1\. a) $\int_0^1 (x^2+xy+y^2)\,dy$

$$\left(y\cdot x^2 + x\cdot\frac{y^2}{2}+\frac{y^3}{3}\right)\Big|_{y=0}^{y=1} = x^2+\frac{x}{2}+\frac{1}{3}$$

b) $$\frac{-(y-x)^5}{5}\Big|_{x=y}^{x=y^2} = \frac{-(y-y^2)^5}{5}$$

2\. a) $$\int_0^1 \frac{x}{(1+x^2+y^2)^{\frac{3}{2}}}\,dx = \frac{1}{2}\int_{2+y^2}^{1+y^2}\frac{dt}{t^{\frac{3}{2}}} = \frac{1}{2}\cdot\frac{t^{-\frac{1}{2}}}{-\frac{1}{2}}\Big|_{1+y^2}^{2+y^2}$$

$t = 1+x^2+y^2$

$dt = 2x\,dx$

$x=0 \Rightarrow t = 1+y^2$

$x=1 \Rightarrow t = 2+y^2$

$$= -\left(\frac{1}{\sqrt{2+y^2}}-\frac{1}{\sqrt{1+y^2}}\right)$$

$$= \int_0^1\left(-\frac{1}{\sqrt{2+y^2}}+\frac{1}{\sqrt{1+y^2}}\right)dy = -\ln\left(y+\sqrt{2+y^2}\right)+\ln\left(y+\sqrt{1+y^2}\right)\Big|_0^1$$

b) $$\int_0^{\frac{1}{x}}\frac{1}{1+x^2y^2}\,dy = \frac{1}{x}\int_0^{\frac{1}{x}}\frac{(x\cdot y)'_y}{1+(x\cdot y)^2}\,dy = \frac{1}{x}\,\mathrm{actg}(x\cdot y)\Big|_{y=0}^{y=\frac{1}{x}}$$

$$= \frac{1}{x}\,\mathrm{arctg}\,1 = \frac{\pi}{4x}$$

$$\int_1^2\frac{\pi}{4x}\,dx = \frac{\pi}{4}\ln x\Big|_1^2$$

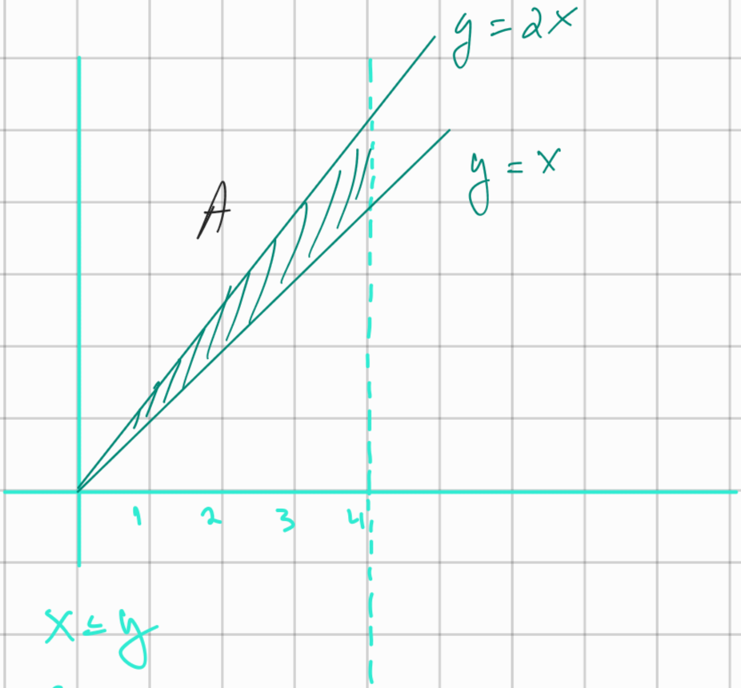
3. a) $\iint_A \frac{x}{1+xy}\,dx\,dy = \int_0^1 \left(\int_0^2 \frac{x}{1+xy}\,dy \right) dx =$

$= \int_0^2 \left(\int_0^1 \frac{x}{1+xy}\,dx \right) dy$

$\int_0^2 \frac{x}{1+xy}\,dy = \ln(1+xy) \Big|_{y=0}^{y=2} = \ln(1+2x)$

$\int_0^1 x' \ln(1+2x)\,dx = \dots$

b)



