Codeforces Round 967 (Div. 2)

A. Make All Equal

1 second, 256 megabytes

You are given a cyclic array $a_1, a_2, \ldots, a_n$.

You can perform the following operation on a at most $n - 1$ times:

- Let m be the current size of a, you can choose any two adjacent elements where the previous one is no greater than the latter one (In particular, a_m and a_1 are adjacent and a_m is the previous one), and delete exactly one of them. In other words, choose an integer i ($1 \le i \le m$) where $a_i \le a_{(i \bmod m)+1}$ holds, and delete exactly one of a_i or $a_{(i \bmod m)+1}$ from a.

Your goal is to find the minimum number of operations needed to make all elements in a equal.

Input

Each test contains multiple test cases. The first line contains the number of test cases t ($1 \le t \le 500$). The description of the test cases follows.

The first line of each test case contains a single integer n ($1 \le n \le 100$) — the length of the array a.

The second line of each test case contains n integers $a_1, a_2, \ldots, a_n$ ($1 \le a_i \le n$) — the elements of array a.

Output

For each test case, output a single line containing an integer: the minimum number of operations needed to make all elements in a equal.

input
7 1 1 3 1 2 3 3 1 2 2 5 5 4 3 2 1 6 1 1 2 2 3 3 8 8 7 6 3 8 7 6 3 6 1 1 4 5 1 4

output
0 2 1 4 4 6 3

In the first test case, there is only one element in a, so we can't do any operation.

In the second test case, we can perform the following operations to make all elements in a equal:

- choose $i = 2$, delete a_3, then a would become $[1, 2]$.
- choose $i = 1$, delete a_1, then a would become $[2]$.

It can be proven that we can't make all elements in a equal using fewer than 2 operations, so the answer is 2.

B. Generate Permutation

1.5 seconds, 256 megabytes

There is an integer sequence a of length n, where each element is initially -1.

Misuki has two typewriters where the first one writes letters from left to right, with a pointer initially pointing to 1, and another writes letters from right to left with a pointer initially pointing to n.

Misuki would choose one of the typewriters and use it to perform the following operations until a becomes a permutation of $[1, 2, \ldots, n]$

- write number: write the minimum **positive** integer that isn't present in the array a to the element a_i, i is the position where the pointer points at. Such operation can be performed only when $a_i = -1$.
- carriage return: return the pointer to its initial position (i.e. 1 for the first typewriter, n for the second)
- move pointer: move the pointer to the next position, let i be the position the pointer points at before this operation, if Misuki is using the first typewriter, $i := i + 1$ would happen, and $i := i - 1$ otherwise. Such operation can be performed only if after the operation, $1 \le i \le n$ holds.

Your task is to construct any permutation p of length n, such that the minimum number of carriage return operations needed to make $a = p$ is the same no matter which typewriter Misuki is using.

Input

Each test contains multiple test cases. The first line of input contains a single integer t ($1 \le t \le 500$) — the number of test cases. The description of the test cases follows.

The first line of each test case contains a single integer n ($1 \le n \le 2 \cdot 10^5$) — the length of the permutation.

It is guaranteed that the sum of n over all test cases does not exceed $2 \cdot 10^5$.

Output

For each test case, output a line of n integers, representing the permutation p of length n such that the minimum number of carriage return operations needed to make $a = p$ is the same no matter which typewriter Misuki is using, or -1 if it is impossible to do so.

If there are multiple valid permutations, you can output any of them.

input
3 1 2 3

output
1 -1 3 1 2

In the first testcase, it's possible to make $a = p = [1]$ using 0 carriage return operations.

