Public debt, Fiscal Policy and Velocity

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Abstract

In this paper, we analyze the relationship between the public debt-to-GDP ratio and the velocity of money zero maturity (MZM) in the United States. We conduct Granger causality tests on the two variables over different time samples. Our findings suggest that the public debt-to-GDP ratio Granger causes velocity from 1959Q1 to 1995Q4. We also observe significant coefficients for other time samples, although the significance of the lag varies. To further investigate these findings and assess the extent to which the level of public debt may be a determinant of velocity, we analyze this relationship through the lens of a New Keynesian model with fiscal policy shocks. NOminal rigidities play a key role in our results. Our contribution to literature lies in two aspects. Firstly, we provide a comprehensive examination of expectations regarding public debt, which we consider highly relevant, particularly during times of historically high levels of public debt. Secondly, our analysis introduces a determinant of money velocity, offering insights into the transmission mechanisms of fiscal policy shocks.

JEL Classification: E51; E62; E63

Keywords: velocity; public debt; fiscal policy

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1 Introduction

The velocity of money zero maturity (MZM) in the US has exhibited three different trends since 1959. Firstly, it rose until 1981Q3, then experienced a steady decline during the recession of 1980 - 1982. After that, it showed a fluctuating trend until 1997Q1, and eventually demonstrated a clear downward trend until 2020. Therefore, as assessed in the literature, the evolution of the money velocity is not constant, although it is often referred to as constant for convenience (Mankiw, 2014). During the 1980s and 1990s, many focused their work on the monetary base velocity, the narrowest measure of money in the economy. In these studies, changes in money velocity are explained as being driven by technological innovation and the creation of substitutes for money (Belongia & Ireland, 2019). Following the approach of Teles & Zhou (2005), Motley (1988), Poole (1991), among others, we focus on the Money Zero Maturity (MZM) aggregate[1]. Teles & Zhou (2005) show that MZM, as a variable for the monetary aggregate, not only represents a more appropriate measure of base money, but is also the money measure necessary to preserve a stable relationship between money supply, the interest rate and GDP[2] MZM includes all assets that can be easily converted into liquid assets without any maturity constraints, making them readily usable for transactions. Therefore, utilizing MZM in our analysis offers the advantage of capturing financial and technological innovations in the nature of money.

[1]Money zero maturity includes currency outside the U.S. Treasury, Federal Reserve Banks, and the vaults of depository institutions, demand deposits at commercial banks (excluding those amounts held by depository institutions, the U.S. government, and foreign banks and official institutions) less cash items in the process of collection and Federal Reserve float, other liquid deposits, consisting of other checkable deposits (or OCDs, which comprise negotiable order of withdrawal, or NOW, and automatic transfer service, or ATS, accounts at depository institutions, share draft accounts at credit unions, and demand deposits at thrift institutions) and savings deposits (including money market deposit accounts), savings deposits (which include money market deposit accounts, or MMDAs), balances in retail money market mutual funds (MMMFs) and institutional money funds.

[2]Teles & Zhou (2005) note how the money demand estimations of Lucas Jr (1988) are valid only until the 1980s. The authors show that the reason for this limitation lies in the instability of the measure used for money supply. When using MZM, instead of other monetary aggregates such as M1, the stability of the money demand is restored, even after the 1980s.

The main focus of this paper is on the relationship between the public debt-to-GDP ratio and changes observed in velocity trends. We aim to disentagle the main transmission mechanisms of the impact of the public debt-to-GDP ratio on velocity by focusing on the public debt expectations channel.

The first varible of interest is the public debt-to-GDP ratio as the increasing trend in the total amount of government debt has become a concern after the Great Financial Crisis. According to the fiscal theory of the price level (Sims (1994), Woodford (1994), Benhabib et al. (2001), Cochrane (2001), Bassetto (2002), Cochrane (2022), Cochrane (2023)) when fiscal policy is active, fiscal policy is one determinant of the price level (thus inflation). However, this paper limits its analysis to the velocity of money, the public debt-to-GDP ratio, and fiscal policy without investigating the specific link between fiscal policy and inflation.

The second variable of interest is velocity of money supply because, by definition, it measures the rate at which money circulates within the economy, and provides a measure of economic wealth. Moreover, following the seminal work of Lucas Jr & Stokey (1987), velocity is recognized as a determinant of inflation, influencing it through the expectations channel. While many studies have analyzed the relationship between velocity and inflation, our specific focus is not on inflation, but rather on exploring the link between velocity and fiscal policy.

Firstly, we present the results obtained from Granger causality tests that we ran between the US time series of velocity of MZM and the public debt-to-GDP ratio.

We consider data from 1959Q1 to 2019Q4. To construct a measure for velocity, Gordon et al. (1998) use real money supply, and real consumption and investments. Following Isard & Rojas-Suarez (1986), Piazzesi et al. (2019) and the FRED database construction of velocity, we use nominal variables for the empirical part of our work, because nominal variables provide a comprehensive view of the transactions in the economy. We find strong evidence against the hypothesis that velocity is not Granger-caused by the public debt-to-GDP ratio, mainly for the entire sample, and between 1959Q1 and 1995Q4. We choose to split our samples based on a graphical analysis of the evolution of the two time series. Looking at Figure 2, from 1994 until 2002, the debt-to-GDP ratio and the inverse velocity take opposite directions. Therefore, we start from 1959Q1 until 1994Q1 and add one year at a time. We then run a Granger causality test on each of these subsamples. We find out that until 1995Q4, there is still a strong Granger causality relationship between the two time series. From 1996Q1 until 2002Q4, the evidence is weaker, the lag significance changes, and the sign of the coefficient is reversed, as shown in Figure 2. From 2003Q1 until 2019Q4 we again obtain evicence of public debt being a Granger cause for MZM velocity. To validate our findings, sensitivity analyses are performed. We conduct sensitivity analyses removing the Great Financial Crisis. For all of the tests we conduct, the initial significant results for the entire sample and the sample period of 1959Q1 - 1995Q4 are confirmed. Interestingly, when testing for reversed causality we fail to reject the hypothesis of non-Granger causality between velocity and public debt-to-GDP.

To explain the dynamics of velocity and the role of public debt, we construct a small New Keynesian model based on the framework developed by Galí (2020). Our model incorporates endogenous money supply and includes a fiscal block comprising government spending and transfers. By incorporating fiscal shocks and money supply that is endogeneously determined, our model captures real effects similar to those observed in previous studies such as Galí et al. (2007) and Davig & Leeper (2011), among others. In this framework, an increase in government spending leads to an expansion in private consumption, reflecting the interaction between fiscal policy and aggregate demand. Finally, through the linearity of the model, we explore the effects of a fiscal shock on velocity.

