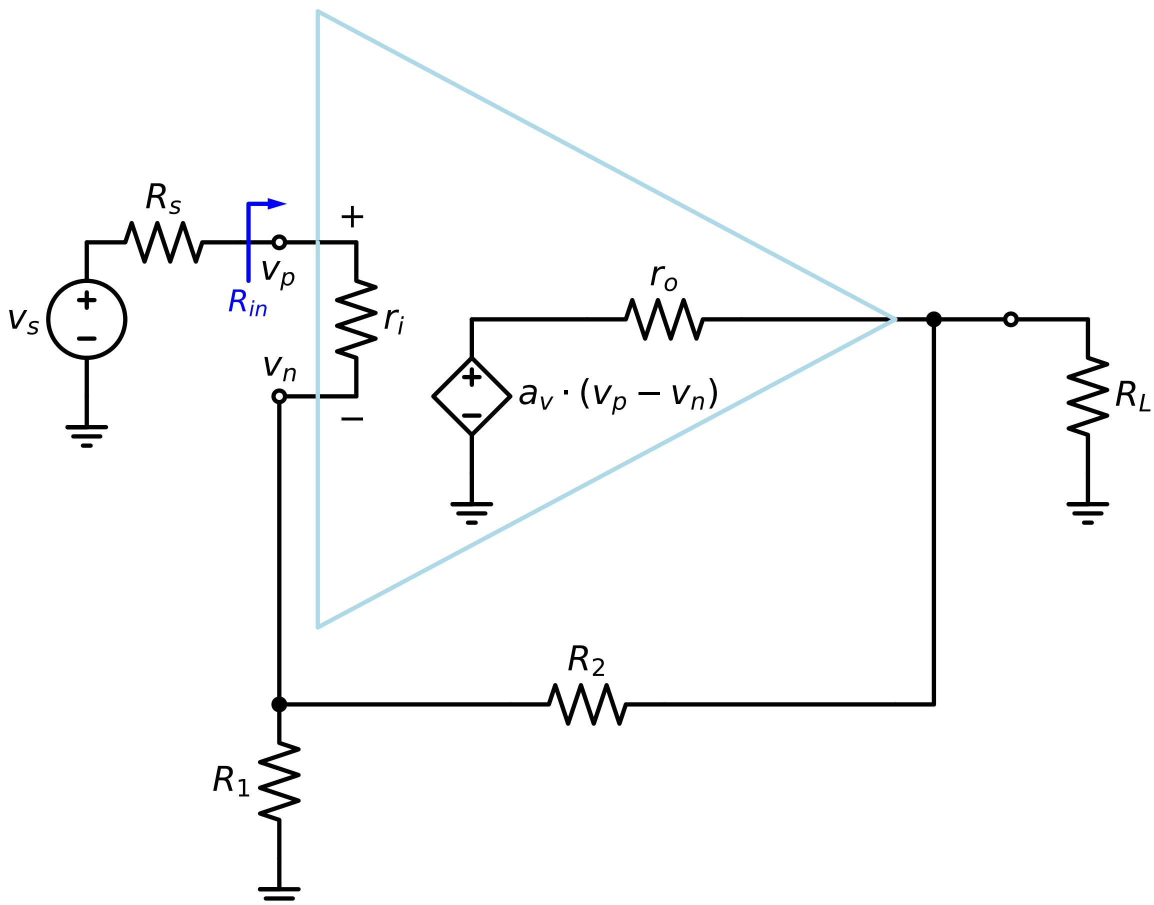


Figure 50: Input resistance of non-inverting op amp circuit

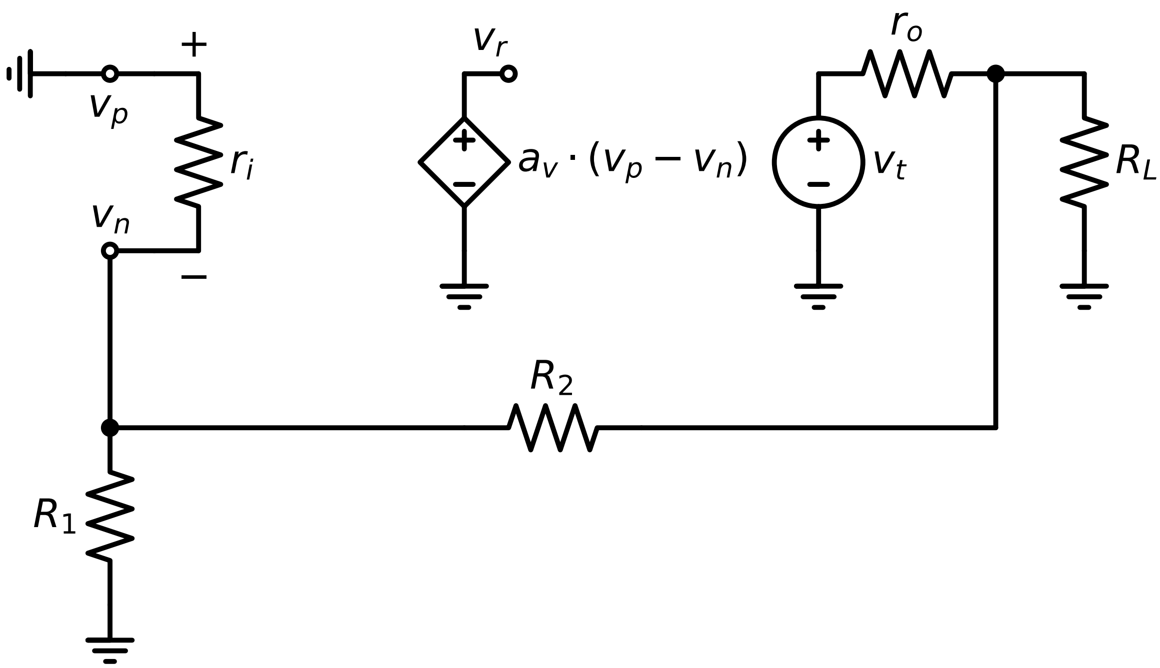


Figure 51: T(with input port shorted) for non inverting configuration

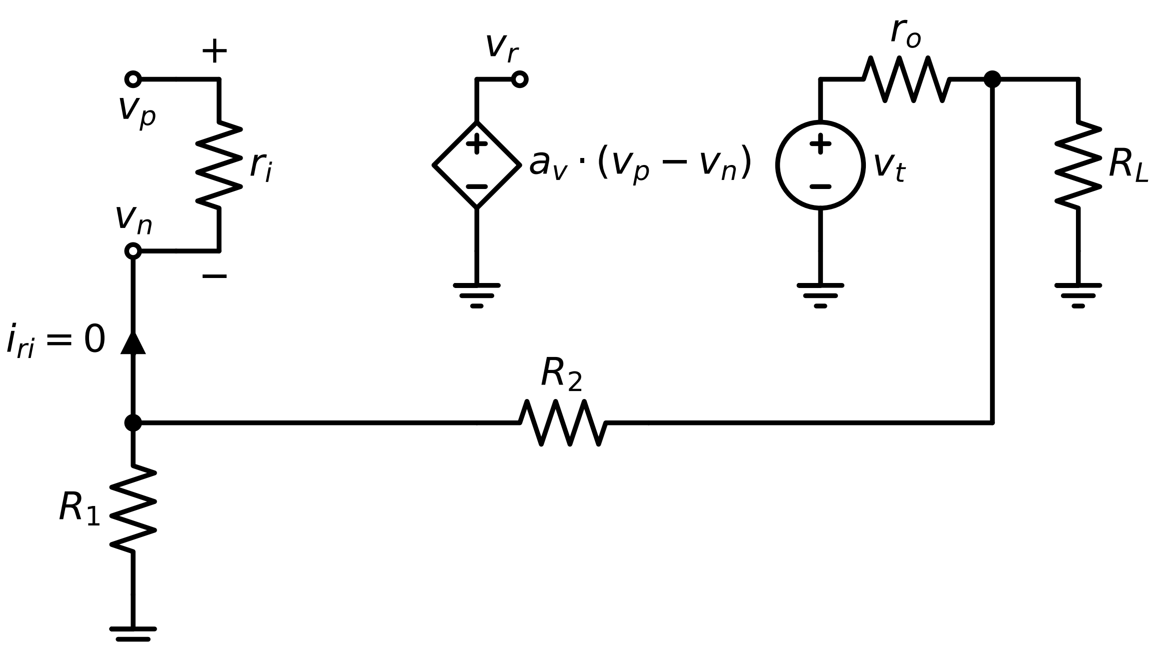


Figure 52: T(with input port open) for non inverting configuration

For $a_v = 10^6$, $R_1 = 20k\Omega$, $R_2 = 80k\Omega$:

