

Homework #1

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Course: *Special Relativity (Physics 301)* – Professor: *Dr. Albert Einstein*
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Question 1

Suppose that

$$\begin{aligned}A &= \{x : x \in \mathbb{N} \text{ and } x \text{ is even}\} \\B &= \{x : x \in \mathbb{N} \text{ and } x \text{ is prime}\} \\C &= \{x : x \in \mathbb{N} \text{ and } x \text{ is a multiple of } 5\}\end{aligned}$$

Answer. Describe each of the following sets:

- a) $A \cap B = \{2\}$ because 2 is the only even prime number
- b) $B \cap C = \emptyset$ because a multiple of 5 cannot be prime
- c) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, \dots\}$, the set of all naturals minus composite odd numbers
- d) $A \cap (B \cup C)$ includes 2 and the even multiples of 5, because 2 is the only even prime.

Question 2

Suppose that $A = \{a, b, c\}$, $B = \{1, 2, 3\}$, $C = \{x\}$, and $D = \emptyset$.

Answer. List all of the elements in each of the following sets

- a) $A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3), (c, 1), (c, 2), (c, 3)\}$
- b) $B \times A = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c), (3, a), (3, b), (3, c)\}$
- c) $A \times B \times C = \{(a, 1, x), (a, 2, x), (a, 3, x), (b, 1, x), (b, 2, x), (b, 3, x), (c, 1, x), (c, 2, x), (c, 3, x)\}$
- d) $A \times D = \emptyset$

Question 8

Prove $A \subset B$ if and only if $A \cap B = A$

Answer. Take any two sets B and $A \subset B$.

$$A \subset B \iff \forall a \in A, a \in B \iff A \cap B = A$$

as required. ■

Question 10Prove $A \cup B = (A \cap B) \cup (A \setminus B) \cup (B \setminus A)$ **Answer.** Take any two sets A, B .

$$\begin{aligned}
A \cup B &= \{x : x \in A\} \cup \{x : x \in B\} \\
&= ((A \setminus B) \cup (A \cap B)) \cup ((B \setminus A) \cup (B \cap A)) \\
&= (A \setminus B) \cup (A \cap B) \cup (B \setminus A) \cup (B \cap A) \\
&= (A \cap B) \cup (B \cap A) \cup (A \setminus B) \cup (B \setminus A) \\
&= (A \cap B) \cup (A \setminus B) \cup (B \setminus A)
\end{aligned}$$

as required. ■

Question 11Prove $(A \cup B) \times C = (A \times C) \cup (B \times C)$ **Answer.** Take any three sets A, B, C .

$$\begin{aligned}
(A \cup B) \times C &= \{(x, c) : x \in (A \cup B), c \in C\} \\
&= \{(x, c) : x \in A, c \in C\} \cup \{(x, c) : x \in B, c \in C\} \\
&= (A \times C) \cup (B \times C)
\end{aligned}$$

as required. ■

Question 14Prove $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$ **Answer.** Take any three sets A, B, C .

$$\begin{aligned}
A \setminus (B \cup C) &= A \cap (B \cup C)' \\
&= A \cap A \cap (B' \cap C') \\
&= A \cap B' \cap A \cap C' \\
&= (A \setminus B) \cap (A \setminus C)
\end{aligned}$$

as required. ■

Question 16

Prove $(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B)$

Answer. Take any two sets A, B .

$$\begin{aligned}
 (A \setminus B) \cup (B \setminus A) &= (A \cap B') \cup (B \cap A') \\
 &= ((A \cap B') \cup B) \cap ((A \cap B') \cup A') \\
 &= ((A \cup B) \cap (B \cup B')) \cap ((A \cup A') \cap (B' \cup A')) \\
 &= (A \cup B) \cap (B' \cup A') \\
 &= (A \cup B) \cap (A \cap B)' \\
 &= (A \cup B) \setminus (A \cap B)
 \end{aligned}$$

as required. ■

Question 18

Determine which of the following functions are one-to-one and which are onto. If the function is not onto, determine its range.

Answer. Functions:

a) $f : \mathbb{R} \rightarrow \mathbb{R} : x \mapsto e^x$

This function is one-to-one but not onto. Its range is $\mathbb{R}_{>0}$

b) $f : \mathbb{Z} \rightarrow \mathbb{Z} : n \mapsto n^2 + 3$

c) $f : \mathbb{R} \rightarrow \mathbb{R} : x \mapsto \sin(x)$

d) $f : \mathbb{Z} \rightarrow \mathbb{Z} : n \mapsto n^2$