2 Brief literature review on fiscal expectations

Several authors have investigated topics related to fiscal expectations in the literature (Calvo (1988), Gordon et al. (1998), Leeper (2009), Bernasconi et al. (2009)). Bernasconi et al. (2009) conducted a laboratory experiment to assess the extent to which fiscal variables affect fiscal expectations. The authors ran the experiment in a controlled environment using real economic data, which the participants were shown and understood through adaptive expectations. The authors found that expectations are affected by the data. However, their work primarily focuses on expectations about fiscal variables through an experimental analysis. Calvo (1988) discusses the credibility of the government and the expectations about future repayment of the public debt. The focus of the analysis is on the non-uniqueness of equilibria, which can arise from the existence of government bonds and tax postponement.

Leeper (2009) addresses the anchoring of fiscal expectations and the differences between monetary policy expectations and fiscal policy expectations. The author emphasises the effects of anticipated tax changes and the transparency of government actions. Given the significant Great Financial Crisis and the increasing focus on the fiscal side, Leeper (2009) argues that expectations about fiscal policy should be addressed similary to expectations about monetary policy, due to their relevance in achieving macroeconomic stability.

Our focus is on the determinants of the velocity related to fiscal policy. To the best of our knowledge, not much work has been done on the relation between the velocity of money supply, fiscal policy and public debt. Our paper shares conceptual similarities with the work of Gordon et al. (1998). The authors focus on the general equilibrium determinants of velocity. The paper explores the trends in the velocity of base money in the US from 1960 to 1997 and whether these trends can be explained by endogenous responses to changing expectations about monetary and fiscal policy. The authors use a model that maps policy expectations into portfolio decisions, making equilibrium velocity a function of expected future money growth, tax rates, and government spending. They find that the observed secular movements in velocity can be accounted for exclusively by endogenous responses to policy expectations. While our empirical results align with those obtained by Gordon et al. (1998), our study differs in that we provide empirical evidence of Granger Causality between public debt-to-GDP ratio and velocity over an extended sample period. Additionally, we analyse periods of time when we find both positive and negative Granger causality between the two variables. Furthermore, we demonstrate evidence of these empirical findings within the framework of a New Keynesian model. The analysis conducted through the lens of a New Keynesian model helps us understand the significance of nominal rigidities in the transmission mechanisms of changes in fiscal policy and expectations regarding future surpluses or deficits.

3 Empirical evidence

We evaluate several US time series representing the public debt-to-GDP ratio and MZM inverse velocity. The inverse velocity is constructed as the ratio between MZM money stock and nominal GDP. We use the inverse of velocity in order to establish a relationship between two comparable ratios. The inverse velocity is given by $\frac{M_t}{P_t Y_t}$, where M_t represents the monetary stock, P_t is the price level, and Y_t is real GDP. Similarly, the public debt-to-GDP ratio is given by $\frac{B_t}{P_t Y_t}$, where and B_t is the public debt. By using the inverse of velocity and the ratio between public debt and nominal GDP, we ensure that the evolution of the two ratios is not influenced by the position of output Y_t in the ratios. This allows us to evaluate the relation between two quatities as a share of output Y_t[3].

To construct the public debt-to-GDP ratio, we follow Bianchi et al. (2022) and other researchers, that use the market value of marketable treasury debt. This approach provides a more reliable measure for quantifying the debt burden of the United States. All the time series used in the analysis are retrieved from the Federal Reserve Economic Data database of the Federal Reserve Bank of St. Louis.

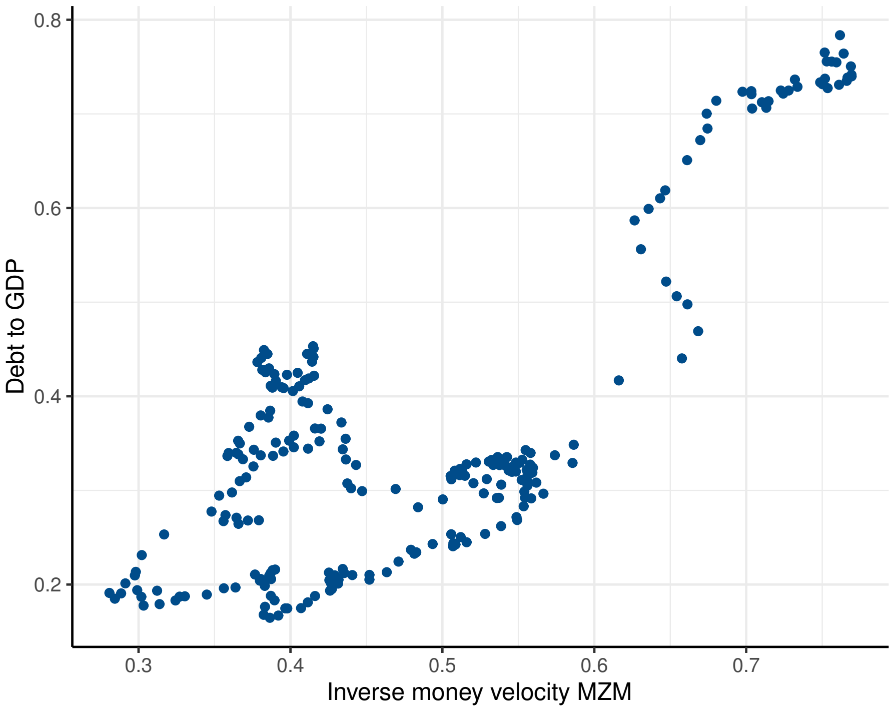
Figure 1 illustrates the relation between quarterly data for public debt-to-GDP and inverse velocity over the period 1959Q1 - 2019Q4. The figure reveals a positive relation between the two variables (hence, a negative correlation between velocity and the public debt-to-GDP ratio). A correlation test conducted on the two stationary time series yields a Pearson correlation coefficient of 0.50.

3.1 Regression results

To test for Granger causality (Granger, 1969), we perform a regression of the inverse of velocity on its own lagged values as well as on lagged values of the public debt-to-GDP ratio. The null hypothesis tested is that the estimated coefficients cor-

[3]A positive Granger causality between the inverse velocity and the public debt-to-GDP ratio implies a negative Granger causality between velocity and the public debt-to-GDP ratio.

