Sampling in the space of image patches with Determinantal Point Processes

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Determinantal Point Processes (DPPs)

- Model negative correlation thanks to the deteminant of a kernel K : avoid bunching effect
- An exact sampling algorithm, based on the spectral decomposition of K
- Natural to apply DPPs to the space of paches, redundant and diverse

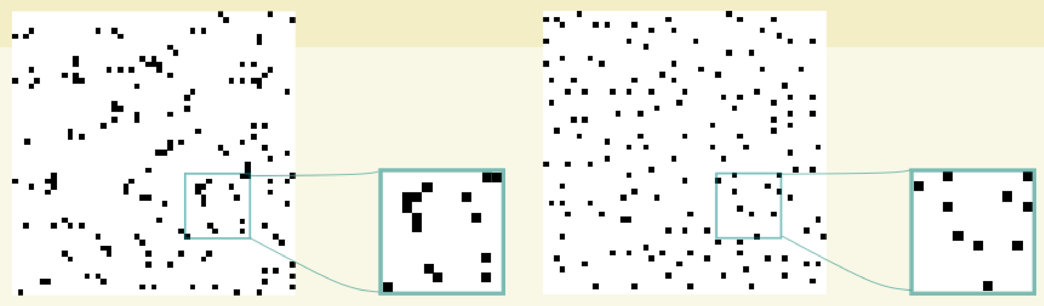


Figure : Realisations of a Poisson Point Process and a DPP

Definition [1]

Let $\mathcal{Y} = \{1, \ldots, N\}$ and K be a hermitian $N \times N$ matrix, such that $0 \preceq K \preceq 1$, then the random subset $Y \subset \mathcal{Y}$, defined by

$$\forall A \subset \mathcal{Y}, \quad \mathbb{P}(A \subset Y) = \det(K_A), \quad \text{is a DPP.}$$

- Marginal probabilities of singletons: $\mathbb{P}(i \in Y) = K_{ii}$.
- Negative correlation between elements :

$$\mathbb{P}(\{i,j\} \subset Y) = \mathbb{P}(i \in Y)\mathbb{P}(j \in Y) - |K_{i,j}|^2.$$

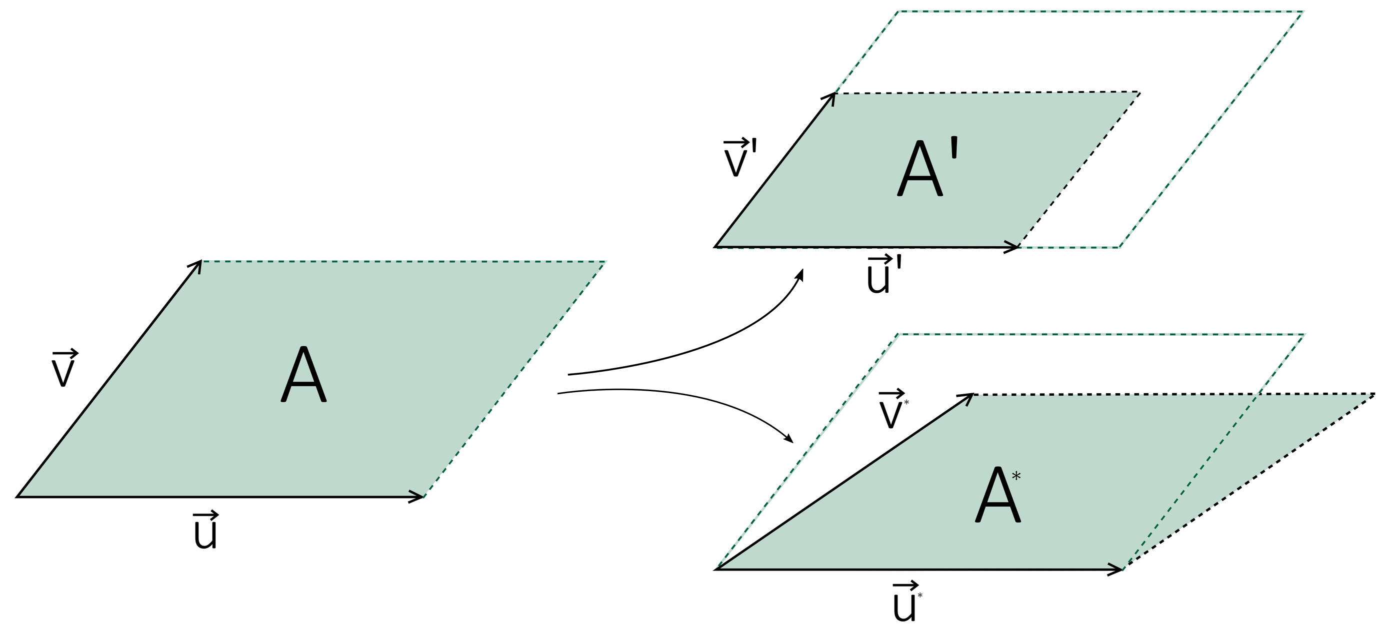


Figure : Analogy with parallelograms : | det (u,v) | = A

The space of image patches and choosing an adapted kernel

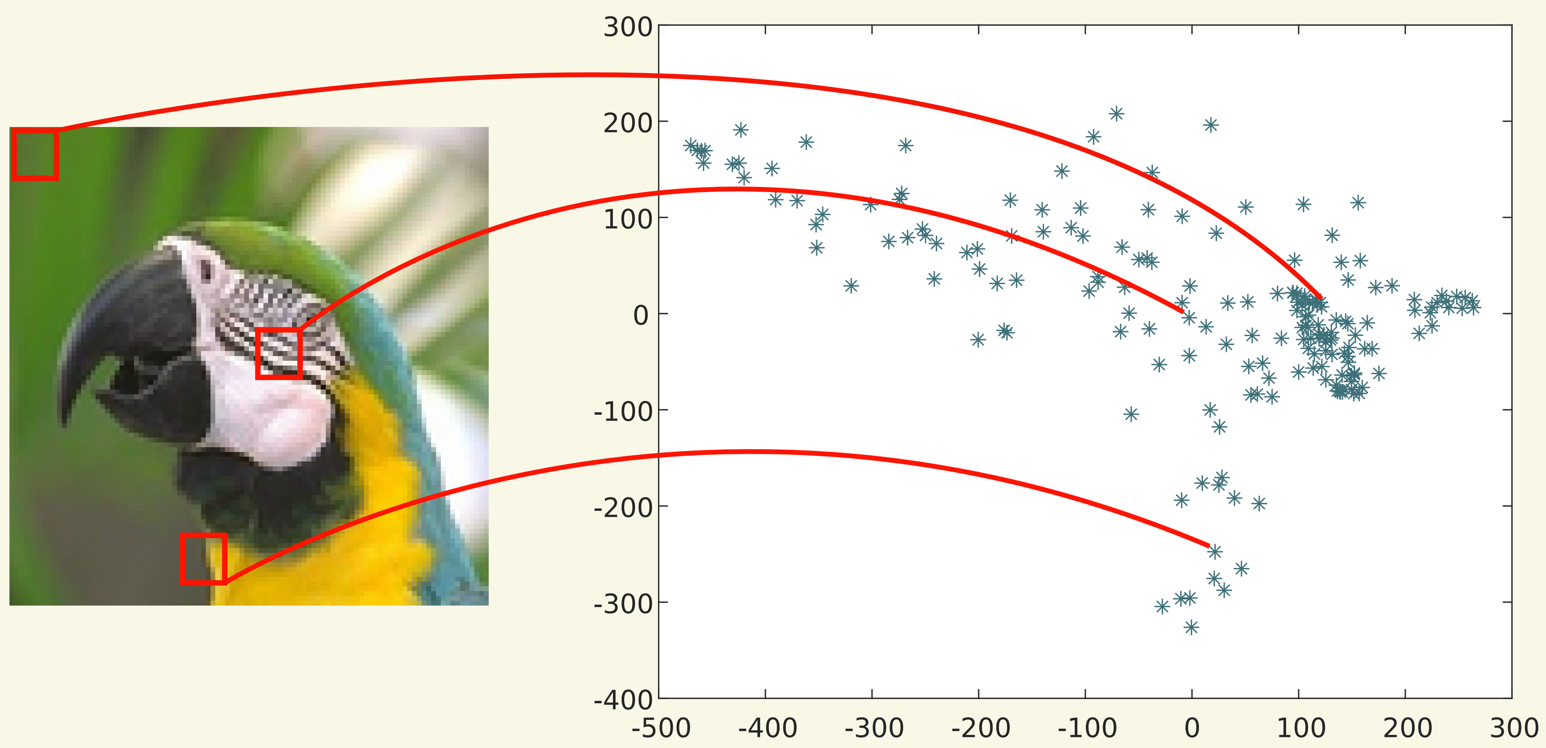


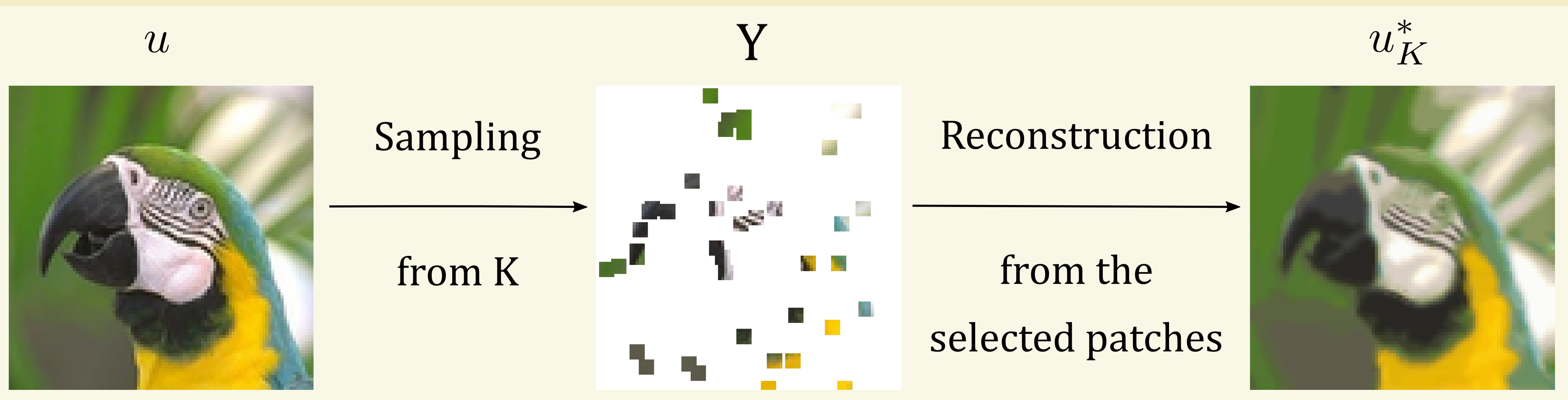
Figure : Representation of the patches from the coordinates on the first two components of the PCA

Goal : Sampling a representative subset Y of $\mathcal{Y}$ the set of all the patches → Define the appropriate kernel K

$K = L(L+I)^{-1}$ with L defined as

- Gaussian kernels [2] from
  - Distance of Intensity + Position,
    $L_{ij} = \exp\left(-\|I_i - I_j\|^2 - \lambda \|Pos_i - Pos_j\|^2\right)$
  - Distance in a PCA reduced space,
    $L_{ij} = \exp\left(-\|PCA_i - PCA_j\|^2\right)$
- Quality/diversity kernel, where $q_i \in \mathbb{R}, \phi_i \in \mathbb{R}^D$,
  $L_{ij} = q_i \phi_i^T \phi_j q_j$

Results from basic image reconstruction



Each patch is replaced by its nearest neighbor in Y

Comparison between kernels and the Poisson process:

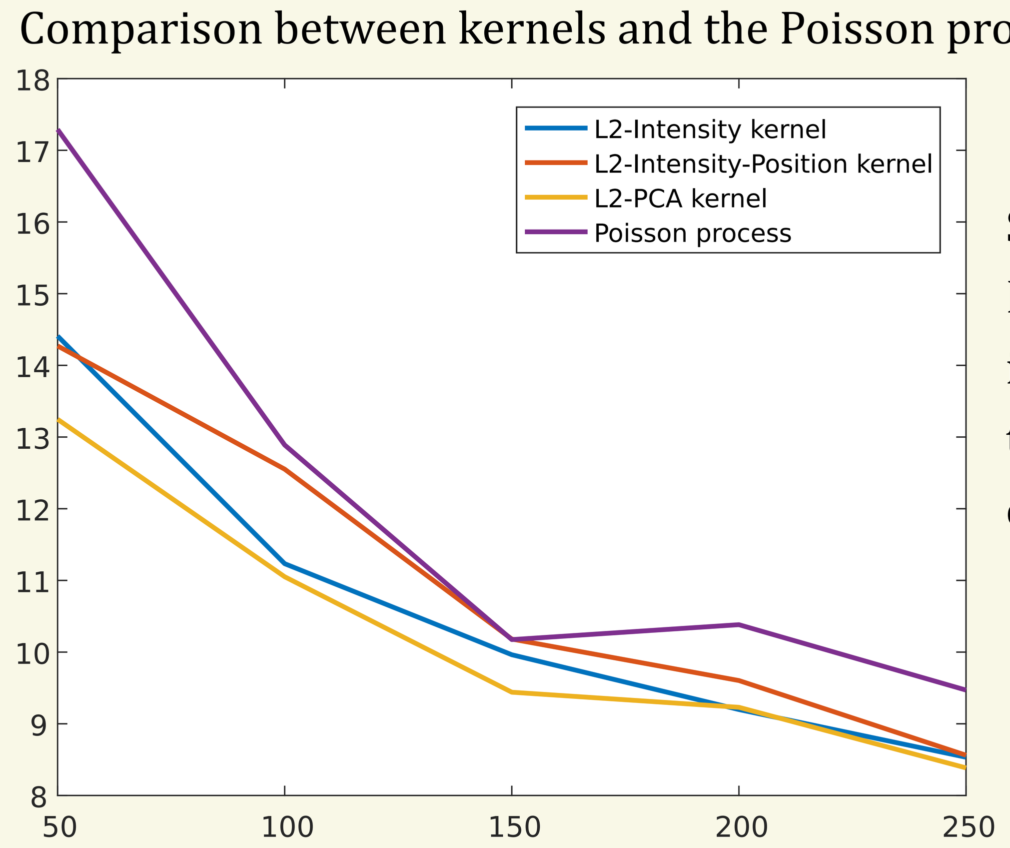


Figure : Square root of the Mean Squared Error (MSE) between the original image of size N and the reconstructed one in function of the number of patches selected

$$\text{MSE} = \frac{1}{N^2}\|u - u^*_K\|_2^2$$

Crops of the best and worst results:

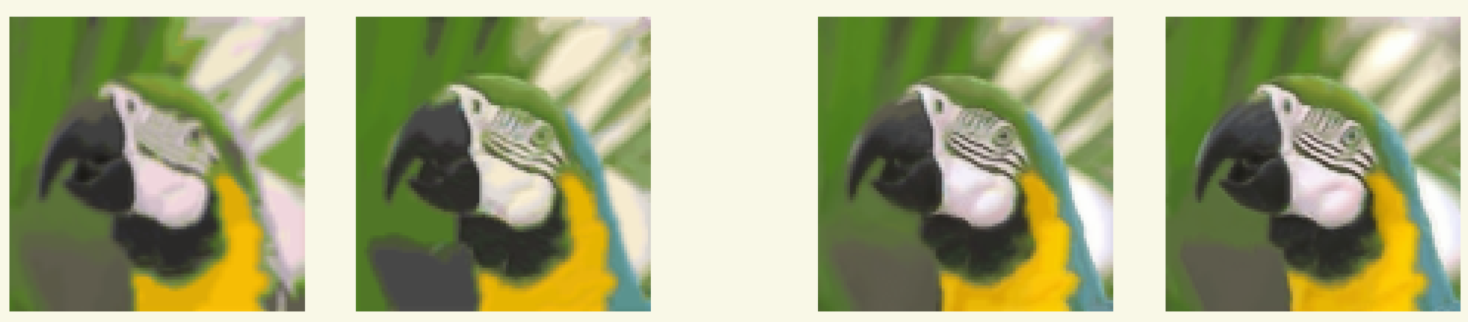


Figure : Reconstructions from a Poisson point process and the DPP L2-PCA kernel,from samples of 50 (two on the left) and 250 (on the right) patches

Applications and further questions

- Possible applications :
  - Image reconstruction or compression
  - Initialization of a k-means algorithm on patches
    ex : A. Coates and A. Y. Ng, Learning Feature Representations with K-means, Springer LNCS 7700, 2012
  - Some studies suppose that patches are distributed as a Gaussian Mixture Model : Need to estimate the parameters.
    Problem : Too many redundant patches
    With DPP : Enlightened subsampling of the set of patches
    ex : A. Houdard, C. Bouveyron, J. Delon, High-Dimensional Models for Unsupervised Image Denoising, preprint, 2017
- Need to study new patch similarity measures to improve the selection :
  ex : C.-A. Deledalle, L. Denis, F. Tupin, How to Compare Noisy Patches ? Patch Similarity Beyond Gaussian Noise, International journal of computer vision 99.1, 2012

Bibliographie

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