$$R_{in} \approx r_i \cdot 200000$$

Example 9 - Output resistance of non-inverting op amp configuration

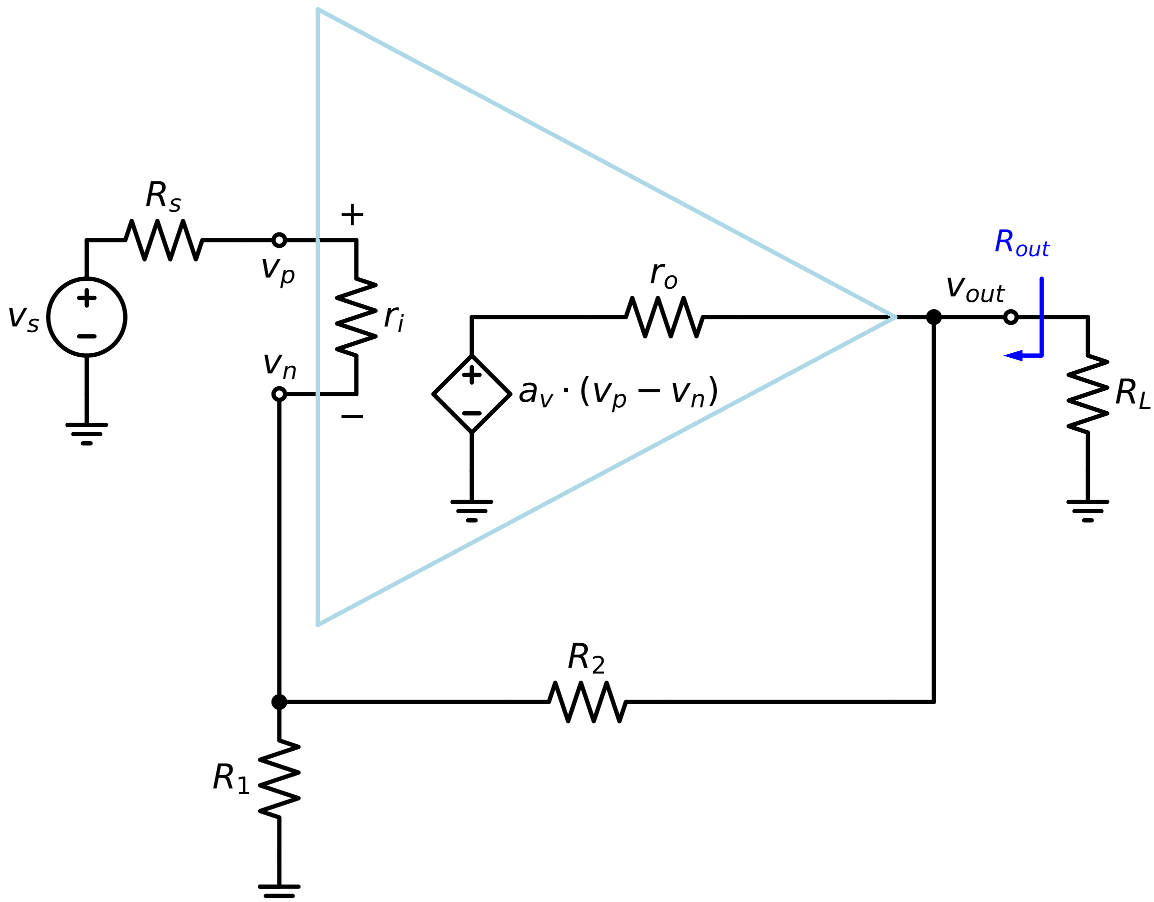


Figure 53: Output resistance of non-inverting op amp circuit

$$R_{out} = R_{out0} \cdot \left[\frac{1 + T(portshorted)}{1 + T(portopen)}\right]$$

- To find R_{out0} set the op amp gain a_v to zero:

$$R_{out0} = r_o || [R_2 + R_1 || (r_i + R_s)] \approx r_o$$

- To compute $T(portshorted)$ short the node v_{out} to ground and find the return ratio:

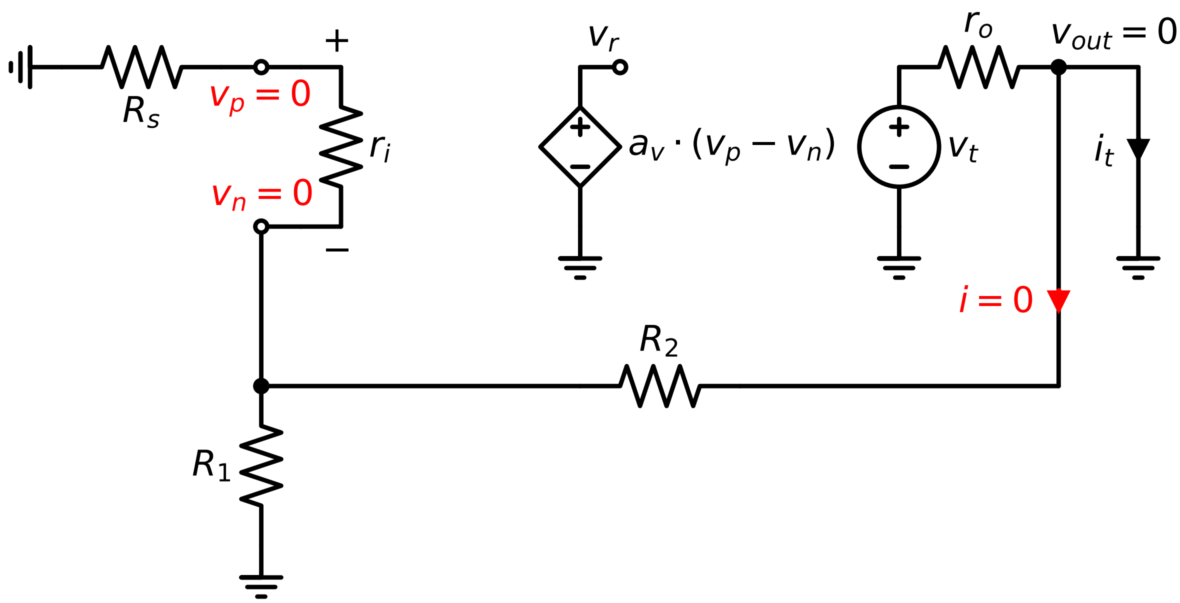


Figure 54: T(with output port shorted) for non-inverting configuration

$$T_{sc} = -\frac{v_r}{v_t}|_{sc} = 0$$

- To compute $T(portopen)$ leave the node v_{out} floating and find the return ratio:

$$v_n = \frac{v_t}{r_o + R_2 + [R_1||(r_i + R_s)]} \cdot R_1||(r_i + R_s) \approx \frac{v_t}{R_2 + R_1} \cdot R_1$$

$$v_p = v_n \frac{R_s}{r_i + R_s} \approx 0$$

$$T_{open} = -\frac{v_r}{v_t}|_{open} = -\frac{a_v(v_p - v_n)}{v_t} \approx a_v \cdot \frac{R_1}{R_1 + R_2}$$

Feedback decreases the output impedance of the circuit significantly.

$$R_{out} \approx r_o \cdot \frac{1 + 0}{1 + a_v \frac{R_1}{R_1 + R_2}} = \frac{r_o}{1 + a_v \frac{R_1}{R_1 + R_2}}$$

For $a_v = 10^6$, $R_1 = 20k\Omega$, $R_2 = 80k\Omega$:

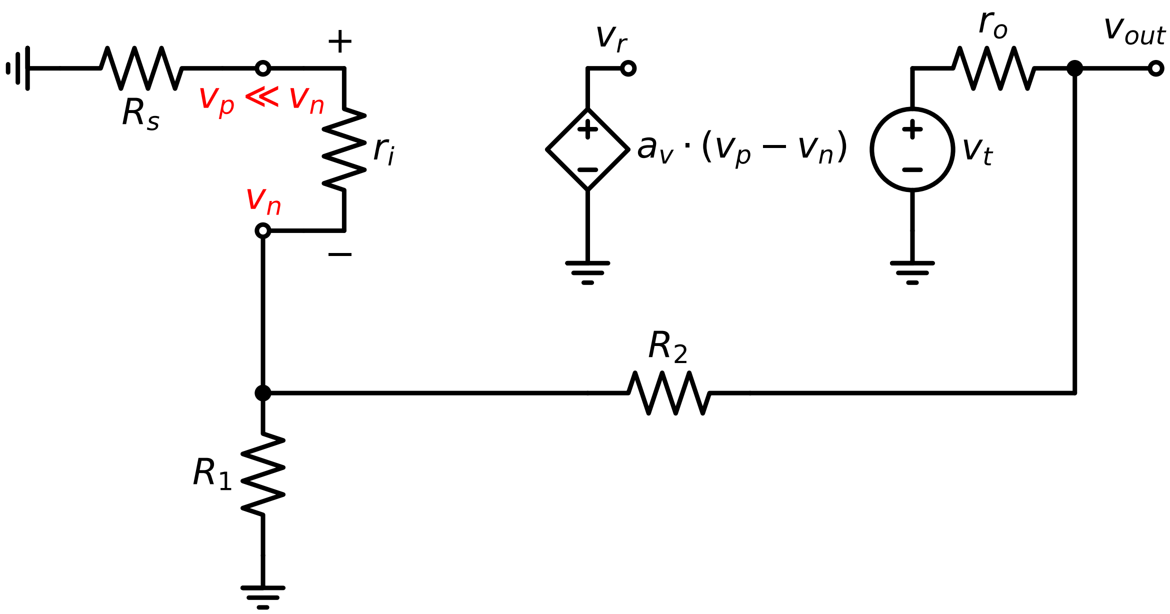


Figure 55: T(with output port open) for non-inverting configuration

$$R_{out} \approx \frac{r_o}{200000}$$

Example 10 - Output resistance of bootstrapped source follower

Consider the bootstrapped source follower stage of Figure 56 and its equivalent small signal circuit to compute the return ratio with the output port shorted (Figure 57) and with the output port left floating (Figure 58)

The output resistance for $k = a_v = 0$ is:

$$R_{out0} \equiv R_{out}(k = 0) \equiv R_{out}(a_v = 0) = \frac{1}{g_m}$$

Output port shorted implies:

$T_{sc} = -\frac{v_r}{v_t}|_{sc} = 0$

Output port open means:

$g_m \cdot v_{gs} = 0 \rightarrow v_{gs} = 0 \rightarrow -v_d = v_t \rightarrow v_r = -a_v \cdot v_t$

$T_{open} = -\frac{v_r}{v_t}|_{open} = a_v$

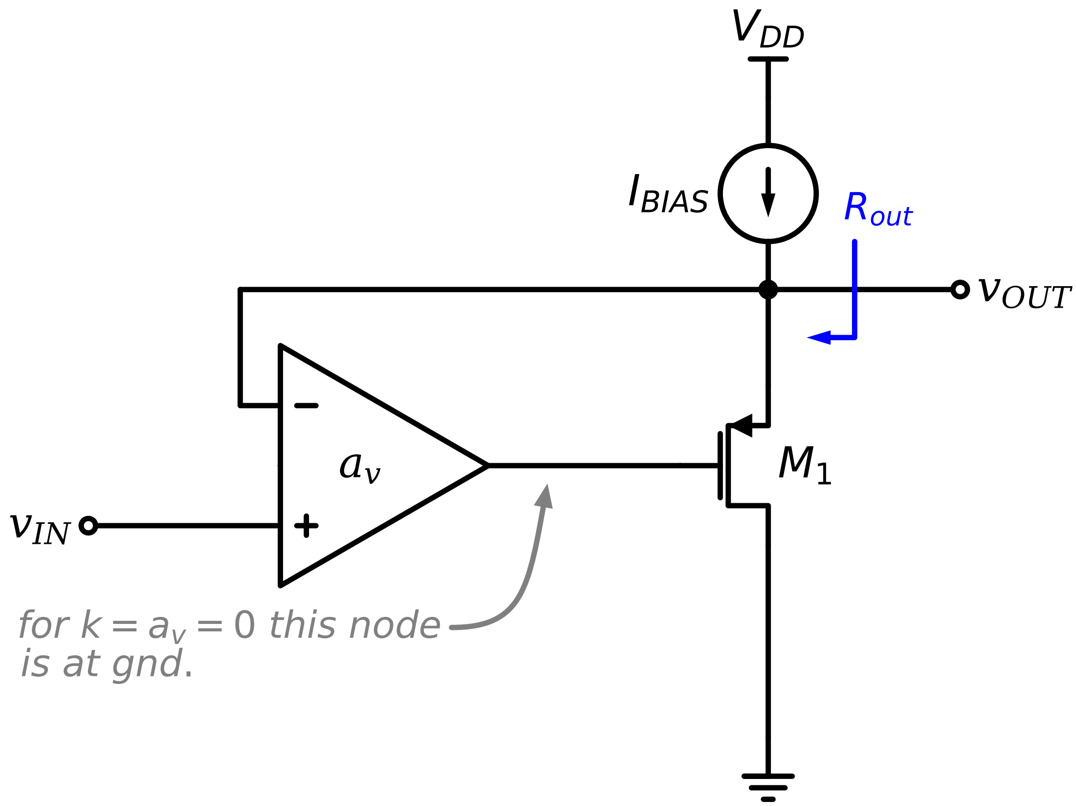


Figure 56: A bootstrapped source follower stage

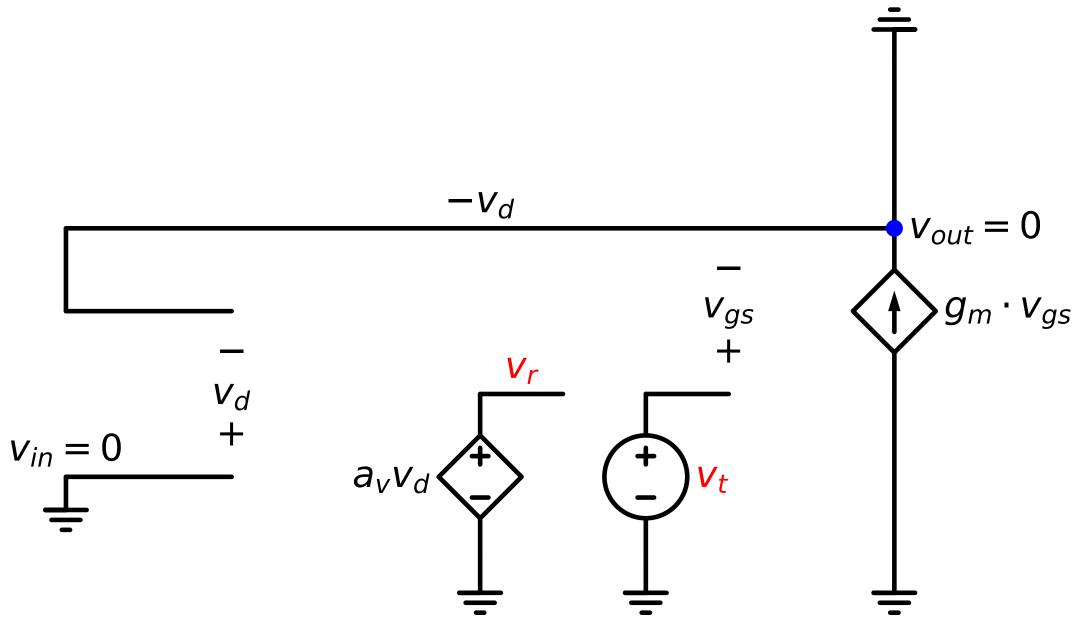


Figure 57: Finding T(with output port shorted), $-v_d = v_{out} = 0$

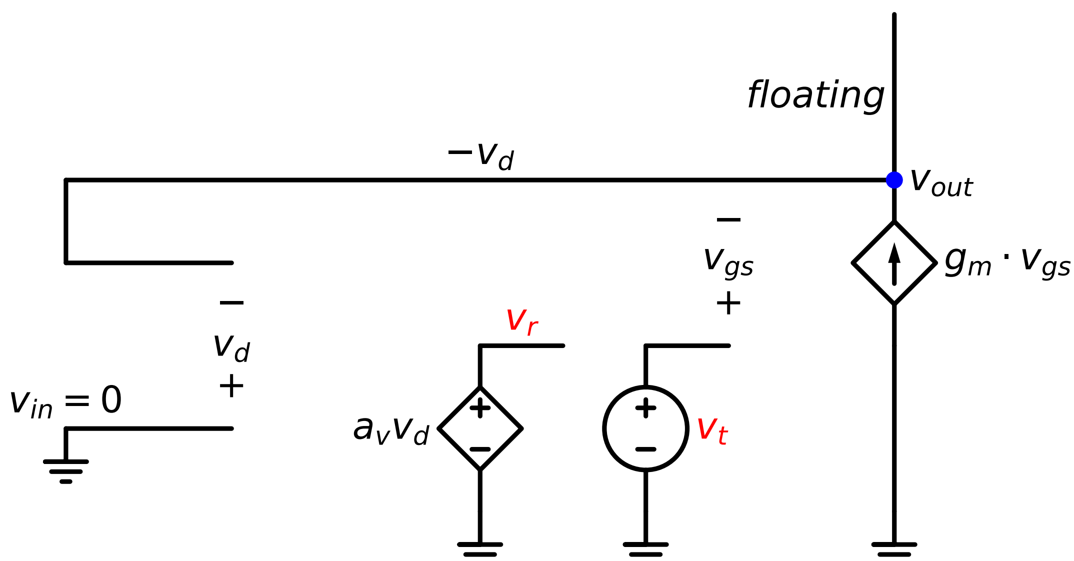


Figure 58: Finding T(with output port open), $v_{out} = -v_d = -v_{gs} + v_t$

Therefore, the closed-loop output resistance is:

$$R_{out} = R_{out0} \cdot \frac{1+T_{sc}}{1+T_{open}} = \frac{1}{g_m} \cdot \left(\frac{1}{1+a_v}\right)$$

Example 11 - Output resistance of super source follower

The super-source follower uses feedback to reduce its output impedance.