Figure 1: Relationship between the public debt-to-GDP ratio and the MZM inverse velocity.



responding to the lagged values of the debt-to-GDP ratio are zero. Failure to reject the null hypothesis is equivalent to failing to reject the hypothesis that the public debt to GDP does not Granger-cause the inverse velocity of MZM. Furthermore, considering that no specific pattern is assumed in the Granger causality analysis, we also regress the public debt-to-GDP ratio on its own lagged values and on the lagged values of the inverse of velocity. Rejection of the null hypothesis in this case indicates rejection of non-Granger causality in the opposite direction: from the inverse of velocity to public debt-to-GDP ratio. Results for this robustness analysis are presented in section 7.2.

We use a VAR(p) model of the type:

$$V_t = \phi + \sum_{i=1}^{p} \phi_i^V V_{t-i} + \sum_{i=1}^{p} \phi_i^B B_{t-i} + \epsilon_t^V$$

$$B_t = \gamma + \sum_{i=1}^{p} \gamma_i^V B_{t-i} + \sum_{i=1}^{p} \gamma_i^B V_{t-i} + \epsilon_t^B$$

where V_t is the MZM inverse velocity and B_t represents the public debt-to-GDP ratio.

In structural form it is represented as:

$$\mathbf{x}_t = c + \sum_{i=1}^{p} A_i \, \mathbf{x}_{t-i} + \epsilon_t \tag{1}$$

The number of lags in the model is choosen based on the Bayesian Information Criterion (BIC), also known as the Schwarz' criterion.

Table 1 shows the results of Granger causality tests between the variables for the entire sample 1959Q1 until 2019Q4, as well as for other subsamples. The table displays the estimated coefficient that is found to be significant based on the VAR model selected using the BIC. We report the significant estimates for each regression, the lag number to which each estimate corresponds, the total number of lags in the model, and the adjusted R-squared of the regression. The analysis indicates that for each of the subsamples, the coefficient for the first lag of the public debt-to-GDP ratio is found to be statistically significant and positive, except for the period between 1996Q1 and 2002Q4. For this period the sign of the coefficient is reversed. It is also worth noting that for the subsamples 1996Q1 - 2002Q4 and 2003Q1 - 2007Q4, the standard errors are larger than for the other subsamples. As mentioned above, table 1 presents the results for the relationship between the the public debt-to-GDP ratio and the inverse velocity of money supply. Therefore, all coefficient signs reported in the table are reversed when considering the relation between public debt-to-GDP ratio and the MZM velocity.

For the entire sample, the results suggest that inverse velocity is Granger caused by the public debt-to-GDP ratio. Additionally, we obtain significant and positive coefficients for the public debt-to-GDP ratio from 1959Q1 to 1995Q4, with high significance levels. The negative sign for the public debt-to-GDP coefficient from 1996Q1 until 2002Q1 can be confirmed by examining the evolution of the two time series in the upper and middle plots of Figure 2. Interestingly, this time period coincides with Bill Clinton's second term, characterized by the largest budget surpluses in the US government history, and the first surpluses since the 1960s. The third part of figure 2 illustrates the evolution of Federal Deficits and Surpluses throughout the analyzed time period.

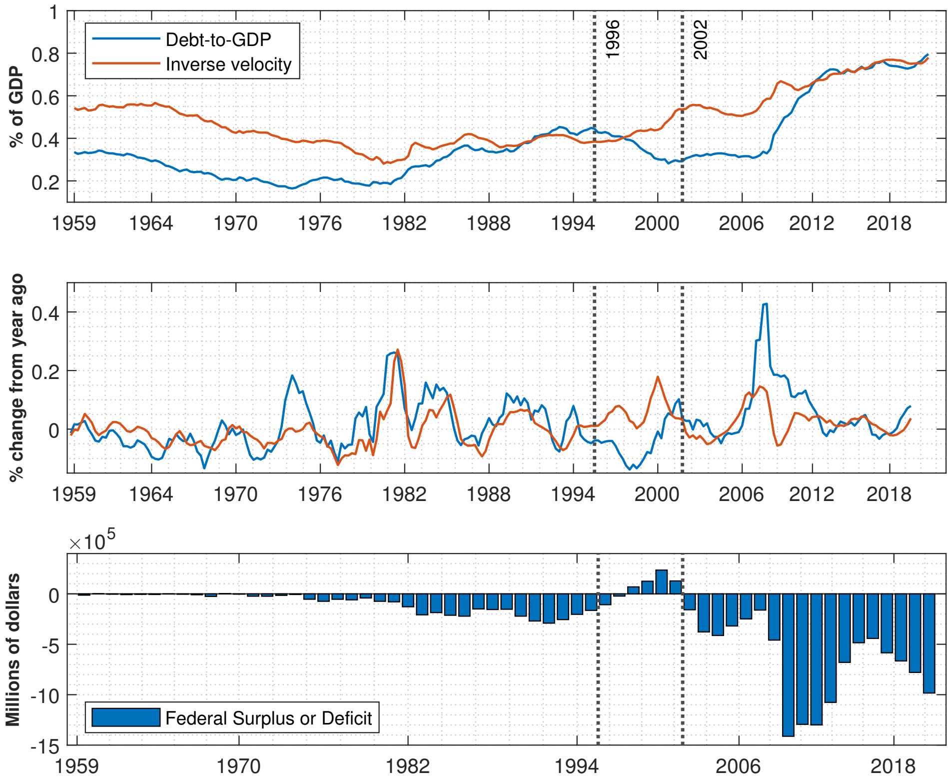
Table 1: The table reports results for Granger causality test with H0: Public debt-to-GDP does not Granger cause inverse velocity. The column "Lag" reports the significant lag for each time subsample. The column "BIC lags" indicates the selected lags for the VAR model. Standard errors are between parentheses. ***, **, * denote rejection of the null hypothesis at the 1, 5 and 10% significance level, respectively.

Time period	Estimate	Lag	BIC lags	Adj. R-squared	p-value
1959Q1 - 2019Q4	0.22***	1	1	0.31	4e-08
	(0.03)				
1959Q1 - 1995Q4	0.34***	1	1	0.32	6e-09
	(0.05)				
1996Q1 - 2002Q4	-0.33*	3	3	0.23	0.11
	(0.15)				
2003Q1 - 2007Q4	0.44*	1	1	0.29	0.02
	(0.18)				
2008Q1 - 2019Q4	0.15*	1	1	0.30	0.02
	(0.06)				

In 2003 the war in Iraq began, leading to an escalation in the fiscal burden and a transition from surpluses to deficits. From 2003Q1 to 2019Q4 our results once again indicate a positive relationship between the public debt-to-GDP ratio and the inverse velocity. It is worth noting that this time period encompasses the Global Financial Crisis, which coincided with substantial changes in the US public debt. Results for robustness analysis excluding the Great Financial Crisis from the entire sample are presented in 3.

