MSBA7003 Decision Analytics

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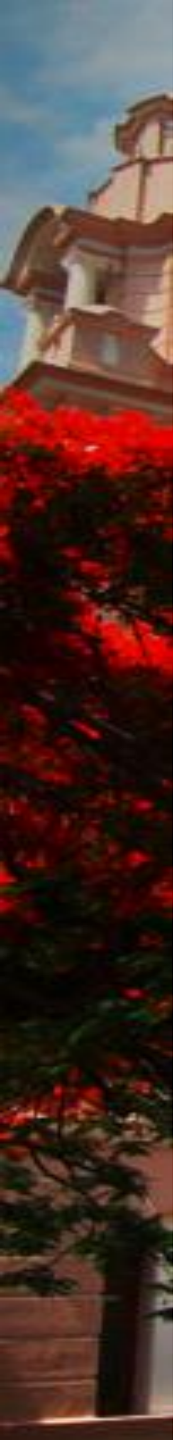
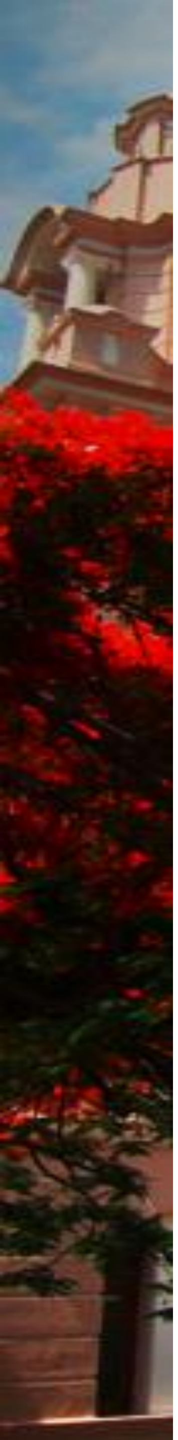
04 Monte Carlo Simulation & Dynamic Decisions



Agenda

- Monte Carlo Simulation
- Risk Assessment
- Inventory Management
- Revenue Management
- Service Management



Monte Carlo Simulation

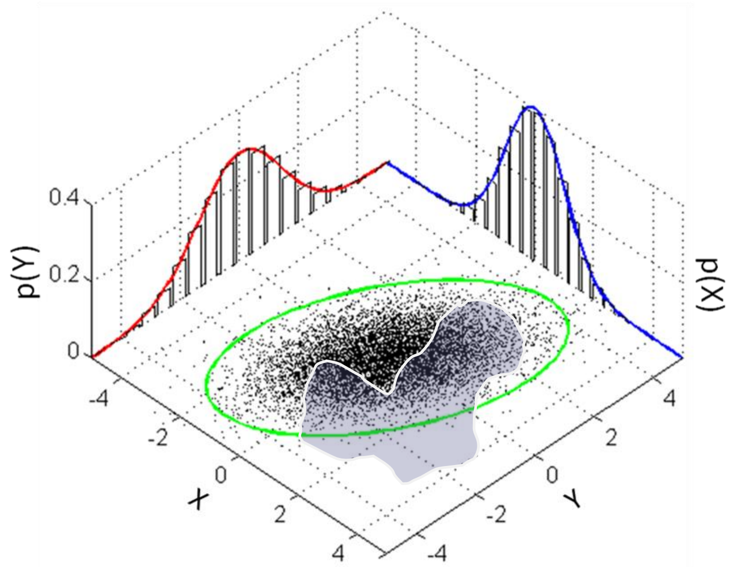
- In practice, we often make decisions under risk for very complicated problems.
  - Too many random variables involved
  - Too many or even infinitely many possible states
  - Too many options
  - Too many stages
  - Cannot be explicitly solved

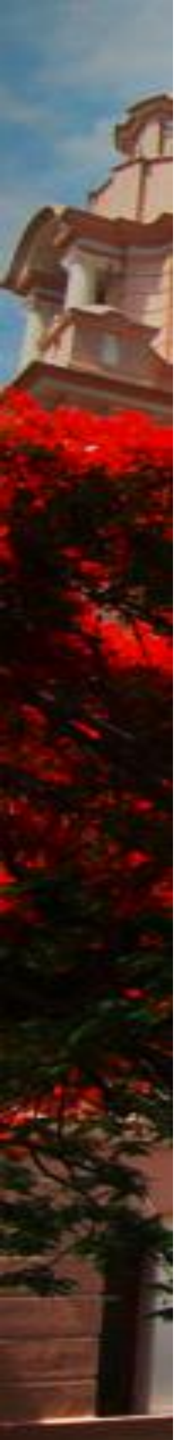
- Simulation is to use a computer to duplicate the features, appearance, and characteristics of a real system, and learn the outcome of a decision.

Monte Carlo Simulation

- Monte Carlo methods rely on repeated random sampling to obtain numerical results.

What is the probability of hitting the shaded area?





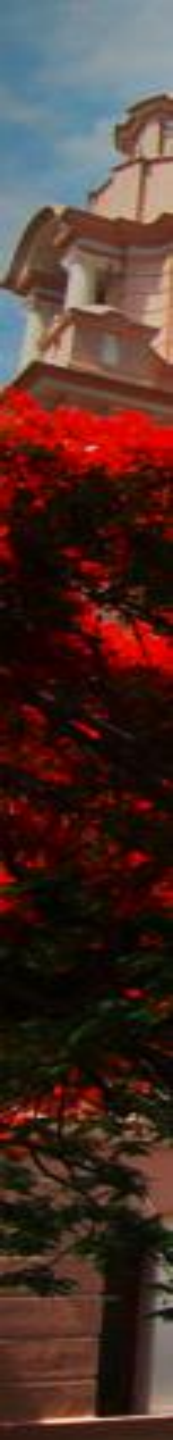
Simulation-Based Decision Making: Single Decision

- Mathematically, if the objective function $V(X, D)$ depends on the value of a random variable (vector) X and a decision (vector) D, then the expected objective value given D can be approximated by randomly sample X independently N times and computing the average:

$$\frac{1}{N}\sum_{i=1}^{N} V(X_i, D)$$

- Then we can try different D to maximize or minimize the objective.

	Sampling State of the World via Simulation					
Options	Sample (1)	Sample (2)	Sample (3)	…	Sample (N)	Average Objective Value
Option (A)	VA1	VA2	VA3	…	VAN	(VA1+VA2+VA3+ … +VAN)/N
Option (B)	VB1	VB2	VB3	…	VBN	(VB1+VB2+VB3+ … +VBN)/N
Option (C)	VC1	VC2	VC3	…	VCN	(VC1+VC2+VC3+ … +VCN)/N



