Atmospheric effects for gorund-based CMB observations

Stefano Mandelli

stefano.mandelli@unimi.it

Università degli Studi di Milano - Istituto Nazionale di Fisica Nucleare

24 - June - 2020

Index

The atmosphere's role in the CMB observation

Atmospheric load in the microwave band

- Due to absorption / emission processes
  - Water vapor
  - Oxygen molecules
- The oxygen is well mixed in the atmosphere
- The water vapor presents highly variable concentrations

Assessment of the scientific impact

We have to create a model of the atmosphere and simulate its observation. We can start from the atmospheric dispersive proprieties

The atmosphere's role in the CMB observation

Atmospheric load in the microwave band

- Due to absorption / emission processes
  - Water vapor
  - Oxygen molecules
- The oxygen is well mixed in the atmosphere
  - The increase of the optical load led to an increasing in the white noise level
- The water vapor presents highly variable concentrations

Assessment of the scientific impact

We have to create a model of the atmosphere and simulate its observation. We can start from the atmospheric dispersive proprieties

The atmosphere's role in the CMB observation

Atmospheric load in the microwave band

- Due to absorption / emission processes
  - Water vapor
  - Oxygen molecules
- The oxygen is well mixed in the atmosphere
  - The increase of the optical load led to an increasing in the white noise level
- The water vapor presents highly variable concentrations

Assessment of the scientific impact

We have to create a model of the atmosphere and simulate its observation. We can start from the atmospheric dispersive proprieties

The atmosphere's role in the CMB observation

Atmospheric load in the microwave band

- Due to absorption / emission processes
  - Water vapor
  - Oxygen molecules
- The oxygen is well mixed in the atmosphere
  - The increase of the optical load led to an increasing in the white noise level
- The water vapor presents highly variable concentrations

Assessment of the scientific impact

We have to create a model of the atmosphere and simulate its observation. We can start from the atmospheric dispersive proprieties

The atmosphere's role in the CMB observation

Atmospheric load in the microwave band

- Due to absorption / emission processes
  - Water vapor
  - Oxygen molecules
- The oxygen is well mixed in the atmosphere
  - The increase of the optical load led to an increasing in the white noise level
- The water vapor presents highly variable concentrations
  - spurious $1/f$-like noise in the time-streams

Assessment of the scientific impact

We have to create a model of the atmosphere and simulate its observation. We can start from the atmospheric dispersive proprieties

The atmosphere's role in the CMB observation

Atmospheric load in the microwave band

- Due to absorption / emission processes
  - Water vapor
  - Oxygen molecules
- The oxygen is well mixed in the atmosphere
  - The increase of the optical load led to an increasing in the white noise level
- The water vapor presents highly variable concentrations
  - spurious $1/f$-like noise in the time-streams

Assessment of the scientific impact

We have to create a model of the atmosphere and simulate its observation. We can start from the atmospheric dispersive proprieties

The atmosphere's role in the CMB observation

Atmospheric load in the microwave band

- Due to absorption / emission processes
  - Water vapor
  - Oxygen molecules
- The oxygen is well mixed in the atmosphere
  - The increase of the optical load led to an increasing in the white noise level
- The water vapor presents highly variable concentrations
  - spurious $1/f$-like noise in the time-streams

Assessment of the scientific impact

We have to create a model of the atmosphere and simulate its observation. We can start from the atmospheric dispersive proprieties

The atmosphere's role in the CMB observation

Atmospheric load in the microwave band

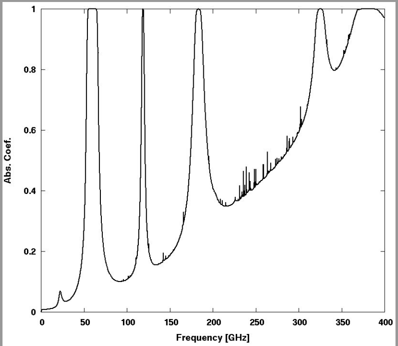
- Due to absorption / emission processes
  - Water vapor
  - Oxygen molecules
- The oxygen is well mixed in the atmosphere
  - The increase of the optical load led to an increasing in the white noise level
- The water vapor presents highly variable concentrations
  - spurious $1/f$-like noise in the time-streams

Assessment of the scientific impact

We have to create a model of the atmosphere and simulate its observation. We can start from the atmospheric dispersive proprieties

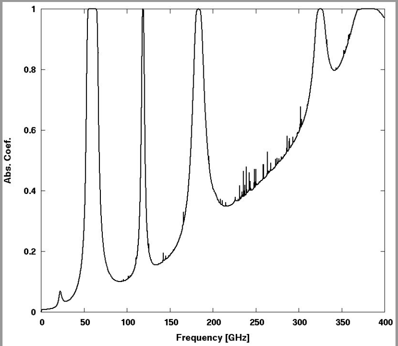
The atmosphere as a dispersive medium

- Dispersive medium: $\varepsilon(\omega) = \varepsilon(\omega)_R + i\,\varepsilon(\omega)_I$
- The contribute to complex permittivity are due mostly to the water vapor and the oxygen molecules that are dissolved in the atmosphere
- $\varepsilon(\omega)_R = \sqrt{n}$, and $\varepsilon(\omega)_I = \lambda\alpha/4\pi$
- The real and complex parts of $\varepsilon(\omega)$ are linked by the Kramers-Krönig relations.
- There is a relation also between the refractive index n and the apsorption coefficient α



