UNIVERSITY OF OKLAHOMA
DEPARTMENT OF MATHEMATICS

Homework #1

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Course: *Abstract Linear Algebra* – Professor: *Dr. Gregory Muller*
Due date: *Sep 5, 2021*

1C: 7

Give an example of a nonempty subset U of $\mathbb{R}^2$ such that U is closed under addition and under taking additive inverses (meaning $-u \in U$ whenever $u \in U$), but U is not a subspace of $\mathbb{R}^2$.

Answer. Will demonstrate by example:

1C: 8

Give an example of a nonempty subset U of $\mathbb{R}^2$ such that U is closed under scalar multiplication, but U is not a subspace of $\mathbb{R}^2$.

Answer. Will demonstrate by example:

1C: 9

A function $f : \mathbb{R} \to \mathbb{R}$ is called periodic if there exists a positive number p such that $f(x) = f(x + p)$ for all $x \in \mathbb{R}$. Is the set of periodic functions from $\mathbb{R}$ to $\mathbb{R}$ a subspace of $\mathbb{R}^{\mathbb{R}}$? Explain.

Answer. Will demonstrate by example:

2A: 1

Suppose v_1, v_2, v_3, v_4 spans V. Prove that the list

$$v_1 - v_2, v_2 - v_3, v_3 - v_4, v_4$$

also spans V.

Answer. Will demonstrate by example:

2A: 3

Find a number t such that

$$(3,1,4),(2,-3,5),(5,9,t)$$

is not linearly independent in $\mathbb{R}^3$.

Answer. Will demonstrate by example:

2A: 9

Prove or give a counterexample: If $v_1,\ldots,v_m$ and $w_1,\ldots,w_m$ are linearly independent lists of vectors in V, then $v_1+w_1+\cdots+v_m+w_m$ is linearly independent.

Answer. Will demonstrate by counter-example:

2B: 5

Prove or disprove: there exists a basis p_0,p_1,p_2,p_3 of $\mathcal{P}_3(\mathbb{F})$ such that none of the polynomials p_0,p_1,p_2,p_3 has degree 2.

Answer. Will demonstrate by counter-example:

2A: 6

Suppose v_1,v_2,v_3,v_4 is a basis of V. Prove that

$$v_1+v_2,v_2+v_3,v_3+v_4,v_4$$

is a basis of V.

Answer. Will demonstrate by counter-example: