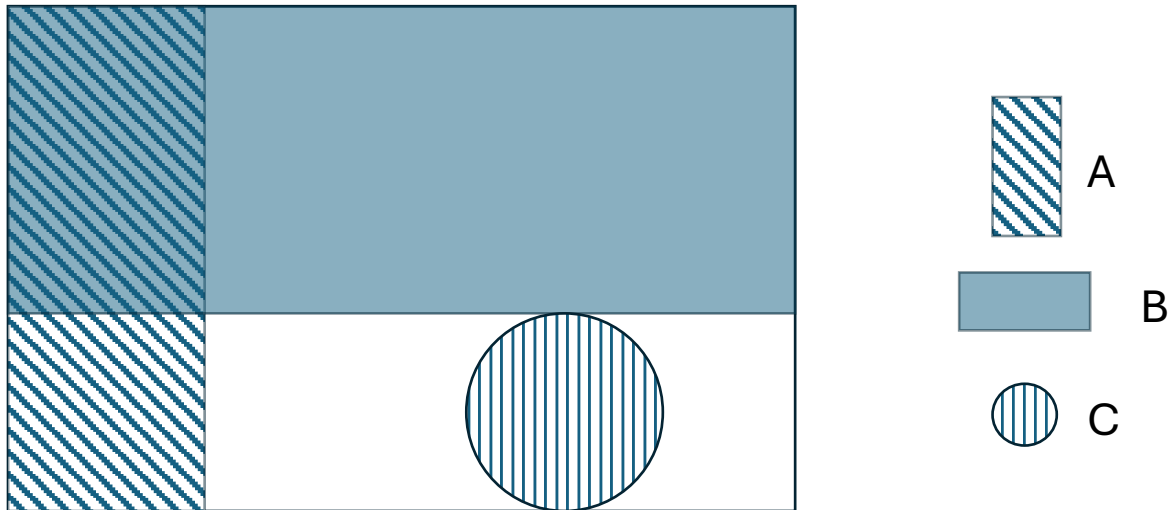


**MSBA7003 Decision Analytics – Assignment 1 (Due Sep 14 noon at 11:55 a.m.)**

**Please review the example answer sheet provided, and use the specified template to submit your solutions. You need to submit a Excel file for Q1-Q3 and an appendix file for Q3.**

Q1.

Which of the following statement(s) is(are) true about the below Venn Diagram?



- (A) Events A, B, and C are collectively exhaustive.
- (B) Events A and C are correlated.
- (C) Events B and C are correlated.
- (D) Events A and B are correlated.

Q2.

Suppose you can see a lawn out of your window. The lawn may be wet or dry in the afternoon. The state depends on the whether it rained earlier today and the previous operation of the sprinkler on the lawn. In the past, you manually recorded the lawn state, the weather, and the sprinkler operation every day. You constructed below table of conditional probabilities of the afternoon lawn state:

Previous operation of the sprinkler		On	On	Off	Off
Whether it rained earlier today		Yes	No	Yes	No
Lawn state	Wet	0.99	0.9	0.8	0.0
	Dry	0.01	0.1	0.2	1.0

Today, you forgot to check the morning weather and the sprinkler operation, and you found the lawn dry in the afternoon. Suppose the sprinkler is operated by a computer program that is independent of the weather and is turned on with probability 0.5. Your prior belief about the morning weather is that it rains with probability 0.2.

Which of the following statement(s) is(are) true?

- (A) Given a dry lawn, the probability that it rained in the morning is 0.0567.
- (B) Given a dry lawn, the probability that it rained in the morning is 0.0456.
- (C) Given it rained in the morning and the lawn dried in the afternoon, the probability that the sprinkler was on in the morning is 0.0852.
- (D) Given it rained in the morning and the lawn dried in the afternoon, the probability that the sprinkler was on in the morning is 0.0476.

Q3.

Random variable  $X$  follows an exponential distribution with parameter  $L$ . The cumulative distribution function is  $F(x) = P(X \leq x) = 1 - \exp(-Lx)$ . We don't know the value of  $L$ , but we have some samples of  $X$  in the list below:

Sample	1	2	3	4	5
Value	0.4256	0.2253	0.0253	0.2126	0.1289

**Please answer the following questions in the answer template and upload your detailed calculations in a separate file as the appendix.**

- (A) Based on the samples, the mean value of  $X$  is \_\_\_\_\_.
- (B) Let  $M$  denote the inversed sample mean rounded to the nearest integer (i.e.,  $M = \lceil 1/\text{Mean}(X) \rceil$ ). We then consider three possible values of  $L$ :  $M$ ,  $M-1$ , and  $M+1$ . Suppose the prior probabilities of them are equal. If we use all the samples and Bayesian rule to update our belief, the probability of  $L = M$  is \_\_\_\_\_. (Hint: Please compute the probability of  $X = x$  as the product of  $f(x) \cdot d$ , wherein  $f(x) = F'(x)$  is the density and  $d$  is a very small positive number.)