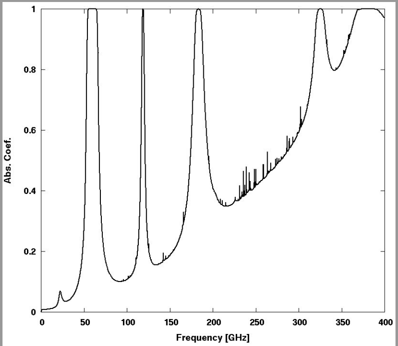
The atmosphere as a dispersive medium

- Dispersive medium: $\varepsilon(\omega) = \varepsilon(\omega)_R + i\,\varepsilon(\omega)_I$
- The contribute to complex permittivity are due mostly to the water vapor and the oxygen molecules that are dissolved in the atmosphere
- $\varepsilon(\omega)_R = \sqrt{n}$, and $\varepsilon(\omega)_I = \lambda\alpha/4\pi$
- The real and complex parts of $\varepsilon(\omega)$ are linked by the Kramers-Krönig relations.
- There is a relation also between the refractive index n and the apsorption coefficient α



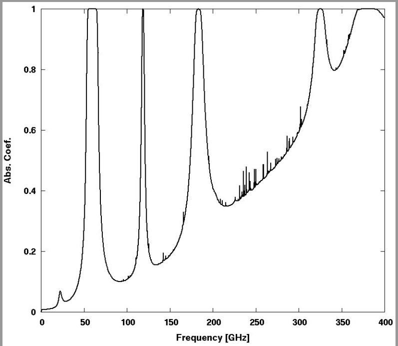
The atmosphere as a dispersive medium

- Dispersive medium: $\varepsilon(\omega) = \varepsilon(\omega)_R + i\,\varepsilon(\omega)_I$
- The contribute to complex permittivity are due mostly to the water vapor and the oxygen molecules that are dissolved in the atmosphere
- $\varepsilon(\omega)_R = \sqrt{n}$, and $\varepsilon(\omega)_I = \lambda\alpha/4\pi$
- The real and complex parts of $\varepsilon(\omega)$ are linked by the Kramers-Krönig relations.
- There is a relation also between the refractive index n and the apsorption coefficient α



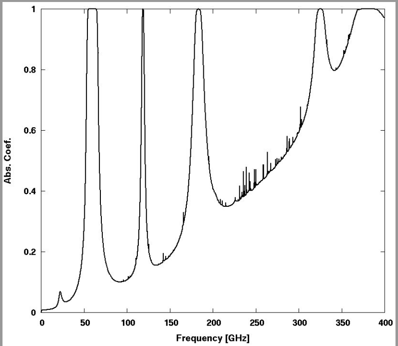
The atmosphere as a dispersive medium

- Dispersive medium: $\varepsilon(\omega) = \varepsilon(\omega)_R + i\,\varepsilon(\omega)_I$
- The contribute to complex permittivity are due mostly to the water vapor and the oxygen molecules that are dissolved in the atmosphere
- $\varepsilon(\omega)_R = \sqrt{n}$, and $\varepsilon(\omega)_I = \lambda\alpha/4\pi$
- The real and complex parts of $\varepsilon(\omega)$ are linked by the Kramers-Krönig relations.
- There is a relation also between the refractive index n and the apsorption coefficient α



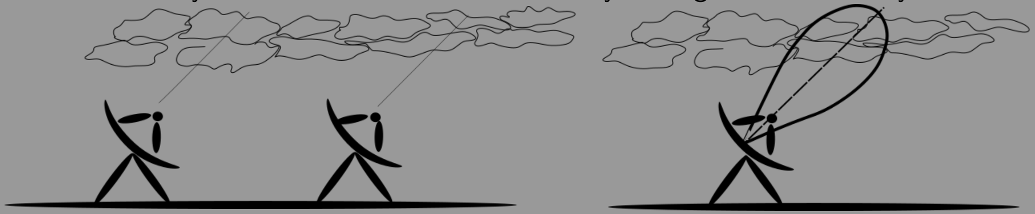
The atmosphere as a dispersive medium

- Dispersive medium: $\varepsilon(\omega) = \varepsilon(\omega)_R + i\,\varepsilon(\omega)_I$
- The contribute to complex permittivity are due mostly to the water vapor and the oxygen molecules that are dissolved in the atmosphere
- $\varepsilon(\omega)_R = \sqrt{n}$, and $\varepsilon(\omega)_I = \lambda\alpha/4\pi$
- The real and complex parts of $\varepsilon(\omega)$ are linked by the Kramers-Krönig relations.
- There is a relation also between the refractive index n and the apsorption coefficient α



Fluctuations in the refractive index

Refractive index fluctuations induce a change in the optical path length, and leds a two different systematic effects for interferometry or single dish, relatively



Interferometer: Phase fluctuations, degrade the interference pattern.

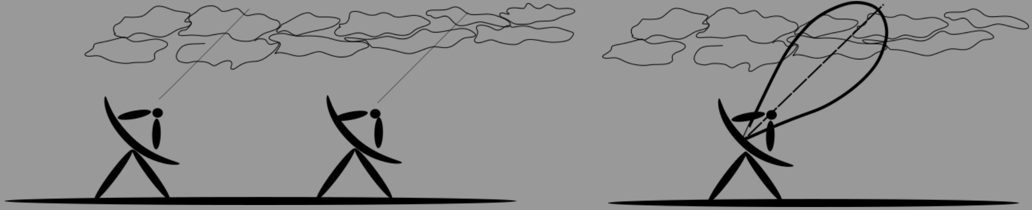
Single-dish: can cause fluctuations in the apparent pointing

An interesting power-low relation

There is a relations between the RMS phase fluctuations and the interference baseline $\langle \phi^2 \rangle \sim D^{5/3}$ [see S.E. Church paper]

Fluctuations in the refractive index

Refractive index fluctuations induce a change in the optical path length, and leds a two different systematic effects for interferometry or single dish, relatively



Interferometer: Phase fluctuations, degrade the interference pattern.

