Atmospheric Effects

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Implementatio

Atmospheric effects for gorund-based CMB observations

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24 - June - 2020



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Atmospheric load in the microwave band

- Due to absorption / emission processes
 - Water vapor
 - Oxygen molecules
- The oxygen is well mixed in the atmosphere
- The water vapor presents highly variable concentrations

Assessment of the scientific impact

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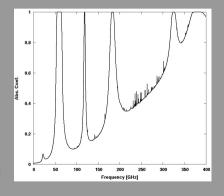
Dispersive medium: $\varepsilon(\omega) = \varepsilon(\omega)_R + i \varepsilon(\omega)_L$

The contribute to complex permittivity are due mostly to the water vapor and the oxygen molecules that are dissolved in the atmosphere

$$arepsilon arepsilon (\omega)_R = \sqrt{n}$$
, and $arepsilon (\omega)_I = \lambda lpha / 4\pi$

The real and complex parts of $\varepsilon(\omega)$ are linked by the Kramers-Krönig relations

There is a relation also between the refractive



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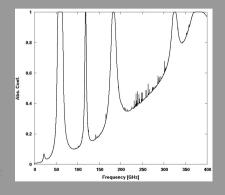
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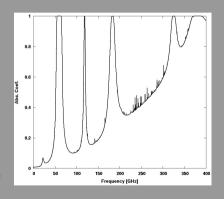
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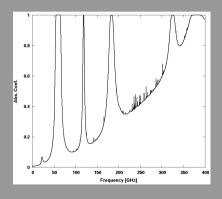
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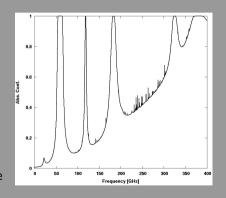
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Fluctuations in the refractive index

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Refractive index fluctuations induce a change in the optical path length, and leds a two different systematic effects for interferometry or single dish, relatively



Interferometer: Phase fluctuations, degrade the interference pattern.



Single-dish: can cause fluctuations in the apparent pointing

An interesting power-low relation

There is a relations between the RMS phase fluctuations and the interference baseline $\langle \phi^2 \rangle \sim D^{5/3}$ [see S.E. Church paper]

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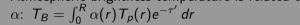
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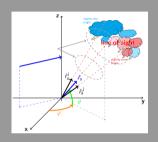
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Atmospheric brightness temperature is related to



$$T_A = \frac{1}{\lambda^2} \int_V A(\hat{r}_s, \vec{r}) T_p(\vec{r}) \frac{dV}{r^2}$$



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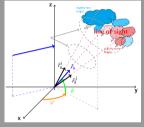
The Model

Implementation

Parametei

- Atmospheric brightness temperature is related to α : $T_B = \int_0^R \alpha(r) T_p(r) e^{-r} dr$
- \circ $e^{- au}\sim$ 1, au fluctuations are small compared to lpha
- Antenna temperature:

$$T_A = \frac{1}{\lambda^2} \int_V A(\hat{r}_s, \vec{r}) T_p(\vec{r}) \frac{dV}{r^2}$$



$$\langle T_A^2 \rangle = \frac{1}{\lambda^4} \int_V \frac{dV}{r^2} \int_{V'} \frac{dV'}{r'^2} A(\hat{r}_s, \vec{r}) A(\hat{r'}_s, \vec{r'}) T_p(\vec{r}) T_p(\vec{r'}) \langle \alpha(\vec{r}), \alpha(\vec{r'}) \rangle$$

Atmosphere structures

The correlation term $\langle \alpha(r), \alpha(r') \rangle$ is unknown.

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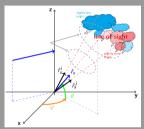
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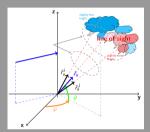
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Interferometric observation of the atmospheric structure

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Parameters

- Structural spatial function $D_n(r_1, r_2) = \langle [n(r_1) n(r_2)]^2 \rangle$
- Inner and outer correlation lengths I_0 and L
- Specific humidity a

Energy injection

Energy transfer

Dissipation

(a) Isotropic turbulent atmosphere

- Ground temperature T_0
- Solar irradiation
- Wind speed and direction
- Structural spatial function for r

$$D(r_1, r_2) \propto L_0^{4/3} \left(\frac{dn}{da} \frac{dq}{dz}\right) |r_1 - r_2|^{2/3} \propto C_0^2 \exp\left(-\frac{z}{z_0}\right)$$