This analysis suggests that the Great Financial Crisis may not have affected the relation between the two indicators. However, it is noteworthy that the coefficient, as well as the standard error, for the public debt-to-GDP coefficient prior to the crisis is higher compared to after the crisis. During the crisis, monetary policy underwent changes that included unconventional measures aimed at managing the effects of the recession. One such measure was Quantitative Easing, through which the Federal Reserve acquired substantial amounts of long-term Treasury debt. Given the objectives of Quantitative easing, these large-scale purchases had an impact on

Figure 2: The plot illustrates the relationship between the public debt-to-GDP ratio and the inverse of the MZM velocity. The second plot shows the percentage change of both variables compared to the previous year. The histogram provides an overview of the evolution of deficits and surpluses in the United States.



both US money supply and the total federal debt.

We develop a small theoretical model to enhance our understanding of the trasmission mechanisms through which fiscal policy affects the velocity of money supply. The model is presented in the next section.

4 Theoretical model

In this section we introduce the theoretical model. The model follows the New Keynesian framework in Galí (2020). The model behaves as its counterparts in the vast

New Keynesian literature, except we introduce velocity and we model money supply such that the quantity of money is endogeneously determined. Such a representation closely resembles the unconventional monetary policies that have characterized central bank actions since the Great Financial Crisis. The increase in money supply within the economy has been driven by the Federal Reserve's purchases of government assets on secondary markets. Additionally, this model enables us to replicate the empirical findings concerning velocity and public debt. Calibration of price rigidity is relevant for the final results of this model.

4.1 Households

The economy is populated by a continuum of households. The households consume, supply labor force to firms, and own and receive dividends from those firms. The also hold one-period riskless bonds. The implicit form of the household's utility is the following:

$$E_0 \sum_{t=0}^{\infty} \beta^t \mathcal{U}\left(C_t, L_t, N_t; Z_t\right)$$

with period $\mathcal{U}(.)$ utility function taking the form: $\mathcal{U} = \left(U\left(C, L\right) - V\left(N\right)\right) Z$ where $L_t \equiv M_t/P_t$ are the real money balances. M_t is the nominal stock of money and P_t is the price level, C_t is consumption, N_t is employment, and Z_t represents a preference shifter. The preference shifter can be represented as:

$$Z_t = \left(Z_{t-1}\right)^{\rho_z} \tag{2}$$

As such, the utility function assumes non separability between consumption and money balances.

The household's budget constraint is standard and writes:

$$P_t C_t + B_t + M_t = B_{t-1}\left(1 - i_{t-1}\right) + M_{t-1} + W_t N_t + D_t + P_t T_t$$

where B_t is a one-period riskless bond, W_t is the nominal wage, N_t represents the hours worked. As owners of the firms, D_t are dividends received. T_t represent the government transfers. Given the discount factor β, the households maximize their utility function.

4.2 Firms

There are two types of firms in the economy: final good firms and intermediate goods firms. Final good firms pack intermediate goods into a final good, that is then sold to households in a perfectly competitive market. Intermediate good firms hire labor from households, that also own the intermediate good firms, and produce intermediate good that are then sold to the final good firms. Intermediate good firms face a cost of changing prices, that produces nominal rigidities in the market. Intermediate goods are produced with the following technology:

$$Y_{it} = N_{it}^{1-\alpha}$$

where N_t represents the labor. The real marginal cost MC_t^r is:

$$MC_t^r = W_t P_t^{-1} A_t^{-1} (1-\alpha)^{-1} N_t^{\alpha}$$

Intermediate firms set their prices following Calvo (1983). A fraction of firms equal to $1-\theta$ can change their price, while the remaining fraction θ have to keep their prices unchanged. For this reason, their FOC is:

$$E_t \sum_{k=0}^{\infty} \beta^k \theta^k Q_{t,t+k} Y_{it+k|t} \left(P_t^* - \mu_{t+k} MC_{t+k|t}^n \right) = 0$$

where $Q_{t,t+k}$ is the nominal discount factor for firms and the discount factor for the households, $Y_{it+k|t}$ is output produced in period $t+k$ given the price set in period t, P_t^* is the optimal price, μ_{t+k} is the price mark-up and $MC_{t+k|t}^n$ is the nominal marginal cost.

The final good firms produce their goods with the following technology:

$$Y_t = \left(\int_0^1 Y_{it}^{\frac{1}{1+\epsilon_t}} \, di \right)^{1+\epsilon_t}$$

4.3 Government and monetary policy

The government and the central bank cooperate to issue joint fiscal and monetary policy. More precisely, in our model, government spending and government transfers are financed either by money growth or by government debt. The coordinated budget constraint of government and the central bank is:

$$P_tG_t + B_{t-1}\,(1 + i_{t-1}) = B_t - P_tT_t + \Delta M_t \tag{3}$$

where G_t is the government spending, T_t are the transfers to households and ΔM_t is the money growth controlled by the central bank, where $\Delta M_t = M_t - M_{t-1}$. Due to the endogeneity of money, set jointly by the central bank and the government, the model exhibits fiscal dominance. Additionally, we make the assumption that the tax rates are kept constant throughout the analysis. However, it is worth noting that instead of the government transfers, we could have alternatively used lump-sum taxes with the opposite sign, and the results would remain unchanged.

4.4 Velocity of money supply

We adopt the definition of velocity as employed in the Federal Reserve Bank of St. Louis FRED database, where it is defined as the ratio of nominal GDP to the monetary aggregate. In our study, the monetary aggregate we consider is the monetary base, MZM.

$$V_t \equiv \frac{P_tY_t}{M_t} \tag{4}$$

In this framework, where households anticipate that future deficits will not be repaid via debt but rather through monetary means, we can now analyze the separate influences of real GDP and money on velocity. We do not make the assumption that GDP or velocity remains constant, as empirical data indicate that velocity exhibits variability. By introducing the linearized identity that defines velocity, we will examine the effects of real GDP and real money supply on velocity.

4.5 Equilibrium

The equilibrium in the economy is given by:

$$Y_t = C_t + G_t \tag{5}$$

To see the model equilibrium consitions and the full set of linearized equations, please refer to the Appendix 8.

Figure 3: Impulse response functions to one standard deviation government transfers shock with different Calvo parameters.

