

Sample Examination

Course: MSBA7003 QUANTITATIVE ANALYSIS METHODS

INSTRUCTIONS

1. Attempt all questions in Section I and Section II and write your answers in the place provided.
2. Answer all questions in Section III and write your answers in the place provided.

SECTION I: TRUE/FALSE. Write 'T' if the statement is true and 'F' if it is false.

- 1) If $P(A|C) > P(B|C)$, then we must have $P(A) > P(B)$. __F__
- 2) If three events A, B, and C are collectively exhaustive, $P(A) + P(B) + P(C) = 1$. __F__
- 3) The shadow price associated with a resource in linear programming tells us how much we should charge at most to give away a unit of that resource. __F__
- 4) We can generate an exponential random variable with given mean by making use of a uniform random variable generator (such as rand() in excel). __T__
- 5) In linear programming, if dropping a constraint does not affect the optimal solution, then this constraint must be redundant. __F__
- 6) If event A and B are independent given C is occurring, then A and B are unconditionally independent. __F__

SECTION II: MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

- 7) If the expected monetary value (EMV) of option 1 is 10 and the expected value of perfect information (EVPI) is 2, then __D__
A) the expected value with perfect information is 12.
B) the expected opportunity loss (EOL) of option 1 less than or equal to 2.
C) the optimal decision is not option 1.
D) there could be an option that can achieve an EMV of larger than 12.
E) None of the above

8) The host has four locked boxes. Three of them contain rocks, and one contains a diamond. The host knows what is inside in each of the boxes. You are asked to randomly select one box (without opening it). After you select a box, the host opens a box which contains a rock from the rest three boxes and shows you. Then, the host asks you whether to switch to another unopened box. If you choose to switch, the probability of getting the diamond is: B

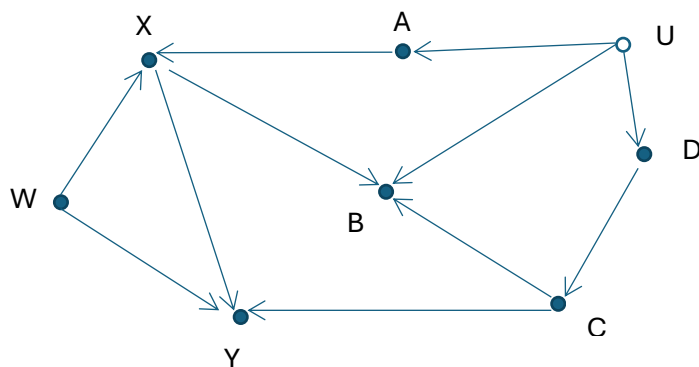
A) 1/8 B) 3/8 C) 5/8 D) 7/8
E) None of the above.

9) Consider the following linear programming problem: B

Maximize $4X + 3Y$
Subject to: $X + Y \leq 1.5$
 $X + 3Y \leq 3$
 $8X + 6Y \leq 7$
 All variables ≥ 0

- A) there are 5 corner points for the feasible region.
B) there is a redundant constraint.
C) there is only a single optimal solution.
D) there is no feasible solution.
E) None of the above

Next two questions are based on the causal graph shown below



10) If we want to estimate how X directly influences Y, using conditioning strategy, we can condition on C

A) W and B B) A and D C) D and W
D) B and D E) None of the above.

11) Assume the causal influences are all linear in this system. Which of the following models can correctly estimate the causal effect of C on Y? B

A) $Y = \alpha + \beta C + \epsilon$ B) $Y = \alpha + \beta C + \gamma A + \theta W + \epsilon$
C) $Y = \alpha + \beta C + \gamma B + \delta W + \epsilon$
D) $Y = \alpha + \beta C + \gamma A + \theta B + \delta W + \epsilon$
E) None of the above.

SECTION III: SHORT ANSWERS. Write your answer in the space provided. Show all necessary steps to get partial credit.

12). Factory ABC produces a seasonal product with a limited capacity of 1,000 units. There are five customers that ask for different quantities and offer different prices, as shown in the table:

Customer	1	2	3	4	5
Quantity	365	470	550	200	340
Price offered	2.70	2.50	2.20	2.80	2.60

Customers cannot be partially satisfied, but it is okay to reject their requests except customer No. 5. The objective of ABC is to maximize the revenue with the given capacity.

- (a) Develop a mixed integer program for ABC. Define the decision variables first, and then write down the objective function and constraints.

Define X_i as whether to accept customer i 's order.

Maximize: $2.7 \cdot 365 \cdot X_1 + 2.5 \cdot 470 \cdot X_2 + 2.2 \cdot 550 \cdot X_3 + 2.8 \cdot 200 \cdot X_4$

Subject to: $365X_1 + 470X_2 + 550X_3 + 200X_4 \leq 660$

X_1, X_2, X_3 , and X_4 are binary.

- (b) How to model the constraint of "at least three customers should be satisfied?"

$X_1 + X_2 + X_3 + X_4 \geq 2$

- (c) How to model the constraint of "if customer 4 is satisfied then at least one of customer 2 and customer 3 should also be satisfied?"