Simulation-Based Decision Making: Multiple Decisions

- When there are multiple stages, our objective is to maximize or minimize $\sum_t V_t(X_t, D_t)$ by choosing a set of decisions $\{D_t\}$. Different from single decision problems, the decision maker can collect information along the process. I_t is the cumulated information up to stage t. We aim to obtain a solution represented by a policy

$$\Pi: I_t \rightarrow D_t.$$

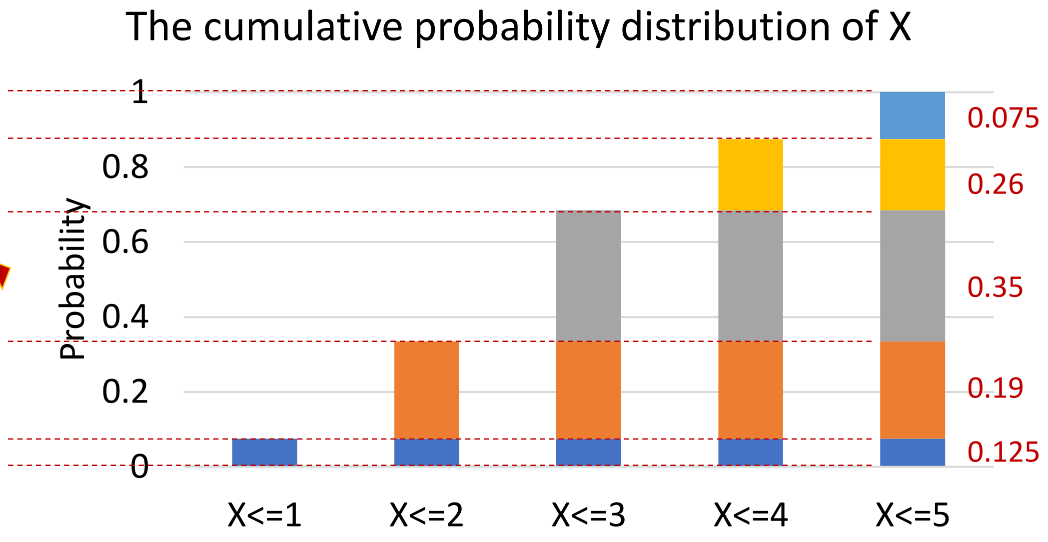
- We can use Monte Carlo simulation to evaluate Π and find the best possible policy.

Fix Π	Sample the value of X_t at different stages and compute $V_t(X_t, D_t)$					
Simulation	Stage 1	Stage 2	Stage 3	…	Stage k	Simulated Objective
Round 1	V11	V12	V13	…	V1k	V1=V11+V12+V13+ … +V1k
Round 2	V21	V22	V23	…	V2k	V2=V21+V22+V23+ … +V2k
…	…	…	…	…	…	…
Round N	VN1	VN2	VN3	…	VNk	VN=VN1+VN2+VN3+ … +VNk
					Average	(V1+V2+ … +VN)/N

Generating a Random Number

- Random sampling is a critical step. Suppose the model is ready (i.e., the distribution of X is known), how to sample the value of X?

2. Generate a random number (uniformly) from [0,1].



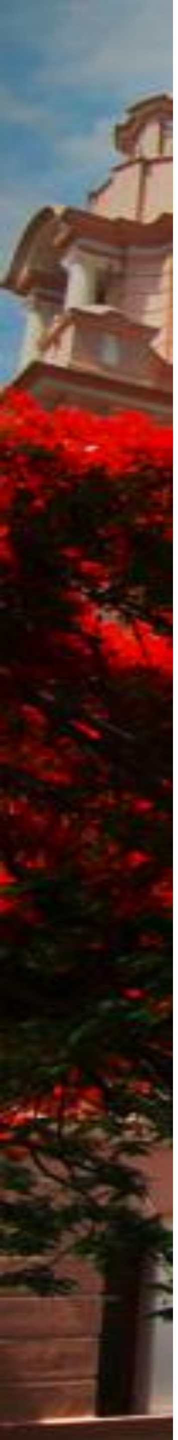
1. Calculate the cumulative probability for each possible value of X and divide the [0,1] interval into segments corresponding to each possible value of X.

3.Return the value of X corresponding to the segment that contains the random number.

Generating a Random Number

- For discrete random variables
  - Step 1: Build a cumulative distribution table with $X_i < X_{i+1}$.
  - Step 2: Assign the interval $(0, F(X_1)]$ to X_1 and $(F(X_{i-1}), F(X_i)]$ to X_i from $i = 2$.
  - Step 3: generate a random probability and find the corresponding X value.

- Generating a discrete random number in Excel:
  - Binomial distribution: BINOM.INV(n, p_s, probability)
  - An arbitrary distribution: VLOOKUP(probability, distribution table, column index for the return value)

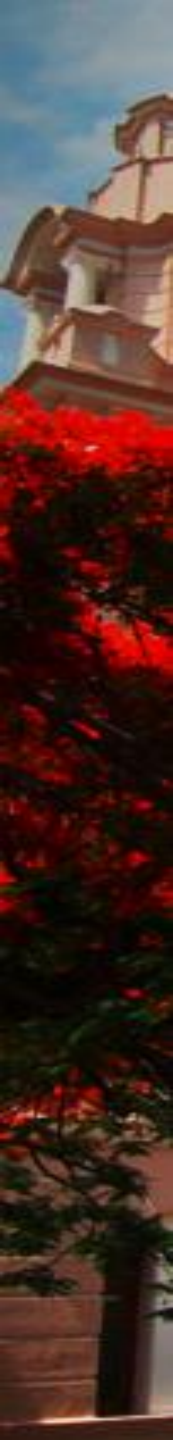


Harry's Auto Tire

- Harry's Auto Tire sells a popular radial tire and replenish its inventory every 10 days. If the daily demand independently follows the distribution shown in the table below and the initial inventory level is 40 units, what is the expected sales in a 10-day period?

Daily Demand	0	1	2	3	4	5
Probability	0.05	0.10	0.20	0.30	0.20	0.15

- Note that 10-day sales = min(10-day demand, total inventory).



Harry’s Auto Tire

First, build a cumulative distribution table.
Next, suppose we decompose the simulation of daily demand into two steps: random number & vlookup

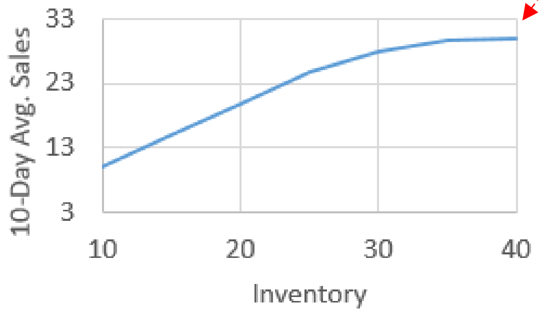
	A	B	C	D	E
1	**Harry's Auto Tire**				
2					
3	**Daily Demand**	**Probability**	**Cumulative Pr**	**Lower**	**Daily Demand**
4	0	0.05	0.05	0	0
5	1	0.1	0.15	0.05	1
6	2	0.2	0.35	0.15	2
7	3	0.3	0.65	0.35	3
8	4	0.2	0.85	0.65	4
9	5	0.15	1	0.85	5
10					
11	**Simulation**				
12	**Days**	**Random Numb**	**Demand**		
13	Day 1	0.283855951	2	=vlookup(B13,$D$4:$E$9,2)	
14	Day 2	0.351300493	3		

The VLOOKUP function looks up the random number in the leftmost column of the defined lookup table. It moves downward through this column until it finds a cell that is *equal to or bigger than* the random number. It then goes to the previous row and gets the value from the specified column.

