

To begin, let's look at the simplest possible model of virulence evolution: an SI model with a classic trade-off between transmission rate and virulence. A quantitative genetics model for virulence evolution is the following:

$$\frac{dS}{dt} = b(S + I) - \beta(v)SI - mS \quad (1)$$

$$\frac{dI}{dt} = \beta(v)SI - (m + v)I \quad (2)$$

$$\frac{dv}{dt} = V \left( \frac{d}{dv} \left( \frac{1}{I} \frac{dI}{dt} \right) \right) = V \left( \frac{\beta_0 h S}{(h + v)^2} - 1 \right) \quad (3)$$

$$\beta(v) = \frac{\beta_0 v}{h + v} \quad (4)$$

This model has