$x \le y$

$y \le 2x$

A simplă în raport cu Ox

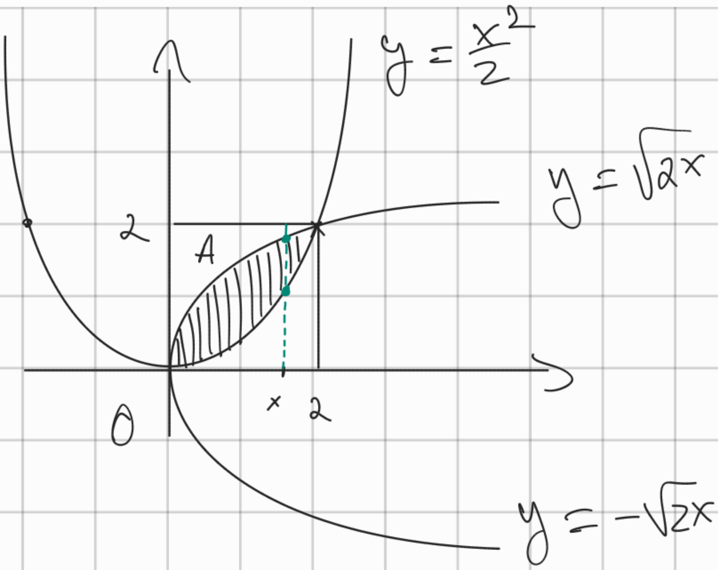
$\iint_A xy\,dx\,dy = \int_0^4 \left(\int_x^{2x} xy\,dy \right) dx = \int_0^4 \frac{xy^2}{2} \Bigg|_{y=x}^{y=2x} dx =$

$= \int_0^4 \frac{4x^3}{2} - \frac{x^3}{2}\,dx = \int_0^4 \frac{3}{2} x^3\,dx = \frac{3}{2} \frac{x^4}{4} \Big|_0^4$

d) $y^2 = 2x$

$x^2 = 2y$

$y = \frac{x^2}{2}$



$$\iint_A dx\,dy = \text{aria}(A)$$

$$A = \left\{ (x,y) \in \mathbb{R}^2 \;\middle|\; 0 \le x \le 2,\ \frac{x^2}{2} \le y \le \sqrt{2x} \right\}$$

$$\text{aria}(A) = \int_0^2 \left(\int_{\frac{x^2}{2}}^{\sqrt{2x}} dy \right) dx = \int_0^2 y \Big|_{\frac{x^2}{2}}^{\sqrt{2x}} dx$$

$$= \int_0^2 \left(\sqrt{2x} - \frac{x^2}{2} \right) dx$$

$$= \int_0^2 (2x)^{\frac{1}{2}} - \frac{x^2}{2}$$

$$= \int_0^2 2^{\frac{1}{2}} \cdot x^{\frac{1}{2}} - \frac{x^2}{2}\, dx$$

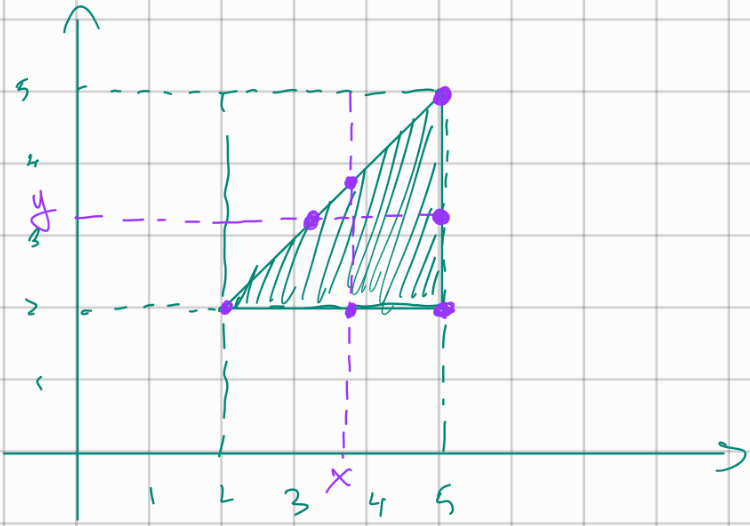
$$= \int 2^{\frac{1}{2}} \cdot x^{\frac{1}{2}}\, dx - \int \frac{x^2}{2}\, dx$$

$$= \left(\frac{2\sqrt{2x} \cdot |x|}{3} - \frac{x^3}{6} \right) \Bigg|_0^2$$

$$= \frac{4}{3}$$

2) A simplă Ox

$$A = \{(x,y) \in \mathbb{R}^2 \mid 2 \le x \le 5,\ 2 \le y \le x\}$$



A simplă în Oy

$$A = \{(x,y) \in \mathbb{R}^2 \mid 2 \le y \le 5,\ y \le x \le 5\}$$

$$\iint_A \frac{y}{x}\,dxdy = \int_2^5 \left(\int_2^x \frac{y}{x}\,dy \right) dx = \int_2^5 \left(\int_y^5 \frac{y}{x}\,dx \right) dy = \ldots$$