Harry’s Auto Tire

A more compact & comprehensive spreadsheet model:

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Harry's Auto Tire										Data Table	Avg. Sales	
2											Inventory	29.96	
3	Demand	Prob	Cumu. Pr.	Lower	Demand		Inventory				10	10	
4	0	0.05	0.05	0	0		40				15	15	
5	1	0.1	0.15	0.05	1						20	19.92	
6	2	0.2	0.35	0.15	2		Average 10-Day Sales				25	24.74	
7	3	0.3	0.65	0.35	3		29.96				30	27.96	
8	4	0.2	0.85	0.65	4						35	29.68	
9	5	0.15	1	0.85	5						40	29.82	
10													
11													
12	Simulatio	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10	Total	Sales
13	1	4	2	2	3	4	1	0	4	3	3	26	26
14	2	5	4	3	0	3	4	3	5	2	2	31	31
15	3	2	3						5	3	3	28	28
16	4	4	2						3	3	4	31	31
17	5	3	4						1	3	4	28	28
18	6	1	2						2	2	2	27	27
19	7	2	4						2	1	5	27	27
20	8	3	3						4	1	2	25	25
21	9	3	1						5	2	5	30	30
22	10	4	5						1	2	3	28	28
23	11	3	2						3	3	4	30	30

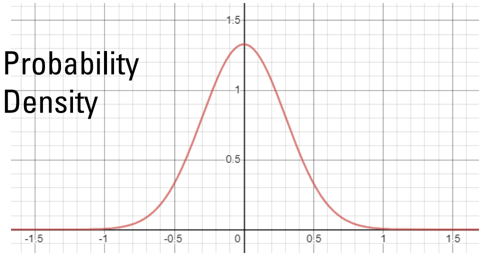


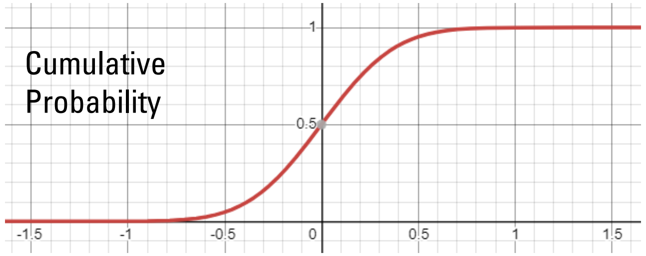
Use data table to try different inventory levels and plot the relationship between sales and inventory.

=VLOOKUP(RAND(),$D$4:$E$9,2,1)

Generating a Random Number

- For continuous random variables, the idea is the same
  - Step 1: derive the inverse function of $Y = F(X)$
    - E.g., for exponential distribution with parameter λ, the cumulative distribution function (CDF) is $Y = F(X) = 1 - e^{-\lambda X}$ and the inverse CDF is $X = F^{-1}(Y) = -\lambda^{-1} \cdot \ln(1 - Y)$.
  - Step 2: generate a random probability (i.e., a random number uniformly from [0,1]) and assign it to Y to get X.



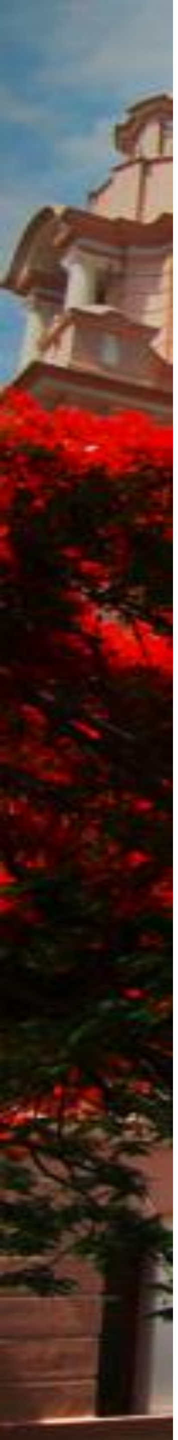


- Generating a continuous random number in Excel:
  - Normal distribution: NORM.INV(probability, mean, st_dev)
  - Standard Normal distribution: NORM.S.INV(probability)
  - Uniform distribution on [0, 1]: RAND()

Generating a Random Number in Python

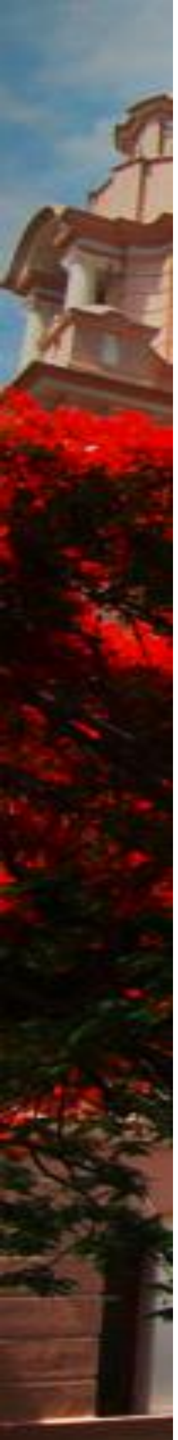
- In Python, we can use numpy to generate various kinds of random numbers.
- For an arbitrary discrete distribution:

```
from numpy import random
dist_table={0:[4,0.05], 1:[5,0.2], 2:[6,0.45], 3:[7,0.65], 4:[8,0.8], 5:[9,0.9], 6:[10,1.0]}
def randnum(d_table, size=None):
    if size==None or size==1:
        temp=random.rand()
        for j in range(len(dist_table)):
            if dist_table[j][1]>temp:
                pointer=j
                break
        return dist_table[pointer][0]
    else:
        numbers=[]
        temp=random.rand(size)
        for i in range(size):
            for j in range(len(dist_table)):
                if dist_table[j][1]>temp[i]:
                    pointer=j
                    break
            numbers.append(dist_table[pointer][0])
        return numbers
```



In-Class Exercises

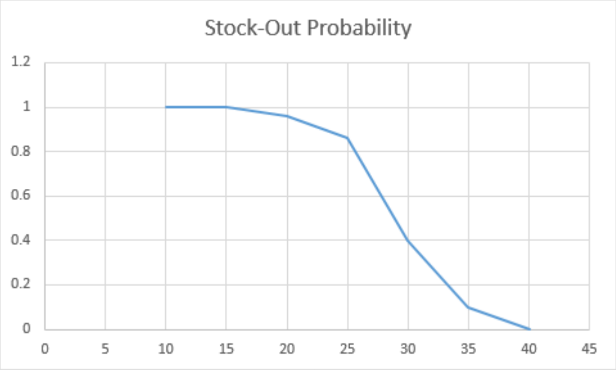
- A jewellery store sells a necklace at the price of $1,100. The number of visits to the store per day follows a binomial distribution with a mean of 300 and maximum value of 1,000. Among the arriving customers, each one has a 0.1 chance of showing interests in this necklace. Customer willingness-to-pay for the product follows a normal distribution with a mean of $1,000 and a standard deviation of $300. What is the average daily sales quantity of this necklace?

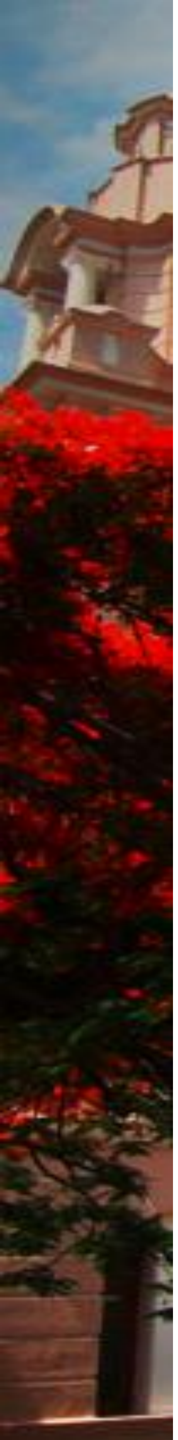


Probability Estimation

- Monte Carlo method can not only assess the average performance of a decision but can also assess the probability of an event.
- For Harry's Auto Tire, what is the probability of running out of stock?