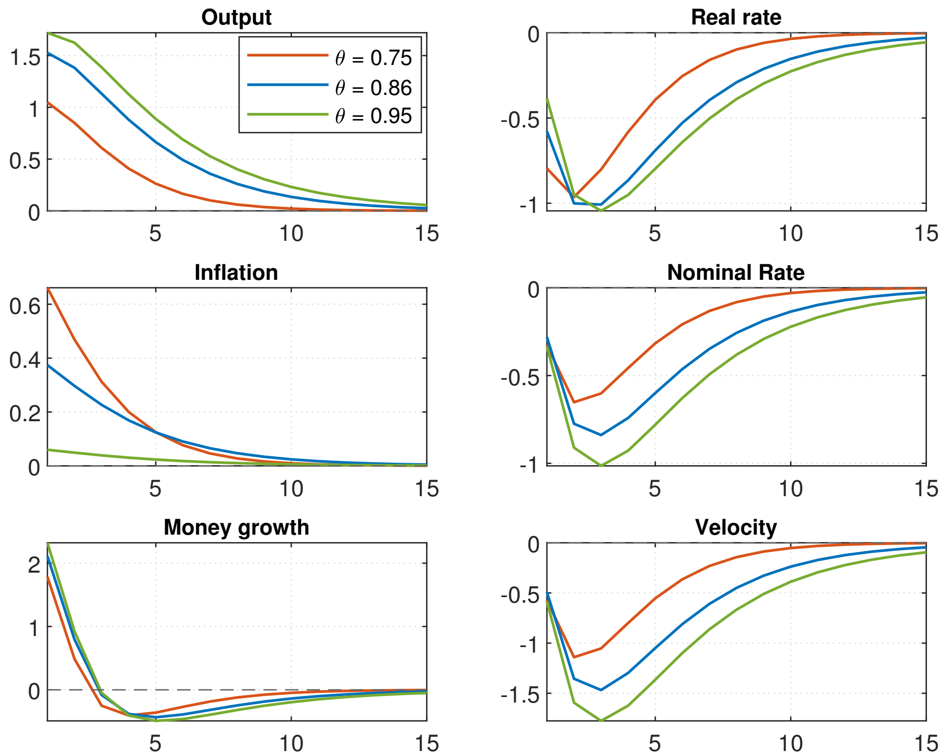
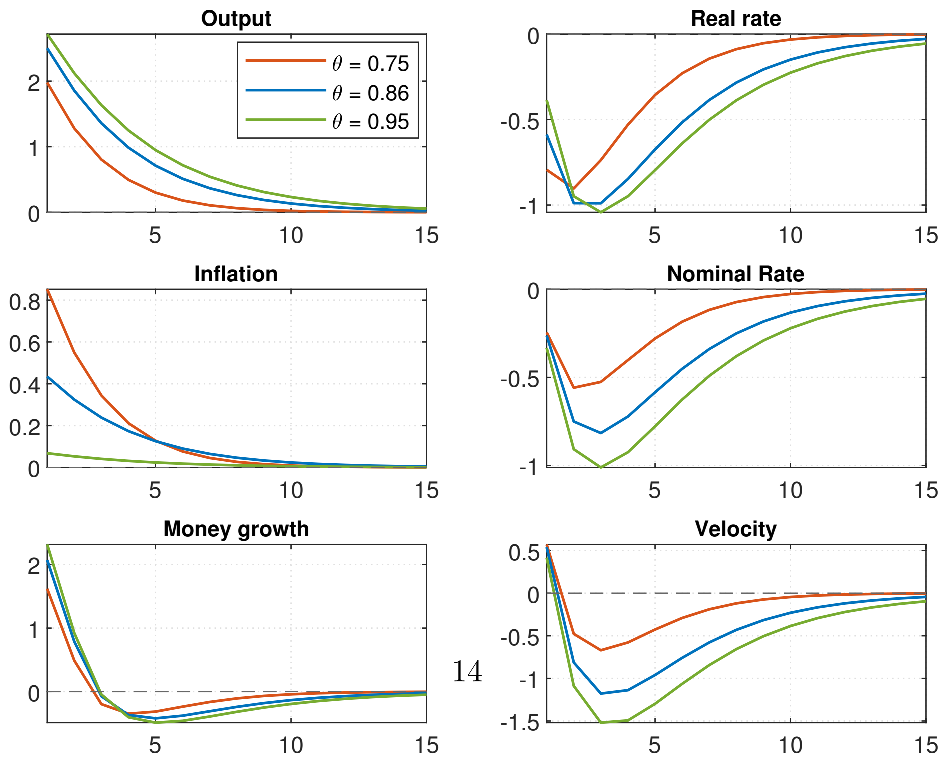


Figure 4: Impulse response functions to one standard deviation government spending shock with different Calvo parameters.



5 Calibration

Our calibration mostly follows Galí (2015) and Galí (2020) and is displayed in table 2. Following Leeper et al. (2010) among others, we model steady state trasfers as a residual from the government bduget constraint, after having defined $\bar{g}$ and $\bar{b}$. We calibrate the parameters for the government transfers and the government spending response to debt as in Leeper et al. (2017), although the impulse response functions for fiscal shocks are robust to different parameterizations.

Table 2: Calibrated parameters and source

Parameter	Value	Source
β Household's discount factor	0.995	Galí (2015, 2020)
α Labor share in Cobb Douglas function	0.25	Galí (2015, 2020)
φ Inverse Frisch elasticity of labor supply	1	Davig & Leeper (2011)
θ Calvo parameter	0.75	Galí (2015, 2020)
η semi-elasticity of money demand to interest rate	7	Galí (2015, 2020)
δ_t AR persistence parameter transfers	0.5	Galí (2020)
δ_g AR persistence parameter government spending	0.5	Galí (2020)
ϵ Elasticity of substitution in CES utility	9	Galí (2015, 2020)
$\bar{b}$ Steady state value of government debt	0.60	Galí (2020)
$\bar{g}$ Steady state value of government spending	0.19	Mean value our sample
χ Steady state inverse velocity of money supply	0.42	Mean value our sample
ψ_{bg} Government spending response to debt	0.2	Leeper et al. (2017)
ψ_{bt} Government transfers response to debt	0.2	Leeper et al. (2017)

Besides the baseline calibration, we run simulations Eichenbaum & Fisher (2005) estimate the Calvo parameter over the period 1959Q1 - 2001Q4. Their estimates range between a low of 0.83 and a high of 0.89. We use the average value of 0.86, that implies a price change each $\frac{1}{1-\theta} = 8.3$ quarters. Rabanal & Rubio-Ramírez (2005) find average values for the Calvo parameter ranging between 0.75 and 0.86. Finally, Leeper et al. (2017) estimate even higher Calvo parameters, ranging from 0.89 to 0.95. Using this evidence, we show impulse response functions for 0.75, 0.86 and 0.95.

6 Fiscal policy shocks and velocity

Figure 3 shows impulse response functions for an increase in transfers. Figure 4 shows impulse response functions for an increase in government spending. The baseline calibration is showed in table 2.

The figure shows the results for different levels of price rigidities as descrived in the previous section.

Consistent with the existing literature, our model demonstrates that a fiscal shock leads to a higher expected future primary deficit. Furthermore, when this shock is financed through money supply, it exerts an expansionary impact on the economy by reducing the real rate. This is possible because the central bank accomodates the inflationary effects resulting from an increase in output. As a result, the nominal interest rate does not increase sufficiently to increase the real rate. This mechanisms applies to both types of fiscal shocks, whether it is an increase in transfers or an increase in government spending.

On the other side, our model follows the mechanisms outlined in the framework of Sargent et al. (1981), where an increase in public debt enhances expectations of debt monetization. These concerns raise inflation expectations and, consequently, inflation itself. As a result, economic agents prefer not to postpone consumption, leading to an increase in both consumption and output in the present.

However, if output and consumption increase more than the increase in the money supply, this will drive up velocity. This is the case of an increase in government spending using the baseline calibration as in Table 2. The orange line in figure 4 represent impulse response functions for this scenario. The impulse response functions obtained using alternative calibrations, show that an increase in the price stickyness parameter change the impact on velocity. With stickier prices, consumption and output increase at a higher pace than the increase in money supply. This allows the velocity to initially slightly increase (which drives the increase in consumption), then to decrease substantially.

With an increase in government transfers velocity decreases for all the scenarios considered. After the increase in fiscal transfers, along with the increse in money supply to finance the rise in future deficits, inflation expectations increase. However, the increase in consumption and in output is not substantial enough to induce an increase in velocity. As a result, velocity decreases.

7 Robustness analysis

7.1 Removing the Great Financial Crisis period

Table 3 presents robustness checks on the entire sample, excluding the period of the Great Financial Crisis (2008Q1 - 2009Q2, which is recognized as a recessionary period according to the Fred database). The results indicate no change in the coefficient lag and significance. Interestingly, the coefficient of the debt-to-GDP first lag, the standard error and the p-value are very similar to the ones obtained including the Great Financial Crisis (GFC) in the sample[4]. These results suggest that the GFC did not have a significant impact on the relationship between velocity and public debt-to-GDP ratio, as mentioned in section 3.

Table 3: The table reports results for Granger causality tests after removing the Great Financial Crisis. H0: Public debt-to-GDP does not Granger cause velocity. The column "Lag" reports the significant lag for each time subsample. The column "BIC lags" indicates the selected lags for the VAR model. Standard errors are between parentheses. ***, **, * denote rejection of the null hypothesis at the 1, 5 and 10% significance level, respectively.

Time period	Estimate	Lag	BIC lags	Adj. R-squared	p-value
1959Q1 - 2019Q4	0.21***	1	1	0.27	1e-06
	(0.04)				

7.2 Inverse velocity → public debt-to-GDP

Table 4 displays the results for Granger causality in the opposite direction, specifically when testing whether velocity Granger causes the public debt-to-GDP ratio. None of the time subsamples yield significant coefficients, suggesting the absence of a Granger causality relationship in this direction.

[4]See table 1 for a comparison.

Table 4: The table reports results for Granger causality test with H0: inverse velocity does not Granger cause public debt-to-GDP ratio. The column ”Lag” reports the significant lag for each time subsample. The column ”BIC lags” indicates the selected lags for the VAR model. Standard errors are between parentheses. ***, **, * denote rejection of the null hypothesis at the 1, 5 and 10% significance level, respectively.

Time period	Estimate	Lag	BIC lags	Adj. R-squared	p-value
1959Q1 - 2019Q4	-	-	1	0.16	0.29
1959Q1 - 1995Q4	-	-	1	0.12	0.66
1996Q1 - 2002Q4	-	-	3	-0.00	0.94
2003Q1 - 2019Q4	-	-	1	0.08	0.91
2008Q1 - 2019Q4	-	-	1	0.22	0.10
Without the Great Financial Crisis					
1959Q1 - 2019Q4	-	-	1	0.15	0.21

8 Conclusions

This paper investigates the relationship between the public debt-to-GDP ratio and the money zero maturity (MZM) inverse velocity in the United States. Our findings provide evidence of Granger causality between the public debt-to-GDP ratio and the inverse of velocity, while we cannot establish the reverse Granger causality. Specifically, we observe a positive relationship between the two variables, indicating a negative association between the public debt-to-GDP ratio and the MZM velocity.

From a theoretical standpoint, we construct a small model to delve deeper into the transmission mechanisms underlying this one-way relationship. In the event of an expansionary fiscal policy characterized by an increase in government spending, the public debt expands, thereby influencing expectations regarding the repayment of future surpluses or deficits. As a result, inflation expectations rise, triggering a rise in inflation. Our results suggest that in a forward-looking model, such as the one employed in this paper, agents' expectations about future money growth driven by a fiscal policy shock influence inflation expectations, thereby affecting inflation in the current period. If current consumption and output increase more than money supply, velocity also increases. Conversely, if they increase less, the model predicts a decline in the velocity of money supply, which aligns with our empirical findings. The degree of nominal rigidities is a key factor influencing these results. When we consider an increase in government transfers, velocity always decreases.

One limit of this paper is worth noting. The empirical analysis conducted with the Granger causality tests only focus on two variables. Both public debt-to-GDP and velocity of money supply can be influenced by a wide range of economic factors, including other type of fiscal policies, monetary policy, and global economic conditions. However, overall preliminary results suggest that the impact of the expansionaty fiscal policy may be transmitted to velocity through the expectations channel.

Appendix

This appendix describes the non-linearized and the linearized version of the theoretical model. The notation is the following: upper case variables with a time subscript are variables in levels (e.g. X_t), steady state values are letters without a time subscript (e.g. X), and lower case variables with a hat and a time subscript are linearized variables (e.g. $\hat{x}_t$).