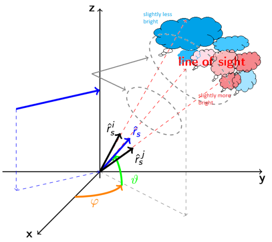
Single-dish: can cause fluctuations in the apparent pointing

An interesting power-low relation

There is a relations between the RMS phase fluctuations and the interference baseline $\langle \phi^2 \rangle \sim D^{5/3}$ [see S.E. Church paper]

Fluctuations in the absorption coefficient

- Atmospheric brightness temperature is related to α: $T_B = \int_0^R \alpha(r) T_p(r) e^{-\tau'} dr$
- $e^{-\tau} \sim 1$, τ fluctuations are small compared to α
- Antenna temperature: $T_A = \frac{1}{\lambda^2} \int_V A(\hat{r}_s, \vec{r}) T_p(\vec{r}) \frac{dV}{r^2}$



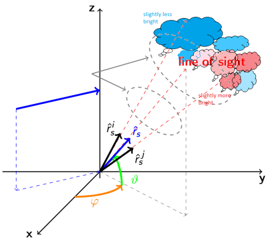
$$\langle T_A^2 \rangle = \frac{1}{\lambda^4} \int_V \frac{dV}{r^2} \int_{V'} \frac{dV'}{r'^2} A(\hat{r}_s, \vec{r}) A(\hat{r}'_s, \vec{r}') T_p(\vec{r}) T_p(\vec{r}') \langle \alpha(\vec{r}), \alpha(\vec{r}') \rangle$$

Atmosphere structures

The correlation term $\langle \alpha(r), \alpha(r') \rangle$ is unknown.

Fluctuations in the absorption coefficient

- Atmospheric brightness temperature is related to α: $T_B = \int_0^R \alpha(r)\, T_p(r) \cancel{e^{-\tau}}\, dr$
- $e^{-\tau} \sim 1$, τ fluctuations are small compared to α
- Antenna temperature: $T_A = \frac{1}{\lambda^2} \int_V A(\hat{r}_s, \vec{r})\, T_p(\vec{r}) \frac{dV}{r^2}$



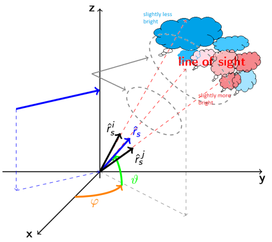
$$\langle T_A^2 \rangle = \frac{1}{\lambda^4} \int_V \frac{dV}{r^2} \int_{V'} \frac{dV'}{r'^2} A(\hat{r}_s, \vec{r}) A(\hat{r'}_s, \vec{r'})\, T_p(\vec{r})\, T_p(\vec{r'}) \langle \alpha(\vec{r}), \alpha(\vec{r'}) \rangle$$

Atmosphere structures

The correlation term $\langle \alpha(r), \alpha(r') \rangle$ is unknown.

Fluctuations in the absorption coefficient

- Atmospheric brightness temperature is related to α: $T_B = \int_0^R \alpha(r)\,T_p(r)\cancel{e^{-\tau}}\,dr$
- $e^{-\tau} \sim 1$, τ fluctuations are small compared to α
- Antenna temperature: $T_A = \frac{1}{\lambda^2}\int_V A(\hat{r}_s,\vec{r})\,T_p(\vec{r})\frac{dV}{r^2}$



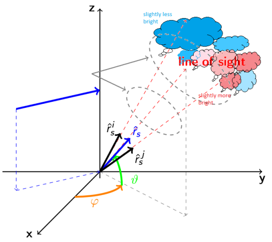
$$\langle T_A^2\rangle = \frac{1}{\lambda^4}\int_V \frac{dV}{r^2}\int_{V'}\frac{dV'}{r'^2}A(\hat{r}_s,\vec{r})A(\hat{r}'_s,\vec{r}')T_p(\vec{r})T_p(\vec{r}')\langle\alpha(\vec{r}),\alpha(\vec{r}')\rangle$$

Atmosphere structures

The correlation term $\langle\alpha(r),\alpha(r')\rangle$ is unknown.

Fluctuations in the absorption coefficient

- Atmospheric brightness temperature is related to α: $T_B = \int_0^R \alpha(r) T_p(r) \cancel{e^{-\tau}}\, dr$
- $e^{-\tau} \sim 1$, τ fluctuations are small compared to α
- Antenna temperature: $T_A = \frac{1}{\lambda^2} \int_V A(\hat{r}_s, \vec{r}) T_p(\vec{r}) \frac{dV}{r^2}$

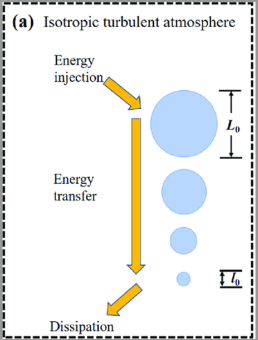


$$\langle T_A^2 \rangle = \frac{1}{\lambda^4} \int_V \frac{dV}{r^2} \int_{V'} \frac{dV'}{r'^2} A(\hat{r}_s, \vec{r}) A(\hat{r'}_s, \vec{r'}) T_p(\vec{r}) T_p(\vec{r'}) \langle \alpha(\vec{r}), \alpha(\vec{r'}) \rangle$$

Atmosphere structures

The correlation term $\langle \alpha(r), \alpha(r') \rangle$ is unknown.

