Linear Algebra Done Right

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1 Vector Spaces

Linear algebra is the study of linear maps on finite-dimensional vector spaces. Vector spaces are defined in this chapter, and their basic properties are developed. Vector spaces are a generalization of the description of a plane using two coordinates, as published by Descartes in 1637.

1.1 $\mathbf{R^n}$, $\mathbf{C^n}$, and $\mathbf{F^n}$

Definition: Complex Number

A **complex number** is an ordered pair $(a, b) \in \mathbf{R}^2$, denoted $a + bi$.

- The set of all complex numbers is denoted by $\mathbf{C}$:

$$\mathbf{C} = \{a + bi : a, b \in \mathbf{R}\}$$

- **Addition and multiplication** on $\mathbf{C}$ are defined by

$$(a + bi) + (c + di) \equiv (a + c) + (b + d)i$$

$$(a + bi) \cdot (c + di) \equiv (ac - bd) + (ad + bc)i$$

Theorem: Complex Number

A **complex number** is an ordered pair $(a, b) \in \mathbf{R}^2$, denoted $a + bi$.