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Paramete

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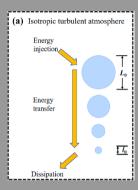
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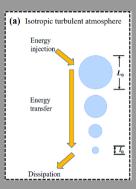
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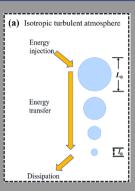
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Structural spatial function $D_n(r_1,r_2)=\langle [n(r_1)-n(r_2)]^2
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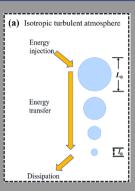
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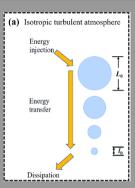
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Structural spatial function for n

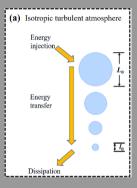
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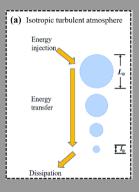


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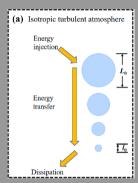
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- The refractive index and the absorption coefficient are linked by the KK relations

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The structural spatial function for the atmospheric absorption coefficient

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- We can neglect second order term and consider $D_n = D_{\alpha}$

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, using the correlation $B_{\alpha}(r_1, r_2) = \langle \alpha(r_1), \alpha(r_2) \rangle$

$$D_{\alpha}(r_1, r_2) = \langle [\alpha(r_1) - \alpha(r_2)]^2 \rangle = B_{\alpha}(r_1, r_1) + B_{\alpha}(r_2, r_2) - 2 B_{\alpha}(r_1, r_2) =$$

$$= C_{\alpha}^2 \left(\frac{r_1 + r_2}{2} \right) |r_1 - r_2|^{2/3}$$

$$B_{\alpha}(r_1, r_2) = \frac{1}{2} C_{\alpha}^2 \left(\frac{r_1 + r_2}{2} \right) L_0^{2/3} \left(1 - \frac{|r_1 - r_2|^{2/3}}{L_0^{2/3}} \right)$$

The structural spatial function for the atmospheric absorption coefficient

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The $\langle \alpha(r_1), \alpha(r_2) \rangle$ expression

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can be written more general

$$B_{\alpha}(r_1, r_2) = \frac{1}{2} C_{\alpha}^2 \left(\frac{r_1 + r_2}{2} \right) L_0^{2/3} b_{\alpha}(|r_1 - r_2|) \tag{1}$$

$$\varrho_{\alpha}(r) \propto \frac{1}{r} \int_{k_0}^{k_M} k \Phi(k) \sin(k \cdot r) dk$$
(2)

The structural spatial function for the atmospheric absorption coefficient

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The $\langle \alpha(r_1), \alpha(r_2) \rangle$ expression

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$$b_{\alpha}(r) \propto \frac{1}{r} \int_{k_{\alpha}}^{k_{M}} k\Phi(k) \sin(k \cdot r) dk$$
 (2)

Different kind of b_{α} spectrum

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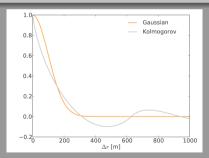
Kolmogorov-Modified power spectrum

$$b_{\alpha}(r) \propto \frac{1}{r} \int_{k_0}^{k_M} k \Phi(k) \sin(k \cdot r) dk$$
 (3)

Kolmogorov-Taylor model

 $\Phi(k) \propto k^{-11/3}$ $k_0 = 1/L_0$ $k_M = 1/l_0$

Gaussian-Like correlation



$$L_0 = 100 m$$

Different kind of b_{α} spectrum

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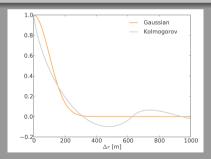
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$$b_{\alpha}(r) \propto \frac{1}{r} \int_{k_0}^{k_M} k \Phi(k) \sin(k \cdot r) dk$$
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- Kolmogorov-Taylor model
 - $\Phi(k) \propto k^{-11/3}$
 - $k_0 = 1/L_0$
 - $k_{\rm M} = 1/h$
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Different kind of b_{α} spectrum

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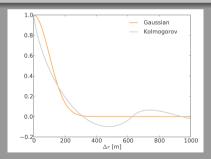
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$$b_{\alpha}(r) \propto \frac{1}{r} \int_{k_0}^{k_M} k \Phi(k) \sin(k \cdot r) dk \tag{3}$$

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Different kind of b_{α} spectrum

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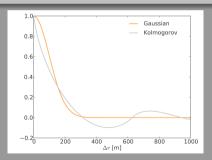
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$$b_{\alpha}(r) \propto \frac{1}{r} \int_{k_0}^{k_M} k \Phi(k) \sin(k \cdot r) dk$$
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Different kind of b_{α} spectrum

