University of Oklahoma
Department of Mathematics

Homework #9

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Course: *Abstract Linear Algebra* – Professor: *Dr. Gregory Muller*
Due date: *Oct 24, 2021*

Problem 5

Suppose $T \in \mathcal{L}(V), m$ is a positive integer, and $v \in V$ is such that $T^{m-1}v \neq 0$ but $T^m v = 0$. Prove that

$$v, Tv, T^2v, \cdots, T^{m-1}v$$

is linearly independent.

Answer. Because $T^m v = 0, T^{m-1}v \in \ker T$ and $V = \ker T \oplus \mathrm{im}\ T$,

First to demonstrate that $\{v\}$ is linearly independent.

$$T^{m-1}v = 0 \implies v \neq 0 \implies \{v\} \text{ is linearly independent}$$

Now to demonstrate that if $T^{m-1}v \neq 0$ and $T^m v = 0$ then .

Problem 6

Suppose $T \in \mathcal{L}(C^3)$ is defined by $T(z_1, z_2, z_3) = (z_2, z_3, 0)$. Prove that T has no square root. More precisely, prove that there does not exist $S \in \mathcal{L}(C^3)$ such that $S^2 = T$.

Answer. .

Problem 7

Suppose $N \in \mathcal{L}(V)$ is nilpotent. Prove that 0 is the only eigenvalue of N.

Answer. .

Problem 9

Suppose $S, T \in \mathcal{L}(V)$ and ST is nilpotent. Prove that TS is nilpotent.

Answer. .

Problem 12

Suppose $N \in \mathcal{L}(V)$ and there exists a basis of V with respect to which N has an upper-triangular matrix with only 0's on the diagonal. Prove that N is nilpotent.

Answer. .