Introductory Statistics with R

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```r
# To submit a line of code, place your cursor in the line and hit Ctrl + Enter
# (Win/Linux) or Cmd + Enter (Mac)


# load packages -------------------------------------------------------------

library(tidyverse)
```

```
## -- Attaching packages --------------------------------------------- tidyverse 1.3.0 --
## v ggplot2 3.2.1     v purrr   0.3.3
## v tibble  2.1.3     v dplyr   0.8.3
## v tidyr   1.0.2     v stringr 1.4.0
## v readr   1.3.1     v forcats 0.4.0
## -- Conflicts ------------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
```

```r
library(binom)

# Supress the numerous start up messages these two packages produce
suppressPackageStartupMessages(library(Hmisc))
suppressPackageStartupMessages(library(mosaic))


# Getting data into R -------------------------------------------------------

# Results from the US Census American Community Survey, 2012.
# https://www.census.gov/programs-surveys/acs
# From the openintro package

# Import the acs12.csv file; the result is a data frame
acs12 <- read.csv("https://github.com/clayford/IntroStatsR/raw/master/data/acs12.csv")

# str() shows us the structure of the data frame
str(acs12)
```

```
## 'data.frame':    2000 obs. of  13 variables:
##  $ income      : int  60000 0 NA 0 0 1700 NA NA NA 45000 ...
##  $ employment  : Factor w/ 3 levels "employed","not in labor force",..: 2 2 NA 2 2 1 NA NA NA 1 ...
##  $ hrs_work    : int  40 NA NA NA NA 40 NA NA NA 84 ...
##  $ race        : Factor w/ 4 levels "asian","black",..: 4 4 4 4 4 3 4 3 1 4 ...
##  $ age         : int  68 88 12 17 77 35 11 7 6 27 ...
```

```
##  $ gender      : Factor w/ 2 levels "female","male": 1 2 1 2 1 1 2 2 2 2 ...
##  $ citizen     : Factor w/ 2 levels "no","yes": 2 2 2 2 2 2 2 2 2 2 ...
##  $ time_to_work: int  NA NA NA NA NA 15 NA NA NA 40 ...
##  $ lang        : Factor w/ 2 levels "english","other": 1 1 1 2 2 2 1 1 2 1 ...
##  $ married     : Factor w/ 2 levels "no","yes": 1 1 1 1 1 2 1 1 1 2 ...
##  $ edu         : Factor w/ 3 levels "college","grad",..: 1 3 3 3 3 3 3 3 3 3 ...
##  $ disability  : Factor w/ 2 levels "no","yes": 1 2 1 1 2 2 1 2 1 1 ...
##  $ birth_qrtr  : Factor w/ 4 levels "apr thru jun",..: 3 2 4 4 3 3 4 3 2 4 ...
```

```
# NOTE: Factors are integers with character labels. Good to think of them as
# categorical variables. By default the read.csv() function automatically
# converts all columns containing text into factors.
#
# Can also click on the name in the Environment window to view the data.
#
# The summary function will provide summaries for all columns in a data frame.
# NA means "Not available", or missing. May not be useful for data with many
# columns.
summary(acs12)
```

```
##      income                   employment     hrs_work        race
##  Min.   :     0   employed         :843   Min.   : 1.00   asian:  87
##  1st Qu.:     0   not in labor force:656   1st Qu.:32.00   black: 206
##  Median :  3000   unemployed       :106   Median :40.00   other: 152
##  Mean   : 23600   NA's             :395   Mean   :37.98   white:1555
##  3rd Qu.: 33700                           3rd Qu.:40.00
##  Max.   :450000                           Max.   :99.00
##  NA's   :377                              NA's   :1041
##       age            gender      citizen     time_to_work       lang
##  Min.   : 0.00   female: 969   no : 118   Min.   :  1     english:1527
##  1st Qu.:19.75   male  :1031   yes:1882   1st Qu.: 10     other  : 368
##  Median :40.00                            Median : 20     NA's   : 105
##  Mean   :40.22                            Mean   : 26
##  3rd Qu.:59.00                            3rd Qu.: 30
##  Max.   :94.00                            Max.   :163
##                                           NA's   :1217
##  married             edu        disability        birth_qrtr
##  no :1167   college    : 359   no :1676   apr thru jun:479
##  yes: 833   grad       : 144   yes: 324   jan thru mar:485
##             hs or lower:1439              jul thru sep:504
##             NA's       :  58              oct thru dec:532
##
##
##
```

```
# What am I typically looking for?
# - Unusual values (too low, too high)
# - missing data (NA's)
# - big differences between mean and median (skewed data?)
# - consistent Factor level names ("male" vs "Male")
# - order of Factor levels
# - excess zeroes, or some other value
# - wrong data types (numbers stored as Factors, etc)
#
# Another option to get all summaries is the describe function from the Hmisc
```

```r
# package
Hmisc::describe(acs12)
```

```
## acs12
##
##  13  Variables      2000  Observations
## ---------------------------------------------------------------------------------
## income
##        n  missing distinct     Info     Mean      Gmd      .05      .10
##     1623      377      266    0.909    23600    35276        0        0
##      .25      .50      .75      .90      .95
##        0     3000    33700    64000    92000
##
## lowest :      0     50    100    110    130, highest: 345000 360000 398000 399000 450000
## ---------------------------------------------------------------------------------
## employment
##        n  missing distinct
##     1605      395        3
##
## Value                 employed not in labor force         unemployed
## Frequency                  843                656                106
## Proportion               0.525              0.409              0.066
## ---------------------------------------------------------------------------------
## hrs_work
##        n  missing distinct     Info     Mean      Gmd      .05      .10
##      959     1041       55    0.919    37.98     13.8       10       20
##      .25      .50      .75      .90      .95
##       32       40       40       50       60
##
## lowest :  1  2  4  5  6, highest: 72 75 80 84 99
## ---------------------------------------------------------------------------------
## race
##        n  missing distinct
##     2000        0        4
##
## Value      asian black other white
## Frequency     87   206   152  1555
## Proportion 0.044 0.103 0.076 0.778
## ---------------------------------------------------------------------------------
## age
##        n  missing distinct     Info     Mean      Gmd      .05      .10
##     2000        0       95        1    40.22    27.24     4.00     8.00
##      .25      .50      .75      .90      .95
##    19.75    40.00    59.00    71.00    79.00
##
## lowest :  0  1  2  3  4, highest: 90 91 92 93 94
## ---------------------------------------------------------------------------------
## gender
##        n  missing distinct
##     2000        0        2
##
## Value      female   male
## Frequency     969   1031
## Proportion  0.484  0.516
```

```
## ---------------------------------------------------------------------------------
## citizen
##        n  missing distinct
##     2000        0        2
##
## Value         no   yes
## Frequency    118  1882
## Proportion 0.059 0.941
## ---------------------------------------------------------------------------------
## time_to_work
##        n  missing distinct     Info     Mean      Gmd      .05      .10
##      783     1217       44    0.989       26    21.49        5        5
##      .25      .50      .75      .90      .95
##       10       20       30       50       60
##
## lowest :   1   2   3   4   5, highest: 145 148 152 157 163
## ---------------------------------------------------------------------------------
## lang
##        n  missing distinct
##     1895      105        2
##
## Value      english   other
## Frequency     1527     368
## Proportion   0.806   0.194
## ---------------------------------------------------------------------------------
## married
##        n  missing distinct
##     2000        0        2
##
## Value         no   yes
## Frequency   1167   833
## Proportion 0.584 0.416
## ---------------------------------------------------------------------------------
## edu
##        n  missing distinct
##     1942       58        3
##
## Value           college        grad hs or lower
## Frequency           359         144        1439
## Proportion        0.185       0.074       0.741
## ---------------------------------------------------------------------------------
## disability
##        n  missing distinct
##     2000        0        2
##
## Value         no   yes
## Frequency   1676   324
## Proportion 0.838 0.162
## ---------------------------------------------------------------------------------
## birth_qrtr
##        n  missing distinct
##     2000        0        4
##
## Value      apr thru jun jan thru mar jul thru sep oct thru dec
```

```
## Frequency            479          485          504          532
## Proportion         0.240        0.242        0.252        0.266
## ---------------------------------------------------------------------------
```

```r
# We can also use summary on individual columns. Use the "$" to access columns
# in a data frame.
summary(acs12$income)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's 
##       0       0    3000   23600   33700  450000     377
```

```r
summary(acs12$employment)
```

```
##           employed not in labor force         unemployed               NA's 
##                843                656                106                395
```

```r
# Notice that RStudio allows autocompletion of column names after you type the
# "$".


# YOUR TURN #0 ------------------------------------------------------------

# Try summary on the time_to_work and edu columns, or any other columns of
# interest.




# Counts and Proportions --------------------------------------------------

# Counts - the most fundamental statistic

# The table() function generates counts of unique values for a given vector,
# such as a column in a data frame. By default is excludes missing data.
table(acs12$employment)
```

```
## 
##           employed not in labor force         unemployed 
##                843                656                106
```

```r
# To see if there are missing data, set exclude = NULL:
table(acs12$employment, exclude = NULL)
```

```
## 
##           employed not in labor force         unemployed               <NA> 
##                843                656                106                395
```

```r
# Can also exclude other values, for example
table(acs12$employment, exclude = "not in labor force")
```

```
## 
##   employed unemployed       <NA> 
##        843        106        395
```

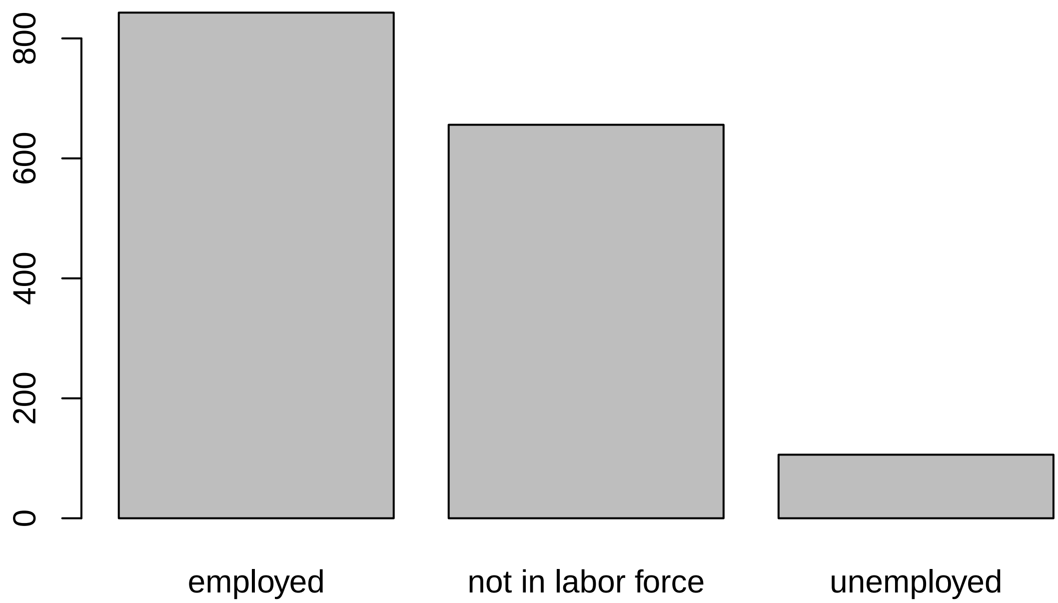
```r
table(acs12$employment, exclude = c("not in labor force", NA))
```

```
## 
##   employed unemployed 
##        843        106
```

```r
# summary() returns NAs by default (for Factors)
summary(acs12$employment)
```

```
##           employed not in labor force         unemployed               NA's
##                843                656                106                395
```

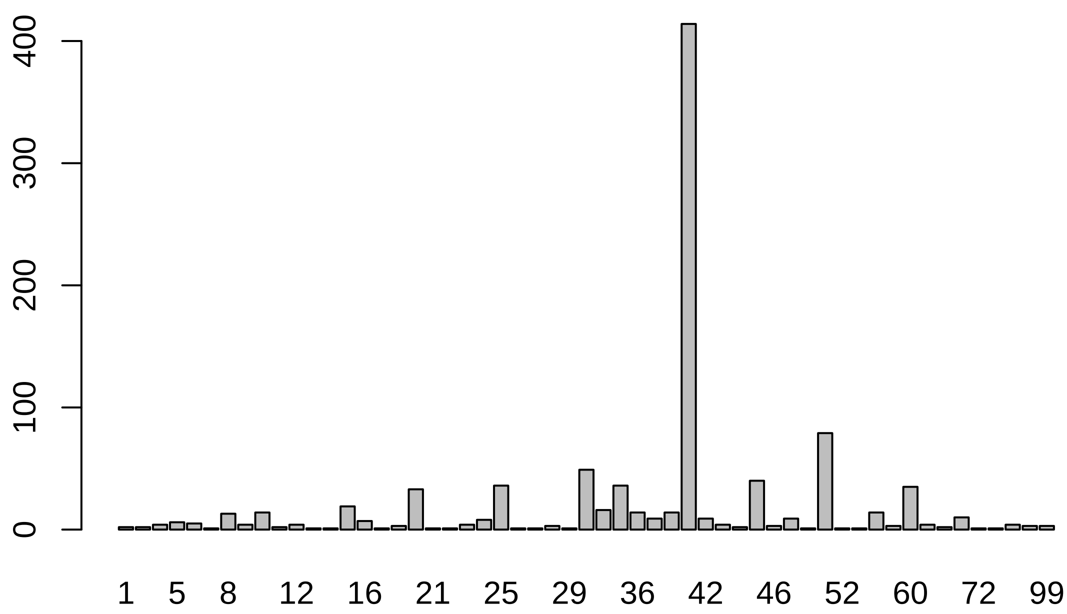
```r
# quick way to visualize counts, as long as the column is a Factor:
plot(acs12$employment)
```




```r
# Or we can use barplot() with table(); table() will count anything that can be
# converted to a factor (integers, numeric, character).

# counts of subjects by number of hours they work each week
table(acs12$hrs_work)
```

```
##
##   1   2   4   5   6   7   8   9  10  11  12  13  14  15  16  17  18  20  21  22
##   2   2   4   6   5   1  13   4  14   2   4   1   1  19   7   1   3  33   1   1
##  23  24  25  26  27  28  29  30  32  35  36  37  38  40  42  43  44  45  46  48
##   4   8  36   1   1   3   1  49  16  36  14   9  14 414   9   4   2  40   3   9
##  49  50  52  53  55  56  60  65  68  70  72  75  80  84  99
##   1  79   1   1  14   3  35   4   2  10   1   1   4   3   3
```

```r
barplot(table(acs12$hrs_work))
```




```
# We can also count things satisfying a condition.
#
#  ==  EQUAL
#  !=  NOT EQUALS
#  >   GREATER THAN
#  <   LESS THAN
#  >=  GREATER THAN OR EQUAL
#  <=  LESS THAN OR EQUAL

# How many people are age 60?
sum(acs12$age == 60)
```

```
## [1] 28
```

```
# How many people are over age 60
sum(acs12$age > 60)
```

```
## [1] 466
```

```
# How many people commute more than 60 minutes to work?
sum(acs12$time_to_work > 60)
```

```
## [1] NA
```

```
# In certain cases, R does not skip missing values unless told to do so.
# Set na.rm = TRUE
sum(acs12$time_to_work > 60, na.rm = TRUE)
```

```
## [1] 36
```

```r
# How many people do NOT have employment = "employed"
sum(acs12$employment != "employed", na.rm=TRUE)
```

```
## [1] 762
```

```r
# Combine conditions
#    & (AND)
#    | (OR)
#
# How many married AND work more than 50 hours
sum(acs12$married == "yes" & acs12$hrs_work > 50, na.rm = TRUE)
```

```
## [1] 59
```

```r
# How many under age 18 OR over age 65
sum(acs12$age < 18 | acs12$age > 65)
```

```
## [1] 759
```

```r
# Proportions
#
# What proportion of people are over age 60? The mean of 0s and 1s is the
# proportion of 1s.
mean(acs12$age > 60)
```

```
## [1] 0.233
```

```r
# What proportion of people commute over 60 minutes to work?
mean(acs12$time_to_work > 60, na.rm = TRUE)
```

```
## [1] 0.04597701
```

```r
# Use prop.table to get proportions from tables
#
# NOTE: these are proportions of non-missing!
prop.table(table(acs12$employment))
```

```
##
##           employed not in labor force         unemployed
##         0.52523364         0.40872274         0.06604361
```

```r
# Notice the difference if we include the missings
prop.table(table(acs12$employment, exclude = NULL))
```

```
##
##           employed not in labor force         unemployed               <NA>
##             0.4215             0.3280             0.0530             0.1975
```

```r
# or using summary
prop.table(summary(acs12$employment))
```

```
##           employed not in labor force         unemployed               NA's
##             0.4215             0.3280             0.0530             0.1975
```

```r
# Quick note about pipes. If the dplyr package is loaded, we can use pipes
# instead of nesting functions. For example:
table(acs12$employment) %>% prop.table()
```

```
##
##           employed not in labor force         unemployed
```

```
##         0.52523364         0.40872274         0.06604361
```

```r
# Use Ctrl + Shift + M or Cmd + Shift + M to enter %>%
#
# Pipes take the output of one function and feed it to the first argument of the
# next function.

table(acs12$employment) %>% prop.table() %>% round(2)
```

```
##
##           employed not in labor force         unemployed
##               0.53               0.41               0.07
```

```r
# Confidence intervals of proportions
#
# These are just estimates. How certain are we? Another sample would yield
# slightly different results.
#
# Confidence intervals help us quantify the uncertainty. They provide an
# estimated lower and upper bound of our estimate.
#
# Example: how certain are we about 0.53 employed?
#
# The prop.test() function returns a 95% confidence interval for an estimated
# proportion.
#
# 95% Confidence Interval theory: sample the data, calculate a 95% confidence
# interval, repeat many times. About 95% of confidence intervals will contain
# the "true" value you're estimating. See Appendix for a demo.
#
# The prop.test() function requires number of "successes" (ie, number employed)
# and total number of "trials" (ie, number of respondents).

# Number of Employed: 843
sum(acs12$employment == "employed", na.rm = TRUE)
```

```
## [1] 843
```

```r
# Total number who responded (ie, total not missing): 1605
sum(table(acs12$employment))
```

```
## [1] 1605
```

```r
# x = number of "successes", n = number of "trials"
prop.test(x = 843, n = 1605)
```

```
##
##  1-sample proportions test with continuity correction
##
## data:  843 out of 1605
## X-squared = 3.9875, df = 1, p-value = 0.04584
## alternative hypothesis: true p is not equal to 0.5
## 95 percent confidence interval:
##  0.5004608 0.5498845
## sample estimates:
##         p
## 0.5252336
```

```r
# 95 percent confidence interval: (0.50, 0.55)
#
# can save into an object
p.out <- prop.test(x = 843, n = 1605)

# access the confidence interval
p.out$conf.int
```

```
## [1] 0.5004608 0.5498845
## attr(,"conf.level")
## [1] 0.95
```

```r
# The mosaic version of prop.test allows us to do the following:
mosaic::prop.test(acs12$employment == "employed")
```

```
##
##  1-sample proportions test with continuity correction
##
## data:  acs12$employment == "employed"  [with success = TRUE]
## X-squared = 3.9875, df = 1, p-value = 0.04584
## alternative hypothesis: true p is not equal to 0.5
## 95 percent confidence interval:
##  0.5004608 0.5498845
## sample estimates:
##         p
## 0.5252336
```

```r
# Note: placing the package name and two colons before a function tells R to use
# the function in that package.
#
# The prop.test function in the stats package that comes with R cannot do this.
# stats::prop.test(acs12$employment == "employed")
#
# The binom.confint() function from the binom package allows us to get
# confidence intervals for all three employment levels. method = "prop.test"
# returns traditional confidence intervals
binom.confint(x = table(acs12$employment),
              n = sum(table(acs12$employment)),
              method = "prop.test")
```

```
##      method   x    n       mean      lower      upper
## 1 prop.test 843 1605 0.52523364 0.50046077 0.54988448
## 2 prop.test 656 1605 0.40872274 0.38461017 0.43327685
## 3 prop.test 106 1605 0.06604361 0.05461547 0.07959697
```

```r
# The binom.confint() function provides a 11 different methods for calculating
# CIs. Why use other methods? When proportion estimate is near boundary (ie, 0
# or 1). Notice these are equal out to 3 decimal places.
binom.confint(x = 843,n = 1605)
```

```
##           method   x    n      mean     lower     upper
## 1  agresti-coull 843 1605 0.5252336 0.5007722 0.5495745
## 2     asymptotic 843 1605 0.5252336 0.5008035 0.5496638
## 3          bayes 843 1605 0.5252179 0.5007974 0.5496209
## 4        cloglog 843 1605 0.5252336 0.5005007 0.5493391
## 5          exact 843 1605 0.5252336 0.5004647 0.5499102
```

```
## 6          logit 843 1605 0.5252336 0.5007625 0.5495842
## 7         probit 843 1605 0.5252336 0.5007713 0.5496012
## 8        profile 843 1605 0.5252336 0.5007778 0.5496090
## 9            lrt 843 1605 0.5252336 0.5007827 0.5496030
## 10     prop.test 843 1605 0.5252336 0.5004608 0.5498845
## 11        wilson 843 1605 0.5252336 0.5007723 0.5495745
```

```
# Sometimes we derive proportions from numeric data. Example: proportion of
# people working more than 40 hours
mean(acs12$hrs_work > 40, na.rm = TRUE)
```

```
## [1] 0.2387904
```

```
# How certain are we about that proportion?
# The mosaic version of prop.test allows us to do the following:
mosaic::prop.test(acs12$hrs_work > 40)
```

```
##
##  1-sample proportions test with continuity correction
##
## data:  acs12$hrs_work > 40  [with success = TRUE]
## X-squared = 260.69, df = 1, p-value < 2.2e-16
## alternative hypothesis: true p is not equal to 0.5
## 95 percent confidence interval:
##  0.2123835 0.2673217
## sample estimates:
##         p
## 0.2387904
```

```
# 95 percent confidence interval: (0.21, 0.27)




# YOUR TURN #1 ------------------------------------------------------------

# (1) What proportion of ACS respondents are married?

# (2) As a population estimate, how certain is it? Calculate a confidence
# interval





# Means and Medians -------------------------------------------------------

# The mean and median are two measures of center.

# calling summary() on a numeric variable returns the mean and median
summary(acs12$hrs_work)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
##    1.00   32.00   40.00   37.98   40.00   99.00    1041
```

```
# The median is the middle of the sorted data
# The mean is the "balance point" of the data
```

```r
# Symmetric data have similar means and medians.


# The mean and median functions; if any data is missing the result is NA
mean(acs12$hrs_work)
```

```
## [1] NA
```

```r
median(acs12$hrs_work)
```

```
## [1] NA
```

```r
# specify na.rm = TRUE to ignore missing data
mean(acs12$hrs_work, na.rm = TRUE)
```

```
## [1] 37.97706
```

```r
median(acs12$hrs_work, na.rm = TRUE)
```

```
## [1] 40
```

```r
# How much data is missing? The is.na() function can help us answer this.
sum(is.na(acs12$hrs_work))   # count
```

```
## [1] 1041
```

```r
mean(is.na(acs12$hrs_work))  # proportion
```

```
## [1] 0.5205
```

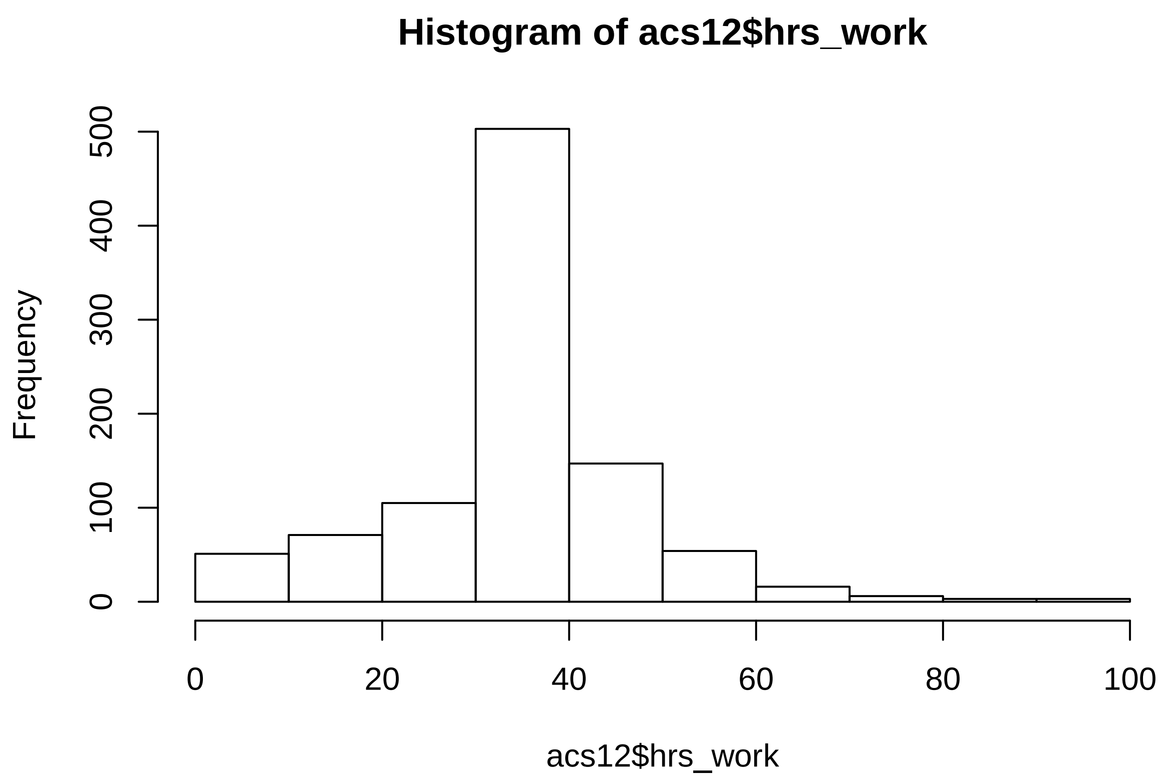
```r
# How much data is NOT missing? Precede is.na() with !
sum(!is.na(acs12$hrs_work))
```

```
## [1] 959
```

```r
mean(!is.na(acs12$hrs_work))
```

```
## [1] 0.4795
```

```r
# histograms are good to visualize the distribution of numeric variables
hist(acs12$hrs_work)
```




```
# Numeric measures of spread include the standard deviation...
sd(acs12$hrs_work, na.rm = TRUE)
```

```
## [1] 13.49768
```

```
# ...the Interquartile Range (difference between 75th and 25th quartiles)
IQR(acs12$hrs_work, na.rm = TRUE)
```

```
## [1] 8
```

```
# The IQR may be preferred with highly skewed data, but can always report both.

# Recall the estimated mean of hrs_work
mean(acs12$hrs_work, na.rm = TRUE)
```

```
## [1] 37.97706
```

```
# The mean of 37.977 is just an estimate. How certain are we about the mean?
# Another sample would yield slightly different results. The t.test() function
# returns a 95% confidence interval for a mean.
# Notice we do not need na.rm = TRUE.
t.test(acs12$hrs_work)
```

```
##
##  One Sample t-test
##
## data:  acs12$hrs_work
## t = 87.131, df = 958, p-value < 2.2e-16
## alternative hypothesis: true mean is not equal to 0
```

```
## 95 percent confidence interval:
##  37.12170 38.83242
## sample estimates:
## mean of x
##  37.97706
```

```
# CI: (37.1, 38.8)

# We can save into an object and access the confidence interval directly
t.out <- t.test(acs12$hrs_work)
t.out$conf.int
```

```
## [1] 37.12170 38.83242
## attr(,"conf.level")
## [1] 0.95
```

```
# The smean.cl.normal() function from the Hmisc package returns just a
# confidence interval:
Hmisc::smean.cl.normal(acs12$hrs_work)
```

```
##     Mean    Lower    Upper
## 37.97706 37.12170 38.83242
```

```
# Confidence intervals for medians are a little tricker.

# The smedian.hilow() function from the Hmisc package returns a confidence
# interval for the median. "computes the sample median and a selected pair of
# outer quantiles having equal tail areas."
Hmisc::smedian.hilow(acs12$hrs_work)
```

```
## Median  Lower  Upper
##  40.00   8.00  65.15
```

```
# The wilcox.test() function returns a CI for medians, with conf.int = TRUE.
# Note the estimate is a "pseudomedian".
wilcox.test(acs12$hrs_work, conf.int = TRUE)
```

```
##
##  Wilcoxon signed rank test with continuity correction
##
## data:  acs12$hrs_work
## V = 460320, p-value < 2.2e-16
## alternative hypothesis: true location is not equal to 0
## 95 percent confidence interval:
##  38.99996 39.99999
## sample estimates:
## (pseudo)median
##       39.99995
```

```
# Bootstrapping is another approach for estimating uncertainty and calculating
# confidence intervals. See Appendix below for more information on basic
# bootstrapping.



# YOUR TURN #2 ------------------------------------------------------------

# (1) What is the mean income of ACS respondents? As a population estimate, how
```

```r
# certain is it? Calculate a confidence interval.


# (2) How many respondents reported an income of 0?


# (3) What proportion of people reported earning more than $100,000? As a
# population estimate, how certain is it? Calculate a confidence interval.




# Comparing two proportions -----------------------------------------------

# We often want to compare two proportions.

# Example: Is there a difference between the proportion of people with
# disabilities between citizens and non-citizens?

# cross tabulation of citizenship and disability using table()
table(acs12$citizen, acs12$disability)
```

```
##
##          no  yes
##   no    105   13
##   yes  1571  311
```

```r
# can also use xtabs(); the table includes row and column labels
# read the "~" as "tabulate the data by"
xtabs(~ citizen + disability, data = acs12)
```

```
##        disability
## citizen   no  yes
##     no   105   13
##     yes 1571  311
```

```r
# prop.table() calculates proportions; margin = 1 means across rows; margin = 2
# means down the columns
xtabs(~ citizen + disability, data = acs12) %>%
  prop.table(margin = 1) %>%
  round(3)
```

```
##        disability
## citizen    no   yes
##     no  0.890 0.110
##     yes 0.835 0.165
```

```r
# About 0.11 of non-citizens have a disability. About 0.17 of citizens have a
# disability. That's a difference of about -0.06. How certain are we about that
# difference? Would another random sample result in a differece in the opposite
# direction?
#
# Can use prop.test() to answer this. The first argument x takes number of
# "successes" for each group (citizens and non-citizens); the second argument n
# takes number of people surveyed in each group (citizens and non-citizens)
prop.test(x = c(13, 311), n = c(105 + 13, 1571 + 311))
```

```
##
##  2-sample test for equality of proportions with continuity correction
##
## data:  c(13, 311) out of c(105 + 13, 1571 + 311)
## X-squared = 2.0923, df = 1, p-value = 0.148
## alternative hypothesis: two.sided
## 95 percent confidence interval:
##  -0.118515145  0.008354659
## sample estimates:
##    prop 1    prop 2
## 0.1101695 0.1652497
```

```
# The p-value says there's about a 15% chance of getting a difference this big,
# or bigger, if there really is no difference between the proportions.
#
# Traditionally, we judge a difference "significant", or not due to chance
# alone, if the p-value is small, say below 0.05.
#
# In this case, we might conclude:
#
# - "with the current sample size the data were unable to overcome the
#    supposition of no difference in the proportions"
#
# The 95 percent confidence interval is on the difference of proportions. Since
# it overlaps 0 (barely) we're uncertain about the direction of the difference.
#
#
# To avoid hard-coding numbers, we can use the mosaic version which allows us to
# use formula notation to specify we want to compare the proportion of people
# with disabilities by citizen. Notice we need to specify success = "yes", which
# means we want to compare proportion of people who answered "yes" to
# disability.
mosaic::prop.test(disability ~ citizen, data = acs12, success = "yes")
```

```
##
##  2-sample test for equality of proportions with continuity correction
##
## data:  tally(disability ~ citizen)
## X-squared = 2.0923, df = 1, p-value = 0.148
## alternative hypothesis: two.sided
## 95 percent confidence interval:
##  -0.118515145  0.008354659
## sample estimates:
##    prop 1    prop 2
## 0.1101695 0.1652497
```

```
# A slightly more complicated example...
#
# cross tabulation of citizenship and education
xtabs(~ citizen + edu, data = acs12)
```

```
##        edu
## citizen college grad hs or lower
##     no       15    8          94
##     yes     344  136        1345
```

```r
# proportion of education level by citizenship
xtabs(~ citizen + edu, data = acs12) %>%
  prop.table(margin = 1) %>%
  round(3)
```

```
##        edu
## citizen college  grad hs or lower
##     no    0.128 0.068       0.803
##     yes   0.188 0.075       0.737
```

```r
# About 0.13 of non-citizens are college graduates. About 0.19 of citizens are
# college graduates. That's a difference of -0.06. How certain are we about that
# difference? Would another random sample result in a differece in the opposite
# direction?
#
# We can use prop.test() to answer this. The first argument x takes number of
# "successes"; the second argument n takes number of trials

# Number of success: 15, 344
xtabs(~ citizen + edu, data = acs12)
```

```
##        edu
## citizen college grad hs or lower
##     no       15    8          94
##     yes     344  136        1345
```

```r
# Number of trials: 117, 1825. The addmargins() function adds margin totals to a
# table in the specified dimension.
xtabs(~ citizen + edu, data = acs12) %>%
  addmargins(margin = 2)
```

```
##        edu
## citizen college grad hs or lower  Sum
##     no       15    8          94  117
##     yes     344  136        1345 1825
```

```r
# Use values with prop.test()
prop.test(x = c(15, 344), n = c(117, 1825))
```

```
##
##  2-sample test for equality of proportions with continuity correction
##
## data:  c(15, 344) out of c(117, 1825)
## X-squared = 2.2671, df = 1, p-value = 0.1321
## alternative hypothesis: two.sided
## 95 percent confidence interval:
##  -0.128015161  0.007439116
## sample estimates:
##    prop 1    prop 2
## 0.1282051 0.1884932
```

```r
# The p-value says there's about a 13% chance of getting a difference this big,
# or bigger, if there really is no difference between the proportions.
#
# The confidence level of the difference in proportions barely overlaps 0.
#
# To avoid entering numbers we can also save table with margins and use with
```

```r
# prop.test
tab <- xtabs(~ citizen + edu, data = acs12) %>%
  addmargins(margin = 2)
tab
```

```
##        edu
## citizen college grad hs or lower  Sum
##     no       15    8         94  117
##     yes     344  136       1345 1825
```

```r
tab[,"college"]
```

```
##  no yes
##  15 344
```

```r
tab[,"Sum"]
```

```
##   no  yes
##  117 1825
```

```r
prop.test(x = tab[,"college"], n = tab[,"Sum"])
```

```
##
##  2-sample test for equality of proportions with continuity correction
##
## data:  tab[, "college"] out of tab[, "Sum"]
## X-squared = 2.2671, df = 1, p-value = 0.1321
## alternative hypothesis: two.sided
## 95 percent confidence interval:
##  -0.128015161  0.007439116
## sample estimates:
##    prop 1    prop 2
## 0.1282051 0.1884932
```

```r
# YOUR TURN #3 ------------------------------------------------------------

# Compare the proportion of people with disabilities between male and female
# (gender). What is the confidence interval of the difference of proportions?

xtabs(~ gender + disability, data = acs12)
```

```
##         disability
## gender    no yes
##   female 817 152
##   male   859 172
```

```r
# Comparing two means -----------------------------------------------------

# We often want to compare means between two groups.

# Example: does mean income differ between gender?

# The aggregate() function allows us to calculate a specified statistic for
# groups.

# Example: Take income, group by gender, and calculate mean for each group
aggregate(income ~ gender, data = acs12, mean)
```

```
##   gender   income
## 1 female 14335.99
## 2   male 32627.30
```

```
# The mosaic version of mean allows a formula and "grouping" variable
mosaic::mean(~ income | gender, data = acs12, na.rm = TRUE)
```

```
##   female     male
## 14335.99 32627.30
```

```
# The mean income of females is about $14,335. The mean income of males is about
# $32,627. That's a difference of -$18,292. How certain are we about that
# difference? Would another random sample result in a differece in the opposite
# direction?

# The t.test() function can help us assess the difference.
t.test(income ~ gender, data = acs12)
```

```
##
##  Welch Two Sample t-test
##
## data:  income by gender
## t = -8.1362, df = 1142.1, p-value = 1.056e-15
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -22702.24 -13880.38
## sample estimates:
## mean in group female   mean in group male
##             14335.99             32627.30
```

```
# The 95% CI on the difference is (-22702.24, -13880.38)
#
# The t-test is testing that the data from the two groups came from the same
# Normal distribution. The t-test is pretty robust to this assumption with large
# sample sizes, say greater than 30. In other words, we can trust the results
# even if the Normality assumption is suspect.
#
# The p-value is virtually 0. There is just about no chance we would see a
# difference in means this large or larger if the two groups came from the same
# normally distributed population.
#
# Save to an object and extract the CI of the difference in means
t.out <- t.test(income ~ gender, data = acs12)
t.out$conf.int
```

```
## [1] -22702.24 -13880.38
## attr(,"conf.level")
## [1] 0.95
```

```
# It's time for a closer look at the data:
# Proportion of zeros
mean(acs12$income == 0, na.rm=TRUE)
```

```
## [1] 0.4491682
```

```
# Proportion of zeros by gender
table(acs12$gender, acs12$income == 0) %>%
  prop.table(margin = 1) %>%
```

```r
  round(2)
```

```
##
##          FALSE TRUE
##   female  0.51 0.49
##   male    0.59 0.41
```

```r
# number of income == by employment, by gender
table(acs12$income == 0, acs12$employment, acs12$gender)
```

```
## , ,  = female
##
##
##         employed not in labor force unemployed
##   FALSE      347                 38         25
##   TRUE        26                335         22
##
## , ,  = male
##
##
##         employed not in labor force unemployed
##   FALSE      440                 19         25
##   TRUE        30                264         34
```

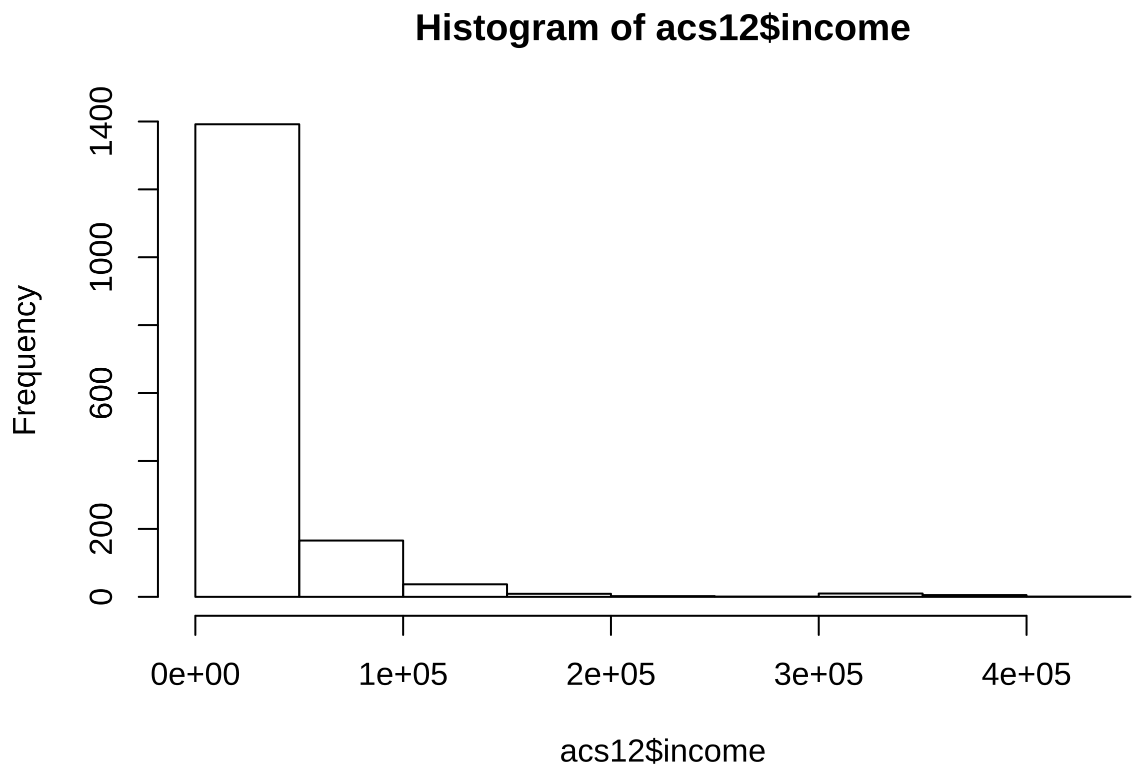
```r
# Maybe we should drop the zeroes? We can use the subset() function to only use
# rows for which income is greater than 0.
t.out <- t.test(income ~ gender, data = subset(acs12, income > 0))
t.out
```

```
##
##  Welch Two Sample t-test
##
## data:  income by gender
## t = -7.9637, df = 700.29, p-value = 6.754e-15
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -34161.16 -20648.53
## sample estimates:
## mean in group female   mean in group male
##             28007.63             55412.48
```

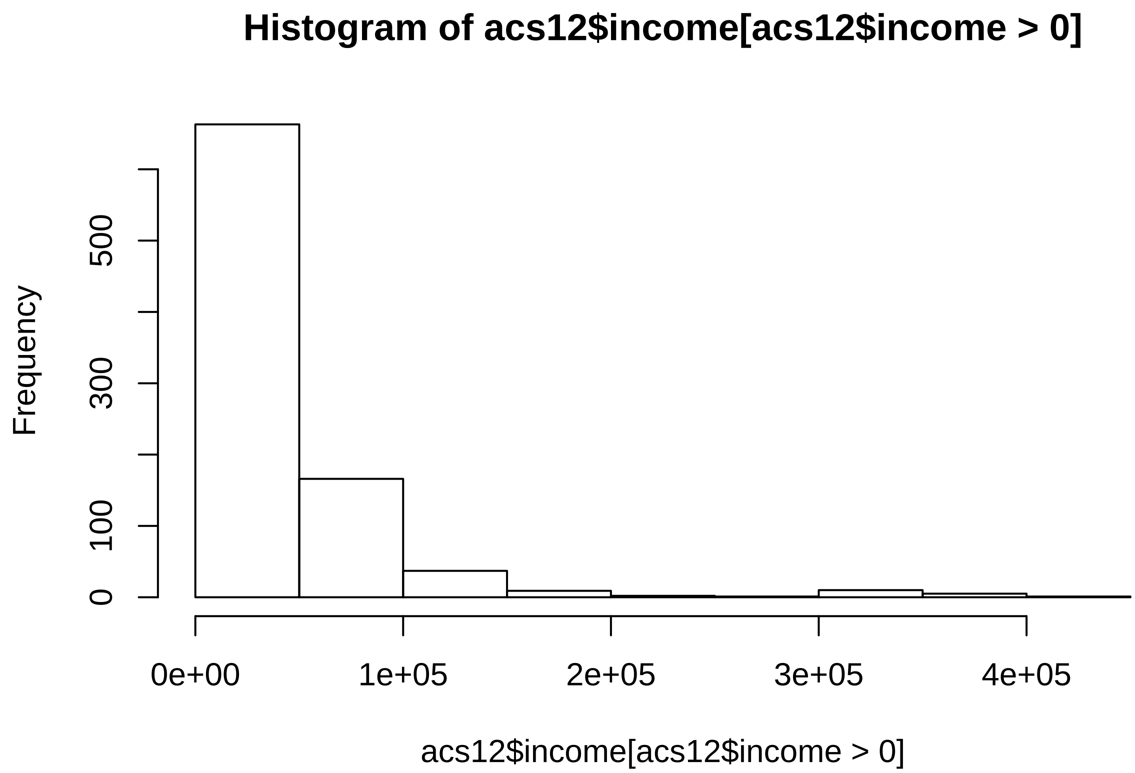
```r
t.out$conf.int
```

```
## [1] -34161.16 -20648.53
## attr(,"conf.level")
## [1] 0.95
```

```r
# The expected income difference for females is about (-$34,161, -$20,648) less
# than males.
#
# It's important to note the data is quite skew.
hist(acs12$income)
```

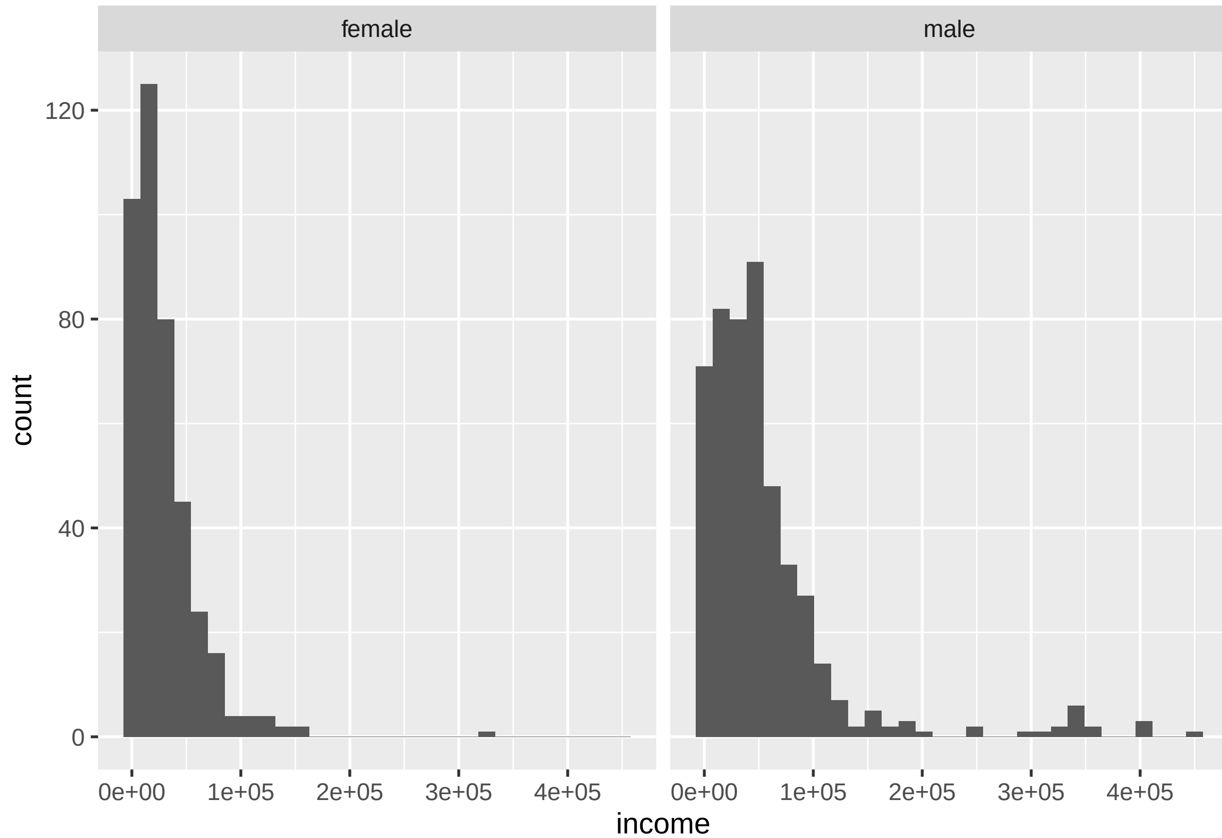



```r
hist(acs12$income[acs12$income > 0])
```

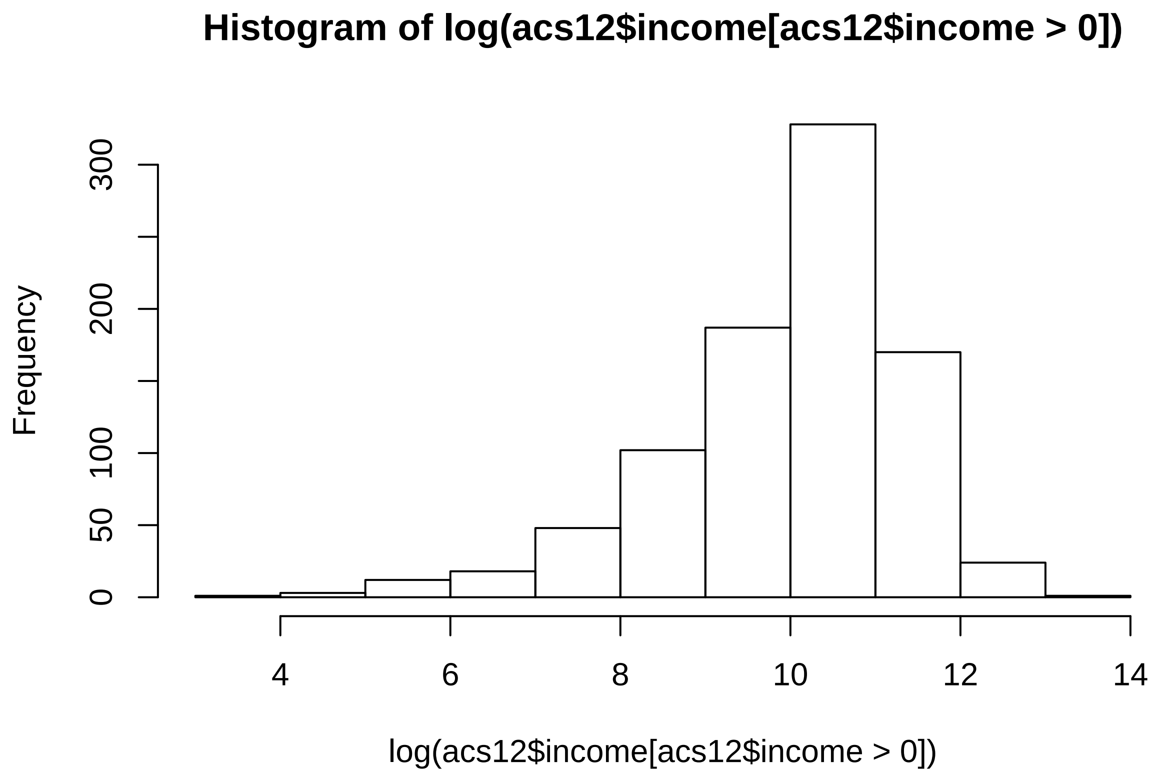



```r
# ggplot makes it pretty easy to see the histograms for both genders
ggplot(subset(acs12, income > 0), aes(x = income)) +
  geom_histogram() +
  facet_wrap(~gender)
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

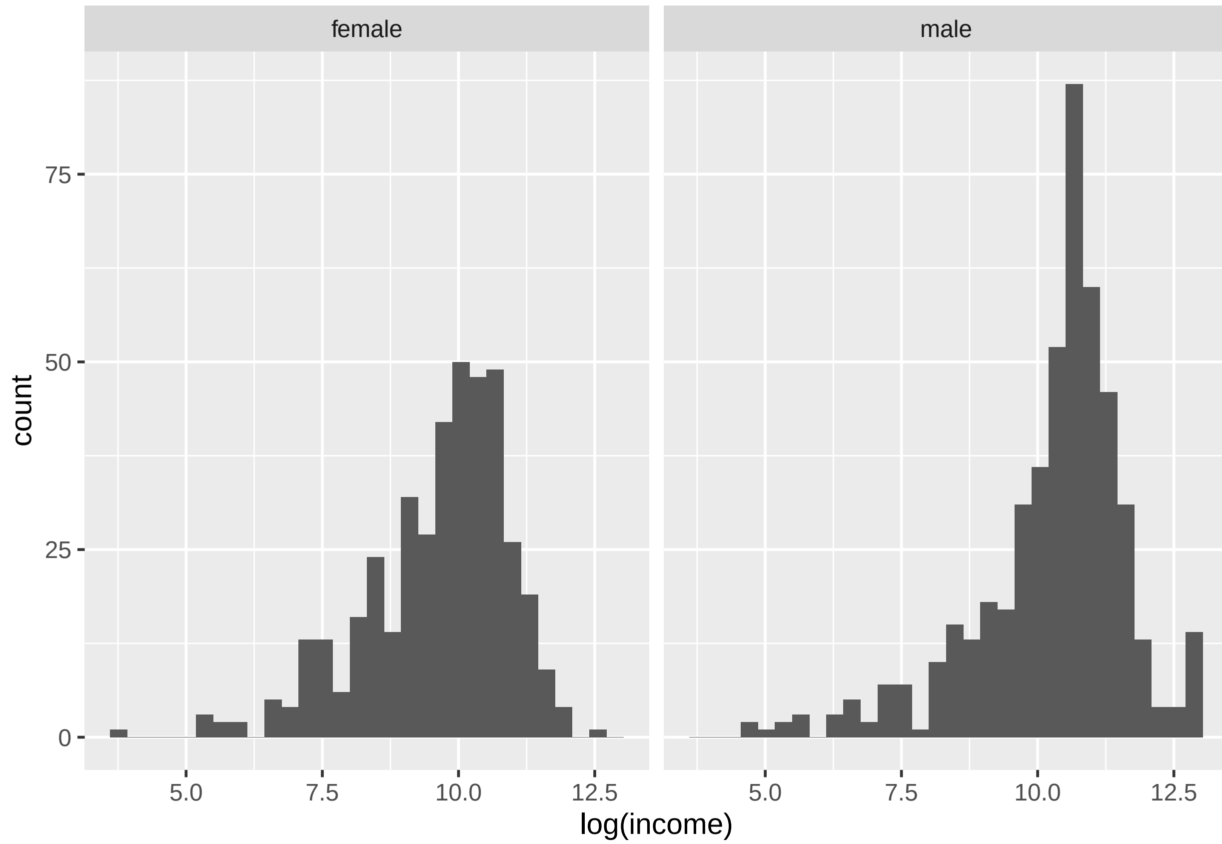



```r
# One alternative approach might be a log transformation. This is useful for
# skewed data such as income. NOTE: you can only log transform values greater
# than 0
#
# after log transformation; natural log (base e)
hist(log(acs12$income[acs12$income > 0]))
```




```r
# for both genders
ggplot(subset(acs12, income > 0), aes(x = log(income))) +
  geom_histogram() +
  facet_wrap(~gender)
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```




```r
# Now skewed the other direction...
#
# t-test for difference in log-transformed means
t.test(log(income) ~ gender, data = subset(acs12, income > 0))
```

```
##
##  Welch Two Sample t-test
##
## data:  log(income) by gender
## t = -7.1765, df = 879.47, p-value = 1.52e-12
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.8333371 -0.4754139
## sample estimates:
## mean in group female   mean in group male
##             9.613135            10.267511
```

```r
# The result is still quite large and signficant, but difficult to interpret. We
# need to exponentiate to get data on original scale. First let's save the
# result.
t.out <- t.test(log(income) ~ gender, data = subset(acs12, income > 0))
t.out
```

```
##
##  Welch Two Sample t-test
##
## data:  log(income) by gender
```

```
## t = -7.1765, df = 879.47, p-value = 1.52e-12
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.8333371 -0.4754139
## sample estimates:
## mean in group female   mean in group male
##             9.613135            10.267511
```

```
t.out$estimate
```

```
## mean in group female   mean in group male
##             9.613135            10.267511
```

```
# Now exponentiate the estimated means
exp(t.out$estimate)
```

```
## mean in group female   mean in group male
##             14960.00             28782.15
```

```
# exponentiating the confidence interval returns the multiplicative effect of
# gender on income.
t.out$conf.int
```

```
## [1] -0.8333371 -0.4754139
## attr(,"conf.level")
## [1] 0.95
```

```
exp(t.out$conf.int)
```

```
## [1] 0.4345966 0.6216277
## attr(,"conf.level")
## [1] 0.95
```

```
# females appear to earn about 0.43 to 0.62 times the income of males
#
# If you doubt the normality assumption, you can try a nonparametric test such
# as the wilcox.test. Instead of assuming the two means come from the same
# Normal distribution, we assume the two means just come from the same
# distribution.
wilcox.test(income ~ gender, data = subset(acs12, income > 0))
```

```
##
##  Wilcoxon rank sum test with continuity correction
##
## data:  income by gender
## W = 66680, p-value < 2.2e-16
## alternative hypothesis: true location shift is not equal to 0
```

```
# To get a confidence interval, set conf.int = TRUE
wilcox.test(income ~ gender, data = subset(acs12, income > 0),
            conf.int = TRUE)
```

```
##
##  Wilcoxon rank sum test with continuity correction
##
## data:  income by gender
## W = 66680, p-value < 2.2e-16
## alternative hypothesis: true location shift is not equal to 0
## 95 percent confidence interval:
```

```
##  -20000 -12000
## sample estimates:
## difference in location
##                 -16000
```

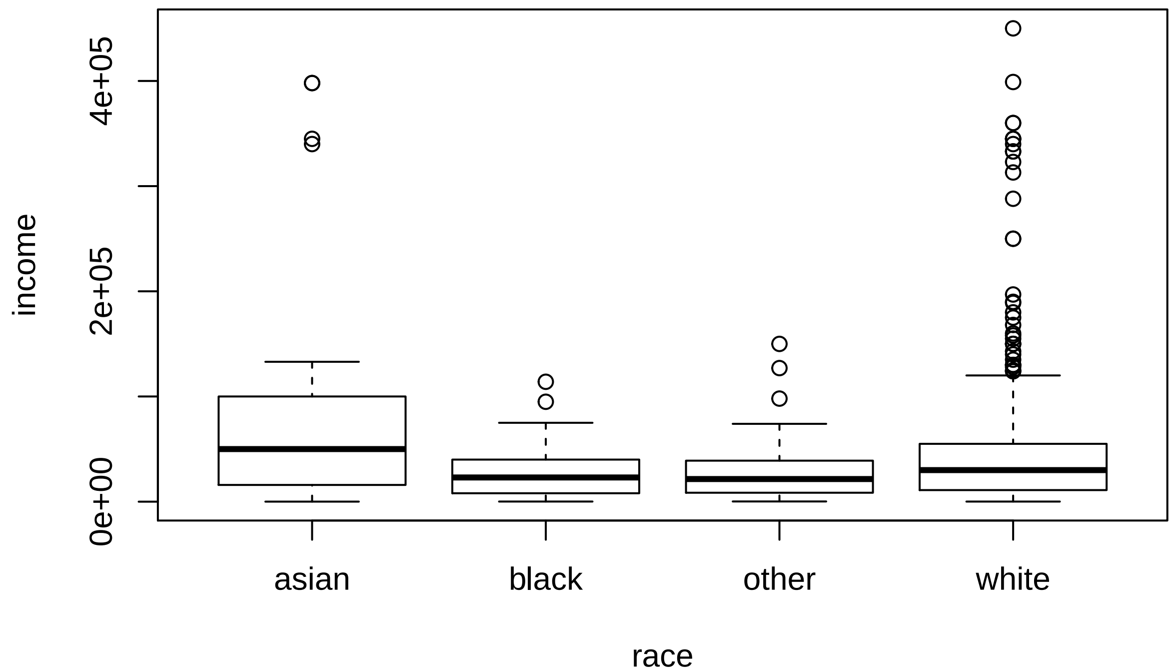
```r
# The "difference in location" estimates the median of the difference between a
# sample from x and a sample from y.


# YOUR TURN #4 ------------------------------------------------------------

# How does mean hrs_work differ between gender? What is the confidence interval
# of the difference?




# Comparing more than 2 means ---------------------------------------------

# We often want to compare means between more than two groups.
#
# Example: does mean income differ between race?
#
# Use aggregate to calculate mean income for each race. Let's drop the rows with
# 0 income.
aggregate(income ~ race, data = subset(acs12, income > 0), mean)
```

```
##    race   income
## 1 asian 79338.37
## 2 black 28003.84
## 3 other 28098.44
## 4 white 43772.60
```

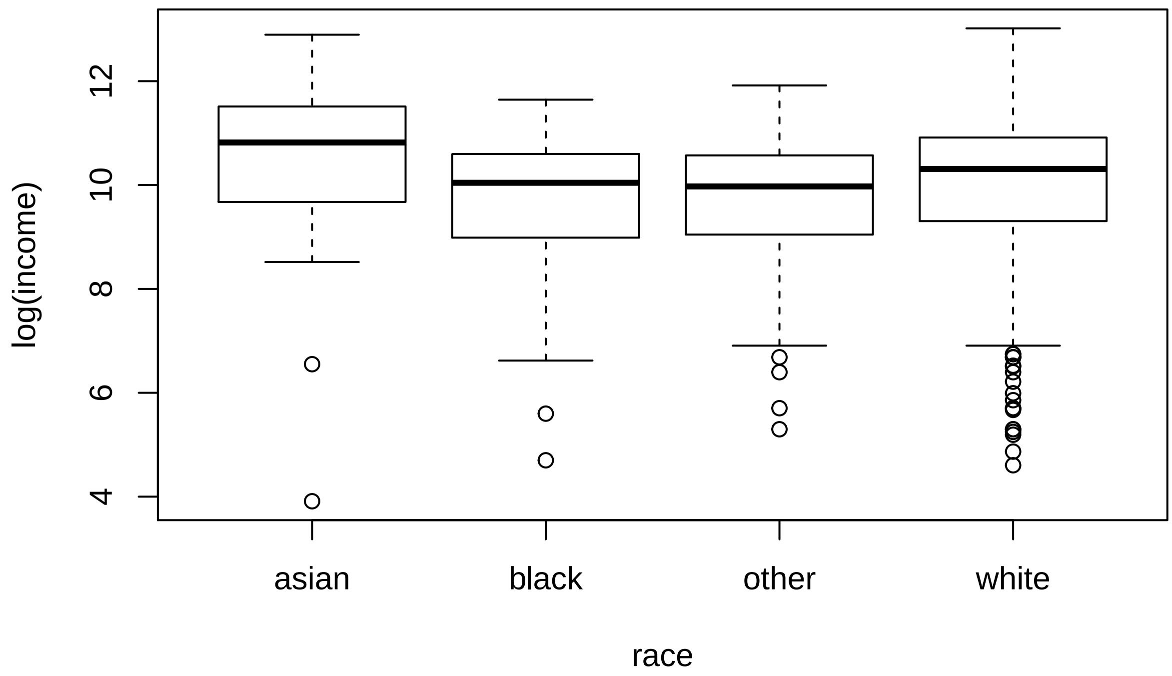
```r
# Can also calculate median
aggregate(income ~ race, data = subset(acs12, income > 0), median)
```

```
##    race income
## 1 asian  50000
## 2 black  23000
## 3 other  21500
## 4 white  30000
```

```r
# boxplots are good for visualizing distribution of numeric variables by a
# grouping variable. The black bar is the median (not the mean). The "box" is
# the middle 50% of the data.
boxplot(income ~ race, data = subset(acs12, income > 0))
```




```
# We can also easily plot log(income)
boxplot(log(income) ~ race, data = subset(acs12, income > 0))
```




```r
# It appears mean income may be different between races. Is this due to chance?
#
# The ANOVA procedure is usually emmployed to determine if there is a
# statistically significant difference between the means.
#
# The aov() function works like the t.test() function. Best to save result and
# use summary()
aov.out <- aov(log(income) ~ race, data = subset(acs12, income > 0))
summary(aov.out)
```

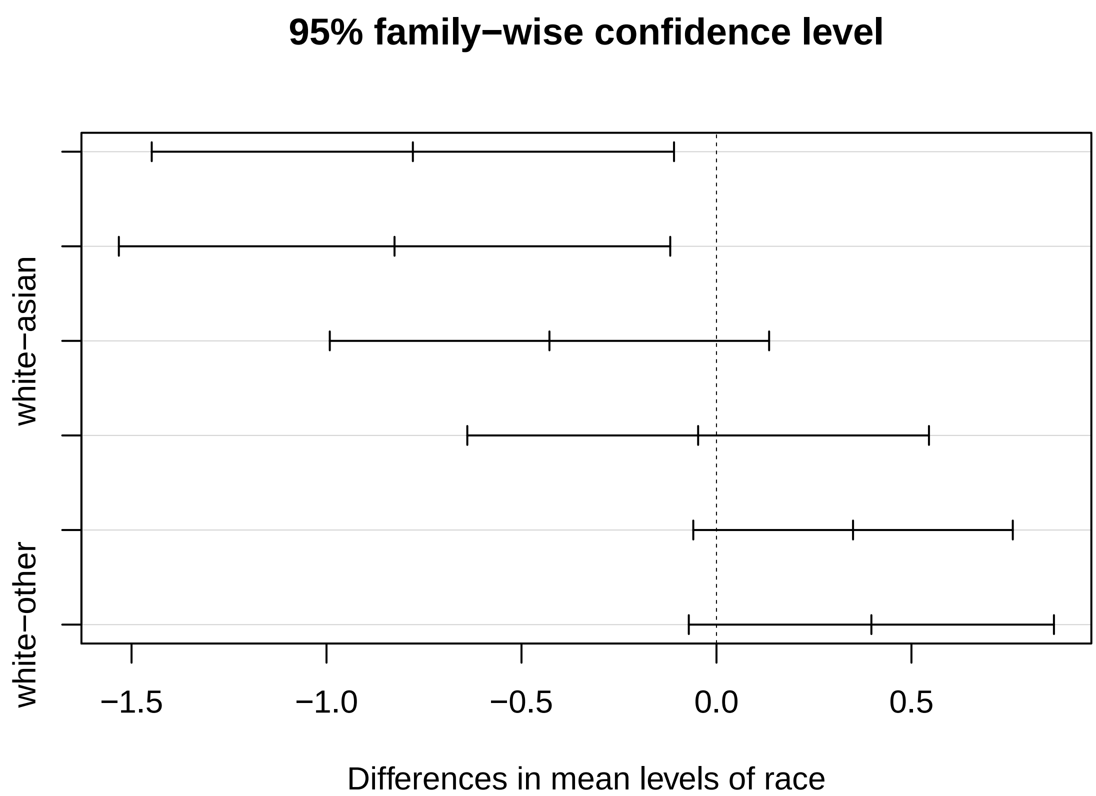
```
##              Df Sum Sq Mean Sq F value  Pr(>F)
## race          3     27   8.998   4.637 0.00317 **
## Residuals   890   1727   1.941
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# A small p-value provides against the null hypothesis that all means are the
# same. So where are the differences? This leads to what are commonly called
# "post-hoc" procedures.
#
# A basic one built into R is Tukey's Honestly Significantly Difference,
# TukeyHSD(). It compares all combinations of pairs and tests whether the
# difference is 0. It also reports 95% confidence intervals on the differences.
#
# Call it on the aov object and save. The result is a list of all pairwise
# comparisons along with the estimated difference and confidence intervals and
# an "adjusted" p-value on the "significance" of the difference.
```

```r
tukey.out <- TukeyHSD(aov.out)
tukey.out
```

```
##   Tukey multiple comparisons of means
##     95% family-wise confidence level
##
## Fit: aov(formula = log(income) ~ race, data = subset(acs12, income > 0))
##
## $race
##                    diff         lwr        upr     p adj
## black-asian -0.77851761 -1.44819702 -0.1088382 0.0150733
## other-asian -0.82553898 -1.53254517 -0.1185328 0.0144690
## white-asian -0.42829314 -0.99160485  0.1350186 0.2052936
## other-black -0.04702137 -0.63893994  0.5448972 0.9969750
## white-black  0.35022447 -0.05944602  0.7598950 0.1238799
## white-other  0.39724584 -0.07096048  0.8654522 0.1285042
```

```r
# There is a plotting method for a TukeyHSD object.
plot(tukey.out)
```




```r
# Exponentiate to get multiplicative effect
exp(tukey.out$race[,"diff"])
```

```
## black-asian other-asian white-asian other-black white-black white-other
##   0.4590861   0.4379989   0.6516204   0.9540670   1.4193861   1.4877216
```

```r
# race=black makes about 46 cents for every dollar race=asian earns.
```

```r
# YOUR TURN #5 ------------------------------------------------------------

# Run an ANOVA of log(income) on edu. Is there a difference in mean log(income)
# values between different levels of education? Go ahead subset acs12 to use
# income greater than 0.