- Structural spatial function $D_n(r_1, r_2) = \langle [n(r_1) - n(r_2)]^2 \rangle$
- Inner and outer correlation lengths l_0 and L_0
- Specific humidity q
  - Liquid (TQL)
  - Ice cristals (TQI)
  - Precipitable water vapor (TQV)
- Ground temperature T_0
- Solar irradiation
- Wind speed and direction



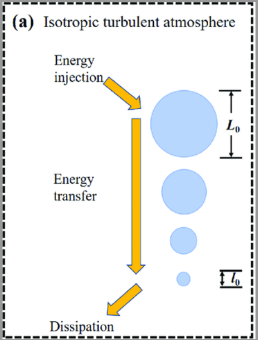
Structural spatial function for n

$$D(r_1, r_2) \propto L_0^{4/3} \left(\frac{dn}{dq} \frac{dq}{dz} \right) |r_1 - r_2|^{2/3} \propto C_0^2 \exp\left(-\frac{z}{z_0} \right)$$

Characterizing the turbulent structure of the atmosphere

Interferometric observation of the atmospheric structures

- Structural spatial function $D_n(r_1, r_2) = \langle [n(r_1) - n(r_2)]^2 \rangle$
- Inner and outer correlation lengths l_0 and L_0
- Specific humidity q
  - Liquid (TQL)
  - Ice cristals (TQI)
  - Precipitable water vapor (TQV)
- Ground temperature T_0
- Solar irradiation
- Wind speed and direction



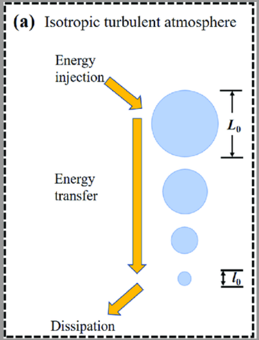
Structural spatial function for n

$$D(r_1, r_2) \propto L_0^{4/3} \left(\frac{dn}{dq} \frac{dq}{dz} \right) |r_1 - r_2|^{2/3} \propto C_0^2 \exp\left(-\frac{z}{z_0} \right)$$

Characterizing the turbulent structure of the atmosphere

Interferometric observation of the atmospheric structures

- Structural spatial function $D_n(r_1, r_2) = \langle [n(r_1) - n(r_2)]^2 \rangle$
- Inner and outer correlation lengths l_0 and L_0
- Specific humidity q
  - Liquid (TQL)
  - Ice cristals (TQI)
  - Precipitable water vapor (TQV)
- Ground temperature T_0
- Solar irradiation
- Wind speed and direction



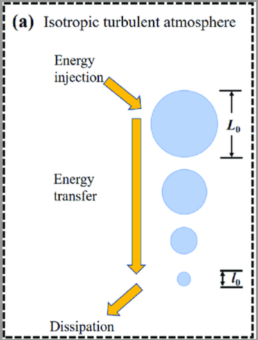
Structural spatial function for n

$$D(r_1, r_2) \propto L_0^{4/3} \left(\frac{dn}{dq} \frac{dq}{dz} \right) |r_1 - r_2|^{2/3} \propto C_0^2 \exp\left(-\frac{z}{z_0} \right)$$

Characterizing the turbulent structure of the atmosphere

Interferometric observation of the atmospheric structures

- Structural spatial function $D_n(r_1, r_2) = \langle [n(r_1) - n(r_2)]^2 \rangle$
- Inner and outer correlation lengths l_0 and L_0
- Specific humidity q
    - Liquid (TQL)
    - Ice cristals (TQI)
    - Precipitable water vapor (TQV)
- Ground temperature T_0
- Solar irradiation
- Wind speed and direction



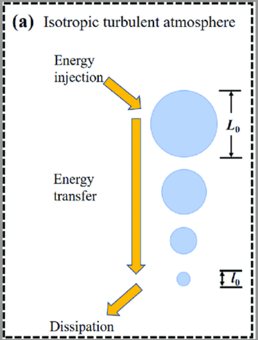
Structural spatial function for n

$$D(r_1, r_2) \propto L_0^{4/3} \left(\frac{dn}{dq} \frac{dq}{dz} \right) |r_1 - r_2|^{2/3} \propto C_0^2 \exp\left(-\frac{z}{z_0} \right)$$

Characterizing the turbulent structure of the atmosphere

Interferometric observation of the atmospheric structures

- Structural spatial function $D_n(r_1, r_2) = \langle [n(r_1) - n(r_2)]^2 \rangle$
- Inner and outer correlation lengths l_0 and L_0
- Specific humidity q
  - Liquid (TQL)
  - Ice cristals (TQI)
  - Precipitable water vapor (TQV)
- Ground temperature T_0
- Solar irradiation
- Wind speed and direction



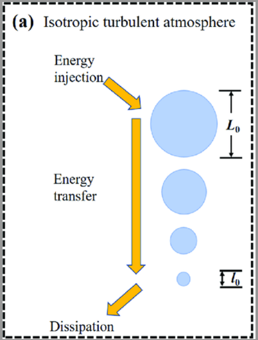
Structural spatial function for n

$$D(r_1, r_2) \propto L_0^{4/3} \left(\frac{dn}{dq} \frac{dq}{dz} \right) |r_1 - r_2|^{2/3} \propto C_0^2 \exp\left(-\frac{z}{z_0} \right)$$

Characterizing the turbulent structure of the atmosphere

Interferometric observation of the atmospheric structures

- Structural spatial function $D_n(r_1, r_2) = \langle [n(r_1) - n(r_2)]^2 \rangle$
- Inner and outer correlation lengths l_0 and L_0
- Specific humidity q
  - Liquid (TQL)
  - Ice cristals (TQI)
  - Precipitable water vapor (TQV)
- Ground temperature T_0
- Solar irradiation
- Wind speed and direction