K	L	M	N	O	P	Q
Data Table	Avg. Sales				Data Table	Stock-out Prob.
Inventory	28.5				Inventory	0.02
10	10				10	1
15	15				15	1
20	19.98				20	0.96
25	24.88				25	0.86
30	28.28				30	0.4
35	29.26				35	0.1
40	29				40	0
				Probability		
				0.02		
Day 10	Total	Sales		Out of stock?		
2	21	21		0		
5	42	40		1		
3	28	28		0		
3	28	28		0		
1	29	29		0		
4	31	31		0		
5	25	25		0		
0	27	27		0		
3	32	32		0		
3	26	26		0		
3	32	32		0		
3	32	32		0		
4	30	30		0		



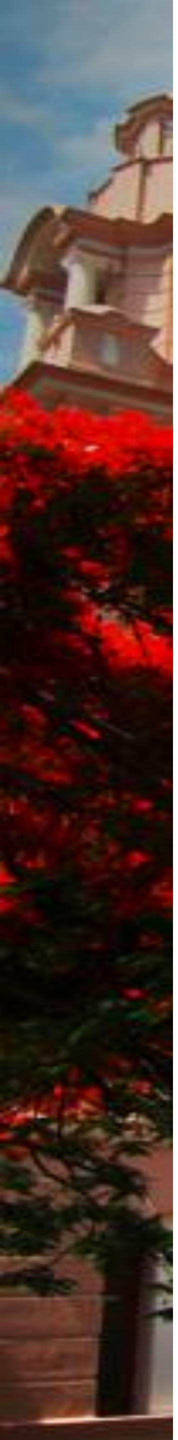


Inventory Management: Simkin's Hardware Store

- The owner of Simkin's hardware store wants to find a good, low cost inventory policy for an electric drill (Ace Model). Because all the sales and inventory are recorded in the computer and the supplier accepts orders at any time, so continuous inventory monitoring is adopted.
- How to optimize the order quantity (Q) and reorder point (r), given the distributions of daily demand and reorder lead time below?

Daily Demand	Frequency (Days)	Prob.	Cumu. Prob.
0	15	0.05	0.05
1	30	0.10	0.15
2	60	0.20	0.35
3	120	0.40	0.75
4	45	0.15	0.90
5	30	0.10	1.00
	300	1.00	

Lead Time (Days)	Frequency (Orders)	Prob.	Cumu. Prob.
1	10	0.20	0.20
2	25	0.50	0.70
3	15	0.30	1.00
	50	1.00	



Inventory Management: Simkin's Hardware Store

- Suppose Simkin's wants to test the policy of **Q=10 and r=5**. The process is simulated manually for a 10-day period.

DAY	UNITS RECEIVED	BEGINNING INVENTORY	RANDOM NUMBER	DEMAND	ENDING INVENTORY	LOST SALES	ORDER	RANDOM NUMBER	LEAD TIME
1	0	10	0.06	1	9	0	No		
2	0	9	0.63	3	6	0	No		
3	0	6	0.57	3	3	0	Yes	0.02	1
4	0	3	0.94	5	0	2	No		
5	10	10	0.52	3	7	0	No		
6	0	7	0.69	3	4	0	Yes	0.33	2
7	0	4	0.32	2	2	0	No		
8	0	2	0.30	2	0	0	No		
9	10	10	0.48	3	7	0	No		
10	0	7	0.88	4	3	0	Yes	0.14	1
				Total	41	2			

Inventory Management: Simkin's Hardware Store

- The objective is to find a low-cost solution so Simkin's must determine the costs associated with carrying inventory, lost sales, and ordering cost.
- Simkin's store is open 200 days a year, and the holding cost is $12 per drill per year. The estimated ordering cost is $10 per order. Lost sales cost $8 per unit.

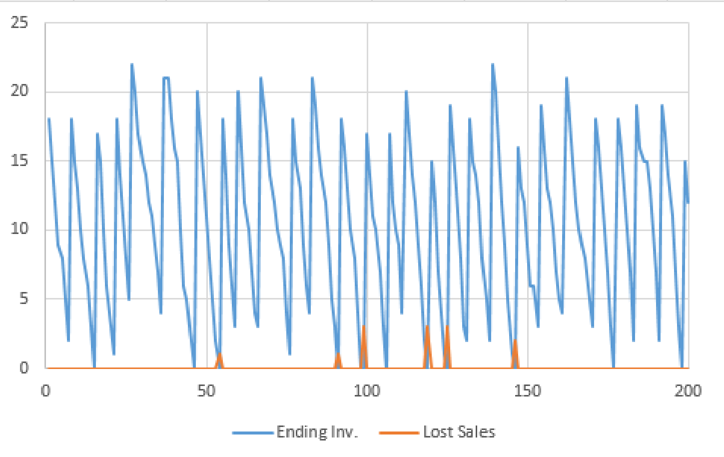
- The average (ending) inventory level = 41/10 = 4.1 units per day
- The rate of lost sales = 2/10 = 0.2 units per day
- The rate of orders = 3/10 = 0.3 times per day
- The average daily cost = 4.1*12/200 + 0.2*8 + 0.3*10 = 4.85

Inventory Management: Simkin's Hardware Store

Search for night of arrival from previous rows

Compare previous total order placements and order arrivals

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1	Inventory Policy	Quantity (Q) :		20		Trigger (r) :	10	Costs:	Holding	Stock-Out	Ordering	Avg. Level	Stock-Out Rate		Reorder Rate			Daily Total Cost
2									0.06	8	10	10.39	0.065		0.135			2.4934
3	Probability Distributions:						Inventory Level Simulation											
4	Demand	Prob.	Cumu. Pr.	Lower	Demand		Day	Order Arrive	Initial Inv.	R.N.	Demand	Ending Inv.	Lost Sales	Inv. Position	Reorder	R.N.	Lead Time	Night of Arr
5	0	0.05	0.05	0	0		1	0	20	0.310074	2	18	0	18	0	-	-	-
6	1	0.1	0.15	0.05	1		2	0	18	0.353592	3	15	0	15	0	-	-	-
7	2	0.2	0.35	0.15	2		3	0	15	0.690177	3	12	0	12	0	-	-	-
8	3	0.4	0.75	0.35	3		4	0								035	3	7
9	4	0.15	0.9	0.75	4		5	0									-	-
10	5	0.1	1	0.9	5		6	0									-	-
11							7	0									-	-
12	Lead Time	Prob.	Cumu. Pr.	Lower	Lead Time		8	1									-	-
13	1	0.2	0.2	0	1		9	0									-	-
14	2	0.5	0.7	0.2	2		10	0									-	-
15	3	0.3	1	0.7	3		11	0									-	-
16							12	0								423	3	15
17	Q|r 2D Data Table						13	0									-	-
18	2.4934	5	10	15	20		14	0									-	-
19	10	7.7218	4.2451	3.5663	3.6843		15	0									-	-
20	20	4.7053	2.2981	2.3421	2.6327		16	1									-	-
21	30	3.2742	1.9592	2.1567	2.4989		17	0									-	-
22	40	3.5457	2.0362	2.2444	2.5402		18	0									-	-
23	50	3.7768	2.[illegible]047	2.4563	2.7703		19	0								772	2	21
24							20	0	6	0.204729	2	4	0	24	0	-	-	-



Use 2D data table to try different（r,Q）combination and find the optimal policy.