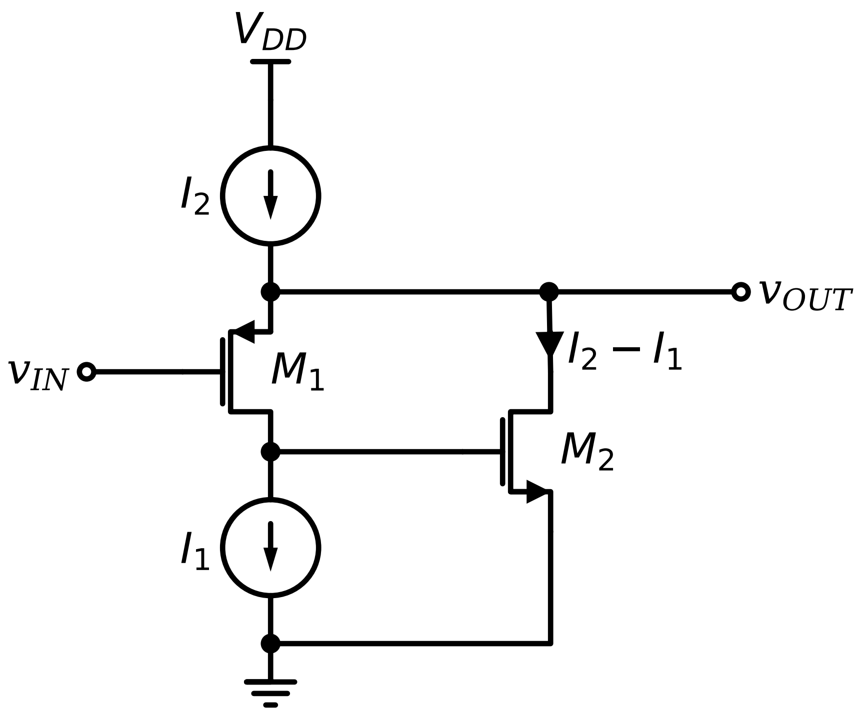


Figure 59: Super-source follower stage

The small signal model of the super-source follower is shown in Figure 60.

- To find R_{out0} we must set $v_{in} = 0$ and $k = 0$. In this circuit we can use, either g_{m1} or g_{m2} as k. We will use, $k = g_{m2}$.

The current in the g_{m1} generator flows only in r_{o1}, so M_1 has no effect on the output resistance when $g_{m2} = 0$

$$R_{out0} \equiv R_{out}(g_{m2} = 0) = r_{o2}$$

- The return ratio for g_{m2} with the output port shorted is:

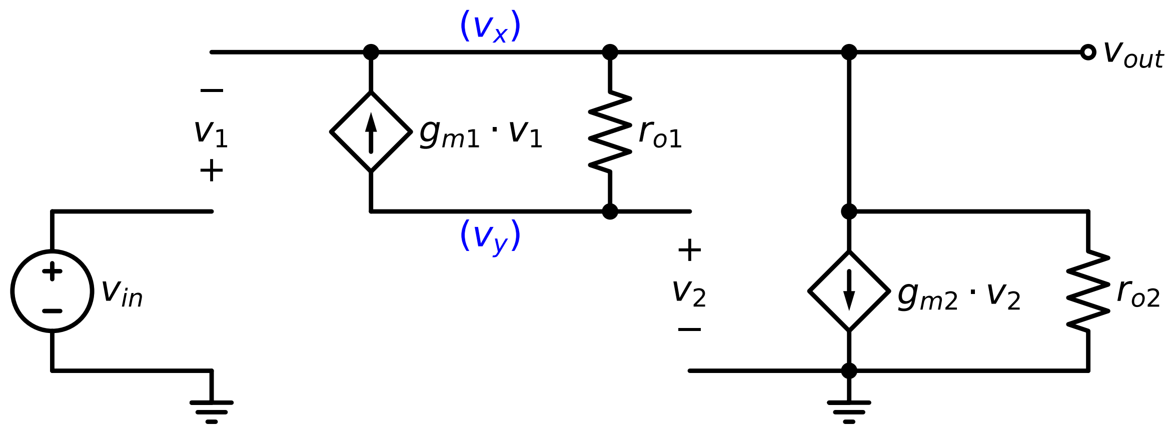


Figure 60: Small-signal model of the super-source follower

$$T_{sc} = -\frac{i_r}{i_t}|_{sc} = 0$$

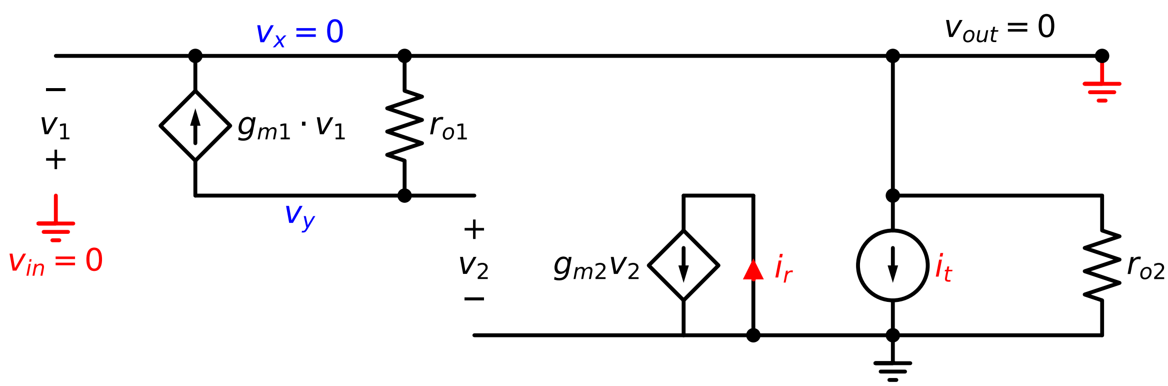


Figure 61: Return Ratio for g_{m2} with the output port shorted

Shorting the output port forces $v_{out} = 0$

$v_{out} \equiv v_x \Leftrightarrow v_x = 0$

$v_x = -v_1 \leftrightarrow v_1 = 0$

$g_{m1} \cdot v_1 = 0 \Leftrightarrow v_y = 0$

$v_2 = x_x - v_y = 0 \Leftarrow g_{m2}v_2 \equiv i_r = 0$

- The return ratio for g_{m2} with the output port open is:

$$T_{open} = -\frac{i_r}{i_t}|_{open} = g_{m2}r_{o2}\left(1 + g_{m1}r_{o1}\right)$$

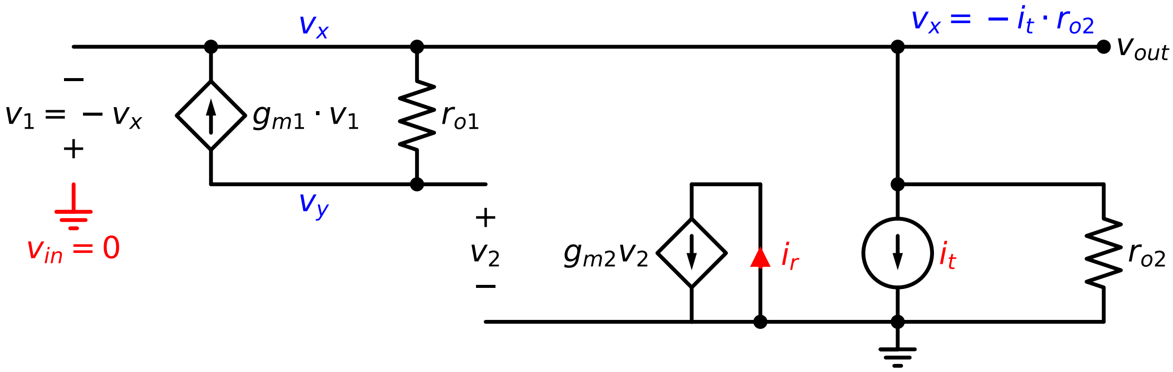


Figure 62: Return Ratio for g_{m2} with the output port open

Opening the output port gives:

$v_x = -i_t \cdot r_{o2} \Leftrightarrow i_t = -\dfrac{v_x}{r_{o2}}$

$v_x = -v_1$

$v_y - v_x = -g_{m1} \cdot v_1 \cdot r_{o1} \Leftrightarrow v_y = v_x + g_{m1} \cdot r_{o1} \cdot v_x \Leftrightarrow v_y = v_x\left(1 + g_{m1}r_{o1}\right)$

$v_2 = v_y$

$i_r \equiv g_{m2}v_2 = g_{m2}\left(1 + g_{m1} \cdot r_{o1}\right) \cdot v_x$

$T_{open} = -\dfrac{i_r}{i_t} = g_{m2}r_{o2}\left(1 + g_{m1}r_{o1}\right)$

- The closed-loop output resistance is:

$$R_{out} = R_{out0} \cdot \frac{1 + T_{sc}}{1 + T_{open}} = \frac{r_{o2}}{1 + g_{m2}r_{o2}\left(1 + g_{m1}r_{o1}\right)}$$

Assuming $g_m r_o \gg 1$

$$R_{out} \approx \frac{1}{g_{m1}g_{m2}r_{o1}}$$

Example 12 - Input and output resistance of shunt-shunt stage (TIA)

Closed-loop input resistance

$R_{in}(g_m = 0) = R_F + r_o$

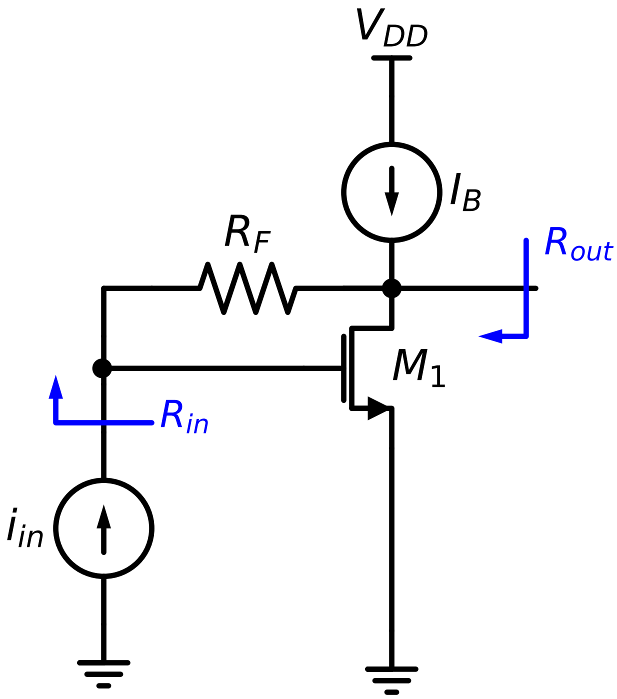


Figure 63: Shunt-shunt stage (Transimpedance Amplifier)