In the second testcase, it is possible to make $a = p = [1, 2]$ with the minimal number of carriage returns as follows:

If Misuki is using the first typewriter:

- Write number: write 1 to a_1, a becomes $[1, -1]$
- Move pointer: move the pointer to the next position. (i.e. 2)
- Write number: write 2 to a_2, a becomes $[1, 2]$

If Misuki is using the second typewriter:

- Move pointer: move the pointer to the next position. (i.e. 1)
- Write number: write 1 to a_1, a becomes $[1, -1]$
- Carriage return: return the pointer to 2.
- Write number: write 2 to a_2, a becomes $[1, 2]$

It can be proven that the minimum number of carriage returns needed to transform a into p when using the first typewriter is 0 and it is 1 when using the second one, so this permutation is not valid.

Similarly, $p = [2, 1]$ is also not valid, so there is no solution for $n = 2$.

In the third testcase, it is possibile to make $a = p = [3, 1, 2]$ with 1 carriage return with both the first and the second typewriter. It can be proven that, for both typewriters, it is impossible to write p with 0 carriage returns.

With the first typewriter it is possible to:

- Move pointer: move the pointer to the next position. (i.e. 2)
- Write number: write 1 to a_2, a becomes $[-1, 1, -1]$
- Move pointer: move the pointer to the next position. (i.e. 3)
- Write number: write 2 to a_3, a becomes $[-1, 1, 2]$
- Carriage return: return the pointer to 1.
- Write number: write 3 to a_1, a becomes $[3, 1, 2]$

With the second typewriter it is possible to:

- Move pointer: move the pointer to the next position. (i.e. 2)
- Write number: write 1 to a_2, a becomes $[-1, 1, -1]$
- Carriage return: return the pointer to 3.
- Write number: write 2 to a_3, a becomes $[-1, 1, 2]$
- Move pointer: move the pointer to the next position. (i.e. 2)
- Move pointer: move the pointer to the next position. (i.e. 1)
- Write number: write 3 to a_1, a becomes $[3, 1, 2]$

C. Guess The Tree

2 seconds, 256 megabytes

This is an interactive problem.

Misuki has chosen a secret tree with n nodes, indexed from 1 to n, and asked you to guess it by using queries of the following type:

- "`? a b`" — Misuki will tell you which node x minimizes $|d(a, x) - d(b, x)|$, where $d(x, y)$ is the distance between nodes x and y. If more than one such node exists, Misuki will tell you the one which minimizes $d(a, x)$.

Find out the structure of Misuki's secret tree using at most $15n$ queries!

Input

Each test consists of multiple test cases. The first line contains a single integer t ($1 \le t \le 200$) — the number of test cases.

Each test case consists of a single line with an integer n ($2 \le n \le 1000$), the number of nodes in the tree.

It is guaranteed that the sum of n across all test cases does not exceed 1000.

Interaction

The interaction begins by reading the integer n.

Then you can make up to $15n$ queries.

To make a query, output a line in the format "`? a b`" (without quotes) ($1 \le a, b \le n$). After each query, read an integer — the answer to your query.

To report the answer, output a line in the format "`!` a_1 b_1 a_2 b_2 ... a_{n-1} b_{n-1}" (without quotes), meaning that there is an edge between nodes a_i and b_i, for each $1 \le i \le n - 1$. You can print the edges in any order.

After $15n$ queries have been made, the response to any other query will be -1. Once you receive such a response, terminate the program to receive the `Wrong Answer` verdict.

After printing each line, do not forget to output the end of line and flush the output buffer. Otherwise, you will receive the `Idleness limit exceeded` verdict. To flush, use:

- `fflush(stdout)` or `cout.flush()` in C++;
- `System.out.flush()` in Java;
- `flush(output)` in Pascal;
- `stdout.flush()` in Python;
- see the documentation for other languages.

Hacks

For hacks, use the following format: The first line contains an integer t ($1 \le t \le 200$) — the number of test cases.

The first line of each test contains an integer n — the number of nodes in the hidden tree.

Then $n - 1$ lines follow. The i-th of them contains two integers a_i and b_i ($1 \le a_i, b_i \le n$), meaning that there is an edge between a_i and b_i in the hidden tree.