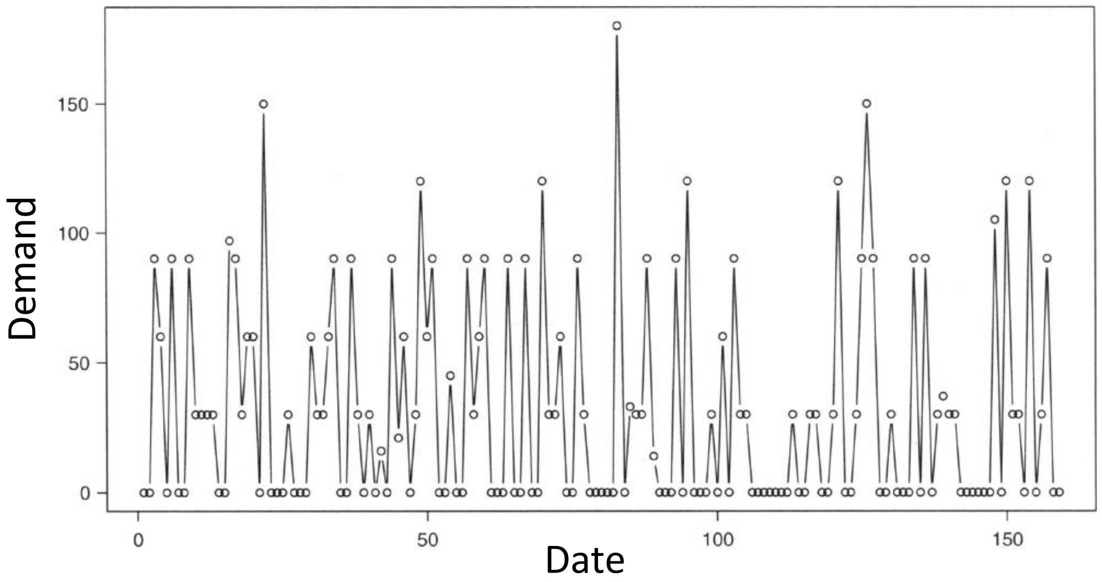
Kroger

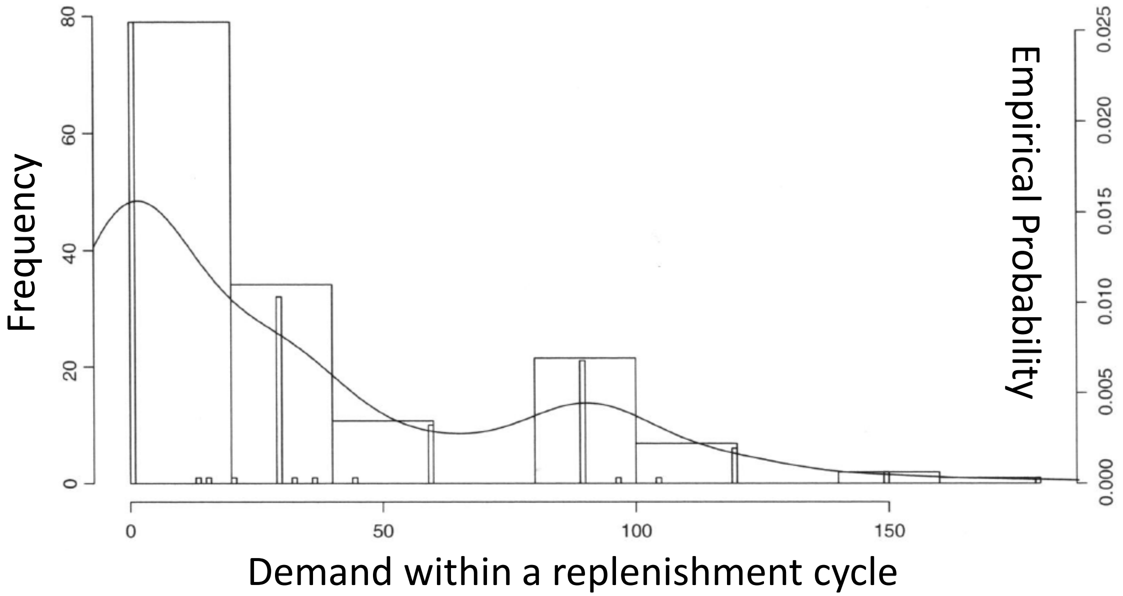
- Company Overview:
- 1,950 in-store pharmacies
- Each stocks about 2 to 3 thousand drugs
- Periodic inventory review: (s,S) policy
- Each pharmacy faces unique demand

- The Challenge of Inventory Management:
- The demand distribution is highly irregular and cannot be captured by standard distributions; complicated models are not acceptable

Kroger

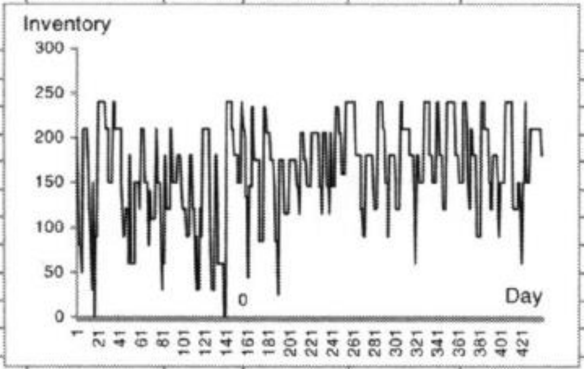
- The historical demand data of a medicine sold at a certain pharmacy store.
- The basic dose is one pill per day and the prescriptions are often written for 30 days.