# Association between categorical variables -------------------------------

# Let's create a cross tabulation of race and employment
xtabs(~ race + employment, data = acs12)
```

```
##        employment
## race    employed not in labor force unemployed
##   asian       39                 31          3
##   black       76                 66         20
##   other       58                 39         11
##   white      670                520         72
```

```r
# proportion of employment by race
xtabs(~ race + employment, data = acs12) %>%
  prop.table(margin = 1) %>%
  round(3)
```

```
##        employment
## race    employed not in labor force unemployed
##   asian    0.534              0.425      0.041
##   black    0.469              0.407      0.123
##   other    0.537              0.361      0.102
##   white    0.531              0.412      0.057
```

```r
# Is there an assocation between race and employment? Does knowing race give us
# additional information about the proportions of employment levels?
#
# A common approach to answering this question is the chi-square test. The
# chisq.test() function simply requires a table
xtabs(~ race + employment, data = acs12) %>%
  chisq.test()
```

```
## Warning in chisq.test(.): Chi-squared approximation may be incorrect
```

```
##
##  Pearson's Chi-squared test
##
## data:  .
## X-squared = 14.182, df = 6, p-value = 0.02767
```

```r
# A small p-value provides evidence against the null hypothesis of no
# association. However it does not tell us where the association is or the
# magnitude of the association.
#
# Also notice the warning. That is due to small expected cell counts. Happens
```

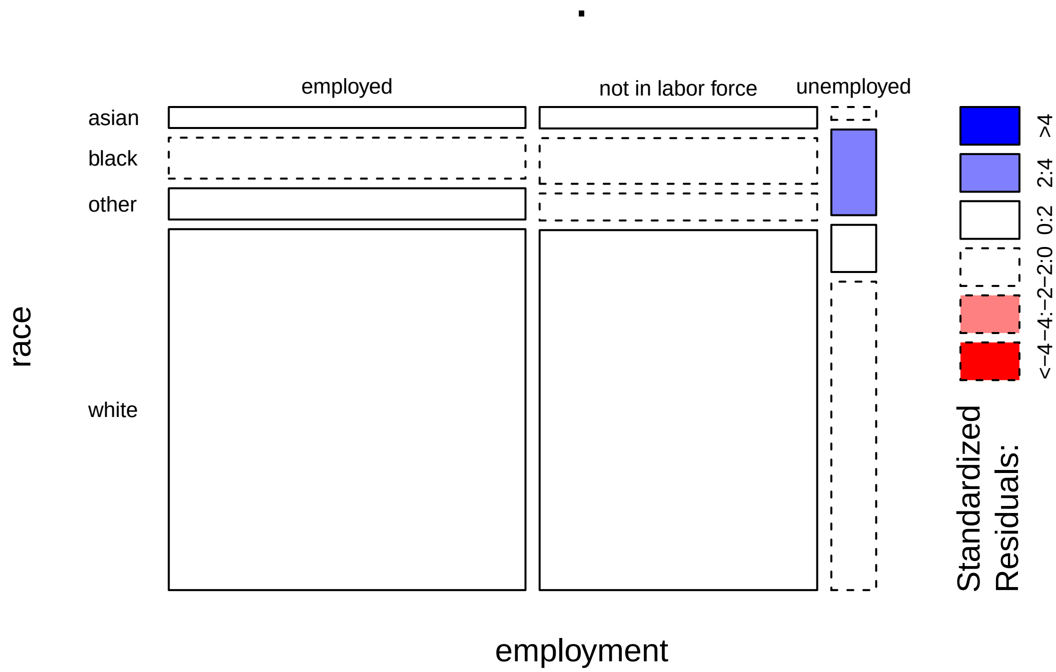
```r
# when the expected count of a cell is less than 5. Set simulate.p.value = TRUE
# to simulate a p-value using Monte Carlo simulation and not get warning.
c.out <- xtabs(~ race + employment, data = acs12) %>%
  chisq.test(simulate.p.value = TRUE)
c.out
```

```
##
##  Pearson's Chi-squared test with simulated p-value (based on 2000
##  replicates)
##
## data:  .
## X-squared = 14.182, df = NA, p-value = 0.02799
```

```r
# Check the residuals of the chisq.test to see where the "unexpected" high/low
# counts are occuring. Residuals over 2 (or less than -2) are usually of
# interest.
c.out$residuals
```

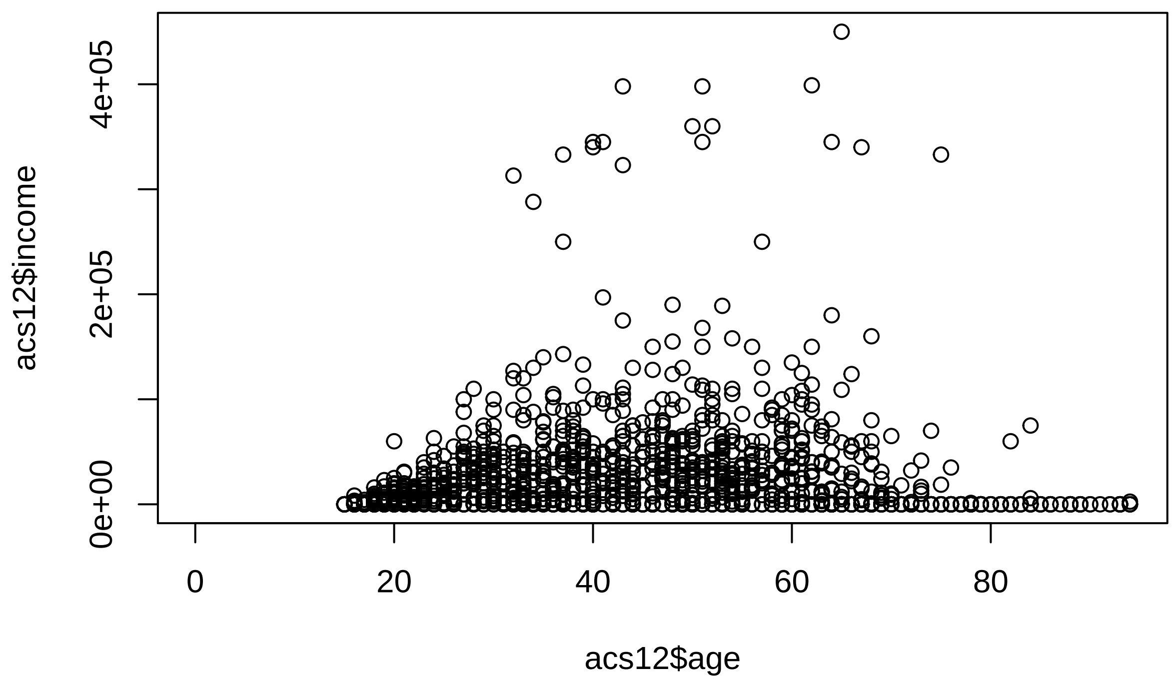
```
##        employment
## race      employed not in labor force unemployed
##   asian  0.1062554          0.2129578 -0.8294246
##   black -0.9852068         -0.0261866  2.8435029
##   other  0.1692554         -0.7739458  1.4480362
##   white  0.2779151          0.1845724 -1.2429038
```

```r
# It appears we have more unemployed for race=black than would be expected if
# there was no association between race and employment
#
# The mosaicplot() function with base R can help visualize residuals from a
# chi-square test. shade = TRUE colors the tiles where the residuals are
# unusually large. las=1 makes the labels horizontal.
xtabs(~ employment + race, data = acs12) %>%
  mosaicplot(shade = TRUE, las = 1)
```

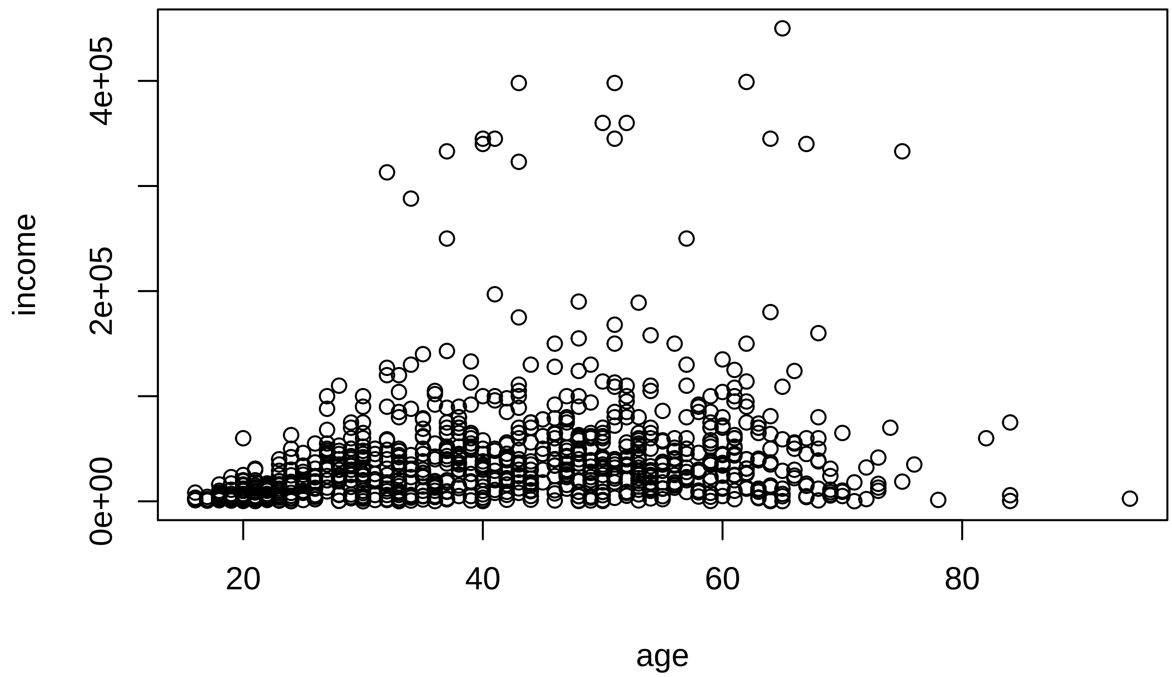



```
# The width of the boxes corresponds to the proportions of employment. The
# height of the boxes correspond to the proportion of race, within each level of
# employment. Within unemployed, we see a higher than expected proportion of
# race=black.


# YOUR TURN #6 ------------------------------------------------------------

# (1) Create a cross tabulation of employment and education


# (2) Run a chi-square test on the cross tabulation. Is there evidence of
# association?


# (3) Create a mosaic plot of the cross tabulation with residual shading




# Linear association between numeric variables ----------------------------

# Correlation summarizes linear association between two numeric variables.
# Ranges from -1 to 1. No correlation is 0.
#
# See an illustration:
```

```r
# https://en.wikipedia.org/wiki/Correlation_and_dependence#/media/File:Correlation_examples2.svg
#
# Is age correlated with income?
#
# Let's first plot age versus income
plot(acs12$age, acs12$income)
```




```r
# Plot income versus age for income > 0
plot(income ~ age, data = subset(acs12, income > 0))
```




```r
# Correlation between age and income; returns NA because we have missing data.
cor(acs12$age, acs12$income)
```

```
## [1] NA
```

```r
summary(acs12$income)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
##       0       0    3000   23600   33700  450000     377
```

```r
# Set use = "pairwise.complete.obs" to calculate correlation using all complete
# pairs of observations
cor(acs12$age, acs12$income, use = "pairwise.complete.obs")
```

```
## [1] -0.03462251
```

```r
# The mosaic version allows us to use the formula interface which allows us to
# subset acs12 for income > 0
mosaic::cor(age ~ income, use = "pairwise.complete.obs",
            data = subset(acs12, income > 0))
```

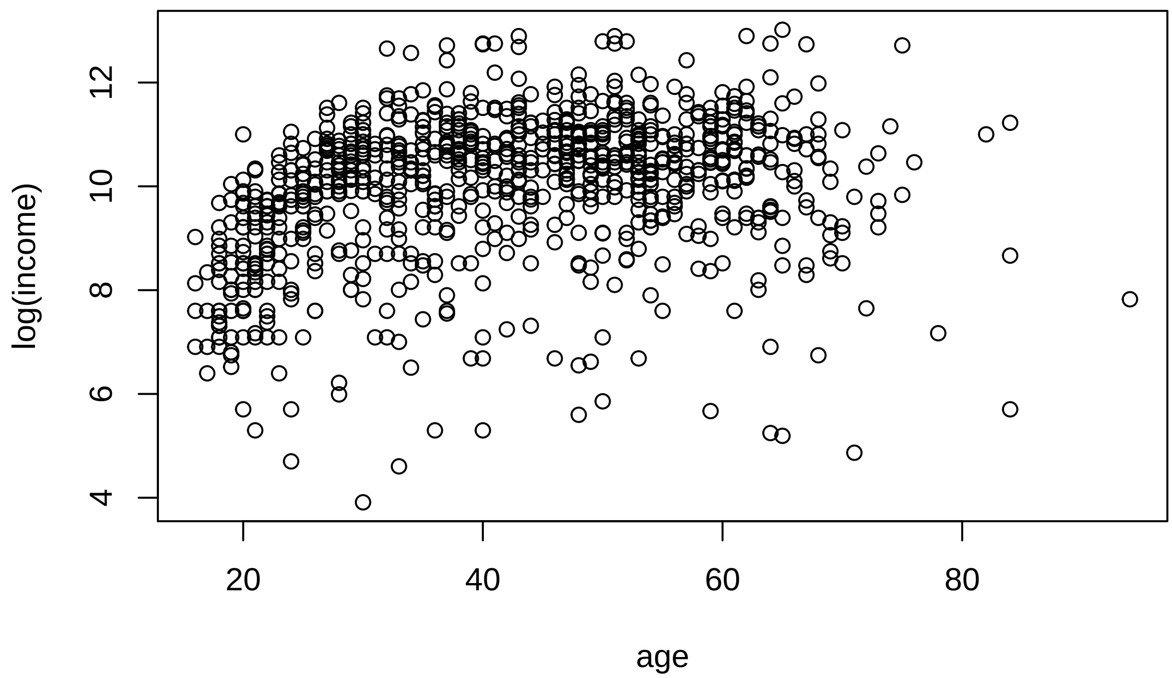
```
## [1] 0.2133743
```

```r
# The cor.test function returns a confidence interval on the correlation
mosaic::cor.test(age ~ income, use = "pairwise.complete.obs",
                 data = subset(acs12, income > 0))
```

```
##
##  Pearson's product-moment correlation
##
```

```
## data:  age and income
## t = 6.5229, df = 892, p-value = 1.154e-10
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
##  0.1499045 0.2750927
## sample estimates:
##       cor
## 0.2133743
```

```
# Plot log(income) versus age for income > 0
plot(log(income) ~ age, data = subset(acs12, income > 0))
```




```
# correlation between age and log(income)
mosaic::cor.test(age ~ log(income), use = "pairwise.complete.obs",
                 data = subset(acs12, income > 0))
```

```
##
##  Pearson's product-moment correlation
##
## data:  age and log(income)
## t = 8.8703, df = 892, p-value < 2.2e-16
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
##  0.2233106 0.3438571
## sample estimates:
##      cor
## 0.284709
```

```r
# YOUR TURN #7 ----------------------------------------------------------

# Calculate the correlation, and associated 95% confidence interval, between age
# and hrs_work. Make a scatterplot as well.




# Simple Linear Regression ----------------------------------------------

# Simple linear regression is basically summarizing the relationship between two
# variables as a straight line, using the familiar slope-intercept formula:
#
# y = a + bx
#
# This implies we can approximate the mean of y for a given value of x by
# multiplying x by some number and adding a constant value.
#
# It allows us to answer the question, "how does the mean of y change as x
# increases?"

# Example: how does mean income change as hrs_work increases? Regress income on
# hrs_work.
mod <- lm(income ~ hrs_work, data = acs12, subset = income > 0)
summary(mod)
```

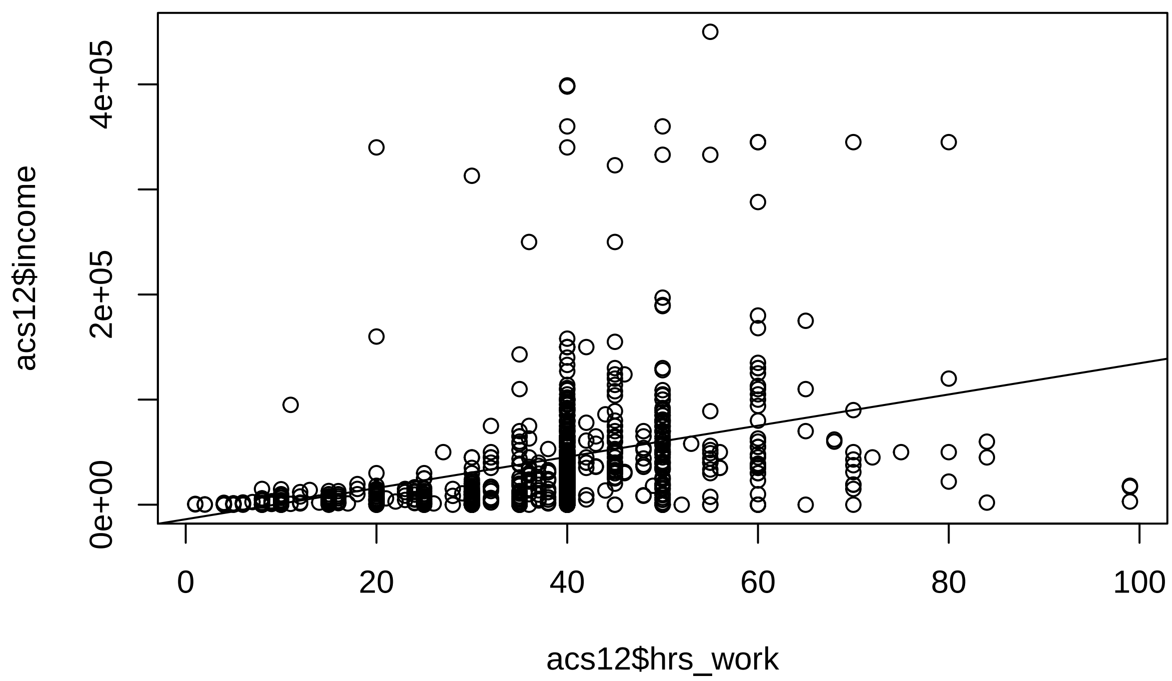
```
##
## Call:
## lm(formula = income ~ hrs_work, data = acs12, subset = income >
##     0)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -130100  -23763  -10526    7092  382210
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -13846.5     5311.9  -2.607  0.00929 **
## hrs_work      1484.3      131.3  11.301  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 52220 on 892 degrees of freedom
##   (377 observations deleted due to missingness)
## Multiple R-squared:  0.1252, Adjusted R-squared:  0.1243
## F-statistic: 127.7 on 1 and 892 DF,  p-value: < 2.2e-16
```