The sum of n over all test cases must not exceed 1000.

input
1 4 1 1 3
output
? 1 2 ? 1 3 ? 1 4 ! 1 2 1 3 3 4

A tree is an undirected acyclic connected graph. A tree with n nodes will always have $n - 1$ edges.

In the example case, the answer to "`? 1 2`" is 1. This means that there is an edge between nodes 1 and 2.

The answer to "`? 1 3`" is 1. This means that there is an edge between nodes 1 and 3.

The answer to "`? 1 4`" is 3. It can be proven that this can only happen if node 3 is connected to both node 1 and 4.

The edges of the tree are hence $(1,2)$, $(1,3)$ and $(3,4)$.

D. Longest Max Min Subsequence

2 seconds, 256 megabytes

You are given an integer sequence $a_1, a_2, \ldots, a_n$. Let S be the set of all possible non-empty subsequences of a without duplicate elements. Your goal is to find the longest sequence in S. If there are multiple of them, find the one that minimizes lexicographical order after multiplying terms at odd positions by -1.

For example, given $a = [3,2,3,1]$, $S = \{[1],[2],[3],[2,1],[2,3],[3,1],[3,2],[2,3,1],[3,2,1]\}$. Then $[2,3,1]$ and $[3,2,1]$ would be the longest, and $[3,2,1]$ would be the answer since $[-3,2,-1]$ is lexicographically smaller than $[-2,3,-1]$.

A sequence c is a subsequence of a sequence d if c can be obtained from d by the deletion of several (possibly, zero or all) elements.

A sequence c is lexicographically smaller than a sequence d if and only if one of the following holds:

- c is a prefix of d, but $c \neq d$;
- in the first position where c and d differ, the sequence c has a smaller element than the corresponding element in d.

Input

Each test contains multiple test cases. The first line contains the number of test cases t ($1 \le t \le 5 \cdot 10^4$). The description of the test cases follows.

The first line of each test case contains an integer n ($1 \le n \le 3 \cdot 10^5$) — the length of a.

The second line of each test case contains n integers $a_1, a_2, \ldots, a_n$ ($1 \le a_i \le n$) — the sequence a.

It is guaranteed that the sum of n over all test cases does not exceed $3 \cdot 10^5$.

Output

For each test case, output the answer in the following format:

Output an integer m in the first line — the length of b.

Then output m integers $b_1, b_2, \ldots, b_m$ in the second line — the sequence b.

input

```
4
4
3 2 1 3
4
1 1 1 1
9
3 2 1 3 2 1 3 2 1
1
1
```

output

```
3
3 2 1
1
1
3
3 1 2
1
1
```

input

```
10
2
1 2
10
5 2 1 7 9 7 2 5 5 2
2
1 2
10
2 2 8 7 7 9 8 1 9 6
9
9 1 7 5 8 5 6 4 1
3
3 3 3
6
1 6 4 4 6 5
6
3 4 4 5 3 3
10
4 1 4 5 4 5 10 1 5 1
7
1 2 1 3 2 4 6
```

output

```
2
1 2
5
5 1 9 7 2
2
1 2
6
2 7 9 8 1 6
7
9 1 7 5 8 6 4
1
3
4
1 4 6 5
3
4 5 3
4
5 4 10 1
5
2 1 3 4 6
```

In the first example, $S = \{[1],[2],[3],[1,3],[2,1],[2,3],[3,1],[3,2],[2,1,3],[3,2,1]\}$. Among them, $[2,1,3]$ and $[3,2,1]$ are the longest and $[-3,2,-1]$ is lexicographical smaller than $[-2,1,-3]$, so $[3,2,1]$ is the answer.

In the second example, $S = \{[1]\}$, so $[1]$ is the answer.