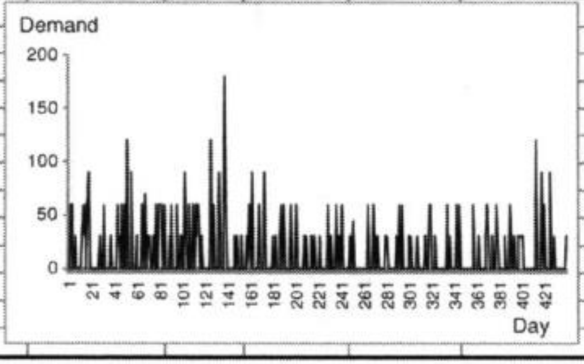


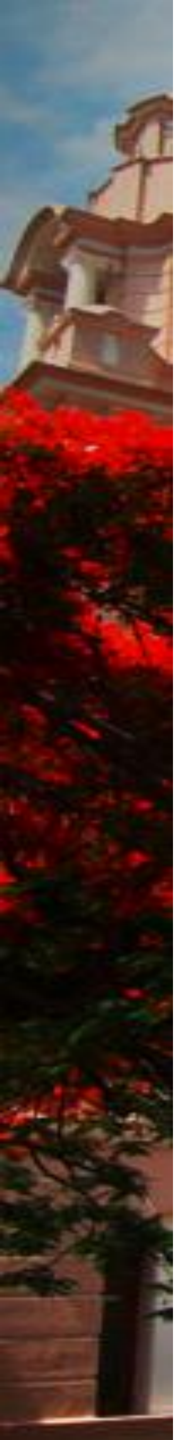


Kroger

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1	Drug information				Decision	variables					Performance metrics			Objectives			Starting	Solution
2	National drug code (DrugID)	DRUG0037		Reorder point		Order-up-to level				Total orders	Average inventory	No. of OOS		Objective		EOQ	Reorder point	Order-up-to-level
3	No. of days with prescriptions	121		165		235				53	163.30	0		11.96		40	180	220
4	Total demand in units	5,980																
5	Package size	10						Simulation process										
6	Query ID	Date	Demand	Days of week	Ordering period	Beginning inventory	Inv position	Order or not	Order quantity	Order arrival	Ending inventory	Out of stock				Graphics		
7	DRUG0037-2009-1-1	1/1/2009	0	4	0	200	200	0	0	0	200	0						
8	DRUG0037-2009-1-2	1/2/2009	60	5	1	200	200	0	0	0	140	0						
9	DRUG0037-2009-1-3	1/3/2009	60	6	0	140	140	0	0	0	80	0						
10	DRUG0037-2009-1-4	1/4/2009	0	7	0	80	80	0	0	0	80	0						
11	DRUG0037-2009-1-5	1/5/2009	0	1	1	80	80	1	160	0	80	0						
12	DRUG0037-2009-1-6	1/6/2009	30	2	0	80	240	0	0	0	50	0						
13	DRUG0037-2009-1-7	1/7/2009	0	3	1	50	210	0	0	160	210	0						
14	DRUG0037-2009-1-8	1/8/2009	0	4	0	210	370	0	0	0	210	0						
15	DRUG0037-2009-1-9	1/9/2009	0	5	1	210	210	0	0	0	210	0						
16	DRUG0037-2009-1-10	1/10/2009	0	6	0	210	210	0	0	0	210	0						
17	DRUG0037-2009-1-11	1/11/2009	0	7	0	210	210	0	0	0	210	0						
18	DRUG0037-2009-1-12	1/12/2009	30	1	1	210	210	0	0	0	180	0						
19	DRUG0037-2009-1-13	1/13/2009	60	2	0	180	180	0	0	0	120	0						
20	DRUG0037-2009-1-14	1/14/2009	30	3	1	120	120	1	120	0	90	0						
21	DRUG0037-2009-1-15	1/15/2009	60	4	0	90	210	0	0	0	30	0						
22	DRUG0037-2009-1-16	1/16/2009	0	5	1	30	150	1	90	120	150	0						
23	DRUG0037-2009-1-17	1/17/2009	60	6	0	150	360	0	0	0	90	0						
24	DRUG0037-2009-1-18	1/18/2009	90	7	0	90	180	0	0	0	0	0						
25	DRUG0037-2009-1-19	1/19/2009	0	1	1	0	90	1	150	90	90	0						
26	DRUG0037-2009-1-20	1/20/2009	0	2	0	90	240	0	0	0	90	0						
27	DRUG0037-2009-1-21	1/21/2009	0	3	1	90	240	0	0	150	240	0						
28	DRUG0037-2009-1-22	1/22/2009	0	4	0	240	390	0	0	0	240	0						
29	DRUG0037-2009-1-23	1/23/2009	0	5	1	240	240	0	0	0	240	0						
30	DRUG0037-2009-1-24	1/24/2009	0	6	0	240	240	0	0	0	240	0						
31	DRUG0037-2009-1-25	1/25/2009	0	7	0	240	240	0	0	0	240	0						
32	DRUG0037-2009-1-26	1/26/2009	0	1	1	240	240	0	0	0	240	0						

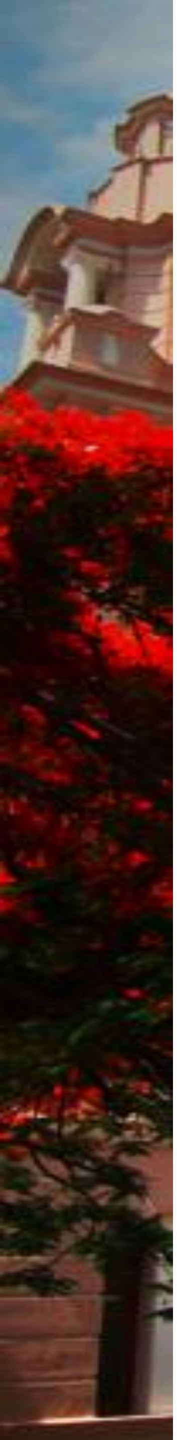






Revenue Management: K-Fashion Revisited

- There are two prices to make. One is the original tag price, and the other is the markdown price for the third month. The store traffic is smaller in the first two months. According to data, the monthly visit numbers follow a normal distribution with a mean of 4,000 and a standard deviation of 800. In the third month, the number of visit also follows a normal distribution with a mean of 8,000 and a standard deviation of 1,500.
- According to experience, 1/30 of the arriving customers will show interests in this style (e.g., asking about the price and/or trying on). These customers only like one of the three SKUs, and the chance is 1/3 for each SKU. The customer willingness-to-pay for this style in the first two months follows a uniform distribution on [$0, $1,000], and in the last month the range becomes [$0, $600].



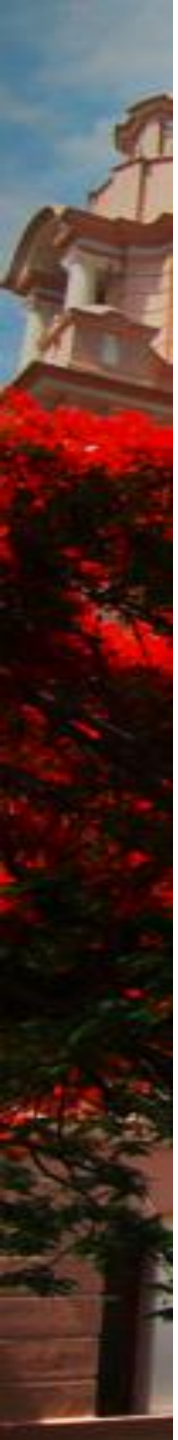
Revenue Management: K-Fashion Revisited

- Consider the tag price p_0 and the markdown policy:

$$p = p_0 - a \sum_{i=A,B,C} n_i \Big/ \left(\epsilon + \sum_{i=A,B,C} I_i \right)$$

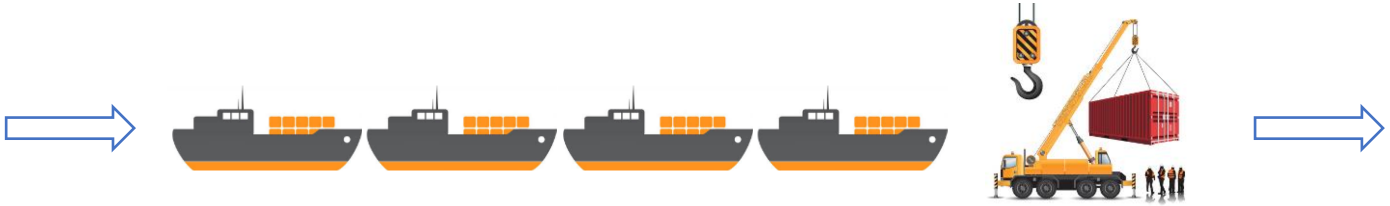
- n_i is the beginning inventory level of SKU i in the third month;
- $I_i = 0$ or 1, which indicates whether the SKU is available in the third month.

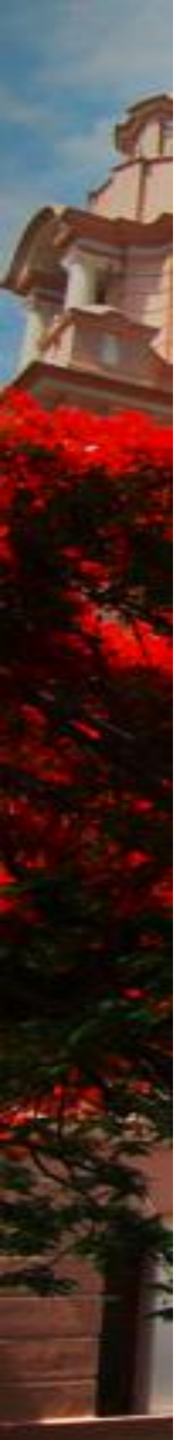
Month	Beginning Inventory A	Beginning Inventory B	Beginning Inventory C	Price	Customer Arrival	Interested	Target A	Target B	Target C	Buy A	Buy B	Buy C	Sales Value
1	10	10	10	869	3059	129	36	52	41	9	5	3	14773
2	1	5	7	869	4313	159	59	51	49	1	4	2	6083
3	0	1	5	543	8386	268	91	84	93	0	1	5	3258
1	10	10	10	869	5263	179	60	57	62	6	8	9	19987
…	…	…	…	…	…	…	…	…	…	…	…	…	…



Service Management: Port of New Orleans

- Fully loaded barges arrive at night for unloading
- The number of barges each night varies from 0 - 5
- The number of barges vary from day to day
- Barges are unloaded first-in, first-out
- The unloading rate also varies from day to day
- Barges must wait for unloading, which is expensive
- The dock manager wants to do a simulation study to enable him to make better staffing decisions



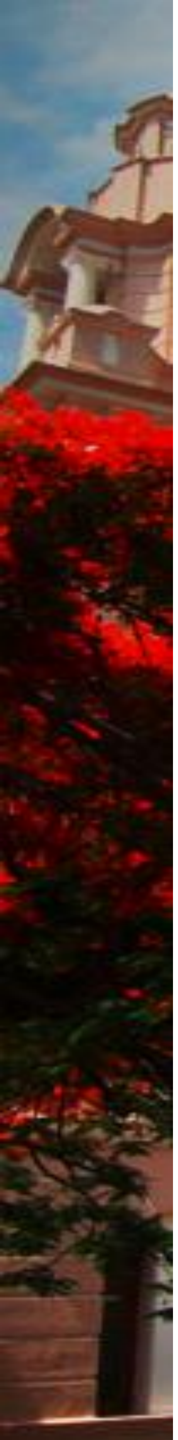


Service Management: Port of New Orleans

- Historical data suggests the following distributions:

NUMBER OF ARRIVALS	PROBABILITY	CUMULATIVE PROBABILITY
0	0.13	0.13
1	0.17	0.30
2	0.15	0.45
3	0.25	0.70
4	0.20	0.90
5	0.10	1.00

DAILY UNLOADING RATE	PROBABILITY	CUMULATIVE PROBABILITY
1	0.05	0.05
2	0.15	0.20
3	0.50	0.70
4	0.20	0.90
5	0.10	1.00

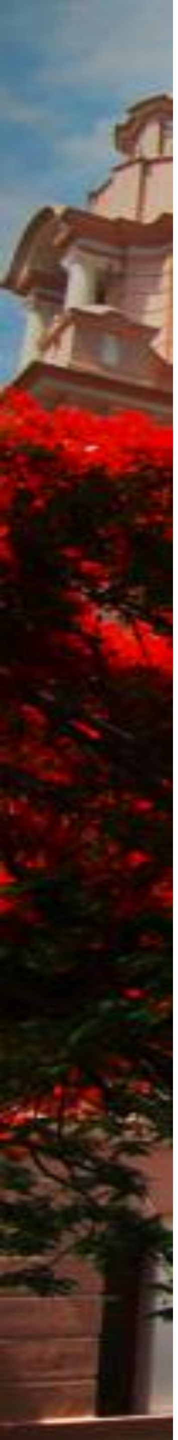


Service Management: Port of New Orleans

DAY	NUMBER DELAYED FROM PREVIOUS DAY	RANDOM NUMBER	NUMBER OF NIGHTLY ARRIVALS	TOTAL TO BE UNLOADED	RANDOM NUMBER	Unloading Rate	NUMBER UNLOADED
1	—	0.52	3	3	0.37	3	3
2	0	0.06	0	0	0.63	3	0
3	0	0.50	3	3	0.28	3	3
4	0	0.88	4	4	0.02	1	1
5	3	0.53	3	6	0.74	4	4
6	2	0.30	1	3	0.35	3	3
7	0	0.10	0	0	0.24	3	0
8	0	0.47	3	3	0.03	1	1
9	2	0.99	5	7	0.29	3	3
10	4	0.37	2	6	0.60	3	3
11	3	0.66	3	6	0.74	4	4
12	2	0.91	5	7	0.85	4	4
13	3	0.35	2	5	0.90	4	4
14	1	0.32	2	3	0.73	4	3
15	0	0.00	5	5	0.59	3	3
	20		41				39
	Total delays		Total arrivals				Total unloaded

Service Management: Pinevalley Bank

- The X branch of Pinevalley Bank has four service counters, but sometimes it is not necessary to open all four counters. Historical data shows that customers arrive every 10 minutes on average between 2 to 3 p.m. on a Thursday afternoon. The inter-arrival time follows exponential distribution. The service time for each customer also follows exponential distribution with a mean of 5 minutes. The question is, in order to make sure the average waiting time for each customer is less than 3 minutes, what is the minimum number of counters needed during this period of time?



