$$\int_0^1 \frac{3x^3 - x^2 + 2x - 4}{\sqrt{x^2 - 3x + 2}}\,dx$$

$$\begin{array}{l|l}
3x^3 - x^2 + 2x - 4 & x-1 \\ \hline
-3x^3 + 3x^2 & 3x^2 + 2x + 4 \\ \cline{1-1}
0 + 2x^2 + 2x - 4 & \\
-2x^2 + 2x & \\ \cline{1-1}
0 \quad 4x - 4 & \\
\quad -4x + 4 &
\end{array}$$

$$3x^3 - x^2 + 2x - 4 = (x-1)(3x^2 + 2x + 4) = 3x^3 + 2x^2 + 4x - 3x^2 - 2x - \ldots$$

$$= 3x^2 - x^2 + 2x - 4$$

$$x^2 - 3x + 2 = (x-1)(x-2)$$

$$\sqrt{x^2 - 3x + 2} = x + t$$

$$\cancel{x^2} - 3x + 2 = (x+t)^2 = \cancel{x^2} + 2xt + t^2$$

$$2 - t^2 = 2xt + 3x = x(2t+3)$$

$$\longrightarrow x = \frac{2 - t^2}{2t+3}$$

$$dx = \frac{-2t(2t+3) - 2(2-t^2)}{(2t+3)^2}\,dt$$

$$= -\frac{4t^2 + 6t + 4 - 2t^2}{(2t+3)^2}\,dt$$

$$= -\frac{2t^2 + 6t + 4}{(2t+3)^2}\,dt = -2\,\frac{t^2 + 3t + 2}{(2t+3)^2}\,dt$$

$$= -2\,\frac{(t+1)(t+2)}{(2t+3)^2}\,dt$$

$$x-1 \longrightarrow \frac{2-t^2-2t-3}{2t+3} = -\frac{t^2+2t+1}{2t+3} = -\frac{(t+}{2t+}$$

$$x-2 \longrightarrow \frac{2-t^2-4t-6}{2t+3} = -\frac{t^2+4t+4}{2t+3} = -\frac{(t+}{2t+}$$

$$3x^2+2x+4 \longrightarrow 3\frac{(2-t^2)^2}{(2t+3)^2} + 2\frac{2-t^2}{2t+3} + 4$$

$$= \frac{3(4+t^4-4t^2)+2(2-t^2)(2t+3)+4(\cdot}{(2t+3)^2}$$

$$= \frac{12+3t^4-12t^2+2(4t+6-2t^3-3t^2)+}{(2t+3)^2}$$

$$\ldots +4(4t^2+9+12t)$$

$$= \frac{12+3t^4-12t^2+8t+12-4t^3-6t^2+16t^2+36+48t}{(2t+3)^2}$$

$$= \frac{3t^4-4t^3-2t^2+56t+60}{(2t+3)^2}$$

$$1 \longrightarrow 2-t^2=2t+3 \rightarrow t^2+2t+1=0$$

$$t_{1,2} = \frac{-2\pm\sqrt{4-4}}{2} = -1$$

$$0 \rightarrow t^2=2 \rightarrow t=\sqrt{2}$$

$$\int_{\sqrt{2}}^{-1} \frac{(t+1)^2}{\cancel{2t+3}} \frac{3t^4-4t^3-2t^2+56t+60}{(2t+3)^2} \frac{\cancel{2t+3}}{\cancel{(t+1)}\cancel{(t+2)}}$$

$$2\,\frac{\cancel{(t+1)}\cancel{(t+2)}}{(2t+3)^2}\,dt$$

$$= \int_{\sqrt{2}}^{-1} 2\,\frac{(t+1)^2\left(3t^4-4t^3-2t^2+56t+60\right)}{(2t+3)^4}\,dt$$

$$2t+3 = u \longrightarrow du = 2dt$$

$$t = \frac{u-3}{2} \qquad t+1 = \frac{u-3+2}{2} = \frac{u-1}{2}$$

$$\cancel{3t^4} \qquad (t+1)^2 = \cancel{\frac{u^2+25-10u}{4}} \qquad \frac{u^2+1-2u}{4}$$

$$t^2 = \frac{u^2+9-6u}{4}$$

$$t^3 = \frac{(u-3)(u^2+9-6u)}{8} = \frac{u^3+9u-6u^2-3u^2-27+18u}{8}$$

$$= \frac{u^3-9u^2+27u-27}{8}$$

$$t^4 = \frac{(u-3)(u^3-9u^2+27u-27)}{16} = \frac{u^4-9u^3+27u^2-27u}{16}$$

$$-3u^3+27u^2-81u+81$$

$$= \frac{u^4-12u^3+54u^2-108u+81}{16}$$

$\frac{3}{16}u^4 - \frac{36}{16}u^3 + \frac{162}{16}u^2 - \frac{324}{16}u + \frac{243}{16}$

$-\frac{u^3}{2} + \frac{9u^2}{2} - \frac{27}{2}u + \frac{27}{2}$

$-\frac{u^2}{2} - \frac{9}{2} + \frac{6u}{2}$

$+\frac{56}{2}u - \frac{168}{2}u + 60$

$= \frac{3}{16}u^4 -$

$\boxed{\begin{array}{l} \sqrt{2} \to 2\sqrt{2}+3 \\ -1 \to 1 \end{array}}$

$3t^4 - 4t^3 - 2t^2 + 56t + 6$

$\frac{3u^4 - 36u^3 + 162u^2 - 324u + 243}{16}$

$-\frac{8u^3 + 72u^2 - 216u + 216}{16}$

$-\frac{8u^2 - 72 + 48u}{16}$

$+\frac{448u - 1344}{16}$

$+60$

$= \frac{3u^4 - 44u^3 + 226u^2 - 44u - 857}{16} + 60$

$= \frac{3}{16}u^4 - \overset{11}{\frac{44}{16_{4}}}u^3 + \overset{113}{\frac{226}{16_{8}}}u^2 - \overset{11}{\frac{44}{16_{4}}}u - \boxed{\frac{857}{16} + 60}$

$+\frac{3}{16}$

$$= \frac{3}{16}u^4 - \frac{11}{4}u^3 + \frac{113}{8}u^2 - \frac{11}{4}u + \frac{3}{16}$$

$$(t+1)^2(3t^4 - 4t^3 - 2t^2 + 56t + 60)$$

$$\to \frac{(u^2+1-2u)}{4}\left(\frac{3}{16}u^4 - \frac{11}{4}u^3 + \frac{113}{8}u^2 - \frac{11}{4}u + \frac{3}{16}\right)$$

$$= \frac{1}{4}\Big[\frac{3}{16}u^6 - \frac{11}{4}u^5 + \frac{113}{8}u^4 - \frac{11}{4}u^3 + \frac{3}{16}u^2$$

$$+ \frac{3}{16}u^4 - \frac{11}{4}u^3 + \frac{113}{8}u^2 - \frac{11}{4}u + \frac{3}{16}$$

$$- \frac{6}{16}u^5 + \frac{22}{4}u^4 - \frac{226}{8}u^3 + \frac{22}{4}u^2 - \frac{6}{16}u\Big]$$

$$= \frac{1}{4}\Big[\frac{3}{16}u^6 - \frac{6+44}{16}u^5 + \frac{226+3+88}{16}u^4$$

$$- \frac{22+22+226}{8}u^3 + \frac{3+226+88}{16}u^2$$

$$- \frac{44+6}{16}u + \frac{3}{16}\Big]$$

$$= \frac{1}{4}\Big[\frac{3}{16}u^6 - \frac{25}{8}u^5 + \frac{317}{16}u^4 - \frac{135}{4}u^3 + \frac{317}{16}u^2$$

$$- \frac{25}{8}u + \frac{3}{16}\Big]$$

$$= \frac{3}{64}u^6 - \frac{25}{32}u^5 + \frac{317}{64}u^4 - \frac{135}{16}u^3 + \frac{317}{64}u^2 - \frac{25}{32}u + \frac{3}{64}$$

$$\Rightarrow \int_{2\sqrt{2}+3}^{1} \cancel{2}\frac{du}{\cancel{2}}\left[\frac{3}{64}u^2 - \frac{25}{32}u + \frac{317}{64} - \frac{135}{16}u^{-1} + \frac{317}{64}u^{-2} - \frac{25}{32}u^{-3} + \frac{3}{64}u^{-4}\right]$$

$$\cancel{= \int_{2\sqrt{2}+3}^{5} du \left(\frac{3}{128}u^2\right.}$$

$$= \left[\frac{u^3}{64} - \frac{25u^2}{64} + \frac{317}{64}u - \frac{135}{16}\log(u) - \frac{317}{64}u^{-1} + \frac{25}{64}u^{-2} - \frac{u^{-3}}{64}\right]_{2\sqrt{2}+3}^{1}$$

5.828

$$\cancel{= \frac{1}{64} - \frac{25}{64} + \frac{317}{64} - \frac{317}{64} + \frac{25}{64} - \frac{1}{64}}$$

$$- \frac{198}{64} + \frac{849.1396}{64} - \frac{1847.476}{64}$$

$$\boxed{+ \frac{103.345}{16}} + \frac{54.3928}{64}$$

$$- \frac{0.736}{64} + \frac{0.005}{64} = \cancel{428.2948}$$

$$14.8776 \quad -17.854 \quad = -2.98143906\ldots$$