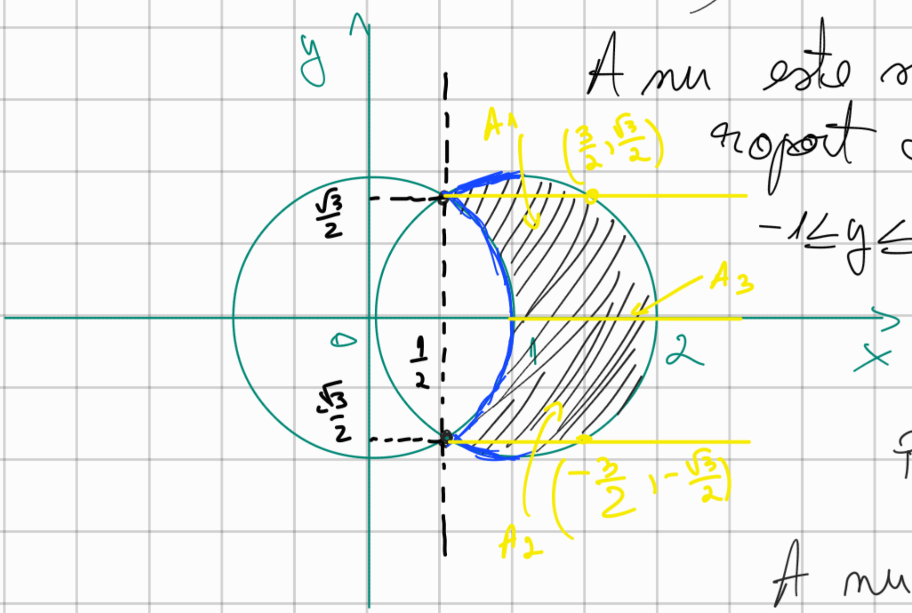
(4) $A = \{(x,y) \in \mathbb{R}^2 \mid 1 \le x^2 + y^2 \le 2x\}$

$x^2 + y^2 = 1$

$x^2 + y^2 = 2x$

$x^2 - 2x + 1 + y^2 = 1$

$(x-1)^2 + y^2 = 1$



A nu este simplă în raport cu Ox

$-1 \le y \le 1,\ \varphi(y) \le x \le \psi(y)$

φ nu este clasă C

A nu este simplă cu Oy

$$A_1 = \left\{ (x,y) \in \mathbb{R}^2 \;\middle|\; \frac{1}{2} \le x \le \frac{3}{2},\ \frac{\sqrt{3}}{2} \le y \le \sqrt{2x - x^2} \right\}$$

$$\begin{cases} y = \frac{\sqrt{3}}{2} \\ x^2 + y^2 = 2x \\ x^2 + \frac{3}{4} - 2x = 0 \\ x^2 - 2x + 1 = \frac{1}{4} \\ (x-1)^2 = \frac{1}{4} \\ x = 1 \pm \frac{1}{2} \end{cases}$$

$$A_2 = \left\{ (x,y) \in \mathbb{R}^2 \;\middle|\; -\frac{\sqrt{3}}{2} \le y \le \frac{\sqrt{3}}{2},\ \sqrt{1-y^2} \le x \le 1 + \sqrt{1-y^2} \right\}$$

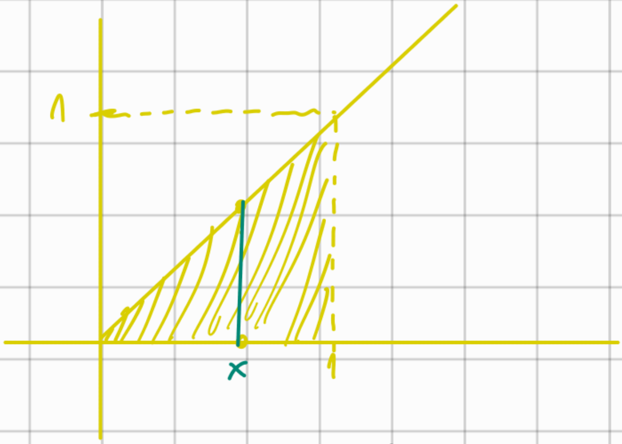
Analog A_3

$$\iint_A f(x,y)\,dx\,dy + \iint_A f\,dx\,dy + \iint_{A_3} f\,dx\,dy$$

(5) $$\int_0^1 \left(\int_y^1 \frac{1}{1+x^4}\,dx \right) dy = \iint_A \frac{1}{1+x^4}\,dx\,dy = *$$

$$A = \left\{ (x,y) \in \mathbb{R}^2 \;\middle|\; 0 \le y \le 1,\ y \le x \le 1 \right\}$$

Mulțime simplă în raport cu Oy



A simplă în raport Ox:

$$A=\{(x,y)\in\mathbb{R}^2 \mid 0\le x\le 1,\ 0\le y\le x\}$$

$$\iint = \int_0^1\left(\int_0^x \frac{1}{1+x^4}\,dy\right)dx = \int_0^1 \frac{y}{1+x^4}\Big|_{y=0}^{y=x} dx =$$

$$= \int_0^1 \frac{x}{1+x^4}\,dx = \frac{1}{2}\int_0^1 \frac{(x^2)'}{1+(x^2)^2}\,dx = \frac{1}{2}\operatorname{arctg}(x^2)\Big|_0^1 =$$

$$= \frac{1}{2}\operatorname{arctg} 1 = \frac{\pi}{8}$$

$\boxed{1,5}$

$\boxed{2} + \boxed{8} = 10$

$\boxed{3}$

$\boxed{1,5}$

$8 \ldots 10$

$3 \ldots x$

$$\frac{8}{3} = \frac{10}{x}$$

$$8x = 30$$

$$x = \frac{30}{8} = \underline{3,75}$$