Household

The household has the following utility function:

$$\max_{C_t,N_t,L_t,B_t} E_t \sum_{t=0}^{\infty} \beta^t \mathcal{U}\left(C_t, L_t, N_t; Z_t;\right)$$

and the budget constraint is:

$$P_t C_t + B_t + M_t = B_{t-1}\left(1 - i_{t-1}\right) + M_{t-1} + W_t N_t + D_t + P_t T_t$$

The maximisation problem is the following:

$$\max_{C_t,N_t,L_t,B_t} E_t \sum_{t=0}^{\infty} \beta^t \mathcal{U}\left(C_t, L_t, N_t; Z_t;\right) - \\ - \lambda_t \left(P_t C_t + B_t + M_t - B_{t-1}\left(1 - i_{t-1}\right) - M_{t-1} - W_t N_t - D_t - P_t T_t\right)$$

The first order conditions are:

$$\lambda_t = \beta^t U_{c,t} Z_t \frac{1}{P_t} \tag{6}$$

$$\lambda_t = \beta^t \, Z_t \frac{V_{n,t}}{W_t} \tag{7}$$

$$\beta^t \frac{1}{P_t} \frac{U_{l,t}}{\zeta_t} Z_t = \lambda_t - \lambda_{t+1} \tag{8}$$

$$\frac{\lambda_t}{\lambda_{t+1}} = 1 + i_t \tag{9}$$

The Euler equation is obtained by substituting $\frac{\lambda_t}{\lambda_{t+1}}$ in equation (9) with the derivation of the lagrangian , λ_t obtained from (6):

$$U_{c,t} = \beta^t \left(1 + i_t\right) \frac{Z_{t+1}}{Z_t} \frac{P_t}{P_{t+1}} U_{c,t+1} \tag{10}$$

The labor supply equation is derived from equations (6) and (7):

$$\frac{W_t}{P_t} = \frac{V_{n,t}}{U_{c,t}} \tag{11}$$

Finally, the money demand is derived from equations (6), (8) and (9).

From (6):

$$\frac{U_{l,t}}{U_{c,t}} = 1 - \frac{P_t \lambda_{t+1}}{P_t \lambda_t}$$

after cancelling out P_t and substituting $\frac{\lambda_{t+1}}{\lambda_t}$ as in (9), we obtain:

$$\frac{U_{l,t}}{U_{c,t}} = 1 - \frac{1}{1+i_t} = \frac{i_t}{1+i_t}$$

As in Galí (2020) we define $\frac{U_{l,t}}{U_{c,t}} = h\left(\frac{L_t}{C_t}\right)$. The money demand can be written as:

$$\frac{U_{l,t}}{U_{c,t}} = h\left(\frac{L_t}{C_t}\right) = \frac{i_t}{1+i_t} \tag{12}$$

Firms

Final good firms

The final good firms produces their goods with the following technology:

$$Y_t = \left(\int_0^1 Y_{it}^{\frac{1}{1+\epsilon_t}}\, di\right)^{1+\epsilon_t}$$

where Y_{it} are intermediate goods. Firms maximise profits subject to:

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\frac{(1+\epsilon_t)}{\epsilon_t}} Y_t$$

where the price index P_t can be written as:

$$P_t = \left(\int_0^1 P_{it}^{-\frac{1}{\epsilon_t}}\, di\right)^{-\epsilon_t}$$

Intermediate firms

Intermediate goods are produced with technology:

$$Y_{it} = A_t N_{it}^{1-\alpha}$$

and the marginal cost MC_t is:

$$MC_t = W_t P_t^{-1} A_t^{-1} (1-\alpha)^{-1} N_t^{\alpha}$$

Intermediate firms adjust their prices according to the Calvo price setting, as outlined in Calvo (1983). This implies that a fraction of firms, denoted as θ, are unable to change their prices over time. On the other hand, a fraction of $1-\theta$ firms have the flexibility to adjust their price over time, and they set P_t^*. P_t^* is the same price for all the firms that are ajusting. The firms that have the ability to change their prices take into account the potential impact on future profits when deciding on a price adjustment today.

The aggregate price dynamics is

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon}$$

and the linearized version of it, doing a first order Taylor expansion:

$$\pi_t = (1-\theta)\left(p_t^* - p_{t-1}\right) \tag{13}$$

The firms' optimizing problem is:

$$\max_{P_t^*} \left\{ \sum_{k=0}^{\infty} \theta^k E_t \left[\left(\beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}} \right) \left(P_t^* Y_{it+k|t} - TC_{it+k|t}^n \left(Y_{it+k|t} \right) \right) \right] \right\} \tag{14}$$

under the following set of demand constraints:

$$Y_{it+k|t} = \left(\frac{P_t}{P_{t+k}}\right)^{-\epsilon} Y_{t+k} \tag{15}$$

where P_t^* is the price that maximizes present value of profits while having that price and it is set by the firms. $Y_{it+k|t}$ is the output produced while having that price and $MC_{it+k|t}^n$ is the marginal cost a firm faces given that price. $\beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}}$ is the discount factor derived from the Euler equation. The first order condition for price P_t^* is

$$E_t \sum_{k=0}^{\infty} \beta^k \theta^k Q_{t,t+k} Y_{it+k|t} \left(P_t^* - \mu_{t+k}\, MC_{t+k|t}^n \right) = 0 \tag{16}$$

where $Q_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}}$ is the nominal discount factor for firms and the discount factor for the households.

Steady state equations

First, the price mark-up has to be derived to obtain the first steady state equation. To do so, the MPN_t^n marginal productivity of labor is defined as:

$$MPN_t^n = \frac{\partial Y_t}{\partial N_t} = A_t \left(1 - \alpha\right) N_t^{-\alpha} \tag{17}$$

As in Galí (2015), the nominal marginal cost using labor is W_t. The nominal marginal gain of firms by using labor is the income increase, that is the price times the marginal increase in production by adding one unit more of labor. Thus, the real marginal cost is the nominal cost relative to the nominal gain:

$$MC_t^r = \frac{W_t}{P_t \, MPN_t^n} \tag{18}$$

Substituting for MPN_t as in equation (17) we obtain:

$$MC_t^r = \frac{W_t}{P_t \left(1 - \alpha\right) A_t N_t^{-\alpha}} \tag{19}$$

As the firms' mark up is equal to the inverse of the real marginal cost:

$$\mu_t = \frac{\left(1 - \alpha\right) P_t A_t}{W_t N_t^{\alpha}} \tag{20}$$

Where μ_t is the price mark-up.