```r
# It appears mean annual income increases by about $1400 for each additional
# hour worked.
#
# The coefficient is just an estimate. Use confint() to get a confidence
# interval. 95% CI: ($1226, $1742)
confint(mod)
```

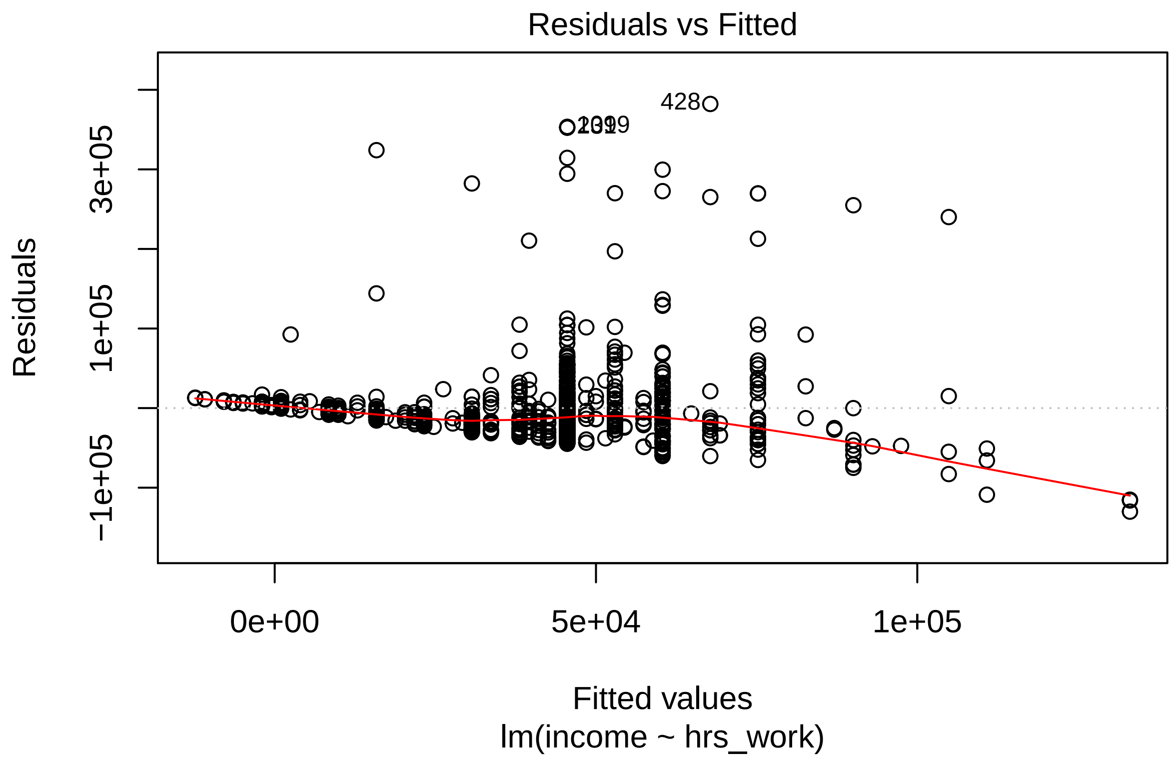
```
##                  2.5 %    97.5 %
```

```
## (Intercept) -24271.75 -3421.341
## hrs_work       1226.52  1742.089
```

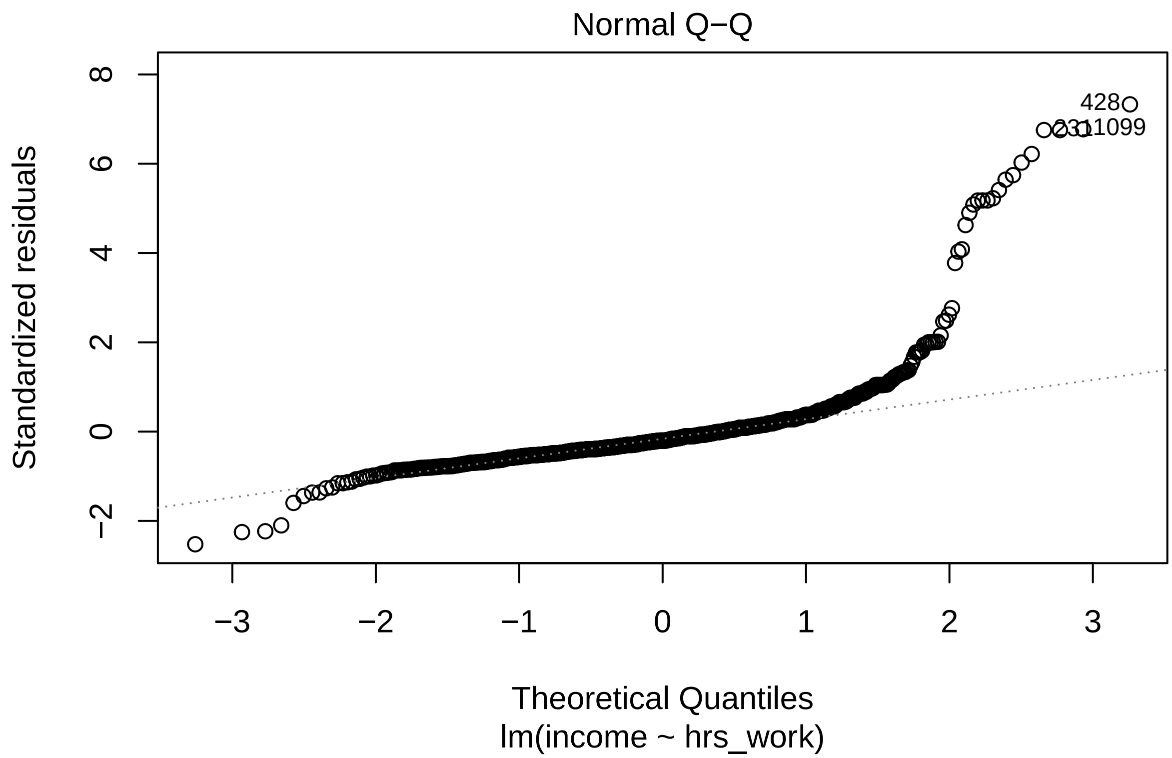
```r
# Is this a "good" estimate? One way to judge is to plot the fitted straight
# line over the raw data.
plot(acs12$hrs_work, acs12$income)

# add fitted straight line using the abline() function
abline(mod)
```




```r
# It doesn't look like a great estimate. The fitted line overpredicts and
# underpredicts over the range of hrs_work.
#
# The differences between the fitted straight line and observed data are called
# the residuals. An assumption of a simple linear model is that these residuals
# are a random sample from a normal distribution with mean 0 and some finite
# standard deviation. This implies we expect an even scatter of raw data around
# the fitted line (ie, constant variance). We can check this assumption by
# calling plot() on the model object.
#
# The first plot assess the constant variance assumption. The second
# assesses the Normality assumption.
plot(mod, which = 1)
```




```r
plot(mod, which = 2)
```




```r
# Neither look very good. It appears the constant variance assumption is
# violated. We like to see a mostly horizontal red line in the first plot, and a
# mostly diagonal line in the second plot.
#
# One approach to address this is to try a log transformation
mod2 <- lm(log(income) ~ hrs_work, data = acs12, subset = income > 0)

# get estimated coefficients
summary(mod2)
```

```
##
## Call:
## lm(formula = log(income) ~ hrs_work, data = acs12, subset = income >
##     0)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.8446 -0.5067  0.1663  0.6880  3.9169
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 7.558321   0.114263   66.15   <2e-16 ***
## hrs_work    0.063076   0.002825   22.32   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 1.123 on 892 degrees of freedom
##   (377 observations deleted due to missingness)
## Multiple R-squared:  0.3585, Adjusted R-squared:  0.3577
## F-statistic: 498.4 on 1 and 892 DF,  p-value: < 2.2e-16
```

```r
# For every extra hour worked, income increases by about (exp(0.063) - 1) * 100
# = 6.5 percent.
exp(0.063)
```

```
## [1] 1.065027
```

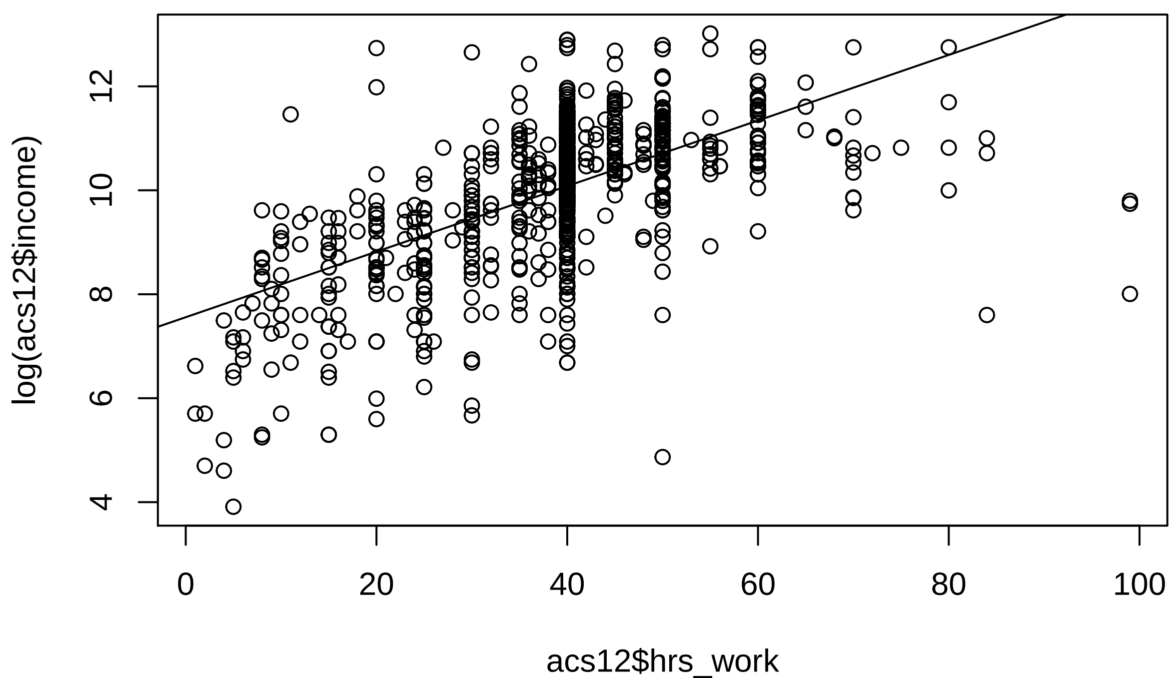
```r
(exp(0.063) - 1) * 100
```

```
## [1] 6.502684
```

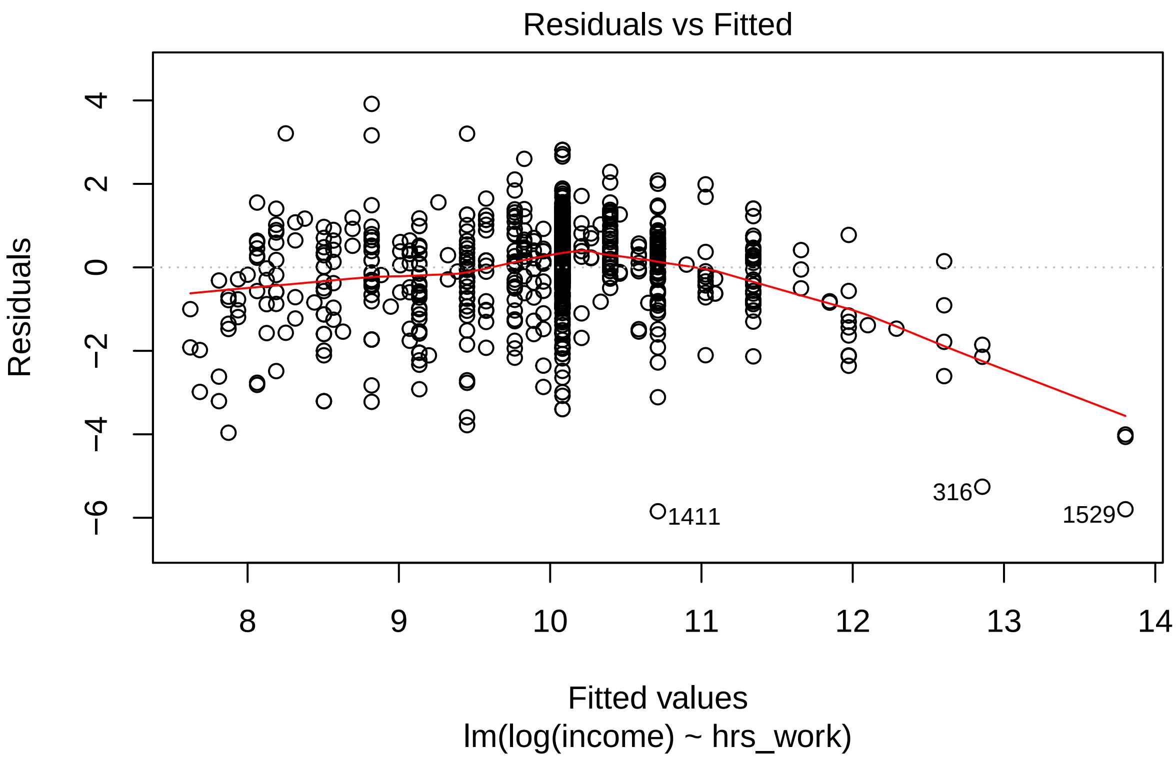
```r
# see confidence intervals
confint(mod2)
```

```
##                  2.5 %    97.5 %
## (Intercept) 7.33406617 7.7825764
## hrs_work    0.05753058 0.0686209
```

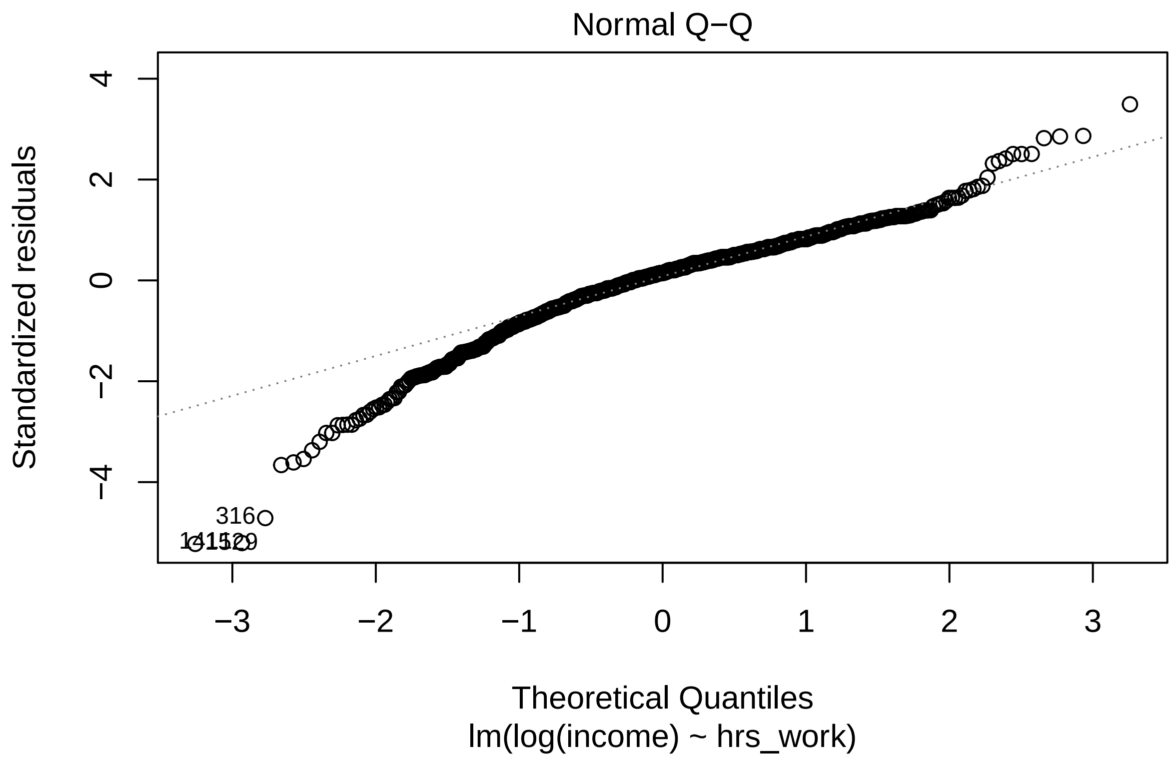
```r
# scatterplot with fitted line
plot(acs12$hrs_work, log(acs12$income))
abline(mod2)
```




```r
# check assumptions
plot(mod2, which = 1)
```




```
plot(mod2, which = 2)
```

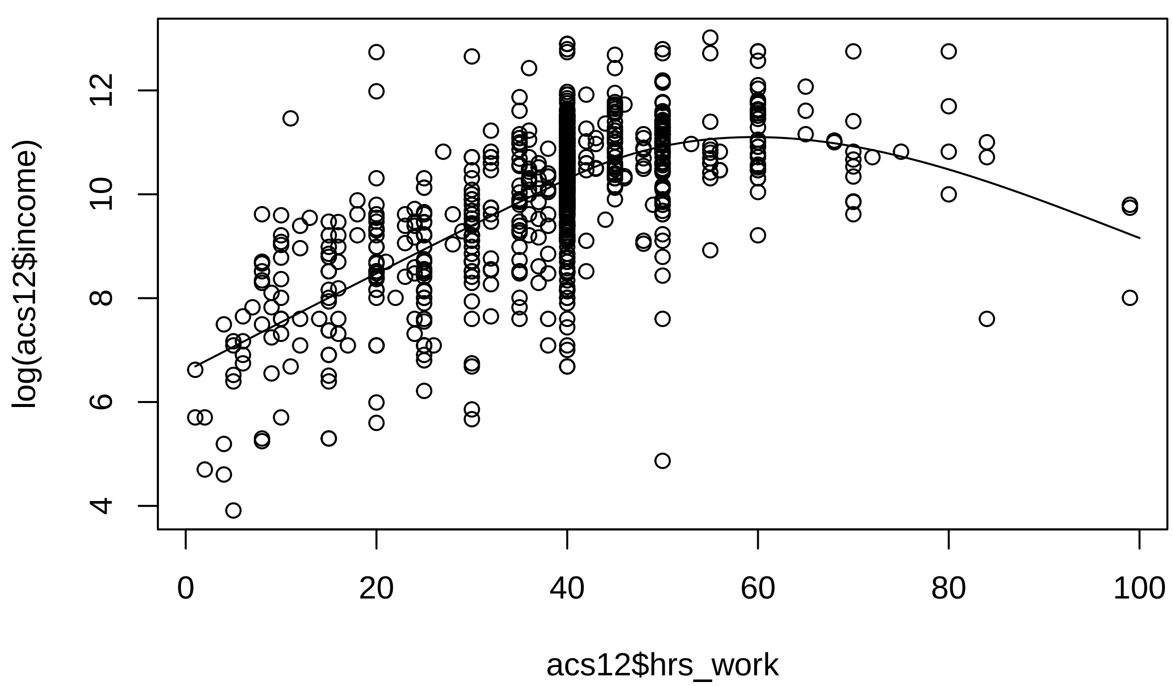



```r
# There still appears to be a violation of the constant variance assumption.
# This is basically because there is a non-linear relationship between hrw_work
# and log(income).
#
# We can try to fit a smooth line that follows the data. One approach is called
# regression splines.
#
# 1) Load the splines package (comes with R)
# 2) use the ns() function and specify the number of times the line can "change
#    direction" (usually 3 - 5) as the degrees of freedom (df).
#    ns = natural spline

library(splines)
mod3 <- lm(log(income) ~ ns(hrs_work, 3), data = acs12, subset = income > 0)
summary(mod3)
```

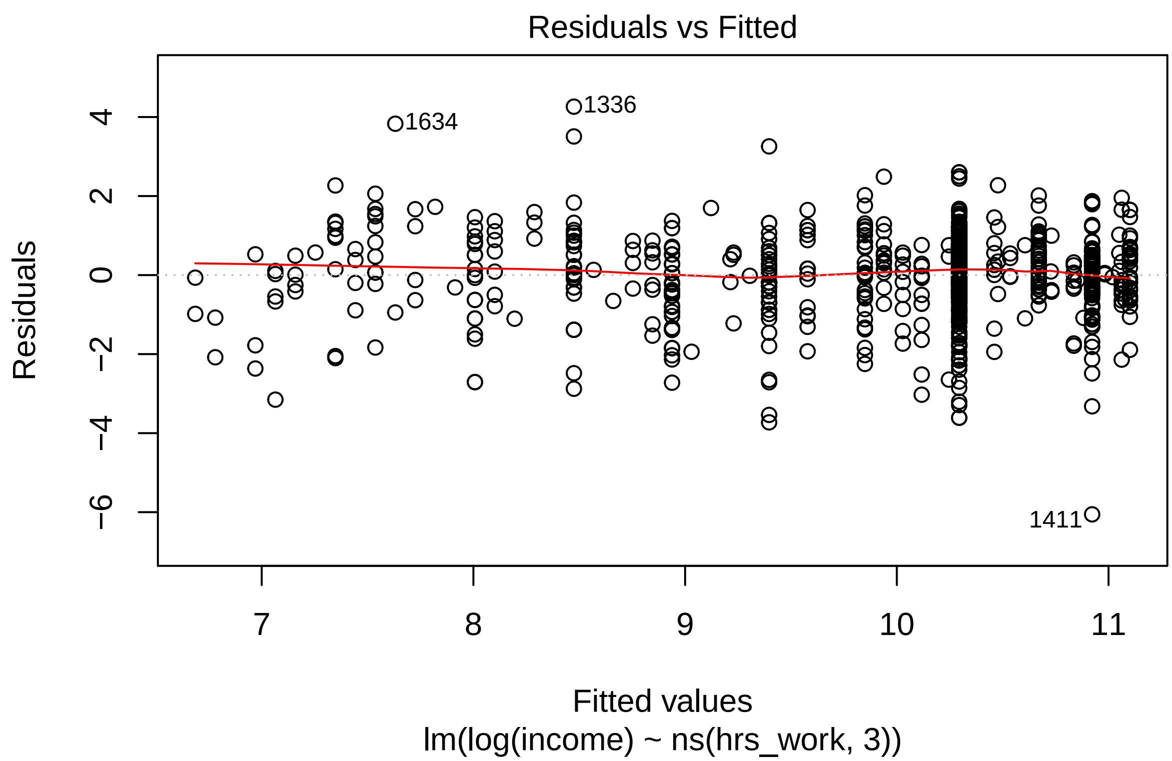
```
##
## Call:
## lm(formula = log(income) ~ ns(hrs_work, 3), data = acs12, subset = income >
##     0)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.0550 -0.4766  0.0788  0.5854  4.2620
##
## Coefficients:
```

```
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)         6.6861     0.1865  35.847  < 2e-16 ***
## ns(hrs_work, 3)1   4.2222     0.1958  21.563  < 2e-16 ***
## ns(hrs_work, 3)2   6.1903     0.4629  13.374  < 2e-16 ***
## ns(hrs_work, 3)3   1.4197     0.4230   3.356 0.000824 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.028 on 890 degrees of freedom
##   (377 observations deleted due to missingness)
## Multiple R-squared:  0.4641, Adjusted R-squared:  0.4623
## F-statistic:   257 on 3 and 890 DF,  p-value: < 2.2e-16
```

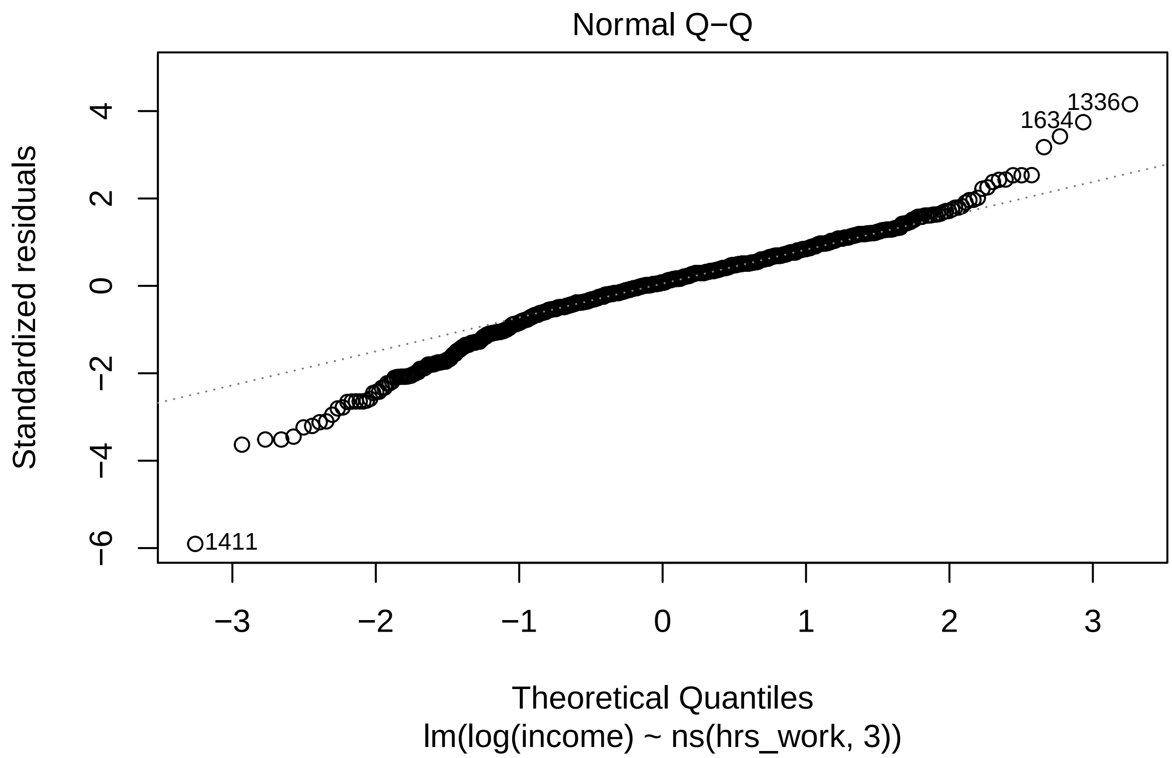
```r
# There is really no model interpretation
#
# Plot the fitted line over the raw data
plot(acs12$hrs_work, log(acs12$income))

# Since the prediction is no longer a straight line we cannot use abline().
# Use the predict() function to get fitted values on the range of 0 - 100.
# Notice the newdata argument requires a data frame
y <- predict(mod3, newdata = data.frame(hrs_work = 1:100))

# add line to plot
lines(1:100, y)
```



```r
# The assumptions appear to be met though row 1411 seems to be unusual
plot(mod3, which = 1)
```




```r
plot(mod3, which = 2)
```




```
# Use model to make a prediction of expected income given hours worked.
predict(mod3, newdata = data.frame(hrs_work = c(20,30,40,50)),
        interval = "confidence") %>%
  exp()
```

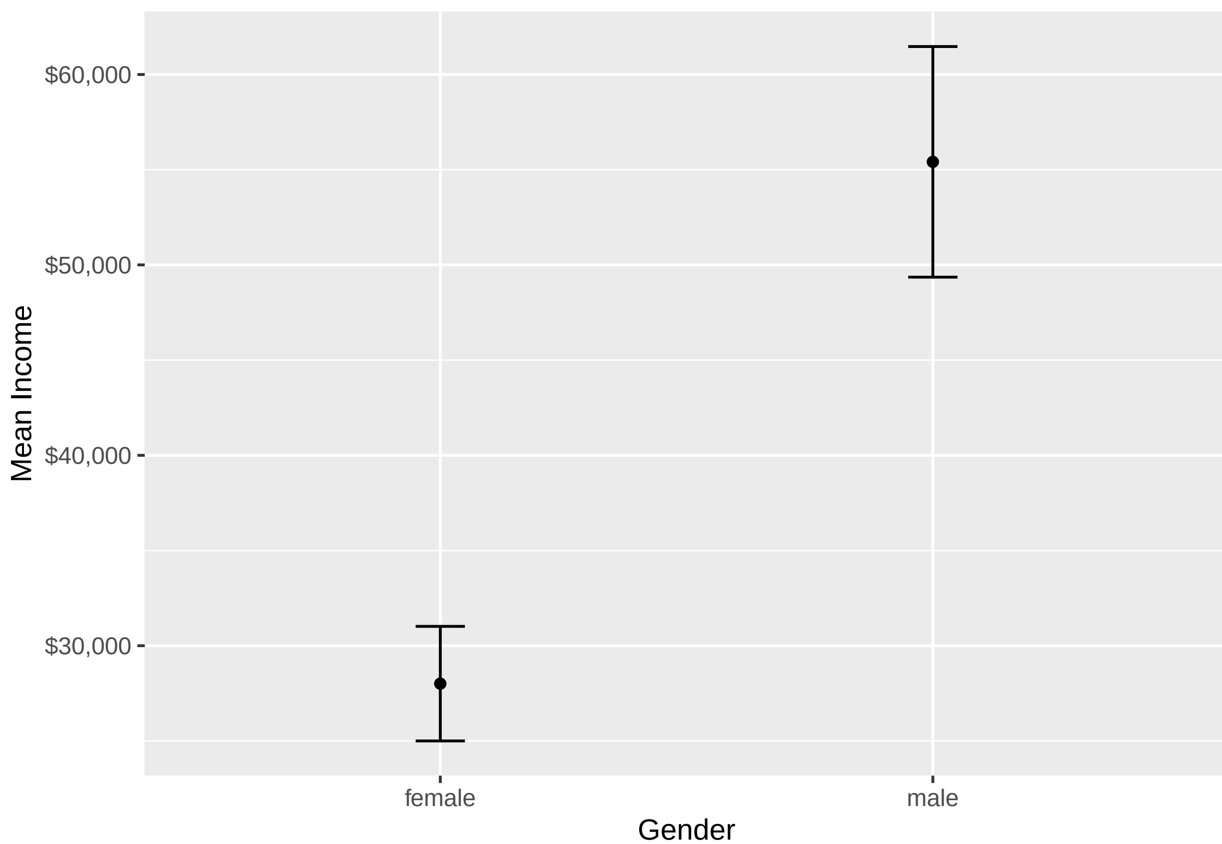
```
##         fit       lwr       upr
## 1  4791.953  4208.156  5456.741
## 2 12049.213 10670.627 13605.905
## 3 29574.489 27377.243 31948.083
## 4 55413.389 48720.156 63026.147
```

```
# YOUR TURN #8 ------------------------------------------------------------

# (1) Regress log(income) on age. That is, fit lm(log(income) ~ age, data =
#     acs12). Be sure to subset the data: subset = income > 0 & age > 17
#     Plot the data and fitted line.



# (2) Regress log(income) on ns(age, 3). Again, Be sure to subset the data:
#     subset = income > 0 & age > 17. Plot the data and fitted line.



# Appendix: Plotting means and confidence intervals -----------------------

# difference in mean income by gender.
```

```r
# create a data frame of means and upper and lower CIs
income.ci <- acs12 %>%
  filter(income > 0) %>%
  group_by(gender) %>%
  summarise(mean = mean(income, na.rm = TRUE),
            lower = smean.cl.normal(income)["Lower"],
            upper = smean.cl.normal(income)["Upper"])
income.ci
```

```
## # A tibble: 2 x 4
##   gender   mean  lower  upper
##   <fct>   <dbl>  <dbl>  <dbl>
## 1 female 28008. 24999. 31016.
## 2 male   55412. 49356. 61468.
```

```r
library(ggplot2)
ggplot(income.ci, aes(x = gender, y = mean)) +
  geom_point() +
  geom_errorbar(aes(ymin = lower, ymax = upper), width = 0.1) +
  scale_y_continuous(labels = scales::dollar) +
  labs(y = "Mean Income", x = "Gender")
```

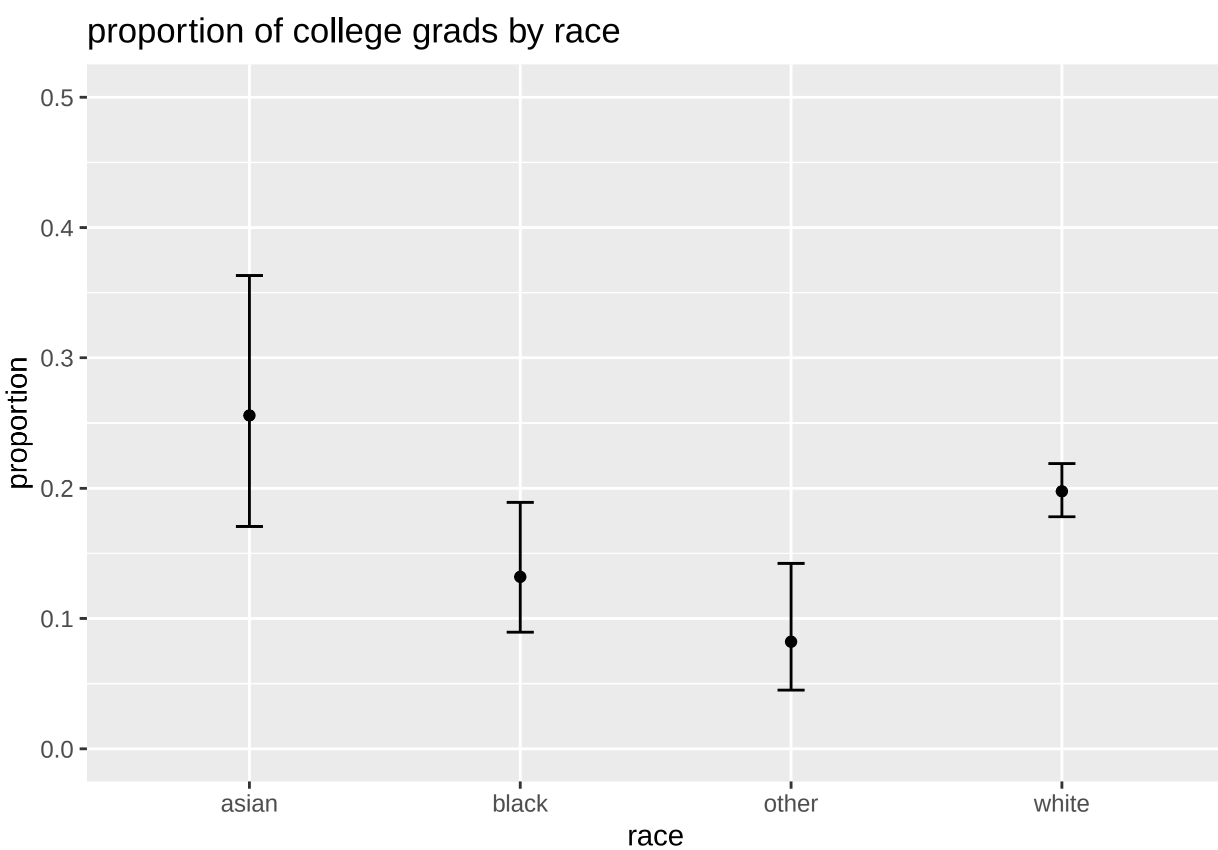