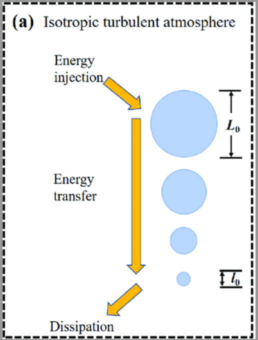
Structural spatial function for n

$$D(r_1, r_2) \propto L_0^{4/3} \left(\frac{dn}{dq} \frac{dq}{dz} \right) |r_1 - r_2|^{2/3} \propto C_0^2 \exp\left(-\frac{z}{z_0} \right)$$

Characterizing the turbulent structure of the atmosphere

Interferometric observation of the atmospheric structures

- Structural spatial function $D_n(r_1, r_2) = \langle [n(r_1) - n(r_2)]^2 \rangle$
- Inner and outer correlation lengths l_0 and L_0
- Specific humidity q
  - Liquid (TQL)
  - Ice cristals (TQI)
  - Precipitable water vapor (TQV)
- Ground temperature T_0
- Solar irradiation
- Wind speed and direction



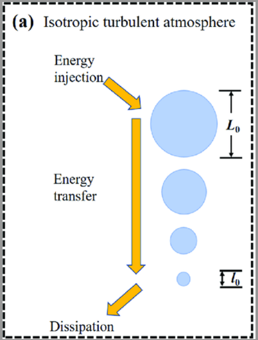
Structural spatial function for n

$$D(r_1, r_2) \propto L_0^{4/3} \left(\frac{dn}{dq} \frac{dq}{dz} \right) |r_1 - r_2|^{2/3} \propto C_0^2 \exp\left(-\frac{z}{z_0} \right)$$

Characterizing the turbulent structure of the atmosphere

Interferometric observation of the atmospheric structures

- Structural spatial function $D_n(r_1, r_2) = \langle [n(r_1) - n(r_2)]^2 \rangle$
- Inner and outer correlation lengths l_0 and L_0
- Specific humidity q
  - Liquid (TQL)
  - Ice cristals (TQI)
  - Precipitable water vapor (TQV)
- Ground temperature T_0
- Solar irradiation
- Wind speed and direction



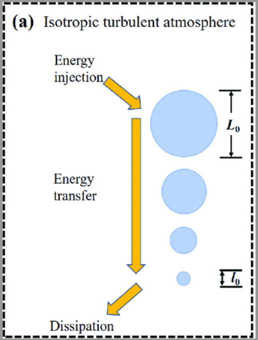
Structural spatial function for n

$$D(r_1, r_2) \propto L_0^{4/3} \left(\frac{dn}{dq} \frac{dq}{dz} \right) |r_1 - r_2|^{2/3} \propto C_0^2 \exp\left(-\frac{z}{z_0} \right)$$

Characterizing the turbulent structure of the atmosphere

Interferometric observation of the atmospheric structures

- Structural spatial function $D_n(r_1, r_2) = \langle [n(r_1) - n(r_2)]^2 \rangle$
- Inner and outer correlation lengths l_0 and L_0
- Specific humidity q
  - Liquid (TQL)
  - Ice cristals (TQI)
  - Precipitable water vapor (TQV)
- Ground temperature T_0
- Solar irradiation
- Wind speed and direction



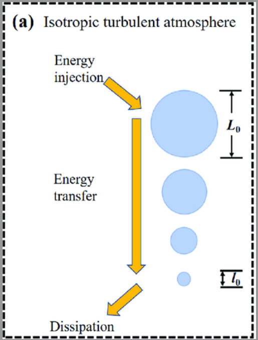
Structural spatial function for n

$$D(r_1, r_2) \propto L_0^{4/3} \left(\frac{dn}{dq} \frac{dq}{dz} \right) |r_1 - r_2|^{2/3} \propto C_0^2 \exp\left(-\frac{z}{z_0} \right)$$

Characterizing the turbulent structure of the atmosphere

Interferometric observation of the atmospheric structures

- Structural spatial function $D_n(r_1, r_2) = \langle [n(r_1) - n(r_2)]^2 \rangle$
- Inner and outer correlation lengths l_0 and L_0
- Specific humidity q
  - Liquid (TQL)
  - Ice cristals (TQI)
  - Precipitable water vapor (TQV)
- Ground temperature T_0
- Solar irradiation
- Wind speed and direction



Structural spatial function for n

$$D(r_1, r_2) \propto L_0^{4/3} \left(\frac{dn}{dq} \frac{dq}{dz} \right) |r_1 - r_2|^{2/3} \propto C_0^2 \exp\left(-\frac{z}{z_0} \right)$$

Characterizing the turbulent structure of the atmosphere

The structural spatial function for the atmospheric absorption coefficient

- The refractive index and the absorption coefficient are linked by the KK relations
- Same dependency from the atmospheric humidity q
- We can neglect second order term and consider $D_n = D_\alpha$

$D_\alpha(r_1, r_2) = \langle[\alpha(r_1) - \alpha(r_2)]^2\rangle$, using the correlation $B_\alpha(r_1, r_2) = \langle\alpha(r_1), \alpha(r_2)\rangle$

$$D_\alpha(r_1, r_2) = \langle[\alpha(r_1) - \alpha(r_2)]^2\rangle = B_\alpha(r_1, r_1) + B_\alpha(r_2, r_2) - 2\,B_\alpha(r_1, r_2) = \\ = C_\alpha^2\left(\frac{r_1 + r_2}{2}\right)|r_1 - r_2|^{2/3}$$

$$B_\alpha(r_1, r_2) = \frac{1}{2}C_\alpha^2\left(\frac{r_1 + r_2}{2}\right)L_0^{2/3}\left(1 - \frac{|r_1 - r_2|^{2/3}}{L_0^{2/3}}\right)$$

Characterizing the turbulent structure of the atmosphere

The structural spatial function for the atmospheric absorption coefficient

- The refractive index and the absorption coefficient are linked by the KK relations
- Same dependency from the atmospheric humidity q
- We can neglect second order term and consider $D_n = D_\alpha$