Service Management: Pinevalley Bank

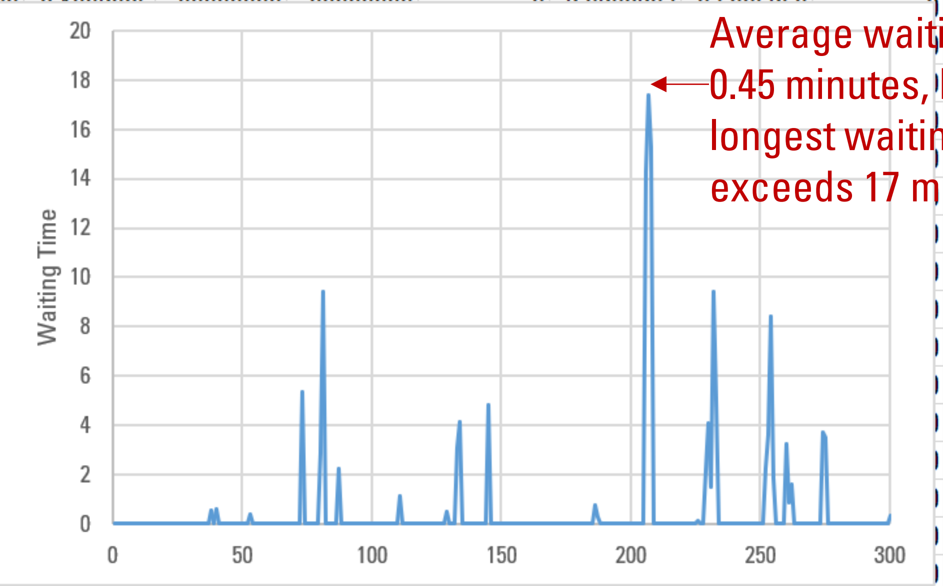
- We first manually simulate the process of customer arrival and service (assuming that two counters are open):

Customer No.	Arrival Interval (Minute)	Arrival Time (Minute)	Counter 1 Available From (Minute)	Counter 2 Available From (Minute)	Assigned Counter	Service Time (Minute)	Service Complete (Minute)	Waiting Time (Minute)
1	6	6	0	0	1	8	14	0
2	9	15	14	0	2	3	18	0
3	1	16	14	18	1	7	23	0
4	1	17	23	18	2	4	22	1
5	7	24	23	22	2	3	27	0
6	12	36	23	27	1	4	40	0

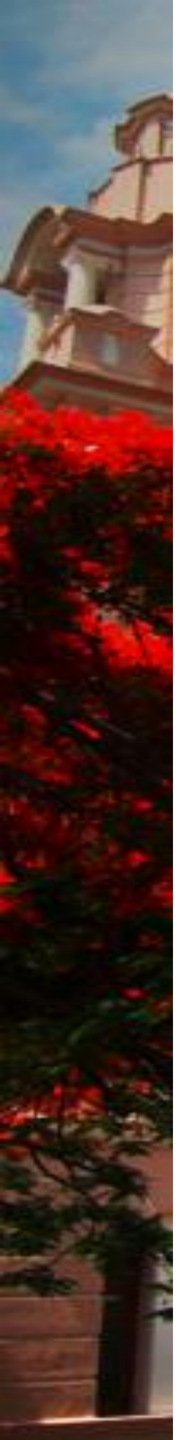
Service Management: Pinevalley Bank

Counters not available will be available at an infinitely far time point.

	A	B	C	D	E	F	G	H	I	J	K
1	Arrival Interval		Service Time			Counter 1	Counter 2	Counter 3	Counter 4		Avg. Waiting
2	Mean	10	Mean	5		1	1	0	0		0.4489164
3											
4											
5	Cust. No.	Interval	Arrival	C1 Avail.	C2 Avail.	C3 Avail.	C4 Avail.	Assigned	Service T.	Completion	Waiting T.
6	1	1.0611212	1.0611212	0	0	99999999	99999999	1	2.6404352	3.7015564	0
7	2	2.0534632	3.1145844	3.7015564	0	99999999	99999999	2	3.0848121	6.1993965	0
8	3	10.302417	13.417002	3.7015564	6.1993965	99999999	99999999	1	14.409288	27.82629	0
9	4	11.269725	24.686726	27.826							
10	5	18.717532	43.404258	27.826							
11	6	22.130622	65.53488	47.3434							
12	7	4.8181124	70.352993	47.3434							
13	8	8.0492574	78.40225	70.8097							
14	9	3.8245924	82.226843	70.8097							
15	10	6.4709148	88.697757	82.9886							
16	11	11.897114	100.59487	82.9886							
17	12	4.4088432	105.00371	104.435							
18	13	0.890287	105.894	104.435							
19	14	27.258104	133.15211	109.366							
20	15	5.2159821	138.36809	109.366							
21	16	12.589318	150.95741	140.023							
22	17	26.687014	177.64442	140.023							
23	18	0.5885117	178.23293	179.019							
24	19	1.2409604	179.47389	179.019							
25	20	16.670045	196.14394	183.91262	179.15221	99999999	99999999	2	11.99618	208.14012	0



Average waiting time 0.45 minutes, but the longest waiting time exceeds 17 minutes.

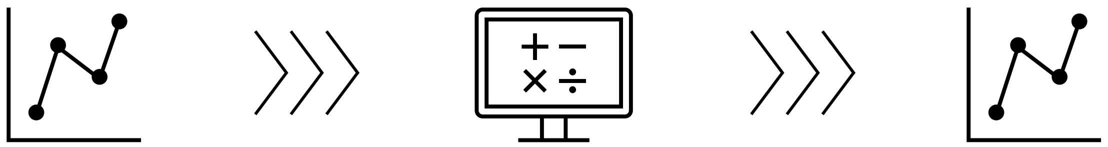


Verification and Validation

- It is important that a simulation model be checked to see that it is working properly and providing good representation of the real-world situation
- *Verification* process involves determining that the computer model is internally consistent and following the logic of the conceptual model
  - Verification answers the question “Did we build the model right?”
- *Validation* is the process of comparing a simulation model to the real system it represents to make sure it is accurate
  - Validation answers the question “Did we build the right model?”

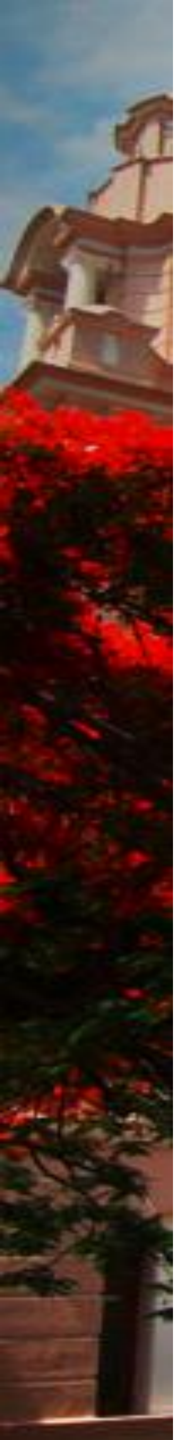
From Data to Model

- In general, this is a difficult task.
- For a random variable, you may consider different probability models and use data and Bayesian inference to compute the posterior.
- Then you may generate a random value from a model according to the posterior probability.
- Or, you may simply do random draws from historical data.



Potential Problems in the Data

- We may encounter two types of problems when we use historical data to build simulation models.
- I. Data truncation.
  - The bias caused by incomplete observation. For example, the observed daily sales quantity of an ice-cream may be less than the true demand due to stockouts. For another example, the observed arrival rate of hourly parking customers may be less than the true demand rate due to the carpark closure when it is full.
- II. Endogeneity.
  - The bias caused by non-random decisions. For example, apparel shops tend to mark down their prices during high traffic periods, which could create an illusion of high price sensitivity among the customers. For another example, automakers and consumer electronics manufacturers would like to cyclically introduce new products, which creates a cyclic pattern of demand.



Quiz

- ***True or False?***
  - Ken is good at basketball. For 3-pointers, his chance of success is 0.85; for 2-pointers, his chance of success is 0.95. In a training session, he is going to have 20 shots. He will start with a 2-pointer. Whenever he succeeds with a shot, he will try a 3-pointer in the next shot; whenever he fails a shot, he will try a 2-pointer in the next shot. The probability of achieving at least 40 points in this round of training is at least 70%.