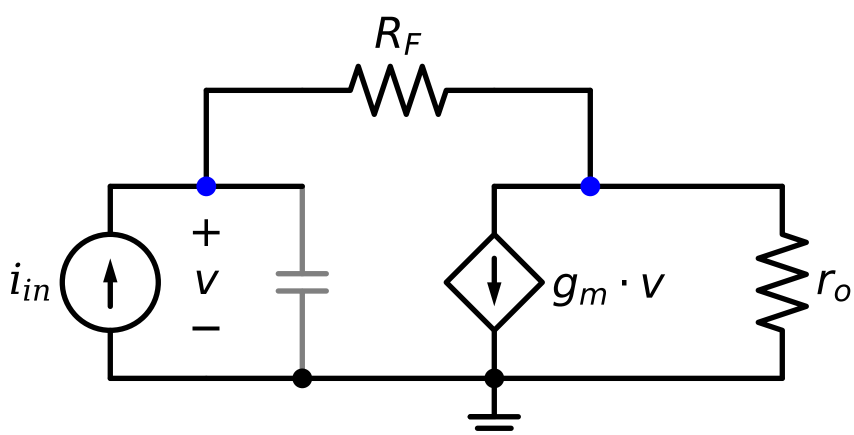


Figure 64: Small signal model of the shunt-shunt stage

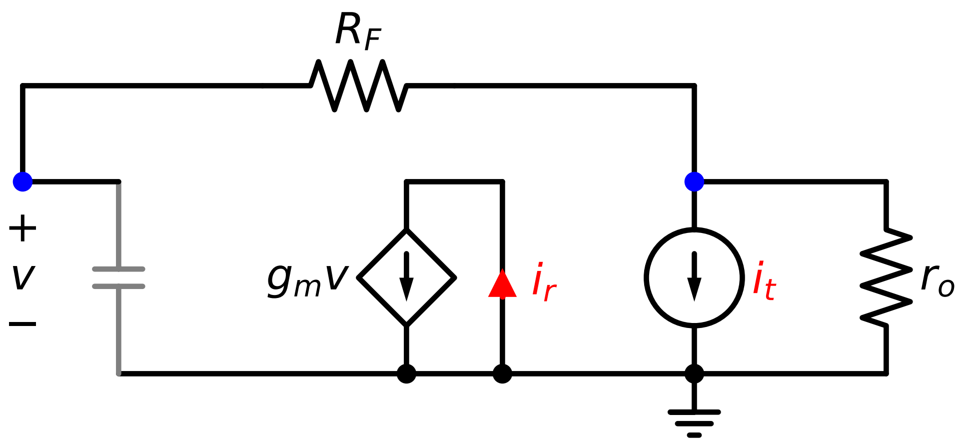


Figure 65: Small signal model to find the return ratio for g_m

$T(inputshorted) = 0$

$T(inputopen) = g_m r_o$

Therefore, the closed-loop input resistance is:

$$R_{in} = (R_F + r_o) \cdot \frac{1}{1 + g_m r_o} \approx \frac{1}{g_m}\left(1 + \frac{R_F}{r_o}\right)$$

Closed-loop output resistance

$R_{out}(g_m = 0) = r_o$

$T(inputshorted) = 0$

$T(inputopen) = g_m r_o$

Therefore, the closed-loop output resistance is:

$$R_{out} = r_o \cdot \frac{1}{1 + g_m r_o} \approx \frac{1}{g_m}$$

Example 13 - Output resistance of active cascode

The "active cascode" circuit is also referred as "regulated cascode" or "gain-boosting" technique.

- To find R_{out0} we set $v_{in} = 0$ and $k = a_v = 0$

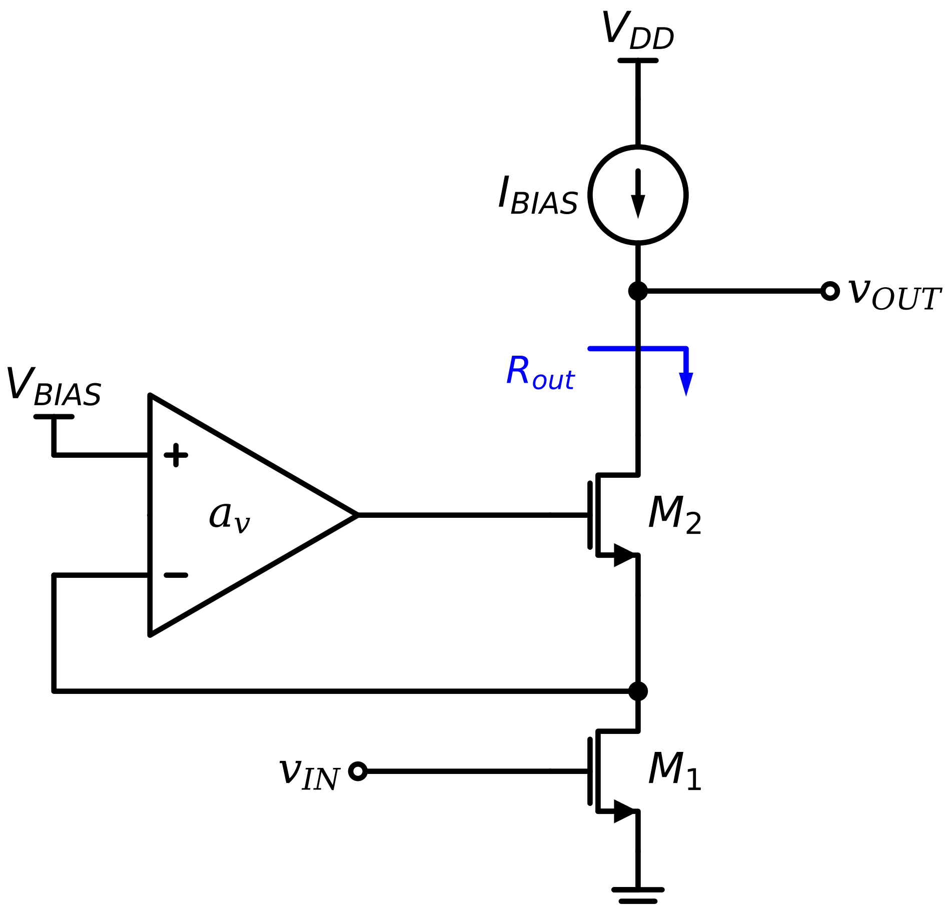


Figure 66: Active cascode gain stage

With $a_v = 0$ the gate of M_2 gets grounded:

$$R_{out0} \equiv R_{out}(a_v = 0) = r_{o1} + r_{o2}\,(1 + g'_{m2}) \approx r_{o1} \cdot r_{o2} \cdot g'_{m2}$$
$$with g'_{m2} = g_{m2} + g_{m2b}$$

- To find the return ratio for a_v with the output port open, we set $v_{in} = 0$ and leave the output node floating

The current of the g_m-generators ($g_{m2}v_{gs2}$ and $g_{mb2}v_{bs2}$) flows only in r_{o1}, changing the drain voltage of M_2, but keeping the source voltage of M_2 to $v_x = 0 \Rightarrow v_d = -v_x = 0$. Therefore, $v_r = a_v \cdot v_d = 0$