In steady state, the mark-up is equal to the desired mark-up:

$$\mu = \frac{\left(1 - \alpha\right) P}{W N^{\alpha}} = \frac{\epsilon}{\epsilon - 1} \tag{21}$$

where A is normalized to 1. Denoting $\frac{\epsilon}{\epsilon-1} = \mathcal{M}$, from (21) it follows that:

$$\left(1 - \alpha\right) P = \mathcal{M} W N^{\alpha} \tag{22}$$

and considering equation (11) evaluated at steady state,

$$\frac{W}{P} = \frac{V_n}{U_c} \tag{23}$$

equation (22) becomes:

$$\left(1 - \alpha\right) = \mathcal{M} \frac{W}{P} N^{\alpha} \tag{24}$$

$$\left(1 - \alpha\right) = \mathcal{M} \frac{V_n}{U_c} N^{\alpha} \tag{25}$$

$$\left(1 - \alpha\right) U_c = \mathcal{M} V_n N^{\alpha} \tag{26}$$

which is equivalent to writing:

$$(1-\alpha)\, U_c\left(N^{1-\alpha}, L\right) = \mathcal{M}\, V_n\,(N)\, N^{\alpha} \tag{27}$$

The second equation describing the steady state is obtained from the money demand:

$$h\left(\frac{L}{C}\right) = \frac{i}{1+i} \tag{28}$$

where, from the definition of β and the Euler equation evaluated at steady state, $i = \rho$. Therefore, equation (28) can be rewritten as:

$$h\left(\frac{L}{N^{1-\alpha}}\right) = \frac{\rho}{1+\rho} \tag{29}$$

From the government

Linearized model

Economic identity

$$\hat{y}_t = \hat{c}_t + \hat{g}_t \tag{30}$$

Euler equation

$$\hat{\xi}_t = \hat{\xi}_{t+1} + \hat{i}_t - \pi_{t+1} - \hat{\rho}_t \tag{31}$$

Non-separable household utility function

This equation describes the two linearized components of the utility function $U(C, L)$.

$$\begin{aligned} \hat{\xi}_t &= ln\left(\frac{U_{c,t}}{U_c}\right) \\ &= \hat{c}_t C \frac{U_{cc}}{U_c} + \hat{l}_t L \frac{U_{cl}}{U_c} \\ &= -\sigma \hat{c}_t + \nu \hat{l}_t \end{aligned} \tag{32}$$

where $U_{c,t} = U(C_t, L_t)$, $\sigma \equiv -C\frac{U_{cc}}{U_c}$ and $\nu \equiv L\frac{U_{cl}}{U_c}$

Price mark-up

$$\hat{\mu}_t = \hat{\xi}_t - \hat{y}_t\left(\frac{\varphi}{1-\alpha} + \frac{\alpha}{1-\alpha}\right) + \hat{a}_t\left(\frac{\alpha+\varphi}{1-\alpha} + 1\right) \tag{33}$$

Money demand

$$\hat{l}_t = \hat{c}_t - \eta \hat{i}_t$$

where:

$$\eta = \frac{\epsilon_{l,c}}{\rho} \text{ and } \epsilon_{l,c} = -\frac{1}{h'}\frac{\rho}{1+\rho}V = \frac{1}{\sigma_l + \nu}$$

New Keynesian Phillips Curve

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \widehat{mc}_t^r + \hat{\lambda}_t \tag{34}$$

where

$$\lambda = \frac{(1-\theta)(1-\theta\beta)}{\theta}$$

Definition of money growth

$$\Delta \hat{m}_t = \hat{l}_t - \hat{l}_{t-1} + \pi_t \tag{35}$$

where $\hat{m}_t - \hat{m}_{t-1} = ln\left(\frac{M_t}{M_{t-1}}\right) = \Delta\hat{m}_t$ and $\hat{p}_t - \hat{p}_{t-1} = ln\left(\frac{P_t}{P_{t-1}}\right) = \pi_t$.

Fiscal rule

We follow Leeper et al. (2010) in setting the fiscal rules, with the exception that in our baseline analysis, we use fiscal rules based on debt only. However, as a robustness analysis, we also show results obtained with fiscal rule based on both output and public debt.

The fiscal rule for transfers is:

$$\hat{t}_t = -\psi_{bt}\,\hat{b}_{t-1} + \hat{t}_t^*$$
$$\hat{t}_t^* = \delta_t\,\hat{t}_{t-1}^* + \epsilon_t$$

and for government spending:

$$\hat{g}_t = -\psi_{bg}\,\hat{b}_{t-1} + \hat{g}_t^*$$
$$\hat{g}_t^* = \delta_g\,\hat{g}_{t-1}^* + \epsilon_g$$

Government budget constraint

$$\frac{\bar{g}}{\bar{y}}\hat{g}_t + \frac{\bar{t}}{\bar{y}}\hat{t}_t + \frac{\bar{b}}{\bar{y}}\bar{r}(\hat{b}_{t-1} + \hat{r}_{t-1} - \hat{\pi}_t) = \frac{\bar{b}}{\bar{y}}\hat{b}_t - \frac{\bar{m}}{\bar{y}}(\hat{l} - \hat{l}_{t-1} + \hat{\pi}_t) \tag{36}$$

Where $\frac{\bar{m}}{\bar{y}}(\hat{l} - \hat{l}_{t-1} + \hat{\pi}_t)$ represents the seignoirage term and has been obtained as a fraction of output from the government budget constraint:

$$\begin{aligned}\frac{\Delta M_t}{P_t} =& \frac{M_t - M_{t-1}}{P_t}\frac{P_{t-1}}{P_{t-1}} \\ =& l_t - \frac{l_{t-1}}{\pi_t}\end{aligned}$$

Linearizing the term above and taking the ratio $\frac{1}{y_t}$:

$$(l_t - \frac{l_{t-1}}{\pi_t})\frac{1}{y_t} \approx \frac{\bar{m}}{\bar{y}}(\hat{l} - \hat{l}_{t-1} + \hat{\pi}_t) \tag{37}$$

the term between parenthesis in equation 37 represents nominal money growth, as defined in equation 35.

Velocity identity

Finally, the linearized velocity equation, transformed in real terms, is the following:

$$\hat{v}_t \equiv \hat{y}_t - \hat{l}_t \tag{38}$$

Data

The data are retrieved from the FRED database of the Federal Reserve Bank of St. Louis. The nominal public debt is the Market Value of Marketable Treasury Debt (MVMTD027MNFRBDAL) expressed in billions of dollars. For the Gross Domestic Product, we use the series Gross Domestic Product (GDP) in billions of dollars. Finally, we use MZM Money Stock (MZM) for the monetary base. The final data we utilize consists of growth rates.

$$x_t = \frac{X_t - X_{t-1}}{X_{t-1}} \tag{39}$$

where $x_t = \frac{MZM}{GDP}, \frac{MVMTD027MNFRBDAL}{GDP}$

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