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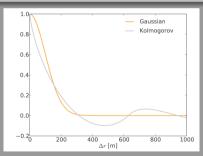
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$$b_{\alpha}(r) \propto \frac{1}{r} \int_{k_0}^{k_M} k \Phi(k) \sin(k \cdot r) dk$$
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$$b_{\alpha}(|r_1 - r_2|) = \exp\left[-\frac{|r_1 - r_2|^2}{2(L_0/3)^2}\right]$$



$$L_0 = 100 m$$

Different kind of b_{α} spectrum

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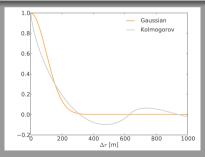
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$$b_{\alpha}(r) \propto \frac{1}{r} \int_{k_0}^{k_M} k \Phi(k) \sin(k \cdot r) dk$$
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- Kolmogorov-Taylor model
 - $\Phi(k) \propto k^{-11/3}$
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$$L_0 = 100 m$$

Temporal fluctuations in antenna temperature

The rigid translation of the atmospheric structures by the wind

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$$T_A(t) = \frac{1}{\lambda^2} \int_{z_1}^{z_u} \iint_{-\infty}^{+\infty} \alpha(x, y, z) T_p(z) A(x - vt, y, z) \frac{dx \, dy \, dz}{z^2}$$
(4)

$$\langle T_{A}(t), T_{A}(t+\tau) \rangle = \frac{L_{0}^{2/3}}{2\pi} \int_{z_{1}}^{z_{u}} \frac{dZ}{w^{2}(Z)} C_{\alpha}^{2}(Z) T_{\rho}^{2}(Z) \iiint_{-\infty}^{+\infty} b_{\alpha}(\xi_{x}, \xi_{y}, \xi_{z}) \cdot \exp \left[-\frac{(\xi_{x} + vt)^{2} + \xi_{y}^{2}}{w^{2}(Z)} \right] d\xi_{x} d\xi_{y} d\xi_{z}$$
(5)

$$S(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \langle T_A(t), T_A(t+\tau) \rangle \exp(-i\omega\tau) d\tau$$
 (6)

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Temporal fluctuations in antenna temperature

The two-slope model

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$$S(\omega) = \frac{1}{2\sqrt{2\pi}} \frac{L_0^{2/3}}{v} \int_{z_1}^{z_u} \frac{dZ}{w(Z)} C_\alpha^2(Z) T_p^2(Z) \exp\left[-\frac{w^2(Z)\omega^2}{4v^2}\right] \cdot \int \int \int_{-\infty}^{+\infty} \exp\left[\frac{i\xi_x \omega}{v}\right] \exp\left[-\frac{\xi_y^2}{w^2(Z)}\right] b_\alpha(\xi_x, \xi_y, \xi_z) d\xi_x d\xi_y d\xi_z$$

$$(7)$$

$$S(\omega) = \Phi(\omega/\nu) \cdot I(\omega/\nu) \tag{8}$$

- If $W(Z) \gg L_0$
 - The exponential in ξ_y vary slowly compared to b_{α}
 - Describe high altitude emission

- If $W(Z) \ll L_0$
 - b_{α} vary slowly compared the exponential in ξ_{ν}
 - Describe low altitude emission

Temporal fluctuations in antenna temperature

The two-slope model

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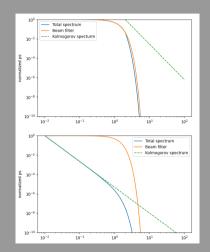
Parameter

• If
$$W(Z) \gg L_0$$

$$\Phi\left(\frac{\omega}{v}\right) = \begin{cases} \left(\frac{\omega}{v}\right)^{-11/3} & \frac{1}{L_0} \ll \left(\frac{\omega}{v}\right) \ll \frac{1}{l_0} \\ const. & \frac{\omega}{v} \ll \frac{1}{L_0} \end{cases}$$

• If
$$W(Z) \ll L_0$$

$$\Phi\left(\frac{\omega}{v}\right) = \begin{cases} \left(\frac{\omega}{v}\right)^{-8/3} & \frac{1}{L_0} \ll \left(\frac{\omega}{v}\right) \ll \frac{1}{l_0} \\ const. & \frac{\omega}{v} \ll \frac{1}{L_0} \end{cases}$$



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The statistical approach based on MERRA and ERA data

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