```r
# plot proportion of college graduates by race
# first get counts of college grads and total race
tab <- xtabs(~ race + edu, data = acs12) %>%
  addmargins(margin = 2)
# get confidence intervals
```

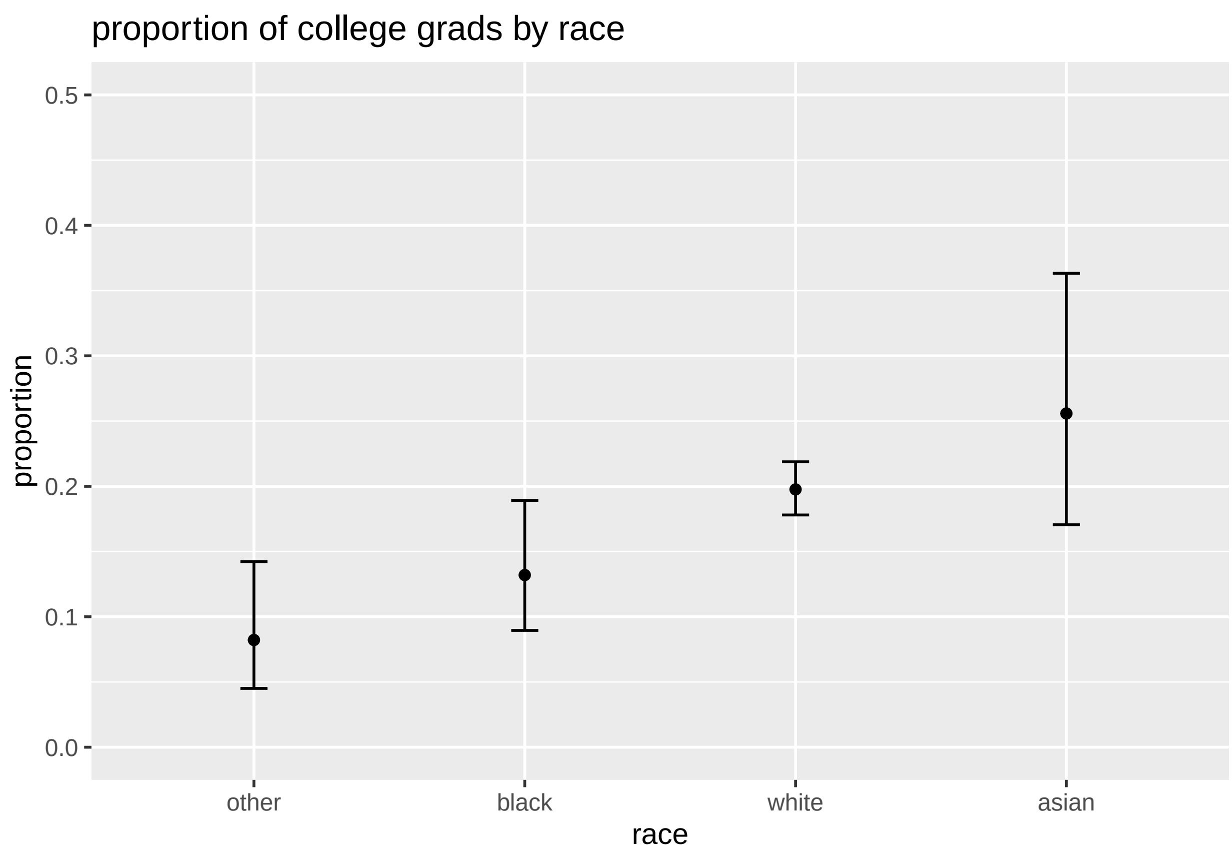
```
binom.out <- binom.confint(x = tab[,"college"],
                           n = tab[,"Sum"],
                           methods = "prop.test")
# add race to the binom.out data frame
binom.out$race <- rownames(tab)
binom.out
```

```
##      method   x    n       mean      lower     upper  race
## 1 prop.test  22   86 0.25581395 0.17051512 0.3632744 asian
## 2 prop.test  26  197 0.13197970 0.08956575 0.1892122 black
## 3 prop.test  12  146 0.08219178 0.04510384 0.1422820 other
## 4 prop.test 299 1513 0.19762062 0.17801538 0.2187826 white
```

```
# create the plot
ggplot(binom.out, aes(x = race, y = mean)) +
  geom_point() +
  geom_errorbar(aes(ymin = lower, ymax = upper), width = 0.1) +
  ylim(0,0.5) +
  labs(y = "proportion", title = "proportion of college grads by race")
```




```
# reorder race by mean
ggplot(binom.out, aes(x = reorder(race, mean), y = mean)) +
  geom_point() +
  geom_errorbar(aes(ymin = lower, ymax = upper), width = 0.1) +
  ylim(0,0.5) +
  labs(y = "proportion", x = "race",
  title = "proportion of college grads by race")
```




```
# Appendix: Demonstration of confidence intervals ------------------------

# 95% Confidence Interval theory: sample the data, calculate a confidence
# interval, repeat many times. About 95% of confidence intervals will contain
# the "true" value you're trying to estimate.

# Simulation to demonstrate

# simulate a population of 10,000 from a Normal distribution with mean 10 and
# standard deviation 1.5.
population <- rnorm(10000, mean = 10, sd = 1.5)

# The TRUE mean value is 10. Pretend we don't know that.

# sample the population (size = 30) and calculate 95% confidence interval of
# mean:
s <- sample(population, size = 30, replace = TRUE)
ci <- t.test(s)$conf.int
ci
```

```
## [1]  9.304268 10.183508
## attr(,"conf.level")
## [1] 0.95
```

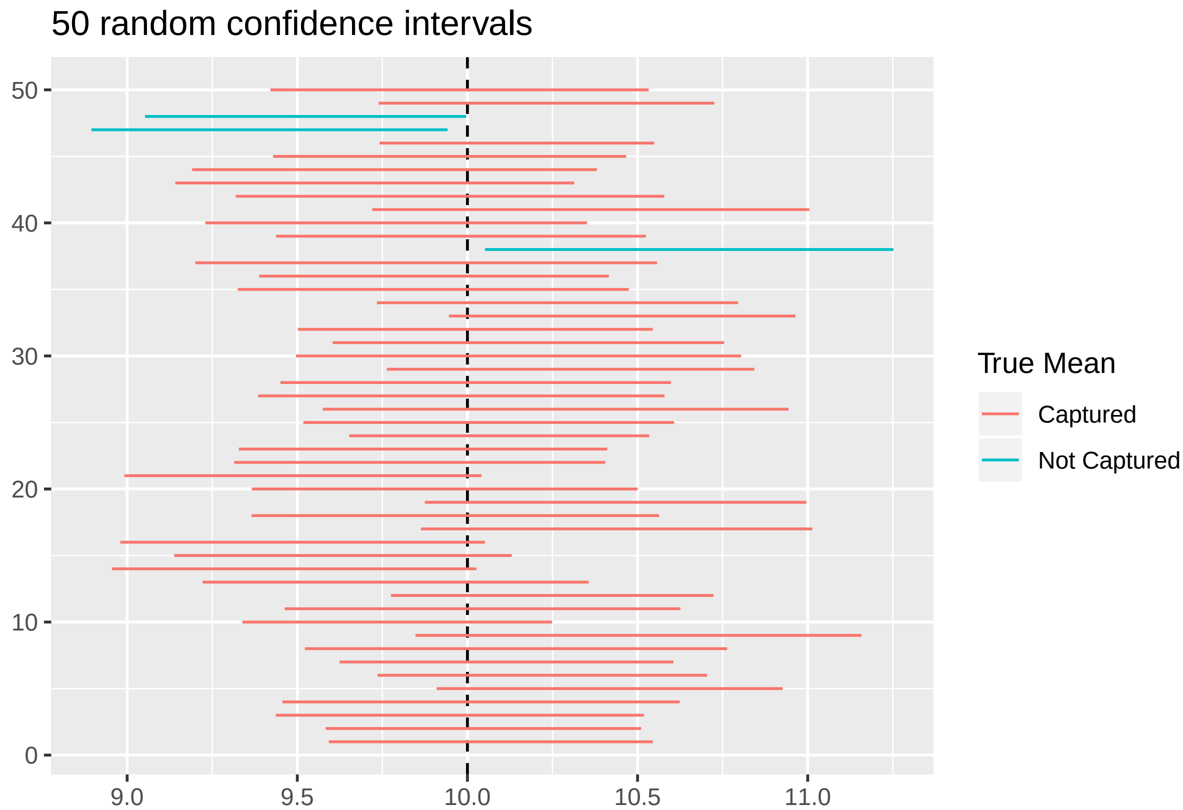
```
# Does confidence interval contain mean?
ci[1] < 10 & ci[2] > 10
```

```
## [1] TRUE
```

```r
# Let's do this 1000 times
result <- logical(length = 1000)  # empty vector to store result (TRUE/FALSE)
for(i in 1:1000){
  s <- sample(population, size = 30, replace = TRUE)
  ci <- t.test(s)$conf.int
  result[i] <- ci[1] < 10 & ci[2] > 10
}

# what proportion of times did confidence interval contain 10?
mean(result)
```

```
## [1] 0.953
```

```r
# The process works about 95% of the time. Hence, the reason we call it a
# "confidence" interval. We're confident the process will return a confidence
# interval that contains the true value.

# Here's another demonstration that produces a plot.

# a random seed that ensures you get the results I got
set.seed(6)

# create an empty data frame containing a counter and upper and lower CI
# values.
dat <- data.frame(i = 1:50, lwr = NA, upr = NA)

# Sample from "population" and calculate CI 50 times
for(i in 1:50){
  s <- sample(population, size = 30, replace = TRUE)
  ci <- t.test(s)$conf.int
  dat[i,"lwr"] <- ci[1]
  dat[i,"upr"] <- ci[2]
}

# create an indicator that indentifies whether or not the CI captured the "True"
# mean of 10.
dat$grp <- ifelse(dat$lwr < 10 & dat$upr > 10, "Captured", "Not Captured")

# Generate plot using the ggplot2 package
library(ggplot2)
ggplot(dat) +
  geom_vline(xintercept = 10, linetype = 2) +
  geom_segment(aes(x = lwr, y = i, xend = upr, yend = i, color = grp)) +
  labs(x = "", y = "", title = "50 random confidence intervals") +
  scale_color_discrete("True Mean")
```




```r
# Appendix: Basic bootstrapping -------------------------------------------

# Bootstrapping means resampling your data with replacement, calculating a
# statistic of choice (such as a mean) and then repeating many times. The result
# is many means which we then can use to get an estimate of uncertainty of our
# original estimated mean. Because it's based on a resampling, your bootstrapped
# CI will be different from mine, but only slightly.

# Bootstrapping is effective for estimating uncertainty when the usual
# assumptions for calculating standard errors are suspect, or when a standard
# error formula is complex or not available.

# Example data:
x <- c(12, 22, 21, 18, 19, 23, 7)
mean(x)
```

```
## [1] 17.42857
```

```r
# Bootstrap "by hand":
# Resample with replacement (ie allow a value to be sampled more than once) and
# estimate mean
mean(sample(x, replace = TRUE))
```

```
## [1] 16.28571
```

```r
# repeat 1000 times and store
b <- replicate(n = 1000, mean(sample(x, replace = TRUE)))
```

```r
# b contains 1000 bootstrapped means
head(b)
```

```
## [1] 18.85714 17.71429 16.28571 18.14286 11.42857 13.28571
```

```r
# We can take the 0.025 and 0.975 percentiles to get an approximate Confidence
# Interval:
quantile(b, probs = c(0.025, 0.975))
```

```
##     2.5%    97.5%
## 13.14286 21.00000
```

```r
# The Hmisc function smean.cl.boot calculates bootstrapped confidence intervals:
Hmisc::smean.cl.boot(x)
```

```
##     Mean    Lower    Upper
## 17.42857 13.00000 21.14286
```

```r
# Calculate a bootstrap CI of hrs_work using the Hmisc function smean.cl.boot.
# If you run it multiple times you will get slightly difference intervals.
smean.cl.boot(acs12$hrs_work, B = 1500)
```

```
##     Mean    Lower    Upper
## 37.97706 37.11509 38.80198
```

```r
# To see the bootstrapped estimates, set reps = TRUE
smean.cl.boot(acs12$hrs_work, B = 100, reps = TRUE)
```

```
##     Mean    Lower    Upper
## 37.97706 37.22101 38.73152
## attr(,"reps")
##   [1] 37.43066 38.73201 38.62982 37.74035 37.83733 38.49739 37.99791 38.43274
##   [9] 37.72888 37.80813 38.34932 38.10532 38.22732 37.63608 38.11783 38.07404
##  [17] 38.30865 38.73097 38.19812 38.59541 37.60584 38.40980 37.49635 38.23670
##  [25] 37.71637 38.44213 39.07821 37.39937 37.86548 37.69239 38.08551 38.32325
##  [33] 38.94891 38.03233 38.09802 37.43379 37.97706 37.59437 38.59437 37.79145
##  [41] 37.75391 37.37539 38.59541 38.29197 38.20751 38.39208 37.07091 38.51408
##  [49] 37.42231 37.78102 38.66111 37.79771 37.89364 38.21064 37.41189 38.01460
##  [57] 38.17831 38.35036 38.59750 37.69447 37.75287 37.66945 37.89260 38.26486
##  [65] 37.88008 38.37435 37.59020 37.76955 37.81960 37.79562 37.95933 37.73514
##  [73] 38.08551 37.63087 38.68300 37.86653 37.38895 38.49426 38.16267 37.72888
##  [81] 37.95933 37.76642 38.37748 37.01251 37.88425 38.18874 38.44943 37.60271
##  [89] 37.80292 37.08133 38.30970 38.28780 38.21481 37.71220 37.83733 37.65902
##  [97] 38.26069 37.66215 37.62044 37.83107
```

```r
# The reps represent the 100 means that resulted from the 100 bootstrapped
# samples.

# The boot package that comes with R provides the boot() function that allows
# you to bootstap just about any statistic as long as you can write a function
# for it.

# Here we bootstrap the median of income. How certain is that?
median(acs12$income, na.rm = TRUE)
```

```
## [1] 3000
```

```r
# load the boot package
library(boot)
```

```
##
## Attaching package: 'boot'
## The following object is masked from 'package:mosaic':
##
##     logit
## The following object is masked from 'package:survival':
##
##     aml
## The following object is masked from 'package:lattice':
##
##     melanoma
```

```
# Write a function that calculates the median of a bootstrapped sample. The [i]
# is required to sample the selected indices (rows) generated by the
# re-sampling.
bootMedian <- function(x, i)median(x[i], na.rm = TRUE)

# Run the bootstrap 999 times
boot.out <- boot(data = acs12$income,
                 statistic = bootMedian,
                 R = 999)

# Get the percentile confidence interval of the bootstrapped estimates using the
# boot.ci() function, also in the boot package
boot.ci(boot.out, type = "perc")
```

```
## BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
## Based on 999 bootstrap replicates
##
## CALL :
## boot.ci(boot.out = boot.out, type = "perc")
##
## Intervals :
## Level     Percentile
## 95%   (1200, 4800 )
## Calculations and Intervals on Original Scale
```

```
# Another example:
# IQR. How to get a confidence interval for the IQR of income?
IQR(acs12$income, na.rm = TRUE)
```

```
## [1] 33700
```

```
# Use the bootstrap. First write a function:
bootIQR <- function(x, i)IQR(x[i], na.rm = TRUE)

# Run the bootstrap.
boot.out <- boot(data = acs12$income,
                 statistic = bootIQR,
                 R = 999)

# Calculate the percentile confidence interval
boot.ci(boot.out, type = "perc")
```

```
## BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
## Based on 999 bootstrap replicates
##
## CALL :
## boot.ci(boot.out = boot.out, type = "perc")
##
## Intervals :
## Level     Percentile
## 95%   (30000, 36000 )
## Calculations and Intervals on Original Scale
```

```
# Another example:
# difference in medians

# How to get a confidence interval on the difference in medians?
mosaic::median(~ income | gender, data = acs12, na.rm = TRUE)
```

```
## female   male
##    680   8000
```

```
# diff() subtracts first element from the second
diff(mosaic::median(~ income | gender, data = acs12, na.rm = TRUE))
```

```
## male
## 7320
```

```
# Use the bootstrap. First write a function:
bootMedianDiff <- function(x, i)diff(mosaic::median(~ income | gender,
                                                    data = x[i,],
                                                    na.rm = TRUE))

# Run the bootstrap.
boot.out <- boot(data = acs12,
                 statistic = bootMedianDiff,
                 R = 999)

# Calculate the percentile confidence interval
boot.ci(boot.out, type = "perc")
```

```
## BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
## Based on 999 bootstrap replicates
##
## CALL :
## boot.ci(boot.out = boot.out, type = "perc")
##
## Intervals :
## Level     Percentile
## 95%   ( 3450, 12800 )
## Calculations and Intervals on Original Scale
```

```
# Another example:
# 75th percentile

# How to get a confidence interval on the 75th percentile of income?
quantile(acs12$income, probs = 0.75, na.rm = TRUE)
```

```
##   75%
```

```
## 33700
```

```r
# Use the bootstrap. First write a function:
boot75p <- function(x, i)quantile(x[i], probs = 0.75, na.rm = TRUE)

# Run the bootstrap.
boot.out <- boot(data = acs12$income,
                 statistic = boot75p,
                 R = 999)

# Calculate the percentile confidence interval
boot.ci(boot.out, type = "perc")
```

```
## BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
## Based on 999 bootstrap replicates
##
## CALL :
## boot.ci(boot.out = boot.out, type = "perc")
##
## Intervals :
## Level     Percentile
## 95%   (30000, 36000 )
## Calculations and Intervals on Original Scale
```