$X_2 + X_3 \geq X_4$

13) Ken is a new basketball player, and his 3-pointer ability in real games is unknown. The school coach is not sure whether Ken is talented or just an average player. For talented players, the chance of success for 3-pointers is 0.9; for average players, the chance of success is 0.6. The coach's prior belief is that Ken is a talented player with a probability of 0.7, but the coach will recruit Ken to the school team only when he is at least 80% sure that Ken is talented. Before the coach makes his decision, he wants to see two of Ken's inner-school games. Suppose in Ken's first inner-school game, he tried 7 three-pointers and succeeded 5 times. (**Binomial Distribution Table is attached**)

- (a) What is the coach's updated belief about Ken based on his first game record? (5 points)

Answer:_____ The updated belief is 0.5257 for Ken being talented.

Steps: By Bayes Theorem:

$P(\text{talented}/5 \text{ success out of } 7)$

$= \frac{P(5S/T) \cdot P(T)}{P(5S/T) \cdot P(T) + P(5S/A) \cdot P(A)} = \frac{0.124 \cdot 0.7}{(0.124 \cdot 0.7 + 0.261 \cdot 0.3)} = 0.5257$

5S: 5 success

T: talent

A: average

- (b) According to the first game record and the coach's prior belief, what is Ken's 3-pointer chance of success in the next game? (5 points)

Answer: _ The chance of success in the next game is 0.7577.

Steps:

P(S in next game/updated belief that

$$P(\text{talent}) = 0.5257 = P(S/T)P(T) + P(S/A)P(A) = 0.9 \cdot 0.5257 + 0.6 \cdot 0.4743 = 0.7577$$

- (c) In the next game, if Ken will try 7 shots again, what is the minimum number of successes needed for the coach to recruit him? (10 points)

Answer: _ Ken must succeed for all 7 shots in the next game. _____

Steps:

If succeed 6 times, $P(\text{talented}/6 \text{ success out of } 7) =$

$$= P(6S/T) \cdot P(T) / [P(6S/T) \cdot P(T) + P(6S/A) \cdot P(A)] = 0.372 \cdot 0.5257 / (0.372 \cdot 0.5257 + 0.131 \cdot 0.4743) = 0.7589 < 0.8$$

If succeed 7 times, $P(\text{talented}/7 \text{ success out of } 7) =$

$$= P(7S/T) \cdot P(T) / [P(7S/T) \cdot P(T) + P(7S/A) \cdot P(A)] = 0.478 \cdot 0.5257 / (0.478 \cdot 0.5257 + 0.028 \cdot 0.4743) = 0.9498 > 0.8$$

14)

Simulate 10 weeks of operations for the retailer, assuming they start with 5 units in inventory. Use the random numbers provided in the table below to generate the demand and lead time values:

Trigger (r):	3	Costs:	Holding	Stock Out	Shipping	Avg. Level	Stock Out Rate		Reorder Rate			Weekly Total Cost
			5	45	40	1.7	0.2		0.2			25.5
Inventory Level Simulation												
Week	Order Arrive	Initial Inv.	R.N.	Demand	Sales	Lost Sales	Ending Inv.	Reorder		R.N.	Lead Time	Weeks of Arr.
1	0	5	0.29	1	1	0	4	0		-	-	-
2	0	4	0.48	2	2	0	2	1		0.53	2	4
3	0	2	0.37	1	1	0	1	0		-	-	-
4	0	1	0.77	3	1	2	0	0		-	-	-
5	1	10	0.22	1	1	0	9	0		-	-	-
6	0	9	0.91	4	4	0	5	0		-	-	-
7	0	5	0.51	2	2	0	3	1		0.45	2	9
8	0	3	0.64	2	2	0	1	0		-	-	-
9	0	1	0.17	1	1	0	0	0		-	-	-
10	1	10	0.42	2	2	0	8	0		-	-	-
Total Sales		17										
Total Lost Sales		2										
Total Number of Orders Placed		2										
Total Cost		\$335										

- (a) Please fill the blanks in the above table
- (b) The total sales during the 10 weeks: __17__.
- (c) The total lost sales during these 10 weeks: __2__.
- (d) The total number of orders placed during these 10 weeks is __2__.
- (e) The total cost, including holding, shipping, and backorder costs, is \$ __335__.

15) We want to investigate how Manning's staffing level affects store sales using Ordinary Least Square (OLS) Regression. What factors must be included in the regression? If the back-door path cannot be completely blocked, what can be a valid instrument variable?

Answer:

To estimate the causal effect with a regression model, we need to condition on at least store size, traffic size, and assortment.

Employee sick leave can be used as an IV.