$D_\alpha(r_1, r_2) = \langle[\alpha(r_1) - \alpha(r_2)]^2\rangle$, using the correlation $B_\alpha(r_1, r_2) = \langle\alpha(r_1), \alpha(r_2)\rangle$

$$D_\alpha(r_1, r_2) = \langle[\alpha(r_1) - \alpha(r_2)]^2\rangle = B_\alpha(r_1, r_1) + B_\alpha(r_2, r_2) - 2\,B_\alpha(r_1, r_2) = = C_\alpha^2\left(\frac{r_1 + r_2}{2}\right)|r_1 - r_2|^{2/3}$$

$$B_\alpha(r_1, r_2) = \frac{1}{2}C_\alpha^2\left(\frac{r_1 + r_2}{2}\right)L_0^{2/3}\left(1 - \frac{|r_1 - r_2|^{2/3}}{L_0^{2/3}}\right)$$

- The refractive index and the absorption coefficient are linked by the KK relations
- Same dependency from the atmospheric humidity q
- We can neglect second order term and consider $D_n = D_\alpha$

$D_\alpha(r_1, r_2) = \langle[\alpha(r_1) - \alpha(r_2)]^2\rangle$, using the correlation $B_\alpha(r_1, r_2) = \langle\alpha(r_1), \alpha(r_2)\rangle$

$$D_\alpha(r_1, r_2) = \langle[\alpha(r_1) - \alpha(r_2)]^2\rangle = B_\alpha(r_1, r_1) + B_\alpha(r_2, r_2) - 2\,B_\alpha(r_1, r_2) =$$
$$= C_\alpha^2\left(\frac{r_1 + r_2}{2}\right)|r_1 - r_2|^{2/3}$$

$$B_\alpha(r_1, r_2) = \frac{1}{2}C_\alpha^2\left(\frac{r_1 + r_2}{2}\right)L_0^{2/3}\left(1 - \frac{|r_1 - r_2|^{2/3}}{L_0^{2/3}}\right)$$

Characterizing the turbulent structure of the atmosphere

The structural spatial function for the atmospheric absorption coefficient

- The refractive index and the absorption coefficient are linked by the KK relations
- Same dependency from the atmospheric humidity q
- We can neglect second order term and consider $D_n = D_\alpha$

$D_\alpha(r_1, r_2) = \langle[\alpha(r_1) - \alpha(r_2)]^2\rangle$, using the correlation $B_\alpha(r_1, r_2) = \langle\alpha(r_1), \alpha(r_2)\rangle$

$$D_\alpha(r_1, r_2) = \langle[\alpha(r_1) - \alpha(r_2)]^2\rangle = B_\alpha(r_1, r_1) + B_\alpha(r_2, r_2) - 2\,B_\alpha(r_1, r_2) =$$

$$= C_\alpha^2\left(\frac{r_1 + r_2}{2}\right)|r_1 - r_2|^{2/3}$$

$$B_\alpha(r_1, r_2) = \frac{1}{2}C_\alpha^2\left(\frac{r_1 + r_2}{2}\right)L_0^{2/3}\left(1 - \frac{|r_1 - r_2|^{2/3}}{L_0^{2/3}}\right)$$

Characterizing the turbulent structure of the atmosphere

The structural spatial function for the atmospheric absorption coefficient

- The refractive index and the absorption coefficient are linked by the KK relations
- Same dependency from the atmospheric humidity q
- We can neglect second order term and consider $D_n = D_\alpha$

$D_\alpha(r_1, r_2) = \langle[\alpha(r_1) - \alpha(r_2)]^2\rangle$, using the correlation $B_\alpha(r_1, r_2) = \langle\alpha(r_1), \alpha(r_2)\rangle$

$$D_\alpha(r_1, r_2) = \langle[\alpha(r_1) - \alpha(r_2)]^2\rangle = B_\alpha(r_1, r_1) + B_\alpha(r_2, r_2) - 2\,B_\alpha(r_1, r_2) =$$
$$= C_\alpha^2\left(\frac{r_1 + r_2}{2}\right)|r_1 - r_2|^{2/3}$$

$$B_\alpha(r_1, r_2) = \frac{1}{2}C_\alpha^2\left(\frac{r_1 + r_2}{2}\right)L_0^{2/3}\left(1 - \frac{|r_1 - r_2|^{2/3}}{L_0^{2/3}}\right)$$

Characterizing the turbulent structure of the atmosphere

The structural spatial function for the atmospheric absorption coefficient

- The refractive index and the absorption coefficient are linked by the KK relations
- Same dependency from the atmospheric humidity q
- We can neglect second order term and consider $D_n = D_\alpha$

$D_\alpha(r_1, r_2) = \langle[\alpha(r_1) - \alpha(r_2)]^2\rangle$, using the correlation $B_\alpha(r_1, r_2) = \langle\alpha(r_1), \alpha(r_2)\rangle$

$$D_\alpha(r_1, r_2) = \langle[\alpha(r_1) - \alpha(r_2)]^2\rangle = B_\alpha(r_1, r_1) + B_\alpha(r_2, r_2) - 2\, B_\alpha(r_1, r_2) = \\ = C_\alpha^2\left(\frac{r_1 + r_2}{2}\right)|r_1 - r_2|^{2/3}$$

$$B_\alpha(r_1, r_2) = \frac{1}{2} C_\alpha^2\left(\frac{r_1 + r_2}{2}\right) L_0^{2/3}\left(1 - \frac{|r_1 - r_2|^{2/3}}{L_0^{2/3}}\right)$$

