

Homework #4

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Course: *Abstract Linear Algebra* – Professor: *Dr. Gregory Muller*
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1C: 24

A function $f : \mathbb{R} \rightarrow \mathbb{R}$ is called even if

$$f(-x) = f(x)$$

for all $x \in \mathbb{R}$. A function $f : \mathbb{R} \rightarrow \mathbb{R}$ is called odd if

$$f(-x) = -f(x)$$

for all $x \in \mathbb{R}$.

Let U_e denote the set of real-valued even functions on \mathbb{R} and let U_o denote the set of real-valued odd functions on \mathbb{R} . Show that

$$\mathbb{R}^{\mathbb{R}} = U_e \oplus U_o$$

Answer. If $f \in \mathbb{R}^{\mathbb{R}}$, then

$$\begin{aligned} f(x) &= \frac{f(x) + f(x)}{2} + 0 \\ &= \frac{f(x) + f(x)}{2} + \frac{f(-x) - f(-x)}{2} \\ &= \frac{f(x) + f(-x)}{2} + \frac{f(x) - f(-x)}{2} \end{aligned}$$

for all $x \in \mathbb{R}$. Notice,

$$\frac{f(x) + f(-x)}{2} \in U_e$$

and

$$\frac{f(x) - f(-x)}{2} \in U_o$$

which implies that f can be uniquely written as the sum of an even function and an odd function. Since any $f \in \mathbb{R}^{\mathbb{R}}$ can be expressed as a unique sum of an even function and odd function, $\mathbb{R}^{\mathbb{R}}$ is a direct sum of U_o and U_e . ■

2C: 11**Answer. ■****3A: 4****Answer. ■****3A: 7****Answer. ■****3A: 14****Answer. ■**