Appendix: Binomial Distribution Table

This table shows the probability of r successes in n independent trials, each with probability of success p .

n	r	p																			
		.01	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95
2	0	.980	.902	.810	.723	.640	.563	.490	.423	.360	.303	.250	.203	.160	.123	.090	.063	.040	.023	.010	.002
	1	.020	.095	.180	.255	.320	.375	.420	.455	.480	.495	.500	.495	.480	.455	.420	.375	.320	.255	.180	.095
	2	.000	.002	.010	.023	.040	.063	.090	.123	.160	.203	.250	.303	.360	.423	.490	.563	.640	.723	.810	.902
3	0	.970	.857	.729	.614	.512	.422	.343	.275	.216	.166	.125	.091	.064	.043	.027	.016	.008	.003	.001	.000
	1	.029	.135	.243	.325	.384	.422	.441	.444	.432	.408	.375	.334	.288	.239	.189	.141	.096	.057	.027	.007
	2	.000	.007	.027	.057	.096	.141	.189	.239	.288	.334	.375	.408	.432	.444	.441	.422	.384	.325	.243	.135
3	3	.000	.000	.001	.003	.008	.016	.027	.043	.064	.091	.125	.166	.216	.275	.343	.422	.512	.614	.729	.857
4	0	.961	.815	.656	.522	.410	.316	.240	.179	.130	.092	.062	.041	.026	.015	.008	.004	.002	.001	.000	.000
	1	.039	.171	.292	.368	.410	.422	.412	.384	.346	.300	.250	.200	.154	.112	.076	.047	.026	.011	.004	.000
	2	.001	.014	.049	.098	.154	.211	.265	.311	.346	.368	.375	.368	.346	.311	.265	.211	.154	.098	.049	.014
	3	.000	.000	.004	.011	.026	.047	.076	.112	.154	.200	.250	.300	.346	.384	.412	.422	.410	.368	.292	.171
4	4	.000	.000	.000	.001	.002	.004	.008	.015	.026	.041	.062	.092	.130	.179	.240	.316	.410	.522	.656	.815
5	0	.951	.774	.590	.444	.328	.237	.168	.116	.078	.050	.031	.019	.010	.005	.002	.001	.000	.000	.000	.000
	1	.048	.204	.328	.392	.410	.396	.360	.312	.259	.206	.156	.113	.077	.049	.028	.015	.006	.002	.000	.000
	2	.001	.021	.073	.138	.205	.264	.309	.336	.346	.337	.312	.276	.230	.181	.132	.088	.051	.024	.008	.001
	3	.000	.001	.008	.024	.051	.088	.132	.181	.230	.276	.312	.337	.346	.336	.309	.264	.205	.138	.073	.021
	4	.000	.000	.000	.002	.006	.015	.028	.049	.077	.113	.156	.206	.259	.312	.360	.396	.410	.392	.328	.204
5	5	.000	.000	.000	.000	.000	.001	.002	.005	.010	.019	.031	.050	.078	.116	.168	.237	.328	.444	.590	.774
6	0	.941	.735	.531	.377	.262	.178	.118	.075	.047	.028	.016	.008	.004	.002	.001	.000	.000	.000	.000	.000
	1	.057	.232	.354	.399	.393	.356	.303	.244	.187	.136	.094	.061	.037	.020	.010	.004	.002	.000	.000	.000
	2	.001	.031	.098	.176	.246	.297	.324	.328	.311	.278	.234	.186	.138	.095	.060	.033	.015	.006	.001	.000
	3	.000	.002	.015	.042	.082	.132	.185	.236	.276	.303	.312	.303	.276	.236	.185	.132	.082	.042	.015	.002
	4	.000	.000	.001	.006	.015	.033	.060	.095	.138	.186	.234	.278	.311	.328	.324	.297	.246	.176	.098	.031
	5	.000	.000	.000	.000	.002	.004	.010	.020	.037	.061	.094	.136	.187	.244	.303	.356	.393	.399	.354	.232
	6	.000	.000	.000	.000	.000	.000	.001	.002	.004	.008	.016	.028	.047	.075	.118	.178	.262	.377	.531	.735
7	0	.932	.698	.478	.321	.210	.133	.082	.049	.028	.015	.008	.004	.002	.001	.000	.000	.000	.000	.000	.000
	1	.066	.257	.372	.396	.367	.311	.247	.185	.131	.087	.055	.032	.017	.008	.004	.001	.000	.000	.000	.000
	2	.002	.041	.124	.210	.275	.311	.318	.299	.261	.214	.164	.117	.077	.047	.025	.012	.004	.001	.000	.000
	3	.000	.004	.023	.062	.115	.173	.227	.268	.290	.292	.273	.239	.194	.144	.097	.058	.029	.011	.003	.000
	4	.000	.000	.003	.011	.029	.058	.097	.144	.194	.239	.273	.292	.290	.268	.227	.173	.115	.062	.023	.004
	5	.000	.000	.000	.001	.004	.012	.025	.047	.077	.117	.164	.214	.261	.299	.318	.311	.275	.210	.124	.041
	6	.000	.000	.000	.000	.000	.001	.004	.008	.017	.032	.055	.087	.131	.185	.247	.311	.367	.396	.372	.257
	7	.000	.000	.000	.000	.000	.000	.001	.002	.004	.008	.015	.028	.049	.082	.133	.210	.321	.478	.698	