E1. Deterministic Heap (Easy Version)

3 seconds, 512 megabytes

This is the easy version of the problem. The difference between the two versions is the definition of deterministic max-heap, time limit, and constraints on n and t. You can make hacks only if both versions of the problem are solved.

Consider a perfect binary tree with size $2^n - 1$, with nodes numbered from 1 to $2^n - 1$ and rooted at 1. For each vertex v ($1 \le v \le 2^{n-1} - 1$), vertex $2v$ is its left child and vertex $2v + 1$ is its right child. Each node v also has a value a_v assigned to it.

Define the operation pop as follows:

1. initialize variable v as 1;
2. repeat the following process until vertex v is a leaf (i.e. until $2^{n-1} \le v \le 2^n - 1$);
    a. among the children of v, choose the one with the larger value on it and denote such vertex as x; if the values on them are equal

(i.e. $a_{2v} = a_{2v+1}$), you can choose any of them;
b. assign a_x to a_v (i.e. $a_v := a_x$);
c. assign x to v (i.e. $v := x$);
3. assign -1 to a_v (i.e. $a_v := -1$).

Then we say the pop operation is deterministic if there is a unique way to do such operation. In other words, $a_{2v} \neq a_{2v+1}$ would hold whenever choosing between them.

A binary tree is called a max-heap if for every vertex v ($1 \le v \le 2^{n-1} - 1$), both $a_v \ge a_{2v}$ and $a_v \ge a_{2v+1}$ hold.

A max-heap is deterministic if the pop operation is deterministic to the heap when we do it **for the first time**.

Initially, $a_v := 0$ for every vertex v ($1 \le v \le 2^n - 1$), and your goal is to count the number of different deterministic max-heaps produced by applying the following operation add exactly k times:

- Choose an integer v ($1 \le v \le 2^n - 1$) and, for every vertex x on the path between 1 and v, add 1 to a_x.

Two heaps are considered different if there is a node which has different values in the heaps.

Since the answer might be large, print it modulo p.

Input

Each test contains multiple test cases. The first line contains the number of test cases t ($1 \le t \le 500$). The description of the test cases follows.

The first line of each test case contains three integers n, k, p ($1 \le n, k \le 500$, $10^8 \le p \le 10^9$, p is a prime).

It is guaranteed that the sum of n and the sum of k over all test cases does not exceed 500.

Output

For each test case, output a single line containing an integer: the number of different deterministic max-heaps produced by applying the aforementioned operation add exactly k times, modulo p.

input

```
7
1 13 998244353
2 1 998244353
3 2 998244853
3 3 998244353
3 4 100000037
4 2 100000039
4 3 100000037
```

output

```
1
2
12
52
124
32
304
```

input

```
1
500 500 100000007
```

output

```
76297230
```

input

```
6
87 63 100000037
77 77 100000039
100 200 998244353
200 100 998244353
32 59 998244853
1 1 998244353
```

output

```
26831232
94573603
37147649
847564946
727060898
1
```

For the first testcase, there is only one way to generate a, and such sequence is a deterministic max-heap, so the answer is 1.

For the second testcase, if we choose $v = 1$ and do the operation, we would have $a = [1, 0, 0]$, and since $a_2 = a_3$, we can choose either of them when doing the first pop operation, so such heap is not a deterministic max-heap.

And if we choose $v = 2$, we would have $a = [1, 1, 0]$, during the first pop, the following would happen:

- initialize v as 1
- since $a_{2v} > a_{2v+1}$, choose $2v$ as x, then $x = 2$
- assign a_x to a_v, then $a = [1, 1, 0]$
- assign x to v, then $v = 2$
- since v is a leaf, assign -1 to a_v, then $a = [1, -1, 0]$

Since the first pop operation is deterministic, this is a deterministic max-heap. Also, if we choose $v = 3$, a would be a deterministic max-heap, so the answer is 2.

E2. Deterministic Heap (Hard Version)

4 seconds, 512 megabytes

This is the hard version of the problem. The difference between the two versions is the definition of deterministic max-heap, time limit, and constraints on n and t. You can make hacks only if both versions of the problem are solved.

Consider a perfect binary tree with size $2^n - 1$, with nodes numbered from 1 to $2^n - 1$ and rooted at 1. For each vertex v ($1 \le v \le 2^{n-1} - 1$), vertex $2v$ is its left child and vertex $2v + 1$ is its right child. Each node v also has a value a_v assigned to it.

Define the operation pop as follows:

1. initialize variable v as 1;
2. repeat the following process until vertex v is a leaf (i.e. until $2^{n-1} \le v \le 2^n - 1$);
   a. among the children of v, choose the one with the larger value on it and denote such vertex as x; if the values on them are equal (i.e. $a_{2v} = a_{2v+1}$), you can choose any of them;
   b. assign a_x to a_v (i.e. $a_v := a_x$);
   c. assign x to v (i.e. $v := x$);
3. assign -1 to a_v (i.e. $a_v := -1$).

Then we say the pop operation is deterministic if there is a unique way to do such operation. In other words, $a_{2v} \neq a_{2v+1}$ would hold whenever choosing between them.

A binary tree is called a max-heap if for every vertex v ($1 \le v \le 2^{n-1} - 1$), both $a_v \ge a_{2v}$ and $a_v \ge a_{2v+1}$ hold.

A max-heap is deterministic if the pop operation is deterministic to the heap when we do it **for the first and the second time**.

Initially, $a_v := 0$ for every vertex v ($1 \le v \le 2^n - 1$), and your goal is to count the number of different deterministic max-heaps produced by applying the following operation add exactly k times:

- Choose an integer v ($1 \le v \le 2^n - 1$) and, for every vertex x on the path between 1 and v, add 1 to a_x.

Two heaps are considered different if there is a node which has different values in the heaps.

Since the answer might be large, print it modulo p.

Input

Each test contains multiple test cases. The first line contains the number of test cases t ($1 \le t \le 50$). The description of the test cases follows.

The first line of each test case contains three integers n, k, p ($2 \le n \le 100$, $1 \le k \le 500$, $10^8 \le p \le 10^9$, p is a prime).

It is guaranteed that the sum of n does not exceed 100 and the sum of k over all test cases does not exceed 500.

Output

For each test case, output a single line containing an integer: the number of different deterministic max-heaps produced by applying the aforementioned operation add exactly k times, modulo p.

input
6 2 1 998244353 3 2 998244853 3 3 998244353 3 4 100000037 4 2 100000039 4 3 100000037
output
2 12 40 100 32 224

input
1 100 500 100000037
output
66681128

input
2 87 63 100000037 13 437 100000039
output
83566569 54517140

For the first testcase, if we choose $v = 1$ and do the operation, we would have $a = [1, 0, 0]$, and since $a_2 = a_3$, we can choose either of them when doing the first pop operation, so such heap is not a deterministic max-heap.

And if we choose $v = 2$, we would have $a = [1, 1, 0]$, during the first pop, the following would happen:

- initialize v as 1
- since $a_{2v} > a_{2v+1}$, choose $2v$ as x, then $x = 2$
- assign a_x to a_v, then $a = [1, 1, 0]$
- assign x to v, then $v = 2$
- since v is a leaf, assign -1 to a_v, then $a = [1, -1, 0]$

And during the second pop, the following would happen:

- initialize v as 1
- since $a_{2v} < a_{2v+1}$, choose $2v + 1$ as x, then $x = 3$
- assign a_x to a_v, then $a = [0, -1, 0]$
- assign x to v, then $v = 3$
- since v is a leaf, assign -1 to a_v, then $a = [0, -1, -1]$

Since both the first and the second pop operation are deterministic, this is a deterministic max-heap. Also, if we choose $v = 3$, a would be a deterministic max-heap, so the answer is 2.