Characterizing the turbulent structure of the atmosphere

The structural spatial function for the atmospheric absorption coefficient

The $\langle \alpha(r_1), \alpha(r_2) \rangle$ expression

$$B_\alpha(r_1, r_2) = \frac{1}{2} C_\alpha^2 \left(\frac{r_1 + r_2}{2} \right) L_0^{2/3} \left(1 - \frac{|r_1 - r_2|^{2/3}}{L_0^{2/3}} \right)$$

can be written more general

$$B_\alpha(r_1, r_2) = \frac{1}{2} C_\alpha^2 \left(\frac{r_1 + r_2}{2} \right) L_0^{2/3} b_\alpha(|r_1 - r_2|) \quad (1)$$

Kolmogorov-Modified power spectrum

$$b_\alpha(r) \propto \frac{1}{r} \int_{k_0}^{k_M} k \Phi(k) \sin(k \cdot r) \, dk \quad (2)$$

Characterizing the turbulent structure of the atmosphere

The structural spatial function for the atmospheric absorption coefficient

The $\langle \alpha(r_1), \alpha(r_2) \rangle$ expression

$$B_\alpha(r_1, r_2) = \frac{1}{2} C_\alpha^2 \left(\frac{r_1 + r_2}{2} \right) L_0^{2/3} \left(1 - \frac{|r_1 - r_2|^{2/3}}{L_0^{2/3}} \right)$$

can be written more general

$$B_\alpha(r_1, r_2) = \frac{1}{2} C_\alpha^2 \left(\frac{r_1 + r_2}{2} \right) L_0^{2/3} b_\alpha(|r_1 - r_2|) \tag{1}$$

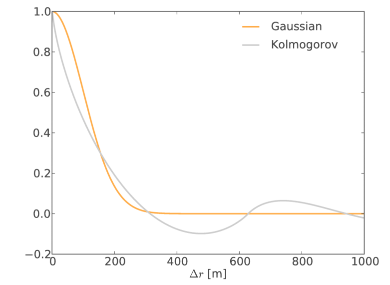
Kolmogorov-Modified power spectrum

$$b_\alpha(r) \propto \frac{1}{r} \int_{k_0}^{k_M} k \Phi(k) \sin(k \cdot r) \, dk \tag{2}$$

Kolmogorov-Modified power spectrum

$$b_\alpha(r) \propto \frac{1}{r} \int_{k_0}^{k_M} k\Phi(k) \sin(k \cdot r)\, dk \quad (3)$$

- Kolmogorov-Taylor model
  - $\Phi(k) \propto k^{-11/3}$
  - $k_0 = 1/L_0$
  - $k_M = 1/l_0$
- Gaussian-Like correlation



$L_0 = 100m$

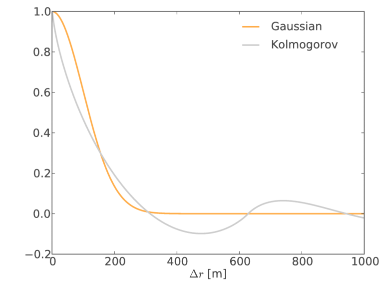
Characterizing the turbulent structure of the atmosphere

Different kind of b_α spectrum

Kolmogorov-Modified power spectrum

$$b_\alpha(r) \propto \frac{1}{r} \int_{k_0}^{k_M} k\Phi(k) \sin(k \cdot r)\, dk \tag{3}$$

- Kolmogorov-Taylor model
  - $\Phi(k) \propto k^{-11/3}$
  - $k_0 = 1/L_0$
  - $k_M = 1/l_0$
- Gaussian-Like correlation



$L_0 = 100m$

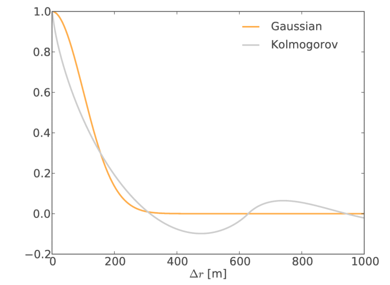
Characterizing the turbulent structure of the atmosphere

Different kind of b_α spectrum

Kolmogorov-Modified power spectrum

$$b_\alpha(r) \propto \frac{1}{r}\int_{k_0}^{k_M} k\Phi(k)\sin(k\cdot r)\,dk \qquad (3)$$

- Kolmogorov-Taylor model
  - $\Phi(k) \propto k^{-11/3}$
  - $k_0 = 1/L_0$
  - $k_M = 1/l_0$
- Gaussian-Like correlation



$L_0 = 100m$

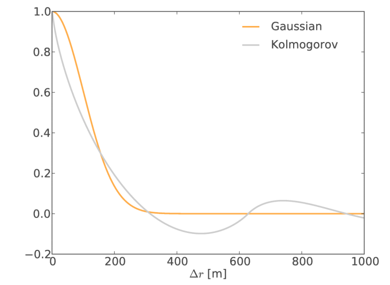
Characterizing the turbulent structure of the atmosphere

Different kind of b_α spectrum

Kolmogorov-Modified power spectrum

$$b_\alpha(r) \propto \frac{1}{r}\int_{k_0}^{k_M} k\Phi(k)\sin(k \cdot r)\,dk \tag{3}$$

- Kolmogorov-Taylor model
  - $\Phi(k) \propto k^{-11/3}$
  - $k_0 = 1/L_0$
  - $k_M = 1/l_0$
- Gaussian-Like correlation
  - $b_\alpha(|r_1 - r_2|) = \exp\left[-\frac{|r_1 - r_2|^2}{2(L_0/3)^2}\right]$



