

MSBA7003 Decision Analytics – Assignment 3 (Due Sep 27 noon at 11:55 a.m.)

Q1.

Jimmy, an aircraft engineer, is designing the passenger cabin for a new aircraft model. This model will be used by an airline for a specific route. Jimmy needs to decide on the number of seats for both business class and economy class. There must be 4 seats per row in business class; the seat length is 1.2 length units; the seat weight is 2.2 weight units. In economy class, there must be 6 seats per row; the seat length is 1 unit; the seat weight is 1 weight unit. The total cabin length is 32 length units, and the seat weight limit is 200 weight units. Additionally, due to limited demand, business class cannot have more than 6 rows. Jimmy's optimization goal is to maximize the expected profit of the aircraft model for its entire lifecycle on this route. It is known that the profit margin of business class seats is twice that of economy class seats.

Which of the following statement(s) is(are) true?

- A) In the optimal design plan, there should be 4 rows for business class and 27 rows for economy class.
- B) In the optimal design plan, there should be 5 rows for business class and 26 rows for economy class.
- C) In the following problem formulation given by GPT4, constraints 1 – 3 are all wrong.



GPT4 (Tokens used 750)

To solve this problem using linear programming, we first need to define the decision variables, constraints, and the objective function.

Let:

x = number of rows of business class seats

y = number of rows of economy class seats

Constraints:

1. Business class seat length constraint: $1.2 * 4x \leq 32$
2. Economy class seat length constraint: $1 * 6y \leq 32$
3. Total length constraint: $4.8x + 6y \leq 32$
4. Weight constraint: $2.2 * 4x + 1 * 6y \leq 200$
5. Demand constraint: $x \leq 6$
6. Non-negativity constraints: $x \geq 0, y \geq 0$

Objective function:

Maximize profit: $P = 2 * (4x) + 1 * (6y)$, where the profit from a business class seat is twice that of an economy class seat.

Now, we can solve this linear programming problem using a solver or any appropriate method.

- D) The total weight constraint is not binding.

Q2.

Abotte Products produces three products, A, B, and C. The company can sell up to 300 pounds of each product at the following prices (per pound): A, \$10; B, \$12; C, \$20. Abotte purchases a raw material at \$5 per pound. Each pound of the raw material can be used to produce either 0.9 pound of A or 0.7 pound of B. For a cost of \$3 per pound of A processed, Abotte can convert one pound of A to 0.6 pounds of B and 0.3 pounds of C. For a cost of \$2 per pound of B processed, Abotte can convert one pound of B to 0.8 pound of C. To determine the best raw material purchase and production plan, Abotte's manager developed the following linear program.

Q_r = amount of raw material purchased

Q_a = amount of A produced from raw material

Q_b = amount of B produced from raw material

X_a = amount of A processed

X_b = amount of B processed

$$\begin{aligned} \text{Maximize Profit} = & 10*(Q_a - X_a) + 12*(Q_b - X_b + 0.6*X_a) + 20*(0.3*X_a + 0.8*X_b) \\ & - 5*Q_r - 3*X_a - 2*X_b \end{aligned}$$

$$\text{s.t.} \quad Q_a - X_a \leq 300 \quad (\text{Constraint 1})$$

$$Q_b - X_b + 0.6*X_a \leq 300 \quad (\text{Constraint 2})$$

$$0.3*X_a + 0.8*X_b \leq 300 \quad (\text{Constraint 3})$$

$$Q_a/0.9 + Q_b/0.7 \leq Q_r \quad (\text{Constraint 4})$$

$$X_a \leq Q_a \quad (\text{Constraint 5})$$

$$X_b \leq Q_b \quad (\text{Constraint 6})$$

$$Q_r, Q_a, Q_b, X_a, X_b \geq 0$$

After solving the problem with Excel, the manager got the following sensitivity report:

Variable Cells

Cell	Name	Final Value	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$2	Quantity Q_r	1297.619048	-5	5	2.985915493
\$C\$2	Quantity Q_a	300	10	0.8412698	4.444444444
\$D\$2	Quantity Q_b	675	12	1E+30	0.862840863
\$E\$2	Quantity X_a	0	0.2	0.8412698	1E+30
\$F\$2	Quantity X_b	375	2	1E+30	2.243386243

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$H\$4	Constraint 1	300	4.444444444	300	1E+30	300
\$H\$5	Constraint 2	300	4.857142857	300	1E+30	300
\$H\$6	Constraint 3	300	8.571428571	300	1E+30	300
\$H\$7	Constraint 4	0	5	0	1297.619048	1E+30
\$H\$8	Constraint 5	-300	0	0	1E+30	300
\$H\$9	Constraint 6	-300	0	0	1E+30	300

(Note: 1E+30 means infinity.)

Which of the following statement(s) is(are) true?

- A) If the demand of A is increased by 100, the total profit can be increased by \$444.44.
- B) If the cost of raw material is increased by \$2, the total profit will not change.
- C) If the selling price of B is reduced by \$5, the optimal production plan will not change.
- D) If the cost of processing B is increased by \$3, the optimal production plan will not change.
- E) None of the above.

Q3.

Suppose in Q2, there is a fixed cost for each production process as listed below:

Process	Raw material to A	Raw material to B	Processing A	Processing B
Cost (\$)	300	500	600	2000

Please answer the following questions in the answer template and upload your detailed calculations in a separate file as the appendix.

Please **round off your final answers to two decimal places.**

- (A) The optimal raw material purchase quantity is _____ pounds.
- (B) The optimal processing quantity of A is _____ pounds.
- (C) The optimal processing quantity of B is _____ pounds.
- (D) The optimal total profit is \$_____.