1 Probability

Probability is a number that quantifies the likelihood of a given event when it is not yet known whether the event will happen or not. This definition is circular because it uses the concept of likelihood, which is a synonym for probability. Nonetheless, we can use it as a starting point. It highlights two important facts:

- probablility refers to an event;
- probablility is a number.

By elaborating on these two facts, we will give an (almost entirely) rigorous definition of probability.

1.1 Sample space, sample points and events

The first thing we do when we start thinking about the probability of an event is to list **a number of things that could possibly happen**. The things in this list form a set, which we denote by Ω.

Definition: Experiment, Sample Space

An **experiment** or **trial** (see below) is any procedure that can be infinitely repeated and has a well-defined set of possible outcomes, known as the **sample space** and usually denoted by Ω. For a set of outcomes Ω to be a sample space, Ω must satisfy the following points

- **Mutually exclusive outcomes.** Only one of the things in Ω can happen. That is, if $\omega \in \Omega$ happens, then none of the things in the set $\{\bar{\omega} \in \Omega : \bar{\omega} \neq \omega\}$ can happen.
- **Exhaustive outcomes.** At least one of the things in Ω will happen.

Random experiments are often conducted repeatedly, so that the collective results may be subjected to statistical analysis. A fixed number of repetitions of the same experiment can be thought of as a composed experiment, in which case the individual repetitions are called **trials**.

Definition: Sample point

An element $\omega \in \Omega$ is called a **sample point**, or possible outcome.

Definition: Realized outcome

When we learn that $\omega \in \Omega$ has happened, ω is called the **realized outcome**.

Definition: Event

A subset $E \subseteq \Omega$ is called an event. E is a *sigma algebra* on Ω, which is defined below.

1.2 Space of Events