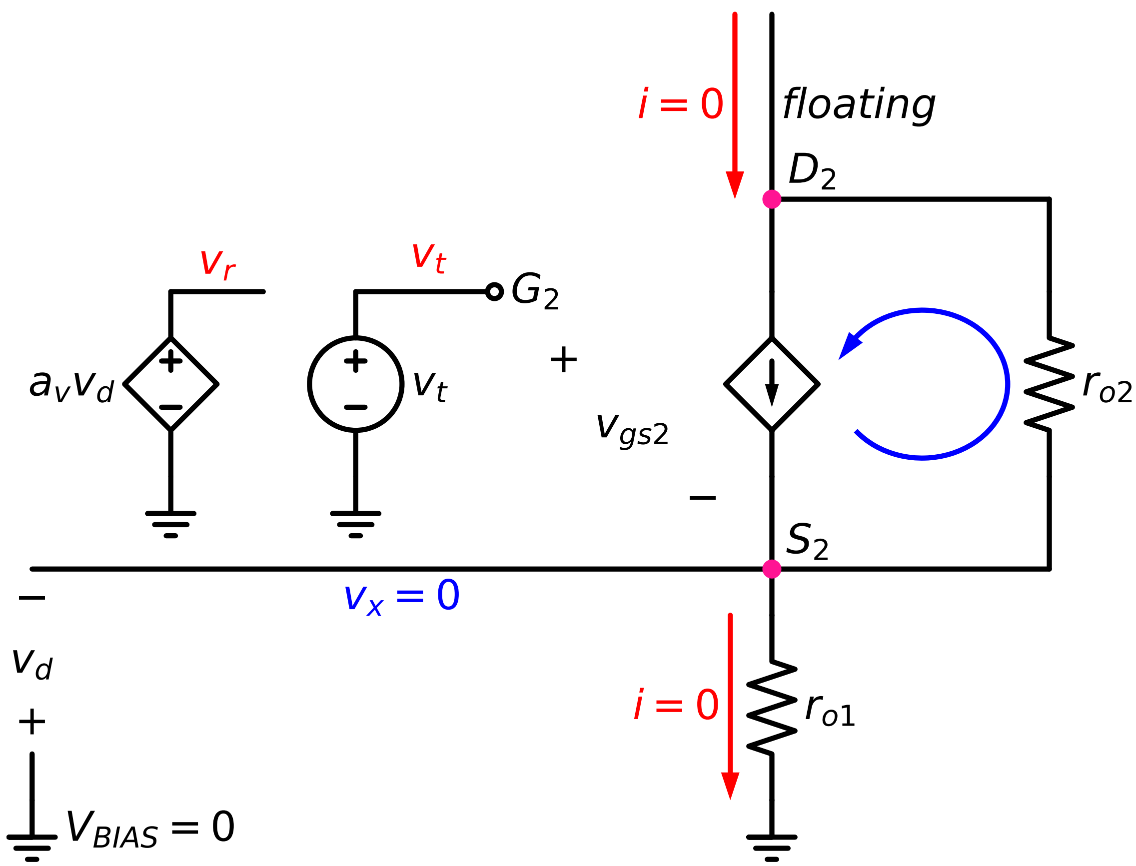


Figure 67: Return ratio for a_v with the output port open

$$T_{open} = -\frac{v_r}{v_t}|_{open} = 0$$

- To find the return ratio for a_v with the output port shorted, we set $v_{in} = 0$ and ground the output node (the output node is the drain of M_2)

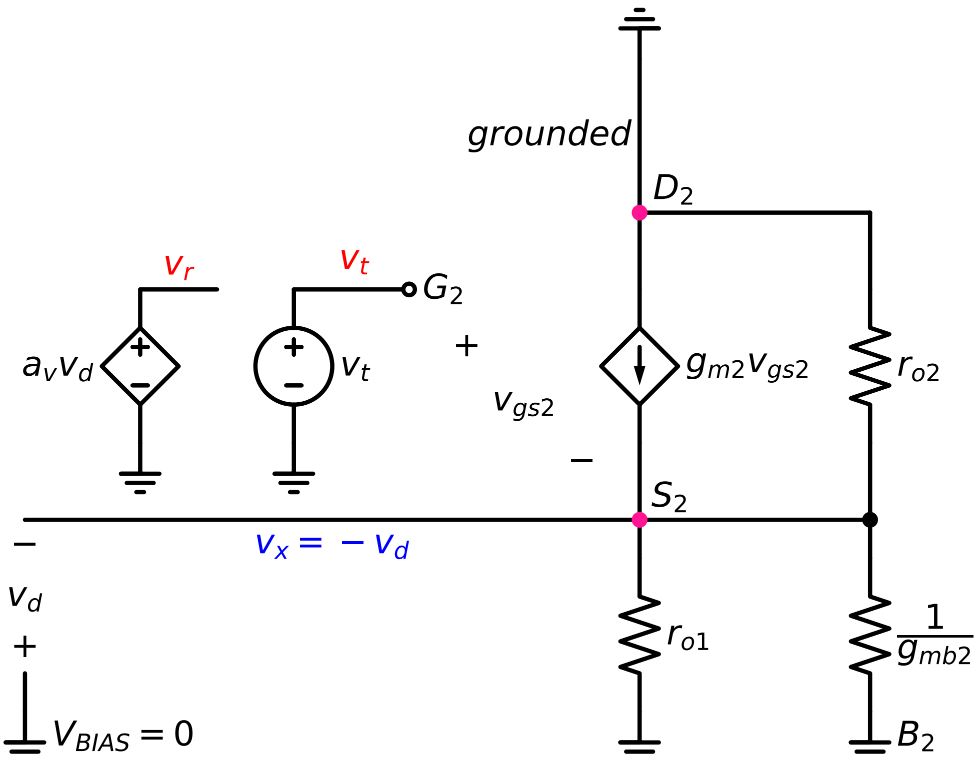


Figure 68: Return ratio for a_v with the output port shorted

With M_2 drain to ground (CD), $v_x = -v_d$ is the output of a CD with v_t as input.

$v_t = v_{gs2} - v_d$

Neglecting the r_o terms w.r.t. the $1/g_{mb2}$ term:

$$-v_d \approx \frac{g_{m2} v_{gs2}}{g_{mb2}} \Rightarrow v_t \approx v_{gs2}\left(1 + \frac{g_{m2}}{g_{mb2}}\right) \Rightarrow -\frac{v_d}{v_t} \approx \frac{g_{m2}}{g_{m2} + g_{mb2}} \approx 1$$

so the return ratio is:

$$T_{sc} = -\frac{v_r}{v_t}|_{sc} \approx a_v \cdot \frac{g_{m2}}{g_{m2} + g_{mb2}} \approx a_v$$

- The closed-loop output resistance is:

$$R_{out} = R_{out0} \cdot \frac{1 + T_{sc}}{1 + T_{open}} \approx r_{o1} \cdot r_{o2} \cdot g'_{m2} \cdot a_v$$

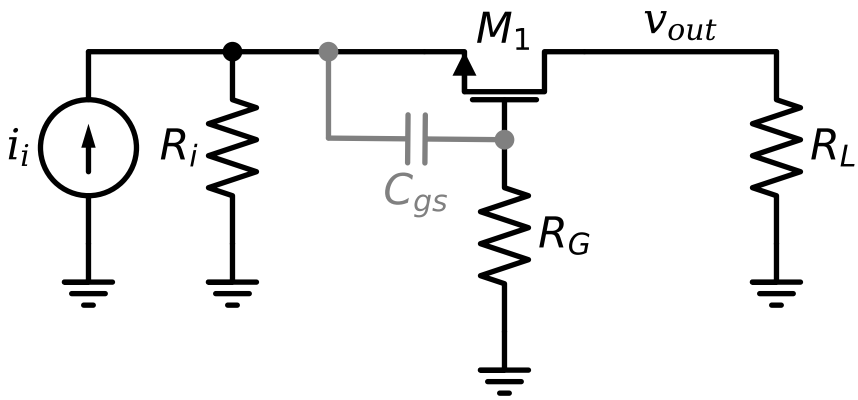


Figure 69: ZVTC calculations

Example 14 - Using Blackman's formula for ZVTC calculations

Using first principles (KVL and KCL) we can derive that:

$$\tau_{Cgs} = C_{gs} \cdot \frac{R_i + R_G}{1 + g_m R_i}$$

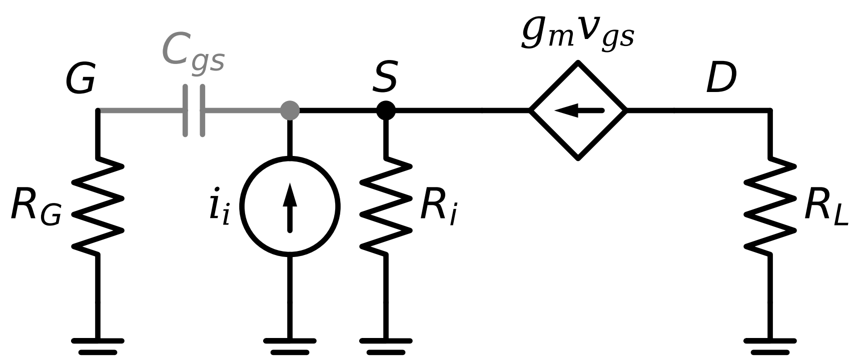


Figure 70: Small signal model for ZVTC calculations, assuming r_o can be ignored

Let's look at the problem, using return ratio.

The port (gate-source) impedance with the return ratio's gain element g_m set to 0 is:

$R_{gs}(g_m = 0) = R_i + R_G$

The return ratio for the gain element g_m with port under consideration shorted is:

$T_{sc} = -\frac{i_r}{i_t}|_{sc} = 0$

Shorting the port forces $v_{gs} = 0$ so it "kills" the g_m generator.

The return ratio for the gain element g_m with port under consideration open is:

$T_{open} = -\frac{i_r}{i_t}|_{open} = g_m \cdot R_i$

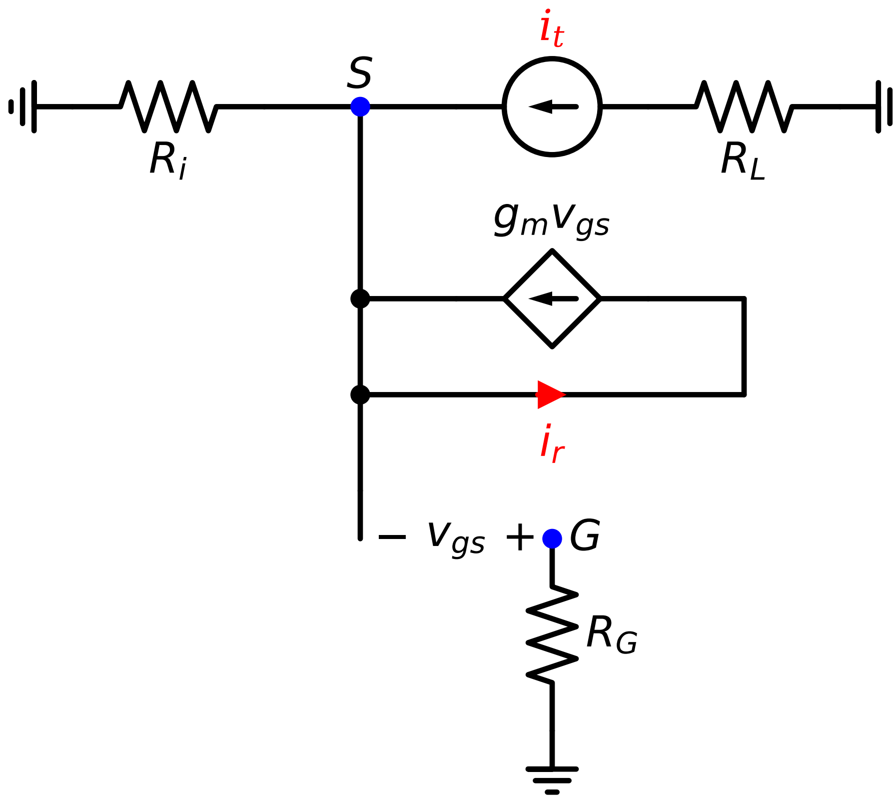


Figure 71: return ratio with port G-S open

$v_s = i_t \cdot R_i$

$v_g = 0 \Rightarrow v_{gs} = -i_t \cdot R_i$

$i_r = g_m v_{gs} = -g_m R_i i_t$

The closed-loop port impedance between G and S is:

$$R_{gs} = R_{gs}(g_m = 0) \cdot \frac{1 + T_{sc}}{1 + T_{open}} = \frac{R_i + R_G}{1 + g_m R_i}$$

Example 15 - Voltage gain and output resistance of common source with degeneration using return ratio

voltage gain (assuming r_o negligible)

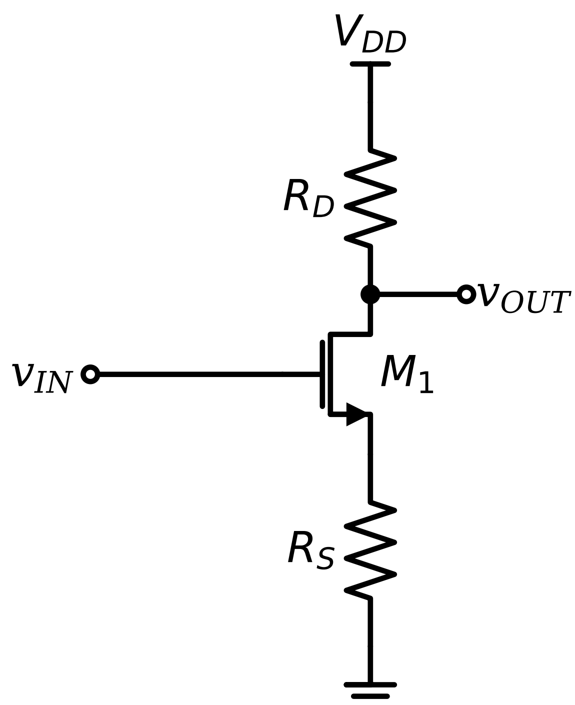


Figure 72: Common source with degeneration

$$A_v = \frac{v_{out}}{v_{in}} = A_\infty \cdot \frac{T}{1+T} + \frac{d}{1+T}$$

A_∞ is the gain of the circuit when the gain element k of the controlled source tends to ∞ (note that if $k = g_m = \infty$, for the current of the dependent source to be finite, it must be $v_{gs} = 0$).

$A_\infty = \frac{v_{out}}{v_{in}}|_{g_m \to \infty} = -\frac{R_D}{R_S}$

d is the gain of the circuit when the gain element k of the controlled source equals 0.

$d = \frac{v_{out}}{v_{in}}|_{g_m = 0} = 0$

T is the return ratio of the dependent source.

To compute the return ratio instead of separating the dependent source from the rest of the circuit and replacing it with an independent source of the same kind as always done so far, we will follow an equivalent procedure consisting of adding an independent source s_x. In practice, this procedure is usually more easily performed, because it does not require tearing the original circuit apart. See (Middlebrook 2006) and (Hajimiri 2023).

The introduction of s_x is completely equivalent to using an independent source s_t in place of the separated dependent source and monitoring the signal s_r that returns to the separated dependent source.

$s_t = s_x + s_r$

$v_{gs} = -i_t R_S$

$i_r = g_m v_{gs} = -g_m R_S i_t$

$T = -\frac{i_r}{i_t} = g_m R_S$

Therefore, as expected:

$$A_v = \frac{v_{out}}{v_{in}} = A_\infty \cdot \frac{T}{1+T} + \frac{d}{1+T} = \frac{-g_m R_D}{1+g_m R_S}$$

Output resistance

To find the output resistance we use Blackman's formula.

The output resistance with $g_m = 0$ is:

$R_{out}(g_m = 0) = R_S = r_o$

With the output port open:

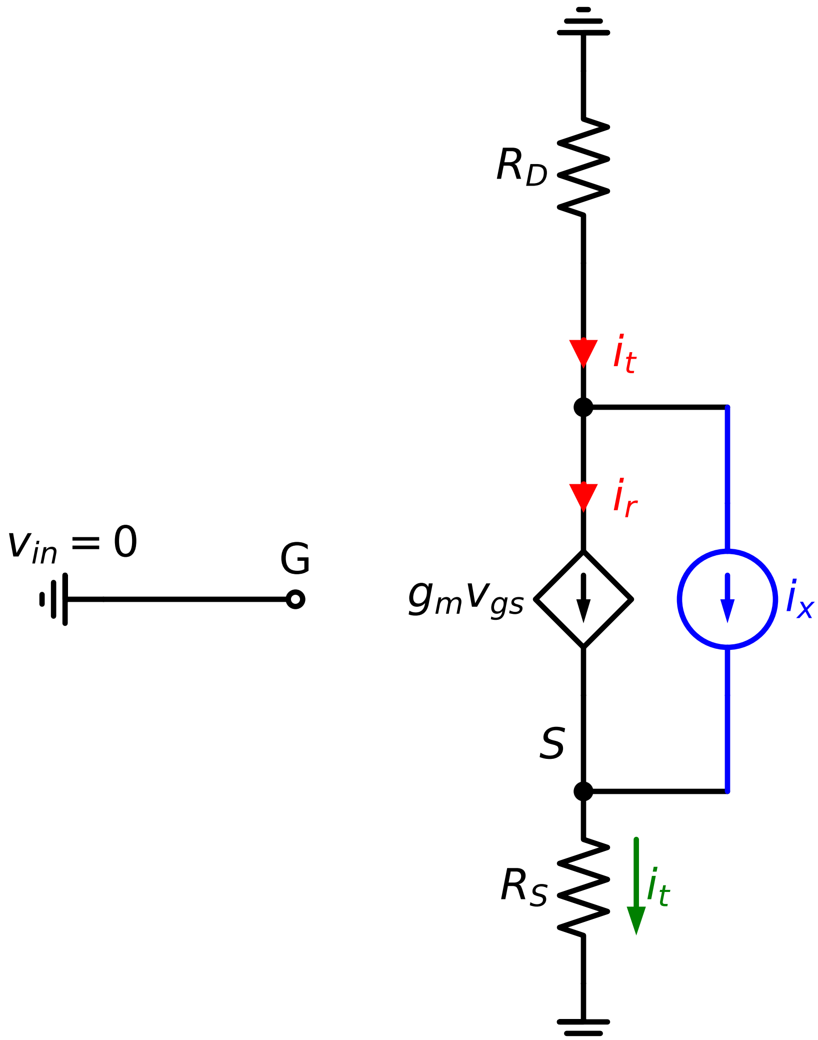


Figure 73: Small signal model to find the return ratio of g_m

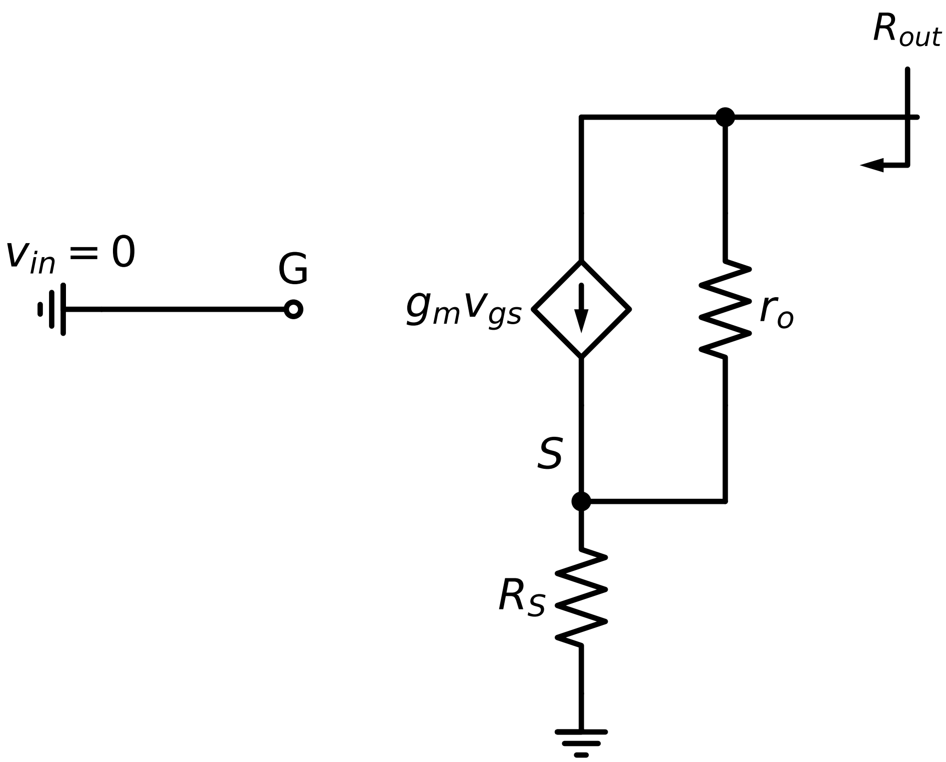


Figure 74: Small signal model to find the output resistance

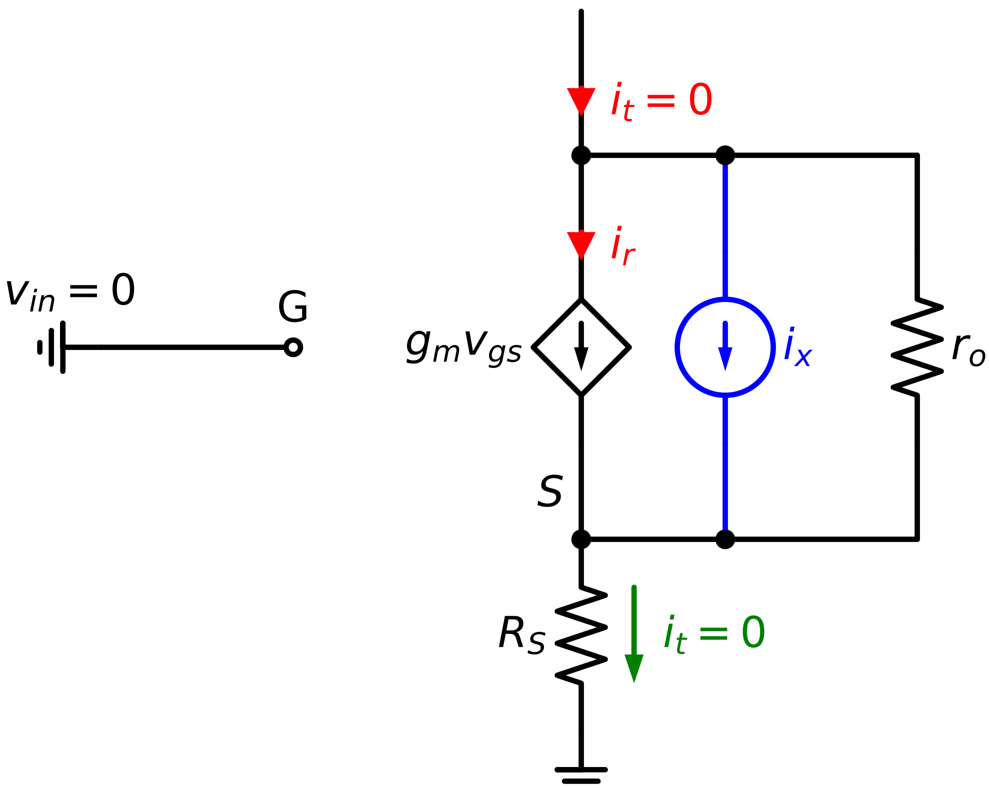


Figure 75: return ratio with output port open

$v_{gs} = -i_t R_S = 0 \Rightarrow i_r = g_m v_{gs} = 0$

So, the return ratio with the output port open is:

$T_{open} = -\frac{i_r}{i_t}|_{open} = 0$

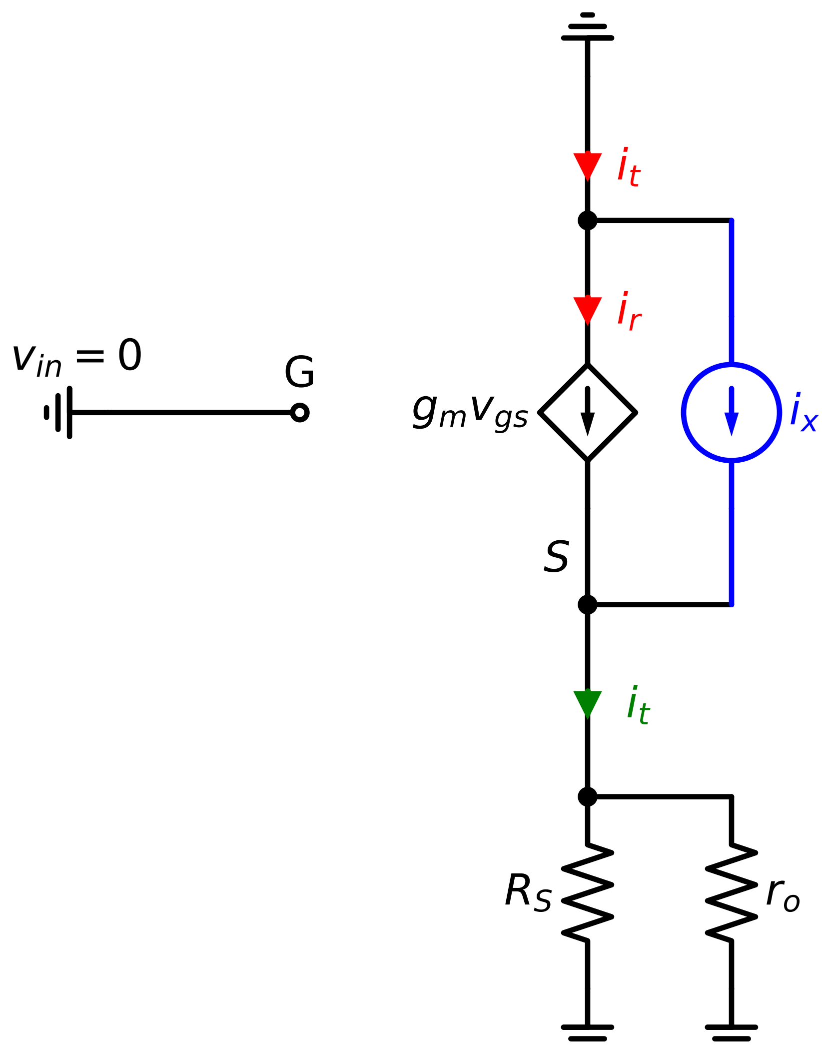


Figure 76: return ratio with output port shorted

With the output port shorted:

$v_{gs} = -(R_S||r_o) \cdot i_t \Rightarrow i_r = g_m v_{gs} = -g_m \cdot (R_S||r_o) \cdot i_t$

So, the return ratio with the output port shorted is:

$T_{sc} = -\frac{i_r}{i_t}|_{sc} = g_m(R_S||r_o)$

And the output resistance is:

$$\begin{aligned}R_{out} &= R_{out}(g_m = 0) \cdot \frac{1 + T_{sc}}{1 + T_{open}} = (R_S + r_o)\left(1 + g_m \frac{R_S r_o}{R_S + r_o}\right) = \\ &= R_S + r_o(1 + g_m R_S)\end{aligned}$$

8.3 Summary - Return Ratio Analysis

- Return-ratio analysis is an alternative approach to two-port feedback analysis.
- Provided the return ratio T is computed for a dependnt source that breaks the feedback loop globally, the return ratio is a measure of loop gain. In general, the return ratio of a controlled source is a measure of how much of the signal generated by that controlled source is returned due to feedback, it is not necessarily the loop gain (Hurst 1992).
- For negative feedback circuits T > 0
- In an ideal feedback system $T \to \infty$ and the closed loop gain is A_∞
- Blackman's formula gives the closed-loop impedance in terms of two return ratios. The formula is the same for any type of feedback circuit, and it applies to any port (not only the input and output ports)
- Return ratio analysis is often simpler than two-port analayis
- Return ratio analysis uses equations that are independent of the type of feedback.
- Two-port analysis uses four-feedback configurations (series-series, series-shunt, shunt-series, and shunt-shunt), therefore, the type of feedback must be correctly identified before starting the analysis
- In practice, all physical networks are multiloop feedback structures. Unwanted local return loops exists around individual transistor through the parasitic capacitances (Tian et al. 2001) (Behmanesh and Andreani 2023). Dealing with multiple feedback mechanisms makes identifying an appropriate controlled source to use for the return ratio analysis more difficult. When dealing with multiloop feedback structures, the controlled source used to compute the return ratio, must be one that breaks the loop globally (Hurst and Lewis 1995).
- One aspect that is often cause of confusion, about return-ratio analysis and two-port analysisis is that the loop gain L computed through two-port analysis and the loop gain T computed using the return ratio, can be different. This is because they are intrinsically two different measures of transmission, however both methods lead to correct closed-loop expressions and both can be used to check stability (Chiu 2012).

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