$L_0 = 100m$

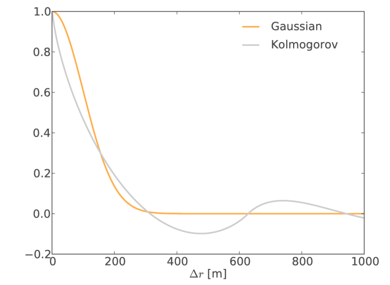
Characterizing the turbulent structure of the atmosphere

Different kind of b_α spectrum

Kolmogorov-Modified power spectrum

$$b_\alpha(r) \propto \frac{1}{r}\int_{k_0}^{k_M} k\Phi(k)\sin(k\cdot r)\,dk \tag{3}$$

- Kolmogorov-Taylor model
  - $\Phi(k) \propto k^{-11/3}$
  - $k_0 = 1/L_0$
  - $k_M = 1/l_0$
- Gaussian-Like correlation
  - $b_\alpha(|r_1 - r_2|) = \exp\left[-\frac{|r_1-r_2|^2}{2(L_0/3)^2}\right]$



$L_0 = 100m$

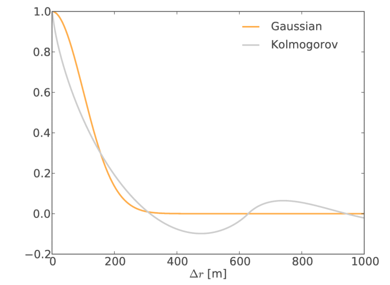
Characterizing the turbulent structure of the atmosphere

Different kind of b_α spectrum

Kolmogorov-Modified power spectrum

$$b_\alpha(r) \propto \frac{1}{r}\int_{k_0}^{k_M} k\Phi(k)\sin(k\cdot r)\,dk \quad (3)$$

- Kolmogorov-Taylor model
  - $\Phi(k) \propto k^{-11/3}$
  - $k_0 = 1/L_0$
  - $k_M = 1/l_0$
- Gaussian-Like correlation
  - $b_\alpha(|r_1 - r_2|) = \exp\left[-\frac{|r_1-r_2|^2}{2(L_0/3)^2}\right]$



$L_0 = 100m$

Temporal fluctuations in antenna temperature

The rigid translation of the atmospheric structures by the wind

$$T_A(t) = \frac{1}{\lambda^2} \int_{z_1}^{z_u} \iint_{-\infty}^{+\infty} \alpha(x,y,z) T_p(z) A(x - vt, y, z) \frac{dx\,dy\,dz}{z^2} \tag{4}$$

$$\begin{aligned} \langle T_A(t), T_A(t+\tau)\rangle = & \frac{L_0^{2/3}}{2\pi} \int_{z_1}^{z_u} \frac{dZ}{w^2(Z)} C_\alpha^2(Z) T_p^2(Z) \iiint_{-\infty}^{+\infty} b_\alpha(\xi_x, \xi_y, \xi_z) \cdot \\ & \cdot \exp\left[-\frac{(\xi_x + vt)^2 + \xi_y^2}{w^2(Z)}\right] d\xi_x\,d\xi_y\,d\xi_z \end{aligned} \tag{5}$$

$$S(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \langle T_A(t), T_A(t+\tau)\rangle \exp(-i\omega\tau)\,d\tau \tag{6}$$

Temporal fluctuations in antenna temperature

The two-slope model

$$S(\omega) = \frac{1}{2\sqrt{2\pi}} \frac{L_0^{2/3}}{v} \int_{z_1}^{z_u} \frac{dZ}{w(Z)} C_\alpha^2(Z)\, T_p^2(Z) \exp\left[-\frac{w^2(Z)\omega^2}{4v^2}\right] \cdot \iiint_{-\infty}^{+\infty} \exp\left[\frac{i\xi_x\omega}{v}\right] \exp\left[-\frac{\xi_y^2}{w^2(Z)}\right] b_\alpha(\xi_x, \xi_y, \xi_z)\, d\xi_x d\xi_y d\xi_z \tag{7}$$

$$S(\omega) = \Phi(\omega/v) \cdot I(\omega/v) \tag{8}$$

- If $W(Z) \gg L_0$
  - The exponential in ξ_y vary slowly compared to b_α
  - Describe high altitude emission
- If $W(Z) \ll L_0$
  - b_α vary slowly compared the exponential in ξ_y
  - Describe low altitude emission

Temporal fluctuations in antenna temperature

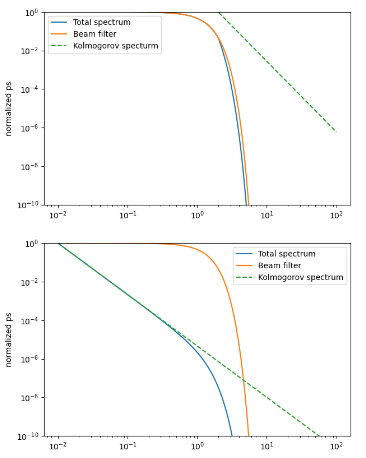
The two-slope model

- If $W(Z) \gg L_0$

$$\Phi\left(\frac{\omega}{v}\right) = \begin{cases} \left(\frac{\omega}{v}\right)^{-11/3} & \frac{1}{L_0} \ll \left(\frac{\omega}{v}\right) \ll \frac{1}{l_0} \\ const. & \frac{\omega}{v} \ll \frac{1}{L_0} \end{cases}$$

- If $W(Z) \ll L_0$

$$\Phi\left(\frac{\omega}{v}\right) = \begin{cases} \left(\frac{\omega}{v}\right)^{-8/3} & \frac{1}{L_0} \ll \left(\frac{\omega}{v}\right) \ll \frac{1}{l_0} \\ const. & \frac{\omega}{v} \ll \frac{1}{L_0} \end{cases}$$



CAL - CMB Atmospheric Library

The statistical approach based on MERRA and ERA data